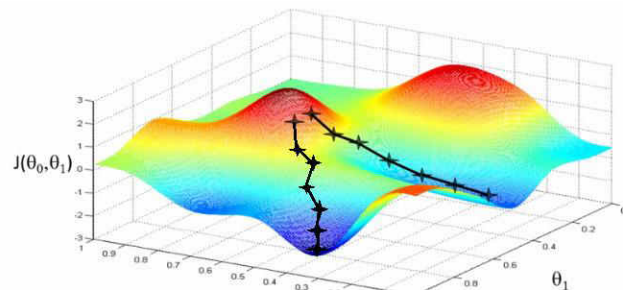


LOCAL SEARCH ALGORITHMS AND OPTIMIZATION PROBLEMS

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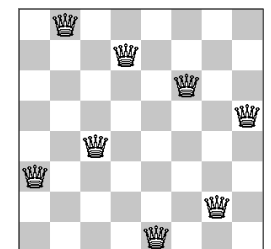
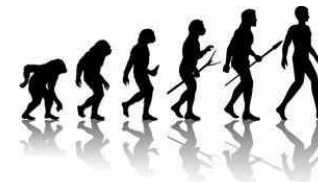
Optimization problems

Outline

- Optimization problems
- Hill-climbing search
- Simulated annealing
- Local beam search
- Genetic algorithm

Global search algorithms

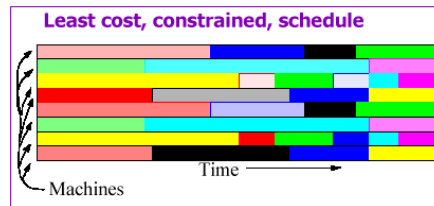
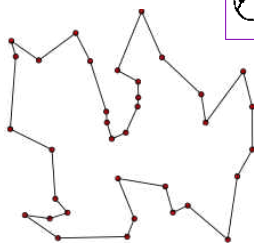
- **Global search:** explore search spaces systematically
 - Keep one or more paths in memory and record which alternatives have been explored at each point along the path
- Many problems do not fit the “standard” search model.
 - The final configuration matters, not the order in which it is formed.
 - No “goal test” and no “path cost”, e.g., [Darwinian evolution](#) with “natural” reproductive fitness



Optimization problems

- Find **the best state** according to an **objective function**.
 - E.g., the Knight's tour, TSP, scheduling, integrated-circuit design, factory-floor layout, automatic programming, vehicle routing, etc.

35	40	47	44	61	08	15	12
46	43	36	41	14	11	62	09
39	34	45	48	07	60	13	16
60	55	42	37	22	17	10	63
33	38	49	54	59	06	23	18
56	51	28	31	26	21	03	04
29	32	53	58	05	02	19	24
52	57	30	27	20	25	04	01

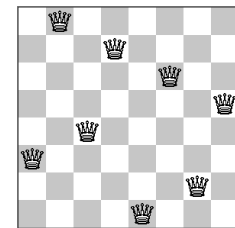


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Local search algorithms

- Operate using a **single current node** and generally move only to **neighbors of that node**
- Not systematic
 - The paths followed by the search are typically not retained.
- Use very little memory, usually a constant amount
- Find reasonable solutions in large or infinite (continuous) state spaces

Global search: $n = 200$
 vs.
 Local search: $n = 1,000,000$



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Local search and Optimization



"Pure optimization" problems

All states have an objective function
 Goal is to find state with max (or min) objective value

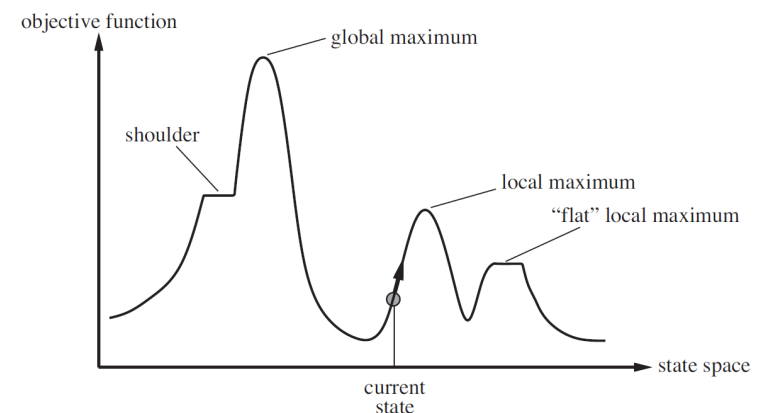


Local search can do quite well on these problems.

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State-space landscape

- A landscape has both **"location"** and **"elevation"**
 - Location \leftarrow state, elevation \leftarrow heuristic cost or objective function



A 1D state-space landscape in which elevation corresponds to the objective function.

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State-space landscape

Global minimum

Elevation corresponds to cost

Find the lowest valley

Global maximum

Elevation corresponds to an objective function

Find the highest peak

- A **complete** local search algorithm finds a goal if one exists.
- An **optimal** local search algorithm finds a global extremum.

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Hill-climbing search

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Hill-climbing search

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  current ← MAKE-NODE(problem.INITIAL-STATE)
  loop do
    neighbor ← a highest-valued successor of current
    if neighbor.VALUE ≤ current.VALUE then return current.STATE
  current ← neighbor
```

A version of HILL-CLIMBING that finds local maximum.

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Hill-climbing search

- A loop that **continually moves** in the direction of **increasing value** and **terminates** when it **reaches a “peak”**.
 - Peaks are where no neighbor has a higher value.
- Not look ahead beyond the immediate neighbors of the current state
- **No search tree** maintained, only the **state and the objective function's value** for the **current node** recorded
- Sometimes called **greedy local search**
 - Grab a good neighbor without thinking ahead about where to go next

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Hill-climbing: 8-queens problem

- Complete-state formulation

- All 8 queens on the board, one per column

- Successor function

- Move a queen to another square in the same column $\rightarrow 8 \times 7 = 56$ successors

- Heuristic cost function $h(n)$

- The number of pairs of queens that are **ATTACKING** each other, either directly or indirectly \rightarrow global minimum has $h(n) = 0$

- Make rapid progress toward a solution

- 8-queens ($8^8 \approx 17$ million states): 4 steps on average when succeeds and 3 when get stuck

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Hill-climbing: 8-queens problem

18	12	14	13	13	12	14	14
14	16	13	15	12	14	12	16
14	12	18	13	15	12	14	14
15	14	14	13	16	13	16	
14	17	15	13	14	16	16	
17	16	18	15	15	15	15	
18	14	15	15	14	16	16	
14	14	13	17	12	14	12	18

The best moves

- Current state $(c_1 c_2 c_3 c_4 c_5 c_6 c_7 c_8) = (5\ 6\ 7\ 4\ 5\ 6\ 7\ 6)$ $h(n) = 17$
- The best successors has $h = 12 \rightarrow$ choose randomly among the set of best successors if there is more than one

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Hill climbing and local maxima

- Often **suboptimal**, due to **local maxima**, **ridges** and **plateau**

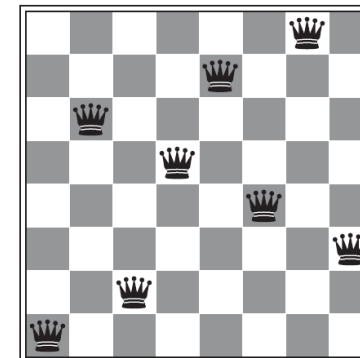
- E.g., 8-queens: steepest-ascent hill climbing gets stuck 86% of the time, solving only 14% of problem instances



The grid of states (dark circles) is superimposed on a ridge rising from left to right, creating a sequence of local maxima that are not directly connected to each other. From each local maximum, all the available actions point downhill.

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Hill climbing and local maxima



- Current state $(8\ 3\ 7\ 4\ 2\ 5\ 1\ 6)$ $h(n) = 1$
- Every successor has a higher cost \rightarrow local minimum

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Solutions: Sideways moves

- If no downhill (uphill) moves, the algorithm can escape with **sideways moves**.
 - A limit on the possible number of sideways moves required to avoid infinite loops
- For example, 8-queens problem
 - Allow sideways moves with a limit of 100 → percentage of problem instances solved raises from 14 to 94%
 - **Success comes at a cost**: the algorithm averages roughly 21 steps for each successful instance and 64 for each failure.

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Solutions: Hill-climbing variants

- **Stochastic hill climbing**
 - Choose at random from among the uphill moves with a probability of selection varied with the moves' steepness
 - Usually converge more slowly than steepest ascent, but find better solutions in some cases
- **First-choice hill climbing**
 - Generate successors randomly until one is generated that is better than the current state
 - Good strategy when a state has many successors (e.g., thousands)

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Solutions: Random-restart hill climbing

- **Random-restart hill climbing** can find a good solution quickly after a small number of restarts.
- That is a series of hill-climbing searches from randomly generated initial states until a goal is found.
 - For each restart: run until termination vs run for a fixed time
 - Run a fixed number of restarts or run indefinitely
- If each search has a probability p of success, the expected number of restarts required is $1/p$
- Very effective indeed for 8-queens

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Quiz 01: 4-queens problem

- Consider the following 4-queen problem

	A	B	C	D
0	♔	♔	♔	♔
1				
2				
3				

- Apply (first-choice) hill-climbing to find a solution, using the heuristic “The number of pairs of queens ATTACKING each other.”

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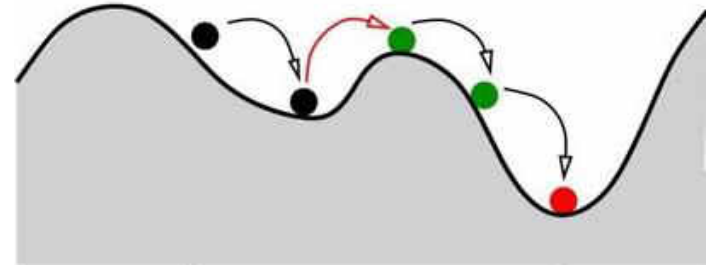


Simulated annealing

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Simulated annealing

- Combine **hill climbing** with a **random walk** in some way that yields both **efficiency** and **completeness**
- Shake hard (i.e., at a high temperature) and then gradually reduce the intensity of shaking (i.e., lower the temperature)



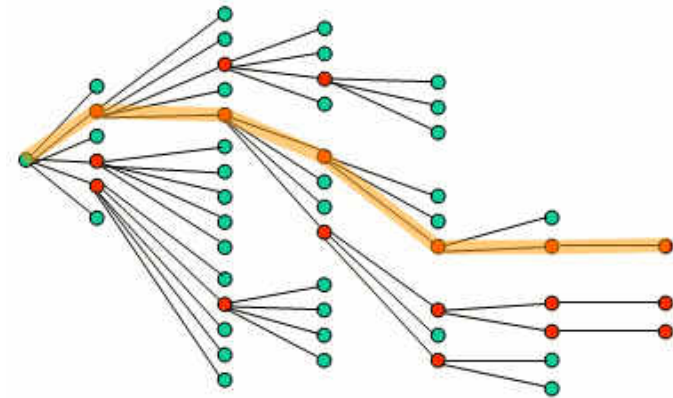
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Simulated annealing

```

function SIMULATED-ANNEALING(problem, schedule)
  returns a solution state
  inputs: problem, a problem
           schedule, a mapping from time to "temperature"
  current  $\leftarrow$  MAKE-NODE(problem.INITIAL-STATE)
  for  $t = 1$  to  $\infty$  do
     $T \leftarrow$  schedule( $t$ )
    if  $T = 0$  then return current
    next  $\leftarrow$  a randomly selected successor of current
     $\Delta E \leftarrow$  next.VALUE - current.VALUE
    if  $\Delta E > 0$  then current  $\leftarrow$  next
    else current  $\leftarrow$  next only with probability  $e^{\Delta E/T}$ 
  
```

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Local beam search

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Local beam search

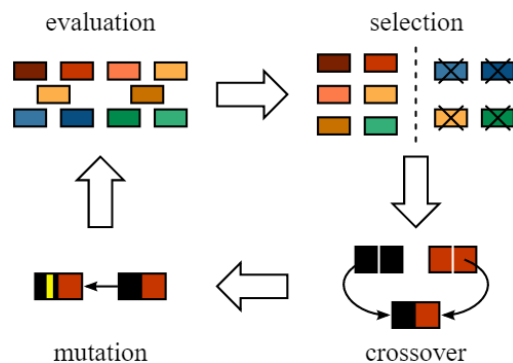
- Keep track of k states rather than just one
- Begin with k randomly generated states
- At each step, all successors of all k states are generated
- If the goal is not found, select the k best successors from the complete list and repeat

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Local beam search

- Useful information is passed among the parallel search threads → major difference from random-restart search
- Possibly suffer from a lack of diversity among the k states
 - Quickly concentrate in a small region of the state space → an expensive version of hill climbing
- **Stochastic beam search**
 - Choose k successors at random, with the probability of choosing a given successor being an increasing function of its value

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Genetic algorithms

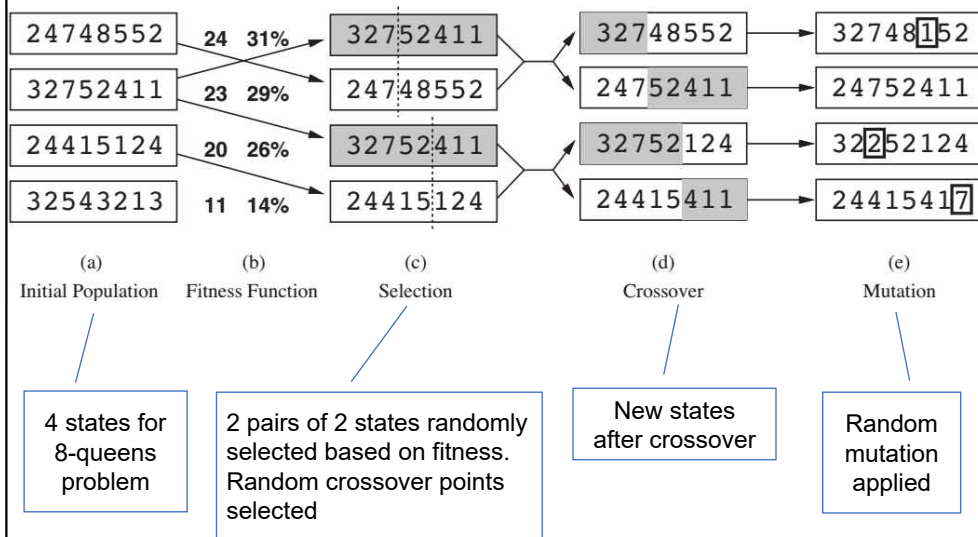
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Genetic algorithms

- A variant of stochastic beam search
- Successor states are generated by combining two parent states rather than by modifying a single state
- The reproduction are sexual rather than asexual

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Genetic algorithms: 8-queens



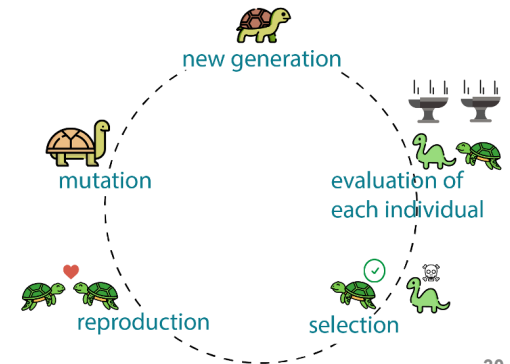
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Genetic algorithms

- **Population:** a set of k randomly generated states to begin with
- **Fitness function:** an objective function that rates each state
 - Higher values for better state
 - E.g., 8-queens: the number of nonattacking pairs of queens (min = 0, max = $8 \times 7/2 = 28$)

- Produce the next generation by "simulated evolution"

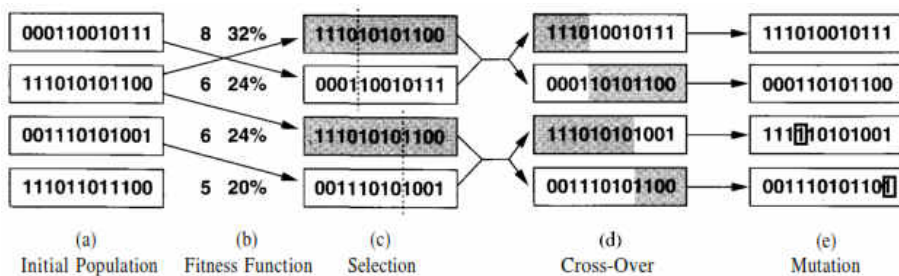
- Random selection
- Crossover
- Random mutation



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Representation of Individuals

- Each state, or **individual**, is represented as a string over a finite alphabet – most commonly, a string of 0s and 1s.
 - E.g., 8-queens: $8 \times \log_2 8 = 24$ bits



- Alternatively, the state could be represented as 8 digits, each in the range from 1 to 8.

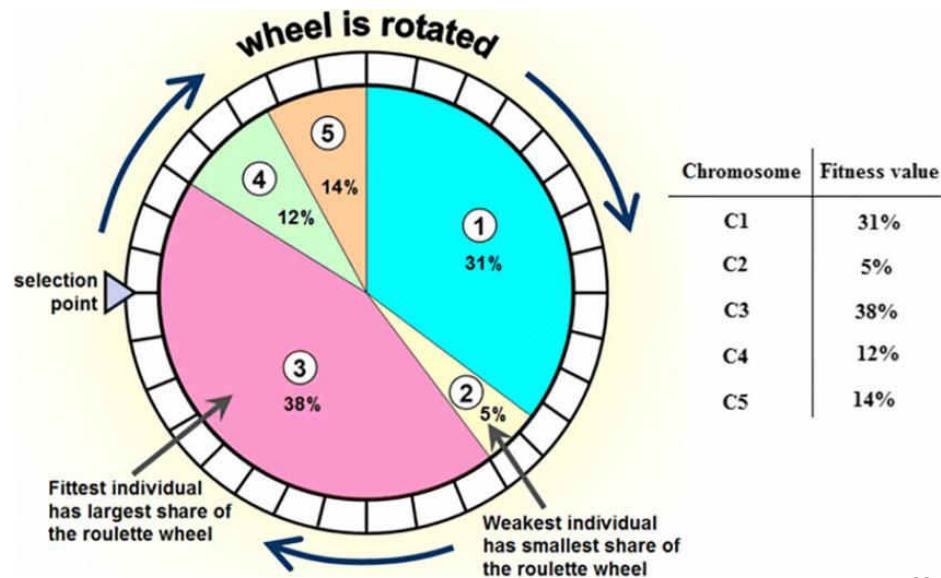
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Reproduction

- Pairs are selected at random for **reproduction**.
- The probability of being chosen for reproducing is directly proportional to the fitness score.
 - E.g., $24/(24+23+20+11) = 31\%$, $23/(24+23+20+11) = 29\%$, etc.
- One individual may be selected several times or not at all.

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Reproduction: Roulette wheel



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Quiz 02: Calculate fitness scores

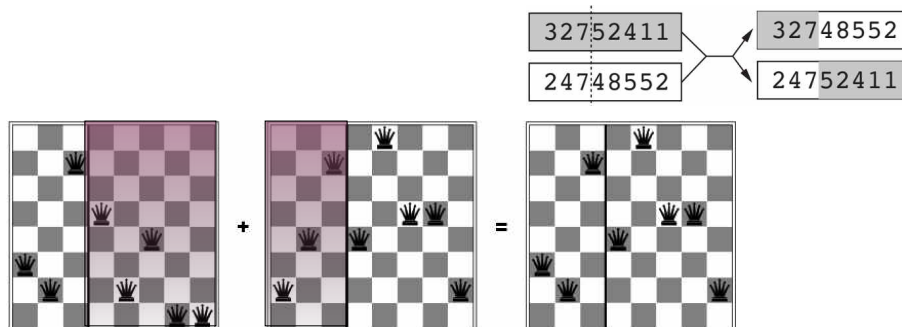
- The current generation include 4 states, S1, S2, S3, and S4.
- Their evaluation functions' values are:

$$\begin{aligned} \text{Eval}(S1) &= 7, & \text{Eval}(S2) &= 15, \\ \text{Eval}(S3) &= 10, & \text{Eval}(S4) &= 18 \end{aligned}$$
- Calculate the probability (the fitness that) each of them will be chosen in the "selection" step.
- Which two of four states have the highest possibilities to be selected for the next generation?

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Crossover operation

- For each pair to be mated, a **crossover point** is chosen **randomly** from the positions in the string.

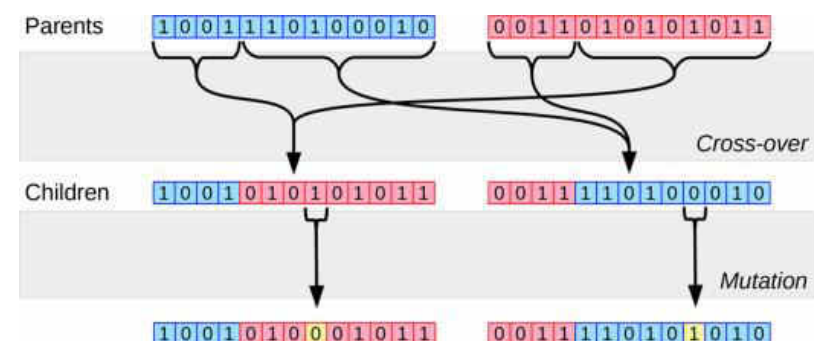


- Effectively "jump" to a completely different new part of the search space (quite non-local)

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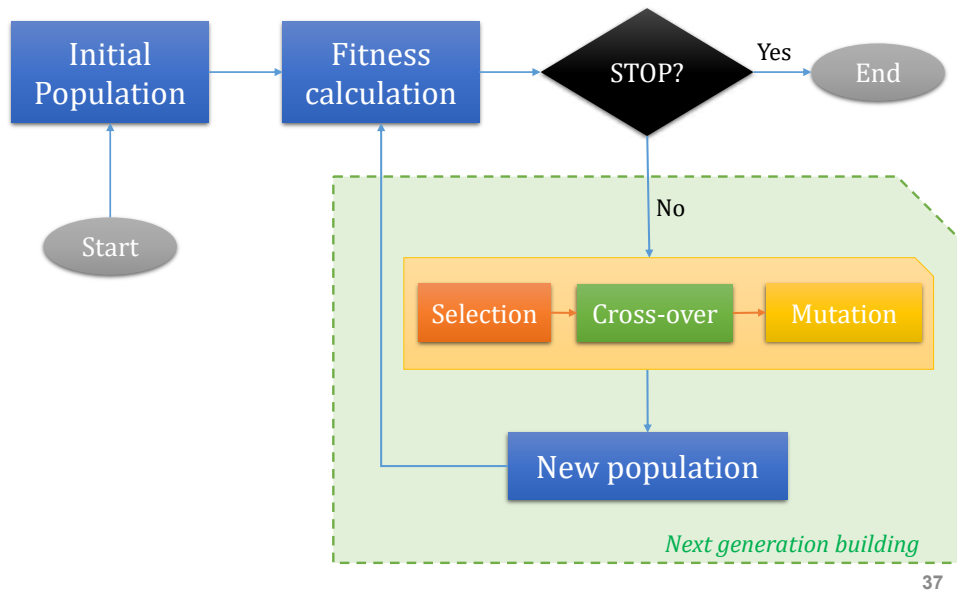
Mutation operation

- Each location is subject to **random mutation** with a small independent probability.
 - E.g., 8-queens: choosing a queen at random and moving it to a random square in its column



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A workflow of Genetic algorithms



Genetic algorithms

```

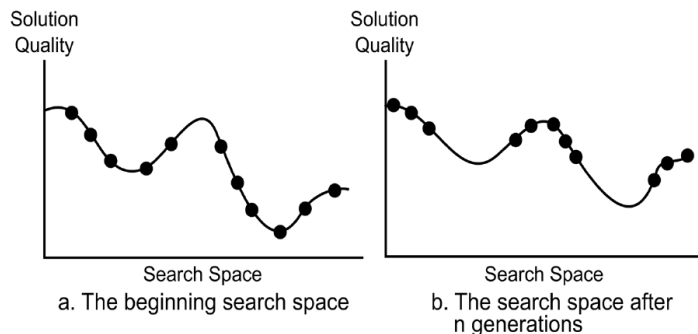
function GENETIC-ALGORITHM(population, FITNESS-FN)
  returns an individual
  inputs: population, a set of individuals
           FITNESS-FN, a function that measures the fitness of an individual
  repeat
    new_population  $\leftarrow$  empty set
    for  $i = 1$  to SIZE(population) do
       $x \leftarrow$  RANDOM-SELECTION(population, FITNESS-FN)
       $y \leftarrow$  RANDOM-SELECTION(population, FITNESS-FN)
      child  $\leftarrow$  REPRODUCE( $x, y$ )
      if (small random probability) then child  $\leftarrow$  MUTATE(child)
      add child to new_population
    population  $\leftarrow$  new_population
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to FITNESS-FN
  
```

The slide number 38 is at the bottom right.

Genetic algorithms

```

function REPRODUCE( $x, y$ ) returns an individual
  inputs:  $x, y$ , parent individuals
   $n \leftarrow$  LENGTH( $x$ );  $c \leftarrow$  random number from 1 to  $n$ 
  return APPEND(SUBSTRING( $x, 1, c$ ), SUBSTRING( $y, c + 1, n$ ))
  
```



Comments on Genetic algorithms

- Random exploration find solutions that local search does not
 - Via crossover primarily
 - Can solve “hard” problem
- Rely on very little domain knowledge
- Appealing connection to human evolution

Comments on Genetic algorithms

- Large number of “tunable” parameters
 - Difficult to replicate performance from one problem to another
- Lack of good empirical studies comparing to simpler methods
- Useful on some (small?) sets of problem, yet no convincing evidence that GAs are better than hill-climbing w/random restarts in general.
- Require careful engineering of the representation
- Application: Genetic Programming!

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THE END

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