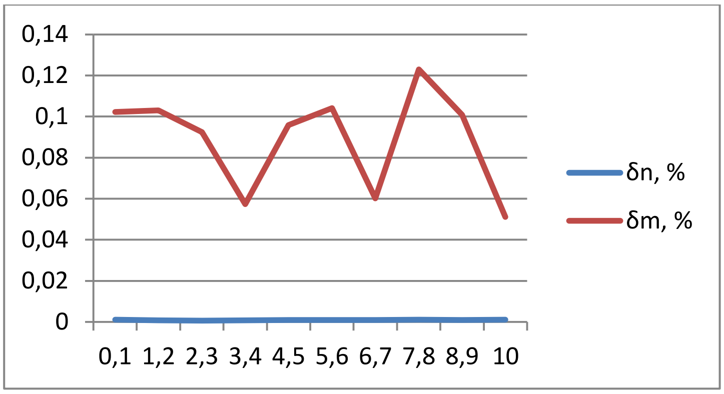
Experiments

We will perform a series of experiments for the sum, product, and ratio of two random variables with different ratios of the standard deviation, and choose variance as the parameter to be certified.

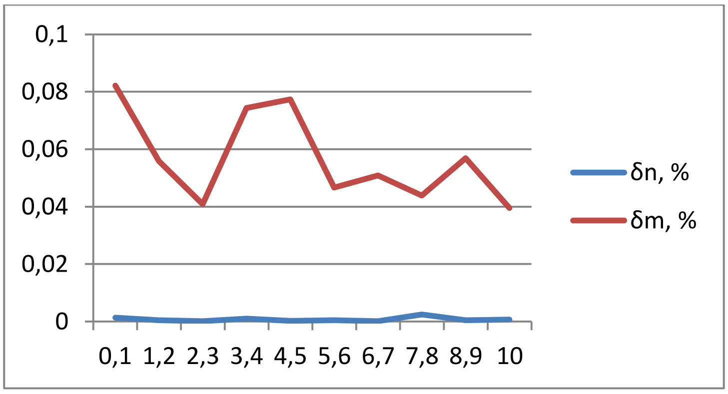
General experiment protocol:

1. Analytically calculate the variance σ^2 for a function of two random variables X and Y with variances σ_X^2 and σ_Y^2, wherein $\sigma_X^2 = 1$, $\sigma_Y^2 = k$, where $k \in [0.1,10]$;
2. Using the method of numerical integration, we find the probability density function of two distributions (the number of samples is 1000), then determine the variance σ_n^2;
3. Perform the propagation of the distributions by the Monte Carlo method and calculate the variance σ_m^2 by GUM-S1 in a series of 10 experiments on 10^6 tests for each, we find the maximum value of variance as $\sigma_m^2 = \sigma^2 + \sigma_{mi} \cdot 1{,}96$, где σ_{mi} –standard deviation for a series;
4. Determine the values of deviations from σ^2, expressed as a percentage for calculated values of variances σ_n^2, σ_m^2, we denote them, respectively, δ_n, δ_m.

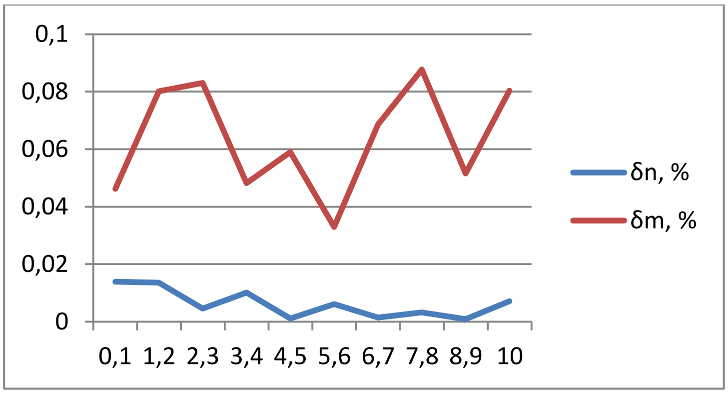
1. $Z = X + Y$
   1.1. $X \sim N(0, 1)$, $Y \sim N(0, k)$



   1.2. $X \sim N(0, 1)$, $Y \sim U[-k\sqrt{3}, k\sqrt{3}]$

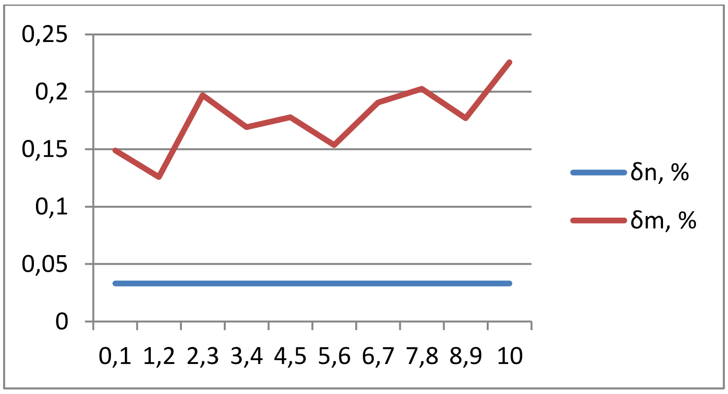


   1.3. $X \sim U[-\sqrt{3}, \sqrt{3}]$, $Y \sim U[-k\sqrt{3}, k\sqrt{3}]$

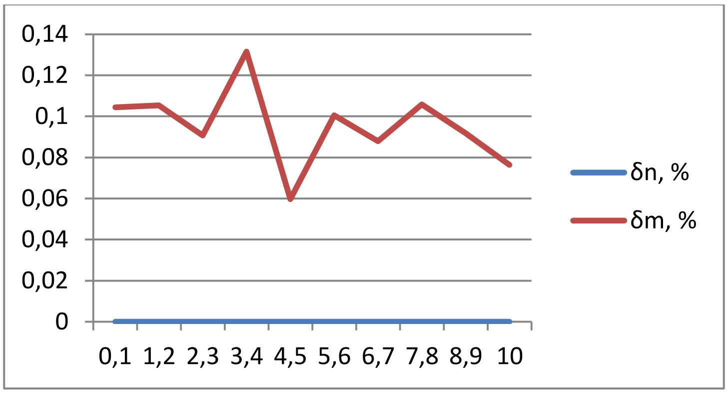


2. $Z = XY$

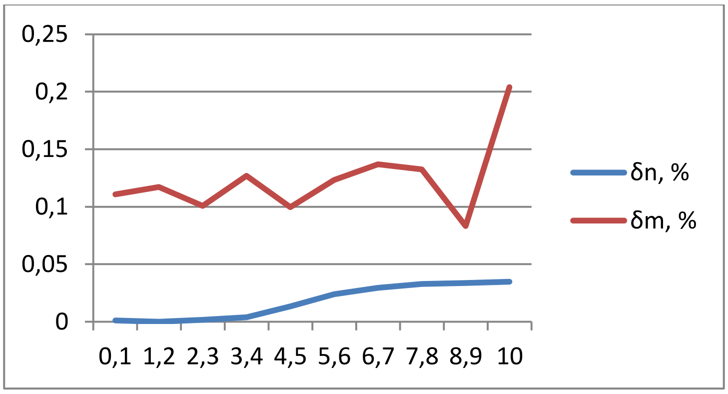
2.1. $X \sim N(0,1)$, $Y \sim N(0,k)$



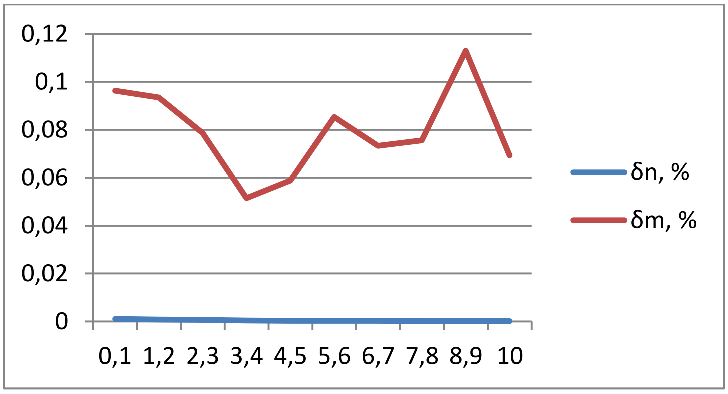
2.2. $X \sim N(10,1)$, $Y \sim N(0,k)$



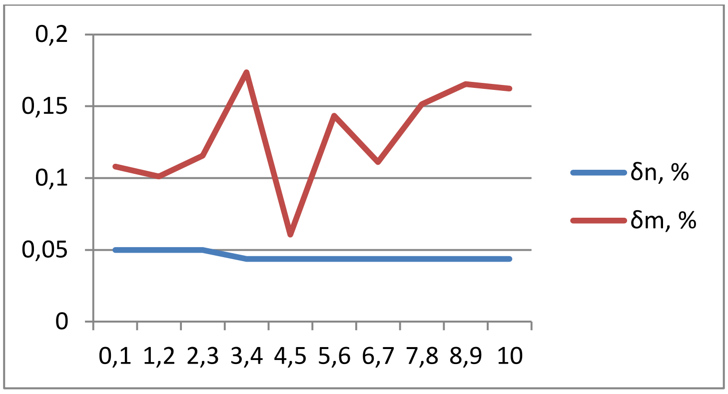
2.3. $X \sim N(0,1)$, $Y \sim N(10,k)$



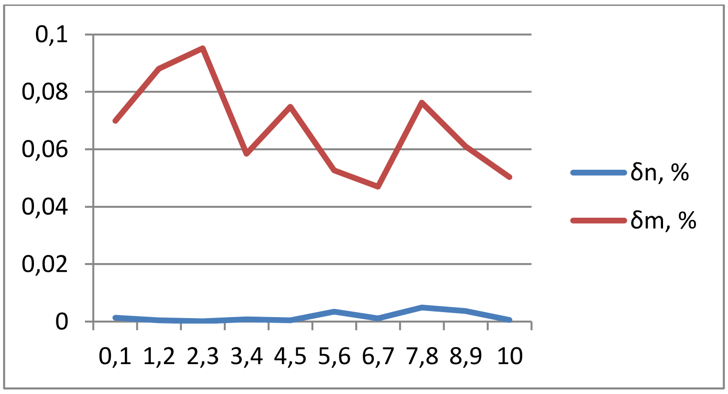
2.4. $X \sim N(10,1)$, $Y \sim N(10,k)$



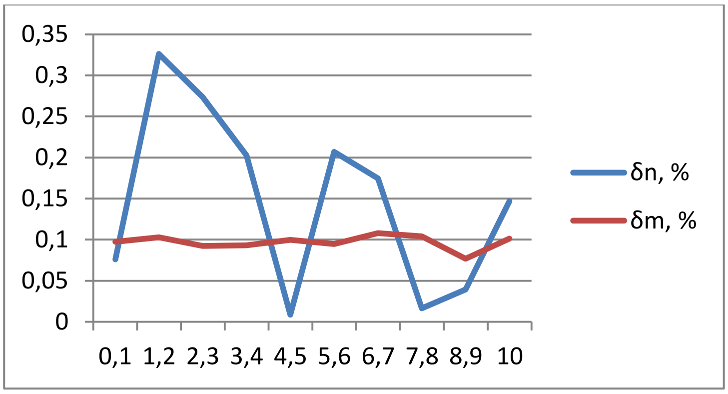
2.5. $X \sim N(0,1)$, $Y \sim U[-k\sqrt{3}, k\sqrt{3}]$



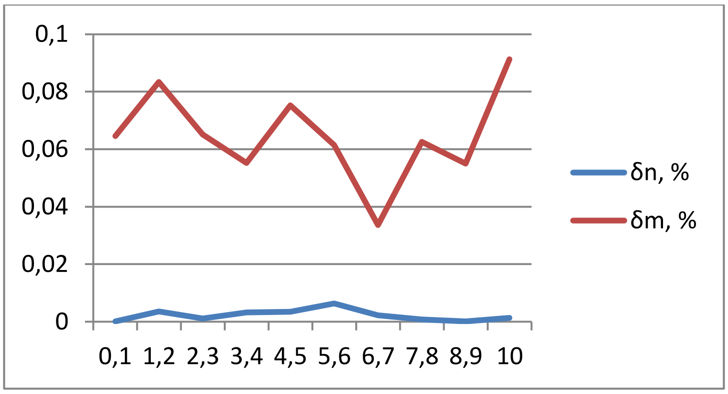
2.6. $X \sim N(10,1)$, $Y \sim U[-k\sqrt{3}+10, k\sqrt{3}+10]$



2.7. $X \sim U[-\sqrt{3}, \sqrt{3}]$, $Y \sim U[-k\sqrt{3}, k\sqrt{3}]$

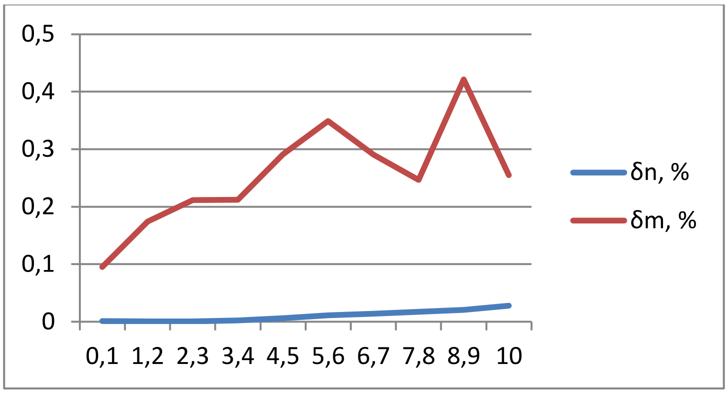


2.8. $X \sim U[-\sqrt{3}+10, \sqrt{3}+10]$, $Y \sim U[-k\sqrt{3}+10, k\sqrt{3}+10]$

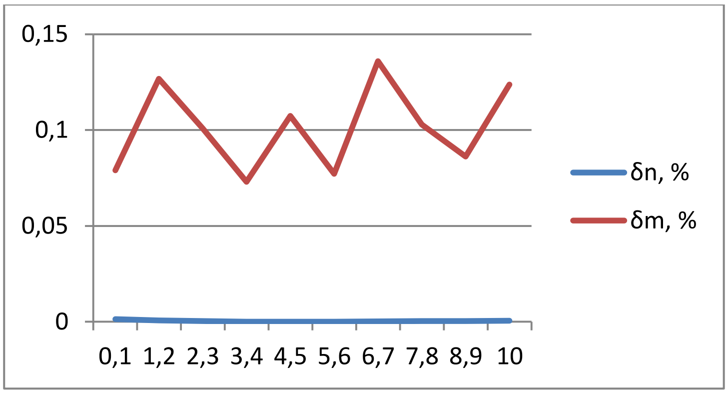


3. $Z = X/Y$

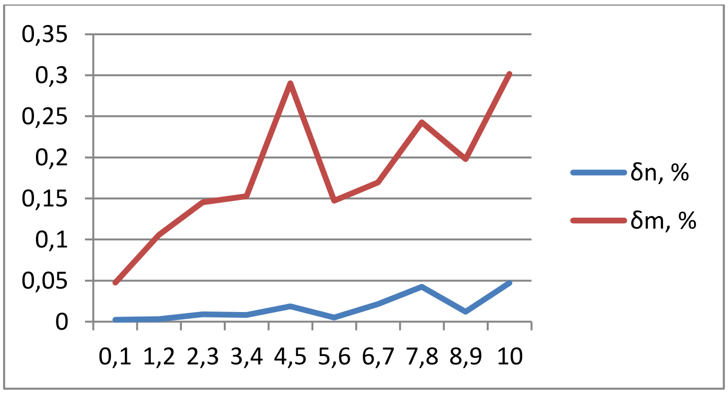
3.1. $X \sim N(0,1)$, $Y \sim U[1,\ 2k\sqrt{3}+1]$



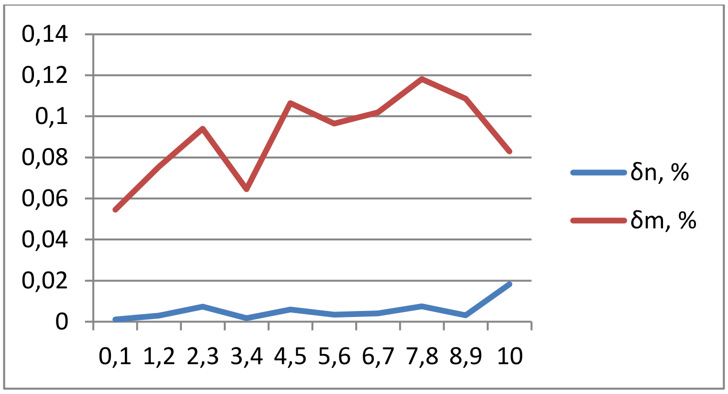
3.2. $X \sim N(10,1)$, $Y \sim U[10,\ 2k\sqrt{3}+10]$



3.3. $X \sim U[-\sqrt{3}, \sqrt{3}]$ $Y \sim U[1,\ 2k\sqrt{3}+1]$



3.4. $X \sim U[-\sqrt{3}+10, \sqrt{3}+10]$, $Y \sim U[10,\ 2k\sqrt{3}+10]$



Conclusions and Remarks:

For all cases, except for the product of random variables, when both of the probability density functions intersect the axis of ordinates, the error of the method of numerical integration is less than the error of the Monte Carlo method. Nevertheless, the methods were set a priori under presumably equal conditions, the integration was carried out with the number of samples 10^3, and in the Monte Carlo method was performed 10^6 experiments, both of the methods should of have error $\delta \approx 0{,}1$ %. In this case in the method of numerical integration, the average computation time is 100 less than in Monte-Carlo. We may also increase the number of samples to 10^4 in the method of numerical integration, then the execution time will be the same as that of Monte-Carlo, but the error will be an order of magnitude less.