A* Search

- **UCS** minimises cost from root - **g(n)**
- **Greedy Search (GS)** minimises estimated cost to the goal (heuristic) - **h(n)**

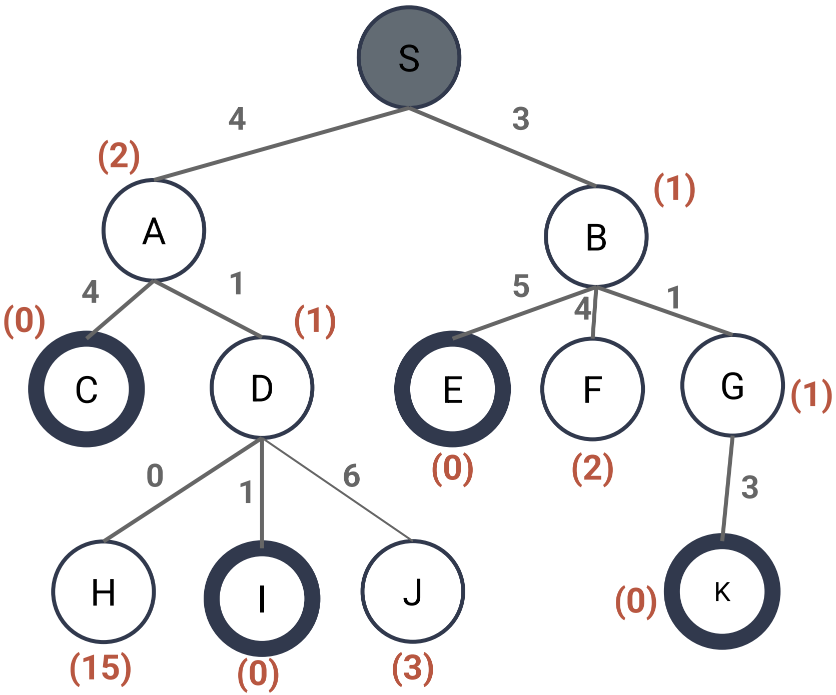
A* combines **UCS** and **GS**, minimises the estimated total cost of path through *n* to the goal:

$$f(n) = g(n) + h(n)$$

Exercise 1 (Homework)

Expanded: S
Fringe: **B[?], A[?]**

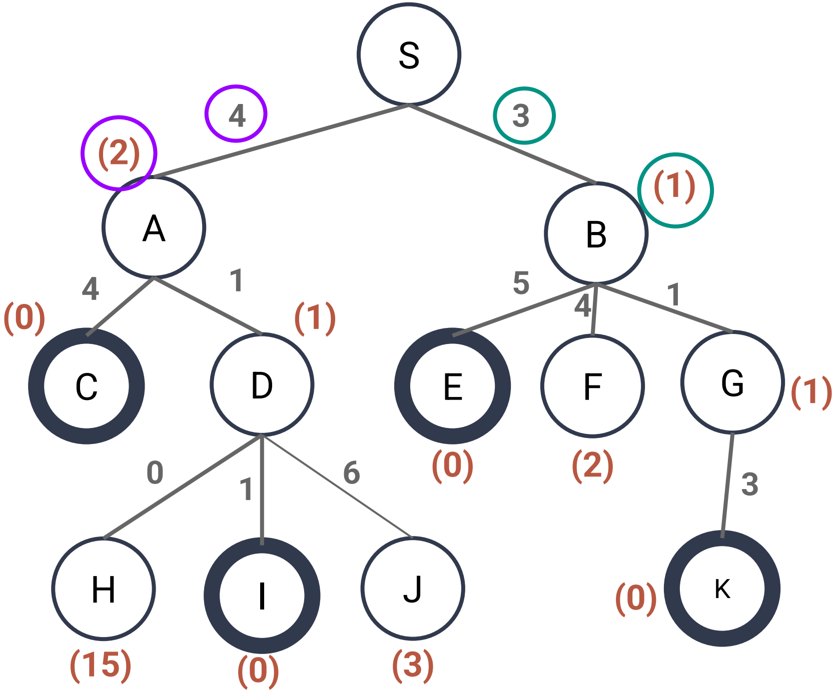
f(A) = ?
f(B) = ?



Exercise 1 (Homework)

Expanded: S
Fringe: **B[4], A[6]**

$f(A) = 4 + 2 = 6$
$f(B) = 3 + 1 = 4$



Exercise 1 (Homework)

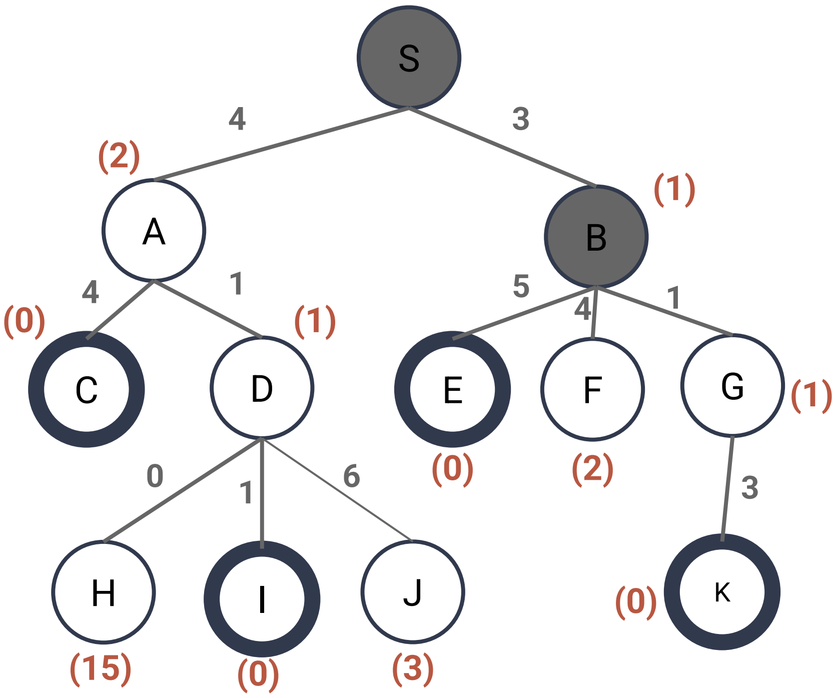
Expanded: S, B[4]

Fringe: **G[5]**, A[6], **E[8]**, **F[9]**

f(G) = 3 + 1 + 1 = 5

f(E) = 3 + 5 + 0 = 8

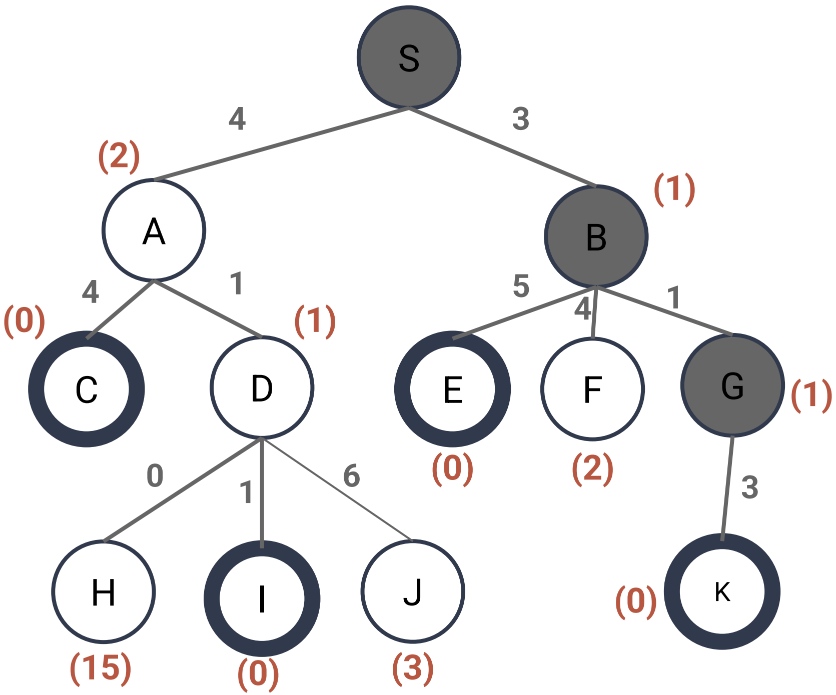
f(F) = 3 + 4 + 2 = 9



Exercise 1 (Homework)

Expanded: S, B[4], G[5]
Fringe: A[6], **K[7]**, E[8], F[9]

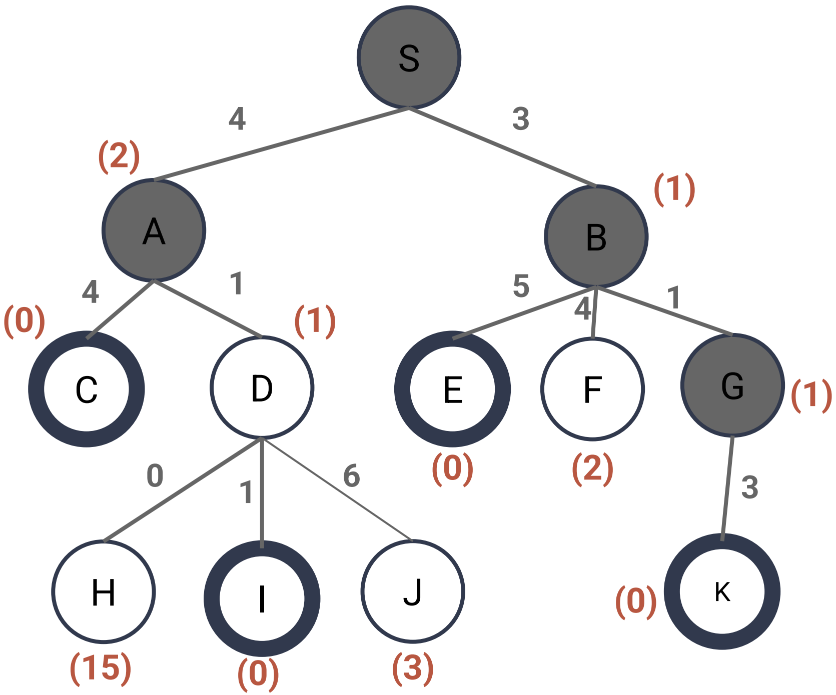
f(K) = 3 + 1 + 3 + 0 = 7



Exercise 1 (Homework)

Expanded: S, B[4], G[5], A[6]
Fringe: **D[6]**, K[7], **C[8]**, E[8], F[9]

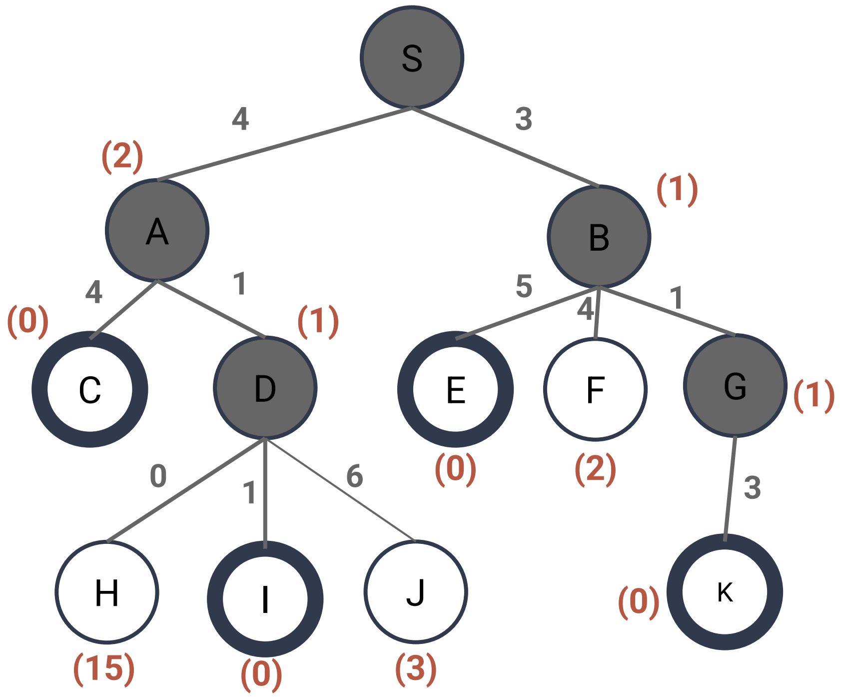
f(C) = 4 + 4 + 0 = 8
f(D) = 4 + 1 + 1 = 6



Exercise 1 (Homework)

Expanded: S, B[4], G[5], A[6], D[6]
Fringe: **I[6]**, K[7], C[8], E[8], F[9], **J[14]**, **H[20]**

f(I) = 4 + 1 + 1 + 0 = 6
f(J) = 4 + 1 + 6 + 3 = 14
f(H) = 4 + 1 + 0 + 15 = 20



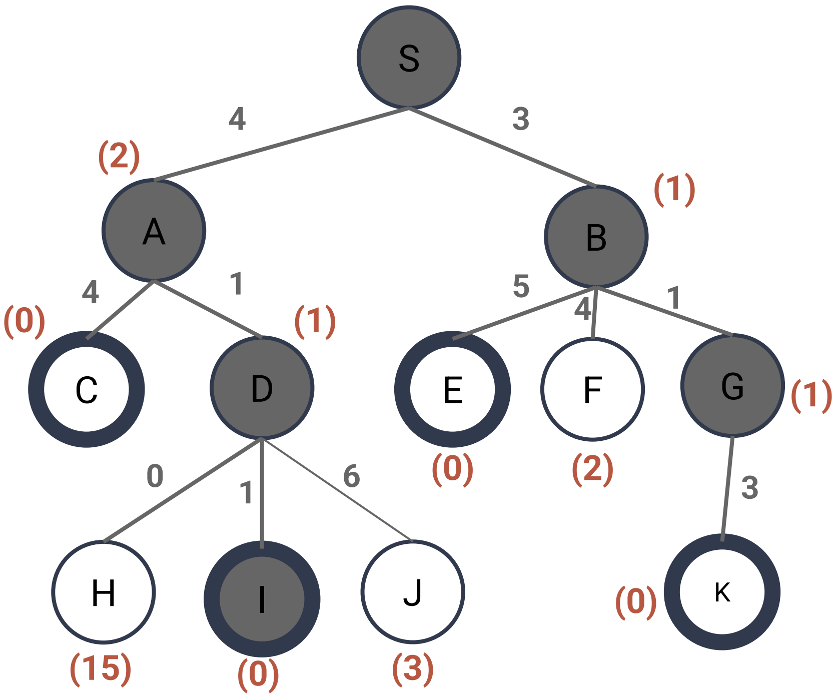
Exercise 1 (Homework)

Solution:

Expanded nodes: SBGADI

Path found: SADI

Path cost: 4 + 1 + 1 = 6



Admissible Heuristic

A heuristic *h(n)* is **admissible** if for every node n:

- ***h(n) ≤ h*(n)*** where ***h*(n)*** is the true cost to reach a goal from n

Admissible heuristics are **optimistic**

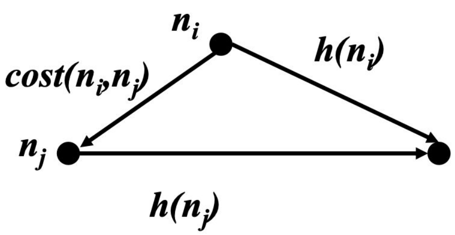
- They think that the cost of solving the problem is less than it actually is

Theorems:

h is admissible => A* is complete and optimal

Consistent (Monotonic) Heuristic

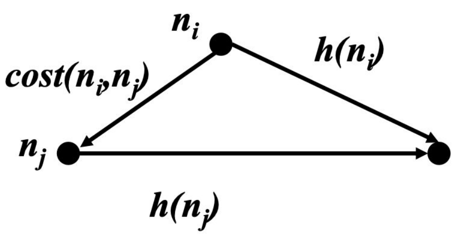
n_i is the parent of n_j.
h is a **consistent (monotonic) heuristic** if for all pairs of n_i and n_j. The following **triangle inequality** is satisfied:

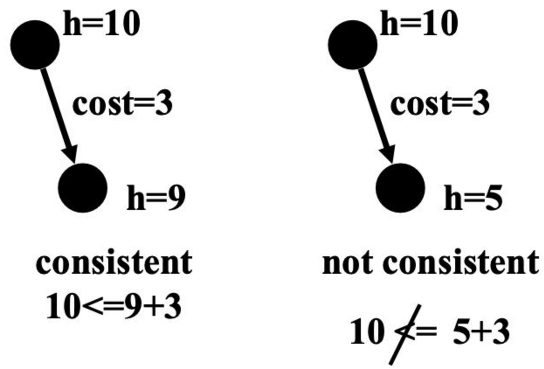


Triangle inequality: The sum of any two sides must be greater than or equal to the remaining side.

Consistent (Monotonic) Heuristic

n_i is the parent of n_j.
h is a **consistent (monotonic) heuristic** if for all pairs of n_i and n_j. The following **triangle inequality** is satisfied:





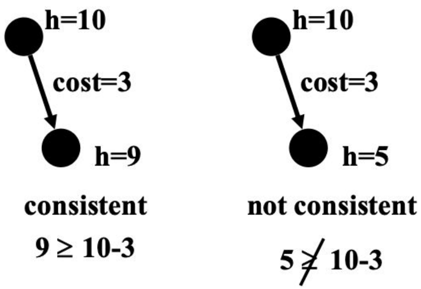
$h(ni) \leq cost(ni,nj)+ h(nj)$ **for all** n

Consistent (Monotonic) Heuristic

<u>Another Interpretation:</u>

=> $h(n_j) \geq h(n_i) - cost(n_i,n_j)$

I.e. Along any path, our **estimation of the remaining cost to the goal** cannot decrease by more than the **arc cost**.



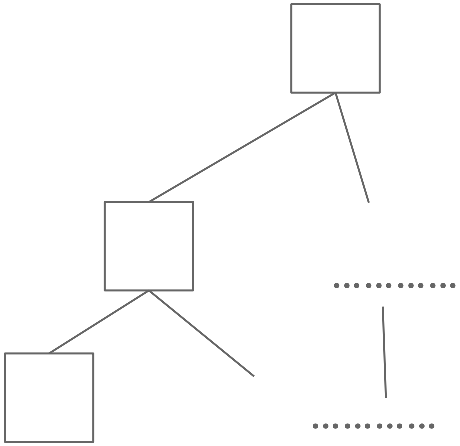
Consistency Theorems

$h(n)$ is consistent ⇔ $f(nj) \geq f(ni)$ i.e. f is non-decreasing along any path

Consistent => admissible

If a heuristic is consistent, it is also admissible (the reverse might not be true!)

Exercise 2 - 4-queens puzzle



Start: Empty board

Depth 1: Placed 1 queen states

Depth 2: Placed 2 queens states

Depth 3: Placed 3 queens states

Depth 4: Placed 4 queens states

Hill-Climbing Algorithm

Two variation:

- Steepest **ascent** - find **maximum**
- Steepest **descent** - find **minimum**

- Store only 1 state - the current state
- Generate children -> Select the best child
- Compare child with parent.
  - Child is not better -> **stop**,
  - else make child the current state and repeat

Beam Search

- Keep track of **k states** rather than just 1 (like in hill-climbing)
- At each level, generates children of the **k states**
  - If any of the children is the goal -> **stop**
  - else select **k best children** and go to next level