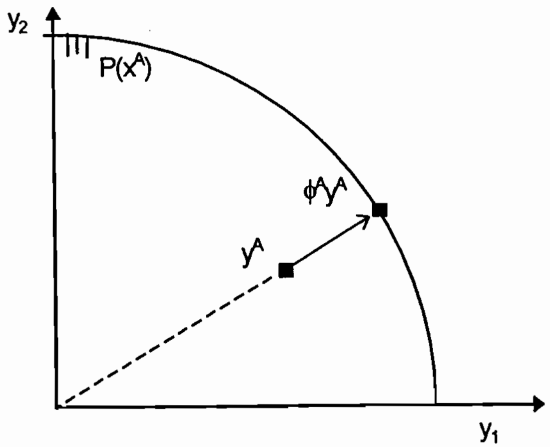

Figure 2.19 An Output-Oriented Measure of Technical Efficiency ($M = 2$)

output–input bundle to the boundary of production technology. The relationships between distance functions and radial efficiency measures are provided in

Proposition 2.3: $TE_I(y, x) = [D_I(y, x)]^{-1}$ and $TE_O(x, y) = D_O(x, y)$.

The input-oriented measure of technical efficiency $TE_I(y, x)$ is the reciprocal of the input distance function $D_I(y, x)$, and the output-oriented measure of technical efficiency $TE_O(x, y)$ coincides with the output distance function $D_O(x, y)$. Thus as we indicated in Section 2.2.3, distance functions are intimately related to the measurement of technical efficiency. This should be clear from a comparison of the properties of $TE_I(y, x)$ and $TE_O(x, y)$ given in Proposition 2.1 with the properties of $D_I(y, x)$ and $D_O(x, y)$ given in Section 2.2.3.

2.4 ECONOMIC EFFICIENCY

In Section 2.3 we introduced a pair of measures of technical efficiency, for both the single-output case and the multiple-output

case. The standards against which technical efficiency is measured are provided by the production frontier in the single-output case and by isoquants in both the single-output case and the multiple-output case. These are fairly weak standards, since no behavioral objective is imposed. If a behavioral objective of cost minimization is appropriate, the cost frontier $c(y, w)$ and its associated system of cost-minimizing input demand equations $x(y, w)$ provide a standard against which to measure cost efficiency. This provides a more exacting input-oriented standard against which to measure producer performance. Alternatively, if a behavioral objective of revenue maximization is appropriate, the revenue frontier $r(x, p)$ and its associated system of revenue-maximizing output supply equations provides a standard against which to measure revenue efficiency. This provides a more exacting output-oriented standard against which to measure producer performance. Finally, if a behavioral objective of profit maximization is appropriate, the profit frontier $\pi(p, w)$ and its associated system of profit-maximizing output supply equations $y(p, w)$ and input demand equations $x(p, w)$ provides a still more exacting standard against which to measure producer performance. We consider each of these economic standards in the next three subsections.

2.4.1 Cost Frontiers and Cost Efficiency

We assume that producers face input prices $w \in R^N_{++}$, and seek to minimize the cost $w^T x$ they incur in producing the outputs $y \in R^M_+$ they choose to produce. The standard against which their performance is evaluated shifts from the production frontier to the cost frontier. We will see that the achievement of input-oriented technical efficiency is necessary, but not sufficient, for the achievement of cost efficiency. This is because a technically efficient producer could use an inappropriate input mix, given the input prices it faces. A measure of cost efficiency is introduced in Definition 2.27 and illustrated in Figures 2.20 and 2.21.

Definition 2.27: A measure of *cost efficiency* is a function $CE(y, x, w) = c(y, w)/w^T x$.

The measure of cost efficiency is given by the ratio of minimum cost to observed cost. In Figure 2.20 the cost efficiency of a producer

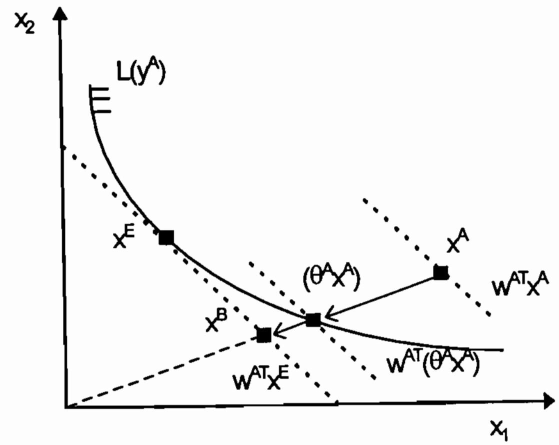

Figure 2.20 The Measurement and Decomposition of Cost Efficiency ($N = 2$)

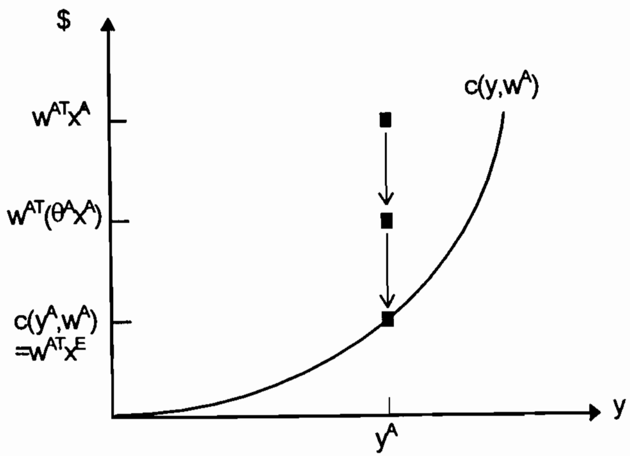

Figure 2.21 The Measurement and Decomposition of Cost Efficiency ($M = 1$)

using inputs x^A, available at prices w^A, to produce output y^A is measured using Isoq $L(y^A)$. The measure of cost efficiency is given by the ratio of minimum cost $c(y^A, w^A) = w^{AT}x^E$ to actual cost $w^{AT}x^A$. The same scenario is depicted in Figure 2.21, using $c(y, w^A)$.

For $x \in L(y)$, the measure of cost efficiency satisfies the following properties:

$CE1$: $0 < CE(y, x, w) \leq 1$, with $CE(y, x, w) = 1 \Leftrightarrow x = x(y, w)$ so that $w^T x = c(y, w)$.

$CE2$: $CE(y, \lambda x, w) = \lambda^{-1} CE(y, x, w)$ for $\lambda > 0$.

$CE3$: $CE(\lambda y, x, w) \geq CE(y, x, w)$ for $\lambda \geq 1$.

$CE4$: $CE(y, x, \lambda w) = CE(y, x, w)$ for $\lambda > 0$.

Thus the measure of cost efficiency is bounded between zero and unity, and achieves its upper bound if, and only if, a producer uses a cost-minimizing input vector. The measure is homogeneous of degree −1 in inputs (e.g., a doubling of all inputs doubles cost and halves cost efficiency), nondecreasing in outputs, and homogeneous of degree 0 in input prices (e.g., a doubling of all input prices has no effect on cost efficiency). Property $CE4$ implies that the measure of cost efficiency depends only on relative input prices.

It is apparent from Figure 2.20 that not all cost inefficiency is necessarily attributable to technical inefficiency. Using Definition 2.20, the input-oriented technical efficiency of the producer being examined is given by $TE_I(y^A, x^A) = \theta^A = w^{AT}(\theta^A x^A)/w^{AT}x^A$. Thus cost efficiency is given by the ratio of expenditure at x^E (which is equal to expenditure at x^B) to expenditure at x^A, whereas input-oriented technical efficiency is given by the ratio of expenditure at $\theta^A x^A$ to expenditure at x^A. The remaining portion of cost inefficiency is given by the ratio of expenditure at x^E to expenditure at $\theta^A x^A$, and is attributable to a misallocation of inputs in light of their relative prices. In Figure 2.20 the input vector $\theta^A x^A$ is technically efficient, but not cost efficient because $(\theta^A x_1^A/\theta^A x_2^A) > x_1^E/x_2^E = x_1(y^A, w^A)/x_2(y^A, w^A)$. The notion of input allocative efficiency is introduced in Definition 2.28 and illustrated in Figures 2.20 and 2.21.

Definition 2.28: A measure of *input allocative efficiency* is a function $AE_I(y, x, w) = CE(y, x, w)/TE_I(y, x)$.

Thus a measure of input allocative efficiency is provided by the ratio of cost efficiency to input-oriented technical efficiency. Since $CE(y^A, w^A) = c(y^A, w^A)/w^{AT}x^A$ and $TE_I(y^A, x^A) = w^{AT}(\theta^A x^A)/w^{AT}x^A$, it follows that $AE_I(y^A, x^A, w^A) = c(y^A, w^A)/w^{AT}(\theta^A x^A)$.

The properties satisfied by $AE_I(y, x, w)$ are

AE_I1: $0 < AE_I(y, x, w) \leq 1$.

AE_I2: $AE_I(y, x, w) = 1 \Leftrightarrow$ there exists a $\lambda \leq 1$ such that $\lambda x = x(y, w)$.

AE_I3: $AE_I(y, \lambda x, w) = AE_I(y, x, w)$ for $\lambda > 0$.

AE_I4: $AE_I(y, x, \lambda w) = AE_I(y, x, w)$ for $\lambda > 0$.

Thus $AE_I(y, x, w)$ is bounded between zero and unity, and attains its upper bound if, and only if, the input vector can be radially contracted to the cost-minimizing input vector. $AE_I(y, x, w)$ is also homogeneous of degree 0 in input quantities and in input prices, being dependent on the input mix and on relative input prices.

The decomposition of cost efficiency into input-oriented technical efficiency and input allocative efficiency illustrated in Figures 2.20 and 2.21 is formalized in Proposition 2.4.

Proposition 2.4: The measure of cost efficiency decomposes as $CE(y, x, w) = TE_I(y, x) \cdot AE_I(y, x, w)$.

Using Proposition 2.4, still another way of expressing the necessary and sufficient condition for cost efficiency is $CE(y, x, w) = 1 \Leftrightarrow TE_I(y, x) = AE_I(y, x, w) = 1$.

2.4.2 Revenue Frontiers and Revenue Efficiency

The structure of this section is essentially the same as the structure of the preceding section; only the orientation changes. We now assume that producers face output prices $p \in R^M_{++}$, and seek to maximize the revenue p^Ty they can generate from the input vector $x \in R^N_+$ they employ. The standard against which their performance is evaluated is provided by the revenue frontier. We will see that failure to maximize revenue can be attributed to either or both of two sources: output-oriented technical inefficiency and production of an inappropriate output mix in light of the prevailing output price vector. A measure of revenue efficiency is introduced in Definition 2.29 and illustrated in Figures 2.22 and 2.23.

Definition 2.29: A measure of *revenue efficiency* is a function $RE(x, y, p) = p^Ty/r(x, p)$.

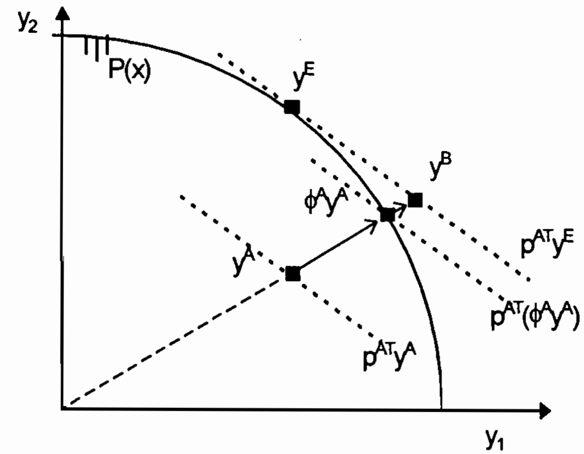

Figure 2.22 The Measurement and Decomposition of Revenue Efficiency ($M = 2$)

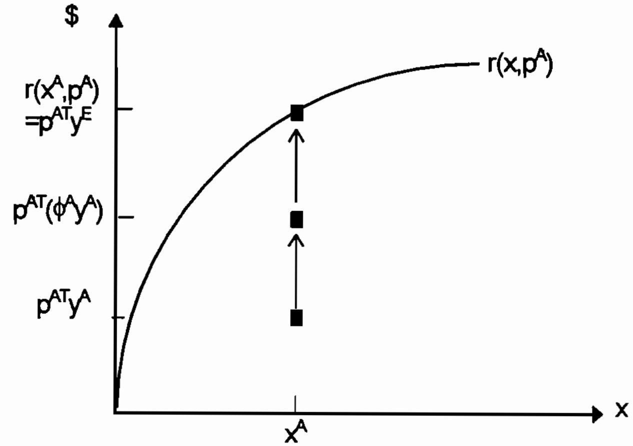

Figure 2.23 The Measurement and Decomposition of Revenue Efficiency ($N = 1$)

The measure of revenue efficiency is given by the ratio of actual revenue to maximum revenue. In Figure 2.22 the revenue efficiency of a producer using inputs x^A to produce outputs y^A for sale at prices p^A is measured using Isoq $P(x^A)$. The measure of revenue efficiency is given by the ratio of observed revenue $p^{AT}y^A$ to maximum revenue $r(x^A, p^A) = p^{AT}y^E$. The same scenario is portrayed in Figure 2.23, using $r(x, p^A)$.

For $y \in P(x)$, the measure of revenue efficiency satisfies the following properties:

$RE1$: $0 < RE(x, y, p) \leq 1$, with $RE(x, y, p) = 1 \Leftrightarrow y = y(x, p)$ so that $p^T y = r(x, p)$.

$RE2$: $RE(x, \lambda y, p) = \lambda RE(x, y, p)$ for $\lambda > 0$.

$RE3$: $RE(\lambda x, y, p) \leq RE(x, y, p)$ for $\lambda \geq 1$.

$RE4$: $RE(x, y, \lambda p) = RE(x, y, p)$ for $\lambda > 0$.

Thus the measure of revenue efficiency is bounded between zero and unity, and achieves its upper bound if, and only if, a producer produces a revenue-maximizing output vector. The measure is homogeneous of degree +1 in outputs (i.e., an equiproportionate change in all outputs generates the same proportionate change in revenue efficiency), nonincreasing in inputs, and homogeneous of degree 0 in output prices (e.g., a doubling of all output prices has no effect on revenue efficiency). Property $RE4$ implies that the measure of revenue efficiency depends only on relative output prices.

It should be clear from Figure 2.22 that not all revenue inefficiency is necessarily due to technical inefficiency. Using Definition 2.22, the output-oriented technical efficiency of the producer being evaluated is given by $TE_O(x^A, y^A) = p^{AT}y^A/p^{AT}(\phi^A y^A) = (\phi^A)^{-1}$. Revenue efficiency is given by the ratio of revenue at y^A to revenue at y^E (which is equal to revenue at y^B), whereas output-oriented technical efficiency is given by the ratio of revenue at y^A to revenue at $\phi^A y^A$. The remaining portion of revenue inefficiency is given by the ratio of revenue at $\phi^A y^A$ to revenue at y^E, and is due to a misallocation of outputs in light of their relative prices. In Figure 2.22 the output vector $\phi^A y^A$ is technically efficient, but not revenue efficient because $(\phi^A y_1^A/\phi^A y_2^A) > y_1^E/y_2^E = y_1(x^A, p^A)/y_2(x^A, p^A)$. The notion of output allocative efficiency is introduced in Definition 2.30 and illustrated in Figures 2.22 and 2.23.

Definition 2.30: A measure of *output allocative efficiency* is a function $AE_O(x, y, p) = RE(x, y, p)/TE_O(x, y)$.

Thus a measure of output allocative efficiency is provided by the ratio of revenue efficiency to output-oriented technical efficiency. Since $RE(x^A, p^A) = p^{AT}y^A/r(x^A, p^A)$ and $TE_O(x^A, y^A) = p^{AT}y^A/p^{AT}(\phi^A y^A)$, it follows that $AE_O(x^A, y^A, p^A) = p^{AT}(\phi^A y^A)/r(x^A, p^A)$.

The properties satisfied by $AE_o(x, y, p)$ are

AE_O1: $0 < AE_O(x, y, p) \leq 1$.

AE_O2: $AE_O(x, y, p) = 1 \Leftrightarrow$ there exists a $\lambda \geq 1$ such that $\lambda y = y(x, p)$.

AE_O3: $AE_O(\lambda y, x, p) = AE_O(x, y, p)$ for $\lambda > 0$.

AE_O4: $AE_O(x, y, \lambda p) = AE_O(x, y, p)$ for $\lambda > 0$.

Thus $AE_O(x, y, p)$ is bounded between zero and unity, and achieves its upper bound if, and only if, the output vector can be radially expanded to the revenue-maximizing output vector. $AE_O(x, y, p)$ is also homogeneous of degree 0 in output quantities and in output prices, being dependent on the output mix and on relative output prices.

The decomposition of revenue efficiency into output-oriented technical efficiency and output allocative efficiency illustrated in Figures 2.22 and 2.23 is formalized in Proposition 2.5.

Proposition 2.5: The measure of revenue efficiency decomposes as $RE(x, y, p) = TE_O(x, y) \cdot AE_O(x, y, p)$.

It follows from Proposition 2.5 that $RE(x, y, p) = 1 \Leftrightarrow TE_O(x, y) = AE_O(x, y, p) = 1$. Thus for a producer to achieve maximum revenue it is necessary and sufficient that the producer be technically efficient and produce the correct mix of outputs. Failure to do either results in less than maximum revenue.

2.4.3 Profit Frontiers and Profit Efficiency

We now assume that producers face output prices $p \in R_{++}^M$ and input prices $w \in R_{++}^N$, and seek to maximize the profit $(p^T y - w^T x)$ they obtain from using $x \in R_+^N$ to produce $y \in R_+^M$. The standard against

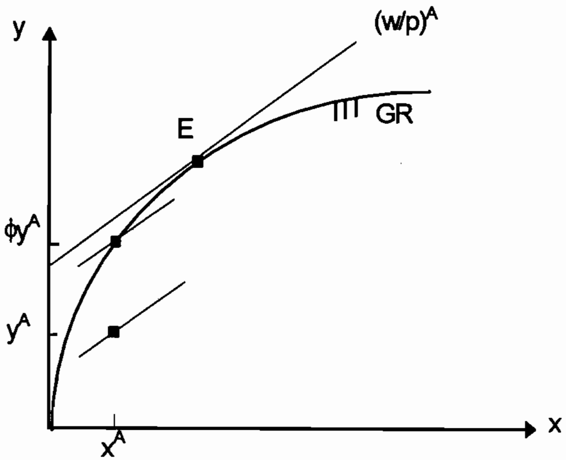

Figure 2.24 The Measurement of Profit Efficiency

which their performance is measured shifts again, from the revenue frontier to the profit frontier. The notion of profit efficiency is introduced in Definition 2.31 and illustrated in Figure 2.24. We will see that several of the previously introduced notions of efficiency are necessary for the achievement of profit efficiency, but that none by itself is sufficient. This is because profit efficiency requires (either input-oriented or output-oriented) technical efficiency and both input allocative efficiency and output allocative efficiency. Moreover, even these efficiencies are not collectively sufficient for profit efficiency, since profit efficiency also requires technical efficiency and both types of allocative efficiency to be achieved at the proper scale. Thus profit efficiency also requires a type of scale efficiency.

Definition 2.31: A measure of *profit efficiency* is a function $\pi\mathrm{E}(y, x, p, w) = (p^T y - w^T x)/\pi(p, w)$, provided $\pi(p, w) > 0$.

Thus a measure of profit efficiency is provided by the ratio of actual profit to maximum profit, and obviously $\pi E(y, x, p, w) \le 1$. In Figure 2.24 a producer facing prices (p^A, w^A) achieves $\pi E(y, x, p, w) = 1$ at output–input combination E, and $\pi E(y, x, p, w) < 1$ for all other feasible output–input combinations.

For $(y, x) \in GR$, the measure of profit efficiency satisfies the following properties:

$\pi E1$: $\pi E(y, x, p, w) \le 1$, with $\pi E(y, x, p, w) = 1 \Leftrightarrow y = y(p, w)$, $x = x(p, w)$ so that $(p^T y - w^T x) = \pi(p, w)$.

$\pi E2$: $\pi E(\lambda y, x, p, w) \ge \pi E(y, x, p, w), \lambda \ge 1$.

$\pi E3$: $\pi E(y, \lambda x, p, w) \le \pi E(y, x, p, w), \lambda \ge 1$.

$\pi E4$: $\pi E(y, x, \lambda p, \lambda w) = \pi E(y, x, p, w), \lambda > 0$.

Thus the measure of profit efficiency is bounded above by unity, and achieves its upper bound if, and only if, a producer actually adopts a profit-maximizing combination of inputs and outputs. Unlike the measures of cost efficiency and revenue efficiency, however, the measure of profit efficiency is not bounded below by zero, since negative actual profit is possible. The measure is nondecreasing in outputs and nonincreasing in inputs. (If $G4$ and $G5$ are replaced with $G6$, then these monotonicity properties are strengthened by replacing λy with $y' \ge y$ and λx with $x' \ge x$, respectively.) Finally, the measure is homogeneous of degree 0 in output prices and input prices collectively (but it is not generally homogeneous in output prices or input prices separately).

A decomposition of profit efficiency into its constituent parts is somewhat arbitrary, depending on whether an input-oriented or an output-oriented measure of technical efficiency is used. Suppose that an output-oriented measure is used, and consider the producer in Figure 2.24 facing prices (p^A, w^A) and using input vector x^A to produce output y^A. Clearly increasing output-oriented technical efficiency by increasing output radially to ϕy^A will increase profit and profit efficiency. But output allocative inefficiency can remain at ϕy^A, and input vector x^A may exhibit input allocative inefficiency, although neither can be depicted in this two-dimensional figure. Figures 2.20 and 2.22 illustrate the possibilities. Both output allocative inefficiency at ϕy^A and input allocative inefficiency at x^A reduce profit beneath $\pi(p^A, w^A)$, and so both contribute to profit inefficiency. Finally, even after accounting for technical inefficiency and both types of allocative inefficiency, scale inefficiency can remain. In Figure 2.24, $(\phi y^A, x^A)$ occurs at an inefficiently small scale to maximize profit, and so scale inefficiency constitutes the final source of profit inefficiency. A similar line of reasoning based on an input-oriented measure of technical efficiency would lead to the conclusion that technical inefficiency, both types of allocative inefficiency, and scale inefficiency contribute to profit inefficiency, although the magnitudes of each component would differ from their magnitudes when an output-oriented measure of technical efficiency is adopted.

The following decomposition of profit efficiency is based on the output-oriented measure of technical efficiency, and makes use of Definitions 2.27 and 2.28 and Propositions 2.3 and 2.4. We leave it to the reader to develop an analogous decomposition based on the input-oriented measure of technical efficiency.

Proposition 2.6: The measure of profit efficiency decomposes as

$$\begin{aligned}\pi E(y,x,p,w) = \{&TE_o(x,y)\cdot AE_o(x,y,p)\\ &\cdot[r(x,p)/p^T y(p,w)]\cdot p^T y(p,w)\\ &-[AE_I(y,x,w)]^{-1}\cdot[c(y/TE_o(x,y),w)/\\ &w^T x(p,w)]\cdot w^T x(p,w)\}/\pi(p,w).\end{aligned}$$

The first two terms in the numerator are less than or equal to unity, and act as a drag on profit-maximizing revenue $p^Ty(p, w)$. The fifth term in the numerator is greater than or equal to unity, and inflates profit-maximizing expenditure $w^Tx(p, w)$. The third and sixth terms in the numerator can be greater than, equal to, or less than unity, depending on the relationships between x and $x(p, w)$ and between $y/TE_o(x, y)$ and $y(p, w)$. Three of these terms have already been identified as measures of output-oriented technical efficiency, output allocative efficiency, and input allocative efficiency. The two new terms constitute our measure of scale efficiency. Clearly $\pi E(y, x, p, w) = [p^Ty(p, w) - w^Tx(p, w)]/\pi(p, w) = 1$ if, and only if, all five terms are unity. Thus the attainment of maximum profit requires technical efficiency, use of the right input mix in light of w, production of the right output mix in light of p, and operation at the right scale in light of (p, w).

2.4.4 Variable Cost Efficiency and Variable Profit Efficiency

In Section 2.2.5 we introduced variable cost frontiers and variable profit frontiers as standards against which to measure producer performance in the presence of fixed inputs. In such a circumstance it would be inappropriate to measure economic efficiency relative to cost and profit frontiers, because producers do not have the flexibility to adjust all inputs. In this section we show how to modify the definitions of cost efficiency and profit efficiency when some inputs are fixed.

Suppose producers use variable inputs $x \in R^N_+$, available at prices $w \in R^N_{++}$, to produce outputs $y \in R^M_+$ in the presence of fixed inputs $z \in R^Q_+$. Suppose also that producers seek to minimize the variable cost w^Tx required to produce y, given technology and (w, z). The standard against which their performance is measured is the variable cost frontier $vc(y, w, z)$. A measure of variable cost efficiency is given by

Definition 2.32: A measure of *variable cost efficiency* is a function $VCE(y, x, w, z) = vc(y, w, z)/w^Tx$.

For given z, $VCE(y, x, w, z)$ satisfies the same properties as $CE(y, x, w)$ does. In addition, since $vc(y, w, z)$ is nonincreasing in the elements of z, so is $VCE(y, x, w, z)$. Finally, for given z, $VCE(y, x, w, z)$ decomposes into a measure of variable input allocative efficiency and a measure of variable input technical efficiency, exactly as $CE(y, x, w)$ does in Proposition 2.4, and the two components satisfy the same properties as $AE_I(y, x, w)$ and $TE_I(y, x)$ do.

Suppose now that producers face output prices $p \in R^M_{++}$ and seek to maximize variable profit $(p^Ty - w^Tx)$, given technology and (p, w, z). The standard against which their performance is measured shifts to the variable profit frontier $v\pi(p, w, z)$. A measure of variable profit efficiency is given by

Definition 2.33: A measure of *variable profit efficiency* is a function $V\pi E(y, x, p, w, z) = (p^Ty - w^Tx)/v\pi(p, w, z)$, provided $v\pi(p, w, z) > 0$.

For given z, $V\pi E(y, x, p, w, z)$ satisfies the same properties as $\pi E(y, x, p, w)$ does. In addition, since $v\pi(p, w, z)$ is nondecreasing in the elements of z, $V\pi E(y, x, p, w, z)$ is nonincreasing in the elements of z. Finally, for given z, $V\pi E(y, x, p, w, z)$ can be decomposed exactly as $v\pi(p, w, z)$ is decomposed in Proposition 2.6.

2.5 A GUIDE TO THE LITERATURE

The first objective of this chapter has been to provide an introduction to the fundamentals of production economics. We make no claim to originality. Our exposition derives ultimately from the pioneering work of Shephard (1953, 1970), who introduced distance functions into the economics profession. We have relied heavily on a more recent treatment of this basic material provided by Färe (1988) and Russell (1998), who provides a comprehensive guide to the role of distance functions in producer and consumer theory. Cost, revenue, and profit frontiers are treated by Debreu (1959), Diewert (1973, 1974, 1982), and McFadden (1978), and Lau (1972, 1976, 1978) provides an extensive analysis of normalized profit frontiers. Diewert (1981a) discusses the properties of variable cost frontiers, and Gorman (1968), Diewert (1973, 1982), Lau (1972, 1976, 1978), and McFadden (1978) discuss the properties of variable profit frontiers. Duality theory is surveyed by Diewert (1982), who also provides an excellent history of thought on the subject, and more recently by Cornes (1992), who provides an exceptionally readable exposition, and by Färe and Primont (1995), who provide a more advanced exposition.

The second objective of this chapter has been to provide an introduction to the fundamentals of efficiency measurement. Here again we make no claim to originality. The measurement of technical and economic efficiency was pioneered by Farrell (1957), who first showed how to measure input-oriented technical efficiency, input allocative efficiency, and cost efficiency. Debreu (1951) introduced a measure of output-oriented technical efficiency, which he called a "coefficient of resource utilization." Detailed analyses of the measurement of all types of efficiency appear in Färe, Grosskopf, and Lovell (1985, 1994), who also provide extensive references to the theoretical and empirical literatures.