6 The Shadow Price Approach to the Estimation and Decomposition of Economic Efficiency

6.1 INTRODUCTION

In Chapter 3 technical inefficiency was modeled as an error component within a stochastic frontier framework. In Chapter 4 and parts of Chapter 5 cost inefficiency and profit inefficiency were also modeled as error components within a stochastic frontier framework. Thus the basic strategy in previous chapters has been to construct a composed error stochastic frontier model, and to extract estimates of inefficiency from the parameters describing the structure of the two error components. This procedure is straightforward in single-equation models, in which the sole objective is to *estimate* technical or economic inefficiency. However this procedure is much less straightforward in simultaneous-equation models, in which the dual objectives are to estimate *and decompose* economic inefficiency.

In this chapter we change our strategy. We do not estimate stochastic frontiers. Instead, we model all types of inefficiency parametrically, through the introduction of additional parameters to be estimated, rather than through an error component. The error structure of the estimating equation (or system of equations) is conventional, the same as that employed in the estimation of cost, revenue, or profit *functions* in the nonfrontier literature. Thus the error term is distributed normally in a single-equation model, and the error terms are distributed as multivariate normal in a system of equations model.

Technical inefficiency is introduced in two alternative ways. Output-oriented technical inefficiency is introduced by allowing the production function intercept to vary across producers; this generates a cost function in which producers' output vectors are scaled differentially, and a profit function in which producers' output price vectors are scaled differentially. Input-oriented technical inefficiency is introduced by scaling producers' input vectors differentially; this generates a cost function whose intercept varies across producers, and a profit function in which producers' input price vectors are scaled differentially. The input orientation is appropriate in a cost minimization framework, in which the objective of producers is to allocate inputs in such a way as to minimize the cost of producing their chosen output vector. Either orientation is appropriate in a profit maximization framework, in which the objective of producers is to allocate inputs and outputs so as to maximize profit. Allocative inefficiency (an inappropriate input mix, an inappropriate output mix, or an inappropriate scale) is introduced by allowing producers to fail to optimize with respect to observed prices, but by assuming that they do optimize with respect to shadow prices, which are parametrically related to observed prices. The usual approach is to model shadow prices by *scaling* observed prices, although it is also possible to model shadow prices by *translating* observed prices. Hypothesis tests concerning magnitudes and directions of various types of inefficiency, and calculations of the costs of various types of inefficiency, are then based on estimated values of these additional parameters.

Inspiration for the shadow price approach goes back at least to Hopper (1965), who reported the results of his 1954 study of the efficiency of resource allocation and crop mix in Indian agriculture. In his study he estimated the parameters of four-input Cobb–Douglas production functions for each of four crops, and compared the estimated value of each input's marginal product across crops. He found the farmers in his sample to be highly efficient in economizing on their scarce resources. Like Tax (1953) before him, Hopper found subsistence farmers to be "poor but efficient," because the penalty for inefficiency was so great. Although Hopper did not introduce inefficiency parameters explicitly into his analysis, his statistical tests of various allocative efficiency hypotheses amounted to essentially the same thing. The vast shadow price literature that has emerged in recent years, beginning with Lau and Yotopoulos (1971), can be viewed as a formalization and extension of Hopper's ideas.

This chapter is organized as follows. In Section 6.2 we consider cross-sectional models. In a cross-sectional context it is not possible to estimate a technical efficiency parameter for every producer, and so the emphasis is on the estimation of allocative efficiency, there being fewer inputs and outputs to be allocated than producers to allocate them. In Section 6.2.1 we consider models of cost efficiency, and in Section 6.2.2 we consider models of profit efficiency. In each section we consider a wider variety of functional forms than we did in previous chapters. We consider the Cobb–Douglas form for its relative simplicity, and we also consider flexible forms because the shadow price approach to the estimation of allocative efficiency is ideally suited to flexible forms. In Section 6.3 we consider panel data models. The obvious advantage of having panel data is the ability to obtain estimates of more parameters than is possible with cross-sectional data. In this case the additional parameters provide information on variation in technical and allocative efficiency across producers. In Section 6.3.1 we consider models of cost efficiency, and in Section 6.3.2 we consider models of profit efficiency. We concentrate on flexible forms in the panel data models. In Sections 6.2.2 and 6.3.2 we focus on the modeling and estimation of profit efficiency rather than variable profit efficiency, as we did in Section 5.2. It is straightforward to convert our profit efficiency models to variable profit efficiency models, and we leave this exercise to the reader. Section 6.4 concludes with a guide to the relevant literature.

6.2 CROSS-SECTIONAL MODELS

Suppose that cross-sectional data on the quantities of N inputs used to produce a single-output are available for each of I producers. The single-output assumption will be relaxed shortly. Then the production relationship can be expressed as

$$y_i \le f(x_i;\beta)\cdot\exp\{v_i\}, \qquad (6.2.1)$$

where y_i is the scalar output of producer i, $i = 1, \ldots, I$, x_i is a vector of N inputs used by producer i, $f(x_i;\beta)$ is the deterministic kernel of

the stochastic production frontier $[f(x_i;\beta)\cdot\exp\{v_i\}]$, β is a vector of technology parameters to be estimated, and v_i is a symmetrically distributed error term with zero mean and constant variance, which captures the effect of random noise on production. The inequality (6.2.1) states that each producer's output cannot exceed maximum possible output, as determined by the stochastic production frontier $[f(x_i;\beta)\cdot\exp\{v_i\}]$.

In the approach adopted in Chapter 3, the inequality (6.2.1) was converted to an equality by introducing an additional error term of the form $\exp\{-u_i\} \le 1$, with $u_i \ge 0$, which was intended to capture the impact of (output-oriented) technical inefficiency on production. Depending on the assumptions imposed on u_i, estimation was based on LSDV, GLS, or MLE techniques. The strategy adopted in this chapter is to convert the inequality (6.2.1) to an equality in a different way. If we specify technical inefficiency as being output oriented, then the inequality (6.2.1) can be converted to the equality

$$y_i = \phi_i f(x_i;\beta)\cdot\exp\{v_i\}, \qquad (6.2.2)$$

where the $0 < \phi_i \le 1$ are producer-specific scalars that provide output-oriented measures of the technical efficiency of each producer. If we specify technical inefficiency as being input oriented, then the inequality (6.2.1) can be converted to the equality

$$y_i = f(\phi_i x_i;\beta)\cdot\exp\{v_i\}, \qquad (6.2.3)$$

where the $0 < \phi_i \le 1$ are producer-specific scalars that provide input-oriented measures of the technical efficiency of each producer. The difference between these two specifications and the error component specification is that the ϕ_i are producer-specific parameters, whereas $\exp\{-u_i\}$ is a random draw from a probability distribution, and the only parameters in $\exp\{-u_i\}$ are those associated with the probability distribution of u_i.

Specifications (6.2.2) and (6.2.3) are equivalent if, and only if, technology satisfies constant returns to scale, since then, and only then, does $f(\phi_i x_i;\beta) = \phi_i f(x_i;\beta)$. Specification (6.2.2) is preferred if the analysis is based on the estimation of a production function, whereas specification (6.2.3) is preferred if the analysis is based on the estimation of a cost function. Either specification is appropriate if the analysis is based on the estimation of a profit function.

In the multiple-output case the production function in equation (6.2.2) would be replaced with an output distance function, which is dual to a revenue function, for which the output-oriented measure of technical efficiency is appropriate. In this event equation (6.2.2) would become

$$\phi_i^{-1} D_o(x_i, y_i;\beta) = D_o\left(x_i, \frac{y_i}{\phi_i};\beta\right) = \exp\{v_i\}, \tag{6.2.4}$$

since the output distance function is homogeneous of degree +1 in outputs. In the multiple-output case the production function in equation (6.2.3) would be replaced with an input distance function, which is dual to a cost function, for which the input-oriented measure of technical efficiency is appropriate. In this event equation (6.2.3) would become

$$D_I(y_i, \phi_i x_i;\beta) = \phi_i D_I(y_i, x_i;\beta) = \exp\{-v_i\}, \tag{6.2.5}$$

since the input distance function is homogeneous of degree +1 in inputs.

There is an obvious problem with equations (6.2.2)–(6.2.5): With I cross-sectional observations it is not possible to obtain estimates of the technology parameter vector β, the variance σ_v^2 of the random-noise error term, and I producer-specific technical inefficiency parameters ϕ_i. The most that can be accomplished is to estimate β and σ_v^2, and to estimate ϕ_i for a limited number of groups of producers (say, by type of ownership), and then to test hypotheses concerning the group technical inefficiency parameters. Thus if the objective of the exercise is to obtain producer-specific estimates of technical efficiency, and only cross-sectional data are available, the stochastic frontier approach outlined in Chapter 3 is preferable to the parametric approach developed in this chapter. By modeling technical inefficiency as an error component, as in Chapter 3, it is possible to obtain estimates of technical efficiency for every producer in a cross section, whereas by modeling technical efficiency parametrically, as in this chapter, it is not possible to do so. The approach developed in this chapter does have some advantages in the estimation of allocative efficiency, however, and we now turn to the estimation and decomposition of cost inefficiency.

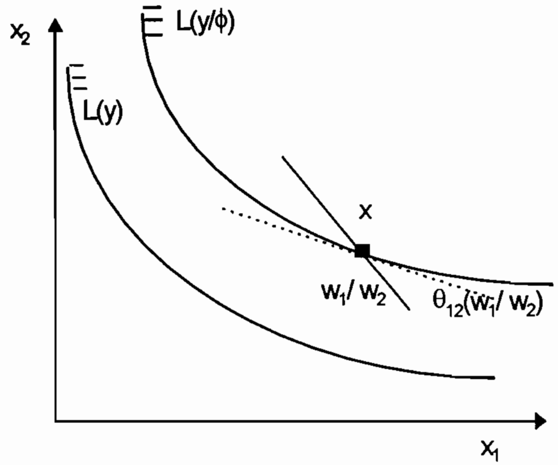

Figure 6.1 The Shadow Price Approach to Estimating Cost Efficiency: Output Orientation

6.2.1 Estimating and Decomposing Cost Inefficiency

In Chapter 4 we saw that estimating cost inefficiency was easy, but decomposing estimated cost inefficiency into its technical and allocative components was not. Decomposition was impossible in a single-equation framework, and while decomposition is theoretically possible in a system of equations framework, it proved to be difficult econometrically. Perhaps the shadow price approach will be more productive.

Suppose that we observe a producer using input vector x, available at input price vector w, to produce scalar output y, with technology characterized by the production function $f(x;\beta)$. The producer is assumed to seek to minimize the cost of producing its chosen rate of output. The problem is illustrated in Figure 6.1, which adopts an output orientation to the measurement of technical efficiency, and in Figure 6.2, which adopts an input orientation to the measurement of technical efficiency. The producer in question is both technically and allocatively inefficient.

In Figure 6.1 an output-oriented measure of the technical efficiency of the producer is provided by $\phi < 1$, since $y/\phi = f(x;\beta)$. The producer is also allocatively inefficient, since the marginal rate of substitution at $x \in \text{Isoq}\, L(y/\phi;\beta)$ diverges from the input price ratio (w_1/w_2).

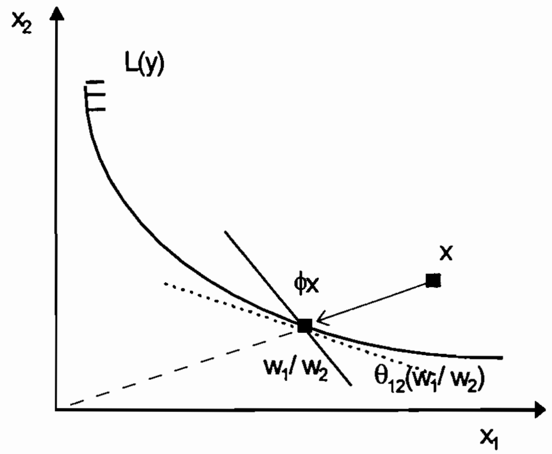

Figure 6.2 The Shadow Price Approach to Estimating Cost Efficiency: Input Orientation

However the producer is allocatively efficient relative to the shadow input price ratio $[\theta_{12}(w_1/w_2)]$, with $\theta_{12} < 1$ because the producer is overutilizing x_1 relative to x_2. Thus, although the producer's input usage is both technically and allocatively inefficient for the observed variables $(y, w_1/w_2)$, it is both technically and allocatively efficient for the shadow variables $[y/\phi, \theta_{12}(w_1/w_2)]$. In this two-input model, the parameters to be estimated are the technology parameters β, the technical ineffeciency parameter ϕ, and the allocative inefficiency parameter θ_{12}. Once the parameters have been estimated, the next step is to calculate the costs of technical and allocative inefficiency.

In Figure 6.2 an input-oriented measure of the technical efficiency of the producer is provided by $\phi < 1$, since $y = f(\phi x; \beta)$. The producer is also allocatively inefficient, since the marginal rate of substitution at $\phi x \in \text{Isoq}\, L(y;\beta)$ diverges from the input price ratio (w_1/w_2). However the producer is allocatively efficient relative to the shadow input price ratio $[\theta_{12}(w_1/w_2)]$, again with $\theta_{12} < 1$ because the producer is overutilizing x_1 relative to x_2. Thus, although the producer's input usage is both technically and allocatively inefficient for the observed variables $(y, w_1/w_2)$, its contracted input vector ϕx is both technically and allocatively efficient for observed y and the shadow

price ratio $[\theta_{12}(w_1/w_2)]$. In this case the parameters to be estimated are $(\beta, \phi, \theta_{12})$.

It should be noted that unless production technology satisfies constant returns to scale, $[\phi, \theta_{12}]$ in the output-oriented framework illustrated in Figure 6.1 will differ from $[\phi, \theta_{12}]$ in the input-oriented framework illustrated in Figure 6.2. This is merely a reflection of the fact, first noted in Chapter 2, that efficiency measures can be sensitive to orientation. Regardless of orientation, however, our strategy becomes one of estimating the additional parameters ϕ and θ_{12}, and then testing the hypotheses, separately or jointly, that $\phi = 1$ and $\theta_{12} = 1$.

The basic idea underlying the output-oriented approach presented in equation (6.2.2) and illustrated in Figure 6.1 can be generalized as follows. The shadow cost function is given by

$$c(y^*, w^*; \beta) = \min_x \{w^{*T} x : f(x;\beta) = y^*\}, \tag{6.2.6}$$

where $y^* = y/\phi$ with $0 < \phi \le 1$, and $w^* = (\theta_1 w_1, \ldots, \theta_N w_N)$ with $\theta_n > 0$, $n = 1, \ldots, N$. Since not all N scalars θ_n can be identified, we arbitrarily specify the first input as numeraire and redefine $w^* = [w_1, (\theta_2/\theta_1)w_2, \ldots, (\theta_N/\theta_1)w_N] = (w_1, \theta_{21}w_2, \ldots, \theta_{N1}w_N)$. Since inputs can be under- or overutilized relative to x_1, $\theta_{n1} \gtreqless 1$, $n = 2, \ldots, N$. Application of Shephard's lemma to $c(y^*, w^*; \beta)$ generates the associated shadow input demand equations and shadow input cost share equations

$$x_n(y^*, w^*; \beta) = \frac{\partial c(y^*, w^*; \beta)}{\partial w_n^*}, \qquad n = 1, \ldots, N, \tag{6.2.7}$$

and

$$S_n(y^*, w^*; \beta) = \frac{w_n^* x_n(y^*, w^*; \beta)}{c(y^*, w^*; \beta)}, \qquad n = 1, \ldots, N, \tag{6.2.8}$$

respectively. Although the producer's actual input usage does not minimize cost, it does minimize shadow cost. Consequently the producer's observed input demands coincide with the shadow input demand equations, and so from equations (6.2.7) and (6.2.8) we have

$$x_n = \frac{c(y^*, w^*; \beta) \cdot S_n(y^*, w^*; \beta)}{w_n^*}, \qquad n = 1, \ldots, N. \tag{6.2.9}$$

The producer's observed expenditure and input cost shares are then obtained from equations (6.2.7)–(6.2.9) as

$$E=\sum_n w_n x_n = c(y^*,w^*;\beta)\cdot\sum_n\left[S_n(y^*,w^*;\beta)\cdot(\theta_{n1})^{-1}\right] \tag{6.2.10}$$

and

$$S_n=\frac{w_n x_n}{E}=\frac{S_n(y^*,w^*;\beta)\cdot(\theta_{n1})^{-1}}{\Sigma_k\left[S_k(y^*,w^*;\beta)\cdot(\theta_{k1})^{-1}\right]},\qquad n=1,\dots,N, \tag{6.2.11}$$

respectively. Equations (6.2.9)–(6.2.11) show that a producer's observed input usage, observed expenditure, and observed input cost shares can be expressed in terms of the corresponding shadow functions. This is important because while (E, y, x_n, w_n, and S_n) are observed, it is the shadow functions that are estimated. All that is required is to assign a functional form to $c(y^*, w^*; \beta)$ and replace y^* and w^* with the observed variables and additional parameters y/ϕ, w_1, and $\theta_{n1}w_n$, $n = 2, \dots, N$.

The basic idea underlying the input-oriented approach presented in equation (6.2.3) and illustrated in Figure 6.2 can be generalized in a similar manner. In this case the shadow cost function becomes

$$c\left(y,\frac{w^*}{\phi};\beta\right)=\min_{\phi x}\left\{\left(\frac{w^*}{\phi}\right)^T(\phi x)\colon f(\phi x;\beta)=y\right\}=\frac{1}{\phi}c(y,w^*;\beta), \tag{6.2.12}$$

where w^* is defined previously and the second equality is a consequence of the homogeneity of degree +1 of $c(y, w^*; \beta)$ in w^*. Since observed input usage minimizes shadow cost,

$$x_n=\frac{1}{\phi}\frac{c(y,w^*;\beta)\cdot S_n(y,w^*;\beta)}{w_n^*},\qquad n=1,\dots,N, \tag{6.2.13}$$

$$E=\sum_n w_n x_n=\frac{1}{\phi}c(y,w^*;\beta)\cdot\sum_n\left[S_n(y,w^*;\beta)\cdot(\theta_{n1})^{-1}\right], \tag{6.2.14}$$

$$S_n=\frac{S_n(y,w^*;\beta)\cdot(\theta_{n1})^{-1}}{\Sigma_k\left[S_k(y,w^*;\beta)\cdot(\theta_{k1})^{-1}\right]},\qquad n=1,\dots,N, \tag{6.2.15}$$

respectively. Equations (6.2.13)–(6.2.15) show that a producer's observed input usage, observed expenditure, and observed input cost shares can be expressed in terms of the corresponding shadow functions when technical efficiency measurement is input oriented. Since observed output y appears in these equations, all that is required is to assign a functional form to $c(y, w^*; \beta)$ and replace w^* with w_1 and $\theta_{n1}w_n$, $n = 2, \dots, N$.

The only difference between the output-oriented system of equations (6.2.9)–(6.2.11) and the input-oriented system of equations (6.2.13)–(6.2.15) is that the technical inefficiency parameter ϕ appears in simpler form in the latter system, disappearing altogether from the share equations (6.2.15). In a single-output context, if constant returns to scale is assumed, it makes no difference which approach is adopted. In all other contexts the input-oriented approach is simpler. Moreover, in a cost-minimizing framework output(s) is (are) exogenous and inputs are endogenous, making it more appropriate to use the input-oriented measure. (However if the economic orientation were toward revenue maximization rather than cost minimization, the opposite would be true. And in the profit maximization context to be considered in Section 6.2.2, the only determining factor is the relationship between the number of inputs and the number of outputs.)

Both the output-oriented approach given in equations (6.2.6)–(6.2.11) and the input-oriented approach given in equations (6.2.12)–(6.2.15) are based on a shadow input price vector satisfying the normalization $w^* = (w_1, \theta_{21}w_2, \dots, \theta_{N1}w_N)$, with the first input being arbitrarily designated as the numeraire. As a result of this normalization, the expressions for observed expenditure and observed input cost shares given in equations (6.2.10)–(6.2.11) and (6.2.14)–(6.2.15) are highly nonlinear. Nonetheless this normalization has been employed in virtually all empirical studies in which allocative inefficiency has been modeled in a nonparametric fashion. Recently Balk (1997) has proposed, and Balk and van Leeuwen (1997) have estimated, a system based on a different normalization. They require w^* to satisfy $w^{*T}x = w^Tx$. As a result of this normalization the expressions for observed expenditure and observed input cost shares simplify considerably, since $\Sigma_k[S_k(y^*, w^*; \beta)\cdot(\theta_{k1})^{-1}] = 1$ in equations (6.2.10)–(6.2.11) and $\Sigma_k[S_k(y, w^*; \beta)\cdot(\theta_{k1})^{-1}] = 1$ in equations (6.2.14)–(6.2.15). However Maietta (1997) has shown that when the cost function is assigned a translog functional form and the normalization $w^{*T}x = w^T x$ is used, neither parameter estimates nor calculated elasticities are invariant to which input cost share equation is deleted. Since no such problem occurs with the normalization $w^* = (w_1, \theta_{21}w_2, \ldots, \theta_{N1}w_N)$, we employ it in the remainder of this chapter.

If cross-sectional data on (y, x) are available, it is possible to estimate equation (6.2.2) to obtain estimates of output-oriented technical efficiency for a limited number of groups of producers. This is not much information, much less than was obtained in Chapter 3, where technical inefficiency was modeled as an error component. However if cross-sectional data on (y, x, w) are available, it is possible to estimate some combination of equations (6.2.9)–(6.2.11) to obtain estimates of output-oriented technical efficiency and input allocative efficiency. However as we noted previously, it is simpler to estimate some combination of equations (6.2.13)–(6.2.15) to obtain estimates of input-oriented technical efficiency and input allocative efficiency. In either case, with cross-sectional data on I producers it is not possible to obtain I estimates of technical efficiency and $(N-1) \times I$ estimates of input allocative efficiency. It is possible to obtain estimates of a limited number of intergroup technical efficiency differentials, and estimates of $(N-1)$ systematic allocative efficiencies that vary across input pairs but are the same for all producers. The estimation of such systematic allocative efficiencies is fairly common in the literature. However if I is large relative to N, it is also possible to obtain estimates of a limited number of intergroup allocative efficiency differentials. This is much less common in the literature, and we delay consideration of the estimation of allocative efficiency differentials to Section 6.3.

We now provide three examples of how the shadow price approach to efficiency estimation works. The Cobb–Douglas example has the pedagogical virtue of simplicity, whereas the two subsequent examples share the practical virtue of flexibility, which reduces the likelihood that input allocative inefficiency will be confounded with a misspecification of the underlying functional form. For the time being we suppress producer subscripts and error terms.

Cobb–Douglas If the production function $f(x;\beta)$ takes the Cobb–Douglas form, estimation of input-oriented technical efficiency and systematic input allocative efficiency is relatively straightforward. At least three approaches are available. In the first approach a system of equations consisting of the production function and $(N-1)$ first-order conditions for shadow cost minimization is written in deterministic form as

$$\ln y = [\beta_o + \Delta D] + \sum_n \beta_n \ln x_n \tag{6.2.16}$$

and

$$\ln x_n - \ln x_1 + \ln w_n - \ln w_1 - \ln\left(\frac{\beta_n}{\beta_1}\right) + \ln \theta_{n1} = 0,$$
$$n = 2, \ldots, N, \tag{6.2.17}$$

respectively, where D is a dummy variable set to unity if a producer belongs to a particular group and set to zero otherwise. (If more than two groups are specified, more than one dummy variable is required.) Thus the input-oriented technical efficiency of producers in that group relative to that of all other producers is $\exp\{\Delta\} \gtreqless 1$ according as $\Delta \gtreqless 0$, where $\Delta = r \ln \phi$ and $r = \Sigma_n \beta_n$. The $(N-1)$ parameters θ_{n1} measure the divergences of the shadow price ratios $[\theta_{n1}(w_n/w_1)]$ from the observed price ratios (w_n/w_1), and because they are common to all producers they provide estimates of the systematic over- or underutilization in the sample of input x_n relative to the numeraire input x_1.

If a random-error vector is appended to the system of N equations (6.2.16) and (6.2.17), this system can be estimated using MLE under the assumption that the random-error vector is distributed as multivariate normal. An alternative would be to use ITSUR, which does not require the normality assumption. Furthermore the ITSUR estimators, when converged, are equivalent to the ML estimators. The output of the exercise consists of estimates of the technology parameters $(\beta_o, \beta_1, \ldots, \beta_N)$, an estimate of the technical efficiency differential Δ, and estimates of the systematic input allocative inefficiencies θ_{n1}, $n = 2, \ldots, N$. Likelihood ratio tests can be performed to test hypotheses concerning any subset of the parameters $(\Delta, \theta_{21}, \ldots, \theta_{N1})$.

The natural logarithm of total expenditure in the presence of input-oriented technical inefficiency and systematic input allocative inefficiency can be derived from the system of equations (6.2.16) and (6.2.17) as

$$\ln E = \left[B + \frac{1}{r}\ln y + \frac{1}{r}\sum_n \beta_n \ln w_n \right] - \frac{1}{r}(\Delta D) + (A - \ln r), \tag{6.2.18}$$

where

$$B = \ln r - \frac{\beta_o}{r} - \frac{1}{r}\sum_n \beta_n \ln \beta_n,$$

$$A = \sum_{n>1} \frac{\beta_n}{r} \ln \theta_{n1} + \ln\left[\beta_1 + \sum_{n>1} \frac{\beta_n}{\theta_{n1}} \right].$$

The first term on the right-hand side of equation (6.2.18) is the Cobb–Douglas cost function. The second term is the cost of the technical efficiency differential; this term is zero if, and only if, $\Delta = 0$. The third term is the cost of systematic input allocative inefficiency; this term is nonnegative and attains its minimum value if, and only if, $\theta_{n1} = 1, n = 2, \ldots, N$. Since the third term is a function of the technology and inefficiency parameters, but not of the data, the cost of systematic input allocative inefficiency is also systematic. As we shall see, however, this is a consequence of using the Cobb–Douglas functional form.

A second approach is to solve the system of equations (6.2.16) and (6.2.17) for the system of shadow cost-minimizing input demand equations. These are given in logarithmic form by

$$\ln x_1 = b_o - \frac{1}{r}\Delta D + \ln \beta_1 + \frac{1}{r}\ln y + \frac{1}{r}\sum_n \beta_n \ln \frac{w_n}{w_1} + \frac{1}{r}\sum_{n>1} \beta_n \ln \theta_{n1},$$

$$\ln x_2 = b_o - \frac{1}{r}\Delta D + \ln \beta_2 + \frac{1}{r}\ln y + \frac{1}{r}\sum_n \beta_n \ln \frac{w_n}{w_2} + \frac{1}{r}\sum_{n>1} \beta_n \ln \frac{\theta_{n1}}{\theta_{21}},$$

$$\vdots$$

$$\ln x_N = b_o - \frac{1}{r}\Delta D + \ln \beta_N + \frac{1}{r}\ln y + \frac{1}{r}\sum_n \beta_n \ln \frac{w_n}{w_N} + \frac{1}{r}\sum_{n>1} \beta_n \ln \frac{\theta_{n1}}{\theta_{N1}}, \tag{6.2.19}$$

where

$$b_o = -\frac{1}{r}\left[\beta_o + \sum_n \beta_n \ln \beta_n \right],$$

$$r = \sum_n \beta_n,$$

$$\Delta = r \ln \phi.$$

The asymmetry of the system of equations (6.2.19) is due to the specification of one input, in this case x_1, as the numeraire. If a random-error vector is appended to the equations in (6.2.19), this system also can be estimated by MLE, under the same distributional assumptions as are maintained in the estimation of the system (6.2.16) and (6.2.17). The system can also be estimated using ITSUR, which does not require a normality assumption on the error vector. The system of equations (6.2.19) generates the same information, and provides the basis for the same hypothesis tests, as does the system (6.2.16) and (6.2.17). The cost implications of the technical efficiency differential and systematic input allocative inefficiency can be inferred from equation (6.2.18).

The third and most straightforward approach is to base the analysis directly on a Cobb–Douglas cost function. The cost function and its associated cost-minimizing input demand equations can be written in deterministic form as

$$c(y, w; \beta) = \gamma_o y^{1/r} \prod_n w_n^{\gamma_n} \tag{6.2.20}$$

and

$$x_1(y, w; \beta) = \gamma_o \gamma_1 y^{1/r} \prod_{n>1} \left(\frac{w_n}{w_1} \right)^{\gamma_n},$$

$$x_2(y, w; \beta) = \gamma_o \gamma_2 y^{1/r} \left(\frac{w_2}{w_1} \right)^{-1} \prod_{n>1} \left(\frac{w_n}{w_1} \right)^{\gamma_n},$$

$$\vdots$$

$$x_N(y, w; \beta) = \gamma_o \gamma_N y^{1/r} \left(\frac{w_N}{w_1} \right)^{-1} \prod_{n>1} \left(\frac{w_n}{w_1} \right)^{\gamma_n}, \tag{6.2.21}$$

respectively, where homogeneity of degree +1 in w of $c(y, w; \beta)$ requires that $\Sigma_n \gamma_n = 1$, and $r \gtreqless 1$ indicates the degree of homogeneity of the underlying production function. The cost function (6.2.20) is posited as a starting point; it is not derived from the production function (6.2.16) and the first-order conditions (6.2.17). Consequently the parameters in equations (6.2.20) and (6.2.21) are not the same as those appearing in equations (6.2.16)–(6.2.19).

An input-oriented technical efficiency differential can be incorporated into equations (6.2.20) and (6.2.21) by replacing γ_o with $\gamma_o \cdot \exp\{-\Delta D\}$. If, in addition, producers are allocatively efficient with respect to shadow input price ratios $[\theta_{n1}(w_n/w_1)]$, $n > 1$, the input demand equations become, in logarithmic form,

$$
\begin{aligned}
\ln x_1 &= \ln\gamma_o - \Delta D + \ln\gamma_1 + \frac{1}{r}\ln y + \sum_{n>1}\gamma_n \ln\left[\theta_{n1}\left(\frac{w_n}{w_1}\right)\right],\\
\ln x_2 &= \ln\gamma_o - \Delta D + \ln\gamma_2 + \frac{1}{r}\ln y\\
&\quad + \sum_{n>1}\gamma_n \ln\left[\theta_{n1}\left(\frac{w_n}{w_1}\right)\right] - \ln\left[\theta_{21}\left(\frac{w_2}{w_1}\right)\right],\\
&\ \ \vdots\\
\ln x_N &= \ln\gamma_o - \Delta D + \ln\gamma_N + \frac{1}{r}\ln y\\
&\quad + \sum_{n>1}\gamma_n \ln\left[\theta_{n1}\left(\frac{w_n}{w_1}\right)\right] - \ln\left[\theta_{N1}\left(\frac{w_N}{w_1}\right)\right],
\end{aligned}
\tag{6.2.22}
$$

respectively. Total expenditure becomes

$$
\ln E = \ln c(y,w;\beta) - \Delta D + \left\{\sum_{n>1}\gamma_n \ln\theta_{n1} + \ln\left[\gamma_1 + \sum_{n>1}\left(\frac{\gamma_n}{\theta_{n1}}\right)\right]\right\},
\tag{6.2.23}
$$

where $c(y, w; \beta)$ is defined in equation (6.2.20). Thus the natural logarithm of total expenditure equals the natural logarithm of minimum cost, plus or minus the percentage cost of the technical efficiency differential, plus the percentage cost of systematic input allocative inefficiency. The percentage cost of the technical efficiency differential is given by $-\Delta D$. The percentage cost of systematic input allocative inefficiency is given by the nonnegative term in braces, which attains its minimum value of zero if, and only if, $\theta_{n1} = 1, n = 2, \ldots, N$. Once again, in a Cobb–Douglas framework the cost of systematic allocative inefficiency is also systematic. After appending classical error terms to the system of input demand equations (6.2.22), the system can be estimated by ITSUR, with cross-equation parameter restrictions imposed, after which the estimated values of Δ and the θ_{n1} can be inserted into equation (6.2.23) to provide estimates of the technical efficiency cost differential and the cost of systematic input allocative inefficiency.

Generalized Leontief This partially flexible functional form for a cost function was introduced by Diewert (1971), and perhaps first used to model systematic input allocative inefficiency by Toda (1976, 1977). The generalized Leontief minimum cost function can be written as

$$
\begin{aligned}
c(y,w;\beta) &= y \cdot \sum_n \sum_k \beta_{nk} w_n^{1/2} w_k^{1/2}\\
&= y \cdot \left[\sum_n \beta_{nn} w_n + 2\sum_{k>n}\sum_n \beta_{kn} w_n^{1/2} w_k^{1/2}\right],
\end{aligned}
\tag{6.2.24}
$$

where $\beta_{nk} = \beta_{kn} \geq 0\ \forall n, k$. $c(y, w; \beta)$ is homogeneous of degree +1 in w, and if at least one $\beta_{nk} > 0$, $c(y, w; \beta)$ is concave in w. An input-oriented technical efficiency differential can be introduced by replacing the input price vector w with $w \cdot \exp\{-\Delta D\}$. Since $c(y, w; \beta)$ is homogeneous of degree +1 in w, this implies that y can be replaced with $y \cdot \exp\{-\Delta D\}$. Note that the underlying technology is assumed to satisfy constant returns to scale, which is why we describe $c(y, w; \beta)$ as being partially flexible.

Application of Shephard's lemma to equation (6.2.24) generates a system of cost-minimizing input demand equations of the form

$$
x_n(y,w;\beta) = y \cdot \exp\{-\Delta D\} \cdot \left[\beta_{nn} + \sum_{k\neq n}\beta_{kn}\left(\frac{w_k}{w_n}\right)^{1/2}\right], \qquad n = 1, \ldots, N.
\tag{6.2.25}
$$

Systematic input allocative inefficiency is introduced by allowing observed input demands to be optimal for shadow price ratios rather than for observed price ratios. The resulting shadow input demand equations can be written as

$$x_n = y \cdot \exp\{-\Delta D\} \cdot \left\{\beta_{nn} + \sum_{k \neq n} \beta_{kn}\left[\theta_{kn}\left(\frac{w_k}{w_n}\right)\right]^{1/2}\right\}, \quad n = 1, \ldots, N, \tag{6.2.26}$$

where by symmetry there are $N(N-1)/2$ shadow price parameters $\theta_{kn} > 0, k > n$, to be estimated. Observed expenditure can be expressed as

$$\begin{aligned} E &= \sum_n w_n x_n \\ &= y \cdot \exp\{-\Delta D\} \cdot \left[\sum_n \beta_{nn} w_n + \sum_{k>n} \sum_n \beta_{kn} (\theta_{kn}^{1/2} + \theta_{kn}^{-1/2}) w_n^{1/2} w_k^{1/2}\right]. \end{aligned} \tag{6.2.27}$$

The term $\exp\{-\Delta D\} \gtreqless 1$ according as producers in the reference group $(D = 0)$ are more, equally, or less technically efficient than those in the comparison group. If the two groups are equally technically efficient, it should be apparent from equations (6.2.24) and (6.2.27) that $E \geq c(y, w; \beta)$ and that $E = c(y, w; \beta) \Leftrightarrow (\theta_{kn}^{1/2} + \theta_{kn}^{-1/2}) = 2 \Leftrightarrow \theta_{kn} = 1 \ \forall k > n$.

Since E is a linear combination of the x_n, estimation and hypothesis testing is based on the system of shadow input demand equations (6.2.26). After converting the shadow input demand equations to shadow input–output demand equations x_n/y, estimation is conducted by means of iterated SUR, with cross-equation parameter restrictions imposed. Several hypothesis tests are of potential interest. Equal technical efficiency across groups can be tested by testing the hypothesis that $\Delta = 0$. Systematic allocative efficiency can be tested by testing the hypothesis that $\theta_{kn} = 1 \ \forall k > n$. Systematic overutilization of one input relative to another input can be tested by testing the hypothesis that $\theta_{kn} = 1$ against the alternative that $\theta_{kn} < 1$. An example is provided by the Averch–Johnson (1962) overcapitalization hypothesis, in which k designates a rate base input and n designates a nonrate base input. Finally, following Lovell and Sickles (1983), the hypothesis that input allocative inefficiency is consistent across inputs can be tested by testing the hypothesis that $\theta_{nk} = \theta_{nm} \cdot \theta_{mk} \ \forall n > m > k$. After estimation and hypothesis testing, the impact on expenditure of the technical efficiency differential can be calculated from equation (6.2.27) as $\exp\{-\Delta D\}$, and the impact on expenditure of systematic allocative inefficiency can be calculated by comparing predicted expenditure with and without the parametric restrictions $\theta_{kn} = 1 \ \forall k > n$. In sharp contrast to the Cobb–Douglas case, the cost of systematic input allocative inefficiency is producer specific, since it depends on input prices as well as parameters.

Translog I Essentially the same procedure can be used to estimate technical and systematic allocative inefficiency within a translog framework. The advantage of the translog form over the generalized Leontief form lies in the ease with which the translog form can incorporate multiple outputs and model nonhomothetic technologies. The single-output translog cost function and its associated cost-minimizing input cost share equations can be written as

$$\begin{aligned} \ln c(y, w; \beta) = \beta_o + \beta_y \ln y + \sum_n \beta_n \ln w_n + \frac{1}{2}\beta_{yy}(\ln y)^2 \\ + \frac{1}{2}\sum_n \sum_k \beta_{nk} \ln w_n \ln w_k + \sum_n \beta_{yn} \ln y \ln w_n \end{aligned} \tag{6.2.28}$$

and

$$S_n(y, w; \beta) = \beta_n + \sum_k \beta_{nk} \ln w_k + \beta_{yn} \ln y, \qquad n = 1, \ldots, N, \tag{6.2.29}$$

respectively, where symmetry and homogeneity of degree +1 in input prices are imposed through the parameter restrictions $\beta_{nk} = \beta_{kn}$, $k \neq n$, $\Sigma_n \beta_n = 1$, $\Sigma_k \beta_{nk} = 0$, $n = 1, \ldots, N$, and $\Sigma_n \beta_{yn} = 0$. Note that the underlying technology is nonhomothetic unless $\beta_{yn} = 0, n = 1, \ldots, N$, which is a testable parametric restriction.

An output-oriented technical efficiency differential can be introduced by replacing y with $y \cdot \exp\{-\Delta D\}$. In this event output-oriented technical inefficiency does not appear simply in an additive fashion in the expression for total expenditure, as it did in the Cobb–Douglas model given in equation (6.2.23). In the translog model output-oriented technical inefficiency interacts with input prices and output quantity, appearing in $(N + 2)$ terms in the cost equation (6.2.28), and in one term in each of the input cost share equations (6.2.29). However if technical inefficiency is specified as being input oriented, it does appear additively in the expression for total expenditure, modifying the intercept term in the cost equation (6.2.28), and it does not appear at all in the input cost share equations (6.2.29). This illustrates the advantage of using the input-oriented specification in a cost minimization framework, an advantage that multiplies in the multiple-output case.

Using the input-oriented approach to technical efficiency and exploiting equations (6.2.14) and (6.2.15), observed expenditure and observed input cost shares can be expressed in terms of shadow cost and shadow input cost shares as

$$\begin{aligned}\ln E &= \beta_o - \Delta D + \beta_y \ln y + \frac{1}{2}\beta_{yy}(\ln y)^2 + \sum_n \beta_n \ln(\theta_{n1} w_n) \\ &+ \frac{1}{2}\sum_n \sum_k \beta_{nk} \ln(\theta_{n1} w_n)\ln(\theta_{k1} w_k) + \sum_n \beta_{yn} \ln y \ln(\theta_{n1} w_n) \\ &+ \ln\left\{\sum_n (\theta_{n1})^{-1}\left[\beta_n + \sum_k \beta_{nk}\ln(\theta_{k1} w_k) + \beta_{yn}\ln y\right]\right\} \qquad (6.2.30)\end{aligned}$$

and

$$S_n = \frac{(\theta_{n1})^{-1}[\beta_n + \Sigma_k \beta_{nk}\ln(\theta_{k1} w_k) + \beta_{yn}\ln y]}{\Sigma_j (\theta_{j1})^{-1}[\beta_j + \Sigma_k \beta_{jk}\ln(\theta_{k1} w_k) + \beta_{yj}\ln y]},$$
$$n = 1, \ldots, N, \qquad (6.2.31)$$

respectively. It follows from equations (6.2.28) and (6.2.30) that

$$\begin{aligned}\ln E &= \ln c(y, w; \beta) - \Delta D + \left\{\sum_n \beta_n \ln\theta_{n1} + \sum_n \sum_k \beta_{nk} \ln w_n \ln\theta_{k1}\right. \\ &\left. + \frac{1}{2}\sum_n \sum_k \beta_{nk} \ln\theta_{n1} \ln\theta_{k1} + \sum_n \beta_{yn} \ln y \ln\theta_{n1}\right\}. \qquad (6.2.32)\end{aligned}$$

Thus the natural logarithm of total expenditure can be expressed as the sum of three components: the translog cost function given in equation (6.2.28), the percentage cost differential due to the input-oriented technical efficiency differential, and the percentage cost of systematic allocative inefficiency. The second term is zero if, and only if, $\Delta = 0$, and the third term is nonnegative and attains its minimum value of zero if, and only if, $\theta_{21} = \cdots = \theta_{N1} = 1$. It is important to note that the percentage cost of systematic allocative inefficiency is not itself systematic in the translog case, since the third term depends on data as well as parameters.

If classical error terms are appended to equations (6.2.30) and (6.2.31), and if one input cost share equation is deleted, the remaining system of N equations can be estimated by ITSUR. After estimation, likelihood ratio tests can be employed to test hypotheses concerning the nature of technical and systematic allocative inefficiency. The effects of the technical efficiency differential and of systematic allocative inefficiency on total expenditure can be calculated from equation (6.2.32).

Generalizing the translog form to accommodate multiple outputs presents no problems, since the impact of input-oriented technical inefficiency appears only in the intercept of the cost function, and the impact of systematic allocative inefficiency modifies only the input prices. All that is required is to increase the number of output-related terms in the cost function and the input cost share equations. Estimation and interpretation of the results are unaffected.

Translog II In the preceding translog framework we created shadow input prices by *scaling* observed prices; we adopted the same approach in the generalized Leontief framework. It is also possible to create shadow input prices by *translating* observed input prices, in either the translog or the generalized Leontief framework. Eakin and Kniesner (1988, 1992), Eakin (1991), and Atkinson and Halvorsen (1992) have done so in a hybrid translog multiple-output cost function framework. A hybrid translog shadow cost function can be written as

$$\begin{aligned}\ln c^H(y, w; \beta, \theta) &= \beta_o + \sum_m \alpha_m y_m + \sum_n \beta_n \ln(w_n + \theta_n) + \frac{1}{2}\sum_m \sum_j \alpha_{mj} y_m y_j \\ &+ \frac{1}{2}\sum_n \sum_k \beta_{nk}[\ln(w_n + \theta_n)][\ln(w_k + \theta_k)] \\ &+ \sum_m \sum_n \gamma_{mn} y_m \ln(w_n + \theta_n), \qquad (6.2.33)\end{aligned}$$

where the qualifier "hybrid" refers to the fact that the natural logarithms of output quantities are replaced with the output quantities themselves. This modification of the translog form allows the introduction of nonpositive output quantities, without disturbing the flexibility of the functional form. The parameter vector θ allows shadow input prices to diverge (additively rather than multiplicatively) from observed input prices. The usual parameter restrictions impose linear homogeneity in shadow input prices. Shephard's lemma generates the shadow input cost share equations

$$S_n^H(y,w;\beta,\theta) = \beta_n + \sum_k \beta_{nk} \ln(w_k + \theta_k) + \sum_m \gamma_{mn} y_m ,$$
$$n = 1,\ldots,N. \tag{6.2.34}$$

The natural logarithm of observed expenditure, and the observed input cost share equations, can be expressed as

$$\ln E = \ln c^H(y,w;\beta,\theta) + \ln\left[\sum_n \left(\frac{w_n}{w_n+\theta_n}\right)\left(\beta_n + \sum_m \gamma_{mn} y_m + \sum_k \beta_{nk} \ln(w_k+\theta_k)\right)\right],$$
$$S_n = \frac{S_n^H(y,w;\beta,\theta)\cdot(w_n/(w_n+\theta_n))}{[\Sigma_j(w_j/(w_j+\theta_j))(\beta_j + \Sigma_m \gamma_{jm} y_m + \Sigma_k \beta_{kj} \ln(w_k+\theta_k))]},$$
$$n = 1,\ldots, N, \tag{6.2.35}$$

where the superscript H indicates that the cost function and its associated input cost share equations have the "hybrid" translog functional form. It is clear that $\ln E = \ln c^H(y, w; \beta, \theta)$ and $S_n = s_n^H(y, w; \beta, \theta)$ if $\theta_n = 0$, $n = 1, \ldots, N$. It is not so clear, but nonetheless true, that $\ln E > \ln c^H(y, w; \beta, \theta)$ and that $S_n \neq S_n^H(y, w; \beta, \theta)$ otherwise. Once again, however, the percentage cost of systematic input allocative inefficiency is producer specific. Finally, as usual, not all shadow price divergence parameters θ_n are identified, and in this additive formulation the only permissible normalization is to set one element of $\theta = 0$.

After substituting equations (6.2.33) and (6.2.34) into the equation system (6.2.35), deleting one input cost share equation, and adding a multivariate normal error vector to the remaining N equations, estimation via nonlinear SUR can proceed. After estimation, likelihood ratio tests of various hypotheses of interest can be conducted. The hypothesis of overall allocative efficiency is tested by testing the hypothesis that $\theta_n = 0\ \forall n$. The hypothesis that the mix of inputs n and k is allocatively efficient is tested by testing the hypothesis that $(w_n/w_k) = (w_n + \theta_n)/(w_k + \theta_k)$. Notice that technical inefficiency is not incorporated into the model, although it could be incorporated in a

restricted fashion in the same manner as it was in the first translog model.

Translog III In the two preceding translog frameworks (and in the Cobb–Douglas and generalized Leontief frameworks as well) we modeled both technical and systematic input allocative inefficiencies parametrically. As a consequence, it was not possible to obtain separate estimates of a technical inefficiency parameter for all I producers, and we settled for an estimate of a single technical efficiency differential parameter Δ. It is possible, however, to combine the parametric approach to the estimation of systematic input allocative inefficiency with a stochastic approach to the estimation of technical inefficiency. Such a procedure enables us to obtain estimates of systematic input allocative inefficiencies and their producer-specific costs, as before, and also to obtain producer-specific estimates of technical inefficiency and its cost. The procedure works as follows.

Writing equations (6.2.14) and (6.2.15) in translog form, we have

$$\begin{aligned}\ln E = {} & \beta_o + \beta_y \ln y + \sum_n \beta_n \ln(\theta_{n1} w_n) + \frac{1}{2}\beta_{yy}(\ln y)^2 \\ & + \frac{1}{2}\sum_n\sum_k \beta_{nk} \ln(\theta_{n1}w_n)\ln(\theta_{k1}w_k) + \sum_n \beta_{yn} \ln y \ln(\theta_{n1}w_n) \\ & + \ln\left\{\sum_n (\theta_{n1})^{-1}\left[\beta_n + \sum_k \beta_{nk} \ln(\theta_{k1}w_k) + \beta_{yn}\ln y\right]\right\} - \ln\phi,\end{aligned} \tag{6.2.36}$$

$$S_n = \frac{(\theta_{n1})^{-1}[\beta_n + \Sigma_k \beta_{nk} \ln(\theta_{k1}w_k) + \beta_{yn}\ln y]}{\Sigma_j(\theta_{j1})^{-1}[\beta_j + \Sigma_k \beta_{jk} \ln(\theta_{k1}w_k) + \beta_{yj}\ln y]}, \qquad n = 1,\ldots, N. \tag{6.2.37}$$

After imposing linear homogeneity on equation (6.2.36) (either parametrically or by subtracting $\ln w_1$ from both sides, since $\theta_{11} = 1$) this system can be estimated by MLE. We assume that $-\ln\phi \sim \text{iid } N^+(0, \sigma_\phi^2)$. This error component does not represent cost inefficiency as it did in Chapter 4; it represents the cost of input-oriented technical inefficiency only. We add a second error component $v \sim \text{iid } N(0, \sigma_v^2)$ to the cost equation and an error vector $v_n \sim \text{iid } N(0, \Sigma)$ to $N - 1$ input cost share equations, and we assume that all three error terms are mutually independent and independent of the regressors. The log likelihood function of the system under these distributional assumptions can be expressed as

$$\ln L = \text{constant} - I \ln \sigma + \sum_i \ln \Phi\left(\frac{\varepsilon_i \lambda}{\sigma}\right) - \frac{1}{2\sigma^2}\sum_i \varepsilon_i^2$$
$$-\frac{I}{2}\ln|\Sigma| - \frac{1}{2}\sum_i v_{ni}'\Sigma^{-1}v_{ni}, \tag{6.2.38}$$

where

$$\varepsilon_i = v_i - \ln \phi_i,$$
$$\sigma^2 = \sigma_v^2 + \sigma_\phi^2,$$
$$\gamma = \frac{\sigma_\phi}{\sigma_v}.$$

Maximization of the preceding log likelihood function generates estimates of all technology parameters and all allocative inefficiency parameters. After estimation, the producer-specific cost of systematic input allocative inefficiency can be determined from the third term on the right-hand side of equation (6.2.32). In addition, the JLMS technique can be used to decompose the cost equation residuals into separate producer-specific estimates of the cost of technical inefficiency and statistical noise. The estimators of $-\ln \phi_i$ are given by

$$E(-\ln \phi_i|\varepsilon_i) = \mu_{*i} + \sigma_*\left[\frac{\phi(-\mu_{*i}/\sigma_*)}{1-\Phi(-\mu_{*i}/\sigma_*)}\right],$$

$$M(-\ln \phi_i|\varepsilon_i) = \begin{cases} \varepsilon_i(\sigma_\phi^2/\sigma^2) & \text{if } \varepsilon_i \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $\mu_{*i} = \varepsilon_i\sigma_\phi^2/\sigma^2$ and $\sigma_*^2 = \sigma_\phi^2\sigma_v^2/\sigma^2$.

Since the model is highly nonlinear, the ML estimators might be difficult to obtain, and one could consider a two-step procedure. In the first step $-\ln \phi_i$ is ignored and ITSUR is used to generate consistent estimates of all remaining parameters except for the intercept, which is biased. One can then use standard asymptotic results, available in Gallant (1987; Chapter 5), to test hypotheses concerning the parameters of the cost frontier. In the second step the cost frontier residuals $\varepsilon_i = \beta_o - \ln \phi_i + v_i$ are obtained and distributional assumptions are made on $-\ln \phi_i$ and v_i, such as $-\ln \phi_i \sim \text{iid } N^+(0, \sigma_\phi^2)$ and $v_i \sim N(0, \sigma_v^2)$, and $-\ln \phi_i$ and v_i are assumed to be distributed independently. Under these assumptions the log likelihood function can be derived, the maximization of which will generate estimates of β_o, σ_ϕ^2, and σ_v^2 (conditional on the parameter estimates obtained in the first step). Finally, the JLMS procedure can be applied to estimate $-\ln \phi_i$.

6.2.2 Estimating and Decomposing Profit Inefficiency

In this section we adopt the same approach to the estimation and decomposition of profit inefficiency as we did to the estimation and decomposition of cost inefficiency in Section 6.2.1. Failure to maximize profit has a technical inefficiency component, which can be output oriented or input oriented. It also has a systematic allocative inefficiency component, which can involve an inappropriate input mix, an inappropriate output mix (in the case of multiple outputs), and an inappropriate scale. As we saw in Section 2.4.3, the magnitudes, and perhaps even the directions, of the scale and mix components of systematic allocative inefficiency can depend on the orientation of the technical efficiency measure. We begin with a graphical illustration. We then generalize the analysis. We derive a shadow profit function, and we then express observed output quantities, observed input quantities, and observed profit in terms of shadow output prices and shadow input prices. We also derive a normalized shadow profit function, and we express observed output quantities, observed input quantities, and observed normalized profit in terms of shadow output prices and shadow input prices. Finally, we illustrate the analysis using Cobb–Douglas and flexible functional forms. All formulations adopt the multiplicative approach of scaling observed prices to create shadow prices, although each formulation could also adopt the additive approach of translating observed prices to create shadow prices. We leave this exercise to the reader.

Suppose we observe a producer using input vector x to produce scalar output y with production frontier $f(x; \beta)$. The observed output price and the observed input price vector are p and w, respectively, and so observed profit is $\pi = py - w^T x$. The measurement and decomposition of profit efficiency are illustrated in Figure 6.3, which takes an output orientation to the measurement of technical efficiency, and in Figure 6.4, which takes an input orientation to the measurement of technical efficiency. Since in both figures it is assumed that a single

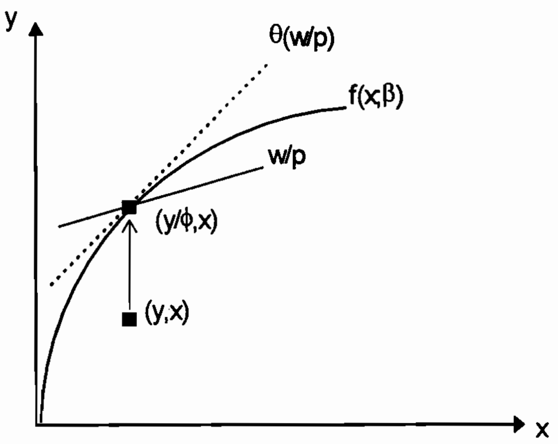

Figure 6.3 The Shadow Price Approach to Estimating Profit Efficiency: Output Orientation

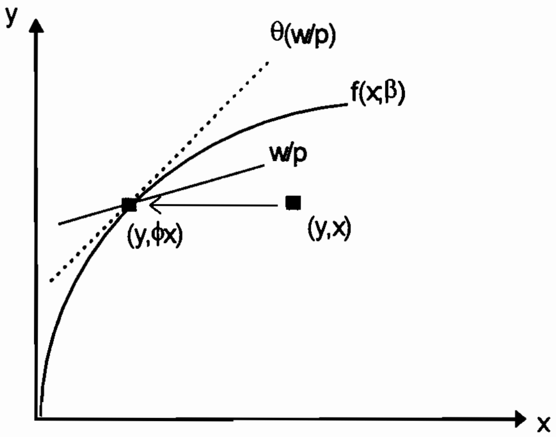

Figure 6.4 The Shadow Price Approach to Estimating Profit Efficiency: Input Orientation

input is used to produce a single output, the only type of allocative inefficiency is scale inefficiency.

In Figure 6.3 the producer is technically inefficient, since $y/\phi = f(x;\beta)$, with $0 < \phi \leq 1$ providing an output-oriented measure of technical

efficiency. In addition, at the technically efficient projection $(y/\phi, x)$ the producer is operating at an inefficiently small scale, since the marginal product of the input exceeds the observed input–output price ratio. However the producer is scale efficient relative to the larger shadow price ratio $[\theta(w/p)]$, $\theta > 1$.

In Figure 6.4 the producer is also technically inefficient, since $y = f(\phi x;\beta)$, with $0 < \phi \leq 1$ providing an input-oriented measure of technical efficiency. In addition, at the technically efficient projection $(y, \phi x)$ the producer is operating at an inefficiently small scale, since the marginal product of the input exceeds the observed input–output price ratio. However the producer is scale efficient relative to the larger shadow price ratio $[\theta(w/p)]$, $\theta > 1$.

It should be apparent that in general neither parametric efficiency measure in the output-oriented Figure 6.3 has the same value as the corresponding measure in the input-oriented Figure 6.4. It should also be noted that the analysis underlying both figures can be generalized to accommodate multiple inputs and multiple outputs, in which case both input allocative inefficiency and output allocative inefficiency can be modeled parametrically, in the same manner as input allocative inefficiency was modeled in Figures 6.1 and 6.2.

The basic idea underlying the output-oriented approach to the estimation and decomposition of profit inefficiency illustrated in Figure 6.3 can be generalized as follows. In the single-output case a shadow profit function is given by

$$\pi(p^*, w^*;\beta) = \max_x\left\{p\phi f(x;\beta) - \sum_n \theta_n w_n x_n\right\}, \tag{6.2.39}$$

where $y = \phi f(x;\beta)$, with $0 < \phi \leq 1$ capturing the effect of output-oriented technical inefficiency. Maximization of shadow profit requires $\partial f(x;\beta)/\partial x_n = (\theta_n w_n/\phi p)$, with $\theta_n \gtreqqless 1$, $n = 1, \ldots, N$, capturing the effects of systematic input allocative inefficiency. Thus $p^* = \phi p$ and $w_n^* = \theta_n w_n$, $n = 1, \ldots, N$. Unlike the shadow cost function model, in the shadow profit function model it is possible to identify all N input allocative inefficiency parameters θ_n, $n = 1, \ldots, N$; a normalization such as $\theta = [1, \theta_{21}, \ldots, \theta_{N1}]$ is not required.

Hotelling's lemma can be applied to equation (6.2.39) to generate the system of observed output supply and input demand equations

$$
\begin{aligned}
y &= \frac{\partial \pi(p^*, w^*; \beta)}{\partial p^*} \\
&= \frac{1}{\phi}\frac{\partial \pi(p^*, w^*; \beta)}{\partial p}, \\
-x_n &= \frac{\partial \pi(p^*, w^*; \beta)}{\partial w_n^*} \\
&= \frac{1}{\theta_n}\frac{\partial \pi(p^*, w^*; \beta)}{\partial w_n}, \qquad n = 1, \ldots, N,
\end{aligned} \tag{6.2.40}
$$

which in turn generate an expression for observed profit

$$
\begin{aligned}
\pi &= p\frac{\partial \pi(p^*, w^*; \beta)}{\partial p^*} + \sum_n w_n \frac{\partial \pi(p^*, w^*; \beta)}{\partial w_n^*} \\
&= \frac{p}{\phi}\frac{\partial \pi(p^*, w^*; \beta)}{\partial p} + \sum_n \frac{w_n}{\theta_n}\frac{\partial \pi(p^*, w^*; \beta)}{\partial w_n}.
\end{aligned} \tag{6.2.41}
$$

Estimation can be based either on the $(N+1)$ equations in (6.2.40) or on a system consisting of the profit equation (6.2.41) and N observed profit share equations replacing the $(N+1)$ observed output supply and input demand equations (6.2.40). All that is required is to impose a functional form on the shadow profit function $\pi(p^*, w^*; \beta)$. However in a shadow profit function framework the linear homogeneity property of $\pi(p^*, w^*; \beta)$ in (p^*, w^*) must be imposed through parametric restrictions. Consequently most empirical analysis has been based on the normalized shadow profit function, following Lau and Yotopoulos (1971). In the single-output case a normalized shadow profit function can be derived from the shadow profit function in equation (6.2.39) as

$$
\begin{aligned}
\frac{\pi(p^*, w^*; \beta)}{p} &= \phi \cdot \max_x \left\{ f(x;\beta) - \sum_n \left(\frac{\theta_n w_n}{\phi p}\right) x_n \right\} \\
&= \phi \cdot \pi[(w/p)^*; \beta],
\end{aligned} \tag{6.2.42}
$$

which is homogeneous of degree 0 in (p^*, w^*). The normalized shadow price ratios $(w/p)_n^* = (\theta_n w_n/\phi p)$, $n = 1, \ldots, N$, contain both technical and systematic allocative inefficiencies.

Application of Hotelling's lemma to equation (6.2.42) generates the system of observed output supply and input demand equations

$$
\begin{aligned}
y &= \phi \cdot \pi[(w/p)^*; \beta] - \phi \cdot \sum_n \left(\frac{w}{p}\right)_n^* \frac{\partial \pi[(w/p)^*; \beta]}{\partial (w/p)_n^*} \\
&= \phi \cdot \pi[(w/p)^*; \beta] - \phi \cdot \sum_n \left(\frac{w}{p}\right)_n \frac{\partial \pi[(w/p)^*; \beta]}{\partial (w/p)_n} \\
-x_n &= \frac{\partial \pi[(w/p)^*; \beta]}{\partial (w/p)_n^*} \\
&= \frac{\phi}{\theta_n}\frac{\partial \pi[(w/p)^*; \beta]}{\partial (w/p)_n}, \qquad n = 1, \ldots, N,
\end{aligned} \tag{6.2.43}
$$

which in turn generate an expression for observed normalized profit

$$
\begin{aligned}
\frac{\pi}{p} &= y - \sum_n \left(\frac{w}{p}\right)_n x_n \\
&= \phi \cdot \pi[(w/p)^*; \beta] + \phi \cdot \sum_n \frac{1-\theta_n}{\theta_n}\left(\frac{w}{p}\right)_n \frac{\partial \pi[(w/p)^*; \beta]}{\partial (w/p)_n}.
\end{aligned} \tag{6.2.44}
$$

Estimation can also be based either on the system of $(N+1)$ equations (6.2.43) or on a system consisting of the normalized profit equation (6.2.44) and N observed profit share equations replacing the $(N+1)$ observed output supply and input demand equations (6.2.43). All that is required is to impose a functional form on the normalized shadow profit function $\pi[(w/p)^*; \beta]$.

The basic idea underlying the input-oriented approach to the estimation and decomposition of profit inefficiency illustrated in Figure 6.4 can be generalized also. In this case the shadow profit function is given by

$$
\pi(p, w^{\otimes}; \beta) = \max_{\phi x} \left\{ p f(\phi x; \beta) - \sum_n \left(\frac{\theta_n w_n}{\phi}\right)(\phi x_n) \right\}, \tag{6.2.45}
$$

where $w_n^{\otimes} = (\theta_n w_n/\phi)$, $n = 1, \ldots, N$. Subsequent derivations follow along lines similar to those developed in equations (6.2.39)–(6.2.44); details are available in Kumbhakar (1996a, c).

We now illustrate the shadow price approach to the estimation and decomposition of profit inefficiency with the Cobb–Douglas functional form, assuming a single output, and with two flexible functional forms, each allowing for multiple outputs.

Cobb–Douglas We follow the strategy of Lau and Yotopoulos (1971, 1979) and Yotopoulos and Lau (1973). Let production technology be characterized by the relationship

$$y = \prod_n x_n^{\beta_n}, \tag{6.2.46}$$

with $\beta_n > 0$, $n = 1, \ldots, N$, and $\Sigma_n \beta_n = r < 1$. The profit function corresponding to this production function is

$$\pi(p,w;\beta) = (1-r)p^{1/(1-r)} \prod_n \left(\frac{w_n}{\beta_n}\right)^{-\beta_n/(1-r)}, \tag{6.2.47}$$

and the normalized profit function is

$$\frac{\pi(p,w;\beta)}{p} = (1-r)p^{r/(1-r)} \prod_n \left(\frac{w_n}{\beta_n}\right)^{-\beta_n/(1-r)}. \tag{6.2.48}$$

Hotelling's lemma can be applied to either equation (6.2.45) or equation (6.2.48) to generate profit-maximizing output supply and input demand equations. They are given by

$$\begin{aligned} y(p,w;\beta) &= p^{r/(1-r)} \prod_n \left(\frac{w_n}{\beta_n}\right)^{-\beta_n/(1-r)}, \\ x_n(p,w;\beta) &= p^{1/(1-r)} \prod_k \left(\frac{w_k}{\beta_k}\right)^{-\beta_k/(1-r)} \left(\frac{w_n}{\beta_n}\right)^{-1}, \qquad n = 1, \ldots, N. \end{aligned} \tag{6.2.49}$$

The corresponding profit-maximizing profit share equations are

$$\begin{aligned} \frac{py(p,w;\beta)}{\pi(p,w;\beta)} &= \frac{1}{1-r}, \\ \frac{w_n x_n(p,w;\beta)}{\pi(p,w;\beta)} &= \left(\frac{\beta_n}{1-r}\right), \qquad n = 1, \ldots, N. \end{aligned} \tag{6.2.50}$$

Now suppose that a producer is technically and allocatively inefficient, and optimizes with respect to shadow price ratios $(\theta_n w_n/\phi p)$, $n = 1, \ldots, N$. Here the θ_ns represent systematic input allocative inefficiency and ϕ represents output-oriented technical inefficiency. Then the producer's observed profit coincides with maximum shadow profit, and so

$$\pi = \pi(p^*, w^*; \beta) = A \cdot \pi(p,w;\beta), \tag{6.2.51}$$

where

$$A = \left[\phi^{1/(1-r)} \cdot (1-r)^{-1}\right] \cdot \left[1 - \sum_n \left(\frac{\beta_n}{\theta_n}\right)\right] \cdot \prod_n \theta_n^{-\beta_n/(1-r)}.$$

Taking the natural logarithm of equation (6.2.51) and using the definition of A shows that the natural logarithm of observed profit is equal to the natural logarithm of maximum profit, plus a term measuring the percentage of maximum profit lost to technical inefficiency, plus a term measuring the percentage of maximum profit lost to systematic input allocative inefficiency. The former loss is nonnegative, and attains its minimum value of zero if, and only if, $\phi = 1$. The latter loss is also nonnegative, and attains its minimum value of zero if, and only if, $\theta_2 = \cdots = \theta_N = 1$ (the normalization $\theta_1 = 1$ is imposed for identification purposes). The same relationship holds between the natural logarithms of observed and maximum normalized profit in equation (6.2.54).

The producer's observed output supply and input demand equations coincide with shadow profit-maximizing output supply and input demand equations, and so

$$\begin{aligned} y &= y(p^*,w^*;\beta) = A_y \cdot y(p,w;\beta), \\ x_n &= x_n(p^*,w^*;\beta) = A_n \cdot x_n(p,w;\beta), \qquad n = 1, \ldots, N, \end{aligned} \tag{6.2.52}$$

where

$$\begin{aligned} A_y &= \left[\phi^{1/(1-r)}\right] \cdot \left[\prod_n \theta_n^{-\beta_n/(1-r)}\right], \\ A_n &= \left[\phi^{1/(1-r)}\right] \cdot \left[\theta_n^{-1} \prod_k \theta_k^{-\beta_k/(1-r)}\right], \qquad n = 1, \ldots, N. \end{aligned}$$

$A_y \gtreqless 1$ and $A_n \gtreqless 1$, $n = 1, \ldots, N$, depending on the magnitude of technical inefficiency and the directions and magnitudes of systematic input allocative inefficiencies. Thus observed output supply and input demand equations coincide with profit-maximizing output supply and input demand equations if, and only if, $\phi = \theta_2 = \cdots = \theta_N = 1$.

The producer's observed profit shares coincide with shadow profit-maximizing profit shares, and so

$$\frac{py}{\pi}=\frac{py(p^*,w^*;\beta)}{\pi(p^*,w^*;\beta)}=\left[1-\sum_n\left(\frac{\beta_n}{\theta_n}\right)\right]^{-1},$$
$$\frac{w_n x_n}{\pi}=\frac{w_n x_n(p^*,w^*;\beta)}{\pi(p^*,w^*;\beta)}=\frac{\beta_n}{\theta_n}\left[1-\sum_k\left(\frac{\beta_k}{\theta_k}\right)\right]^{-1},\qquad n=1,\dots,N. \tag{6.2.53}$$

These observed profit shares coincide with the profit-maximizing profit shares if, and only if, $\theta_2=\cdots=\theta_N=1$. Note that although output-oriented technical inefficiency ϕ appears in the observed output supply and input demand equations (6.2.52), it washes out of the observed profit share equations (6.2.53).

The natural logarithm of observed normalized profit is

$$\ln\frac{\pi}{p}=\ln A^*+\sum_n\beta_n^*\ln\frac{w_n}{p}, \tag{6.2.54}$$

and the corresponding observed normalized input profit share equations are

$$\frac{(w_n/p)x_n}{\pi/p}=\beta_n^{**},\qquad n=1,\dots,N, \tag{6.2.55}$$

where

$$\ln A^*=(1-r)^{-1}\ln\phi+\ln\left(1-\sum_n\frac{\beta_n}{\theta_n}\right)-\sum_n\beta_n(1-r)^{-1}\ln\theta_n$$
$$+\sum_n\beta_n(1-r)^{-1}\ln\beta_n,$$
$$\beta_n^*=-\beta_n(1-r)^{-1},\qquad n=1,\dots,N,$$
$$\beta_n^{**}=\frac{\beta_n/\theta_n}{1-\Sigma_k(\beta_k/\theta_k)},\qquad n=1,\dots,N.$$

Empirical analysis is based on the system of $(N+1)$ equations (6.2.54) and (6.2.55), which are estimated jointly using ITSUR. A number of hypothesis tests are available. Two groups of producers are equally efficient with respect to their allocation of inputs ($\theta_n^1=\theta_n^2$, $n=1,\dots,N$) if, and only if, $\beta_n^{**1}=\beta_n^{**2}$, $n=1,\dots,N$. Producers are systematically efficient with respect to their allocation of inputs ($\theta_1=\cdots=\theta_N=1$) if, and only if, ($\beta_n^*=\beta_n^{**}$, $n=1,\dots,N$). Two groups of producers achieve equal relative profit efficiency if, and only if, ($\ln A^{*1}=\ln A^{*2}$). Finally, two groups of producers achieve equal technical efficiency and equal systematic input allocative efficiency if, and only if, ($\ln A^{*1}=\ln A^{*2}$ and $\beta_n^{**1}=\beta_n^{**2}$, $n=1,\dots,N$). These and other hypothesis tests are described in Lau and Yotopoulos (1971, 1979) and Yotopoulos and Lau (1973).

Generalized Leontief Suppose that the profit function takes the generalized Leontief form, with $M=N=2$ for ease of exposition. Then, following Lovell and Sickles (1983),

$$\begin{aligned}\pi(p,w;\beta)=&\beta_{11}p_1+\beta_{12}p_1^{1/2}p_2^{1/2}+\beta_{13}p_1^{1/2}w_1^{1/2}+\beta_{14}p_1^{1/2}w_2^{1/2}\\&+\beta_{21}p_2^{1/2}p_1^{1/2}+\beta_{22}p_2+\beta_{23}p_2^{1/2}w_1^{1/2}+\beta_{24}p_2^{1/2}w_2^{1/2}\\&+\beta_{31}w_1^{1/2}p_1^{1/2}+\beta_{32}w_1^{1/2}p_2^{1/2}+\beta_{33}w_1+\beta_{34}w_1^{1/2}w_2^{1/2}\\&+\beta_{41}w_2^{1/2}p_1^{1/2}+\beta_{42}w_2^{1/2}p_2^{1/2}+\beta_{43}w_2^{1/2}w_1^{1/2}+\beta_{44}w_2,\end{aligned} \tag{6.2.56}$$

where $\beta_{ij}=\beta_{ji}\ \forall j\neq i$. The generalized Leontief profit function is homogeneous of degree +1 in (p,w) by construction, and it is convex in (p,w) if $\beta_{ij}\leq 0\ \forall j\neq i$. Application of Hotelling's lemma to equation (6.2.56) generates the system of profit-maximizing output supply and input demand equations

$$\begin{aligned}y_1(p,w,\beta)&=\beta_{11}+\beta_{12}\left(\frac{p_1}{p_2}\right)^{-1/2}+\beta_{13}\left(\frac{p_1}{w_1}\right)^{-1/2}+\beta_{14}\left(\frac{p_1}{w_2}\right)^{-1/2},\\y_2(p,w,\beta)&=\beta_{22}+\beta_{21}\left(\frac{p_1}{p_2}\right)^{1/2}+\beta_{23}\left(\frac{p_2}{w_1}\right)^{-1/2}+\beta_{24}\left(\frac{p_2}{w_2}\right)^{-1/2},\\-x_1(p,w,\beta)&=\beta_{33}+\beta_{31}\left(\frac{p_1}{w_1}\right)^{1/2}+\beta_{32}\left(\frac{p_2}{w_1}\right)^{1/2}+\beta_{34}\left(\frac{w_1}{w_2}\right)^{-1/2},\\-x_2(p,w;\beta)&=\beta_{44}+\beta_{41}\left(\frac{p_1}{w_2}\right)^{1/2}+\beta_{42}\left(\frac{p_2}{w_2}\right)^{1/2}+\beta_{43}\left(\frac{w_1}{w_2}\right)^{1/2}.\end{aligned} \tag{6.2.57}$$

Technical inefficiency can be introduced into equations (6.2.57) by replacing the four intercepts with $(\beta_{jj}-\phi_j)$, $j=1,\dots,4$. However in this event technical inefficiency would be nonneutral and difficult to interpret. Consequently we model only allocative inefficiency, as usual by assuming that a producer optimizes with respect to shadow price ratios. Replacing observed price ratios in equations (6.2.57) with shadow price ratios generates the system of observed output supply and input demand equations

$$y_1 = \beta_{11} + \beta_{12}\left[\theta_{12}\left(\frac{p_1}{p_2}\right)\right]^{-1/2} + \beta_{13}\left[\theta_{13}\left(\frac{p_1}{w_1}\right)\right]^{-1/2} + \beta_{14}\left[\theta_{14}\left(\frac{p_1}{w_2}\right)\right]^{-1/2},$$

$$y_2 = \beta_{22} + \beta_{12}\left[\theta_{12}\left(\frac{p_1}{p_2}\right)\right]^{1/2} + \beta_{23}\left[\theta_{23}\left(\frac{p_2}{w_1}\right)\right]^{-1/2} + \beta_{24}\left[\theta_{24}\left(\frac{p_2}{w_2}\right)\right]^{-1/2},$$

$$-x_1 = \beta_{33} + \beta_{13}\left[\theta_{13}\left(\frac{p_1}{w_1}\right)\right]^{1/2} + \beta_{23}\left[\theta_{23}\left(\frac{p_2}{w_1}\right)\right]^{1/2} + \beta_{34}\left[\theta_{34}\left(\frac{w_1}{w_2}\right)\right]^{-1/2},$$

$$-x_2 = \beta_{44} + \beta_{14}\left[\theta_{14}\left(\frac{p_1}{w_2}\right)\right]^{1/2} + \beta_{24}\left[\theta_{24}\left(\frac{p_2}{w_2}\right)\right]^{1/2} + \beta_{34}\left[\theta_{34}\left(\frac{w_1}{w_2}\right)\right]^{1/2} \quad (6.2.58)$$

and the expression for observed profit

$$\pi = \sum_i \beta_{ii} q_i + \sum_i \sum_{j>i} \beta_{ij}\left(\theta_{ij}^{-1/2} + \theta_{ij}^{1/2}\right) q_i^{1/2} q_j^{1/2}, \quad (6.2.59)$$

where $q = (p_1, p_2, w_1, w_2)$.

The system (6.2.58) of observed output supply and input demand equations can be estimated by nonlinear SUR, after which statistical tests of various allocative inefficiency hypotheses can be conducted. The effect of systematic allocative inefficiency on profit is zero if, and only if, all $\theta_{ij} = 1$. If some $\theta_{ij} \neq 1$, the effect of systematic allocative inefficiency (and of each of its components) on profit is producer specific, depending on the prices producers face. The partial effect of systematic output allocative inefficiency on profit is given by the expression

$$\pi(p, w; \beta) - (\pi|\theta_{12} \neq 1) = \beta_{12} p_1^{1/2} p_2^{1/2}\left[2 - \left(\theta_{12}^{-1/2} + \theta_{12}^{1/2}\right)\right], \quad (6.2.60)$$

which is positive unless $\theta_{12} = 1$. If $\theta_{12} \neq 1$, the observed output mix does not maximize profit. The partial effect of systematic input allocative inefficiency on profit is given by the expression

$$\pi(p, w, \beta) - (\pi|\theta_{34} \neq 1) = \beta_{34} w_1^{1/2} w_2^{1/2}\left[2 - \left(\theta_{34}^{-1/2} + \theta_{34}^{1/2}\right)\right], \quad (6.2.61)$$

which is positive unless $\theta_{34} = 1$. If $\theta_{34} \neq 1$ the observed input mix does not maximize profit. The partial effect of systematic scale inefficiency on profit is given by the expression

$$\pi(p, w; \beta) - (\pi|\theta_{13} \neq 1,\ \theta_{14} \neq 1,\ \theta_{23} \neq 1,\ \theta_{24} \neq 1) = \sum_i \sum_j \beta_{ij} q_i^{1/2} q_j^{1/2}\left[2 - \left(\theta_{ij}^{-1/2} + \theta_{ij}^{1/2}\right)\right], \quad (6.2.62)$$

where $i = 1, 2$, $j = 3, 4$. This expression is also positive unless $\theta_{13} = \theta_{14} = \theta_{23} = \theta_{24} = 1$. If $(\theta_{13}, \theta_{14}, \theta_{23}, \theta_{24}) \neq (1, 1, 1, 1)$ the observed output–input ratios do not maximize profit.

Producers face four market prices and three independent market price ratios. However we have used six parameters θ_{ij} to model systematic allocative inefficiency. Although the market price ratios can be expected to be consistent, in the sense that any three independent market price ratios can be used to determine the remaining three market price ratios, it is not clear whether the shadow price ratios $[\theta_{ij}(q_i/q_j)]$ can be expected to be consistent also. That is, it is unclear whether or not allocatively inefficient producers can be expected to be consistent in their misperceptions of market price ratios. The preceding analysis, with three independent market price ratios and six independent shadow price ratios, allows for inconsistent systematic allocative inefficiency. Consistent systematic allocative inefficiency can be modeled as a constrained version of the preceding model, by imposing the testable parametric restrictions

$$\theta_{ik} = \theta_{ij} \cdot \theta_{jk}, \qquad i < j < k, \quad (6.2.63)$$

which reduces the number of independent allocative inefficiency parameters from six to three.

Translog The translog profit function was perhaps first used to analyze allocative efficiency by Atkinson and Halvorsen (1980). The translog functional form is particularly well suited to the parametric analysis of technical and systematic allocative inefficiency. We simply apply a translog functional form to a multiple-output generalization of equations (6.2.39)–(6.2.44), in which technical inefficiency is output oriented, or to a similar system of equations built around equation (6.2.45), in which technical inefficiency is input oriented. In both approaches, which are discussed in Kumbhakar (1996a), the first output arbitrarily serves as a numeraire for the measurement of systematic allocative inefficiency. With multiple outputs, systematic allocative inefficiency can exist between output pairs and between input–output pairs, as well as between input pairs. Using an output orientation to the measurement of technical inefficiency, observed normalized profit can be expressed as

$$\frac{\pi}{p_1} = y_1 + \sum_{m>1}\left(\frac{p_m}{p_1}\right)y_m - \sum_n\left(\frac{w_n}{p_1}\right)x_n$$
$$= \phi\cdot\pi[(p,w)^*;\beta]\cdot\left\{1+\sum_m\left(\frac{1-\kappa_m}{\kappa_m}\right)R_m^* + \sum_n\left(\frac{1-\theta_n}{\theta_n}\right)S_n^*\right\}, \tag{6.2.64}$$

where $\pi[(p,w)^*;\beta]$ is the normalized shadow profit function, $(p,w)^* = [\kappa_m(p_m/p_1), (\theta_n/\phi)(w_n/p_1)]$ is a normalized shadow price vector incorporating both output-oriented technical inefficiency $(0 < \phi \le 1)$ and systematic allocative inefficiency (κ_m, $m = 2, \ldots, M$, and θ_n, $n = 1, \ldots, N$), and

$$R_m^* = \frac{\partial \ln \pi[(p,w)^*;\beta]}{\partial \ln p_m^*}, \qquad m = 2,\ldots, M,$$
$$S_n^* = \frac{\partial \ln \pi[(p,w)^*;\beta]}{\partial \ln w_n^*}, \qquad n = 1,\ldots, N,$$

are output and input shadow profit shares, respectively. Expression (6.2.64) collapses to equation (6.2.44) in the case of a single output.

The relationship between observed normalized profit and shadow normalized profit is given by

$$\ln\frac{\pi}{p_1} = \ln \pi[(p,w)^*;\beta] + \ln H + \ln\phi, \tag{6.2.65}$$

where

$$H = \left\{1+\sum_m\left(\frac{1-\kappa_m}{\kappa_m}\right)R_m^* + \sum_n\left(\frac{1-\theta_n}{\theta_n}\right)S_n^*\right\}.$$

The relationship between observed profit shares and shadow profit shares is given by

$$R_m = \frac{p_m y_m}{\pi} = \frac{1}{H}\cdot\frac{1}{\kappa_m}R_m^*, \qquad m = 2,\ldots, M,$$
$$S_n = \frac{w_n x_n}{\pi} = -\frac{1}{H}\cdot\frac{1}{\theta_n}\cdot S_n^*, \qquad n = 1,\ldots, N. \tag{6.2.66}$$

All that is required now is to assign a translog functional form to the normalized shadow profit function $\pi[(p,w)^*;\beta]$ and exploit

equations (6.2.64)–(6.2.66), which establish the relationships between observed normalized profit and shadow normalized profit and between observed profit shares and shadow profit shares. A translog normalized shadow profit function can be written as

$$\ln \pi[(p,w)^*;\beta]$$
$$= \beta_o + \sum_m \beta_m \ln p_m^* + \sum_n \gamma_n \ln w_n^* + \frac{1}{2}\sum_j\sum_m \beta_{jm}\ln p_j^* \ln p_m^*$$
$$+\frac{1}{2}\sum_k\sum_n \gamma_{kn}\ln w_k^* \ln w_n^* + \sum_m\sum_n \delta_{mn}\ln p_m^* \ln w_n^*, \tag{6.2.67}$$

and the associated shadow profit shares can be written as

$$R_m^* = \beta_m + \sum_j \beta_{jm}\ln p_j^* + \sum_n \delta_{mn}\ln w_n^*, \qquad m = 1,\ldots, M,$$
$$S_n^* = \gamma_n + \sum_k \gamma_{kn}\ln w_k^* + \sum_m \delta_{mn}\ln p_m^*, \qquad n = 1,\ldots, N, \tag{6.2.68}$$

The system of equations to be estimated consists of

$$\ln\frac{\pi}{p_1} = \ln \pi[(p,w)^*;\beta] + \ln H + \ln\phi,$$
$$R_m = \frac{R_m^*}{H\cdot\kappa_m}, \qquad m = 2,\ldots, M,$$
$$S_n = \frac{-S_n^*}{H\cdot\theta_n}, \qquad n = 1,\ldots, N, \tag{6.2.69}$$

where $\ln \pi[(p,w)^*;\beta]$ is defined in equation (6.2.67) and H is defined beneath equation (6.2.65) after substitution of R_m^* and S_n^* from equations (6.2.68).

It is clear, but misleading, from equation (6.2.69) that technical inefficiency enters the expression for observed normalized profit additively. However it also appears in the $\ln H$ term through the w^* terms in its R_m^* and S_n^* components, and it also appears in the $\ln \pi[(p,w)^*;\beta]$ term through the w^* terms. Consequently the effect of technical inefficiency on profit is not additive as it was in the translog cost minimization framework (with input-oriented technical inefficiency). In fact, profit loss due to technical inefficiency depends on prices, regardless of the orientation of the technical efficiency measurement. Consequently technical inefficiency also enters each of the $M+N-1$ profit share equations, through the R_m^* and S_n^* components of the $\ln H$ term. One implication of the complicated form in which technical inefficiency appears in the system of equations (6.2.69) is that its omission will lead to biased and inconsistent estimates of all parameters in the model. Thus estimates of the systematic allocative inefficiency parameters, and of the profit loss due to systematic allocative inefficiency, will be biased and inconsistent. So too will estimates of the structure of the underlying technology, such as substitution and scale elasticities.

It is possible to illustrate this point by rewriting equation (6.2.69) as

$$\ln\frac{\pi}{p_1} = \ln\pi^o + \ln \mathrm{H}_o + \ln \mathrm{H}_{\phi\theta}, \tag{6.2.70}$$

where

$$\begin{aligned}
\ln\pi^o &= \beta_o + \sum_m \beta_m \ln\frac{\kappa_m p_m}{p_1} + \sum_n \gamma_n \ln\frac{\theta_n w_n}{p_1} \\
&\quad + \frac{1}{2}\sum_j\sum_m \beta_{jm} \ln\frac{\kappa_j p_j}{p_1}\ln\frac{\kappa_m p_m}{p_1} \\
&\quad + \frac{1}{2}\sum_k\sum_n \gamma_{kn} \ln\frac{\theta_k w_k}{p_1}\ln\frac{\theta_n w_n}{p_1} \\
&\quad + \sum_m\sum_n \beta_{mn} \ln\frac{\kappa_m p_m}{p_1}\ln\frac{\theta_n w_n}{p_1}, \\
\ln H_o &= \ln\left\{\sum_m\left[(\kappa_m)^{-1}-1\right]\left[\beta_m + \sum_j \beta_{jm}\ln\frac{\kappa_j p_j}{p_1} + \sum_n \delta_{mn}\ln\frac{\theta_n w_n}{p_1}\right]\right. \\
&\quad \left. + \sum_n\left[(\theta_n)^{-1}-1\right]\left[\gamma_n + \sum_k \alpha_{kn}\ln\frac{\theta_k w_k}{p_1} + \sum_m \delta_{mn}\ln\frac{\kappa_m p_m}{p_1}\right]\right\}, \\
\ln H_{\phi\theta} &= \ln\phi - \ln\phi\cdot\left\{\sum_n \gamma_n + \sum_n\sum_k \gamma_{kn}\ln\frac{\theta_n w_n}{p_1}\right. \\
&\quad \left. + \sum_m\sum_n \delta_{mn}\ln\frac{\kappa_m p_m}{p_1} - \frac{1}{2}\ln\phi\cdot\sum_k\sum_n \gamma_{kn}\right\} + \ln\left\{1-\left(\frac{\psi}{H_o}\right)\ln\phi\right\}, \\
\psi &= \sum_m\left[\left((\kappa_m)^{-1}-1\right)\sum_n \delta_{mn}\right] + \sum_n\left[\left((\theta_n)^{-1}-1\right)\sum_k \gamma_{kn}\right].
\end{aligned}$$

The first two terms on the right-hand side of equation (6.2.70) define observed normalized profit in the presence of systematic allocative inefficiency, but in the absence of technical inefficiency.

The third term represents the profit loss due to technical inefficiency, and so the expression for $\ln H_{\phi\theta}$ provides an exact relationship between the magnitude of technical inefficiency and the profit loss it causes. This term is zero if, and only if, $\phi = 1$. Thus neglecting technical inefficiency is equivalent to omitting $\ln H_{\phi\theta}$ from equation (6.2.70). Since $\ln H_{\phi\theta}$ includes all input and output prices, its omission leads to biased and inconsistent parameter estimates.

We conclude that it is not generally possible to decompose profit loss into a component reflecting the cost of systematic allocative inefficiency and a component reflecting the cost of technical inefficiency, since the term $\ln H_{\phi\theta}$ includes both ϕ and θ. This is to be expected in light of Proposition 2.5. However it is easy to see that the term $\ln H_{\phi\theta}$ is independent of θ, and represents a pure technical inefficiency effect, if $\Sigma_n\delta_{mn} = \Sigma_k\gamma_{kn} = 0$. This is a testable restriction, and it corresponds to the condition that the underlying technology be almost homogeneous of degrees +1 and $0 < \Sigma_n\gamma_n < 1$; see Lau (1972) and Hasenkamp (1976).

After $[(\kappa_m p_m/p_1), (\theta_n/\phi)(w_n/p_1)]$ are substituted for $(p, w)^*$, and a multivariate normal error vector is added to the system of equations (6.2.69), the system can be estimated by nonlinear ITSUR. However due to the complicated way in which technical inefficiency enters into the normalized profit function, it is preferable to base estimation on just the $M+N-1$ profit share equations; the only parameter not estimated in this approach is the normalized profit function intercept. Estimation of the profit share equations proceeds as follows. First, rewrite $R_m^* = R_m^o - \delta_{om}\cdot(\ln\phi)$ and $S_n^* = S_n^o - \gamma_{on}\cdot(\ln\phi)$, where $\delta_{om} = \Sigma_n\delta_{nm}$ and $\gamma_{on} = \Sigma_k\gamma_{kn}$, and

$$\begin{aligned}
R_m^o &= \beta_m + \sum_j \beta_{jm}\ln\left[\kappa_j\left(\frac{p_j}{p_1}\right)\right] + \sum_n \delta_{mn}\ln\left[\theta_n\left(\frac{w_n}{p_1}\right)\right], \\
S_n^o &= \gamma_n + \sum_k \gamma_{kn}\ln\left[\theta_k\left(\frac{w_k}{p_1}\right)\right] + \sum_m \delta_{mn}\ln\left[\kappa_m\left(\frac{p_m}{p_1}\right)\right].
\end{aligned}$$

The observed profit share equations can now be rewritten as

$$\begin{aligned}
R_m &= \frac{R_m^*}{H\cdot\kappa_m} \Leftrightarrow \frac{\kappa_m\cdot R_m\cdot H_o - R_m^o}{\delta_{om} - \kappa_m\cdot\Psi\cdot R_m} = -\ln\phi, \\
S_n &= -\frac{S_n^*}{H\cdot\theta_n} \Leftrightarrow -\frac{\theta_n\cdot S_n\cdot H_o - S_n^o}{\gamma_{on} + \theta_n\cdot\Psi\cdot S_n} = -\ln\phi,
\end{aligned} \tag{6.2.71}$$

where H_o and ψ are defined beneath equation (6.2.70). If we assume that $-\ln\phi$ is distributed as multivariate $N^{+}(0,\sigma_\phi^2)$, and if we add a random error vector v distributed as multivariate $N(0,\sigma_v^2)$, the system of $M + N - 1$ equations in (6.2.71) is simply a multiple-equation system of equations analogous to the composed error stochastic production frontier model first encountered in Chapter 3. This system of equations can be estimated by MLE to obtain estimates of all technology parameters and all allocative inefficiency parameters, after which the Kumbhakar (1987) generalization of the JLMS decomposition technique can be employed to obtain estimates of technical inefficiency, which can be inserted into the expression for $\ln H_{\phi\theta}$ to obtain producer-specific estimates of the percentage profit loss due to technical inefficiency. Details of the estimation procedure are available in Kumbhakar (1996a).

6.3 PANEL DATA MODELS

In this section we assume that we have panel data on the prices and the quantities of N inputs used and M outputs produced by each of I producers over T time periods. Producers are allowed to be technically and allocatively inefficient. In Section 6.3.1 we show how to estimate and decompose cost inefficiency in a panel data context, and in Section 6.3.2 we show how to estimate and decompose profit inefficiency in a panel data context. Since the estimation and decomposition of cost inefficiency proceeds independently of the allocative efficiency of producers in output markets, we maintain the single-output assumption in Section 6.3.1, for simplicity and without loss of generality. However the estimation and decomposition of profit inefficiency does involve output allocative inefficiency, and so in Section 6.3.2 we assume that producers produce and market multiple outputs.

In a cross-sectional context information concerning inefficiency is limited. In a strictly parametric framework (e.g., **Translog I** and **II**) it is possible to obtain estimates of technical efficiency differentials among a small number of groups of producers, and it is also possible to obtain estimates of systematic allocative efficiency parameters, which vary across pairs of variables but which are invariant across producers, although the cost of systematic allocative inefficiency is

data dependent and so does vary across producers. In a partly parametric, partly stochastic framework (**Translog III**) it is possible to obtain producer-specific estimates of both technical efficiency and the cost of systematic allocative efficiency, although the former estimates are not consistent. In a panel data context both limitations can be relaxed. It is possible to obtain consistent estimates of producer-specific technical efficiency, using either fixed- or random-effects procedures or MLE. It is also possible to obtain consistent estimates of allocative efficiency parameters, which vary across pairs of variables and either across producers (if T is large relative to I) or through time (if I is large relative to T). This is another illustration of the value of having access to panel data.

6.3.1 Estimating and Decomposing Cost Inefficiency

In this section we extend the analysis of Section 6.2.1 to the panel data context. We begin by showing how the basic framework given by equations (6.2.12)–(6.2.15) is generalized when panel data are available. Although a variety of functional forms can be applied to this generalization, we concentrate on the translog form and leave the application to other forms to the reader. We begin by following a procedure originally suggested by Atkinson and Cornwell (1994a).

Adopting an input orientation to the measurement of technical efficiency, a shadow cost function can be written as

$$c\left(y_{it},\frac{w_{it}^*}{\phi_i};\beta\right)=\min_{\phi_i x_{it}}\left\{\left(\frac{w_{it}^*}{\phi_i}\right)^T(\phi_i x_{it}): f(\phi_i x_{it};\beta)=y_{it}\right\}$$
$$=\frac{1}{\phi_i}c\left(y_{it},w_{it}^*;\beta\right), \tag{6.3.1}$$

where $i = 1, \ldots, I$ indexes producers and $t = 1, \ldots, T$ indexes time periods. The parameters $0 < \phi_i \le 1$, $i = 1, \ldots, I$, provide producer-specific but time-invariant measures of input-oriented technical efficiency. The shadow input prices are defined as $w_{it}^* = [w_{1it}, \theta_{21i}w_{2it}, \ldots, \theta_{N1i}w_{Nit}]$, i = 1, . . . , I, $t = 1, \ldots, T$. Thus allocative inefficiency is no longer systematic, as it was in a cross-sectional framework; it is now producer specific as well as specific to pairs of inputs, although it remains time invariant. In addition to the technology parameter vector β, there are I technical efficiency parameters and $(N-1)\times I$ allocative efficiency parameters to be estimated.

The corresponding shadow input demand equations and shadow input cost share equations are given by

$$x_{nit}\left(y_{it}, w_{it}^*;\beta\right)=\frac{1}{\phi_i}\cdot\frac{\partial c\left(y_{it}, w_{it}^*;\beta\right)}{\partial w_{nit}^*},\qquad n=1,\dots,N, \tag{6.3.2}$$

and

$$S_{nit}\left(y_{it}, w_{it}^*;\beta\right)=\frac{w_{nit}^* x_{nit}\left(y_{it}, w_{it}^*;\beta\right)}{(1/\phi_i)c\left(y_{it}, w_{it}^*;\beta\right)}$$
$$=\frac{w_{nit}^*\left[\partial c\left(y_{it}, w_{it}^*;\beta\right)/\partial w_{nit}^*\right]}{c\left(y_{it}, w_{it}^*;\beta\right)},\qquad n=1,\dots,N, \tag{6.3.3}$$

respectively. Note that the technical efficiency parameters appear in the shadow input demand equations, but they do not appear in the shadow input cost share equations.

Since observed input usage minimizes shadow cost, observed input demand equations can be written as

$$x_{nit}=\frac{1}{\phi_i}\frac{\partial c\left(y_{it}, w_{it}^*;\beta\right)}{\partial w_{nit}^*}$$
$$=\frac{1}{\phi_i}\cdot\frac{c\left(y_{it}, w_{it}^*;\beta\right)\cdot S_{nit}\left(y_{it}, w_{it}^*;\beta\right)}{w_{nit}^*},\qquad n=1,\dots,N. \tag{6.3.4}$$

Observed total expenditure becomes

$$E_{it}=\sum_n w_{nit}x_{nit}=\frac{1}{\phi_i}\cdot c\left(y_{it}, w_{it}^*;\beta\right)\cdot\sum_n\left[S_{nit}\left(y_{it}, w_{it}^*;\beta\right)\cdot\left(\theta_{n1i}\right)^{-1}\right], \tag{6.3.5}$$

and observed input cost share equations become

$$S_{nit}=\frac{w_{nit}x_{nit}}{E_{it}}=\frac{S_{nit}\left(y_{it}, w_{it}^*;\beta\right)\cdot\left(\theta_{n1i}\right)^{-1}}{\sum_k\left[S_{kit}\left(y_{it}, w_{it}^*;\beta\right)\cdot\left(\theta_{k1i}\right)^{-1}\right]},\qquad n=1,\dots,N. \tag{6.3.6}$$

If the shadow cost function $c(y_{it}, w_{it}^*/\phi_i;\beta)$ takes on a translog form, it can be written as

$$\ln c\left(y_{it},\frac{w_{it}^*}{\phi_i};\beta\right)=\beta_o+\beta_y\ln y_{it}+\sum_n\beta_n\ln w_{nit}^*$$
$$+\frac{1}{2}\beta_{yy}(\ln y_{it})^2+\frac{1}{2}\sum_k\sum_n\beta_{kn}\left(\ln w_{kit}^*\right)\left(\ln w_{nit}^*\right)$$
$$+\sum_n\beta_{ny}\left(\ln w_{nit}^*\right)(\ln y_{it})+\ln\frac{1}{\phi_i}, \tag{6.3.7}$$

and the corresponding shadow input cost share equations can be written as

$$S_{nit}\left(y_{it}, w_{it}^*;\beta\right)=\beta_n+\sum_k\beta_{kn}\ln w_{kit}^*+\beta_{ny}\ln y_{it},$$
$$n=1,\dots,N. \tag{6.3.8}$$

From equations (6.3.5), (6.3.7), and (6.3.8), observed expenditure is

$$\ln E_{it}=\beta_o+\beta_y\ln y_{it}+\sum_n\beta_n\ln w_{nit}^*+\frac{1}{2}\beta_{yy}(\ln y_{it})^2$$
$$+\frac{1}{2}\sum_k\sum_n\beta_{kn}\left(\ln w_{kit}^*\right)\left(\ln w_{nit}^*\right)+\sum_n\beta_{ny}\left(\ln w_{nit}^*\right)(\ln y_{it})$$
$$+\ln\left\{\sum_n(\theta_{n1i})^{-1}\left[\beta_n+\sum_k\beta_{kn}\left(\ln w_{kit}^*\right)+\beta_{ny}\ln y_{it}\right]\right\}+\ln\frac{1}{\phi_i}, \tag{6.3.9}$$

and observed input cost shares are

$$S_{nit}=\frac{(\theta_{n1i})^{-1}\left[\beta_n+\Sigma_k\beta_{kn}\ln w_{kit}^*+\beta_{ny}\ln y_{it}\right]}{\Sigma_j(\theta_{j1i})^{-1}\left[\beta_j+\Sigma_k\beta_{kj}\ln w_{kit}^*+\beta_{jy}\ln y_{it}\right]},\qquad n=1,\dots,N. \tag{6.3.10}$$

Estimation of the system consisting of equations (6.3.9) and (6.3.10) can be accomplished in three alternative ways, depending on the assumptions one is willing to make. Atkinson and Cornwell used a random-effects approach, which allows the effects [the $\ln(1/\phi_i)$] to be random without imposing a distributional assumption on them, and which allows for the inclusion of time-invariant regressors. After deleting one input cost share equation and adding a multivariate normal error vector to the remaining N equations, all parameters in the model can be estimated by iterative feasible GLS. Not all ϕ_i are identified, and so one producer is called technically efficient, just as in the random-effects approach outlined in Section 3.3.1. It is also possible to follow a fixed-effects approach based on LSDV, which imposes no distributional assumption on the effects, and which allows the effects to be correlated with the regressors. At the cost of considerable degrees of freedom, it is also possible to allow technical efficiency to be time varying by adapting the Cornwell, Schmidt, and Sickles (1990) approach discussed in Section 3.3.2. Finally, if one is willing to make distributional and independence assumptions, it is possible to estimate the system using MLE techniques. After estimation, the JLMS decomposition technique can be used to obtain estimates of the $\ln(1/\phi_i)$ for each producer, although these estimates are not consistent. In all three approaches the output of the exercise includes estimates of the technology parameter vector β, estimates of the technical efficiency parameter ϕ_i (or perhaps ϕ_{it}) for each producer, and estimates of the allocative inefficiency parameters θ_{n1i}, $n = 2, \dots, N$, for each producer. Potential cost savings resulting from eliminating either technical or allocative inefficiency can be estimated by comparing fitted expenditure with and without the appropriate parameter restrictions imposed.

6.3.2 Estimating and Decomposing Profit Inefficiency

Any of the three functional forms for a normalized profit function – Cobb–Douglas in the single-output case, generalized Leontief or translog in the multiple-output case – studied in Section 6.2.2 can be adapted to the panel data context. Here we consider only the translog form, for its complete flexibility. The analysis follows the translog analysis of Section 6.2.2, with the addition of time subscripts where appropriate. The estimating equations consist of intertemporal versions of equations (6.2.69), which we write as

$$\begin{aligned}
\ln\left(\frac{\pi}{p_1}\right)_{it} &= \ln \pi\left[(p,w)_{it}^*;\beta\right] + \ln H_{it} + \ln \phi_i, \\
R_{mit} &= \frac{R_{mit}^*}{H_{it}\cdot\kappa_{mi}}, \qquad m = 2, \dots, M, \\
S_{nit} &= \frac{-S_{nit}^*}{H_{it}\cdot\theta_{ni}}, \qquad n = 1, \dots, N,
\end{aligned} \tag{6.3.11}$$

where all variables and parameters are defined, apart from time subscripts, in Section 6.2.2. Like the translog cost system considered in Section 6.3.1, this system can be estimated in three ways. Since the techniques require so little modification, we refer the reader to Kumbhakar (1996a, b) for details.

6.4 A GUIDE TO THE LITERATURE

In this chapter we have studied a parametric shadow price approach to the estimation of technical and allocative efficiency, and we have used an inflexible Cobb–Douglas functional form and flexible generalized Leontief and translog functional forms to conduct our analyses. An excellent collection of papers based on the Cobb–Douglas form appears in *Food Research Studies* (1979), and Kumbhakar (1996a, b, c) provides detailed discussions of modeling and estimation procedures for the translog form.

Other papers employing these techniques to estimate cost efficiency in a cross-sectional context include Atkinson and Halvorsen (1984, 1986) (translog) and Bhattacharyya, Parker, and Raffiee (1994) (translog). Studies employing these techniques to estimate profit efficiency in a cross-sectional context include Sidhu (1974) (Cobb–Douglas), Trosper (1978) (Cobb–Douglas), Hollas and Stansell (1988) (translog), Atkinson and Kerkvliet (1989) (translog), Kumbhakar and Bhattacharyya (1992) (translog), and Ali, Parikh, and Shah (1994) (translog).

Atkinson and Cornwell (1993, 1994b) (translog), Balk and van Leeuwen (1997) (translog), and Maietta (1997) (translog) have used these techniques to estimate cost efficiency in a panel data context, and Sickles, Good, and Johnson (1986) (generalized Leontief) and

Kumbhakar (1996b) (translog) have used these techniques to estimate profit efficiency in a panel data context. Finally Atkinson and Cornwell (1998a) (translog) have used these techniques to estimate cost efficiency in a panel data context in which our equations (6.3.9) and (6.3.10) are augmented with an additional equation allowing output price to diverge from shadow marginal cost.