REVIEW EXERCISE 06

Question 1. Convert the following English sentences into FOL sentences, using only the predicates given inside the square brackets.

1. All green apples are sour. [Green[1], Apple[1], Sour[1]]
   $\forall x\ Green(x) \land Apple(x) \rightarrow Sour(x)$
2. All babies love some green apples. [Baby[1], Loves[2], Apple[1], Green[1]]
   $\forall x\ Baby(x) \rightarrow [\exists y\ Green(y) \land Apple(y) \land Loves(x, y)]$
3. Some babies do not love any sour apple. [Baby[1], Loves[2], Apple[1], Sour[1]]
   $\exists x\ Baby(x) \land [\forall y\ Sour(y) \land Apple(y) \rightarrow \neg Loves(x, y)]$
4. Mary eats only one apple. [Apple[1], Eat[2]]
   $\exists x\ Apple(x) \land Eat(Mary, x)\ ) \land [\forall y\ Apple(y) \land \neg(x = y) \rightarrow \neg Eat(Mary, y)]$

Question 2. Find (if it were possible) the Most General Unifier (MGU) of

a) $P(\ g(h(x))\ ,\ f(g(h(b)))\ ,\ f(x)\ )$	and	$P(\ y\ ,\ f(y),\ z)$
b) $P(\ g(h(x))\ ,\ f(h(y))\ ,\ y\ )$	and	$P(\ g(z)\ ,\ f(z)\ ,\ h(a)\ )$
c) $P(\ x\ ,\ h(b)\ ,\ h(x)\ )$	and	$P(\ f(g(y))\ ,\ y\ ,\ h(f(g(h(a))))\ )$
d) $P(\ x\ ,\ g(x)\ ,\ z\ )$	and	$P(\ f(y)\ ,\ g(f(b))\ ,\ h(y)\ )$
e) $P(\ f(g(x))\ ,\ g(b)\ ,\ h(x)\ )$	and	$P(\ f(y)\ ,\ y\ ,\ h(c)\ )$
f) $P(\ x\ ,\ h(x)\ ,\ h(y)\ )$	and	$P(\ f(g(z))\ ,\ h(f(g(b)))\ ,\ h(z)\ )$

a) $\theta = \{y\ /\ g(h(b))\ ,\ x\ /\ b\ ,\ z\ /\ f(b)\ \}$
b) $\theta = \{\ x\ /\ h(a)\ ,\ y\ /\ h(a)\ ,\ z\ /\ h(h(a))\ \}$
c) No MGU
d) $\theta = \{\ x\ /\ f(b)\ ,\ y\ /\ b\ ,\ z\ /\ h(b)\}$
e) No MGU
f) $\theta = \{\ x\ /\ f(g(b))\ ,\ z\ /\ b\ ,\ y\ /\ b\ \}$

Question 3. Consider the following text. *"Anyone passing his history exam and winning the lottery is happy. But anyone who studies or is lucky can pass all his exams. John did not study but John is lucky. Anyone who is lucky wins the lottery."*

a) For each of the axiom above, write the FOL sentence that best expresses its intended meaning, using only the following predicates
   PASS(x, y): "x passes the y exam" HAPPY(x): "x is happy" LUCKY(x): "x is lucky"
   STUDY(x): "x studies" WINLOT(x): "x wins the lottery"

1) $\forall x$ PASS(x, history exams) $\wedge$ WINLOT(x) $\Rightarrow$ HAPPY(x)

2) $\forall x$ STUDY(x) $\vee$ LUCKY(x) $\Rightarrow$ $\forall y$ PASS(x, y)

3) $\neg$STUDY(John) $\wedge$ LUCKY(John)

4) $\forall x$ LUCKY(x) $\Rightarrow$ WINLOT(x)

b) Convert the above FOL clauses to clausal form

1) $\neg$PASS(x, history exams) $\vee$ $\neg$WINLOT(x) $\vee$ HAPPY(x)

2) $\neg$STUDY(x) $\vee$ PASS(x,y)

3) $\neg$LUCKY(x) $\vee$ PASS(x,y)

4) $\neg$STUDY(John)

5) LUCKY(John)

6) $\neg$ LUCKY(x) $\vee$ WINLOT(x)

c) Use resolution to answer the question "Is John happy?"

7) $\neg$HAPPY(John) Negation of conclusion

8) $\neg$PASS(John, history exams) $\vee$ $\neg$WINLOT(John) from 1 and 7 θ = { x/John}

9) WINLOT(John) from 5 and 6 θ = { x/John}

10) $\neg$PASS(John, history exams) from 8 and 9 θ = { x/John}

11) PASS(John, y) from 3 and 5 θ = { x/John}

12) • from 10 and 11 θ = { x/John, y/history}

Conclusion: Therefore, John is happy.

Question 4. Consider the following KB.

1. Buffalo(x) $\wedge$ Pig(y) $\rightarrow$ Faster(x,y)	4. Buffalo(Bob)
2. Pig(y) $\wedge$ Slug(z) $\rightarrow$ Faster (y,z)	5. Pig(Pat)
3. Faster(x,y) $\wedge$ Faster (y, z) $\rightarrow$ Faster(x, z)	6. Slug(Steve)

Use forward chaining in first-order logic to prove **Faster(Bob, Steve)**. If several rules apply, use the one with the smallest number. Do not forget to indicate the unification at every step.

7. Faster(Bob, Pat) from 1 and 4-5 θ = {x / Bob, y / Pat}
8. Faster(Pat, Steve) from 2 and 5-6 θ = {x / Bob, y / Pat, z / Steve}
9. Faster(Bob, Steve) from 3 and 7-8 θ = {x / Bob, y / Pat}

Thus, KB entails Faster(Bob, Steve) using forward chaining.