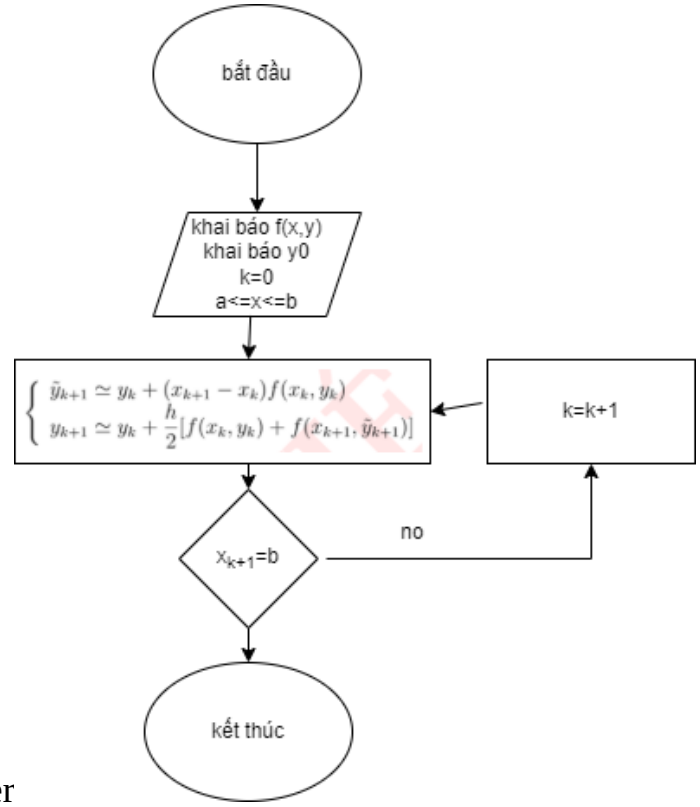
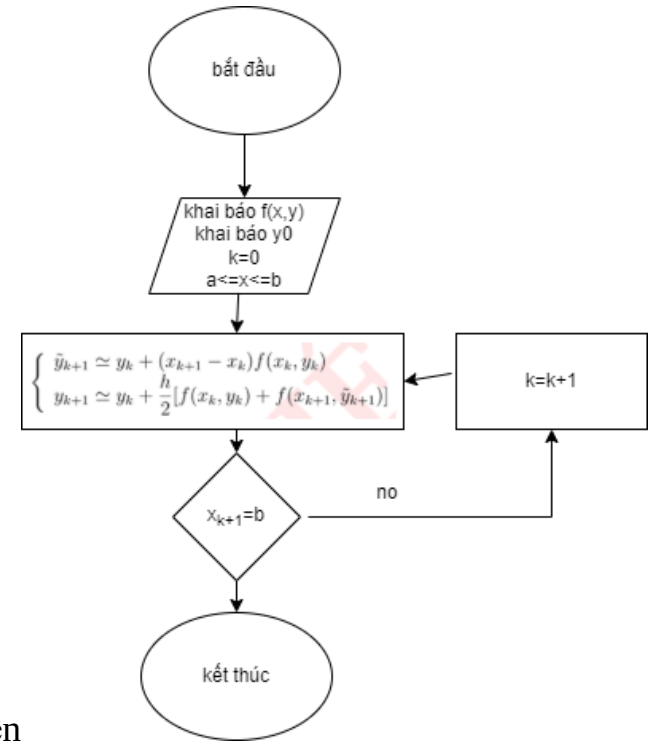
CHƯƠNG 8

8.1

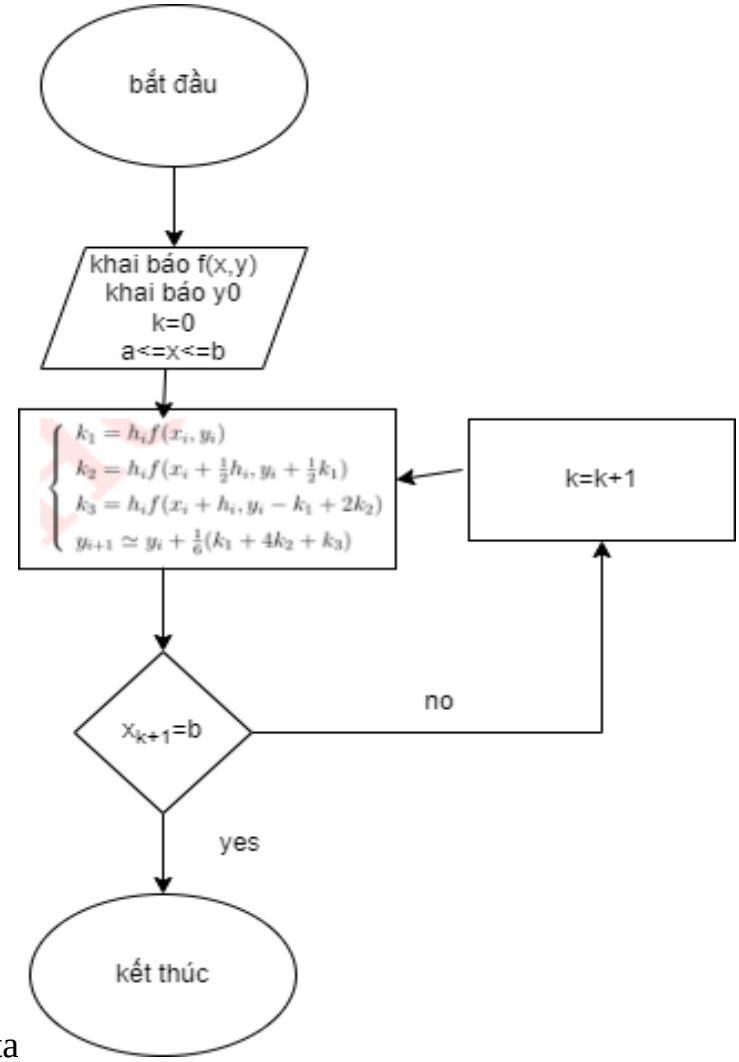
19120325 Đinh Huỳnh Tiến Phú



Euler

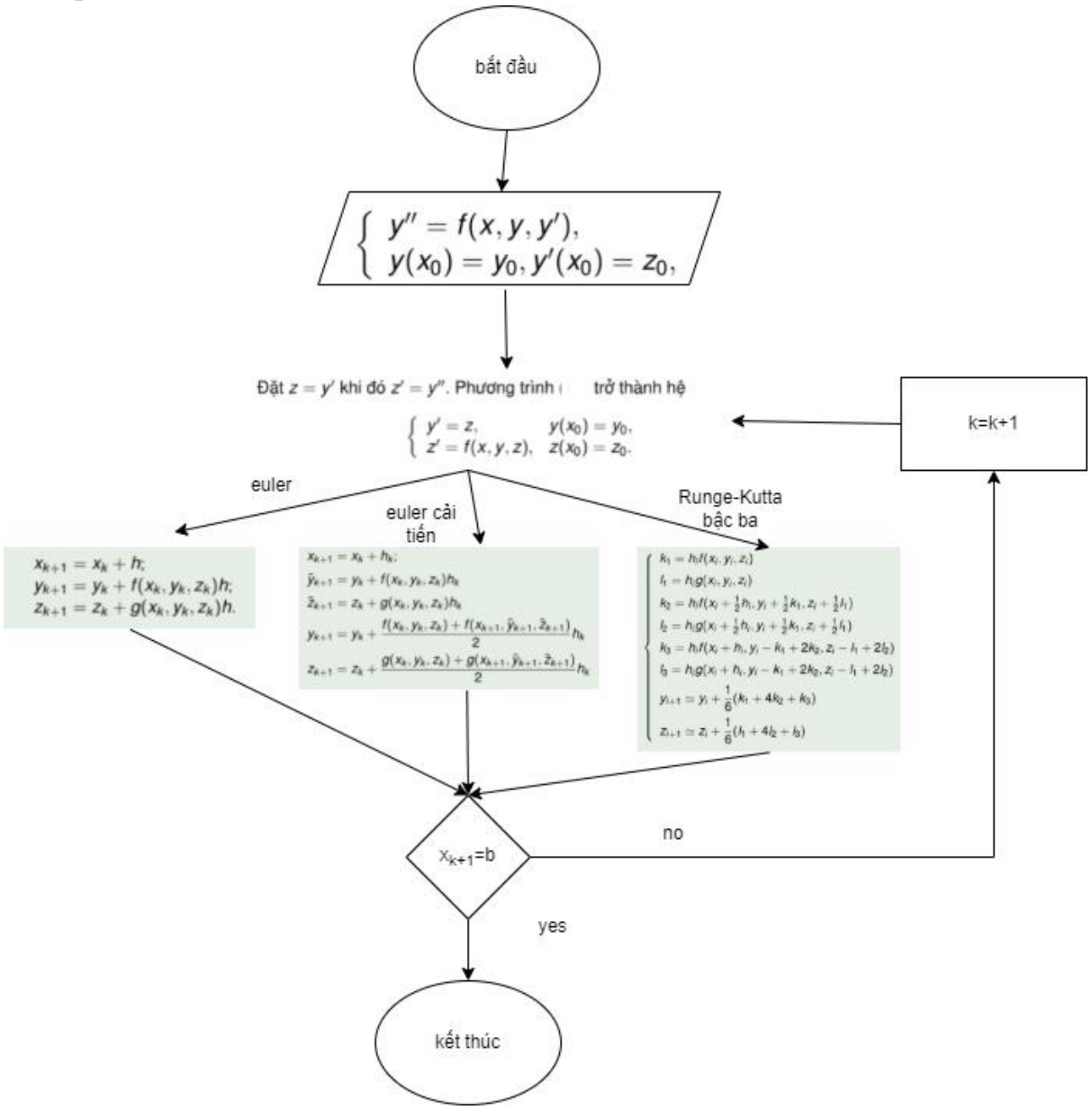


Euler cải tiến


bắt đầu
khai báo f(x,y)
khai báo y0
k=0
a<=x<=b
$k_1 = h_i f(x_i, y_i)$
$k_2 = h_i f(x_i + \frac{1}{2}h_i, y_i + \frac{1}{2}k_1)$
$k_3 = h_i f(x_i + h_i, y_i - k_1 + 2k_2)$
$y_{i+1} \simeq y_i + \frac{1}{6}(k_1 + 4k_2 + k_3)$
k=k+1
x_{k+1}=b
no
yes
kết thúc

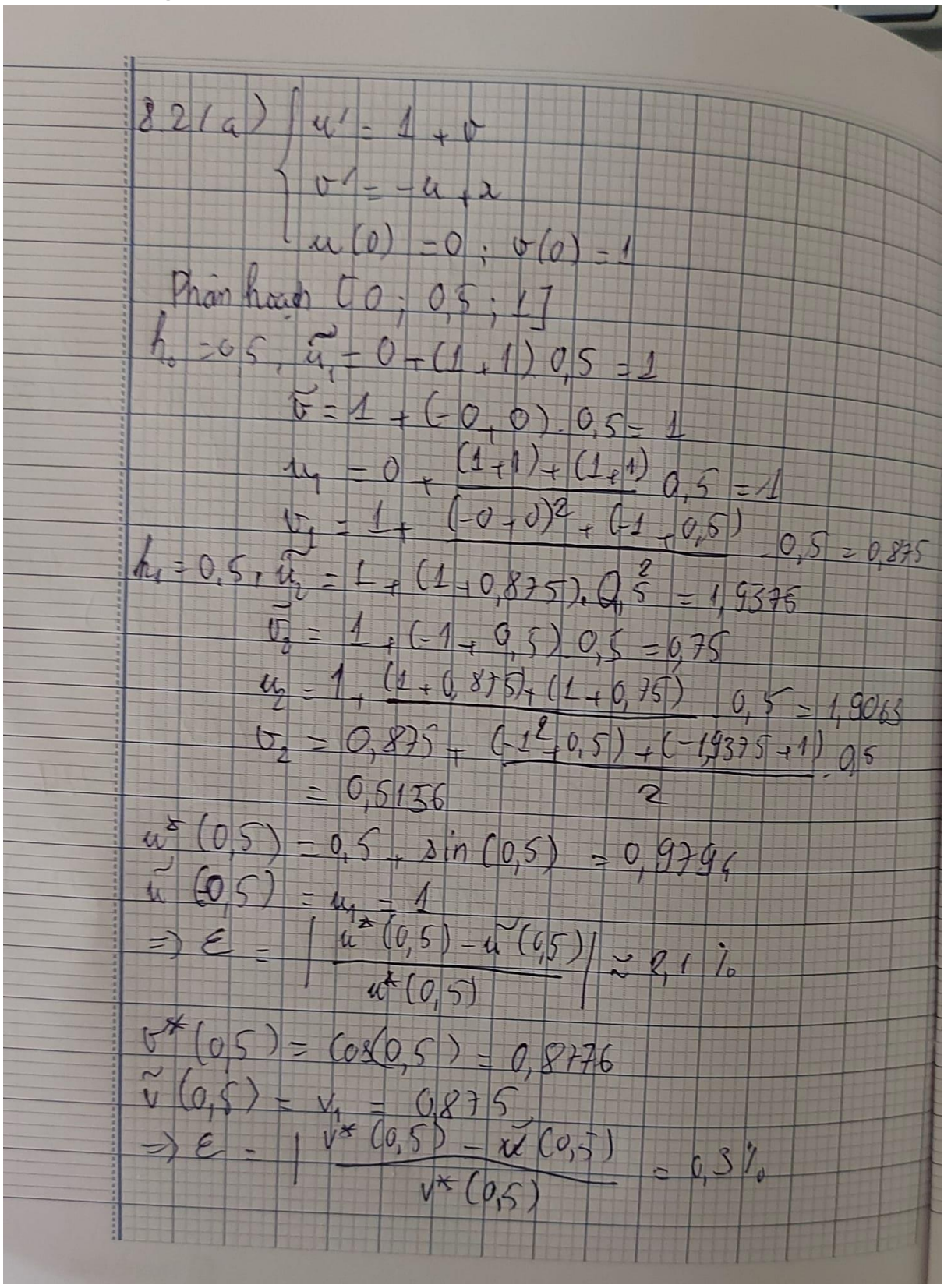
Runge-Kutta

Pt vi phân bậc 2



8.2

Hà Bảo Khang - 19120252



b) $\begin{cases} u' = v - x^3 + 2x + 1 \\ v' = 3xu - 3x^2 - 3x \\ u(0) = 1, \ v(0) = 0 \end{cases}$

Phân hoạch $[0; 0,5; 1]$

$h_0 = 0,5$ ~~$x_1 = 0,5$~~

$\tilde{u}_1 = 1 + (0 - 0^3 + 2.0 + 1).0,5 = 1,5$

$\tilde{v}_1 = 0 + (3.0.1 - 3.0^2 - 3.0).0,5 = 0$

$u_1 = 1 + \dfrac{1 + (0 - 0,5^3 + 2.0,5 + 1).0,5}{2} = 1,7188$

$v_1 = 0 + 0,5.\dfrac{(3.0,1 - 3.0^2 - 3.0^2) + (3.0,5.1,5 - 3.0,5^2 - 3.0,5)}{2}$

$= 0$

$h_1 = 0,5$

$\tilde{u}_2 = 1,7188 + (0 - 0,5^3 + 2.0,5 + 1).0,5 = 2,6563$

$\tilde{v}_2 = 0 + (3.0,5.1,7188 - 3.0,5^2 - 3.0,5).0,5$

$= 0,3282$

$u_2 = 1,7188 + 0,5.\dfrac{(0 - 0,5^3 + 2.0,5 + 1) + (0,3282 - 1^3 + 2.1 + 1)}{2}$

$= 2,7697$

$v_2 = 0 + 0,5.\dfrac{(3.0,5.1,7188 - 3.0,5^2 - 3.0,5) + (3.1.2,6563 - 3.1^2 - 3.1)}{2}$

$= 0,5743$

8.3

Đoàn Thế Huy - 19120079

a. $\begin{cases} y'' + 2y' + y = e^{-x} \\ y(0) = 1,\ y'(0) = 2 \end{cases}$, *Tìm sai số tại x = 0,75 biết y(0,75) = 1,6678,*

Đặt z= y', z' = y''

$$\Rightarrow \begin{cases} y' = z \\ z' = e^{-x} - 2z - y \\ y(0) = 1, \quad z(0) = 2 \end{cases}$$

$h_0 = 0{,}25 \Rightarrow x_1 = 0{,}25$

$\widetilde{y_1} = 1 + 0{,}25(2) = 1{,}5$

$\widetilde{z_1} = 2 + 0{,}25(e^0 - 2.2 - 1) = 1$

$y_1 \approx 1 + \frac{0{,}25}{2}(2 + 1) = 1{,}375$

$z_1 \approx 2 + \frac{0{,}25}{2}(e^0 - 4 - 1 + e^{-0{,}25} - 2 - 1{,}5) = 1{,}1599$

$h_1 = 0{,}25 \Rightarrow x_2 = 0{,}5$

$\widetilde{y_2} = 1{,}375 + 0{,}25(1{,}1599) = 1{,}6650$

$\widetilde{z_2} = 1{,}1599 + 0{,}25(e^{-0{,}25} - 2.1{,}1599 - 1{,}375) = 0{,}4310$

$y_2 \approx 1{,}375 + \frac{0{,}25}{2}(1{,}1599 + 0{,}4310) = 1{,}5739$

$z_2 \approx 1{,}1599 + \frac{0{,}25}{2}(e^{-0{,}25} - 2.1{,}1599 - 1{,}375 + e^{-0{,}5} - 2.0{,}4310 - 1{,}6650) = 0{,}5553$

$h_2 = 0{,}25 \Rightarrow x_3 = 0{,}75$

$\widetilde{y_3} = 1{,}5739 + 0{,}25(0{,}5553) = 1{,}7127$

$\widetilde{z_3} = 0{,}9264 + 0{,}25(e^{-0{,}5} - 2.0{,}5553 - 1{,}5739) = 0{,}4069$

$y_3 \approx 1{,}5739 + \frac{0{,}25}{2}(0{,}5553 + 0{,}4069) = 1{,}6942$

$z_3 \approx 0{,}5553 + \frac{0{,}25}{2}(e^{-0{,}5} - 2.0{,}5553 - 1{,}5739 + e^{-0{,}75} - 2.0{,}4069 - 1{,}7127) = 0{,}0388$

$$\delta y(0{,}75) = \frac{1{,}6942 - 1{,}6678}{1{,}6678} \approx 0{,}01583 = 1{,}583\%$$

b. $\begin{cases} 2y'' - 3y' + y = x^2 \\ y(0) = 1,\ y'(0) = 0 \end{cases}$, *Tìm sai số tại x = 0,75 biết y(0,75) = 1,6678,*

Đặt $z = y', z' = y'' \Rightarrow \begin{cases} y' = z \\ z' = \dfrac{x^2 - y + 3z}{2} \\ y(0) = 1, \quad z(0) = 0 \end{cases}$

$h_0 = 0{,}25 \Rightarrow x_1 = 0{,}25$

$\widetilde{y_1} = 1 + 0{,}25(0) = 1$

$\widetilde{z_1} = 0 + 0{,}25\left(\dfrac{0^2 - 1 + 0}{2}\right) = -0{,}125$

$y_1 \approx 1 + \dfrac{0{,}25}{2}(0 - 0{,}125) = 0{,}9844$

$z_1 \approx 0 + \dfrac{0{,}25}{2}\left(\dfrac{0^2 - 1 + 0}{2} + \dfrac{0{,}25^2 - 1 + 3.(-0{,}125)}{2}\right) = -0{,}1445$

$h_1 = 0{,}25 \Rightarrow x_2 = 0{,}5$

$\widetilde{y_2} = 0{,}9844 + 0{,}25(-0{,}1445) = 0{,}9483$

$\widetilde{z_2} = -0{,}1445 + 0{,}25\left(\dfrac{0{,}25^2 - 0{,}9844 + 3.(-0{,}1445)}{2}\right) = -0{,}3139$

$y_2 \approx 0{,}9844 + \dfrac{0{,}25}{2}(-0{,}1445 - 0{,}3139) = 0{,}9271$

$z_2 \approx -0{,}1445 + \dfrac{0{,}25}{2}\left(\dfrac{0{,}25^2 - 0{,}9844 + 3.(-0{,}1445)}{2} + \dfrac{0{,}5^2 - 0{,}9483 + 3.(-0{,}3139)}{2}\right) = -0{,}3317$

$$\delta y(0{,}5) = \frac{0{,}9271 - 0{,}9224}{0{,}9224} \approx 5{,}0954.10^{-3} = 0{,}5095\%$$

8.4

Huỳnh Tấn Thọ - 19120383

$\begin{cases} y'(t) = 0{,}08y - 0{,}001zy \\ z'(t) = -0{,}02z + 0{,}00002zy \end{cases}$ $\qquad y(0) = 1000; z(0) = 40; h = 1; t = [0\ 1\ 2\ 3]$

Dùng phương pháp Euler cải tiến:

$h_0 = 1; x_1 = 0 + 1 = 1$

$\widetilde{y_1} = 1000 + (0{,}08.1000 - 0{,}001.40.1000) = 1000 + 40 = 1040$

$\widetilde{z_1} = 40 + (-0{,}02.40 + 0{,}00002.40.1000) = 40 + 0 = 40$

$y_1 = 1000 + 0.5[40 + (0{,}08.1040 - 0{,}001.40.1040)] = 1040{,}8$

$z_1 = 40 + 0.5(0 - 0{,}02.40 + 0{,}00002.40.1040) = 40{,}016$

$h_1 = 1; x_2 = 1 + 1 = 2$

$\widetilde{y_2} = 1040{,}8 + (0{,}08.1040{,}8 - 0{,}001.40{,}016.1040{,}8) = 1040{,}8 + 41.6153 = 1082{,}4153$

$\widetilde{z_2} = 40{,}016 + (-0{,}02.40{,}016 + 0{,}00002.40{,}016.1040{,}8) = 40{,}016 + 0{,}0327 = 40{,}0487$

$y_2 = 1040{,}8 + 0.5[41{,}6153 + (0{,}08.1082{,}4153 - 0{,}001.40{,}0487.1082{,}4153)] = 1083{,}2296$

$z_2 = 40{,}016 + 0.5(0{,}0327 - 0{,}02.40{,}0487 + 0{,}00002.40{,}0487.1082{,}4153) = 40{,}0487$

$h_2 = 1; x_3 = 2 + 1 = 3$

$\widetilde{y_3} = 1083{,}2296 + (0{,}08.1083{,}2296 - 0{,}001.40{,}0487.1083{,}2296)$

$= 1083{,}2296 + 43{,}2764 = 1126{,}506$

$\widetilde{z_3} = 40{,}0487 + (-0{,}02.40{,}0487 + 0{,}00002.40{,}0487.1083{,}2296)$

$= 40{,}0487 + 0{,}0667 = 40{,}1153$

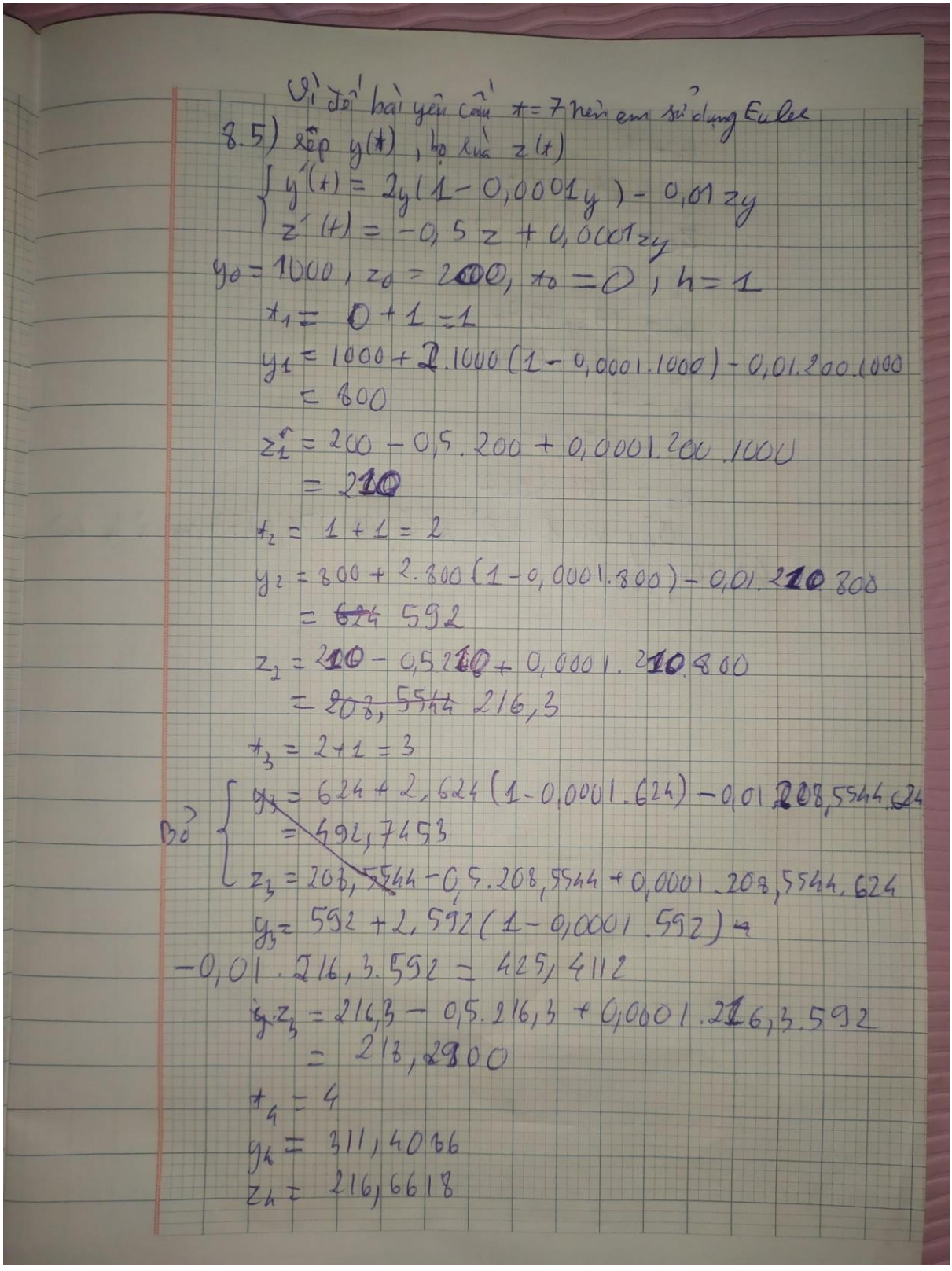
$y_3 = 1083{,}2296 + 0.5[43{,}2764 + (0{,}08.1126{,}506 - 0{,}001.40{,}1153.1126{,}506)] = 1127.333$

$z_3 = 40{,}0487 + 0.5(0{,}0667 - 0{,}02.40{,}1153 + 0{,}00002.40{,}1153.1126{,}506) = 40{,}1328$

Vậy số thỏ sau 3 tháng là 1126,506 con và số sói sau 3 tháng là 40,1328 con

8.5

Lưu Đức Vũ - 19120433



$t_5 = 5$

$y_5 = 240,1273$

$z_5 = 212,5757$

$t_6 = 6$

$y_6 = 198,3974$

$z_6 = 207,0514$

$t_7 = 7$

$y_7 = 176,5353$

$z_7 = 200,8067$

Vậy sau 1 tuần số sếp là 176,5353 con và bọ là là 200,8067 con

8.6

Trần Vũ Việt Cường - 19120465

$P''(t) = kP'(t)\left(1 - \frac{2P}{M}\right)$ *với* $k = 0{,}2$ *và* $M = 3000$

Đặt $y = P(t), z = y'$

$Ta\ có: z' = kz\left(1 - \left(\frac{2y}{M}\right)\right) = 0{,}2z\left(1 - \frac{2y}{3000}\right)$

a) $\begin{cases} y' = z \\ z' = 0{,}2z(1 - \frac{2y}{3000}) \\ y(0) = 2400, z(0) = 100 \end{cases}$

Phân hoạch [0 1 2 3], sử dụng phương pháp Euler, ta được:

Với h = 1, ta có:

$x_1 = x_0 + h = 1$

$y_1 = y_0 + f(t_0, y_0, z_0)h = 2400 + 100 = 2500$

$z_1 = z_0 + g(t_0, y_0, z_0)h = 100 - 12 = 88$

$x_2 = x_1 + h = 2$

$y_2 = y_1 + f(t_1, y_1, z_1)h = 2500 + 88 = 2588$

$z_2 = z_1 + g(t_1, y_1, z_1)h = 76{,}2667$

$x_3 = x_2 + h = 3$

$y_3 = y_2 + f(t_2, y_2, z_2)h = 2588 + 76{,}2667 = 2664{,}2667$

$z_3 = z_2 + g(t_2, y_2, z_2)h = 66{,}2198$

b) $\begin{cases} y' = z \\ z' = 0{,}2z(1 - \frac{2y}{3000}) \\ y(0) = 3500, z(0) = -120 \end{cases}$

Phân hoạch [0 1 2 3], sử dụng phương pháp Euler, ta được:
Với h = 1, ta có:

$x_1 = x_0 + h = 1$
$y_1 = y_0 + f(t_0, y_0, z_0)h = 2500 - 120 = 3380$
$z_1 = z_0 + g(t_0, y_0, z_0)h = -88$

$x_2 = x_1 + h = 2$
$y_2 = y_1 + f(t_1, y_1, z_1)h = 2280 - 88 = 3292$
$z_2 = z_1 + g(t_1, y_1, z_1)h = -85{,}77$

$x_3 = x_2 + h = 3$
$y_3 = y_2 + f(t_2, y_2, z_2)h = 3292 - 85{,}77 = 3206{,}23$
$z_3 = z_2 + g(t_2, y_2, z_2)h = -84{,}1$

8.7

Đoàn Thu Ngân - 19120302

Bài 8.7

$$\theta''(t)+\frac{g}{L}\sin\theta(t)=0$$

Tìm góc quay sau 5 giây

a) Giả sử tại thời điểm bắt đầu, góc quay là 0,2 rad và vận tốc góc quay là 1 rad/s
Ta có góc quay $\theta(0)=0.2$ rad, vận tốc góc $\theta'(0)=1$
Thế g = 9.8, L = 1 và đặt $\theta(t)=y(t)$, ta được $y''(t)+9.8\sin y(t)=0$

Đặt $y'=z\Rightarrow y''=z'$

Ta có hệ phương trình

$$\begin{cases} y'=z \\ z'=-9.8\sin y \\ y(0)=0.2, y'(0)=1 \end{cases}$$

Ta có phân hoạch $[0,1,2,3,4,5]$

$$h_0 = 1 \Rightarrow x_1 = 1$$

$$\tilde{y}_1 = y_0 + z_0 h_0 = 0.2 + 1 = 1.2$$

$$\tilde{z}_1 = z_0 + (-9.8\sin y_0)h_0 = 1 + (-9.8\sin 0.2) \approx -0.9469$$

$$y_1 = y_0 + \frac{z_0 + \tilde{z}_1}{2} h_0 = 0.2 + \frac{1 - 0.9469}{2} \approx 0.2265$$

$$z_1 = z_0 + \frac{(-9.8\sin y_0) + (-9.8\sin \tilde{y}_1)}{2} h_0 = 1 + \frac{(-9.8\sin 0.2) + (-9.8\sin 1.2)}{2} \approx -4.5404$$

$$h_1 = 1 \Rightarrow x_2 = 2$$

$$\tilde{y}_2 = y_1 + z_1 h_1 = 0.2265 + (-4.5404) = -4.3139$$

$$\tilde{z}_2 = z_1 + (-9.8\sin y_1)h_1 = -4.5404 + (-9.8\sin 0.2265) \approx -6.7411$$

$$y_2 = y_1 + \frac{z_1 + \tilde{z}_2}{2} h_1 = 0.2 + \frac{-4.5404 - 6.7411}{2} \approx -5,4407$$

$$z_2 = z_1 + \frac{(-9.8\sin y_1) + (-9.8\sin \tilde{y}_2)}{2} h_1 = -4.5404 + \frac{(-9.8\sin(0.2265)) + (-9.8\sin(-4.3139))}{2} \approx -10.1568$$

$$h_2 = 1 \Rightarrow x_3 = 3$$

$$\tilde{y}_3 = y_2 + z_2 h_2 = -5,4407 + (-10.1568) = -15.5975$$

$$\tilde{z}_3 = z_2 + (-9.8\sin y_2)h_2 = -10.1568 + (-9.8\sin(-5,4407)) \approx -17.4705$$

$$y_3 = y_2 + \frac{z_2 + \tilde{z}_3}{2} h_2 = -5,4407 + \frac{-10.1568 - 17.4705}{2} \approx -19.2543$$

$$z_3 = z_2 + \frac{(-9.8\sin y_2) + (-9.8\sin \tilde{y}_3)}{2} h_2 = -10.1568 + \frac{(-9.8\sin(-5,4407)) + (-9.8\sin(-15.5975))}{2} \approx -13.2734$$

$$h_3 = 1 \Rightarrow x_4 = 4$$

$$\tilde{y}_4 = y_3 + z_3 h_3 = -19.2543 + (-13.2734) = -32.5277$$

$$\tilde{z}_4 = z_3 + (-9.8\sin y_3)h_3 = (-13.2734) + (-9.8\sin(-19.2543)) \approx -9.414$$

$$y_4 = y_3 + \frac{z_3 + \tilde{z}_4}{2} h_3 = -19.2543 + \frac{-13.2734 - 9.414}{2} \approx -30.5981$$

$$z_4 = z_3 + \frac{(-9.8\sin y_3) + (-9.8\sin \tilde{y}_4)}{2} h_3 = -13.2734 + \frac{(-9.8\sin(-19.2543)) + (-9.8\sin(-32.5277))}{2} \approx -6.9510$$

$$h_4 = 1 \Rightarrow x_5 = 5$$

$$\tilde{y}_5 = y_4 + z_4 h_4 = -30.5981 + (-6.9510) = -37.5491$$

$$\tilde{z}_5 = z_4 + (-9.8\sin y_4)h_4 = (-6.9510) + (-9.8\sin(-30.5981)) \approx -14.1016$$

$$y_5 = y_4 + \frac{z_4 + \tilde{z}_5}{2} h_4 = -30.5981 + \frac{-6.9510 - 14.1016}{2} \approx -41.1244$$

$$z_5 = z_4 + \frac{(-9.8\sin y_4) + (-9.8\sin \tilde{y}_5)}{2} h_4 = -6.9510 + \frac{(-9.8\sin(-30.5981)) + (-9.8\sin(-37.5491))}{2} \approx -11.2586$$

Góc quay sau 5s: y(5)=-41.1244 rad

b) Giả sử tại thời điểm bắt đầu, góc quay là 0 rad và vận tốc góc quay là 2 rad/s.

Ta có góc quay $\theta(0) = 0$ rad, vận tốc góc $\theta'(0) = 2$

Thế g = 9.8, L = 1 và đặt $\theta(t) = y(t)$, ta được $y''(t) + 9.8\sin y(t) = 0$

Đặt $y' = z \Rightarrow y'' = z'$

Ta có hệ phương trình

$$\begin{cases} y' = z \\ z' = -9.8\sin y \\ y(0) = 0, y'(0) = 2 \end{cases}$$

Ta có phân hoạch $[0,1,2,3,4,5]$

$h_0 = 1 \Rightarrow x_1 = 1$

$\tilde{y}_1 = y_0 + z_0 h_0 = 0 + 2 = 2$

$\tilde{z}_1 = z_0 + (-9.8\sin y_0)h_0 = 2 + (-9.8\sin 0) = 2$

$y_1 = y_0 + \frac{z_0 + \tilde{z}_1}{2} h_0 = 0 + \frac{2+2}{2} = 2$

$z_1 = z_0 + \frac{(-9.8\sin y_0) + (-9.8\sin \tilde{y}_1)}{2} h_0 = 2 + \frac{(-9.8\sin 0) + (-9.8\sin 2)}{2} \approx -2.4555$

$h_1 = 1 \Rightarrow x_2 = 2$

$\tilde{y}_2 = y_1 + z_1 h_1 = 2 + (-2.4555) = -0.4555$

$\tilde{z}_2 = z_1 + (-9.8\sin y_1)h_1 = -2.4555 + (-9.8\sin 2) \approx -11.3666$

$y_2 = y_1 + \frac{z_1 + \tilde{z}_2}{2} h_1 = 2 + \frac{-2.4555 - 11.3666}{2} \approx -4.9110$

$z_2 = z_1 + \frac{(-9.8\sin y_1) + (-9.8\sin \tilde{y}_2)}{2} h_1 = -2.4555 + \frac{(-9.8\sin(2)) + (-9.8\sin(-0.4555))}{2} \approx -4.7554$

$h_2 = 1 \Rightarrow x_3 = 3$

$\tilde{y}_3 = y_2 + z_2 h_2 = -4.9110 + (-4.7554) = -9.6664$

$\tilde{z}_3 = z_2 + (-9.8\sin y_2)h_2 = -4.7554 + (-9.8\sin(-4.9110)) \approx -14.3627$

$y_3 = y_2 + \frac{z_2 + \tilde{z}_3}{2} h_2 = -4.9110 + \frac{-4.7554 - 14.3627}{2} \approx -14.4700$

$z_3 = z_2 + \frac{(-9.8\sin y_2) + (-9.8\sin \tilde{y}_3)}{2} h_2 = -4.7554 + \frac{(-9.8\sin(-4.9110)) + (-9.8\sin(-9.6664))}{2} \approx -10.7315$

$h_3 = 1 \Rightarrow x_4 = 4$

$\tilde{y}_4 = y_3 + z_3 h_3 = -14.4700 + (-10.7315) = -25.2015$

$\tilde{z}_4 = z_3 + (-9.8\sin y_3)h_3 = (-10.7315) + (-9.8\sin(-14.4700)) \approx -1.4693$

$y_4 = y_3 + \frac{z_3 + \tilde{z}_4}{2} h_3 = -14.4700 + \frac{-10.7315 - 1.4693}{2} \approx -19.1010$

$z_4 = z_3 + \frac{(-9.8\sin y_3) + (-9.8\sin \tilde{y}_4)}{2} h_3 = -10.7315 + \frac{(-9.8\sin(-14.4700)) + (-9.8\sin(-25.2015))}{2} \approx -5.7637$

$h_4 = 1 \Rightarrow x_5 = 5$

$\tilde{y}_5 = y_4 + z_4 h_4 = -19.1010 + (-5.7637) = -24.8647$

$\tilde{z}_5 = z_4 + (-9.8\sin y_4)h_4 = (-5.7637) + (-9.8\sin(-19.1010)) \approx -3.3254$

$y_5 = y_4 + \frac{z_4 + \tilde{z}_5}{2} h_4 = -19.1010 + \frac{-5.7637 - 3.3254}{2} \approx -23.6455$

$z_5 = z_4 + \frac{(-9.8\sin y_4) + (-9.8\sin \tilde{y}_5)}{2} h_4 = -5.7637 + \frac{(-9.8\sin(-23.64551)) + (-9.8\sin(-24.8647))}{2} \approx -11.9443$

Góc quay sau 5s: y(5) = -23.6455 rad