DATA252/DATA551: Modeling and Simulation

Lecture 9: a very basic intro to MCMC

April 13, 2020

Markov Chain

What is a stochastic process?

- X: one random variable
- $\tilde{X} = (X_1, X_2, \ldots, X_{10})$: multivariate analysis
- $\{X_1, X_2, \ldots\}$: discrete time stochastic process
- $\{X_t\}_{t \in [0,\infty]}$: continuous time stochastic process

What is a Markov chain?

- A Markov chain is a discrete time stochastic process $X_1, X_2, \ldots$ with the *Markov property*:

$$[X_{n+1}|X_1, \ldots, X_n] = [X_{n+1}|X_n]$$

  ("conditioning on the present, the future and the past are independet")
- An example: (one dimensional) random walk:

$$X_{n+1} = X_n + 1 \textit{ with prob. } p$$

$$X_{n+1} = X_n - 1 \textit{ with prob. } 1 - p$$

Markov Chain Monte Carlo (MCMC)

Recall, "Monte Carlo" means that the method relies on random sampling from a target distribution.

What is the purpose of MCMC?

- To randomly sample from a target distribution

How?

- *Carefully* construct a Markov chain, such that, *in the long run*, draws from the chain "stabilize" and follow the target distribution.

$$X_1, X_2, X_3, X_4, X_5, \ldots\ldots\ldots, X_B, X_{B+1}, X_{B+2}, \ldots$$

- We'll discart $X_1, \ldots, X_B$ and keep $X_{B+1}, X_{B+2}, \ldots$ as a random sample from the target distribution. B is called the "burn-in" length.

Markov Chain Monte Carlo (MCMC)

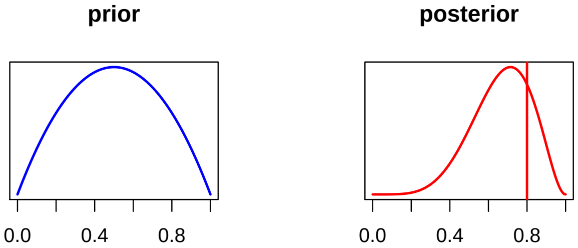
How to construct such a Markov chain?

- Metropolis-Hastings algorithm
- Gibbs sampling
- Hamiltonian MCMC
- Many other modifications and algorithms

A simple demo of MCMC and Bayesian inference

Goal: estimating $p = P(head)$ of a biased coin. Data: HHHTH
What is your estimate of p?

What if you have some prior belief of p?



We **update** our prior distribution of p using data (4 out 5 heads) to obtain a posterior distribution. Since the parameter p is considered **random**, this is an example of Bayesian inference. MCMC is used a lot in Bayesian inference because it allows us to sample from the posterior distribution.