

Introduction to Deep Learning MIT 6.S191

Alexander Amini

January 28, 2019





'Deep Voice' Software Can Clone Anyone's Voice With Just 3.7 Seconds of Audio

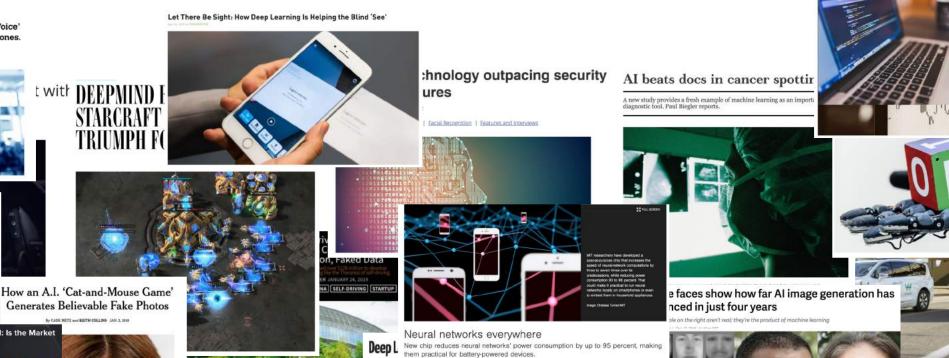
Using snippets of voices, Baidu's 'Deep Voice'

can generate new speech, accents, and tones. 'Creative' AlphaZero leads way for

chess computers and, maybe, science

Former chess world champion Garry Kasparov likes what he

The Rise of Deep Learning





To create the final image in this set, the system generated 10 million revisions over 1

After Millions of Trials, These Simulated Humans Learned to Do Perfect Backflips and Cartwheels

Researchers introduce a deep learning method that converts mono audio recordings into 3D sounds using video scenes

Automation And Algorithms: De-Risking Manufacturing With Artificial Intelligence

Al Can Help In Predicting Cryptocurrency

Value



The two key applications of AI in manufacturing are pricing and

By Robert F. Service | Dec. 6, 2018, 12:05 PM

Google's DeepMind aces protein folding

What is Deep Learning?

ARTIFICIAL INTELLIGENCE

Any technique that enables computers to mimic human behavior



MACHINE LEARNING

Ability to learn without explicitly being programmed



DEEP LEARNING

Extract patterns from data using neural networks

313472

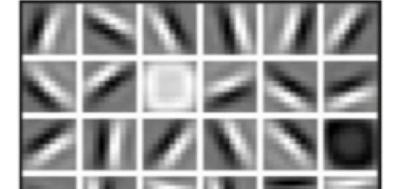
Why Deep Learning and Why Now?

Why Deep Learning?

Hand engineered features are time consuming, brittle and not scalable in practice

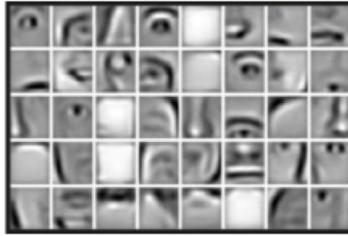
Can we learn the **underlying features** directly from data?

Low Level Features



Lines & Edges

Mid Level Features



Eyes & Nose & Ears

High Level Features



Facial Structure

Why Now?

Neural Networks date back decades, so why the resurgence?

Stochastic Gradient Descent

Perceptron

• Learnable Weights

Backpropagation

• Multi-Layer Perceptron

Deep Convolutional NN

Digit Recognition

I. Big Data

- Larger Datasets
- Easier Collection& Storage







2. Hardware

- Graphics Processing Units (GPUs)
- Massively
 Parallelizable



3. Software

- Improved Techniques
- New Models
- Toolboxes





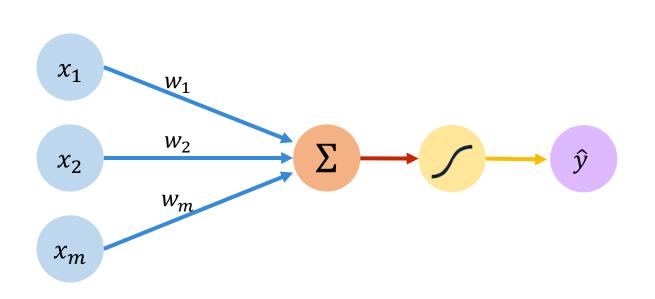
1958

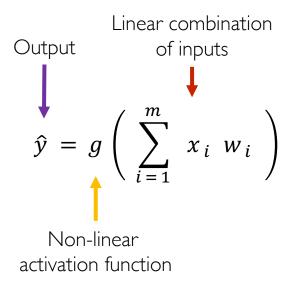
1986

1995

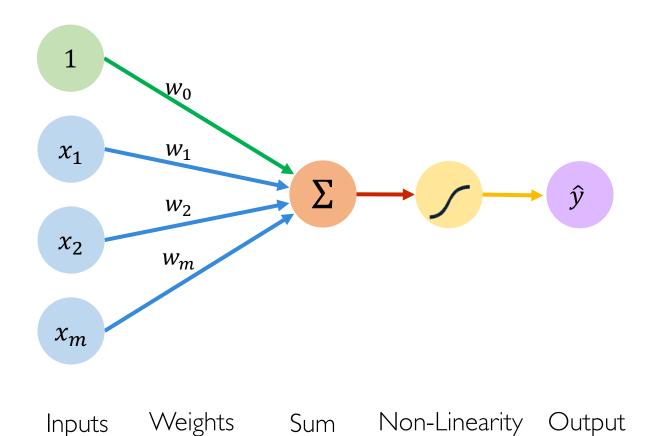


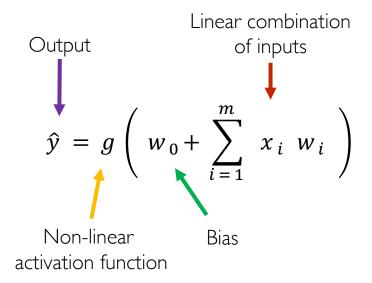
The Perceptron The structural building block of deep learning

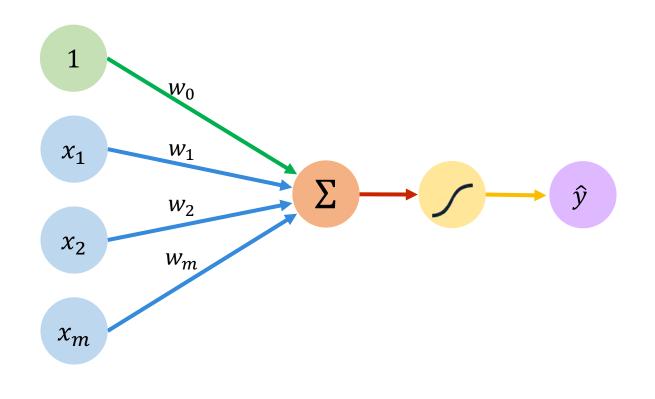




Inputs Weights Sum Non-Linearity Output







Sum

$$\hat{y} = g \left(w_0 + \sum_{i=1}^m x_i w_i \right)$$

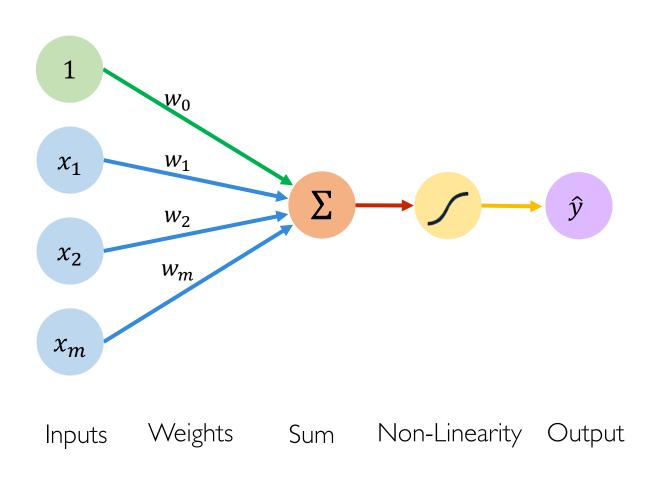
$$\hat{y} = g \left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$

where:
$$\boldsymbol{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix}$$
 and $\boldsymbol{W} = \begin{bmatrix} w_1 \\ \vdots \\ w_m \end{bmatrix}$

Inputs

Weights

Non-Linearity Output

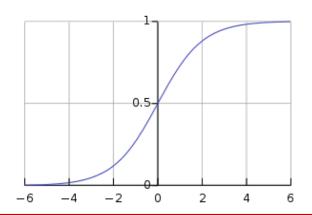


Activation Functions

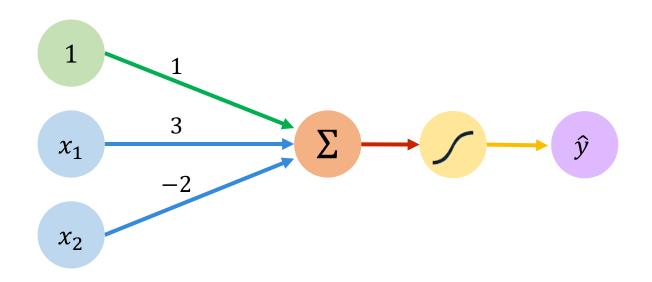
$$\hat{y} = g(w_0 + X^T W)$$

• Example: sigmoid function

$$g(z) = \sigma(z) = \frac{1}{1 + e^{-z}}$$



 \boldsymbol{Z}



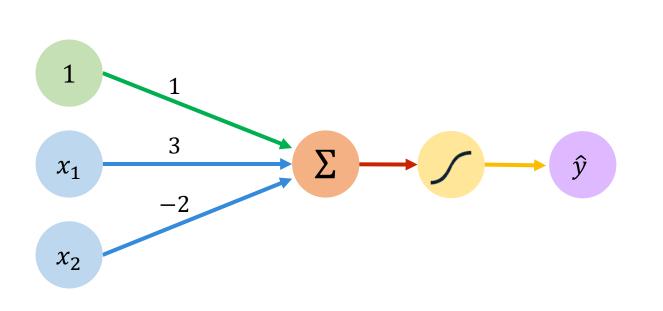
We have:
$$w_0 = 1$$
 and $\boldsymbol{W} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

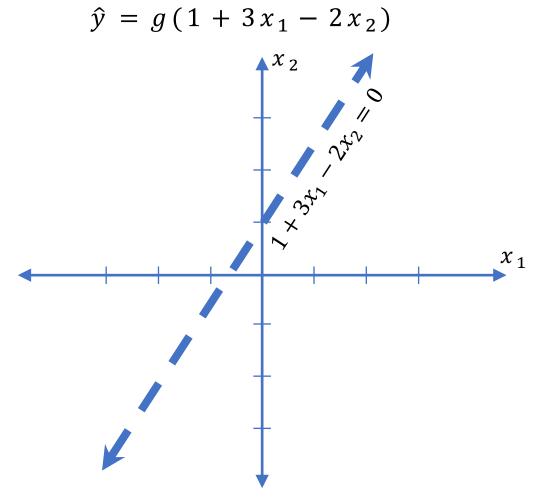
$$\hat{y} = g(w_0 + X^T W)$$

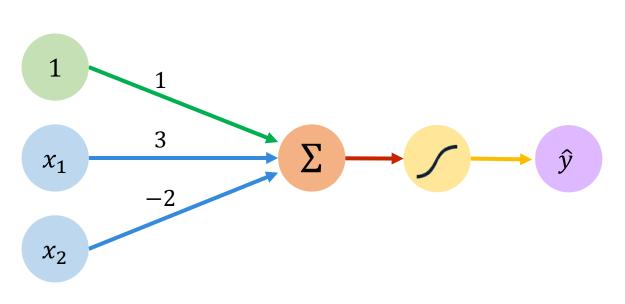
$$= g\left(1 + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 3 \\ -2 \end{bmatrix}\right)$$

$$\hat{y} = g(1 + 3x_1 - 2x_2)$$

This is just a line in 2D!



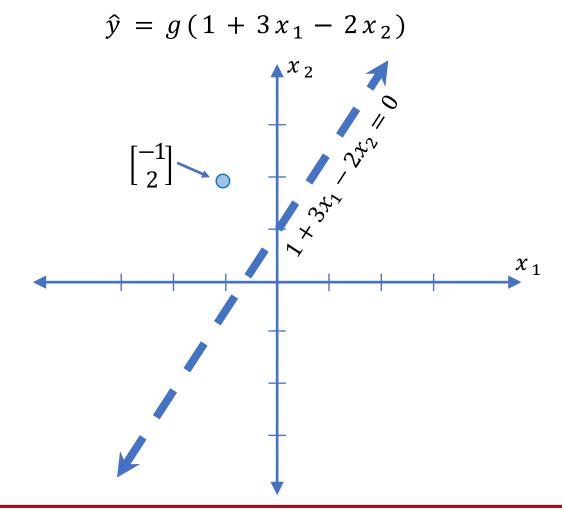


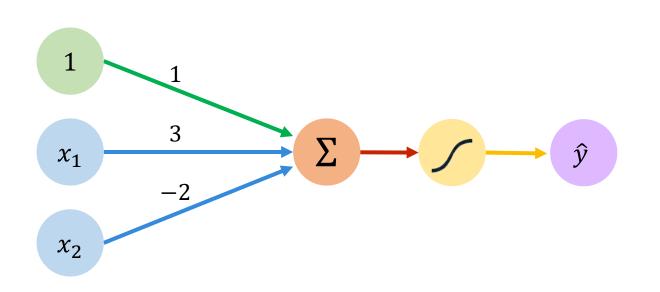


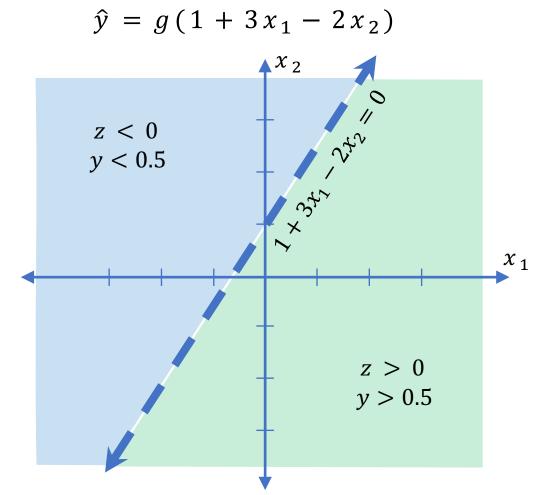
Assume we have input: $X = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$$\hat{y} = g (1 + (3*-1) - (2*2))$$

= $g (-6) \approx 0.002$

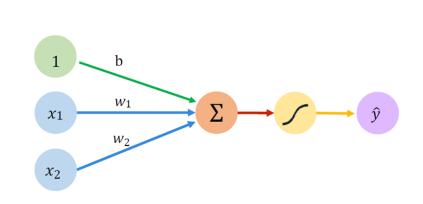


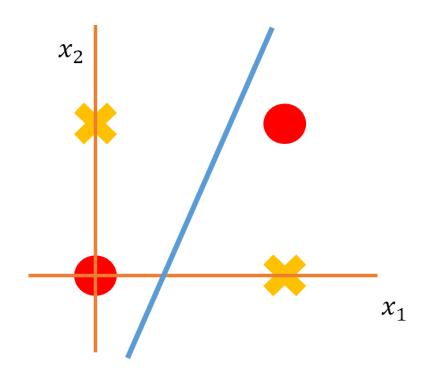




The Perceptron: XOR

Input	Target
x_1 x_2	XOR
0 0	0
0 1	1
1 0	1
1 1	0



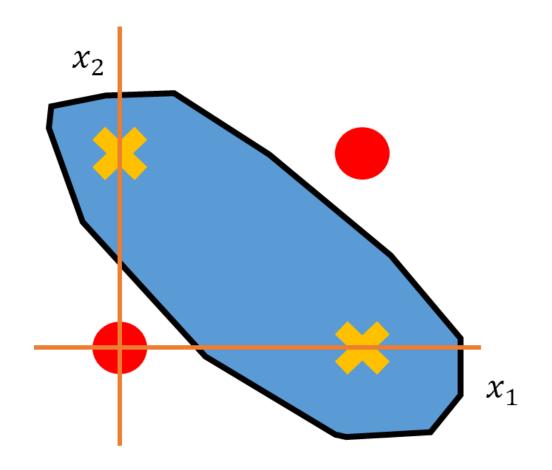


non-separable

Exercise 05-1.

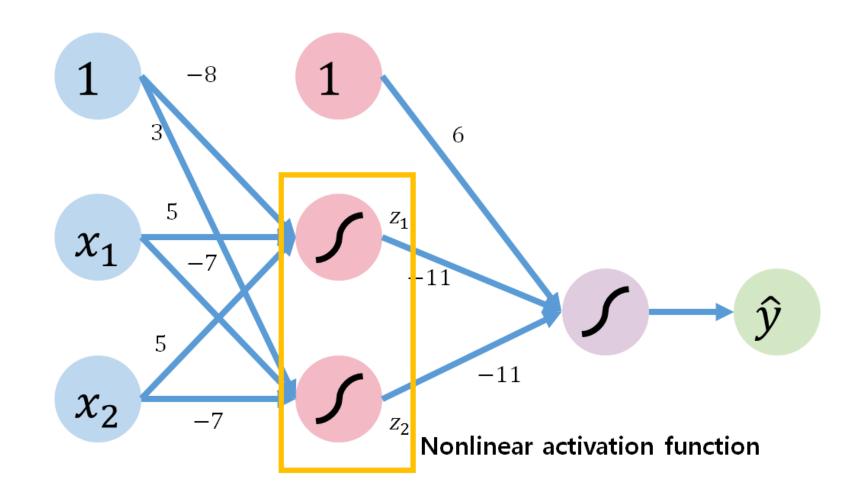
tf2-05-I-xor_perceptron.py

How to sovle this problem?



Nonlinear decision boundary is required!

Solution I: Multilayer + Nonlinear function

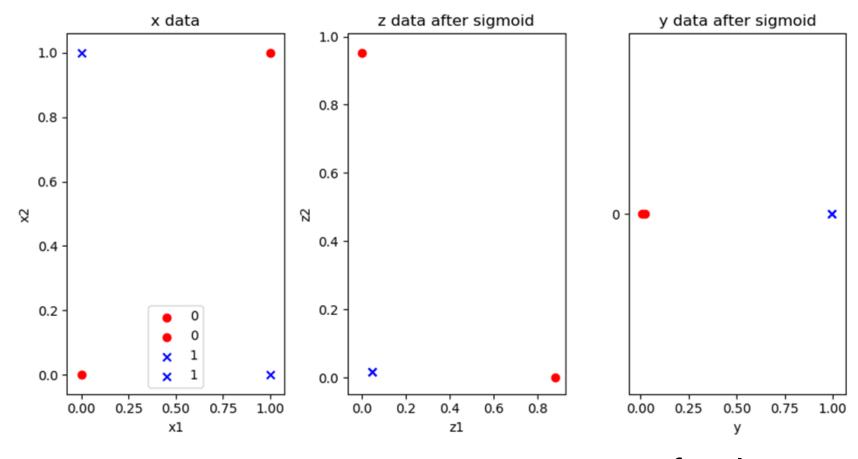


- sigmoid activation function

Exercise 05-2.

tf2-05-2-xor_sigmoid.py

Solution I: Multilayer + Nonlinear function

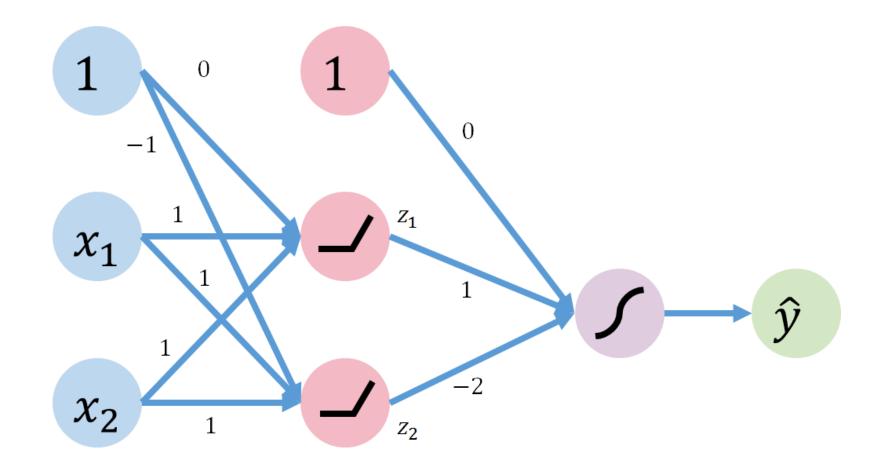


- original data

- after nonlinear transformation

after linear classification

Solution 2: Multilayer + Nonlinear function

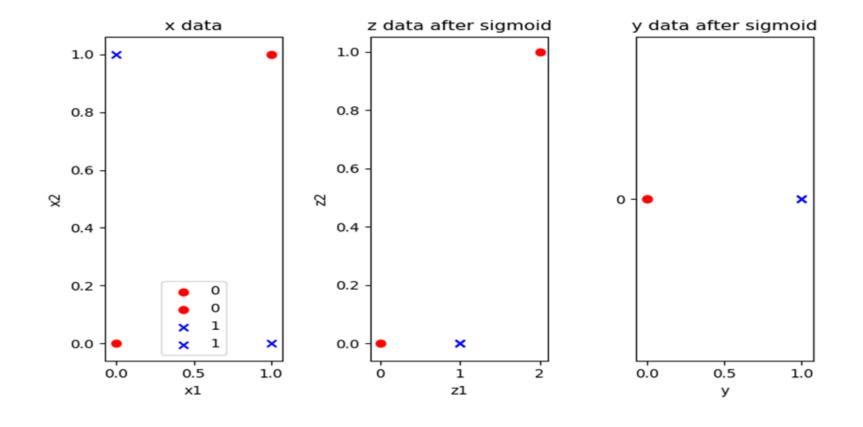


- relu activation function

Exercise 05-3.

tf2-05-3-xor_relu.py

Solution 2: Multilayer + Nonlinear function

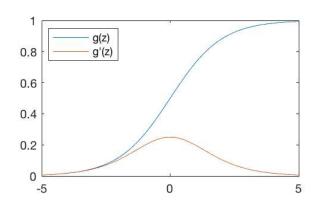


- original data

- after nonlinear transformation
- after linear classification

Common Activation Functions

Sigmoid Function

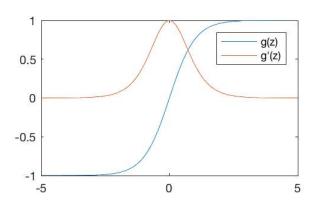


$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$



Hyperbolic Tangent

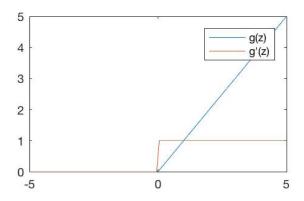


$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$



Rectified Linear Unit (ReLU)



$$g(z) = \max(0, z)$$

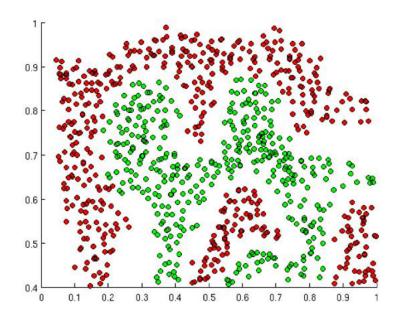
$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$



NOTE: All activation functions are non-linear

Importance of Activation Functions

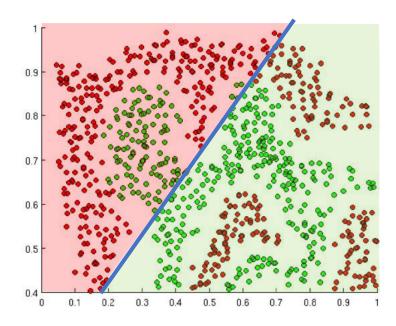
The purpose of activation functions is to **introduce non-linearities** into the network



What if we wanted to build a Neural Network to distinguish green vs red points?

Importance of Activation Functions

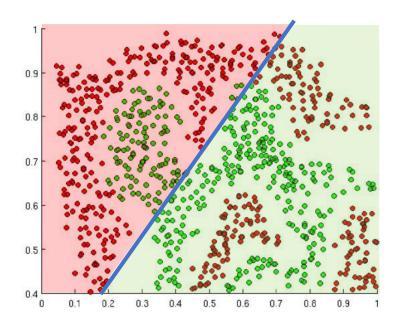
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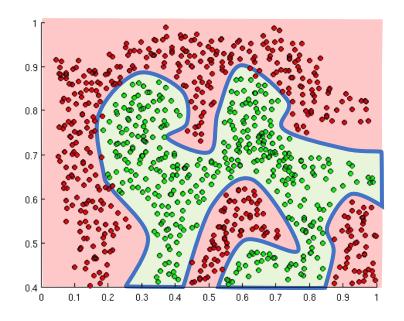
Linear Activation functions produce linear decisions no matter the network size

Importance of Activation Functions

The purpose of activation functions is to **introduce non-linearities** into the network



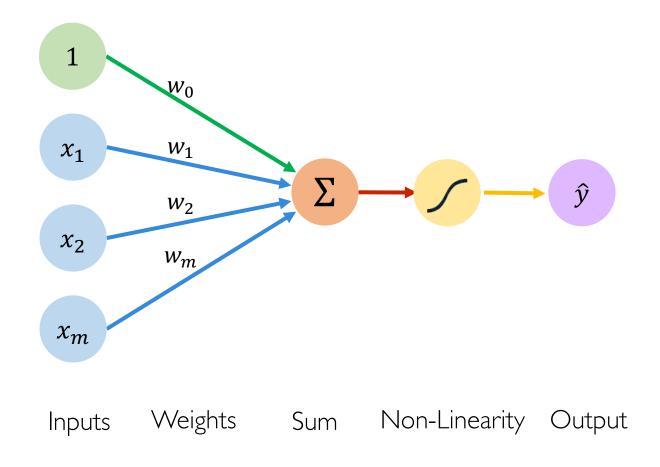
Linear Activation functions produce linear decisions no matter the network size



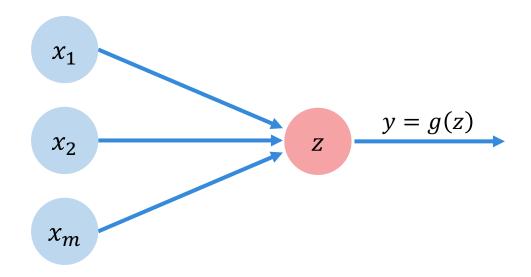
Non-linearities allow us to approximate arbitrarily complex functions

Building Neural Networks with Perceptrons

The Perceptron: Simplified

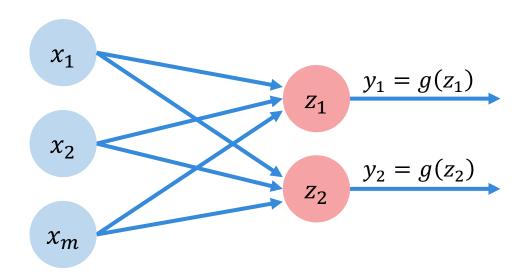


The Perceptron: Simplified



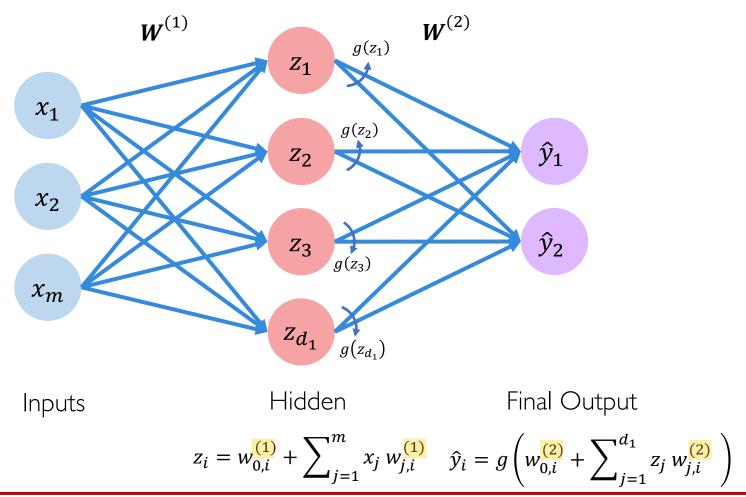
$$z = w_0 + \sum_{j=1}^m x_j w_j$$

Multi Output Perceptron



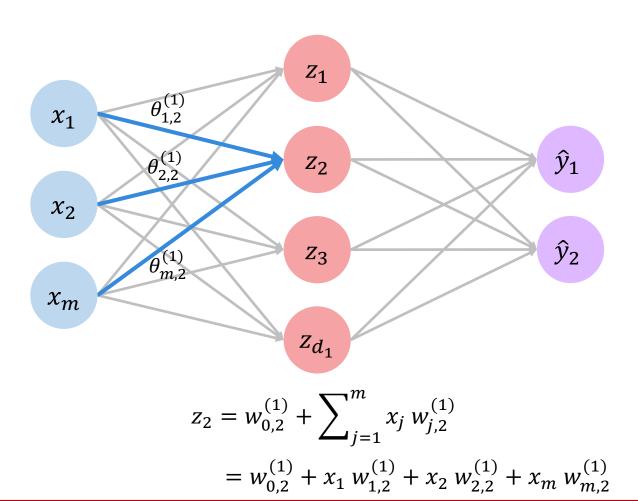
$$z_{\underline{i}} = w_{0,\underline{i}} + \sum_{j=1}^{m} x_j w_{j,\underline{i}}$$

Single Layer Neural Network



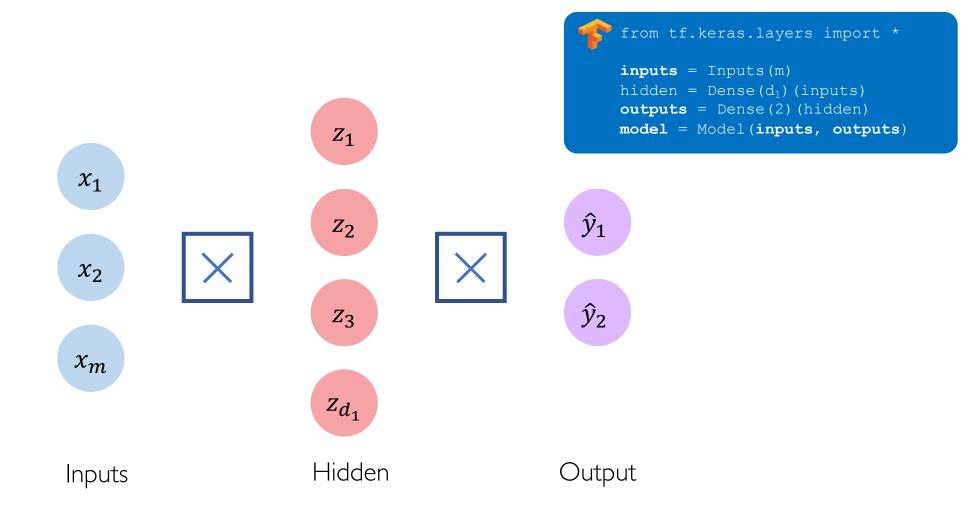


Single Layer Neural Network

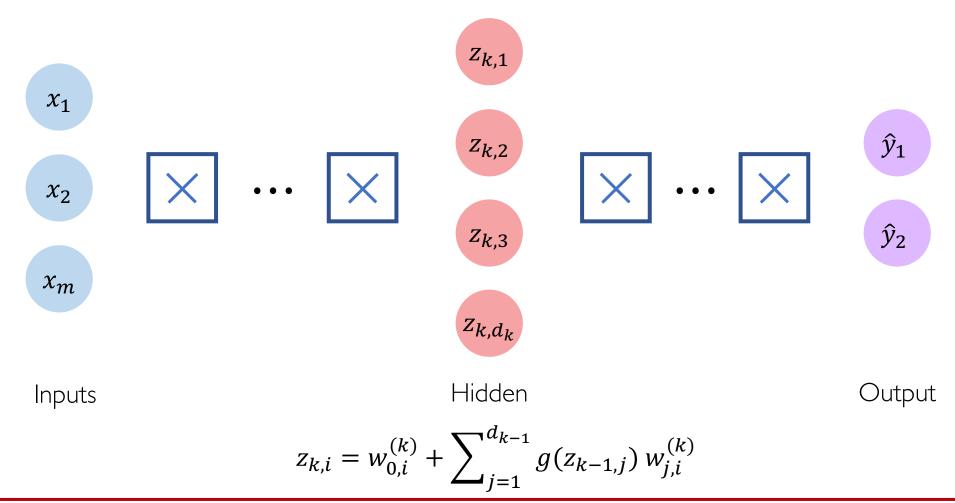




Multi Output Perceptron



Deep Neural Network



Applying Neural Networks

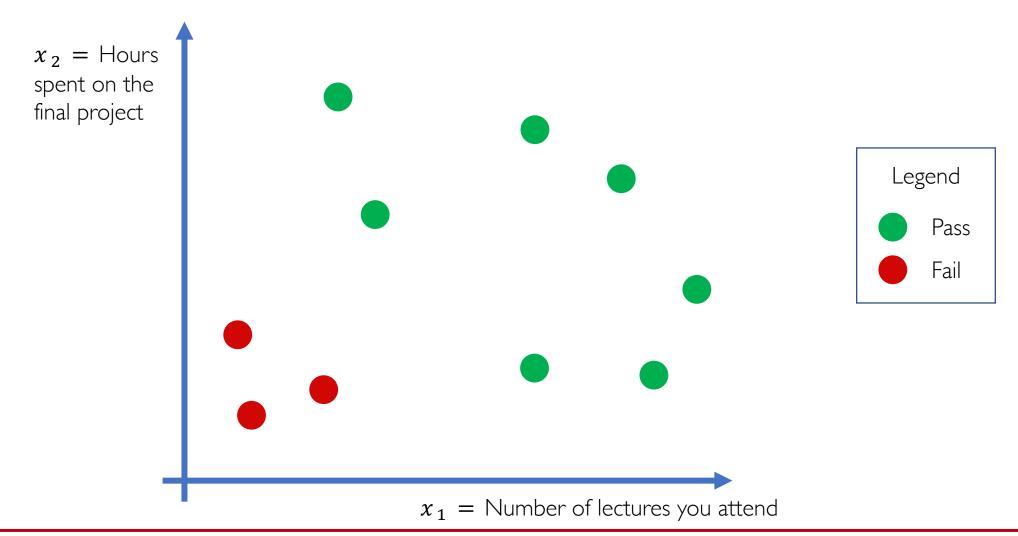
Example Problem

Will I pass this class?

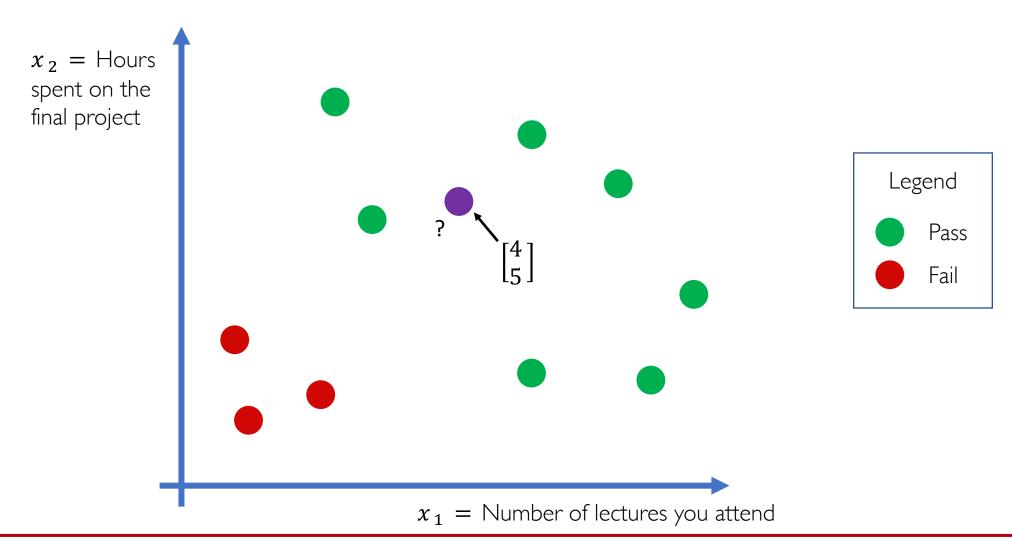
Let's start with a simple two feature model

 x_1 = Number of lectures you attend

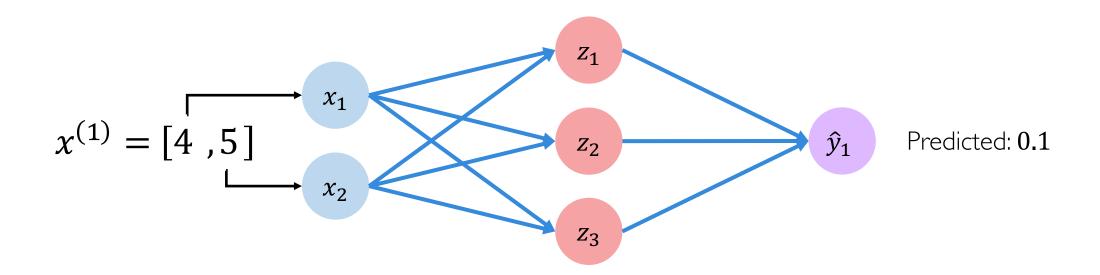
 x_2 = Hours spent on the final project

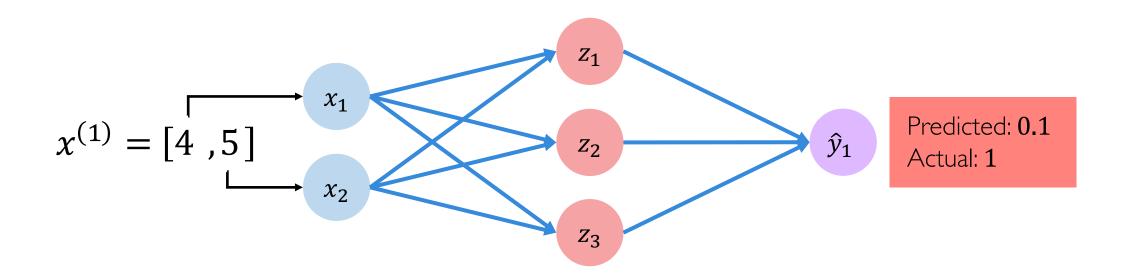






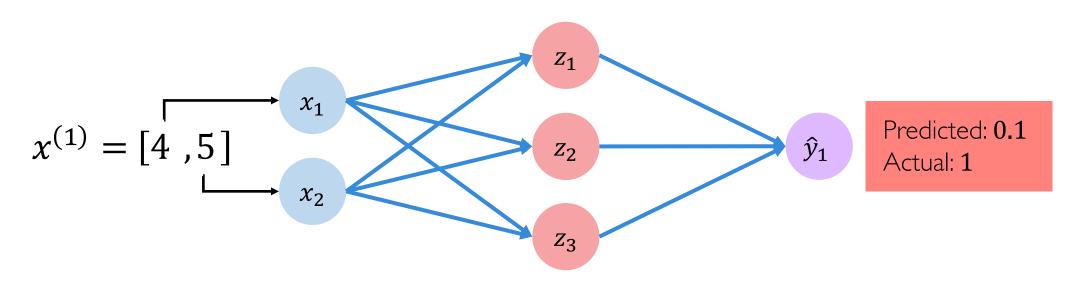






Quantifying Loss

The **loss** of our network measures the cost incurred from incorrect predictions



$$\mathcal{L}\left(f\left(x^{(i)};W\right),y^{(i)}\right)$$
Predicted Actual



Empirical Loss

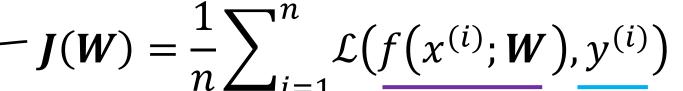
The **empirical loss** measures the total loss over our entire dataset

$$X = \begin{bmatrix} 4 & 5 \\ 2 & 1 \\ 5 & 8 \\ \vdots & \vdots \end{bmatrix} \qquad \begin{array}{c} x_1 \\ x_2 \\ \hline \end{array}$$

$$\begin{array}{c} f(x) & y \\ \hline 0.1 \\ 0.8 \\ 0.6 \\ \vdots \end{array}$$

Also known as:

- Objective function
- Cost function
- Empirical Risk



Predicted

Actual

Binary Cross Entropy Loss

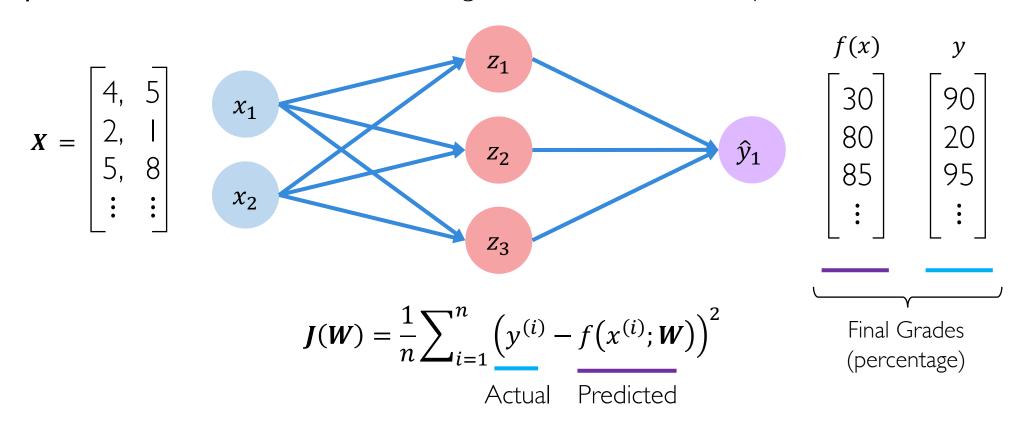
Cross entropy loss can be used with models that output a probability between 0 and 1

$$\mathbf{X} = \begin{bmatrix} 4, & 5 \\ 2, & 1 \\ 5, & 8 \\ \vdots & \vdots \end{bmatrix} \qquad \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \qquad \begin{array}{c} f(x) \\ 0.1 \\ 0.8 \\ 0.6 \\ \vdots \end{array} \qquad \begin{bmatrix} 1 \\ 0 \\ 0.6 \\ \vdots \end{bmatrix}$$

$$J(\mathbf{W}) = \frac{1}{n} \sum_{i=1}^{n} y^{(i)} \log \left(f(x^{(i)}; \mathbf{W}) \right) + (1 - y^{(i)}) \log \left(1 - f(x^{(i)}; \mathbf{W}) \right)$$
Actual Predicted Actual Predicted

Mean Squared Error Loss

Mean squared error loss can be used with regression models that output continuous real numbers



Training Neural Networks

We want to find the network weights that achieve the lowest loss

$$W^* = \underset{W}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}(f(x^{(i)}; W), y^{(i)})$$
$$W^* = \underset{W}{\operatorname{argmin}} J(W)$$

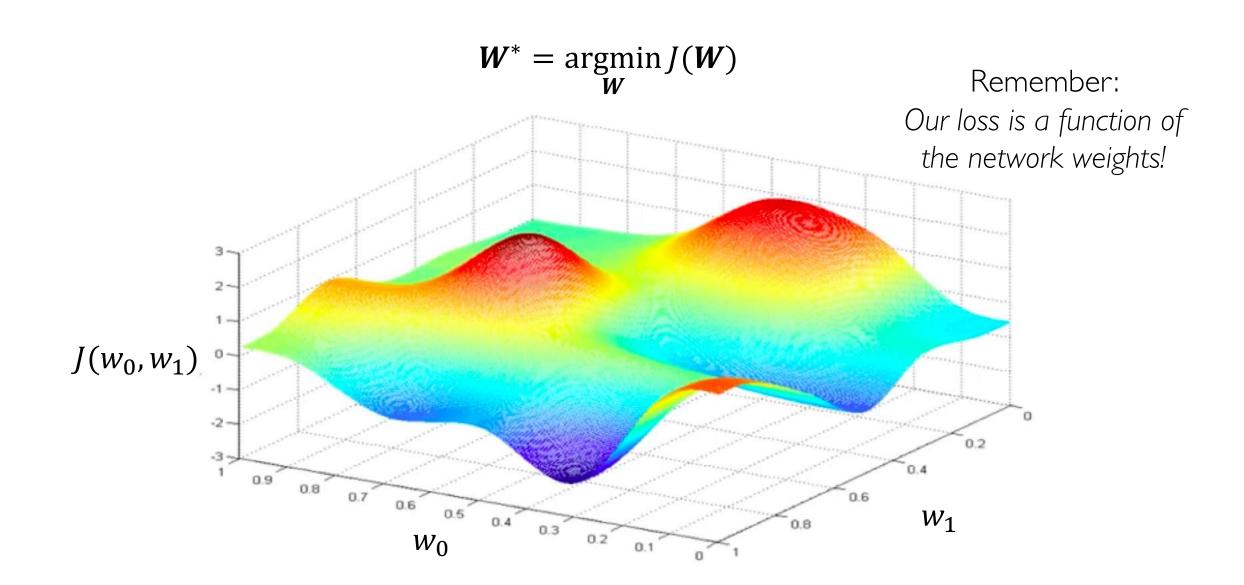
We want to find the network weights that achieve the lowest loss

$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}(f(\boldsymbol{x}^{(i)}; \boldsymbol{W}), \boldsymbol{y}^{(i)})$$

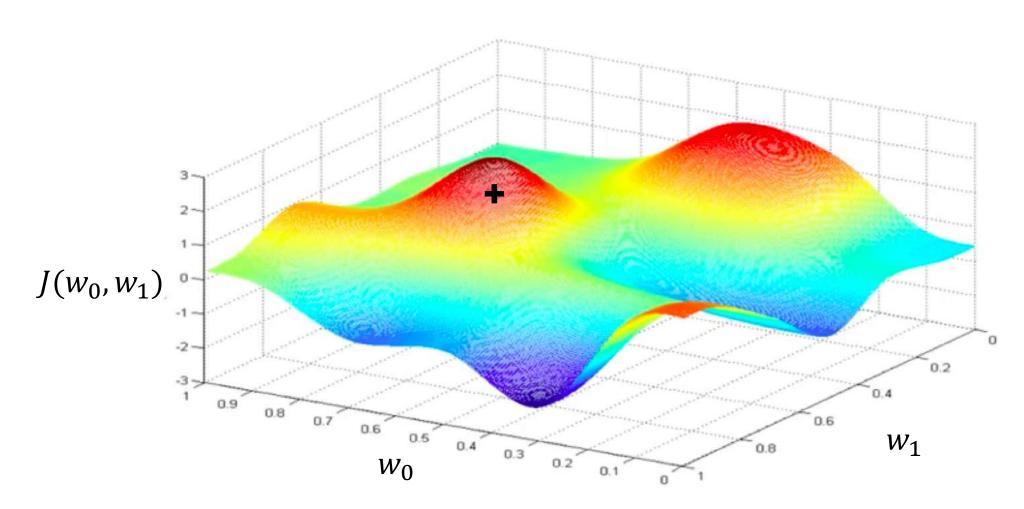
$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}} J(\boldsymbol{W})$$

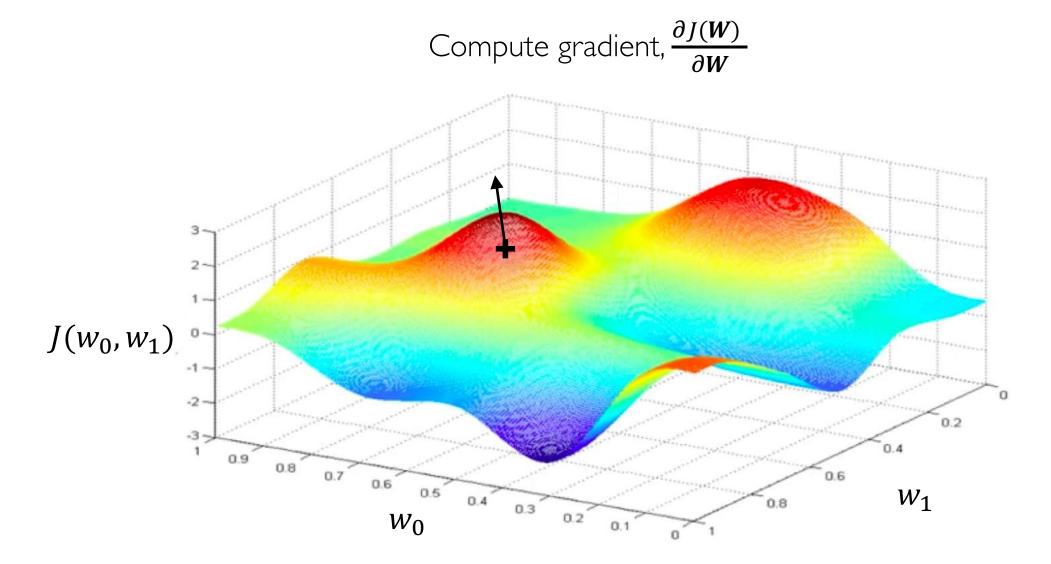
$$\overset{\text{Remember:}}{\boldsymbol{W}}$$

$$\boldsymbol{W} = \{\boldsymbol{W}^{(0)}, \boldsymbol{W}^{(1)}, \dots\}$$

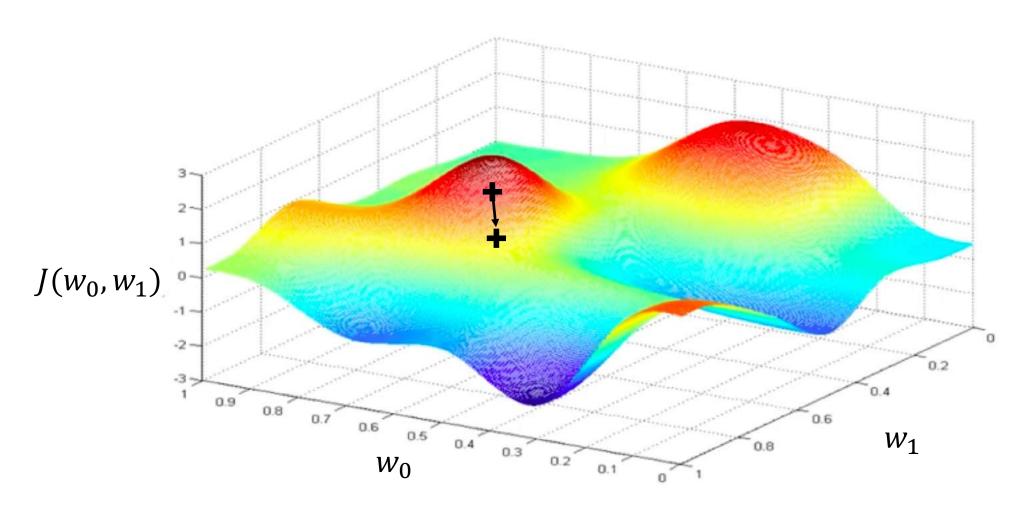


Randomly pick an initial (w_0, w_1)



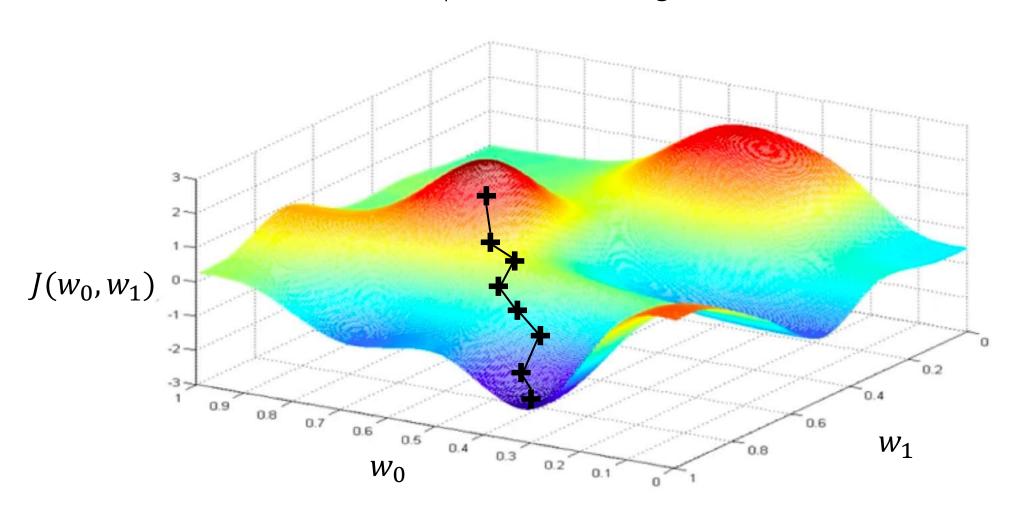


Take small step in opposite direction of gradient



Gradient Descent

Repeat until convergence



Gradient Descent

Algorithm

Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$



- Loop until convergence:
- Compute gradient, $\frac{\partial J(W)}{\partial W}$ 3.
- Update weights, $W \leftarrow W \eta \frac{\partial J(W)}{\partial W}$
- grads = tf.gradients(ys=loss, xs=weights)
- weights_new = weights.assign(weights lr * grads)

5. Return weights

Gradient Descent

Algorithm

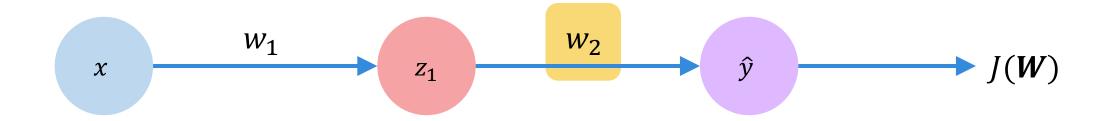
Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$



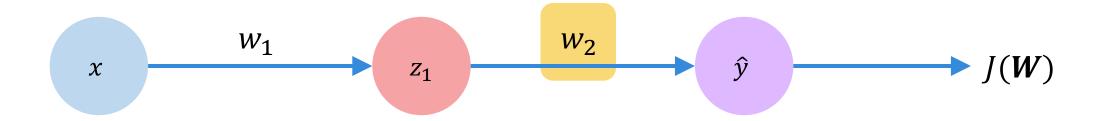
- Loop until convergence:
- 3.
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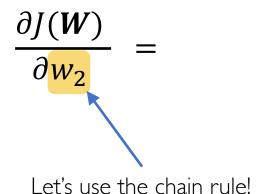
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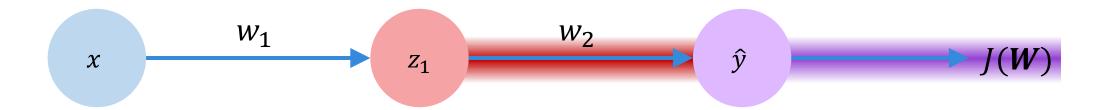


How does a small change in one weight (ex. w_2) affect the final loss J(W)?

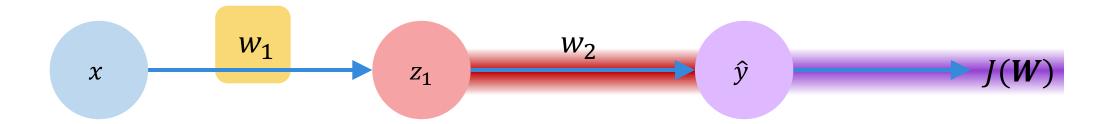


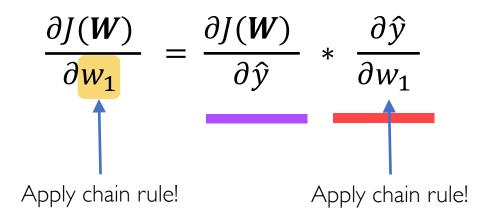




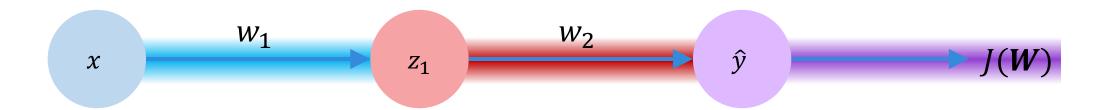


$$\frac{\partial J(\mathbf{W})}{\partial w_2} = \frac{\partial J(\mathbf{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial w_2}$$

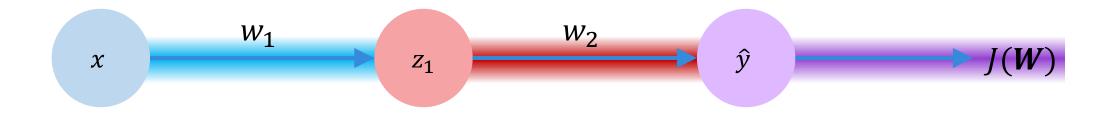








$$\frac{\partial J(\mathbf{W})}{\partial w_1} = \frac{\partial J(\mathbf{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial w_1}$$

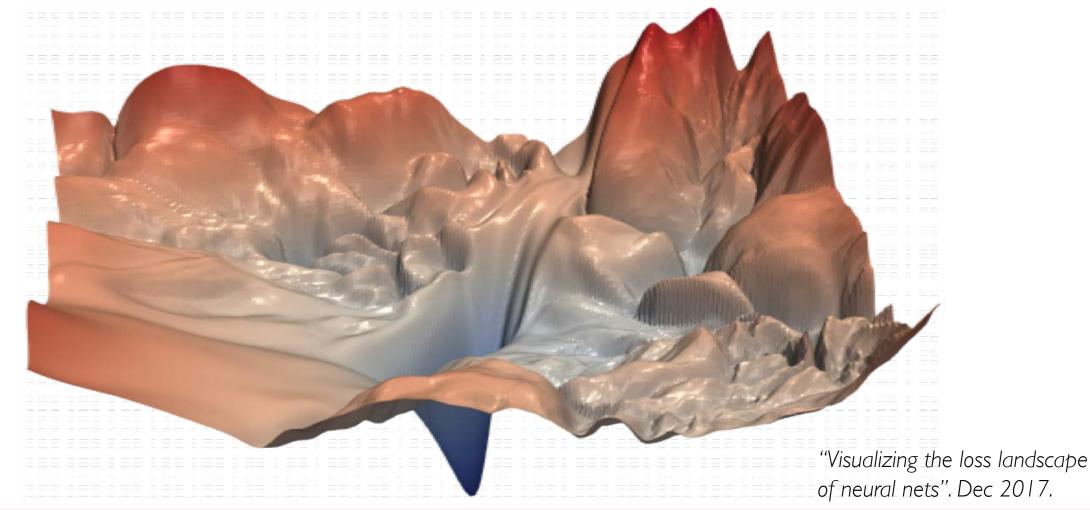


$$\frac{\partial J(\mathbf{W})}{\partial w_1} = \frac{\partial J(\mathbf{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial w_1}$$

Repeat this for every weight in the network using gradients from later layers

Neural Networks in Practice: Optimization

Training Neural Networks is Difficult



Loss Functions Can Be Difficult to Optimize

Remember:

Optimization through gradient descent

$$\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \, \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$$

Loss Functions Can Be Difficult to Optimize

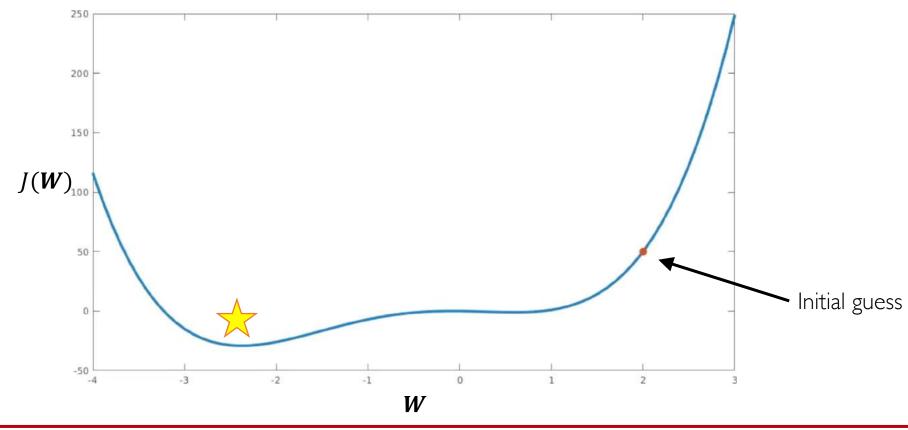
Remember:

Optimization through gradient descent

$$W \leftarrow W - \frac{\partial J(W)}{\partial W}$$
How can we set the learning rate?

Setting the Learning Rate

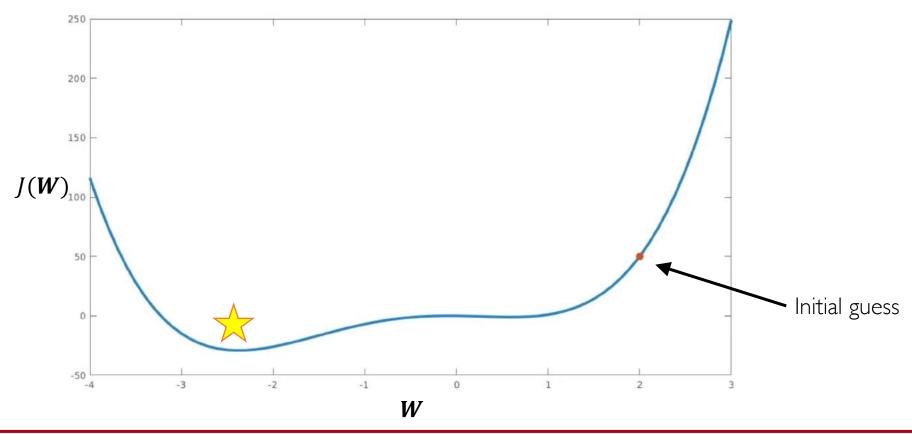
Small learning rate converges slowly and gets stuck in false local minima





Setting the Learning Rate

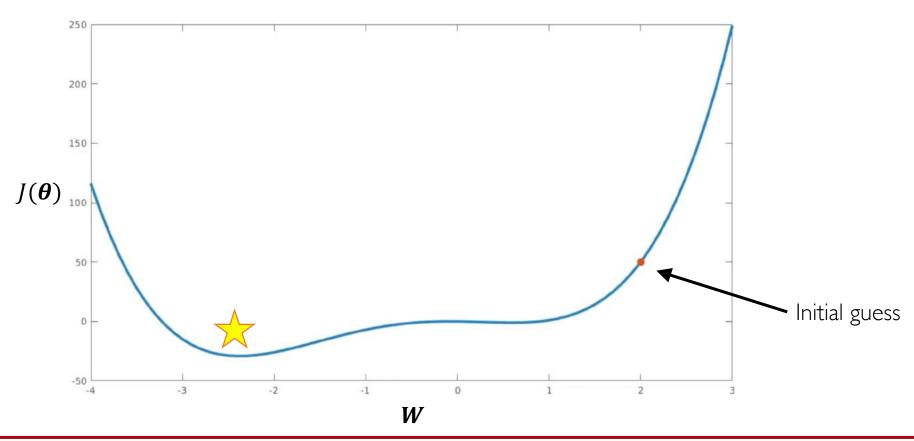
Large learning rates overshoot, become unstable and diverge





Setting the Learning Rate

Stable learning rates converge smoothly and avoid local minima



How to deal with this?

Idea I:

Try lots of different learning rates and see what works "just right"

How to deal with this?

Idea I:

Try lots of different learning rates and see what works "just right"

Idea 2:

Do something smarter!

Design an adaptive learning rate that "adapts" to the landscape



Adaptive Learning Rates

- Learning rates are no longer fixed
- Can be made larger or smaller depending on:
 - how large gradient is
 - how fast learning is happening
 - size of particular weights
 - etc...

Adaptive Learning Rate Algorithms

- Momentum
- Adagrad
- Adadelta
- Adam
- RMSProp









🎓 tf.train.RMSPropOptimizer

Qian et al. "On the momentum term in gradient descent learning algorithms." 1999.

Duchi et al. "Adaptive Subgradient Methods for Online Learning and Stochastic Optimization." 2011.

Zeiler et al. "ADADELTA: An Adaptive Learning Rate Method." 2012.

Kingma et al. "Adam: A Method for Stochastic Optimization." 2014.

Additional details: http://ruder.io/optimizing-gradient-descent/

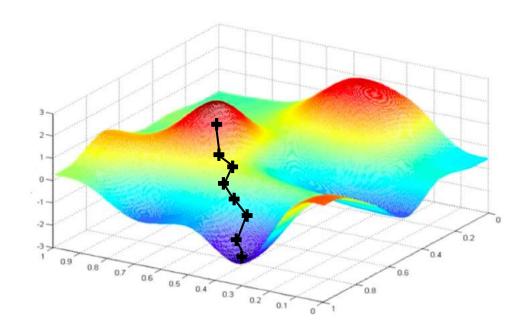


Neural Networks in Practice: Mini-batches

Gradient Descent

Algorithm

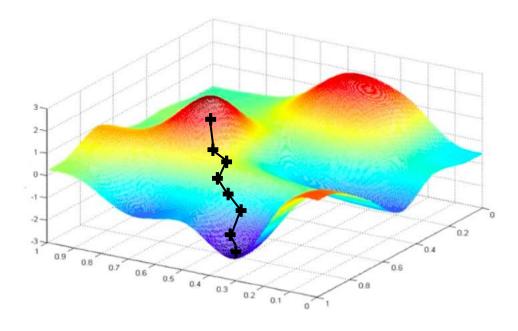
- I. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Compute gradient, $\frac{\partial J(W)}{\partial W}$
- 4. Update weights, $\mathbf{W} \leftarrow \mathbf{W} \eta \frac{\partial J(\mathbf{W})}{\partial \mathbf{W}}$
- 5. Return weights



Gradient Descent

Algorithm

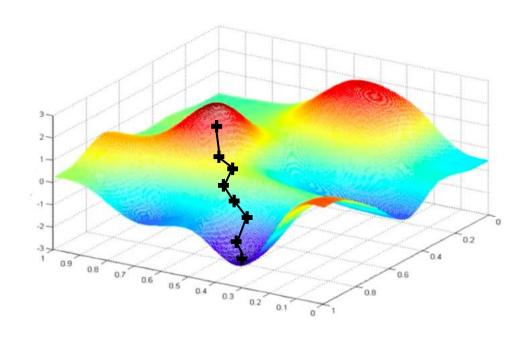
- 1. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Compute gradient, $\frac{\partial J(W)}{\partial W}$
- 4. Update weights, $\mathbf{W} \leftarrow \mathbf{W} \eta \frac{\partial J(\mathbf{W})}{\partial \mathbf{W}}$
- 5. Return weights



Can be very computational to compute!

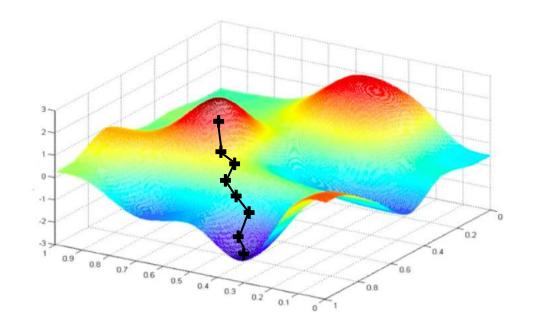
Algorithm

- 1. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Pick single data point i
- 4. Compute gradient, $\frac{\partial J_i(W)}{\partial W}$
- 5. Update weights, $\mathbf{W} \leftarrow \mathbf{W} \eta \frac{\partial J(\mathbf{W})}{\partial \mathbf{W}}$
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Algorithm

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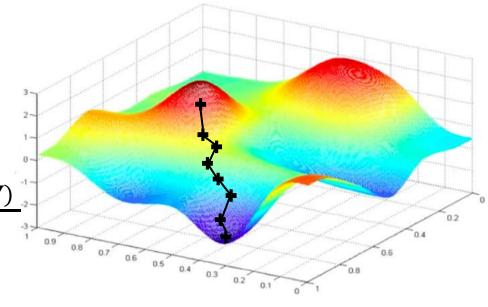
Easy to compute but very noisy (stochastic)!

Algorithm

- I. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
- 2. Loop until convergence:
- 3. Pick batch of B data points

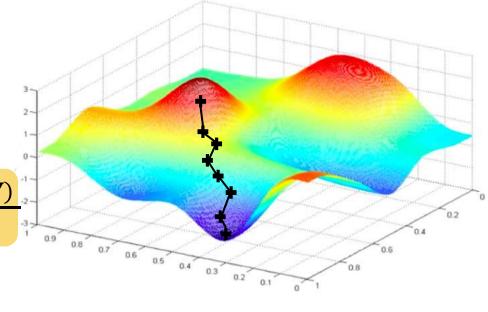
4. Compute gradient,
$$\frac{\partial J(W)}{\partial W} = \frac{1}{B} \sum_{k=1}^{B} \frac{\partial J_k(W)}{\partial W}$$

- 5. Update weights, $\mathbf{W} \leftarrow \mathbf{W} \eta \frac{\partial J(\mathbf{W})}{\partial \mathbf{W}}$
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Algorithm

- I. Initialize weights randomly $\sim \mathcal{N}(0, \sigma^2)$
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Fast to compute and a much better estimate of the true gradient!

Mini-batches while training

More accurate estimation of gradient

Smoother convergence Allows for larger learning rates

Mini-batches while training

More accurate estimation of gradient

Smoother convergence
Allows for larger learning rates

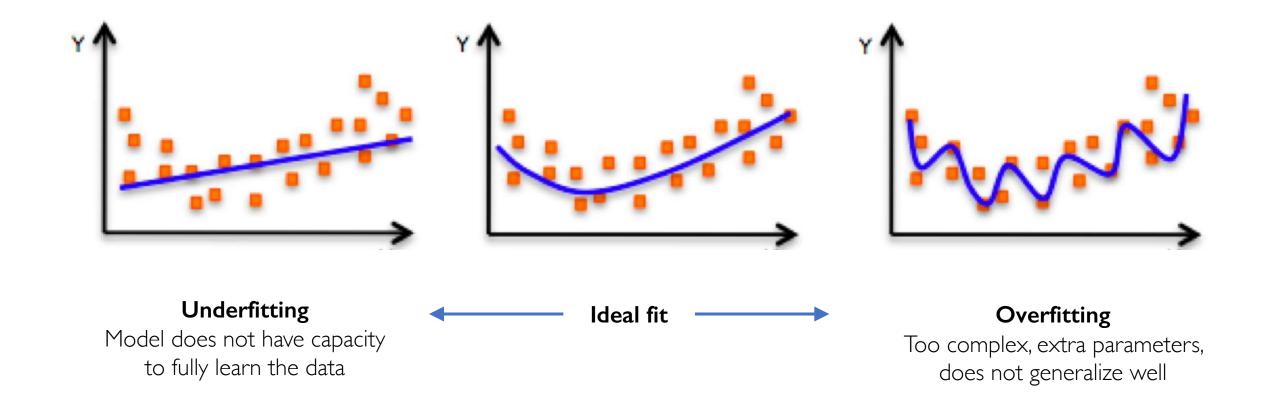
Mini-batches lead to fast training!

Can parallelize computation + achieve significant speed increases on GPU's



Neural Networks in Practice: Overfitting

The Problem of Overfitting



Regularization

What is it?

Technique that constrains our optimization problem to discourage complex models

Regularization

What is it?

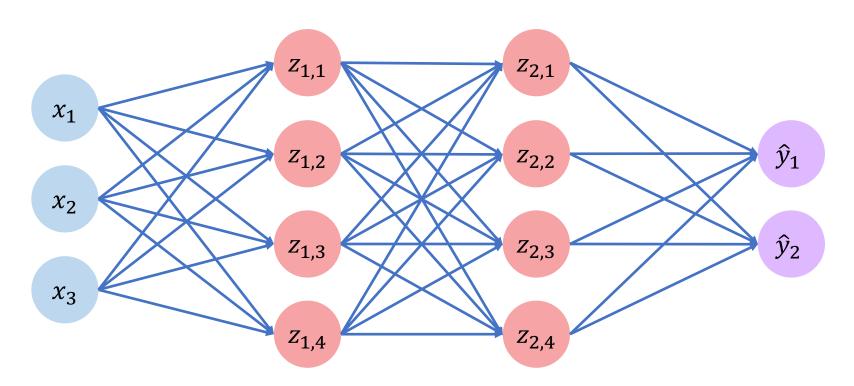
Technique that constrains our optimization problem to discourage complex models

Why do we need it?

Improve generalization of our model on unseen data

Regularization 1: Dropout

During training, randomly set some activations to 0

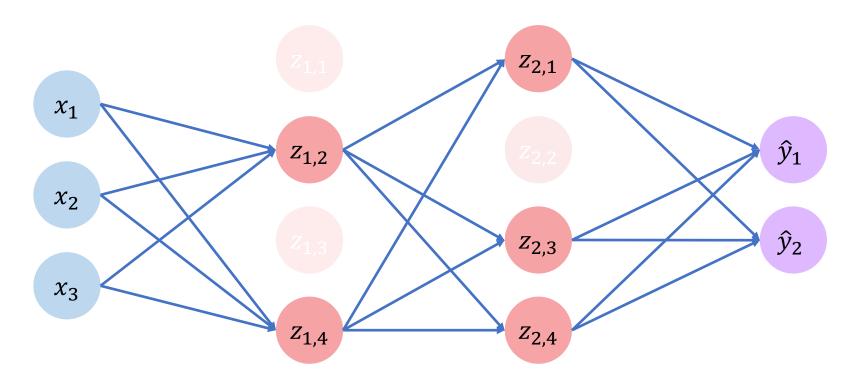




Regularization 1: Dropout

- During training, randomly set some activations to 0
 - Typically 'drop' 50% of activations in layer
 - Forces network to not rely on any I node



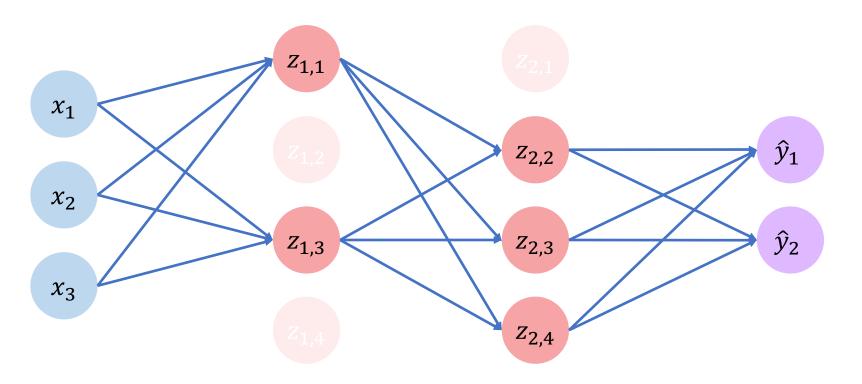




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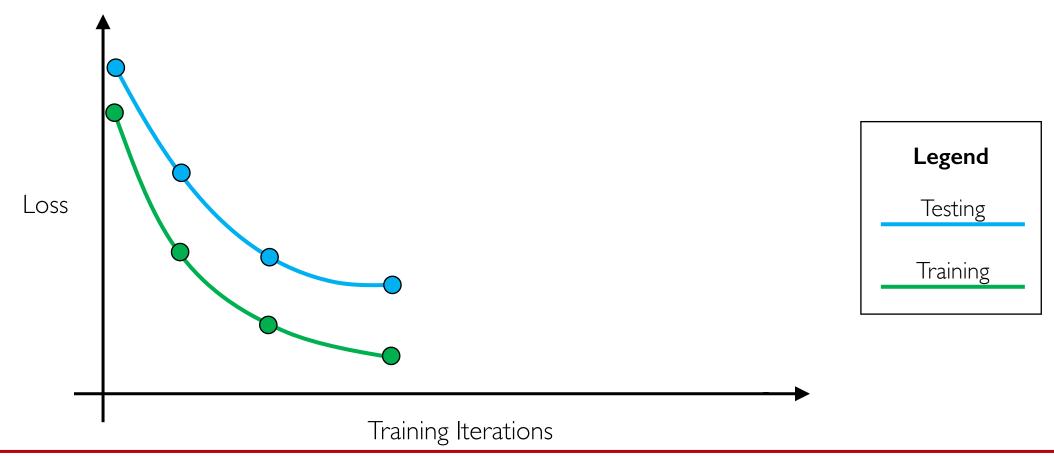


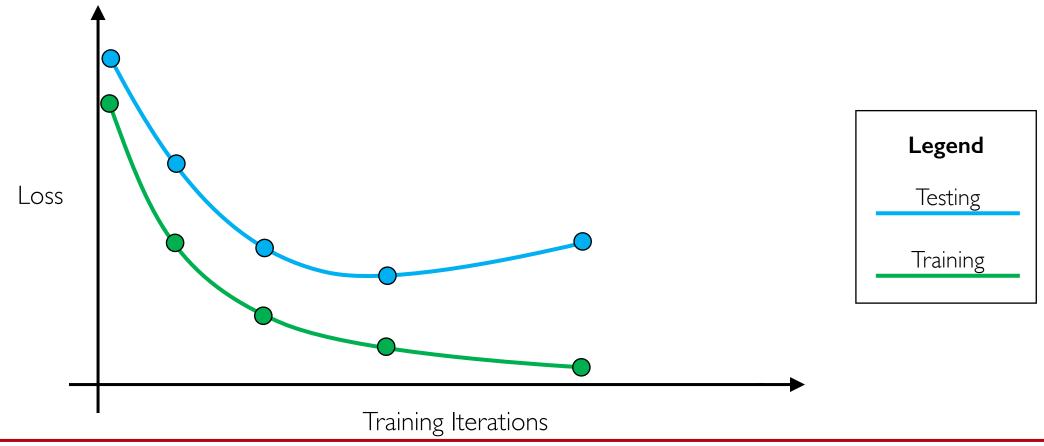




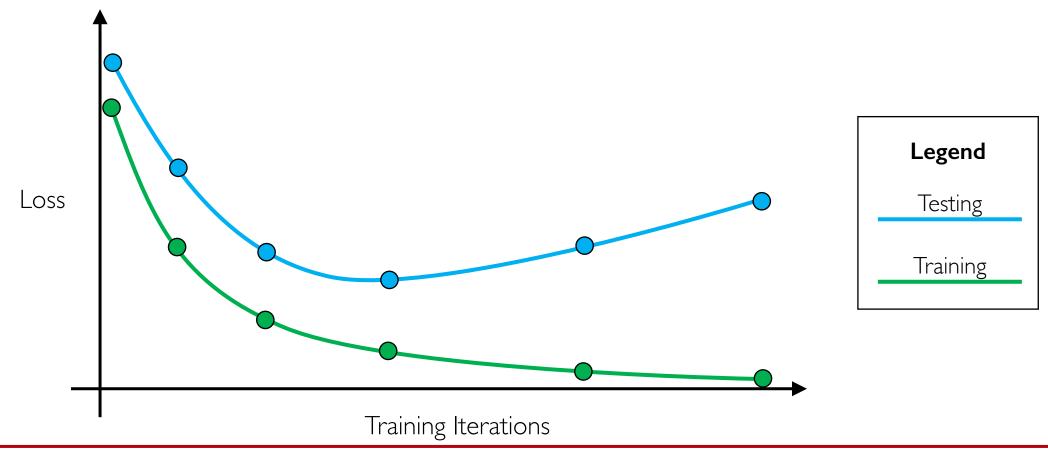


















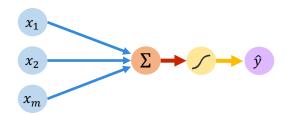




Core Foundation Review

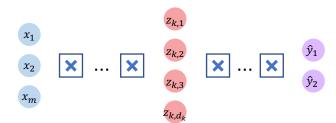
The Perceptron

- Structural building blocks
- Nonlinear activation functions



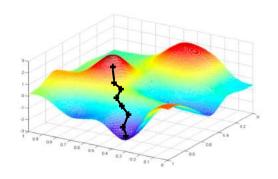
Neural Networks

- Stacking Perceptrons to form neural networks
- Optimization through backpropagation



Training in Practice

- Adaptive learning
- Batching
- Regularization



Questions?