Probabilistic Reasoning
Bayesian Networks

Conditional Probability, Product and Chain Rules

1) Definition of *conditional probability*:

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

NB: , means $\wedge$, so this is the same:

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}, if\ P(b) \neq 0$$

2) The *product rule* (follows from 1):

$$P(a,b) = P(a \mid b)P(b)$$

$$P(a,b) = P(b,a) = P(b \mid a)P(a)$$

<- It works the other way too

commutative property

3) *Chain rule* – derived by successive application of the product rule:

$$P(x_1,\ldots,x_n) = \prod_{i=1}^{n} P(x_i \mid x_{i-1},\ldots,x_1), e.g.$$

The decomposition holds for any order of the variables

$$P(a,b) = P(a \mid b)P(b)$$

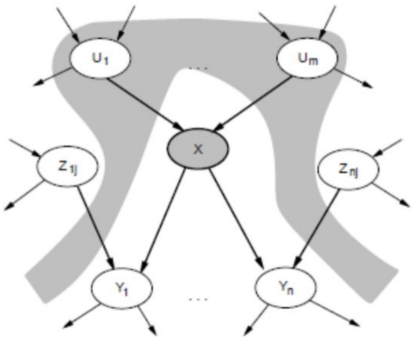
$$P(a,b,c) = P(a \mid b,c)P(b \mid c)P(c)$$

$$P(a,b,c,d) = P(a \mid b,c,d)P(b \mid c,d)P(c \mid d)P(d)$$

Assumption of Bayesian Networks (2)

- **Bayesian Network (BN) assumption: A node is conditionally independent of its *non-descendants*, given its parents**
- **In other words, if we know the values of the parents, then knowing the values of any other non-descendant does not change the probability values of the nodes**
- **This can be written as:**

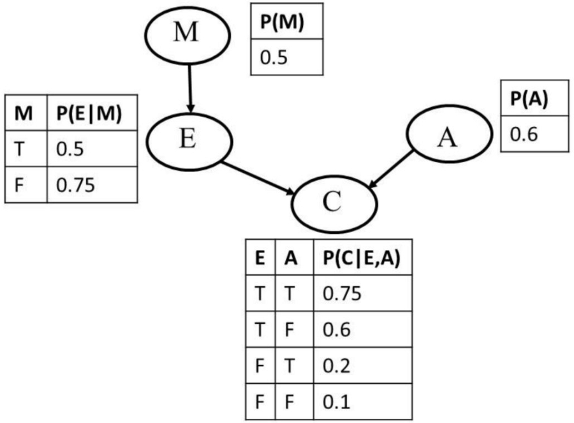
P(node | parents plus any other non-descendants)=P(node | parents)



X is conditionally independent of its non-descendants (nodes Z_{ij}) given its parents (nodes U_i – the gray area)

Exercise 1. ***Bayesian network (Homework)***

Consider the Bayesian network below where all variables are binary:



P(M)
0.5

M	P(E|M)
T	0.5
F	0.75

P(A)
0.6

E	A	P(C|E,A)
T	T	0.75
T	F	0.6
F	T	0.2
F	F	0.1

Compute the following probability and show your calculations: P(M=T, E=T, A=F, C=T).

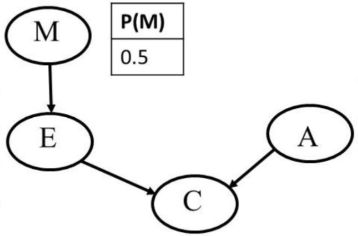
Bayesian Network
Joint Probability Rule

- **Any joint probability can be computed as:**

$$P(x_1,\dots,x_n) = \prod_{i=1}^{n} P(x_i \mid Parents(x_i))$$

Exercise 1. ***Bayesian network (Homework)***

Consider the Bayesian network below where all variables are binary:



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P(A)
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E	A	P(C|E,A)
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T	F	0.6
F	T	0.2
F	F	0.1

(simplified)
Conditional Probability
Tables (CPT)

Compute the following probability and show your calculations: P(M=T, E=T, A=F, C=T).

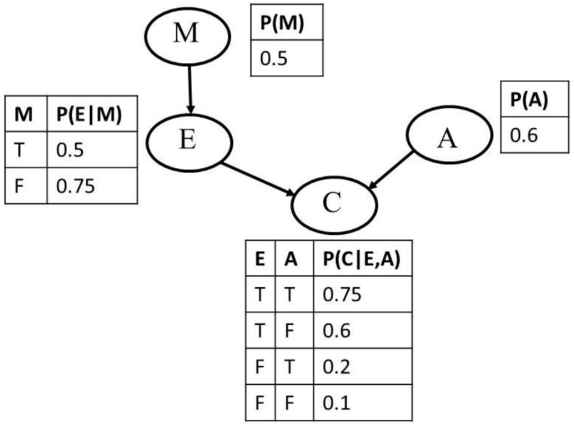
Bayesian Network
Joint Probability Rule

- **Any joint probability can be computed as:**

$$P(x_1,\ldots,x_n) = \prod_{i=1}^{n} P(x_i \mid Parents(x_i))$$

<u>Exercise 1.</u> Bayesian network (Homework)

Consider the Bayesian network below where all variables are binary:



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- **Any joint probability can be computed as:**

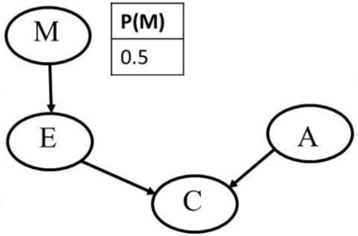
$$P(x_1,\ldots,x_n) = \prod_{i=1}^{n} P(x_i \mid Parents(x_i))$$

<u>Notation:</u>
m denotes M=T
e denotes E=T
a denotes A=T => ~a denote A=F
c denotes C=T

Compute the following probability and show your calculations: P(M=T, E=T, A=F, C=T).

Exercise 1. ***Bayesian network (Homework)***

Consider the Bayesian network below where all variables are binary:



P(M)
0.5

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P(A)
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- **Any joint probability can be computed as:**

$$P(x_1,\ldots,x_n) = \prod_{i=1}^{n} P(x_i \mid Parents(x_i))$$

<u>Notation:</u>
m denotes M=T
e denotes E=T
a denotes A=T   => ~a denote A=F
c denotes C=T

Compute the following probability and show your calculations: P(M=T, E=T, A=F, C=T).

Solution:

P(M=T, E=T, A=F, C=T) = P(m, e, ~a, c)=P(m)P(e|m)P(~a)P(c|e,~a)=0.5*0.5*(1-0.6)*0.6=0.06

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

Calculate the following:

a) P(toothache)
b) **P**(Cavity)
c) **P**(Toothache |cavity)
d) **P**(Cavity| toothache $\vee$ catch)

Probability Distribution

- **P(Meningitis)=<0.3, 0.7>** the values should sum to 1

bold

1^{st} value is for true, 2^{nd} is for false

<u>Exercise 2.</u> Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

Calculate the following:

a) P(toothache)
b) **P**(Cavity)
c) **P**(Toothache |cavity)
d) **P**(Cavity| toothache ∨ catch)

a) P(toothache)=P(Toothache=true)=0.108+0.012+0.016+0.064=0.2

Exercise 2. Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

Probability Distribution

- **P**(Meningitis)=<0.3, 0.7> ← the values should sum to 1

bold

1st value is for true, 2nd is for false

Calculate the following:

a) P(toothache)
b) **P**(Cavity)
c) **P**(Toothache |cavity)
d) **P**(Cavity| toothache ∨ catch)

b) **P**(Cavity) = < P(Cavity=True), P(Cavity=False) >

<u>Exercise 2.</u> Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
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Calculate the following:

a) P(toothache)
b) **P**(Cavity)
c) **P**(Toothache |cavity)
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a) P(toothache)=P(Toothache=true)=0.108+0.012+0.016+0.064=0.2

b) **P**(Cavity) = < P(Cavity=True), P(Cavity=False) >

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
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Calculate the following:

a) P(toothache)
b) **P**(Cavity)
c) **P**(Toothache |cavity)
d) **P**(Cavity| toothache ∨ catch)

a) P(toothache)=P(Toothache=true)=0.108+0.012+0.016+0.064=0.2

b) **P**(Cavity) = < P(Cavity=True), P(Cavity=False) >
=< (0.108+0.012+0.072+0.008), (0.016+0.064+0.144+0.576) >=<0.2, 0.8>

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, \text{if } P(b) \neq 0$$

c) **P**(Toothache |cavity) = **<** P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) **>**

= **<** P(toothache | cavity), P(~toothache | cavity) **>**

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

denominator

c) **P**(Toothache |cavity) = < P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) >
= < P(toothache | cavity), P(~toothache | cavity) >

Two calculation methods:

- **Method 1: Without $\boldsymbol{\alpha}$** (i.e. by calculating the denominator)
- **Method 2: With $\boldsymbol{\alpha}$** (i.e. without calculating the denominator)

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
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$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

denominator

c) **P**(Toothache |cavity) = < P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) >
= < P(toothache | cavity), P(~toothache | cavity) >

Method 1: Without $\boldsymbol{\alpha}$ (i.e. by calculating the denominator)

P(toothache | cavity) = P(toothache, cavity)/P(cavity)

Exercise 2. *Probabilistic reasoning*

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
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$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

denominator

c) **P**(Toothache |cavity) = < P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) >
= < P(toothache | cavity), P(~toothache | cavity) >

Method 1: Without α (i.e. by calculating the denominator)

P(toothache | cavity) = P(toothache, cavity)/P(cavity)
= (0.108+0.012)/(0.108+0.012+0.072+0.008)= 0.12/0.2 = 0.6

Exercise 2. Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

denominator

c) **P**(Toothache |cavity) = < P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) >
= < P(toothache | cavity), P(~toothache | cavity) >

Method 1: Without $\boldsymbol{\alpha}$ (i.e. by calculating the denominator)

P(toothache | cavity) = P(toothache, cavity)/P(cavity)
= (0.108+0.012)/(0.108+0.012+0.072+0.008)= 0.12/0.2 = **0.6**

P(~toothache | cavity) = P(~toothache, cavity)/P(cavity)
= (0.072+0.008)/(0.108+0.012+0.072+0.008)= 0.08/0.2 = **0.4**

=> **P**(Toothache |cavity) = **<0.6, 0.4>**

Exercise 2. Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, \text{ if } P(b) \neq 0$$

denominator

c) **P**(Toothache |cavity) = < P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) >
= < P(toothache | cavity), P(~toothache | cavity) >

Method 2: With α (i.e. without calculating the denominator)

P(toothache | cavity) = α P(toothache, cavity)
P(~toothache | cavity) = α P(~toothache, cavity)

α = 1/ P(cavity)

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, \text{if } P(b) \neq 0$$

denominator

c) **P**(Toothache |cavity) = < P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) >
= < P(toothache | cavity), P(~toothache | cavity) >

Method 2: With α (i.e. without calculating the denominator)

P(toothache | cavity) = α P(toothache, cavity) = α (0.108 + 0.012) = α * 0.12
P(~toothache | cavity) = α P(~toothache, cavity) = α (0.072 + 0.008) = α * 0.08

α = 1/ P(cavity)

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, \textit{if } P(b) \neq 0$$

denominator

c) **P**(Toothache |cavity) = < P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) >
= < P(toothache | cavity), P(~toothache | cavity) >

Method 2: With *α* (i.e. without calculating the denominator)

P(toothache | cavity) = ***α*** P(toothache, cavity) = ***α*** (0.108 + 0.012) = ***α*** * 0.12
P(~toothache | cavity) = ***α*** P(~toothache, cavity) = ***α*** (0.072 + 0.008) = ***α*** * 0.08

α = 1/ P(cavity)

Next we calculate ***α*** **- the normalisation term**

<u>Exercise 2.</u> Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, \text{if } P(b) \neq 0$$

denominator

c) **P**(Toothache |cavity) = < P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) >
= < P(toothache | cavity), P(~toothache | cavity) >

Method 2: With $\boldsymbol{\alpha}$ (i.e. without calculating the denominator)

P(toothache | cavity) = α P(toothache, cavity) = α (0.108 + 0.012) = α * **0.12**

P(~toothache | cavity) = α P(~toothache, cavity) = α (0.072 + 0.008) = α * **0.08**

α = 1/ P(cavity)

α = 1 / (0.12 + 0.08) = 1/0.2

Why?

***Exercise 2.* Probabilistic reasoning**

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

denominator

c) **P**(Toothache |cavity) = < P(Toothache=true | Cavity=true), P(Toothache=false | Cavity=true) >
= < P(toothache | cavity), P(~toothache | cavity) >

Method 2: With α (i.e. without calculating the denominator)

P(toothache | cavity) = α P(toothache, cavity) = α (0.108 + 0.012) = α * **0.12**
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α = 1/ P(cavity)

α = 1 / (0.12 + 0.08) = 1/0.2

=> **P**(Toothache |cavity) =< 0.12/0.2 , 0.08/0.2 > =<0.6, 0.4>

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, \text{if } P(b) \neq 0$$

d) **P**(Cavity| toothache v catch) = < P(cavity| toothache v catch), P(~cavity| toothache v catch) >

Let's compute the <u>first term</u> (without using α):

P(cavity| toothache v catch) = P(cavity, (toothache v catch)) / P(toothache v catch)

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		$\neg$ *toothache*	
	catch	$\neg$ *catch*	*catch*	$\neg$ *catch*
cavity	.108	.012	.072	.008
$\neg$ *cavity*	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

d) **P**(Cavity| toothache v catch) = < P(cavity| toothache v catch), P(~cavity| toothache v catch) >

Let's compute the first term (without using α):

P(cavity| toothache v catch) = P(cavity, (toothache v catch)) / P(toothache v catch)

P(toothache v catch) =

Exercise 2. Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

d) **P**(Cavity| toothache v catch) = < P(cavity| toothache v catch), P(~cavity| toothache v catch) >

Let's compute the first term (without using α):

P(cavity| toothache v catch) = P(cavity, (toothache v catch)) / P(toothache v catch)

P(toothache v catch) = 0.108+0.012+0.016+0.064+0.072+0.144 = 0.416

Exercise 2. Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		¬ toothache	
	catch	¬ catch	catch	¬ catch
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

d) **P**(Cavity| toothache v catch) = < P(cavity| toothache v catch), P(~cavity| toothache v catch) >

Let's compute the first term (without using α):

P(cavity| toothache v catch) = P(cavity, (toothache v catch)) / P(toothache v catch)

P(toothache v catch) = 0.108+0.012+0.016+0.064+0.072+0.144 = 0.416

P(cavity, (toothache v catch)) =

Exercise 2. ***Probabilistic reasoning***

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		¬ toothache	
	catch	¬ catch	catch	¬ catch
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, if\ P(b) \neq 0$$

d) **P**(Cavity| toothache v catch) = < P(cavity| toothache v catch), P(~cavity| toothache v catch) >

Let's compute the first term (without using α):

P(cavity| toothache v catch) = P(cavity, (toothache v catch)) / P(toothache v catch)

P(toothache v catch) = 0.108+0.012+0.016+0.064+0.072+0.144 = 0.416

P(cavity, (toothache v catch)) =

Exercise 2. Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, \textit{if } P(b) \neq 0$$

d) **P**(Cavity| toothache v catch) = < P(cavity| toothache v catch), P(~cavity| toothache v catch) >

Let's compute the first term (without using α):

P(cavity| toothache v catch) = P(cavity, (toothache v catch)) / P(toothache v catch)

P(toothache v catch) = 0.108+0.012+0.016+0.064+0.072+0.144 = 0.416

P(cavity, (toothache v catch)) = 0.108+0.012+0.072 = 0.192

Exercise 2. Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		¬ toothache	
	catch	¬ catch	catch	¬ catch
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, \text{if } P(b) \neq 0$$

d) **P**(Cavity| toothache v catch) = < P(cavity| toothache v catch), P(~cavity| toothache v catch) >

Let's compute the first term (without using α):

P(cavity| toothache v catch) = P(cavity, (toothache v catch)) / P(toothache v catch)

P(toothache v catch) = 0.108+0.012+0.016+0.064+0.072+0.144 = 0.416

P(cavity, (toothache v catch)) = 0.108+0.012+0.072 = 0.192

=> P(cavity| toothache v catch) = 0.192 / 0.416 = 0.4615

Exercise 2. Probabilistic reasoning

Consider the dental example discussed at the lecture; its full joint probability distribution is shown below:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

$$P(a \mid b) = \frac{P(a,b)}{P(b)}, \textit{if } P(b) \neq 0$$

d) **P**(Cavity| toothache v catch) = < P(cavity| toothache v catch), P(~cavity| toothache v catch) >

Let's compute the first term (without using α):

P(cavity| toothache v catch) = P(cavity, (toothache v catch)) / P(toothache v catch)

P(toothache v catch) = 0.108+0.012+0.016+0.064+0.072+0.144 = 0.416

P(cavity, (toothache v catch)) = 0.108+0.012+0.072 = 0.192

=> P(cavity| toothache v catch) = 0.192 / 0.416 = 0.4615

Your task: Compute the second term: P(~cavity| toothache v catch)

Exercise 3. ***Bayesian networks***

(Based on J. Kelleher, B. McNamee and A. D'Arcy, Fundamentals of ML for Predictive Data Analytics, MIT, 2015)

Given is the following description of the causal relationships between storms, burglars, cats and house alarms:

1. Stormy nights are rare.
2. Burglary is also rare, and if it is a stormy night, burglars are likely to stay at home (burglars don't like going out in storms).
3. Cats don't like storms either, and if there is a storm, they like to go inside.
4. The alarm on your house is designed to be triggered if a burglar breaks into your house, but sometimes it can be triggered by your cat coming into the house, and sometimes it might not be triggered even if a burglar breaks in (it could be faulty or the burglar might be very good).

a) Define the topology of a Bayesian network that encodes these relationships.

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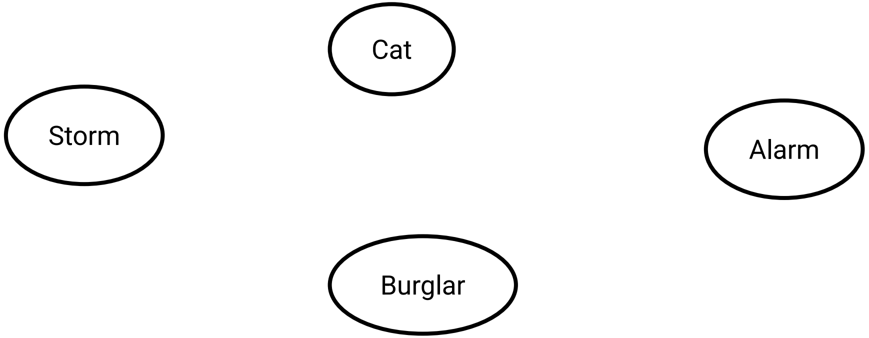

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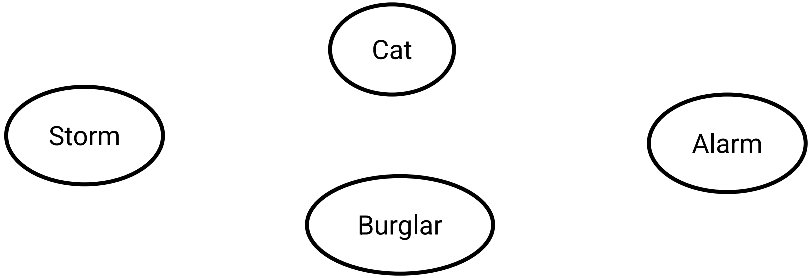

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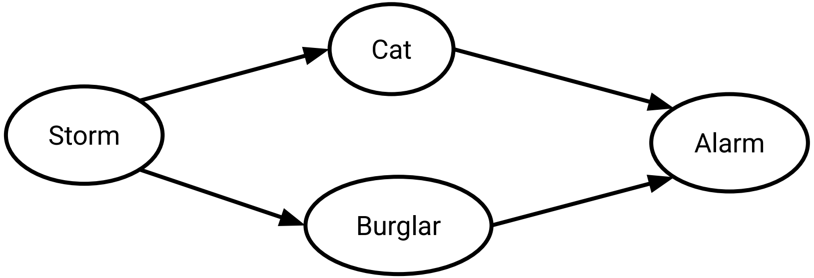
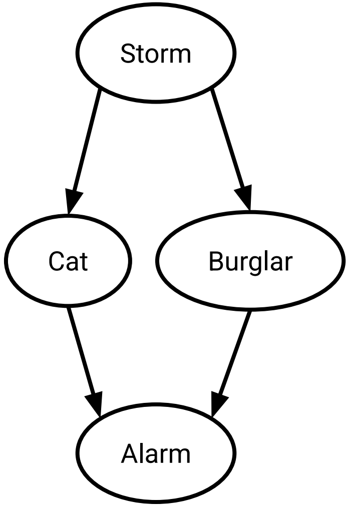


<u>Exercise 3.</u> Bayesian networks

b) Using the data from the table below, create the Conditional Probability Tables (CPTs) for the Bayesian network from the previous step.

ID	STORM	BURGLAR	CAT	ALARM
1	false	false	false	false
2	false	false	false	false
3	false	false	false	false
4	false	false	false	false
5	false	false	false	true
6	false	false	true	false
7	false	true	false	false
8	false	true	false	true
9	false	true	true	true
10	true	false	true	true
11	true	false	true	false
11	true	false	true	false
13	true	true	false	true



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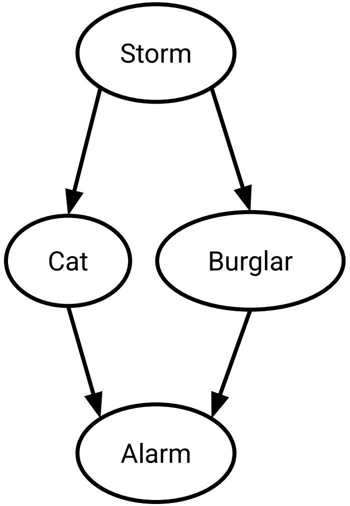
11	true	false	true	false
13	true	true	false	true

s	~s
0.31	0.69



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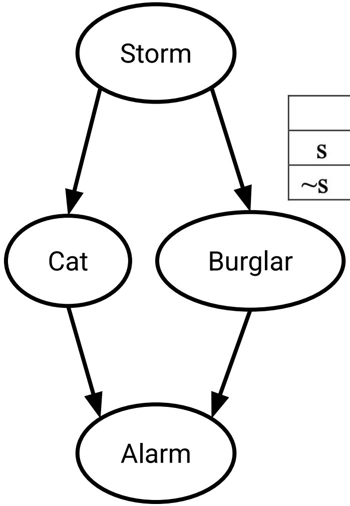
	c	~c
s	0.75	0.25
~s	0.22	0.78



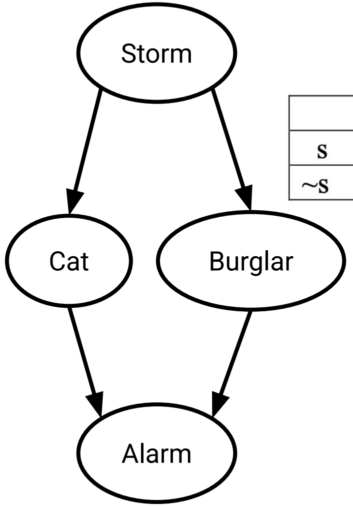
	b	~b
s	0.25	0.75
~s	0.33	0.67

		a	~a
b	c	1	0
b	~c	0.67	0.33
~b	c	0.25	0.75
~b	~c	0.2	0.8

Exercise 3. ***Bayesian networks***

c) Compute the probability that the alarm will be on, given that there is a storm but we don't know if a burglar has broken in or where the cat is.

s	~s
0.31	0.69



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We need to compute P(A=true| S=true).

$$P(a \mid s) = \frac{P(a,s)}{P(s)} = \frac{\sum_{b,c} P(a,s,B_b,C_c)}{P(s)}$$

B and C are the hidden variables; $\sum_b B_b$ means summation over the different values B can take.

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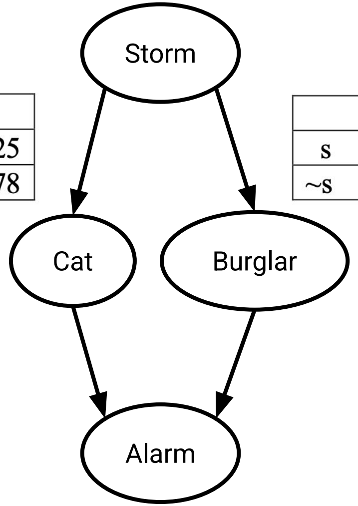

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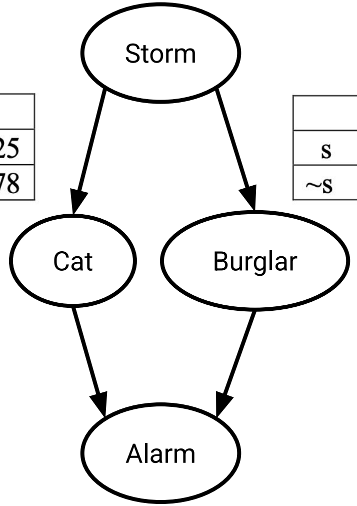

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