Distributions

Quick Guide to the test application

1. **Model expression:**

- The arguments for the formula are the distributions that are specified in the right-hand part in the distributions list;

- It is prohibited to set two operators in a row, even if it follows the generally accepted logic:

Example:

A^-B is incorrect, **A^(-B)** is correct;

- Parenthesis operators are also supported:

Examples:

(A+5)/(B+3)

-(A*B)^(C/3);

- Supported operators. Note that all of the binary operators can be executed with method of random algebra:

Operator	Description	Example
+	Addition	A+5
-	Substitution	A-5
*	Multiplication	A*5
/	Division	A/5
^	Rising to the power	A^5
_	Logarithm to base	A_5
sqrt()	Square root	sqrt(A)
abs()	Absolute value	abs(A)
ln(), lg()	Natural/common logarithm	ln(A)
sin(), cos(), tg()	Sine / cosine / tangent of value in radians	sin(A)

- When calculating by the method of algebra of random variables, it is prohibited to use the same parameter in the formula twice. In case it is impossible to rewrite the expression, it is necessary to use the Monte Carlo method.

Examples:

A+A*2+B – incorrect, should rewrite to **A*3+B**

A*(A+5) – incorrect, Monte-Carlo only

2. **Distribution settings**

A table consists of three columns, an argument, a type of distribution, and distribution settings.

- An argument must be the model parameter;

- Distribution type is selected from the drop-down list, the list of available distribution types will be updated;

- In order to change the distribution settings, double-click the corresponding cell in the distribution settings column with the left mouse button.

- For multidimensional normal distribution parameters are set as variance-covariance matrix and vector of means, the resulting distribution is calculated by the formula:

$$c_1 \cdot D_1 + c_2 \cdot D_2 + \cdots + c_n \cdot D_n,$$

where n – number of dimensions, D –normal distributions, c– scale coefficients from the coefficients table.

3. Multivariate distribution settings

Set the number if dimensions and press “Build tables”. Note that:

- Arguments must be parameters of the model;
- Random algebra approach may be used only on case of bivariate distribution;
- For properly use of random algebra it is necessary to group parameters with parenthesis (as correlated pairs). For example, when you add two bivariate distributions with arguments A, B and C, D respectively:

(A*B)+(C+D) - this model will process correctly;

A*B+C+D - this model will throw error.

- Operations on low correlated t-distributions will give slightly different result under random algebra approach in comparison with Monte-Carlo approach by reason of common multivariate t-distribution random generator generates samples independent t-random values on zero correlation, but at real zero correlated marginal distributions do not agree with statistical independence.

4. Calculations

After the model and distributions settings have been defined in accordance with the previous sections, it is necessary to set the calculation parameters.

For the method of random variable algebra, the most important parameter is the number of **samples**, this parameter indicates the number of samples with which the probability density will be sampled to perform subsequent operations.

- The number of **pockets** only affects plotting;

- **Probability** used when finding quantiles;

- The greater the number of samples, the more accurate, but slower the parameters of the resulting distribution will be calculated;

- In the case of an additive measurement model, convolution is performed by the fast Fourier transform method, its complexity is $O(Nlog(N))$. In case, if the number of samples is up to 100,000, it will take less than 1 second to perform convolution of two random variables. For other cases, for example, the product of two random variables, numerical integration will be performed, its complexity is $O(N^2)$, then less than 1 second will be spent for 10,000 samples, and about a minute (depending on processor speed) for 100,000 samples;

- Recommended values: 1,000 samples for regular measurements and 10,000 for precision;

- If only analytical conversions are performed, this parameter is irrelevant.

For the Monte-Carlo method, the main parameter is the **number of experiments**. The recommended by GUM value is 10^6. The larger the value, the greater the accuracy, but the dependency is back-root, which means that in order to achieve an accuracy of 10 times more, we will need to perform 100 times more experiments.

After all the parameters are set, you should click the **evaluate** button. After that, charts of probability density and distribution functions will be constructed and moments and quantiles in the “results” block will be determined.

5. Results

Parameter	**Description**
t, ms	Time spent on calculations
μ	Expected value
σ	Standard deviation
σ^2	Dispersion
U^+	The upper bound of the coverage interval for a given probability p.
U^-	The lower bound of the coverage interval for a given probability p.
γ	Skewness
U±	$(U^+ - U^-)/2$. If the skewness tends to 0, this value can be used to express the expanded uncertainty $Y = \mu \pm U$, otherwise suggested entry $Y \in [U^-, U^+]$.