

# Parallel Congruence Closure SAT solver

Enrico Martini, VR445204

**Abstract**—In this report is presented a parallel implementation of a satisfiability solver for the congruence closure, able to solve a set of clauses in the quantifiers free fragment of first order logic, based on equality among variables, constants, function applications, recursive data structures with their elements and elements of arrays.

## I. INTRODUCTION

The first theory considered is the theory of equality with uninterpreted functions (EUF). It is the most important theory because its congruence closure algorithm is the core of the entire algorithm.

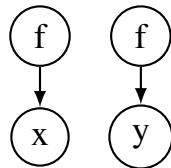
## II. METHODOLOGY

### A. Algorithm

```
#include <iostream>
int main() {
    // print hello to the console
    std::cout << "Hello, world!" << std::endl;
    return 0;
}
```

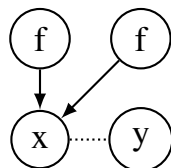
### B. Equality theory congruence closure example

$x = y \& f(x) \neq f(y)$



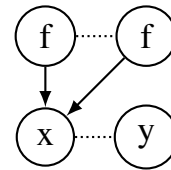
node	find	ccpar
0x	0	2
1y	1	3
2f->0	2	-
3f->1	3	-

```
MERGE 0 1
UNION 0 1
MERGE 2 3 ?
CONGRUENT 2 3 = 1
```

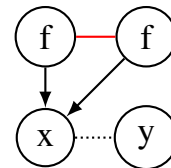


node	find	ccpar
0x	0	23
1y	0	-
2f->0	2	-
3f->1	3	-

```
MERGE 2 3
UNION 2 3
```



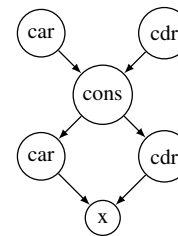
node	find	ccpar
0x	0	23
1y	0	-
2f->0	2	-
3f->1	2	-



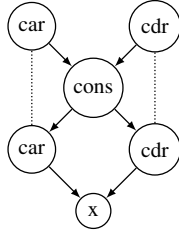
UNSAT

### C. List theory congruence closure example

$\text{atom}(x) \& \text{cons}(\text{car}(x), \text{cdr}(x)) = x$



```
MERGE 4 1
UNION 4 1
MERGE 5 2
UNION 5 2
```

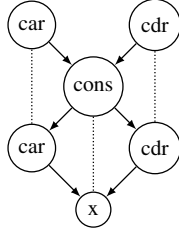


node	find	ccpar
0x	0	12
1car→0	4	–
2cdr→0	5	–
3cons→12	3	45
4car→3	4	3
5cdr→3	5	3

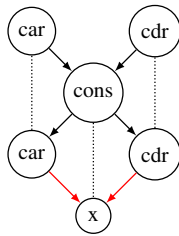
```

MERGE 3 0
UNION 3 0
MERGE 4 1 ?
MERGE 4 2 ?
CONGRUENT 4 2 = 0
MERGE 5 1 ?
CONGRUENT 5 1 = 0
MERGE 5 2 ?

```



node	find	ccpar
0x	3	–
1car→0	4	–
2cdr→0	5	–
3cons→12	3	4512
4car→3	4	3
5cdr→3	5	3



Equality theory passed  
UNSAT

#### D. Array theory congruence closure example

$e = \text{select}(\text{store}(a, i, e), j) \& \text{select}(a, j) \neq e$

detected store

```

1: e=e&j=i&select(a,j)!=e
2: e=select(a,j)&j!=i&select(a,j)!=e

```

$e = e \& j = i \& \text{select}(a, j) \neq e$   
 $e = \text{select}(a, j) \& j \neq i \& \text{select}(a, j) \neq e$

node	find	ccpar
0e	0	–
1j	1	4
2i	2	–
3a	3	4
4select→31	4	–

```

MERGE 0 0
MERGE 1 2
UNION 1 2

```

node	find	ccpar
0e	0	–
1j	1	4
2i	1	–
3a	3	4
4select→31	4	–

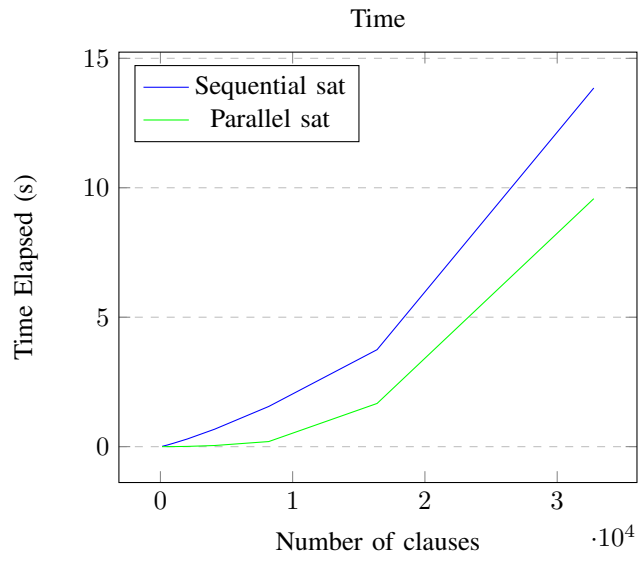
Equality theory passed  
SAT

### III. VALIDATION

### IV. BENCHMARKS

Test	#Formulas	Sequential (s)	Parallel (s)	Speedup
7	128	0,0117	0,0001	82,1
8	256	0,0290	0,0003	100,1
9	512	0,0612	0,0008	78,8
10	1024	0,1374	0,0026	52,1
11	2048	0,2988	0,0103	29,1
12	4096	0,6736	0,0442	15,3
13	8192	1,5526	0,1968	7,9
14	16384	3,7465	1,6690	2,2
15	32768	13,8530	9,5786	1,4

## V. PERFORMANCE ANALYSIS



## VI. CONCLUSION