Artificial Intelligence

# INFORMED SEARCH STRATEGIES

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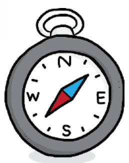
## Outline

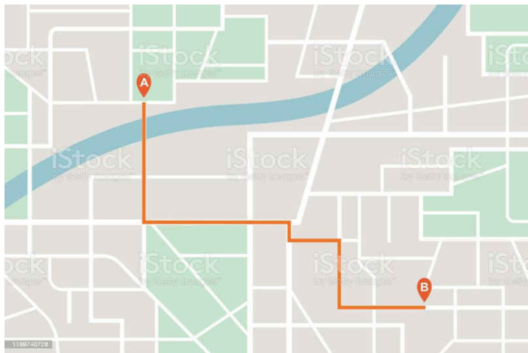
- Informed (Heuristic) search strategies
- Best-first search
- Greedy best-first search
- A* search
- Memory-bounded heuristic search
- Heuristic functions



## Informed (Heuristic) search strategies

- Use problem-specific knowledge beyond the definition of the problem itself
- Find solutions more efficiently
- Provide significant speed-up in practice



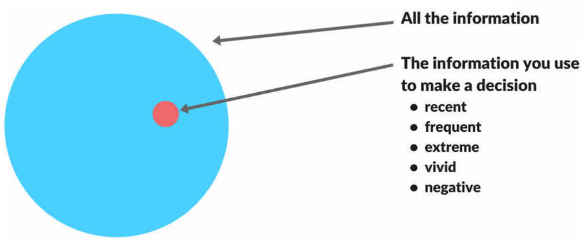



## What are heuristics?

- Additional knowledge of the problem is imparted to the search algorithm using **heuristics**.
- A heuristic is any practical approach to problem solving sufficient for reaching an immediate goal where an optimal solution is usually impossible.
  - Not guaranteed to be optimal, perfect, logical, or rational
  - Speed up the process of finding a satisfactory solution
  - Ease the cognitive load of making a decision

# Heuristics: An example

- Availability heuristic: what comes to mind quickly seems to be significant




# Heuristics: An example

- Representativeness heuristic: estimate the likelihood of an event by comparing it to a prototype already exists in mind.

The portrait of an old woman who is warm and caring with a great love of children



# Informed search strategies

- Best-first search
- Greedy best-first search
- A*
- RBFS
- SMA*



# Best-first search

- A best-first search algorithm can be either a TREE-SEARCH or GRAPH-SEARCH instance.
- A node is selected for expansion based on an **evaluation function**, $f(n)$.
  - Node with the lowest $f(n)$ is expanded first
- The choice of $f$ determines the search strategy.

# Heuristic function

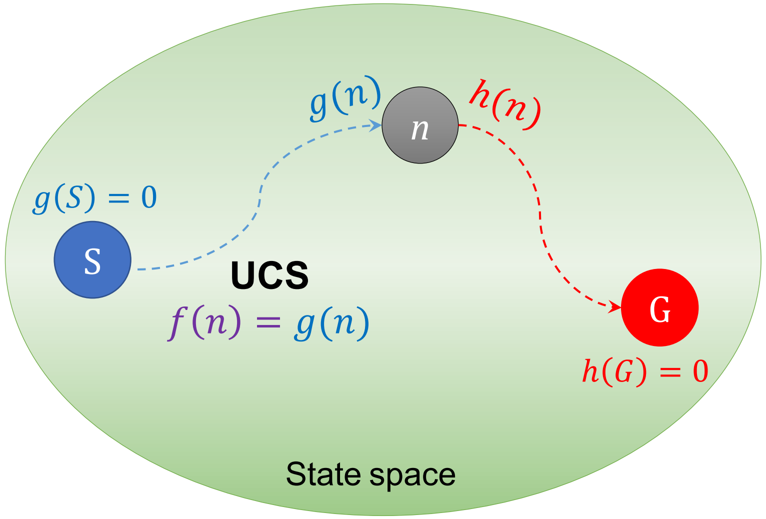
- Most best-first algorithms include a **heuristic function $h(n)$** as a component of $f$.

| $h(n)$ | estimated cost of the cheapest path from the state at node $n$ to a goal |
|---|---|

- Unlike $g(n)$, $h(n)$ depends only on the state at that node
- Assumption of $h(n)$
  - Arbitrary, nonnegative, problem-specific functions
  - Constraint: if $n$ is a goal node, then $h(n) = 0$



# Cost function vs. Heuristic function





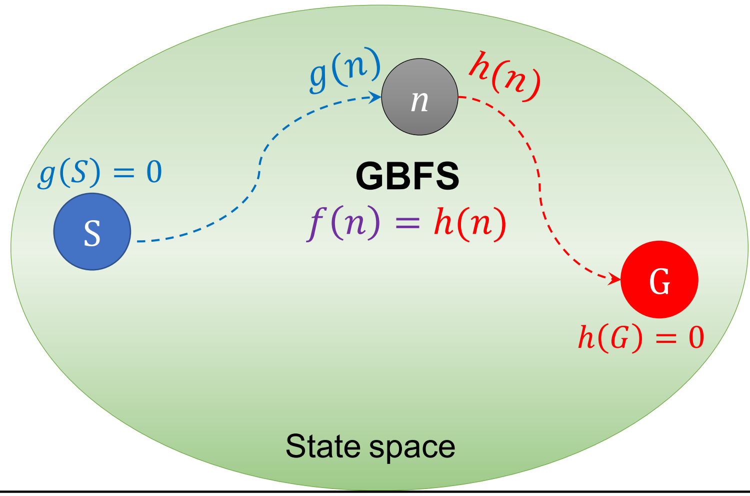

# Greedy best-first search



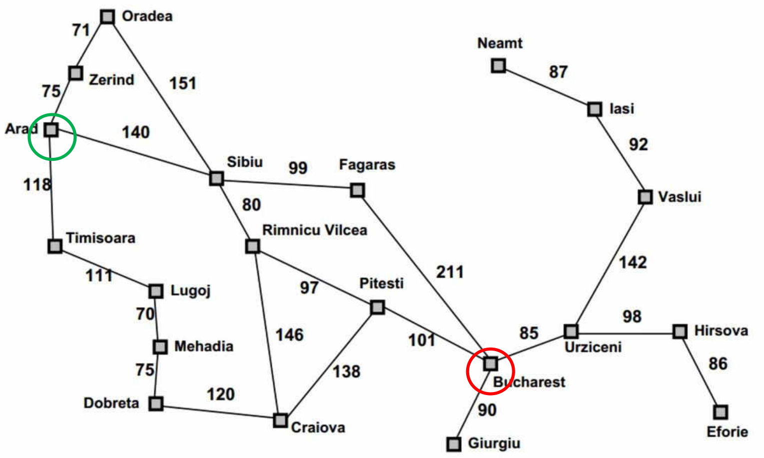
# Greedy best-first search

- Expand the node that appears to be closest to goal using

$$f(n) = h(n)$$



# Straight-line distance heuristic $h_{SLD}$

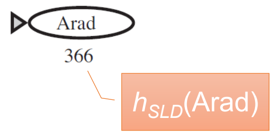



| Straight–line distance to Bucharest | |
|---|---|
| **Arad** | 366 |
| **Bucharest** | 0 |
| **Craiova** | 160 |
| **Dobreta** | 242 |
| **Eforie** | 161 |
| **Fagaras** | 178 |
| **Giurgiu** | 77 |
| **Hirsova** | 151 |
| **Iasi** | 226 |
| **Lugoj** | 244 |
| **Mehadia** | 241 |
| **Neamt** | 234 |
| **Oradea** | 380 |
| **Pitesti** | 98 |
| **Rimnicu Vilcea** | 193 |
| **Sibiu** | 253 |
| **Timisoara** | 329 |
| **Urziceni** | 80 |
| **Vaslui** | 199 |
| **Zerind** | 374 |



# Greedy best-first search: An example

**(a) The initial state**

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|---|---|---|---|
| **Arad** | 366 | **Mehadia** | 241 |
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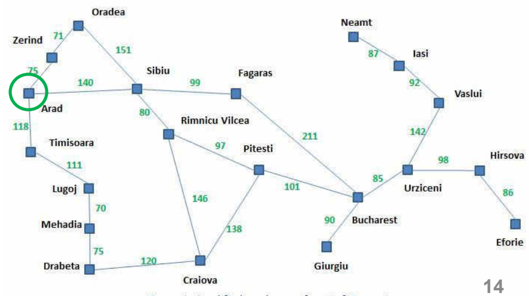
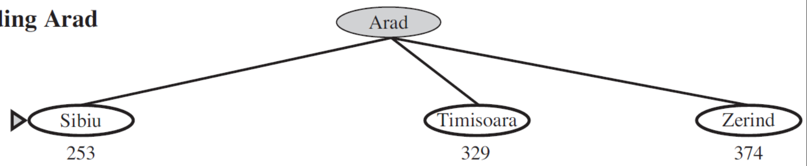


**Figure** A simplified road map of part of Romania.



# Greedy best-first search: An example

**(b) After expanding Arad**

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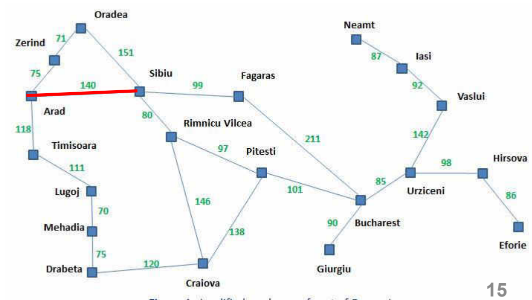
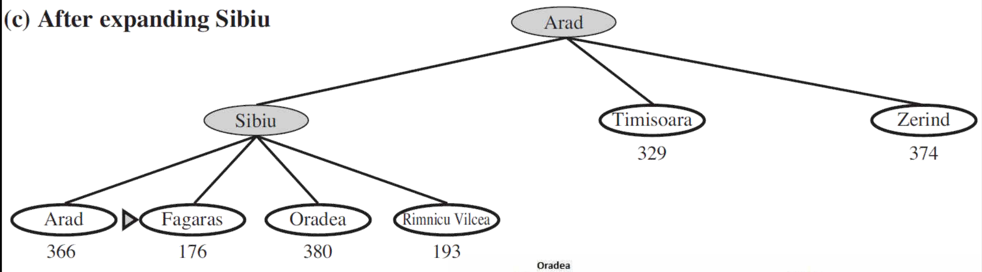


**Figure** A simplified road map of part of Romania.



# Greedy best-first search: An example

**(c) After expanding Sibiu**

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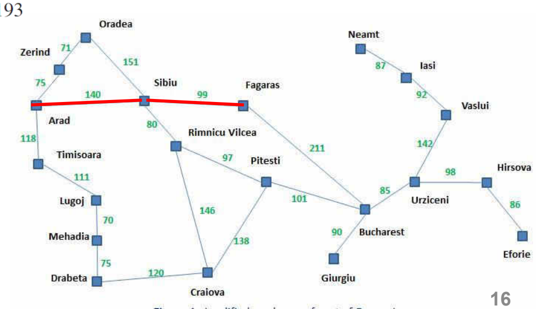
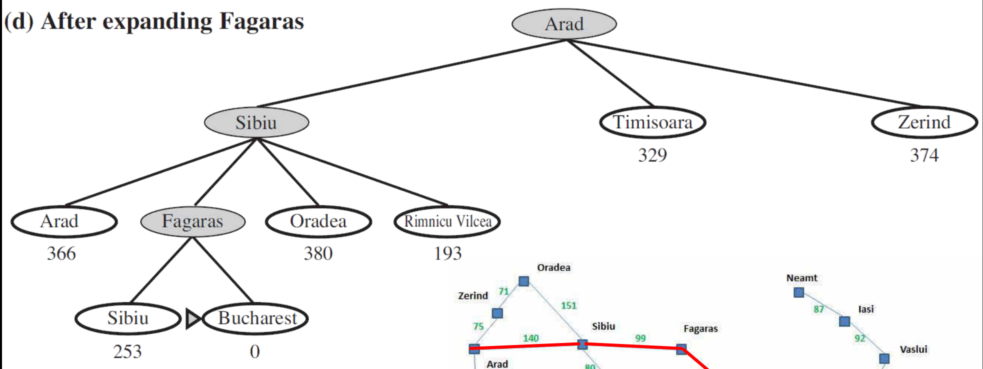


**Figure** A simplified road map of part of Romania.

# Greedy best-first search: An example




| | | | |
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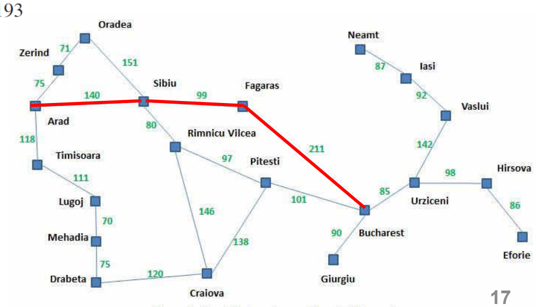



**Figure** A simplified road map of part of Romania.

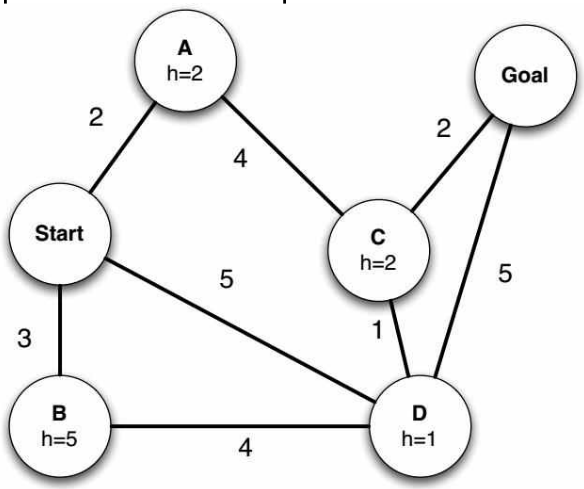


# An evaluation of GBFS (graph-search)

- Completeness
  - YES – if it is a graph-search instance in finite state spaces
- Time complexity
  - $O(b^m) \rightarrow$ reduced substantially with a good heuristic
- Space complexity
  - $O(b^m)$ – keeps all nodes in memory
- Optimality
  - NO

# Quiz 01: Greedy best-first search

- Work out the order in which states are expanded, as well as the path returned by graph search. Assume ties resolve in such a way that states with earlier alphabetical order are expanded first.






# A* Search

# A* search

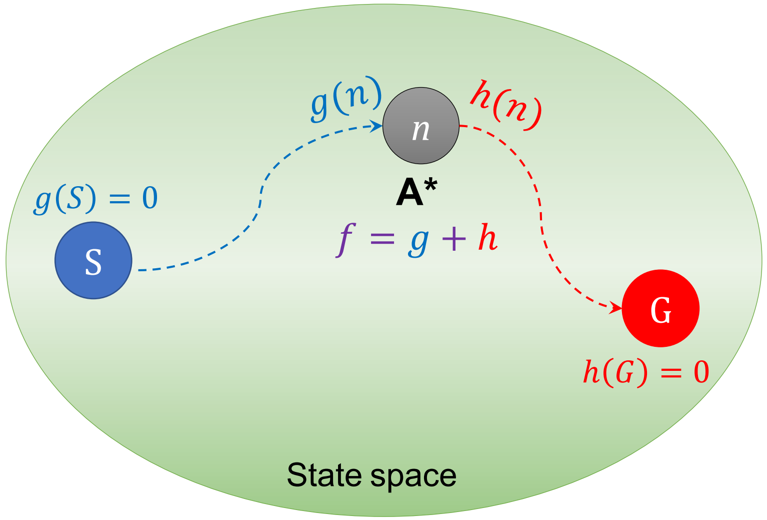
- The most widely known form of best-first search
- Use heuristic to guide search, but not only
- Avoid expanding paths that are already expensive
- Ensure to compute a path with minimum cost

- Evaluate nodes by $\boldsymbol{f(n) = g(n) + h(n)}$
  - where $g(n)$ is the cost to reach the node $n$ and $h(n)$ is the cost to get from $n$ to the goal
  - $f(n)$ = estimated cost of the cheapest solution through $n$

.

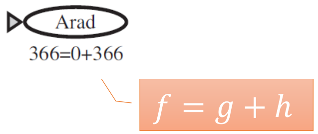


# A* search






**(a) The initial state**

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|---|---|---|---|
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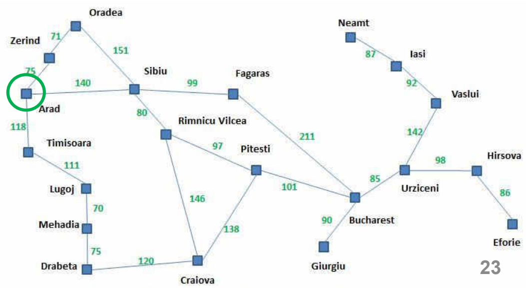
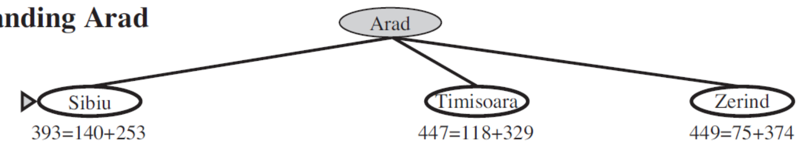



**Figure** A simplified road map of part of Romania.



**(b) After expanding Arad**

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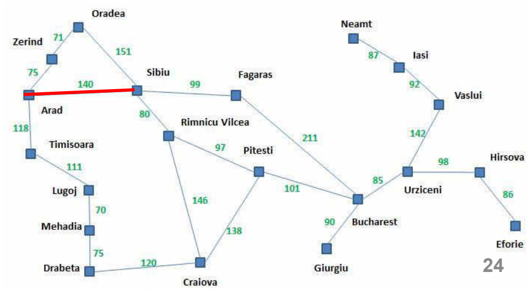
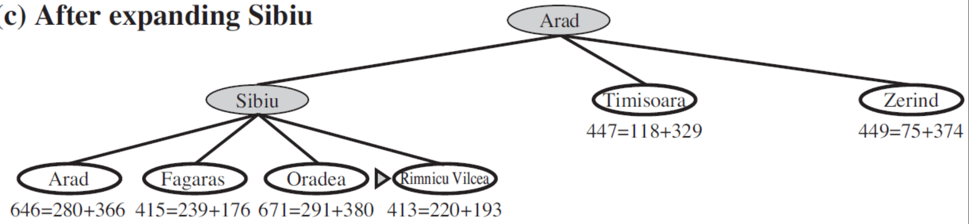



**Figure** A simplified road map of part of Romania.

## (c) After expanding Sibiu

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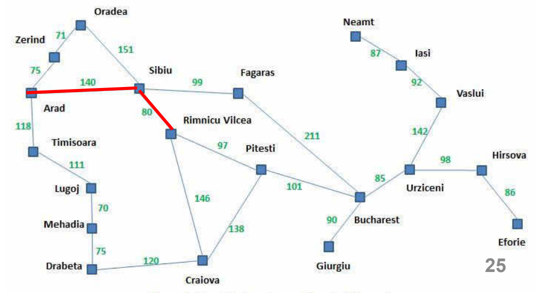
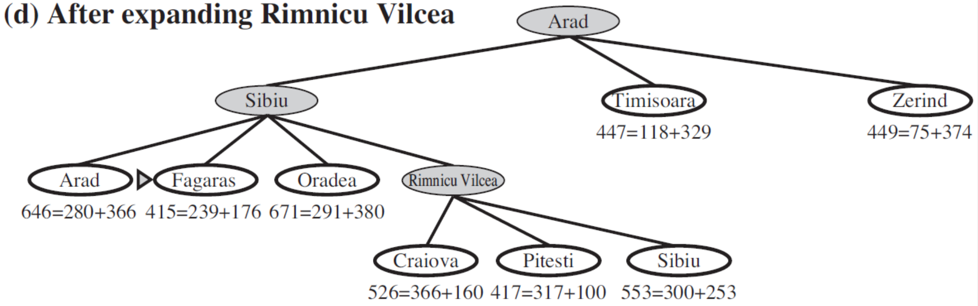



**Figure** A simplified road map of part of Romania.



## (d) After expanding Rimnicu Vilcea

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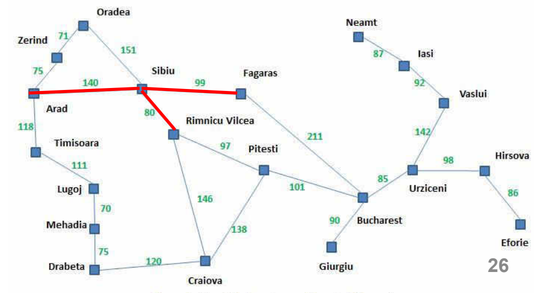
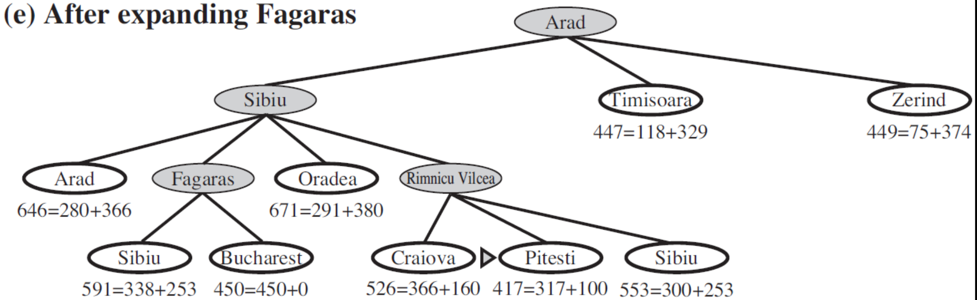


**Figure** A simplified road map of part of Romania.



## (e) After expanding Fagaras

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| **Fagaras** | 176 | **Sibiu** | 253 |
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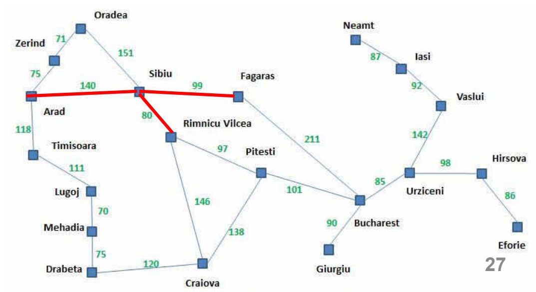
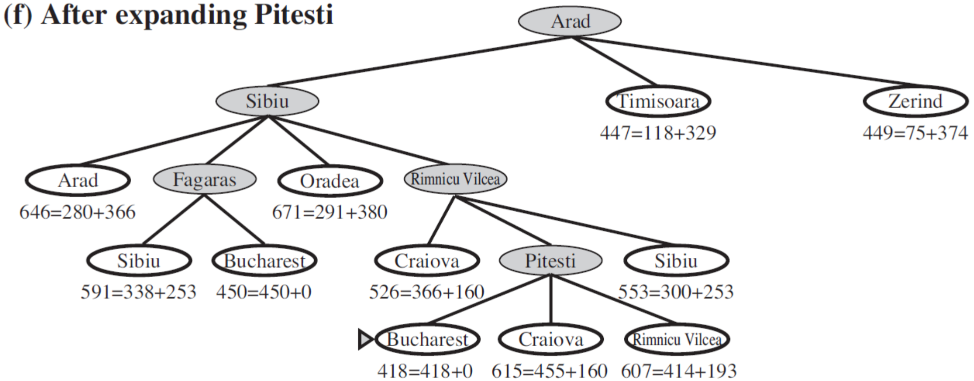


**Figure** A simplified road map of part of Romania.



## (f) After expanding Pitesti

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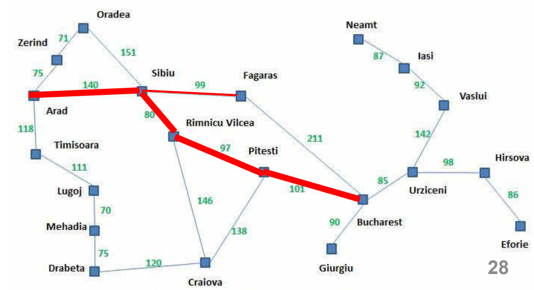


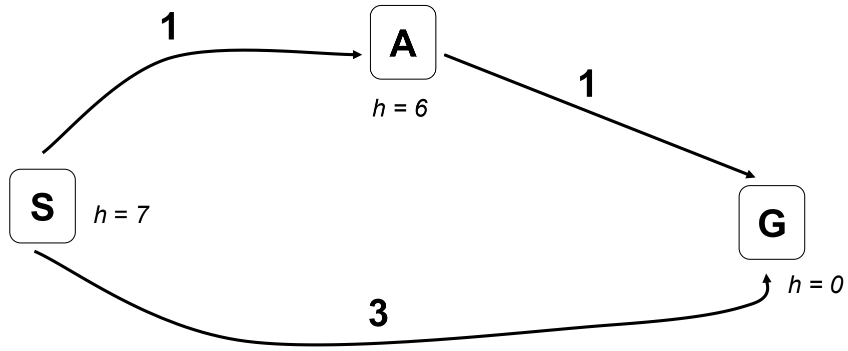
**Figure** A simplified road map of part of Romania.

# An evaluation of A* (graph-search)

- Completeness
  - YES if all step costs exceed some finite $\epsilon$ and if $b$ is finite
  - (review the condition for completeness of UCS)
- Optimality
  - YES – with conditions on heuristic being used
- Time complexity
  - Exponential
- Space complexity
  - Exponential (keep all nodes in memory)



# A* is not always optimal...

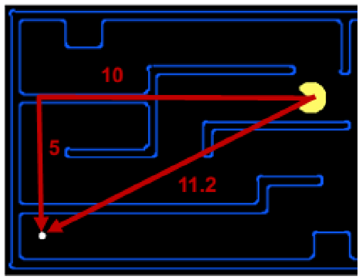


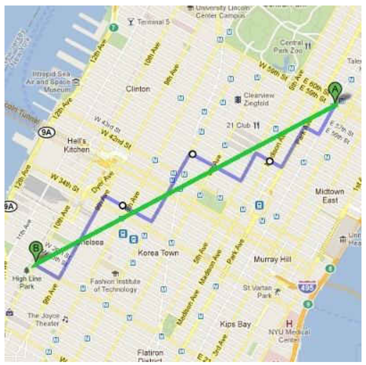

In what conditions, A* is optimal?



# Conditions for optimality: Admissibility

- $h(n)$ must be an admissible heuristic
  - Never overestimate the cost to reach the goal → optimistic
  - E.g., the straight-line distance $h_{SLD}$

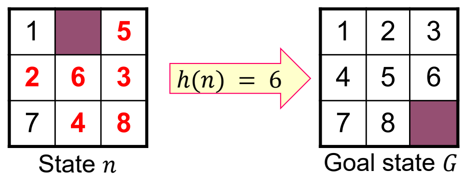


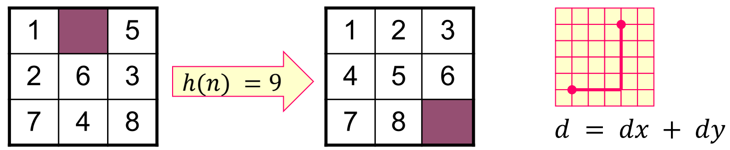


# Admissible heuristics for 8-puzzle

- $h(n) =$ number of misplaced numbered tiles



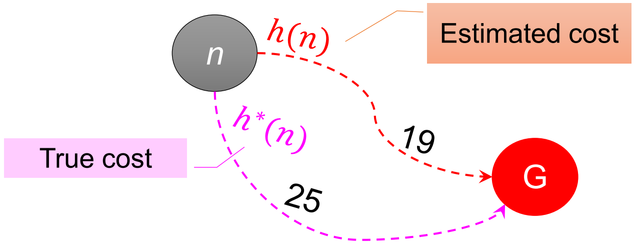

- $h(n) =$ sum of the (Manhattan) distance of every numbered tile to its goal position




$h = 0 + 2 + 1 + 2 + 2 + 1 + 0 + 1$

# Conditions for optimality: Admissibility

- $h(n)$ is admissible if for every node $n$, $\boldsymbol{h(n) \leq h^*(n)}$
  - where $h^*(n)$ is the true cost to reach the goal state from $n$




- Hence, $f(n)$ never overestimates the true cost of a solution along the current path through $n$.
  - $g(n)$ is the actual cost to reach $n$ along the current path



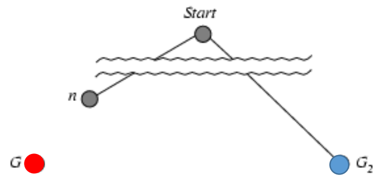
# Conditions for optimality: Admissibility

If $h(n)$ is admissible, A* using **TREE-SEARCH** is optimal

- Suppose some suboptimal goal $G_2$ has been generated and is in the frontier.
- Let $n$ be an unexpanded node in the frontier such that $n$ is on a shortest path to an optimal goal $G$.
- $f(G_2) = g(G_2)$ since $h(G_2) = 0$
- $g(G_2) > g(G)$ since $G_2$ is suboptimal
- $f(G) = g(G)$ since $h(G) = 0$

  $f(G_2) > f(G)$ (1)
- $h(n) \leq h^*(n)$ since $h$ is admissible
- $g(n) + h(n) \leq g(n) + h^*(n)$

  $f(n) \leq f(G)$ (2)
- From (1), (2): $f(G_2) > f(n) \rightarrow$ A* will never select $G_2$ for expansion



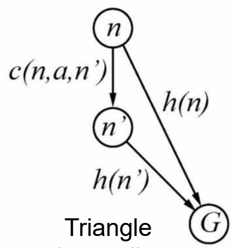



# Conditions for optimality: Consistency

- Admissibility is insufficient for graph search.
  - The optimal path to a repeated state could be discard if it is not the first one selected.
- $h(n)$ is consistent if for every node $n$, every successor $n'$ of $n$ generated by any action $a$,

$$\boldsymbol{h(n) \leq c(n, a, n') + h(n')}$$

- Every consistent heuristic is also admissible.



Triangle inequality



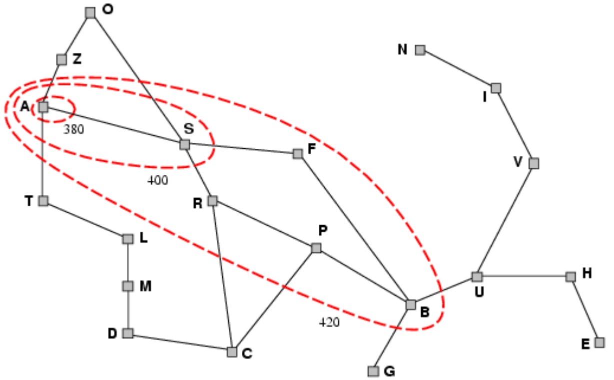
# Conditions for optimality: Consistency

If $h(n)$ is consistent, A* using **GRAPH-SEARCH** is optimal

- If $h(n)$ is consistent, the values of $f(n)$ along any path are non-decreasing.
  - Suppose $n'$ is a successor of $n \rightarrow g(n') = g(n) + c(n, a, n')$
  - $f(n') = g(n') + h(n') = g(n) + c(n, a, n') + h(n') \geq g(n) + h(n) = f(n)$
- Whenever A* selects a node $n$ for expansion, the optimal path to that node has been found.
  - Proof by contradiction: There would have to be another frontier node $n'$ on the optimal path from the start node to $n$ (by the graph separation property)
  - $f$ is nondecreasing along any path $\rightarrow f(n') < f(n) \rightarrow n'$ would have been selected first

# Contours of A* search

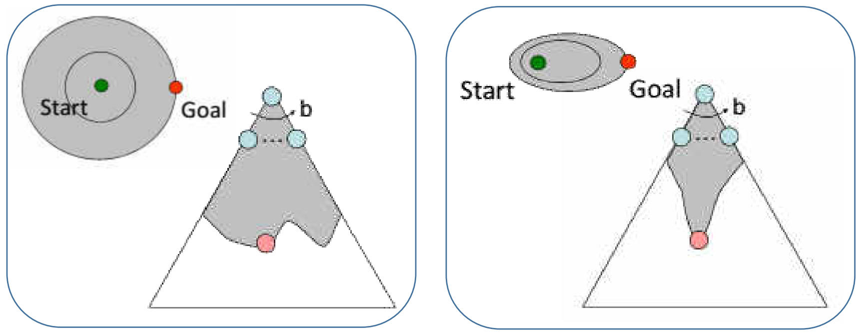
- A* expands nodes in order of increasing $f$-value
- Gradually adds "$f$-contours" of nodes such that contour $i$ has all nodes with $f = f_i$ where $f_i < f_{i+1}$
- A* will expand all nodes with costs $f(n) < C^*$






# A* contours vs. UCS contours

- The bands of UCS will be "circular" around the start state.




- The bands of A*, with more accurate heuristics, will stretch toward the goal state and become more narrowly focused around the optimal path.



# Comments on A*: The good

- Never expand nodes with $f(n) > C^*$
  - All nodes like these are **pruned** while still guaranteeing optimality
- Optimally efficient for any given consistent heuristic
  - No other optimal algorithm is guaranteed to expand fewer nodes



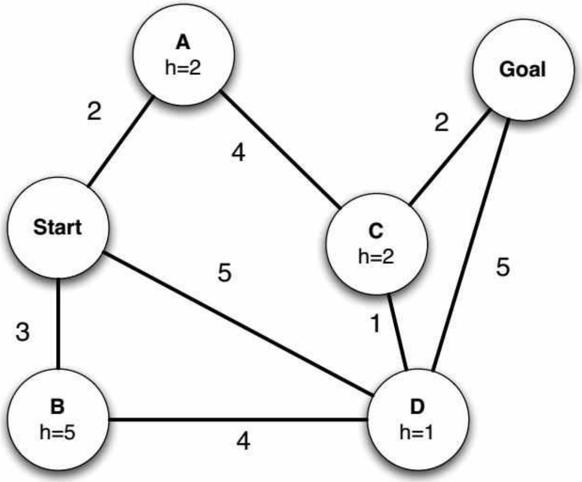
# Comments on A*: The bad

- A* expands all nodes with $f(n) < C^*$ (and possibly some nodes with $f(n) = C^*$) with before selecting a goal node.
  - This can still be exponentially large
  - A* usually runs out of space before it runs out of time
- Exponential growth will occur unless error in $h(n)$ grows no faster than log(true path cost)
  - In practice, error is usually proportional to true path cost (not log)
  - So exponential growth is common

  $\rightarrow$ Not practical for many large-scale problems

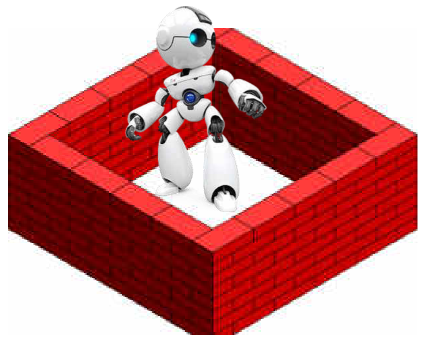
# Quiz 02: A*

- Work out the order in which states are expanded, as well as the path returned by graph search. Assume ties resolve in such a way that states with earlier alphabetical order are expanded first.





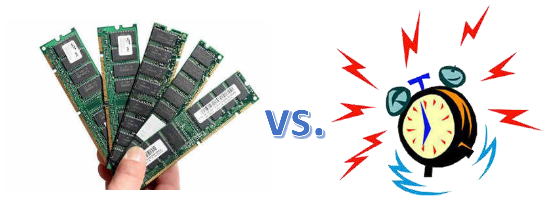

# Memory-bounded heuristic search





# Memory-bound heuristic search

- In practice, A* usually runs out of space long before it runs out of time.




- **Idea:** try something like DFS, but not forget everything about the branches we have partially explored



# Iterative-deepening A* (IDA*)

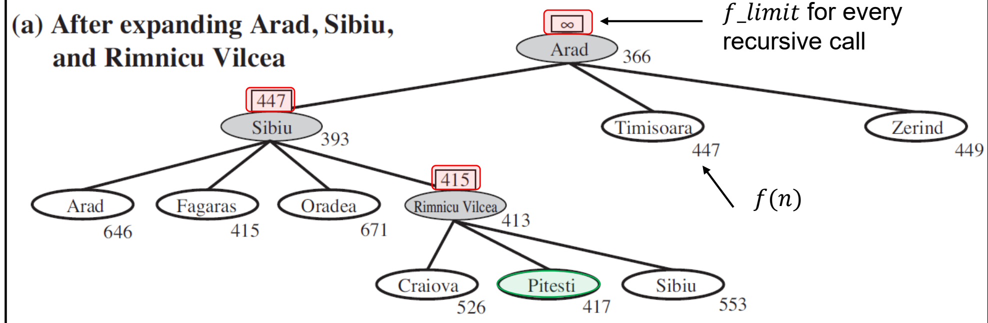
- The main difference with IDS
  - Cut-off use the **$f$-value** $(g + h)$ rather than the depth
  - At each iteration, the cutoff value is the smallest $f$-value of any node that exceeded the cutoff on the previous iteration
- Avoid the substantial overhead associated with keeping a sorted queue of nodes.
- Practical for many problems with unit step costs, yet difficult with real valued costs

# Recursive best-first search (RBFS)

- Keep track of the $f$-value of the best alternative path available from any ancestor of the current node
  - $\rightarrow$ backtrack when the current node exceeds $f\_limit$
- As it backtracks, replace the $f$-value of each node along the path with the best $f(n)$ value of its children



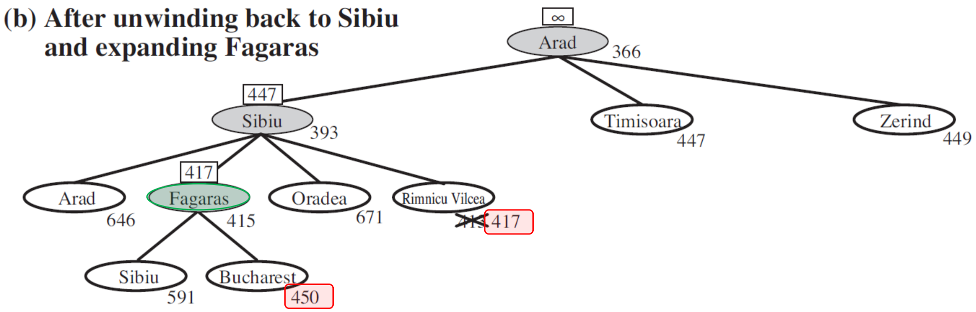
# RBFS: An example




- Path until Rimnicu Vilcea is already expanded
- The path is followed until Pitesti, whose *f*-value worse than the $f\_limit$

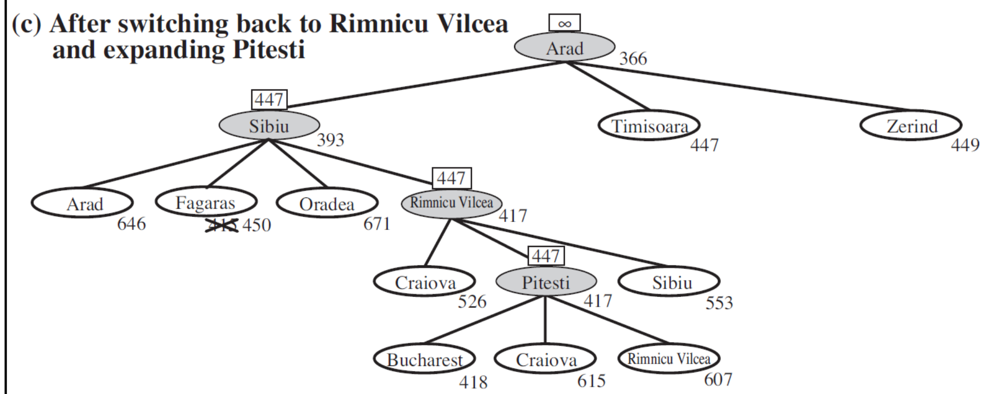


# Recursive best-first search (RBFS)




- Unwind recursion and store best $f$-value for current best leaf Rimnicu Vilcea
  - *result, best.f ← RBFS(problem, best, min(f_limit, alternative))*
- *best* is now Fagaras. Call RBFS for new *best*
  - *best* value is now 450






- Unwind recursion and store best $f$-value for current best leaf of Fagaras
  - *result, best.f ← RBFS(problem, best, min(f_limit, alternative))*
- *best* is now Rimnicu Viclea (again). Call RBFS for new *best*
  - Subtree is again expanded
  - Best alternative subtree is now through Timisoara
- Solution is found since because 447 > 418.

# Recursive best-first search (RBFS)

```
function RECURSIVE-BEST-FIRST-SEARCH(problem) returns a solution, or failure
  return RBFS(problem, MAKE-NODE(problem.INITIAL-STATE),∞)

function RBFS(problem, node, f_limit) returns a solution, or failure and a new f-cost limit
  if problem.GOAL-TEST(node.STATE) then return SOLUTION(node)
  successors ← [ ]
  for each action in problem.ACTIONS(node.STATE) do
    add CHILD-NODE(problem, node, action) into successors
  if successors is empty then return failure, ∞
  for each s in successors do /* update f with value from previous search, if any */
    s.f ← max(s.g+s.h, node.f))
  loop do
    best ← the lowest f-value node in successors
    if best.f > f_limit then return failure, best.f
    alternative ← the second-lowest f-value among successors
    result, best.f ← RBFS(problem, best, min(f_limit, alternative))
    if result ≠ failure then return result
```



# Evaluation of RBFS

- Optimality
  - Like A*, optimal if $h(n)$ is admissible
- Time complexity
  - Difficult to characterize
  - Depends on accuracy of $h(n)$ and how often best path changes
  - Can end up "switching" back and forth
- Space complexity
  - Linear time: *O(bd)*
  - Other extreme to A* - uses **too little** memory even if more memory were available



# (Simplified) Memory-bound A* – (S)MA*

- Like A*, but delete the worst node (largest *f*-value) when memory is full
- Also backs up the value of the forgotten node to its parent
  - If there is a tie (equal $f$-values), delete the oldest nodes first
- Find an optimal reachable solution given memory constraint
  - The depth of the shallowest goal node is less than the memory size (expressed in nodes).
- Time can still be exponential.



# Learning to search better

- *Could an agent learn how to search better?* **YES**
- **Metalevel state space:** in which each state captures the internal (computational) state of a program that is searching in an object-level state space.
- For example, the map of Romania problem,
  - The internal state of the A* algorithm is the current search tree.
  - Each action in the metalevel state space is a computation step that alters the internal state, e.g., [expands a leaf node and adds its successors to the tree]

# Learning to search better

- The expansion of Fagaras is not helpful → harder problems may even include more such missteps

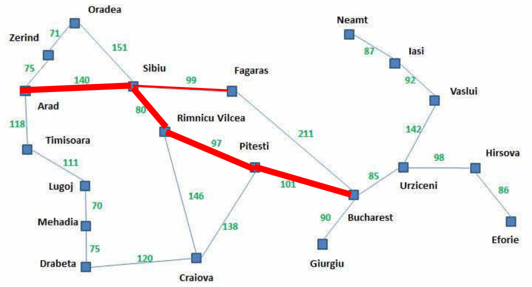


**Figure** A simplified road map of part of Romania.

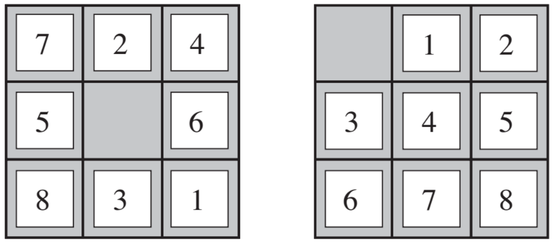
- A metalevel learning algorithm gains from these experiences to avoid exploring unpromising subtrees
  → reinforcement learning



# Heuristic functions



# The 8-puzzle problem



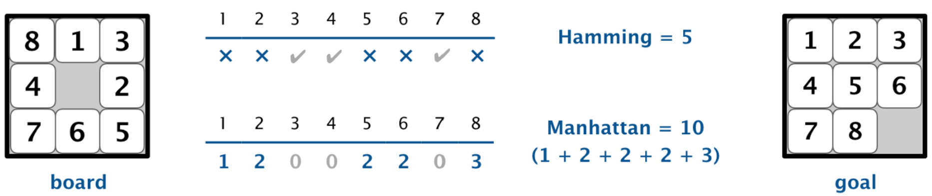
A typical instance of the 8-puzzle. The solution is 26 steps long.

- Average solution cost: about 22 steps, branching factor ~ 3.
- 8-puzzle: 9!/2 = 181,440 reachable states
- 15-puzzle: 1.05 x $10^{13}$ possible states



# Admissible heuristics for 8-puzzle

- $h_1(n)$ = *number of misplaced numbered tiles (Hamming distance)*




- $h_2(n)$ = *sum of the (Manhattan) distance of every numbered tile to its goal position*

# Quiz 03: Admissible heuristics

- Knowing that $h(n) \leq h^*(n)$
- For 8-puzzle, which of the following heuristics is admissible?
  - $h_1(n) = \text{total number of misplaced tiles}$
  - $h_2(n) = \text{total Manhattan distance}$
  - $h_3(n) = 0$
  - $h_4(n) = 1$
  - $h_5(n) = h^*(n)$
  - $h_6(n) = \min(2, h^*(n))$
  - $h_7(n) = \max(2, h^*(n))$



# The effect of heuristic on performance

- Effective branching factor $b^*$: the factor that a uniform tree of depth $d$ would have to contain $N+1$ nodes

$$\boldsymbol{N + 1 = 1 + b^* + (b^*)^2 + \cdots + (b^*)^d}$$

  - where $N$ is the total number of nodes generated by A* for a particular problem and $d$ is the solution depth
  - E.g., A* finds a solution at depth 5 using 52 nodes $\rightarrow b^* = 1.92$
- $b^*$ varies across problem instances, but fairly constant for sufficiently hard problems
- A well-designed heuristic would have a value of $b^*$ close to 1

$\rightarrow$ fairly large problems solved at reasonable cost



# Search cost vs. Branching factor

| | Search Cost (nodes generated) | | | Effective Branching Factor | | |
|---|---|---|---|---|---|---|
| $d$ | IDS | A*($h_1$) | A*($h_2$) | IDS | A*($h_1$) | A*($h_2$) |
| 2 | 10 | 6 | 6 | 2.45 | 1.79 | 1.79 |
| 4 | 112 | 13 | 12 | 2.87 | 1.48 | 1.45 |
| 6 | 680 | 20 | 18 | 2.73 | 1.34 | 1.30 |
| 8 | 6384 | 39 | 25 | 2.80 | 1.33 | 1.24 |
| 10 | 47127 | 93 | 39 | 2.79 | 1.38 | 1.22 |
| 12 | 3644035 | 227 | 73 | 2.78 | 1.42 | 1.24 |
| 14 | – | 539 | 113 | – | 1.44 | 1.23 |
| 16 | – | 1301 | 211 | – | 1.45 | 1.25 |
| 18 | – | 3056 | 363 | – | 1.46 | 1.26 |
| 20 | – | 7276 | 676 | – | 1.47 | 1.27 |
| 22 | – | 18094 | 1219 | – | 1.48 | 1.28 |
| 24 | – | 39135 | 1641 | – | 1.48 | 1.26 |

Comparison of the search costs and effective branching factors for the ITERATIVE-DEEPENING-SEARCH and A∗ algorithms with $h_1$, $h_2$. Data are averaged over 100 instances of the 8-puzzle for each of various solution lengths $d$.



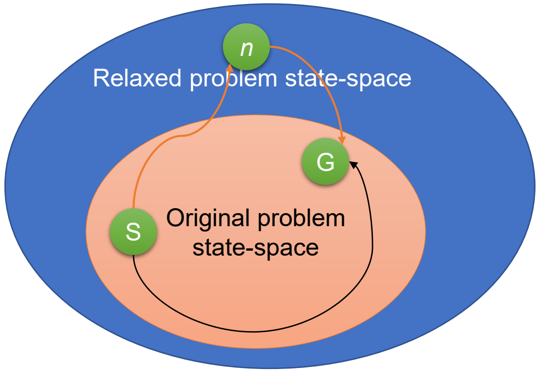
# Heuristic dominance

- Given two admissible heuristics, $h_1$ and $h_2$
- If $h_2(n) \geq h_1(n)$, for all $n$, then $h_2$ **dominates** $h_1$
  - A* using $h_2$ will never expand more nodes than A* using $h_1$
- Better to use a heuristic function with higher values, provided it is consistent and its computation time is not too long.

*How might one have come up with $h_2$?*
*Is it possible to invent such a heuristic mechanically?*

# Relaxed problems

- Problems with fewer restrictions on the actions




The state-space graph of the relaxed problem is a *supergraph* of the original state space

- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem



# Relaxed problems of the 8-puzzle

- Original problem:
  - **A tile can move from square A to square B** if A is horizontally or vertically adjacent to B and B is blank
- Relaxed problems are generated by removing one or both conditions
  - A tile can move from square A to square B if A is adjacent to B.
  - A tile can move from square A to square B if B is blank.
  - A tile can move from square A to square B.

Manhattan distance

Misplaced tiles

It is crucial that the relaxed problems generated by this technique can be solved essentially *without search*



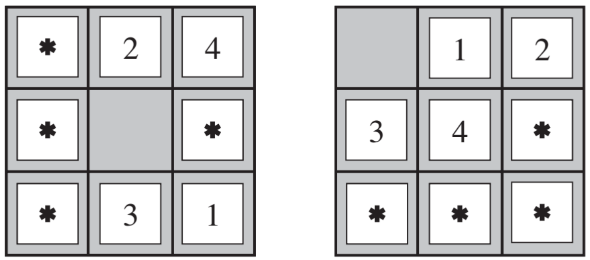
# Relaxed problems

- Given a collection of admissible heuristics, $h_1$, $h_2$,…, $h_m$, available for a problem and none of them dominates any of the others.
- The composite heuristic function is defined as

$$h(n) = \max\{h_1(n), h_2(n), \ldots, h_m(n)\}$$

- $h(n)$ is consistent and dominates all component heuristics

# A subproblem of the 8-puzzle instance




Start State    Goal State

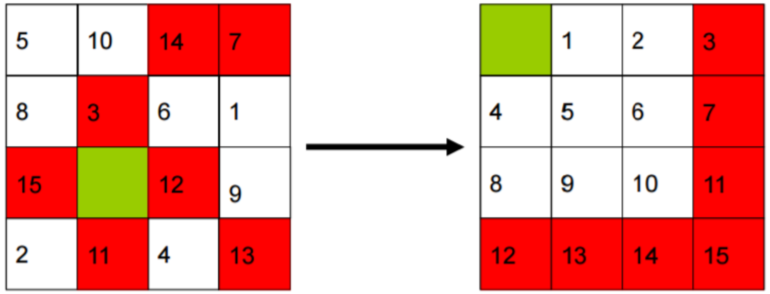
- Get tiles 1, 2, 3, and 4 into their correct positions, without worrying about what happens to the other tiles
  - Optimal cost of this subproblem ≤ cost of the original problem
  - More accurate than Manhattan distance in some cases

# Pattern databases

- Admissible heuristics can also be derived from the **solution cost** of a subproblem of the given problem.
  - This cost is a lower bound on the cost of the complete problem.
- Pattern databases (PDB): store the exact solution costs for every possible subproblem instances
  - E.g., every possible configuration of the four tiles and the blank
- The **complete heuristic** is constructed using the patterns in the databases.



# Heuristic from Pattern databases

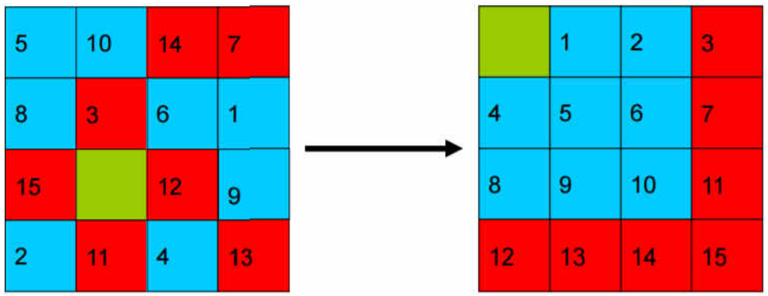



31 moves is a lower bound on the total number of moves needed to solve this particular state

https://courses.cs.washington.edu/courses/cse473/12sp/slides/04-heuristics.pdf



# Heuristic from Pattern databases




31 moves needed to solve red tiles

22 moves needed to solve blue tiles

→ Overall heuristic is maximum of 31 moves

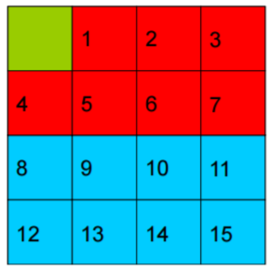
https://courses.cs.washington.edu/courses/cse473/12sp/slides/04-heuristics.pdf



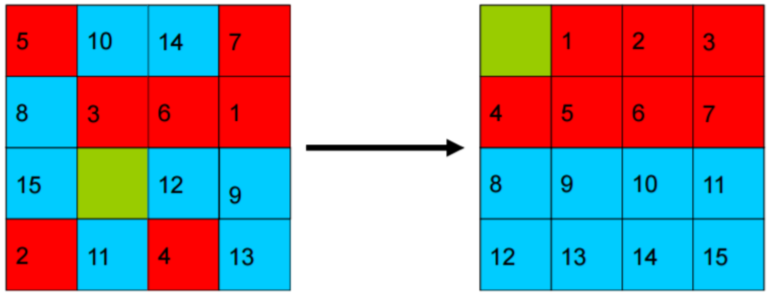
# Additive pattern databases

- Limitation of traditional PDB: Take max → diminish returns on additional DBs
- Disjoint pattern databases: Count only moves of the pattern tiles, ignoring non-pattern moves.
  - If no tile belongs to more than one pattern, add their heuristic values.

The 7-tile database contains 58 million entries.
The 8-tile database contains 519 million entries.



# Additive pattern databases




20 moves needed to solve red tiles

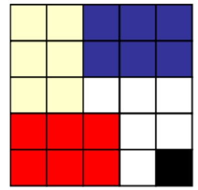
25 moves needed to solve blue tiles

→ Overall heuristic is 20 + 25 = 45 moves



# Performance of PDB

- **15 Puzzle**
  - 2000× speedup vs. Manhattan distance
  - IDA* with the two DBs solves 15-puzzles optimally in 30 milliseconds
- **24 Puzzle**
  - 12 million × speedup vs. Manhattan
  - IDA* can solve random instances in 2 days.
  - Requires 4 DBs as shown
    - Each DB has 128 million entries
  - Without PDBs: 65,000 years





# Learning heuristics from experience

- Experience means solving a lot of instances of a problem.
  - E.g., solving lots of 8-puzzles
- Each optimal solution to a problem instance provides examples from which $h(n)$ can be learned
- Learning algorithms
  - Neural nets
  - Decision trees
  - Inductive learning
  - …




# THE END