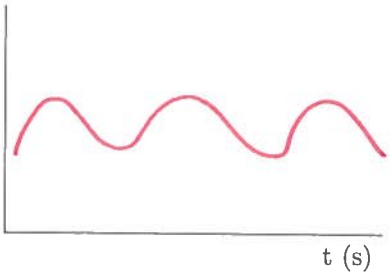
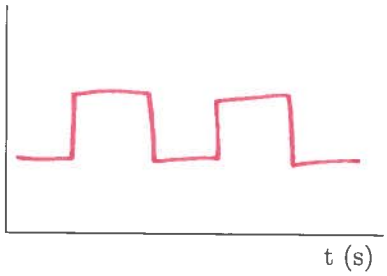
Name: SOLUTIONS

Read each question carefully before answering. Answer all parts. Show all work, calculations, and/or reasoning, otherwise no points will be awarded. K-maps may be used to double check your work, but may NOT be used as your actual work. Point values are as indicated.

1. (5 points) Draw an example of an analog and a digital signal on the axes below. Clearly indicate which is which.

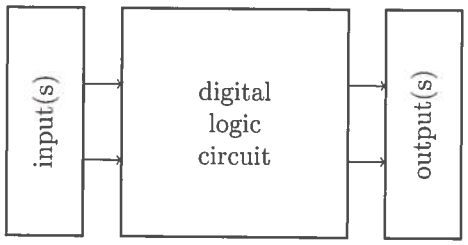


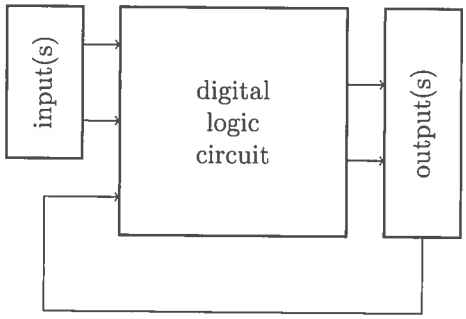




label: analog

label: digital

2. (5 points) Label which of the circuits is sequential, and which is combinational.




label: combinational

label: sequential

3. Express the following decimal numbers as 8-bit signed binary numbers. Use **2's complement** for all negative numbers.

(a) (5 points) 106

$$
\begin{array}{r}
3 \text{ r0} \\
2\overline{)6} \text{ r1} \\
2\overline{)13} \text{ r0} \\
2\overline{)26} \text{ r1} \\
2\overline{)53} \text{ r0 (LSB)} \\
2\overline{)106}
\end{array}
$$

$$
\begin{array}{r}
0 \text{ r1 MSB} \\
2\overline{)1} \text{ r1} \\
2\overline{)3}
\end{array}
$$

$0\ 1\ 1\ 0\ 1\ 0\ 1\ 0$

(b) (5 points) −120

$$
\begin{array}{r}
3 \text{ r1} \\
2\overline{)7} \text{ r1} \\
2\overline{)15} \text{ r0} \\
2\overline{)30} \text{ r0} \\
2\overline{)60} \text{ r0} \\
2\overline{)120}
\end{array}
\qquad
\begin{array}{r}
0 \text{ r1} \\
2\overline{)1} \text{ r1} \\
2\overline{)3}
\end{array}
$$

$01111000_2 = +120_{10}$

$10000111_{1's} = -120_{10}$

$$
\begin{array}{r}
+ \quad\quad 1 \\
\hline
10001000_{2's}
\end{array}
$$

$1\ 0\ 0\ 0\ 1\ 0\ 0\ 0$

(c) (5 points) 110

$$
\begin{array}{r}
3 \text{ r0} \\
2\overline{)6} \text{ r1} \\
2\overline{)13} \text{ r1} \\
2\overline{)27} \text{ r1} \\
2\overline{)55} \text{ r0 LSB} \\
2\overline{)110}
\end{array}
$$

$$
\begin{array}{r}
0 \text{ r1 MSB} \\
2\overline{)1} \text{ r1} \\
2\overline{)3}
\end{array}
$$

$0\ 0\ 1\ 0\ 1\ 1\ 1\ 0$

4. Express the following decimal numbers as **unsigned** binary numbers, rounding the non-integer component to fit into the spaces below.

(a) (5 points) 27.77

0 r1
2|1 r1
2|3 r0
2|6 r1
2|13 r1
2|27

0.77 × 2 = [1].54 MSB

0.54 × 2 = [1].08

0.08 × 2 = [0].16 LSB

| 1 | 1 | 0 | 1 | 1 | . | 1 | 1 | 0 |
|---|---|---|---|---|---|---|---|---|

(b) (5 points) 15.53

0 r1
2|1 r1
2|3 r1
2|7 r1
2|15

0.53 × 2 = [1].06 MSB

0.06 × 2 = [0].12

0.12 × 2 = [0].24

| 0 | 1 | 1 | 1 | 1 | . | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|

(c) (5 points) 18.61

0 r1
2|1 r0
2|2 r0
2|4 r1
2|9 r0
2|18

0.61 × 2 = [1].22 MSB

0.22 × 2 = [0].44

0.44 × 2 = [0].88

| 1 | 0 | 0 | 1 | 0 | . | 1 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|

5. Express the following as 5-bit signed binary numbers and then add them, using **1's complement** for negative numbers. Indicate if there is an overflow in any of the answers.

(a) (5 points) $1 + 13$

```
  0 0 0 0 1
+ 0 1 1 0 1
-----------
  0 1 1 1 0
```

01110

(b) (5 points) $-13 + 5$

```
  0 0 0 0 0
    0 0 1 0 1
+   1 0 0 1 0
-------------
    1 0 1 1 1
```

10111

(c) (5 points) $-9 + -5$

```
  1 1 1 1 0
    1 0 1 1 0
+   1 1 0 1 0
-------------
  1 0 0 0 0
              1   end-around carry
-------------
  1 0 0 0 1
```

10001

6. Express the following as 4-bit signed binary numbers and then multiply them, using **2's complement** for negative numbers. Indicate if there is an overflow in any of the answers.

(a) (5 points) $4 \times 4$

```
  0 0 0 0 0 1 0 0
x 0 0 0 0 0 1 0 0
-----------------
  0 0 0 1 0 0 x x
```

overflow because bits are not identical!

(b) (5 points) $-7 \times 5$

```
  1 1 1 1 1 0 0 1
x 0 0 0 0 0 1 0 1
-----------------
  1 1 1 1 1 0 0 1
+ 1 1 1 0 0 1 x x
-----------------
  1 1 0 1 1 1 0 1
```

overflow: bits are not identical!

(c) (5 points) $2 \times -4$

```
  1 1 1 1 1 1 0 0
x 0 0 0 0 0 0 1 0
-----------------
  1 1 1 1 1 0 0 x
```

ok!

1 1 1 1 1 0 0 0

7. Use the truth table to answer the following questions.

| $A$ | $B$ | $C$ | $F$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | $A'B'C'$ |
| 0 | 0 | 1 | 0 | |
| 0 | 1 | 0 | 1 | $A'BC'$ |
| 0 | 1 | 1 | 1 | $A'BC$ |
| 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 1 | $AB'C$ |
| 1 | 1 | 0 | 0 | |
| 1 | 1 | 1 | 0 | |

(a) (5 points) Find the reduced **sum of products** equation.

$A'B'C' + A'BC' + A'BC + AB'C$

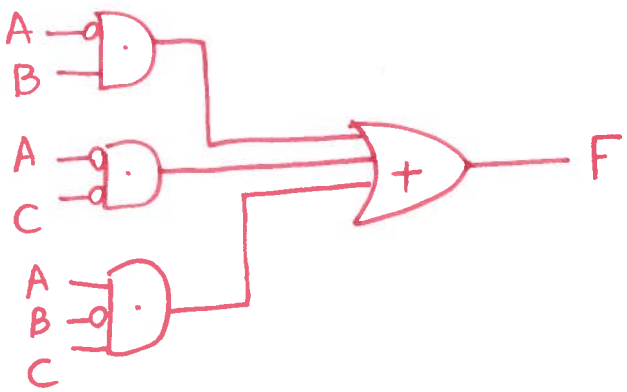
uniting → $A'B$

$A'B'C' + A'B + AB'C$

$A'(B+B'C') + AB'C$

elimination: $\boxed{A'B + A'C' + AB'C = F}$

(b) (5 points) Draw the simplified circuit diagram.



8. Use the truth table to answer the following questions.

| $A$ | $B$ | $C$ | $F$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $(A+B+C)$ |
| 0 | 0 | 1 | 1 | |
| 0 | 1 | 0 | 1 | |
| 0 | 1 | 1 | 1 | $(A'+B+C)$ |
| 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 1 | $(A'+B'+C)$ |
| 1 | 1 | 0 | 0 | $(A'+B'+C')$ |
| 1 | 1 | 1 | 0 | |

(a) (5 points) Find the reduced **product of sums** equation.

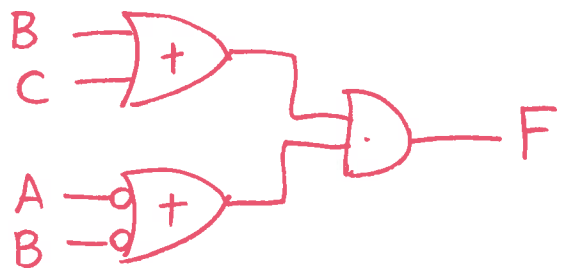
$(A+B+C)(A'+B+C)(A'+B'+C)(A'+B'+C')$

uniting: $(A+B+C)(A'+B+C) \rightarrow (B+C)$

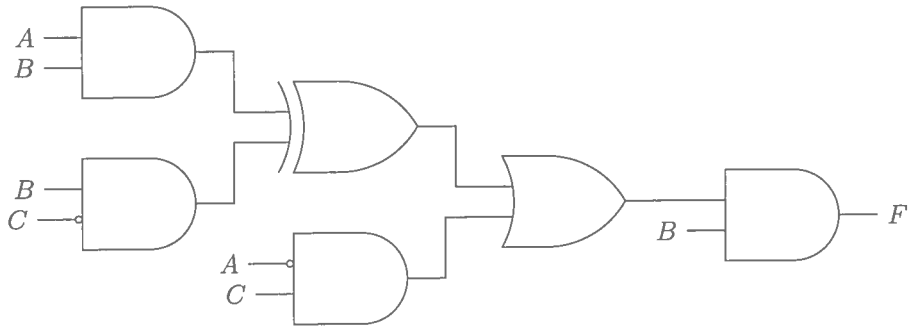
uniting: $(A'+B'+C)(A'+B'+C') \rightarrow (A'+B')$

$$F = (B+C)(A'+B')$$

(b) (5 points) Draw the simplified circuit diagram.



9. Use the circuit diagram to answer the following questions.




(a) (5 points) Fill out the truth table.

| A | B | C | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

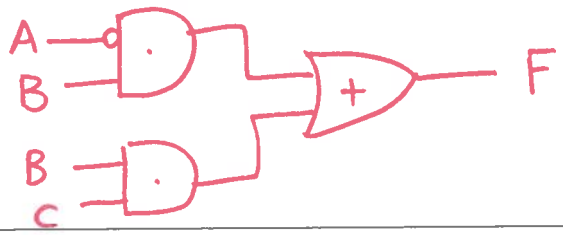
see next page for expanded truth table!

(b) (5 points) Find the reduced **sum of products** equation.

$A'BC' + A'BC + ABC$ — uniting → $BC$

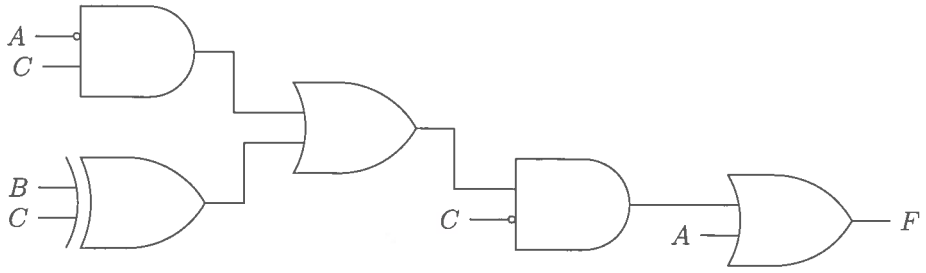
$B(A'C' + C) =$ $A'B + BC = F$

(c) (5 points) Draw the simplified circuit diagram.



| A B C | AB | BC′ | AB ⊕ BC′ | A′C | A′C + | B . | ⊕ |
|---|---|---|---|---|---|---|---|
| 0 0 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 0 0 1 | 0 | 0 | 0 | 1 | 1 | 0 | |
| 0 1 0 | 0 | 1 | 1 | 0 | 1 | 1 | |
| 0 1 1 | 0 | 0 | 0 | 1 | 1 | 1 | |
| 1 0 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 1 0 1 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 1 1 0 | 1 | 1 | 0 | 0 | 0 | 0 | |
| 1 1 1 | 1 | 0 | 1 | 0 | 1 | 1 | |

10. Use the circuit diagram to answer the following questions.




(a) (5 points) Fill out the truth table.

| $A$ | $B$ | $C$ | $F$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

see next page for expanded truth table

(b) (5 points) Find the reduced **product of sums** equation.

$(A+B+C)(A+B+C')(A+B'+C')$

uniting

$(A+B)$

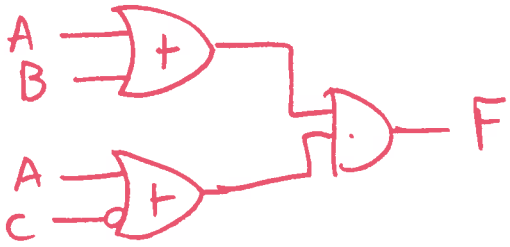
$(A+B)(A+B'+C')$

$= A + B(B'+C')$ 2nd dis

$= A + BC'$

$= (A+B)(A+C')$ 2nd dis *

(c) (5 points) Draw the simplified circuit diagram.






* indicates use of 2nd distributive law

| A B C | $A'C$ | $B \oplus C$ | $A'C + (B \oplus C)$ | $\cdot C'$ | $+A$ (F) |
|---|---|---|---|---|---|
| 0 0 0 | 0 | 0 | 0 | 0 | 0 |
| 0 0 1 | 1 | 1 | 1 | 0 | 0 |
| 0 1 0 | 0 | 1 | 1 | 1 | 1 |
| 0 1 1 | 1 | 0 | 1 | 0 | 0 |
| 1 0 0 | 0 | 0 | 0 | 0 | 1 |
| 1 0 1 | 0 | 1 | 1 | 0 | 1 |
| 1 1 0 | 0 | 1 | 1 | 1 | 1 |
| 1 1 1 | 0 | 0 | 0 | 0 | 1 |

11. Use the minterm expression $F(A, B, C) = \Sigma m(0, 1, 3, 4, 5)$ to answer the following questions.

(a) (5 points) Fill out the truth table.

| $A$ | $B$ | $C$ | $F$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |

(b) (5 points) Find the reduced **sum of products** equation.

$A'B'C' + A'B'C + A'BC + AB'C' + AB'C$ } uniting

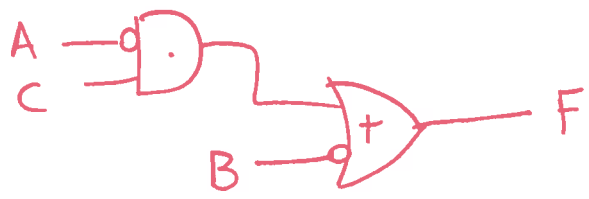
$A'B' + A'BC + AB'$

$A'(B' + BC) + AB'$ } elimination

$= A'B' + A'C + AB'$ } uniting

$F = B' + A'C$

(c) (5 points) Draw the simplified circuit diagram.



12. Use the maxterm expression $F(A,B,C) = \Pi M(2,3,5,7)$ to answer the following questions.

(a) (5 points) Fill out the truth table.

| $A$ | $B$ | $C$ | $F$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 |

(b) (5 points) Find the reduced **sum of products** equation.

$A'B'C' + A'B'C + AB'C' + ABC'$ } uniting

$F = A'B' + AC'$

(c) (5 points) Draw the simplified circuit diagram.