Lecture 4

Logistic (Regression) classification

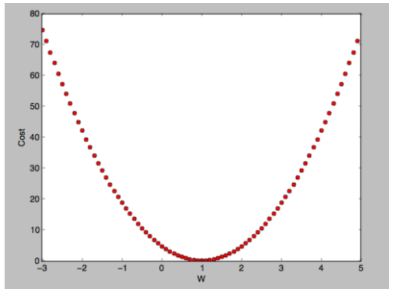
Dong Kook Kim

Regression : Recap

x1 (hours)	x2 (attendance)	y (score)
10	5	90
9	5	80
3	2	50
2	4	60
11	1	40

- Hypothesis: $H(X) = WX$

- Cost: $cost(W) = \frac{1}{m}\sum(WX - y)^2$



- Gradient decent:

$$W := W - \alpha \frac{\partial}{\partial W} \mathrm{cost(W)}$$

Step 1: Binary Classification Data

- Target : 0 or 1 (Binary)

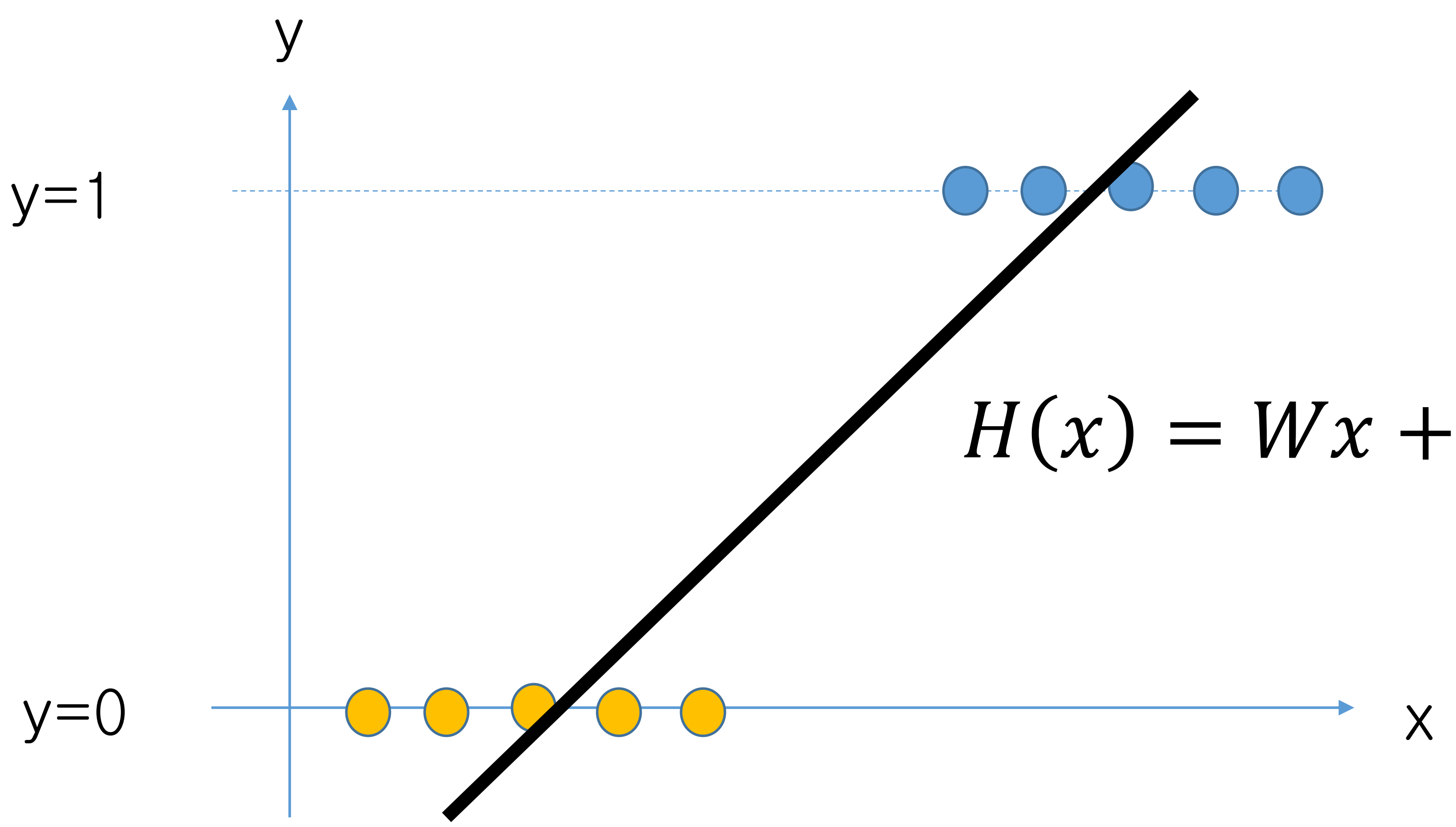
x1 (hours)	x2 (attendance)	y (P or F)	y (target)
10	5	Pass (90)	1
9	5	Pass (80)	1
3	2	Fail (50)	0
2	4	Fail (60)	0
11	1	Fail (40)	0

Target : 0, 1 encoding

- Target : 0 or 1 (binary)

- Spam Detection: Spam (1) or Ham (0)
- Tumor Detection: Malignant(1) or Not (0)
- Facebook feed: show(1) or hide(0)
- Credit Card Fraudulent Transaction detection: legitimate(0) or fraud (1)

Linear Regression Model



$$H(x) = Wx + b$$

Logistic Regression

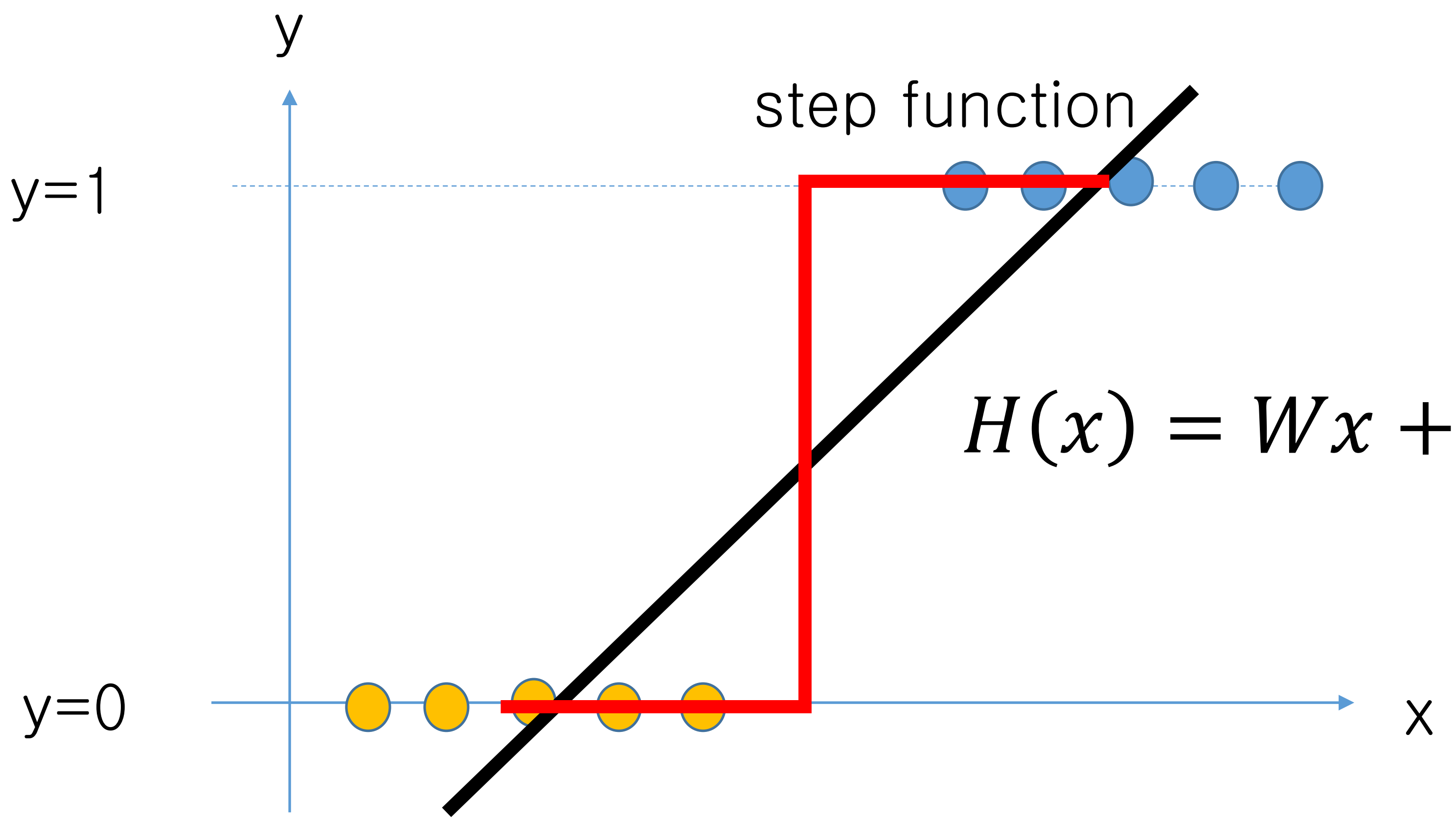
- We know y is 0 or 1

$$H(x) = Wx + b$$: linear regression

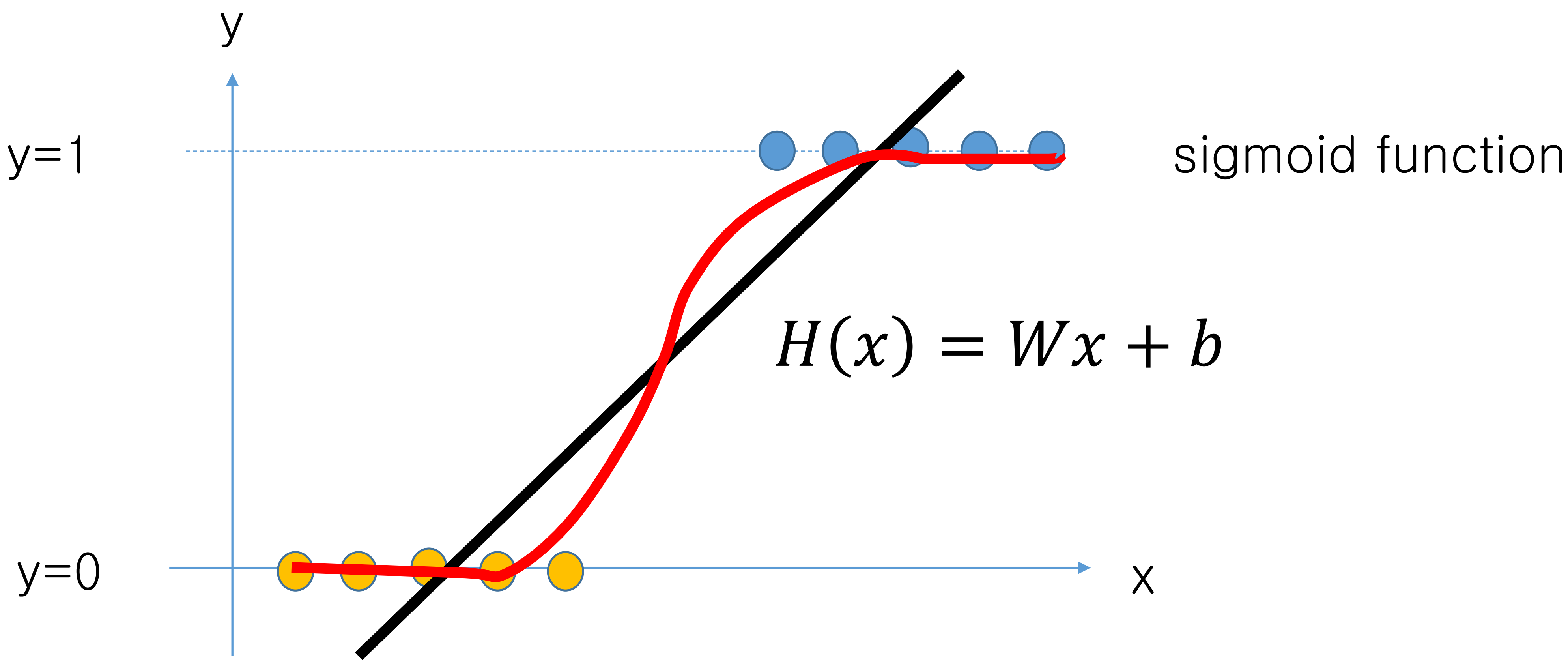
- Hypothesis can give values larger than 1 or less than 0

- A nonlinear function is need to represent 0 or 1 hypothesis

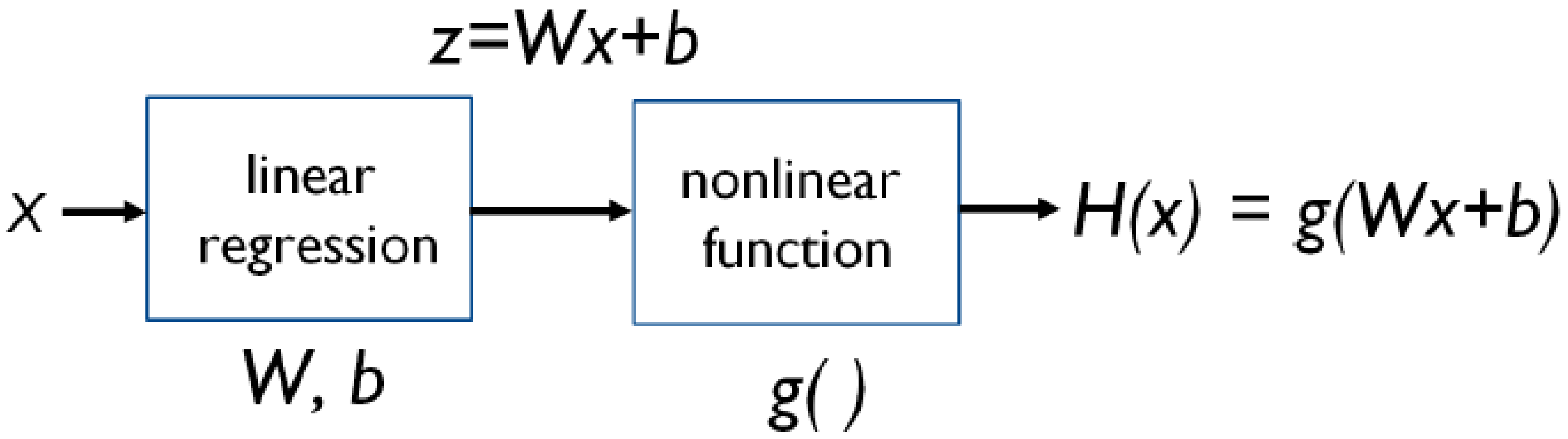
Nonlinear function : Step function



Nonlinear function : Sigmoid function



Step 2: Logistic Regression Model

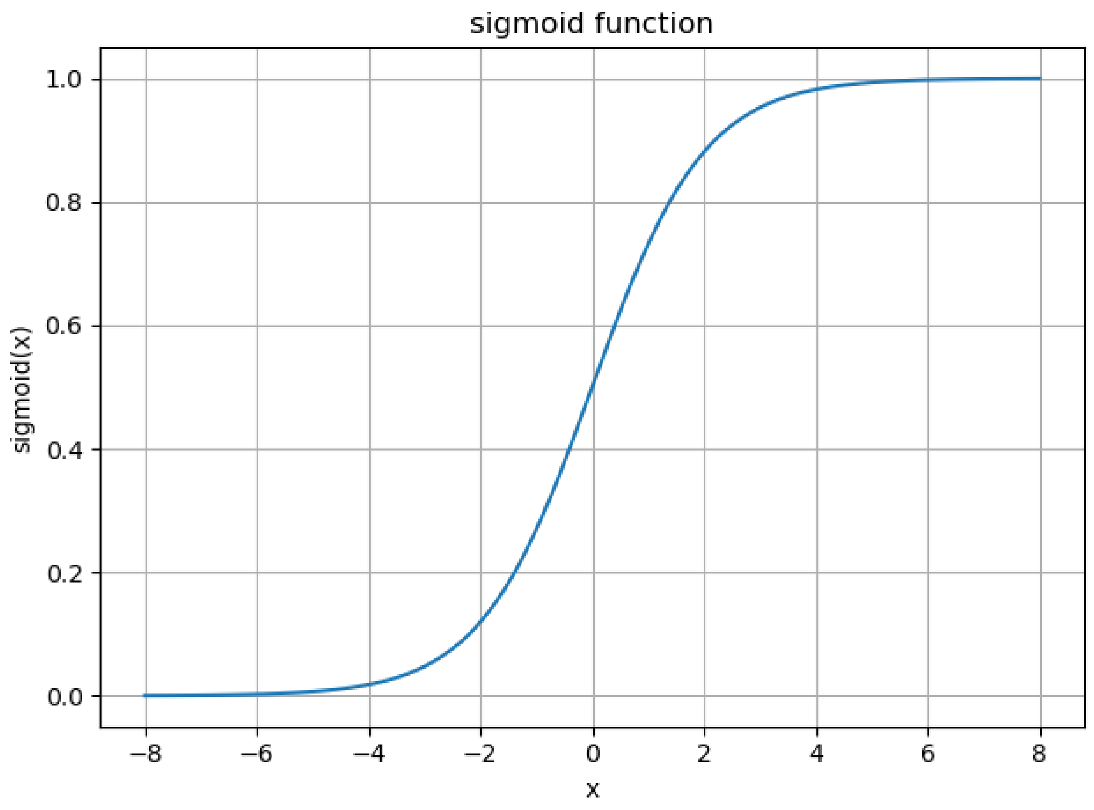


$$z = Wx + b \qquad\qquad H(x) = g(z)$$

Sigmoid Function

- Logistic function or Sigmoid function
- Curved in two directions, liked the letter 'S'

$$y = \sigma(x) = \frac{1}{1 + e^{-x}}$$

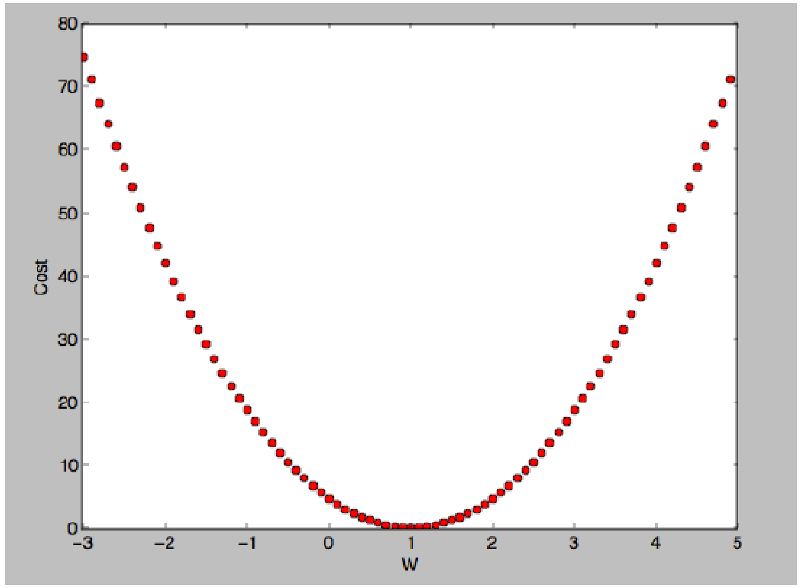


Step 2: Logistic Regression Model

$$H(X) = \sigma(Wx + b) = \frac{1}{1 + e^{-(Wx+b)}}$$

Loss Function : LR

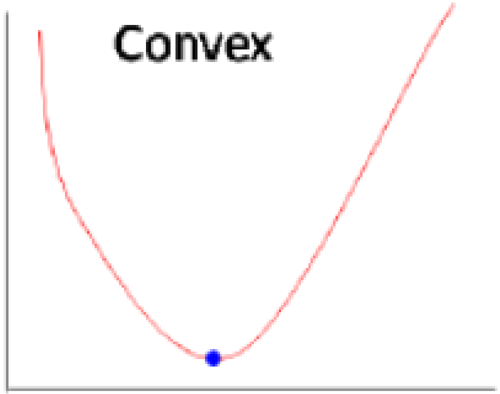
$$cost(W,b) = \frac{1}{m}\sum_{i=1}^{m}\left(H\left(x^{(i)}\right) - y^{(i)}\right)^2 \ \ when \ H(x) = Wx + b$$

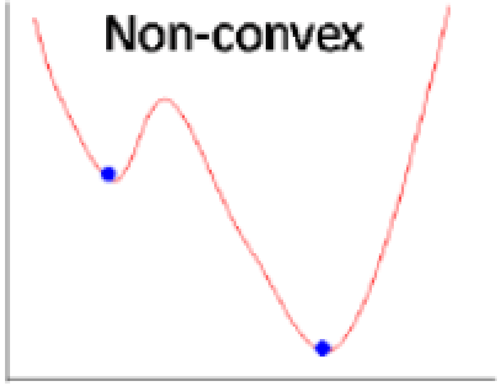


Loss Function : MSE

$$cost(W,b) = \frac{1}{m}\sum_{i=1}^{m}\left(H\left(x^{(i)}\right) - y^{(i)}\right)^2$$

$$H(x) = Wx + b \qquad H(X) = \frac{1}{1+e^{-W^T X}}$$





Step 3: New Loss Function for Logistic

$$cost(W,b) = \frac{1}{m}\sum c(H(x),y)$$

$$C(H(x),y) = \begin{cases} -\log\left(H(x)\right) & : \; y = 1 \\ -\log\left(1 - H(x)\right) & : \; y = 0 \end{cases}$$

Cross – entropy function

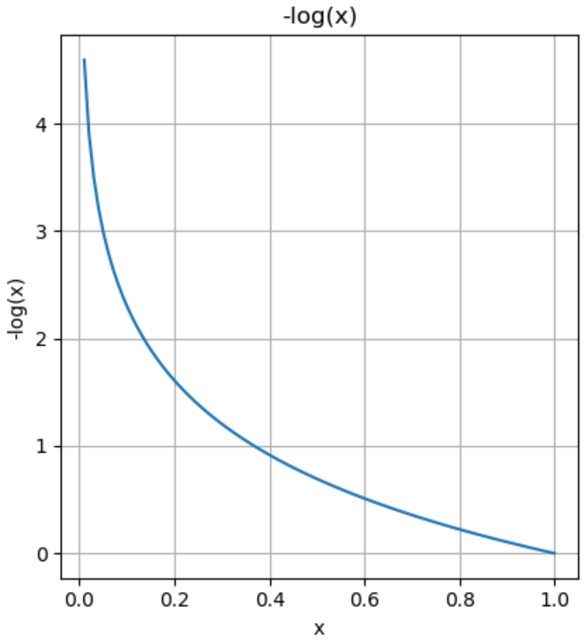
Understanding Loss Function

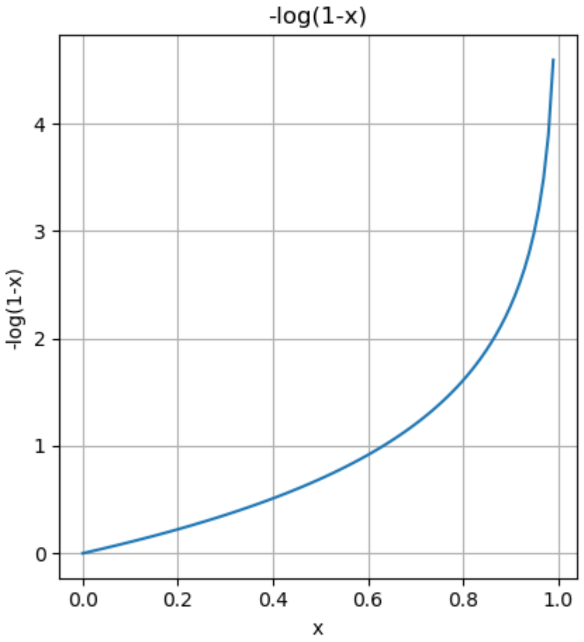
$$C(H(x), y) = \begin{cases} -\log\big(H(x)\big) & : \ y = 1 \\ -\log\big(1 - H(x)\big) & : y = 0 \end{cases}$$

If $y = 1,\ C(H(x), y) = -\log\big(H(x)\big)$

If $y = 0,\ C(H(x), y) = -\log\big(1 - H(x)\big)$

$H(x) = 1 \ \rightarrow cost = 0$
$H(x) = 0 \ \rightarrow cost = \infty$





$H(x) = 1 \ \rightarrow cost = \infty$
$H(x) = 0 \ \rightarrow cost = 0$

Step 3: Loss Function - Cross-Entropy

$$cost(W,b) = \frac{1}{m}\sum c(H(x),y)$$

$$C(H(x),y) = \begin{cases} -\log\big(H(x)\big) & : \; y = 1 \\ -\log\big(1 - H(x)\big) & : \; y = 0 \end{cases}$$

$$C(H(x),y) = -ylog\big(H(x)\big) - (1-y)\log(1-H(x))$$

Step 4: Optimization – GD algorithm

$$cost(W,b) = -\frac{1}{m}\sum ylog\big(H(x)\big) + (1-y)\log\,(1-H(x))$$

$$W := W - \alpha\frac{\partial}{\partial W}cost(W)$$

- Derivative cost(W) wrt W

$$\frac{\partial}{\partial W}cost(W,b) = \sum\big(y - H(x)\big)x$$

Optimization – GD algorithm

$$cost(W,b) = -\frac{1}{m}\sum ylog\big(H(x)\big) + (1-y)\log(1-H(x))$$

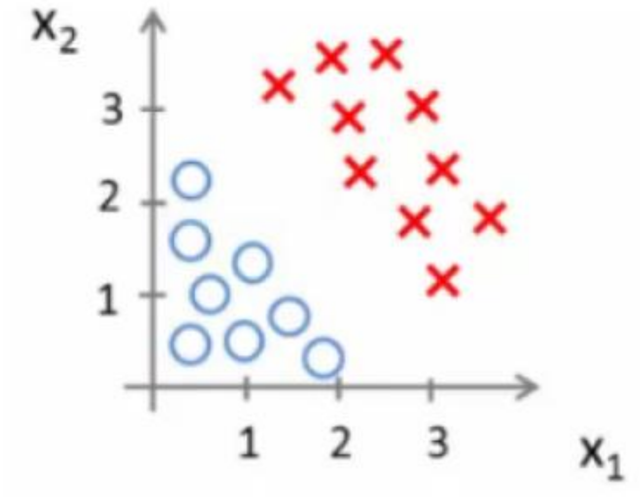
$$W := W - \alpha\frac{\partial}{\partial W}cost(W)$$

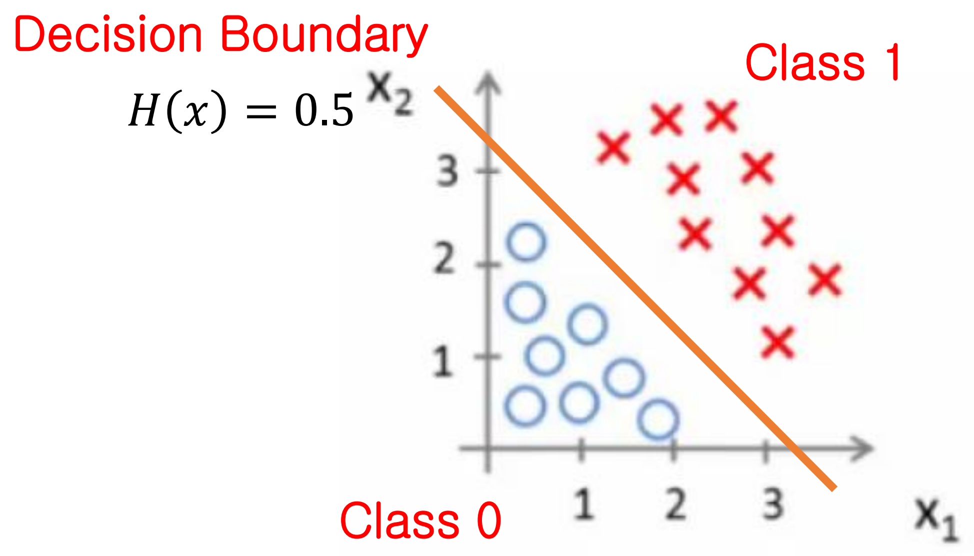
```
# cost function
cost = tf.reduce_mean(-tf.reduce_sum(Y*tf.log(hypothesis) + (1-Y)*tf.log(1-hypothesis)))

# Minimize
a = tf.Variable(0.1) # Learning rate, alpha
optimizer = tf.train.GradientDescentOptimizer(a)
train = optimizer.minimize(cost)
```

Step 5: Testing - Decision Boundary

$$H(x) = \frac{1}{1 + e^{-(Wx+b)}}$$

$$Decision: \quad if\ H(x) > 0.5, y = 1, else\ y = 0$$



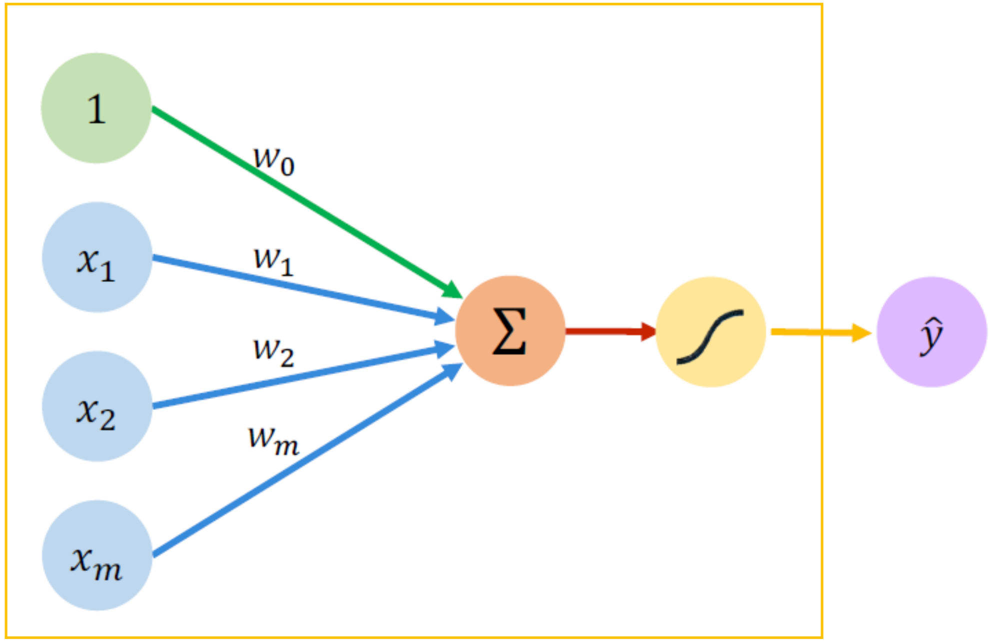


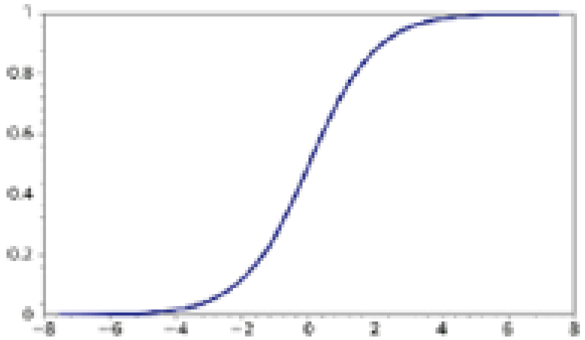
Graph for Logistic Regression

- x : n-dim vector, y : scalar
- W : n-dim vector, b : scalar

$$y = H(X) = \sigma(Wx + b)$$

$$\sigma(a) = \frac{1}{1 + \exp(-a)}$$





- Sigmoid function

Keras : Dense(1, input_dim=n, activation='sigmoid')

Logistic Regression : Summary

- Muti-variate Input/Scalar Target:

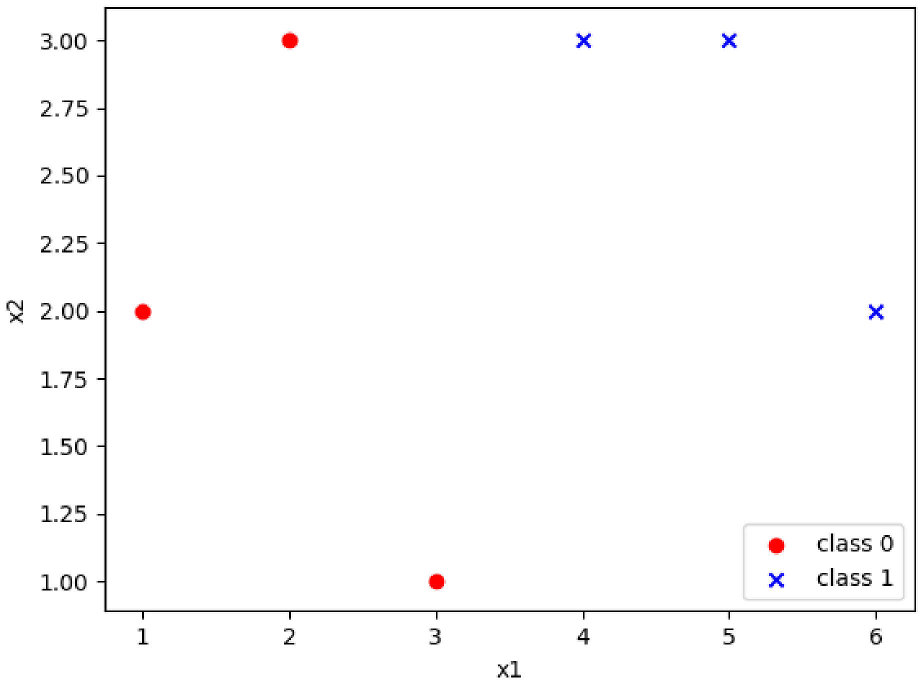
Data Set :	$x^{(i)}, y^{(i)}, i = 1, \dots, m$	Input vector & scalar target (0 or 1)
Model :	$H(X) = \dfrac{1}{1 + e^{-(Wx+b)}}$	Linear model W : weight vector b : bias
Cost Function	$cost(W, b)$ $= -\dfrac{1}{m} \sum y log(H(x)) + (1$	CE
Optimization	$W := W - \alpha \dfrac{\partial cost(W, b)}{\partial W}$	GD
Testing	$Given\ x\ \&\ W, b\ decide\ y = 1\ if\ H(x)$ $> 0.5, else\ y = 0$	Metric : Accuracy

Exercise 03-1.

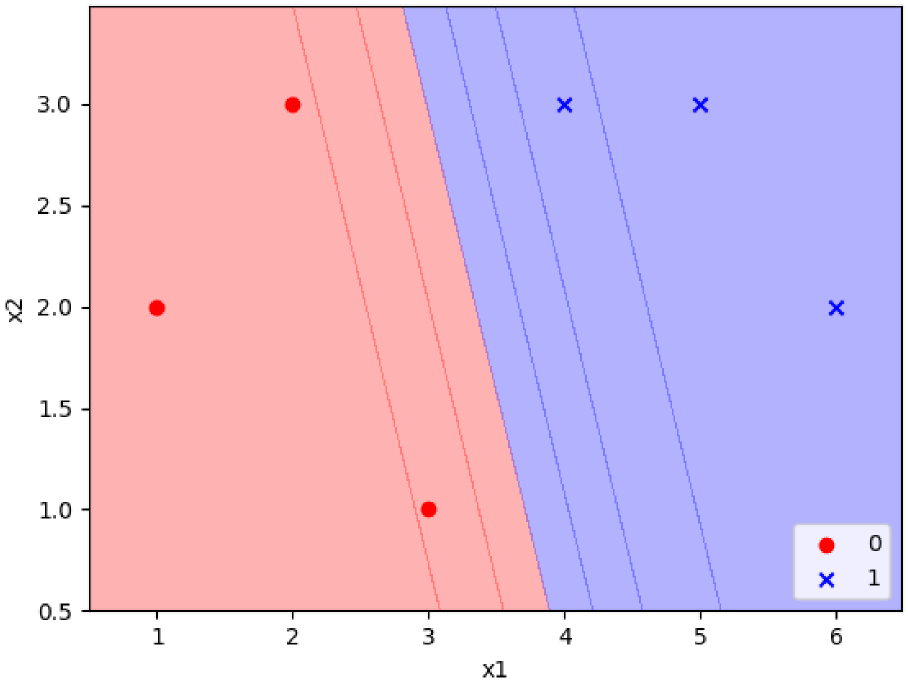
tf2-03-1-logistic_classification.py

Exercise 03-1.

- Data: input and target



- Classification results



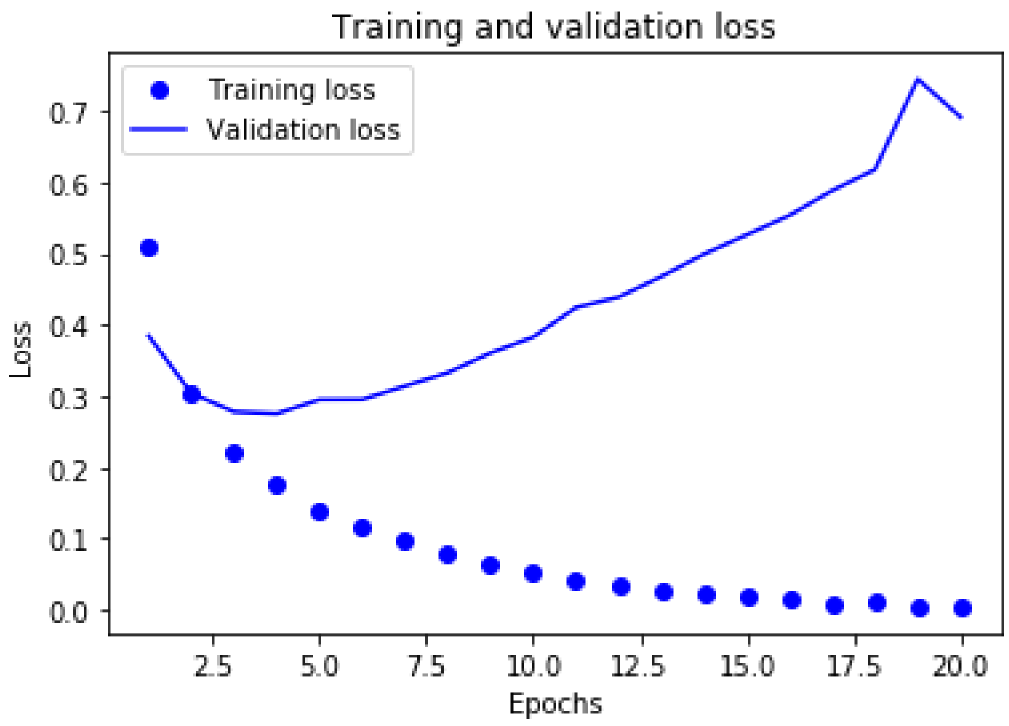
Exercise 03-2.

tf2-03-2-imdb_classification.py

- movie review classification

Exercise 03-2.

- Training & validation loss



- Training & validation accuracy