Journal of Productivity Analysis, 12, 249–267 (1999)

Incorporating the Operating Environment Into a Nonparametric Measure of Technical Efficiency

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Abstract

The ability of a production unit to transform inputs into outputs is influenced by its technical efficiency and external operating environment. This paper introduces a nonparametric, linear programming, frontier procedure for obtaining a measure of managerial efficiency that controls for exogenous features of the operating environment. The approach also provides statistical tests of the effects of external conditions on the efficient use of each individual input (for an input oriented model) or for each individual output (for an output oriented model). The procedure is illustrated for a sample of nursing homes.

Keywords: External operating environment, nonparametric linear programming, composite managerial efficiency

1. Introduction

In 1957, Farrell suggested that the technical efficiency of an individual production unit could be measured by the equiproportionate reduction in current inputs to produce predetermined levels of outputs. This can be empirically implemented in a non-parametric, non-stochastic, mathematical programming framework and in a parametric, stochastic, statistical framework. The essential exercise is to obtain an efficient comparison point for the observation to be evaluated.[1]

This literature has been extended to explore the reasons for efficiency differences across production units by relating measures of inefficiency to features of the external operating environment.[2] Examples of external variables include the form of ownership, location characteristics, labor relations, and government regulations.[3] The characteristics of the external environment could influence the ability of management to transform inputs into outputs. In this context, inefficiency has various components: managerial inefficiency, ownership inefficiency, and regulatory inefficiency.[4] The nature of inefficiency is important for designing policies to improve resource allocation.

This paper introduces an empirical technique to separate the management component of inefficiency from the other components, defined to be outside the control of management within the time frame of the analysis. The discussion is based on the non-parametric, mathematical programming framework.[5] The specific technique is Data Envelopment Analysis (DEA).[6]

The solution to the DEA problem yields the Farrell radial measure of technical efficiency plus additional non-radial input savings (slacks) and output expansions (surpluses).[7] In typical DEA studies, slacks and surpluses are neglected at worst and relegated to the background at best. Our approach utilizes the input slacks or the output surpluses (depending upon the orientation of the model) and re-calculates a measure of technical efficiency that adjusts for differences in the operating environment across production units.

The paper is organized as follows. Section 2 relates our approach to the current literature. Section 3 explains the procedure. Section 4 is a brief empirical illustration. And Section 5 is a conclusion.

2. The Literature

Previous work on the external operating environment and measures of technical efficiency can be broadly classified into three categories: the frontier separation approach, the all-in-one approach and the two-stage approach.

The frontier separation approach stratifies the data set according to a single categorical variable that characterizes the different external environments, e.g., ownership structure. Reference frontiers are calculated for each sub-sample and for the pooled data set. Units are evaluated relative to their sub-sample and the pooled frontier. The impact of the operating environment on inefficiency is determined by comparing sub-sample and pooled efficiency scores, where all inefficient units have been projected onto their respective frontiers.[8] This procedure requires *a priori* selection of the single most important feature of the operating environment and it can be implemented only for a categorical variable.

The all-in-one approach includes the external operating environment variables directly in the linear programming (LP) formulation along with the traditional inputs and outputs. Unlike the frontier separation approach, it permits including more than one feature of the operating environment, and it is not restricted to a categorical variable. The resulting efficiency measure takes differences in the external environment into account by treating them exactly like conventional inputs or outputs. However, this approach requires that the external variable be classified as an input or an output prior to the analysis. An additional input enables production of more outputs, holding efficiency constant. An additional output requires the use of more inputs, holding efficiency constant. If the operating environment enters the LP formulation as an input, this implies that more outputs could be produced, suggesting that the operating environment is favorable. If the operating environment enters the LP formulation as an output, this implies that more inputs are required, suggesting that the operating environment is unfavorable. This prior input or output classification is unsuitable if the objective is to test whether a particular operating environment is favorable or unfavorable. In addition, the radial score is based on the assumption that all the inputs are shrinkable (if input-oriented), or all the outputs are expandable (if output-oriented). This assumption may make little sense for an external feature of the operating environment.

A variant of this approach is to classify variables in the LP problem as controllable and uncontrollable. Controllable variables are adjustable by management; uncontrollable variables are fixed and semi-fixed. The uncontrollable variables influence the position of the frontier, but they are held constant in the calculation of the radial efficiency measure, since they are of each uncontrollable variable as an input or an output, which precludes testing the relationship assumed to be unshrinkable or unexpandable.[9] Although this is appealing, it still requires a prior classification between the external variables and efficiency in terms of both sign and significance.[10]

The two-stage approach includes the traditional inputs and outputs in the LP formulation used to compute radial technical efficiency, which is then used as the dependent variable in a second stage regression, where the explanatory variables measure the external environment.[11] Some studies use ordinary least squares (OLS) to estimate the second stage equation,[12] others use a tobit model. An advantage of the two-stage approach is that the influence of the external variables on the production process can be tested in terms of both sign and significance. However, a disadvantage is that the second stage regression ignores the information contained in the slacks and surpluses. This may bias the parameter estimates and give misleading conclusions regarding the impact of each external variable on efficiency.[13] The two-stage procedure does not provide a separate measure of managerial efficiency.

Two variations of the two-stage approach are the work by Fried, Lovell and Vanden Eeckaut (1993) and McCarty and Yaisawarng (1993). The former includes both radial and non-radial slacks and surpluses as dependent variables in a seemingly unrelated regression (SUR) system in the second stage, instead of a single equation. Their focus is to explain the distribution of efficiency scores by features of the operating environment. McCarty and Yaisawarng (1993) estimate a single equation tobit model in the second stage and interpret the residuals from the second stage as a separate measure of managerial efficiency. These managerial indicators (the residuals), however, are no longer bounded between zero and one and therefore do not provide a measure of potential improvement.

This paper introduces a *four-stage procedure*. The management component of inefficiency is separated from the influences of the external operating environment. The result is a radial measure of managerial efficiency. This approach has the following four advantages. One, the end result is a radial measure of managerial efficiency with the conventional interpretation. Two, it is not necessary to classify the external variables into input and output categories prior to the analysis. Three, the influence of the external variables on the efficient use of each input (for an input-oriented model) or the efficient production of each output (for an output-oriented model) can be tested. And four, information on slacks or surpluses generated by the initial model is used in the calculations.

Our procedure can be viewed as an extension of the two-stage and frontier separation approaches, where the essential contribution is to generate a pure measure of managerial inefficiency. For example, suppose the problem is to measure the inefficiency of a sample of public and private hospitals, some of which are teaching hospitals. Ownership structure and teaching function may constrain the ability of management to transform inputs into outputs and should be accounted for in a measurement of managerial efficiency. The frontier separation approach cannot be applied since there is more than one category that characterizes the operating environment. The two-stage approach can identify the more efficient operating environment on average, but it cannot identify the level of inefficiency attributable to management, either overall, or at the level of the individual firm. It would be useful to isolate the managerial component of inefficiency for individual firms, in order to compare the performance of managers across units that operate under different external regimes. This problem also applies to firms operating in different regions, under different regulatory regimes, and under different labor relations.

3. The Four Stage Procedure

3.1. An Overall Summary

Our procedure for incorporating the operating environment into a measure of technical efficiency to obtain a separate measure of managerial inefficiency has four stages. The first stage is to calculate a DEA frontier using the traditional inputs and outputs according to standard production theory. The external variables are excluded. Farrell radial technical efficiency scores as well as input slacks and output surpluses are computed for each observation. In the second stage, a system of equations is specified in which the dependent variable for each equation is the sum of radial and non-radial input slack for an input-oriented model or radial plus non-radial output surplus for an output-oriented model. The independent variables measure features of the external operating environment and are not restricted to be identical across equations.[14] This equation system identifies the variation in total by-variable measures of inefficiency attributable to factors outside the control of management. The third stage is to use the parameter estimates from the second stage to predict the total input slack or output surplus, depending upon model orientation. These predicted values represent the "allowable" slack or surplus, due to the operating environment, and are used to calculate adjusted values for the primary inputs or outputs. The fourth stage is to re-run the DEA model under the initial-output specification, using the adjusted data set. The new radial efficiency measures incorporate the influences of the external variables on the production process, and isolate the managerial component of inefficiency.

3.2. The Formal Procedure[15]

The *first stage* begins with a specification of production technology. Following Afriat (1972), the piecewise linear input requirement set under variable returns to scale is defined as:

$$L(y) = \{x : Yz \geq y, Xz \leq x, Iz = 1, z \in R_{+}^{K}\}, \tag{1}$$

where y is an $(M \times 1)$ vector of M outputs, x is an $(N \times 1)$ vector of N inputs used to produce output y, Y is an $(M \times K)$ matrix of outputs, X is an $(N \times K)$ matrix of inputs, z is a $(K \times 1)$ vector of activities or weights, I is a $(1 \times K)$ vector of ones, K is the number of producing units, M is the number of outputs, N is the number of inputs. Given output vector y, all input vectors that are feasible for producing output vector y are in the input requirement set. All convex combinations of input vectors which are less than or equal to the input bundle x and are feasible to produce at least output vector y establish the isoquant or reference frontier for output y and is the basis for calculating Farrell technical efficiency. For further details, see Färe, Grosskopf and Lovell (1985).

Given the piecewise linear input requirement set in (1), the DEA model used to compute Farrell technical efficiency in the *first stage*, for unit k, $k = 1, \ldots, K$, is formulated as the following linear programming (LP) problem.

$$\begin{aligned} TE^k &= \min_{z,\lambda} \lambda \\ \text{s.t.}\ \ Yz &\geq y^k \\ Xz &\leq \lambda x^k \\ Iz &= 1 \\ z &\in R_+^K, \end{aligned} \tag{2}$$

where y^k and x^k are output and input vectors for unit k, respectively, λ is a scalar value representing a proportional contraction of all inputs, holding input ratios and output level constant. The minimum value of λ that satisfies all constraints is the Farrell radial technical efficiency measure.

TE is a measure of efficiency under the restriction that a linear combination of efficient units produces the same *or more* of all outputs and that the reduction in inputs is *equiproportionate*. The first condition establishes a best-practice frontier. The second condition is the result of the input-oriented radial efficiency measure. The radial measure is appealing because it is a composite efficiency index that provides a definitive evaluation of units in the sample and is easy to interpret. However, the radial measure is only a partial measure of efficiency; it neglects output surpluses and input slacks. In particular, it could be that were an inefficient unit to produce efficiently it could reduce *all* N inputs by the fraction $(1 - TE)$ and $N - 1$ inputs by additional amounts, and also expand its outputs.[16]

To illustrate the concept of radial and non-radial input slack, consider Figure 1. Suppose that there are four units: A, B, C and D. Each uses two inputs: x_1 and x_2 to produce the same quantity of output y. AB is an efficient frontier which is created by a linear combination of input vectors A and B (i.e., Xz in eq. (2)). It represents the trade off between inputs x_1 and x_2 that is feasible to produce output y. The vertical extension AA' is the result of free disposability of input x_2 (inequality on the constraint of x_2); holding input x_1 constant at x_1^A, any amount of input x_2 which is at least as large as x_2^A is feasible. There is no trade off between x_1 and x_2 along this range. Similarly, the horizontal extension BB' reflects free disposability of input x_1, holding input x_2 constant at x_2^B. $A'ABB'$ is the isoquant used as the reference frontier to measure Farrell radial technical efficiency.

Units A and B are technically efficient, C and D are not. The radial technical efficiencies for C and D (TE^C and TE^D) are OC^*/OC and OD^*/OD, respectively. $TE^C < 1$ indicates that unit C could use the fraction TE^C of its current level of x_1 and x_2 to produce output y were it to operate efficiently. The amount $(1 - TE^C)x^C$ is *radial input slack*, which is the same proportion for all inputs by definition. Once unit C proportionally reduces its

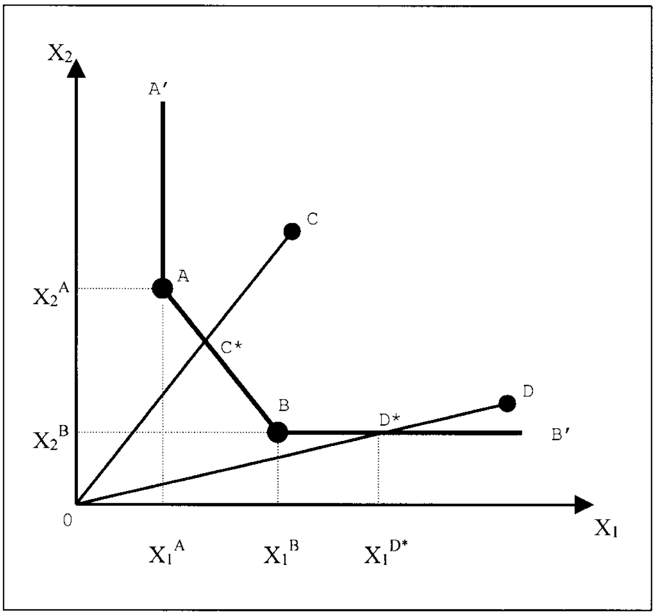

Figure 1. Illustration of radial and non-radial input slack

inputs to C^*, no further reduction is possible without sacrificing output. Unit D, on the other hand, can further reduce input x_1 from $x_1^{D^*}$ to x_1^B after proportionally reducing its current inputs by $(1 - TE^D)$ to D^* to become radially efficient. The potential additional reduction in input x_1 equal to $x_1^B x_1^{D^*}$ is referred to as *non-radial slack* in input x_1. The amount of non-radial slack varies across inputs.

The radial measure computed in the *first stage* provides an evaluation of the performance of a productive unit relative to best practice, predicated upon the inputs and the outputs included in the model, and ignoring the possible additional inefficiencies implied by non-radial slacks and surpluses. There are other variables, however, which influence the ability of a manager to transform the inputs into outputs, but which are outside the manager's control. We refer to these variables as the external environment. This could be ownership structure, location, or regulatory regime, for example. Some firms operate under favorable external conditions; other firms operate under unfavorable external conditions. Unfavorable external conditions mean that additional inputs are required to produce the same level of output in order to overcome the external disadvantage. Since external conditions are not constant across production units, the radial efficiency score generated by the initial model (under)overstates the efficiency of producers operating under (un)favorable conditions.

The *second stage* is to estimate the N input equations using an appropriate econometric technique. The dependent variables are radial plus non-radial input slack; the independent variables are measures of external conditions applicable to the particular input. The objective is to quantify the effect of external conditions on the excessive use of inputs. The N equations are specified as:

$$ITS_j^k = f_j(Q_j^k, \beta_j, u_j^k), \qquad j = 1, \dots, N \qquad k = 1, \dots, K \tag{3}$$

where ITS_j^k is unit k's total radial plus non-radial slack for input j based on the DEA results from stage 1, Q_j^k is a vector of variables characterizing the operating environment for unit k that may affect the utilization of input j, β_j is a vector of coefficients and u_j^k is a disturbance term.[17] These equations explain the variation in total by-variable measures of inefficiency.[18] Note that the explanatory variables characterizing the operating environment in (3) are not restricted to be the same across equations, need not have a linear relationship with the dependent variables and can be a mixture of continuous and categorical variables.[19]

The third stage is to use the estimated coefficients from the regression to predict total input slack for each input and for each unit based on its external variables:

$$I\hat{T}S_j^k = f_j(Q_j^k, \hat{\beta}_j), \qquad j = 1, \dots, N \qquad k = 1, \dots, K. \tag{4}$$

These predictions are used to adjust the primary input data for each unit according to the difference between maximum predicted slack and predicted slack:

$$x_j^{k\,\mathrm{adj}} = x_j^k + [\mathrm{Max}^k\{I\hat{T}S_j^k\} - I\hat{T}S_j^k] \qquad j = 1, \dots, N \qquad k = 1, \dots, K. \tag{5}$$

This creates a new pseudo data set where the inputs are adjusted for the influence of external conditions.[20]

The final stage is to use the adjusted data set to re-run the DEA model under the initial input-output specification and generate new radial measures of inefficiency. These radial scores measure the inefficiency that is attributable to management.

3.3. Some Questions and Some Answers

Why do we omit output surplus in the second-stage regression for an input oriented model? An input oriented model takes output as given and measures inefficiency by the potential reduction in inputs. Output surplus exists in empirical applications because the data set is sparse for some output vectors. Where it does exist, it is likely to be composed mostly of zeros and have insufficient variation to be useful in the estimation.

Why do we adjust the primary input data by the difference between maximum predicted slack and predicted slack? The purpose of maximum predicted slack is to establish a base equal to the least favorable set of external conditions. A firm with external variables corresponding to this base level would not have its input vector adjusted at all. A firm with external variables generating a lower level of predicted slack would have its input vector adjusted to put it on the same basis as the firm with the least favorable external environment.

We choose to use the least favorable operating environment as the base for practical and technical reasons.[21] The practical reason is to provide a performance target that managers can attain regardless of their operating environment. For example, the manager of a firm that operates under favorable circumstances should be able to reduce the inputs required to produce the given level of output by at least the amount indicated by their efficiency score. A manager of a firm that operates under the least favorable circumstances should also be able to reduce inputs required to produce the given level of output by the amount indicated by the efficiency score. Managers cannot use the operating environment as an excuse for failing to achieve the performance target. However, if we had chosen to use the most favorable operating environment as a base, then the performance targets would not necessarily be attainable by managers of firms that operate under less favorable circumstances.

There is also a technical advantage to choosing the least favorable environment as the base. In this case, the data are adjusted by increasing the input levels for firms in more favorable circumstances. If the most favorable environment is used as the base, the data are adjusted by reducing the input levels for firms in less favorable circumstances. In empirical applications, this introduces the possibility of a negative value for an adjusted input, rendering the DEA problem for that unit without a solution.[22]

Why does the adjustment take the form of an increase in the original input vector? Predicted slack below the maximum predicted slack is attributable to external conditions more favorable than the least favorable conditions prevailing in the sample for that input. The purpose of the adjustment is to penalize the firm for the fewer inputs required to operate under favorable external conditions. By increasing the input vector and leaving the output vector unchanged, the firm's performance is purged of the external advantage. This makes it possible to isolate managerial inefficiency by re-running the DEA model on the adjusted data set.

Are the estimated values of the coefficients in the regression of any interest besides generating predicted slack? These coefficients provide information on the effect of external conditions on inefficiency for each input. It could be, for example, that public ownership is associated with inefficient use of labor and efficient use of capital. This procedure permits the signs and statistical significance of the coefficients on the external variables to differ across inputs.

Is it possible to use this procedure to answer a question like the overall effect on inefficiency of ownership type? Consider the previous example of hospitals in which the external environment is described by public/private ownership and teaching/non-teaching. The question is addressed by constructing a new pseudo data set from the second stage regression results, where the coefficient on the ownership term is omitted in the construction of predicted slacks.[23] DEA efficiency scores are calculated and grouped by ownership type. A difference of means test between the two groups reveals the average effect of ownership on managerial efficiency.[24]

Can this procedure be used where external conditions are characterized by both categorical and continuous variables? A particular strength of this procedure is its flexibility. Multiple measures of external conditions are readily incorporated as additional independent variables in the regression equations.

What is the interpretation of input slack and output surplus in the DEA model based upon the adjusted data? These slacks and surpluses have the same interpretation as in any conventional DEA study. The radial score is an incomplete measure of the technical efficiency of an individual unit since it fails to include the slacks and surpluses. The performance of an individual unit, controlling for differences in the external environment, is fully characterized by the radial score, input slacks and output surpluses.

What does the efficiency score of a unit measure after controlling for the external environment? The procedure controls for the external environment by leveling the playing field for all firms. For an input oriented model, this level playing field is the least favorable operating environment (maximum predicted slack). The new efficiency score should be interpreted with care. It represents the reduction in inputs possible if the firm operated in the worst environment and performed up to best practice. Firms operating in more favorable environments should be able to decrease inputs further. The adjusted efficiency score is a measure of minimum potential improvement. The adjusted scores also provide a performance ranking of managers.

4. An Empirical Example

4.1. Data and Variables

The four-stage procedure is illustrated for a sample of hospital-affiliated nursing homes[25] in 1993. The data set is from the American Hospital Association. These nursing homes provide skilled nursing care and/or immediate care over the entire year and have an accounting unit that is distinct from the hospital. Nursing homes that provide residential care are excluded as well as those with missing or inconsistent data. Nursing homes in the sample fall into the following overlapping categories: for-profit, not-for-profit, attached to a hospital, freestanding units located on a hospital campus, and freestanding units located off a hospital campus.

The first stage DEA model includes physical inputs and outputs in the strict production theory sense. There are two outputs: inpatient days of skilled care (SKD) and inpatient days of intermediate care (ICD). Four inputs are used to produce nursing services: registered nurses (RN), licensed practical nurses (LPN), other personnel (OEMP), and non-payroll expenses (NEXP). Labor inputs are measured in number of full time equivalent persons. Non-payroll expenses are measured in dollars.

There are only four for-profit-on-campus homes and two for-profit-off-campus homes. These two categories are omitted from the analysis due to insufficient observations. Potential outliers are identified and removed from the sample based upon a boxplot of output-input ratios.[26] The final sample consists of 990 nursing homes in the United States. Table 1 displays descriptive statistics of the raw data.

Table 1. Descriptive statistics of hospital-affiliated nursing homes, 1993 (sample size = 990).

Variables	Mean	Standard Error	Minimum	Maximum
Outputs:				
SKD	13162.1	15356.7	0	166517
ICD	2876.2	8184.9	0	64970
Inputs:				
RN	6.3	5.5	0.5	43
LPN	7.4	6.1	0.5	47
OEMP	30.3	31.2	1.0	271
NEXP	912513.9	1046032.1	4774.0	8395873

Ownership form and location describe the external environment. There are two forms of ownership and three forms of location. We construct four dummy variables: (1) for-profit and attached to a hospital (FA), (2) not-for-profit and attached to a hospital (NFA), (3) not-for-profit and located on campus (NFON), and (4) not-for-profit and located off campus (NFOFF). Table 2 presents the frequency distribution of nursing homes in the sample by ownership-location type.

4.2. Results

Stage One: Initial DEA

The DEA model includes two outputs and four inputs. Efficiency scores for all nursing homes relative to a best practice frontier are computed using an input orientation and variable returns to scale technology. The initial results show a large variation in efficiency scores across nursing homes. The average efficiency score is 0.52. On average, a nursing home could deliver the same level of services with 52 percent of the current inputs. Or, it could reduce current inputs by 48 percent, were it to perform according to best practice. However, potential input savings are much higher if non-radial input slacks are included. Average total

Table 2. Frequency distribution of nursing homes by ownership-location (sample size = 990).

		Location		
		Attached	On Campus	Off Campus
Ownership	For-Profit	87 (FA)		
	Not-For-Profit	836 (NFA)	27 (NFON)	40 (NFOFF)

radial plus non-radial input slack ranges from 53.8 percent (OEMP) to 57.1 percent (RN). Part of these potential input savings may not be achievable if some nursing homes operate under unfavorable conditions that prevent them from fully utilizing their available resources.

The initial DEA model does not provide a good measure of managerial performance. It may penalize 'good' performers who operate in an unfavorable external environment and reward 'poor' performers who operate in a favorable external environment.

Stage Two: Quantifying the Effect of the Operating Environment

There are four regression equations, one for each input. The dependent variables are the total radial plus non-radial input slacks. The independent variables are dummies for ownership-location combinations and bed-size categories. The purpose of the bed-size dummies is to allow for scale effects in managing the external environment.[27] There are four bed-size categories: smallest (less than 30 beds), next to smallest (30 to 59 beds), next to largest (60–89 beds), and largest (at least 90 beds). The NFA dummy and the smallest bed-size category are omitted from the regressions and serve as the reference case. Single equation tobits[28] are estimated since the independent variables are the same across the four input slack equations. The parameter estimates and standard errors are summarized in Table 3.

A (positive) negative coefficient on an ownership-location dummy suggests that the environment is (un)favorable compared to not-for-profit attached (the omitted dummy), since it is associated with (greater) less excess use of the input. The same interpretation applies to the bed-size dummies, where the base case is the smallest bed-size category.

For-profit attached (FA) has a significant negative coefficient in one out of four input equations and insignificant coefficients on the other three input equations. This suggests that it is a favorable operating environment. This may be the result of the cost discipline imposed by for-profit status combined with possible scope economies associated with being physically attached to a hospital. The nursing facility and the hospital can share specialized equipment, medical personnel and coordinate schedules.

Not-for-profit on campus (NFON) has significant positive coefficients in three out of four input slack equations and not-for-profit off campus (NFOFF) in one out of four input equations. The other coefficients are insignificant. This suggests that not-for-profit on campus and not-for-profit off campus are unfavorable operating environments compared to not-for-profit attached. The reason may be that physical separation from the hospital requires the nursing facility to maintain some specialized functions that could otherwise be shared with the hospital.

The bed-size dummies have significantly positive (9 cases) or insignificant (3 cases) coefficients in all four input equations. This suggests that the smallest bed-size category is the most favorable environment in terms of size.

Stage Three: Data Adjustment

The parameter estimates presented in Table 3 are used to adjust the initial data set according to equation (5). Table 4 summarizes predicted slacks and maximum predicted slacks for

Table 3. Tobit regression results (sample size = 990).

	Dependent Variable			
Independent Variable	RN	LPN	OEMP	NEXP
Constant	2.926** (0.246)	2.991** (0.248)	9.481** (1.088)	353513.0** (38701.0)
For-profit-attached (FA)	−0.575 (0.524)	−0.464 (0.528)	−4.622** (2.320)	−50021.5 (82494.8)
Not-for-profit-on-campus (NFON)	0.692 (0.895)	1.750* (0.898)	12.406** (3.938)	585049.4** (140209.2)
Not-for-profit-off-campus (NFOFF)	0.508 (0.749)	1.091 (0.753)	12.644** (3.230)	166993.4 (117684.4)
Next to smallest (BED2)	0.047 (0.347)	0.625* (0.349)	2.805* (1.532)	287.7 (54536.8)
Next to largest (BED3)	0.341 (0.469)	1.378** (0.471)	11.056** (2.068)	168698.7** (73683.3)
Largest (BED4)	2.716** (0.466)	3.238** (0.469)	21.006** (2.058)	452634.5** (73398.2)
σ	4.407** (0.105)	4.435** (0.106)	19.473** (0.466)	692875.5** (16542.6)
Log likelihood function	−2711.7	−2719.6	−4057.7	−13525.7

Notes: Dependent variables are total radial plus non-radial slacks. Standard errors of the parameter estimates are in parenthesis. The '**' and '*' indicate that the parameter estimate is significantly different from zero at the 5 percent and 10 percent level, respectively.

all inputs by ownership-location-size.[29] The adjusted data controls for the influence of the external operating environment; namely, ownership structure and location.

The maximum predicted slack (least favorable external environment) is not-for-profit on campus for three out of four input slack equations. This holds for all bed-size categories. By and large, not-for-profit off campus has the second highest predicted slack. For-profit attached has the lowest predicted slack (most favorable environment) in all four input slack equations for all bed-size categories. These results are consistent with the discussion of the parameter estimates above.

Stage Four: Re-compute Radial Efficiency Measures

The final stage is to re-run the initial DEA model using the adjusted data. This produces a composite efficiency index (the radial score) which incorporates the effects of the external environment.

Table 5 presents descriptive statistics of efficiency scores from the first and the fourth stages. As a result of controlling for the external environment, the average efficiency score increased, the number of efficient nursing homes decreased, and the standard deviation of

Table 4. Predicted slacks and maximum predicted slacks by ownership-location-size (sample size =990).

Size	Ownership-Location	#	Predicted Slack $\hat{E}(ITS_j^k \mid Q_j^k)$ for RN	LPN	OEMP	NEXP
Smallest	For profit – Attached	77	2.35	2.53	4.86	303491.5
	Not for profit – Attached	314	2.93	2.99	9.48	353513.0
	Not for profit – Off campus	2	3.43	4.08	22.13	520506.4
	Not for profit – On campus	2	3.62	4.74	21.89	938562.4
Maximum predicted slack [max$^k\{I\hat{T}S_j^k\}$]		395	**3.62**	**4.74**	**22.13**	**938562.4**
Next to smallest	For profit – Attached	6	2.40	3.15	7.66	303779.2
	Not for profit – Attached	304	2.97	3.62	12.29	353800.7
	Not for profit – Off campus	7	3.48	4.71	24.93	520794.1
	Not for profit – On campus	5	3.67	5.37	24.69	938850.1
Maximum predicted slack max$^k\{I\hat{T}S_j^k\}$]		322	**3.67**	**5.37**	**24.93**	**938850.1**
Next to largest	For profit – Attached	1	2.69	3.91	15.92	472190.2
	Not for profit – Attached	113	3.27	4.37	20.54	522211.7
	Not for profit – Off campus	7	3.78	5.46	33.18	689205.1
	Not for profit – On campus	5	3.96	6.12	32.94	1107261.1
Maximum predicted slack [max$^k\{I\hat{T}S_j^k\}$]		126	**3.96**	**6.12**	**33.18**	**1107261.1**
Largest	For profit – Attached	3	5.07	5.77	25.87	756126.0
	Not for profit – Attached	105	5.64	6.23	30.49	806147.5
	Not for profit – Off campus	24	6.15	7.32	43.13	973140.9
	Not for profit – On campus	15	6.33	7.98	42.89	1391196.9
Maximum predicted slack [max$^k\{I\hat{T}S_j^k\}$]		147	**6.33**	**7.98**	**43.13**	**1391196.9**

Table 5. Comparison of stage 1 and stage 4 results (sample size = 990).

	Stage 1	Stage 4
Average efficiency scores	0.522	0.682
Standard Deviation	0.247	0.179
Minimum	0.054	0.135
Maximum	1.000	1.000
Number of efficient nursing homes	85	57

the efficiency scores decreased. The increase in average efficiency suggests that without controlling for the operating environment, the penalty to nursing homes operating under unfavorable circumstances was greater than the benefit to nursing homes operating under favorable circumstances. The decrease in the number of efficient nursing homes suggests that without controlling for the operating environment, nursing homes that operate in favorable circumstances are judged efficient as a result of being compared to nursing homes that operate in unfavorable circumstances. The decrease in the standard deviation of the efficiency scores may reflect the fact that without controlling for the external environment, the efficiency scores of nursing homes that operate in favorable circumstances are biased up, and the efficiency scores of nursing homes that operate in unfavorable circumstances are biased down. By adjusting the data so as to put all nursing homes in the same operating environment, this bias to the efficiency scores is removed, and the spread is narrowed.

The Kendall rank correlation coefficient[30] between stage one and stage four efficiency scores is 0.67; the 95 percent confidence interval is 0.60 to 0.73. Adjusting for the external operating environment does make a difference in terms of efficiency rankings. Table 6 illustrates this result. For-profit attached is the most favorable operating environment. The average rank of nursing homes in this category is lower in stage 1 compared to stage 4 in all bed-size categories. Not-for-profit on campus is the least favorable operating environment. The average rank of nursing homes in this category is higher in stage 1 compared to stage 4 for three out of four bed-size categories. This confirms the importance of controlling for the external operating environment.

5. Conclusion

In evaluating performance, it is often useful to obtain measures of managerial inefficiency for firms operating under different external conditions. Standard approaches either incorporate measures of the external environment directly in the DEA model, or a single equation second stage regression is run with the radial score as the dependent variable.

This paper demonstrates an alternative four-stage procedure. The first stage is to obtain traditional DEA results based upon conventional inputs and outputs and excluding

Table 6. Average rank by ownership-location-size for stage 1 and stage 4.

Size	Ownership-Location	#	Average Rank	
			Stage 1	Stage 4
Smallest	For profit-Attached	77	553	611
	Not for profit-Attached	314	545	433
	Not for profit-Off campus	2	735	9
	Not for profit-On campus	2	847	277
Next to smallest	For profit-Attached	6	551	590
	Not for profit-Attached	304	476	500
	Not for profit-Off campus	7	546	386
	Not for profit-On campus	5	470	192
Next to largest	For profit-Attached	1	227	697
	Not for profit-Attached	113	447	560
	Not for profit-Off campus	7	567	557
	Not for profit-On campus	5	691	629
Largest	For profit-Attached	3	232	462
	Not for profit-Attached	105	390	515
	Not for profit-Off campus	24	487	535
	Not for profit-On campus	15	542	546

the external variables. The second stage is to specify a system of equations with total input slack (for an input oriented model) as the dependent variables and the appropriate external variables on the right hand side. The third stage is to use the results of the econometric estimation to calculate maximum predicted total slack and adjust the original data. The fourth stage is to re-run DEA on the new data set and generate adjusted radial efficiency scores that remove the influences of the external variables on inefficiency.

The four-stage approach has some intriguing advantages. It uses the information contained in the slacks or surpluses of the original model. It does not require imposing a sign on the effect of an external variable on inefficiency. It provides tests of significance of each external variable on inefficiency in each individual input (for an input-oriented model) or output dimension (for an output-oriented model). It can provide an overall effect on inefficiency for a categorical variable. Finally, it produces a single summary measure of firm inefficiency that isolates managerial performance.

Acknowledgments

We are grateful for exceptionally thoughtful comments from three anonymous referees. They forced us to think hard about the issues and the paper is much better as a result. We also benefited from comments received from participants at the 1994 Georgia Productivity Workshop and 1995 seminars at the University of Sydney and the University of Melbourne. Bob Bartels, Tim Coelli, Joe Hirschberg and Jesus Pastor made many useful suggestions. Thank you everyone. We take responsibility for any remaining errors.

Notes

1. For a survey of the techniques, see Lovell (1993).
2. See Timmer (1971) for an early example.
3. For example, Holmstrom and Tirole (1992) suggest that private firms may operate under different discipline mechanisms than public firms. Pencavel and Craig (1994) compare conventional firms and worker cooperatives and find empirical evidence of differences in resource allocation. For extensive reviews of the effects of economic regulation and industrial structure on firm behavior see Joskow and Rose (1989) and Schmalensee (1989). For an application to the divestiture of the Bell system, see Krouse et al. (1999).
4. As a referee suggested, managerial inefficiency may be classified as internal inefficiency while ownership inefficiency and regulatory inefficiencies may be classified as exogeneous inefficiencies. The latter is external to the firm but internal to society. There is another type of inefficiency caused by factors that are external to society such as topographic and climatic differences. These factors are beyond the scope of this paper.
5. See Charnes, Cooper and Rhodes (1978) for a formulation of linear programming problems in the operation research literature. For a review of a consistent model formulation based on an axiomatic approach to production theory, see Färe, Grosskopf and Lovell (1985). For a treatment of external variables and measuring efficiency in the context of a stochastic frontier, see Simar, Lovell and Vanden Eeckaut (1994).
6. However the approach is applicable without modification to the free disposal hull (FDH) alternative. The FDH technique was introduced by Deprins, Simar and Tulkens (1984).
7. The discussion is in the context of static efficiency studies. In a dynamic context, it is always possible that the inefficient units in the static model are positioning themselves for the future.
8. See Banker, Kaufman and Morey (1990), Byrnes (1985), Charnes, Cooper and Rhodes (1981), Grosskopf and Valdmanis (1987) and Fizel and Nunnikhoven (1992).
9. Depending on the variable classification criteria, this can be viewed as a short-run approach in which the efficiency score is a sub-vector efficiency measure.
10. See Banker and Morey (1986), McCarty and Yaisawarng (1993).
11. For an example, see Ali and Flinn (1989).
12. The OLS technique does not recognize the censoring of efficiency scores.
13. For further discussion, see Lovell (1993). McCarty and Yaisawarng (1993) compare the all-in-one approach and the two-stage approach when an uncontrollable variable in the all-in-one approach is treated as an independent variable in the second stage equation of the two-stage approach. They find that both approaches give similar efficiency rankings under the condition that there is no strong correlation between variables in the first stage and the independent variable in the second stage.
14. An appropriate econometric technique should be used to estimate the set of equations. Depending upon the data, this could be OLS, SUR or tobit regression.
15. The discussion is in the context of an input-oriented model.
16. Total radial plus nonradial slack and surplus could also be used to rank the performance of a unit. However, these $M + N$ separate input and output efficiency measures potentially provide ambiguous indicators of performance.

17. The specification is analogous for an output-oriented model. Total input slacks are replaced by total output surpluses.
18. Fried, Lovell and Vanden Eeckaut (1993) introduced the idea of using an SUR system to explain radial plus non-radial input slack and non-radial output surplus.
19. If total slack for each input is influenced by different measures of external conditions, the N equations should be estimated as a system. Since the total slack for each input is also censored at zero, a system of tobit regressions is appropriate. If the total slack for each input is influenced by the same measures of external conditions, each equation can be estimated separately using a tobit.

    The magnitude of radial plus non-radial slack may depend upon unit size as well as the external environment. If this is the case, one solution is to express the dependent variables in the second stage as percentages. Alternatively, units can be grouped into size categories and maximum slack calculated for each category. The data is then adjusted for each unit according to the difference between its predicted slack and the maximum for its size category.
20. We are grateful to an anonymous referee for suggesting that we adjust the inputs by comparing predicted slack to maximum predicted slack for the particular input. See section 3.3 below for a discussion.
21. There are other alternatives, such as the average operating environment, or the most favorable operating environment. Since the choice is arbitrary, the absolute magnitude of the managerial efficiency measures should be interpreted with care. For example, where the base is the least favorable operating environment, average managerial efficiency for the sample measures the average performance of management if all firms operated under the least favorable external conditions. Clearly, the average level of efficiency for the sample would be different if the most favorable set of external conditions had been chosen as the base. However, comparisons of managerial efficiency across firms, or groups of firms, are valid regardless of the base selected.
22. For an output-oriented model, the data could be adjusted to represent all firms in terms of the least favorable environment by following an analogous procedure. The least favorable environment corresponds to the maximum predicted surplus for an output. Firm operating under more favorable circumstances would have their outputs reduced by the difference between the maximum predicted surplus and their predicted surplus. An advantage of this procedure is that the output targets would be achievable for managers of all firms regardless of their external environment. A disadvantage of this procedure is that the adjusted data could include negative outputs, rendering the DEA problem infeasible. The latter problem is avoided by choosing the most favorable environment as the base, but then the output targets are not necessarily realistic for firms that operate under less favorable circumstances.
23. This assumes that there is more than one measure of external conditions. If ownership is the only measure of external conditions, then the procedure collapses to comparing the average efficiency by ownership type based upon the original DEA model.
24. An alternative is to apply the frontier separation model to the pseudo data set.
25. For additional treatments of nursing homes, see Fizel and Nunnikoven (1992), Kooreman (1992), Nyman and Bricker (1989) and Vitaliano and Toren (1994).
26. All possible output-input ratios are computed and a boxplot is constructed for each ratio. Fifty percent of the sample lies within the box that is centered at its median. The median ± 3.5IR represents an extended boxplot, where IR is the inter-quartile range, or the difference between the value at the third quartile and the value at the first quartile. Any unit outside the extended boxplot is considered an outlier. This procedure is similar using the mean ± 3SD, where SD is the sample standard deviation. The advantage of centering the plot at the median rather than the mean is that the median is not influenced by outliers.
27. Thanks to the referees for urging us to account for possible scale effects.
28. See Greene (1990) p. 727 for a discussion of the tobit specification.
29. The small number of observations in some of the bed-size categories is not a concern. The coefficients on the bed-size dummies are estimated based upon the total number of nursing homes in the size category, regardless of their external circumstances. For example, although there are only two not-for-profit-off-campus units in the smallest category, the coefficient on the smallest bed-size is estimated based upon 395 observations.
30. The calculation of the rank correlation is heavily influenced by the procedure for handling ties since both models include a number of nursing homes with efficiency scores equal to one. Where possible, ties between efficient homes are broken by calculating the number of times an efficient home appears in the reference set for inefficient homes. Remaining ties receive an average rank. Ranks potentially range from one (best) to 990 (worst).

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