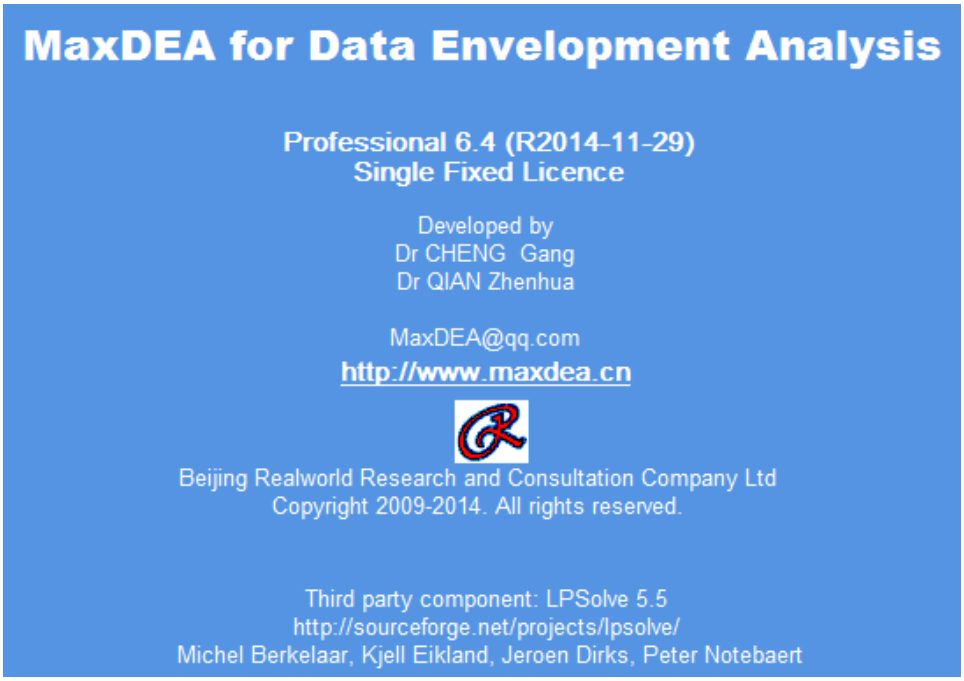
MaxDEA Pro 6.4 Manual



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Contents

1 Overview

Data envelopment analysis (DEA), originally developed by Charnes A, et al. (1978), is a linear programming methodology for evaluating the relative technical efficiency for each member of a set of peer decision making units (DMUs) with multiple inputs and multiple outputs. It has been widely used to measure performance in many areas.

MaxDEA Pro is an easy-to-use but powerful and professional DEA software. It has the most extensive range of the up-to-date DEA models.

Features of MaxDEA Pro

- ✧ Easy to use. It needn't installation and has user-friendly interface. It is very easy to prepare the dataset. You needn't indicate what are the inputs and outputs by field (variable) names or special arrangement of your data. (Tutorial Video)
- ✧ Easy to backup your DEA models and dataset. Everything is saved in a single file. The software, your dataset and the settings for your DEA model are all integrated into a single Access database file (.mdb), and it is the only file needed for MaxDEA Pro, so it is very convenient to backup. After closing and reopening MaxDEA Pro, your database and model settings are still there unchanged.
- ✧ No limitation on the number of DMUs and most comprehensive DEA models.
- ✧ Multiple models can be run at the same time. You can rename or copy the MaxDEA Pro file freely. Each copy of the file contains one DEA model with all your data and settings saved in the file. You can open and run multiple files simultaneously, taking full advantage of your multi-core CPU. It's very useful for time-consuming analysis such as bootstrapping. (Tutorial Video)
- ✧ Most important of all, MaxDEA Pro provides all the possible combinations of up-to-date DEA models. To use a combination of multiple DEA models, just choose all the relevant options. For example, Network-Malmquist model with weakly disposable bad outputs can be achieved by choosing the settings for Network, Undesirable outputs, Weak disposability and Malmquist, at the same time.

DEA Models available in MaxDEA Pro

MaxDEA Pro has the most comprehensive DEA models and **all their possible combinations**, such as the combination of "Undesirable Outputs" and "Malmquist" (**Malmquist-Luenberger** Productivity Index).

1) Distance to measure efficiency
   - ✓ Radial
   - ✓ Maximum distance to frontier：Slack based measure (SBM)
   - ✓ Modified SBM
   - ✓ Minimum distance to weak efficient frontier
   - ✓ Minimum distance to strong efficient frontier
   - ✓ Directional distance function,
   - ✓ Range directional model (RDM)
   - ✓ Hybrid: mixture of radial and SBM measure (EBM)
   - ✓ Hybrid: mixture of radial and non-radial fields
   - ✓ Cost /Revenue / Profit / Revenue-cost ratio
2) Orientation to measure efficiency
   - ✓ Input-oriented
   - ✓ Output-oriented
   - ✓ Non-oriented
   - ✓ Input-oriented (modified)
   - ✓ Output-oriented (modified)
   - ✓ Non-oriented (input-prioritized)
   - ✓ Non-oriented (output-prioritized)
   - ✓ Non-oriented (generalized priority)
3) RTS to measure efficiency
   - ✓ Constance returns to scale (CRS)
   - ✓ Variable returns to scale (VRS)
   - ✓ Non-increasing returns to scale (NIRS)
   - ✓ Non-decreasing returns to scale (NDRS)
   - ✓ Generalized returns to scale (GRS)
4) FDH model
5) Window model
6) Malmquist model
   - ✓ Adjacent Malmquist
   - ✓ Fixed Malmquist

- ✓ Global Malmquist
- ✓ Sequential Malmquist
- ✓ Window-Malmquist (Adjacent)
- ✓ Window-Malmquist (Fixed)

7) Network model
8) Parallel model
9) Dynamic model
10) Context-dependent model
11) Super-efficiency model
12) Cross efficiency model
13) Game cross efficiency model
14) Undesirable output model
15) Nondiscretionary input/output model (non-controllable model, measure specific model)
16) Bounded input/output model
17) Preference (weighted) model
18) Restricted projection model
19) Weak disposability model
20) Restricted multiplier model (assurance region model, trade-offs between inputs and outputs)
21) Cluster model
22) Customized reference ret model
    - ✓ Variable-benchmark model
    - ✓ Fixed-benchmark model
    - ✓ Minimum efficiency model
23) MetaFrontier DEA and MetaFrontier Malmquist
24) Bootstrap
    - ✓ Bootstrap of DEA Score
    - ✓ Bootstrap of Malmquist Index

2 A Quick Guide

System Requirements

MaxDEA runs under Windows systems in any language, including:

Windows XP

Windows 2003

Windows Vista

Windows 7

Windows 8

Windows 10

MaxDEA is developed with VBA for Access, so Microsoft **Access** is required.

If the program file (MaxDEA.mdb) cannot open, it indicates that Microsoft Office Access is not installed in your computer, and you must install MS Access 2003 or higher version (professional edition **or** runtime edition) first. (Tutorial Video)

Access 2013 Runtime can be downloaded **free** at Microsoft website:

http://www.microsoft.com/en-us/download/details.aspx?id=39358

（32-bit & 64-bit）

Access 2010 Runtime can be downloaded **free** at Microsoft website:

http://www.microsoft.com/en-us/download/details.aspx?id=10910

（32-bit & 64-bit）

Access 2007 Runtime can be downloaded **free** at Microsoft website:

http://www.microsoft.com/en-us/download/details.aspx?id=4438

（32-bit）

If you use 32-bit Office, run the program file “MaxDEA 6.mdb”, no matter whether your Windows system is 32-bit or 64-bit;
If you use 64-bit Office, run the program file “MaxDEA 6_x64.mdb”.
If you don’t know what type of Office you are using, just have a try, only one of the two program files can open.

Prepare Data

MaxDEA Pro can import data from **Excel, Access, dBase and comma delimited** text files. Preparing your data in Excel is the most convenient way.

Data format

The first and only the first row must contain field (column, variable) names. The field names can be anything you want.

Table 2-1(A) An example of correct data format

Company	Capital	Labor	Product
A	4323	875	93608
B	2295	469	225559
C	379	1286	327068
D	6644	1339	201354
E	1436	297	188926
F	6281	1266	413738
G	7459	1502	114022

Table 2-1(B) An example of wrong data format

DMU	Input		Output
Company	Capital	Labor	Product
A	4323	875	93608
B	2295	469	225559
C	6379	1286	327068
D	6644	1339	201354
E	1436	297	188926
F	6281	1266	413738
G	7459	1502	114022

DMU Name

DMU Name is the identifier for each DMU, so it must be **unique**. The DMU name can be anything, such as letters, characters, numbers or mixture of them. Note that the DEA results are sorted by DMU name, so if you expect the results to be displayed in the order as that in the dataset, your dataset should be sorted by DMU name. If you use numbers as DMU name, such as 1,2,…,100, MaxDEA

will automatically format the numbers by adding leading zeros, like 001, 002,…,100, so that the results are displayed in numerical order. The number of leading zeros added depends on the length of the maximum number.

Panel Data

For **panel data**, there must be an additional column indicating the time of the data. DMU Name must be **unique** within each period. Panel data are prepared for Malmquist, Window, and Dynamic models. If you try to run a cross-sectional DEA model using panel data, there will be an error message indicating that the DMU names are not unique. Panel data can be sorted by DMU name or by period, but not necessary.

Panel data for Malmquist model and Window model are not necessary to be balanced, i.e., missing values at some periods are permitted. But panel data for Dynamic model must be balanced.

Table 2-2(A) An example of correct panel data format (balanced)

Period	Company	Capital	Labor	Product
1	A	4323	875	93608
1	B	2295	469	225559
1	C	6379	1286	327068
2	A	6644	1339	201354
2	B	1436	297	188926
2	C	6281	1266	413738
3	A	7459	1502	114022
3	B	4464	903	212444
3	C	4524	915	462677

Table 2-2(B) A second example of correct panel data format (balanced)

Period	Company	Capital	Labor	Product
1	A	4323	875	93608
2	B	1436	297	188926
1	B	2295	469	225559
3	A	7459	1502	114022
3	B	4464	903	212444
1	C	6379	1286	327068

2	A	6644	1339	201354
2	C	6281	1266	413738
3	C	4524	915	462677

Table 2-2(C) An example of correct panel data format (unbalanced)

Period	Company	Capital	Labor	Product
1	A	4323	875	93608
1	B	2295	469	225559
1	C	6379	1286	327068
2	A	6644	1339	201354
2	B	1436	297	188926
3	A	7459	1502	114022
3	B	4464	903	212444
3	C	4524	915	462677

The **period field** must be integer numbers, such as

1, 2, 3……

2001, 2002, 2003……

But they needn't to be continuous. The following time series are permitted:

1, 2, 5, 8……

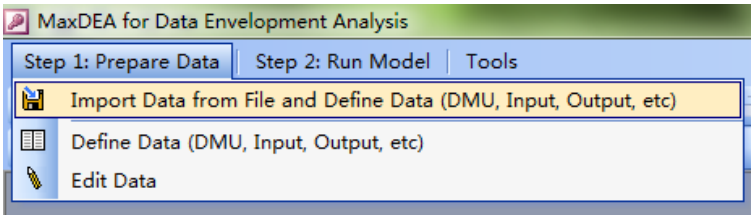
2001, 2005, 2009……

200101, 200102, 200302……

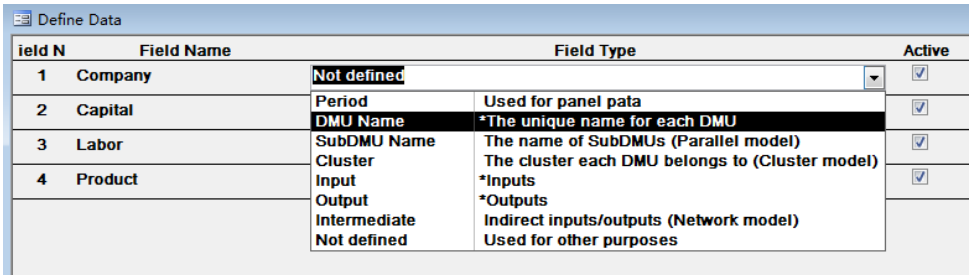
Negative Data

MaxDEA Pro permits negative values in inputs or outputs. The default method of dealing with negative values is the variant radial measure (VRM) proposed by Cheng, et al. (2013). This method of dealing with negative values in the VRM is also applied to other types of distances, such as the non-radial (SBM) model.

Import and Define Data



After the data are imported, the “Data Define” window will open automatically. Defining data is to tell MaxDEA Pro which column is the DMU name, which columns are inputs, and which columns are outputs. At least the fields for **DMU Name**, **Inputs** and **Outputs** must be defined.



Missing values are not permitted. If a record in the data has missing values, you must either delete this record, or replace the missing values.

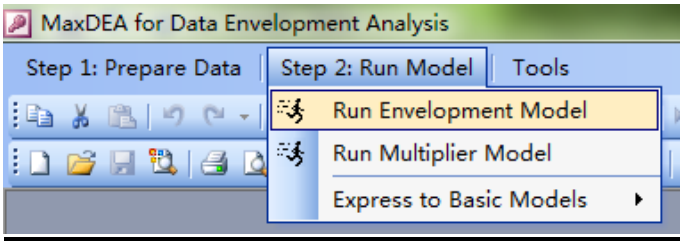
Table 2-3 Missing values not permitted

DMU	Input1	Input2	Output
A	4323	875	93608
B	2295	469	225559
C	6379	✗	327068
D	6644	1339	201354
E	1436	297	188926
F	6281	1266	413738
G	7459	1502	114022

If you want to eliminate some inputs or outputs from the model, you can wither set these fields “Not defined”, or deactivate them. Only those active fields will be included

The imported data and their definitions are permanently saved, so after MaxDEA Pro is closed, they will not be lost.

Run Model



There are two types of DEA models. One is called multiplier model, i.e., the

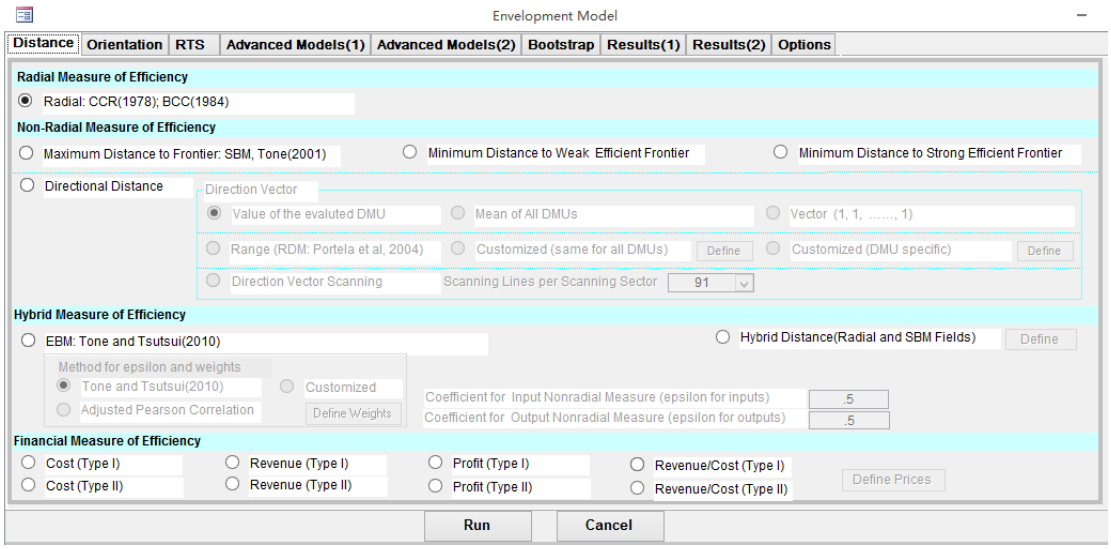
primal model, and the other is called envelopment model, i.e., the dual model. The menu “Run Envelopment Model” is for envelopment models, “Run Multiplier Model” is for multiplier models, and “Express to Basic Models” is an express way to basic models (CCR and BCC), which is easy to use for new DEA leaners.

The settings (options) for the DEA model are permanently saved, so after MaxDEA Pro is closed, they will not be lost. If you want to save a backup of your DEA model, just copy the “MaxDEA X.mdb” file and rename it as you want, such as CCR.mdb, BCC.mdb.

3 Envelopment DEA model

Distance (method of measuring efficiency)

Including radial measure, non-radial measure, hybrid measure and financial measure



Radial

It measures the necessary proportional improvements of relevant factors (inputs / outputs) for the evaluated DMU to reach the frontier.

Ref. to (Banker, et al., 1984; Charnes A, et al., 1978) for CCR model and BCC model.

Maximum distance to frontier: SBM

The SBM model maximizes the average improvements of relevant factors (inputs / outputs) for the evaluate DMU to reach the frontier(Tone Kaoru, 2001). The target (projected point) determined by this method is a strong efficient point on the frontier which is the farthest to the evaluated DMU. This method seems unreasonable in that for an inefficient unit much more efforts may be needed to reach the much farther target.

Minimum distance to weak efficient frontier

A closer target means less effort for an inefficient unit to reach the efficient frontier. The minimum distance to weak efficient frontier finds a closest target located on the weak efficient frontier.

Ref. to (Briec, 1999; Charnes A., et al., 1996)

Minimum distance to strong efficient frontier

The minimum distance to strong efficient frontier determines a closest target located on the strong efficient frontier.

Ref. to (Aparicio, et al., 2007; Jahanshahloo, et al., 2012)

Directional Distance Function

Directional distance function is a generalized form of radial model (Chambers, et al., 1996; Chung, et al., 1997).

MaxDEA Pro provides a further generalized form of the directional distance function. Ref. to Cheng & Zervopoulos (2014) for more discussion.

The directional distance function model in MaxDEA Pro is expressed as

$$\text{Efficiency score} = \min \frac{1-\frac{1}{m}\sum_{i=1}^{m} w_i \beta g_i / x_{io}}{1+\frac{1}{\omega_y+\omega_b}\left(\omega_y \frac{1}{\sum w_r}\frac{1}{s}\sum_{r=1}^{s} w_r \beta g_r / y_{ro} + \omega_b \frac{1}{\sum w_u}\frac{1}{p}\sum_{u=1}^{p} w_u \beta(-g_u)/b_{uo}\right)}$$

$$\text{s.t. } X\lambda + \beta g_x \le x_0$$
$$Y\lambda - \beta g_y \ge y_0$$
$$B\lambda - \beta g_b \le B_0$$

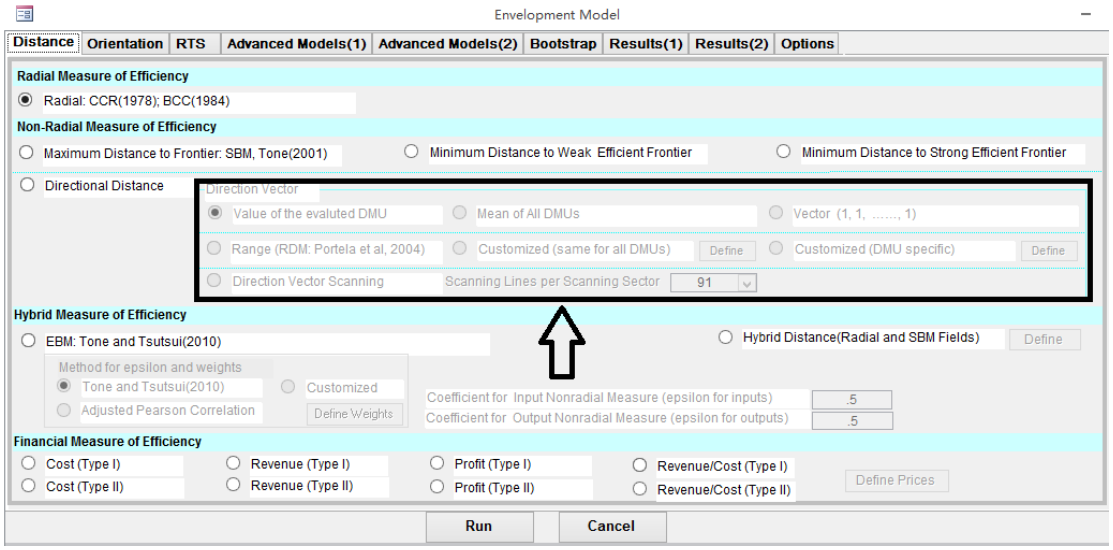
$g_x \ge 0$, $g_y \ge 0$, $g_b \le 0$: the direction vector for inputs, good outputs and bad outputs
ω_y: the general weight for all good outputs
ω_b: the general weight for all bad outputs
w_i, w_r, w_u: the weight for each input, good output and bad output
Note that the direction vector values for bad outputs are non-positive ($g_b \le 0$).

Or equivalently

$$\max \beta$$
$$\text{s.t. } X\lambda + \beta g_x \le x_0$$
$$Y\lambda - \beta g_y \ge y_0$$
$$B\lambda - \beta g_b \le B_0$$

$$\text{Efficiency score} = \frac{1-\frac{1}{m}\sum_{i=1}^{m} w_i \beta^* g_i / x_{io}}{1+\frac{1}{\omega_y+\omega_b}\left(\omega_y \frac{1}{\sum w_r}\frac{1}{s}\sum_{r=1}^{s} w_r \beta^* g_r / y_{ro} + \omega_b \frac{1}{\sum w_u}\frac{1}{p}\sum_{u=1}^{p} w_u \beta^* |g_u|/b_{uo}\right)}$$

There are six types of direction vectors for the directional distance function in MaxDEA.



1) The values of the evaluated DMU, i.e., using x_0 and y_0 as the direction vector;

2) The mean of all DMUs;

3) Vector (1, 1, 1…);

4) Gap between the evaluated value and best value, RDM (Portela, et al., 2004)

5) Customized (same for all DMUs);

6) Customized (DMU specific).

The directional distance function model will be equivalent to the radial model with the first type of direction vector, i.e. $g_x = x_0$, and $g_y = y_0$.

Note that in the non-user-defined direction vector types, i.e.,

1) Value of the evaluted DMU,

2) Mean of All DMUs,

3) Vector (1, 1, ……, 1),

4) Gap between the evaluated value and best value (RDM: Portelal et al, 2004), and

7) Direction Vector Scanning,

the direction vector values for bad outputs will automatically be changed into

negative values.

In the user-defined direction vector types, i.e.,

5) Customized (same for all DMUs), and

6) Customized (DMU specific),

if the user defines positive values for the direction vector of bad outputs, they will not be changed into negative values in the model.

Also See Weak Disposability Model

Hybrid distance: EBM

EBM is a hybrid distance prosed by Tone K. & Tsutsui (2010b).

Also See An epsilon-based measure of efficiency in DEA - An alternative method for the affinity index.

http://www.maxdea.cn/Sharing/EBM_Note.pdf

Hybrid distance(Radial and SBM fields)

This is another type of hybrid model. It is a mixture of radial model and Non-radial model, and it is a generalized form of radial and non-radial models(Cooper, et al., 2007). If Hybrid model is selected, you should define the distances of inputs/outputs. Click the "Define" button right to the Hybrid check box, the distance definition form will open.

If all the inputs/outputs are set to be radial in Hybrid model, it is equivalent to radial model, and if all set to be non-radial, it is equivalent to SBM model.

Cost, Revenue, Profit, and Revenue/Cost Ratio Models

These models deal with financing aspects of DEA models in the case that prices of inputs/outputs are known. It is related to the topic of allocative efficiency(Cooper, et al., 2007).

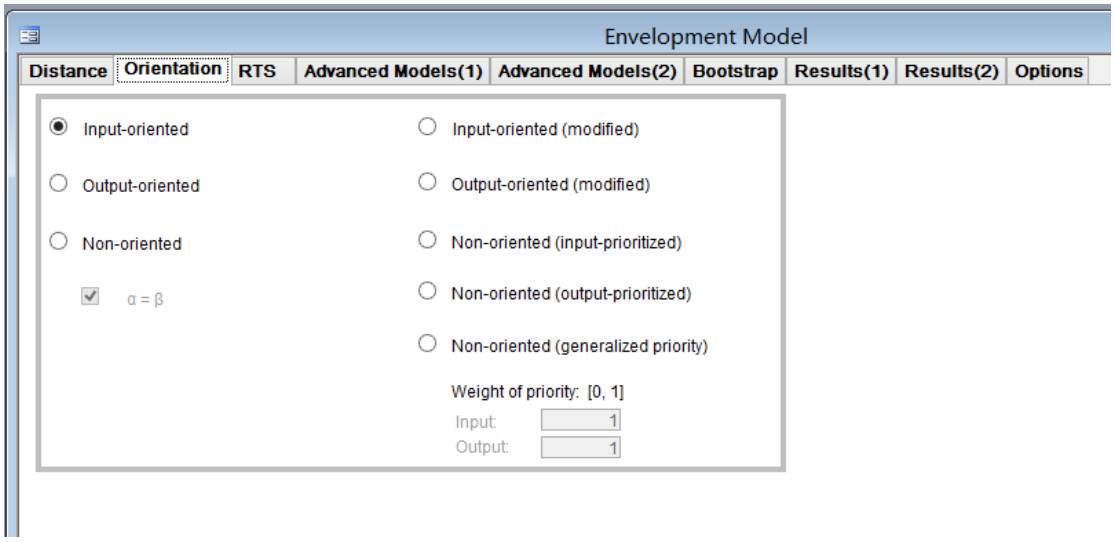
There are two types models, and MaxDEA names them as "Type I "and "Type II" respectively. The difference between type I and type II is that type I models use the

original inputs/outputs values in constraints, while type II models use cost/revenue values of inputs/outputs in constraints. (Type I is traditional and commonly used.)

To run the above models, price information must be set first using the "Define" button on the right side. Cost model needs input prices, Revenue model needs output prices, and Profit and Revenue/Cost Ratio models need both input and output prices. The price fields must be kept "Not define" at the stage of "Define Data".

Note that efficiency score of profit model might be **negative**.

Orientation



Traditional orientations include

1) input-orientation,
2) output-orientation and
3) non-orientation.

MaxDEA Pro provides five new orientations:

4) input-orientation(modified),
5) output-orientation(modified),
6) non-orientation (input-prioritized),
7) non-orientation (output-prioritized),
8) non-orientation (generalized priority).

The significance of the new orientations is that the modified input-oriented and

modified output-oriented super-efficiency models overcome the infeasibility problem in the traditional super-efficiency models.

See Overcoming the infeasibility of the super-efficiency DEA model: A model with generalized orientation.

http://www.maxdea.cn/Sharing/Generalized_Orientation.pdf

Note that if you choose "**Non-oriented**" radial model in MaxDEA, the LP of the model is

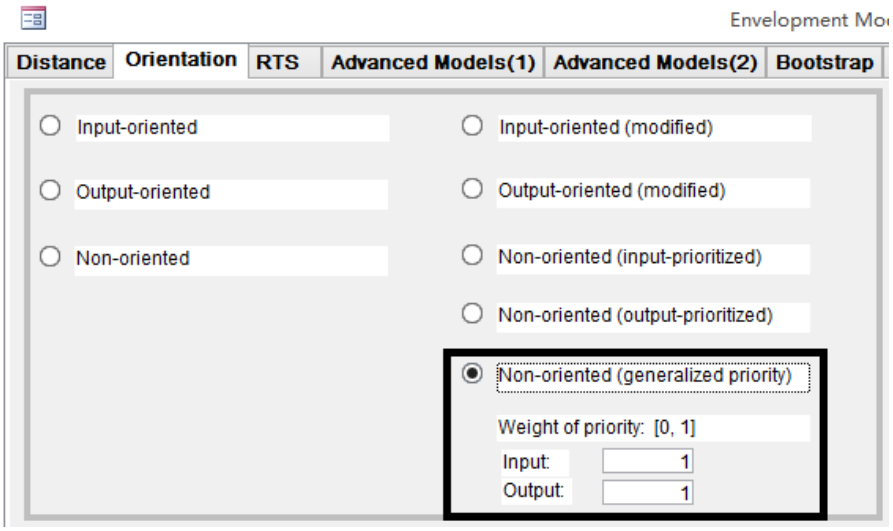
$$\min \quad \frac{1-\beta}{1+\beta}$$

$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le (1-\beta) x_{ik}, \quad i = 1, 2, ..., m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge (1+\beta) y_{rk}, \quad r = 1, 2, ..., s$$

$$\lambda_j \ge 0, \quad j = 1, 2, ..., n$$

If you want to use the following model, in which input movement and output movement are separated and measured as α and β respectively, you should choose "**Non-oriented (generalized priority)"**, and set the weights of the input and the output to 1.



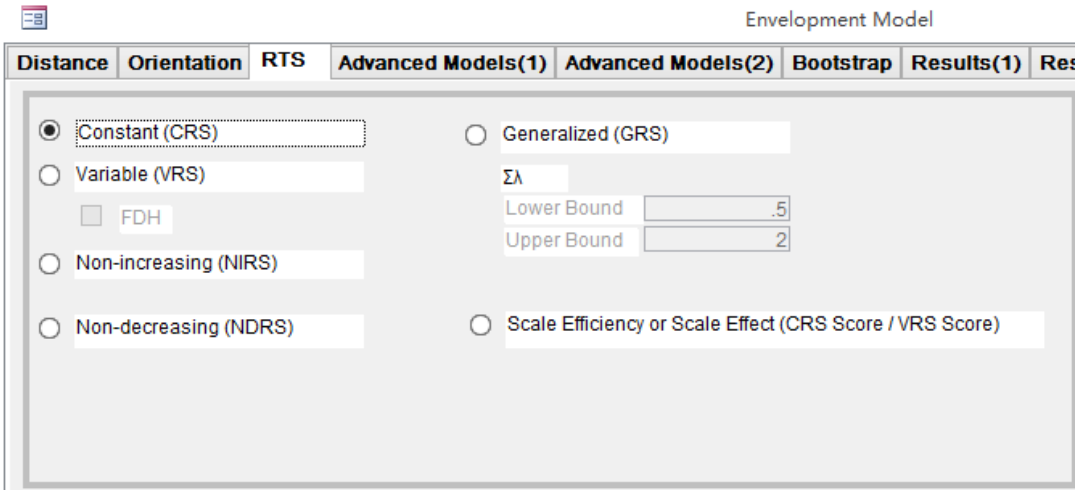
$$\min \frac{1-\alpha}{1+\beta}$$

$$s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le (1-\alpha) x_{ik}, \quad i = 1, 2, ..., m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge (1+\beta) y_{rk}, \quad r = 1, 2, ..., s$$

$$\lambda_j \ge 0, \quad j = 1, 2, ..., n$$

Returns to scale (RTS)



Five types of RTS (returns to scale) are available with MaxDEA: constant(CRS), variable(VRS), non-increasing(NIRS), non-decreasing(NDRS) and generalized(GRS). If GRS is selected, you should also set the lower bound (L) and upper bound (U) of Σλ. GRS is a generalized form of the other four types of RTS, with the following relationship (Cooper, et al., 2007):

Table 3-1 Relationship between GRS and other RTS

	GRS	
	L	**U**
CRS	0	+∞
VRS	1	1

NIRS	0	1
NDRS	1	+∞

The option “Scale Efficiency or Scale Effect (CRS Score / VRS Score)” is a special case in which both CRS and VRS models will be solved, and the scale efficiency or scale effect will be computed.

In Radial model, Scale Efficiency = CRS efficiency / VRS efficiency.

In non-radial models, Scale Effect = CRS efficiency / VRS efficiency.

RTS Estimation is also provided with this option.

For Malmquist model, the relationship between scale factors are as follows

- Scale Effect on Malmquist Index = Malmquist Index (CRS) / Malmquist Index (VRS)
- Scale Effect on Efficiency Change = Efficiency Change (CRS) / Efficiency Change (VRS)

  In Radial model, Scale Effect on Efficiency Change is “**Scale Efficiency Change**”.
- Scale Effect on Technological Change = Technological Change (CRS) / Technological Change (VRS)
- Scale Effect on Malmquist Index = Scale Effect on Efficiency Change * Scale Effect on Technological Change

Note that when “Scale Efficiency or Scale Effect (CRS Score / VRS Score)” is selected, the result window only reports the scale efficiency scores or scale effect scores, but all the detailed results under CRS and VRS can be obtained through the Menu “Tools – Browse Results”.

CCR model

See Distance-Radial

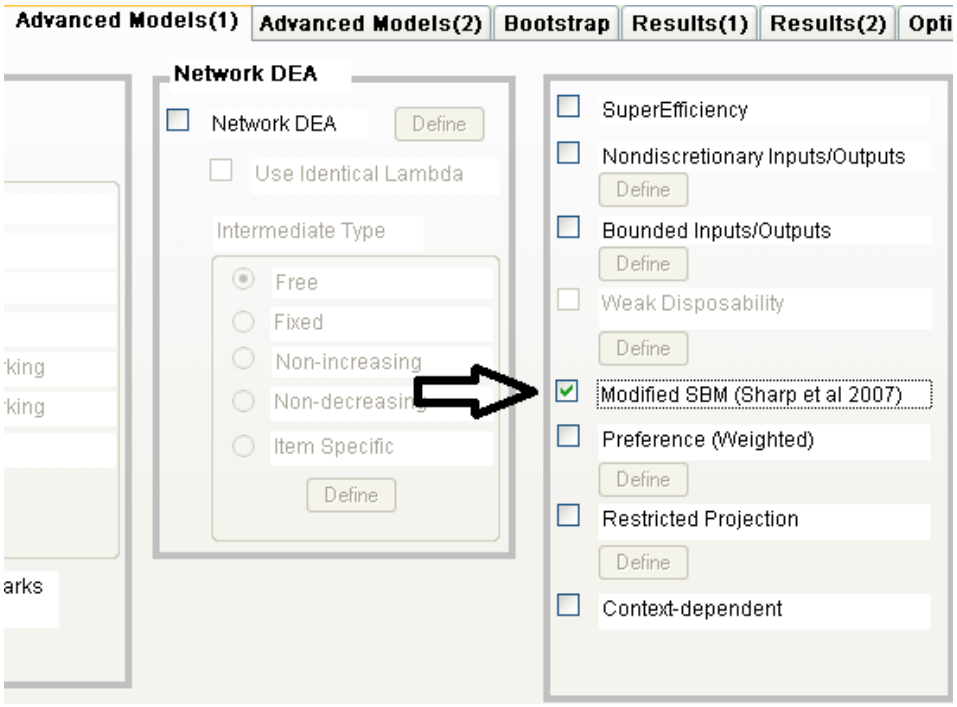
BCC model

See Distance-Radial

SBM model

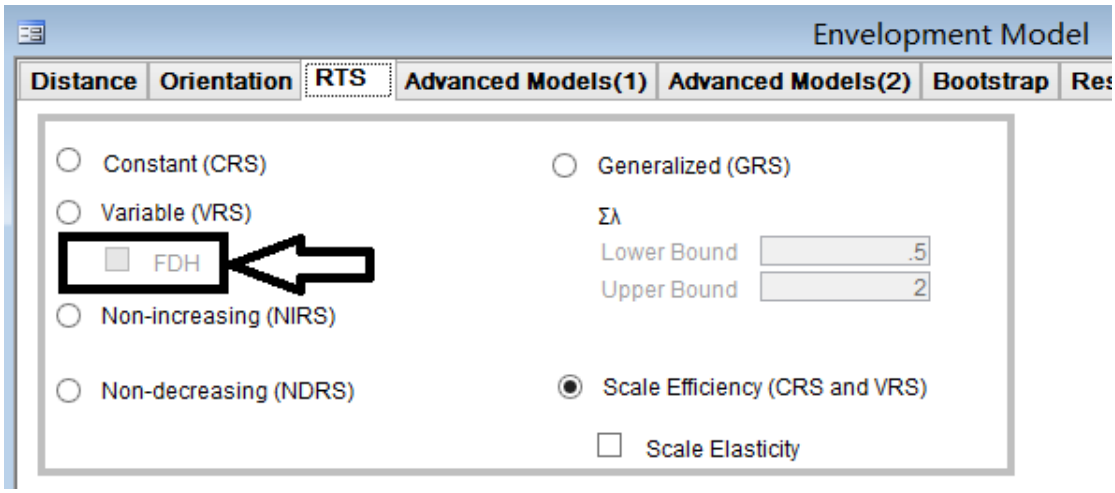
See Distance- Maximum distance to frontier: SBM

Modified SBM



Ref. to Sharp et al (2007)

FDH Model



The purpose of Free Disposal Hull (FDH) model is to ensure that efficiency evaluation is based on only actually observed performances. The input-oriented FDH model can be expressed using the following mixed integer LP,

$\min \ \theta$

$$st \quad \theta x_0 - X\lambda - s^- = 0$$

$$Y\lambda - y_0 - s^+ = 0$$

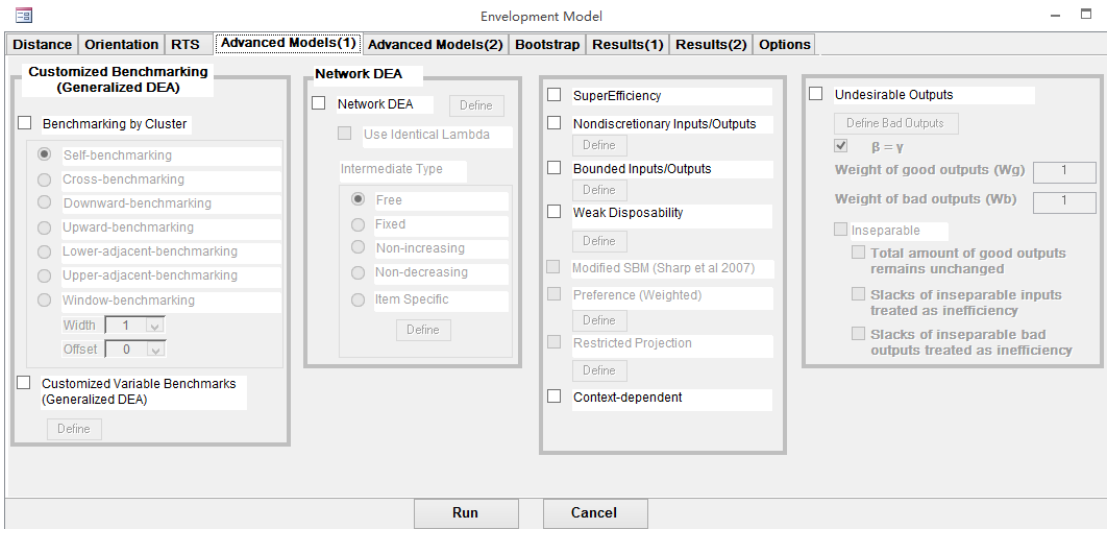
$$s^-, s^+ \geq 0$$

$$e\lambda = 1,\ \lambda \in \{0,1\}$$

Note that the FDH model is located under “Returns to scale (RTS)”.

Ref. to (Deprins, et al., 1984; Tulkens, 1993)

Advanced models



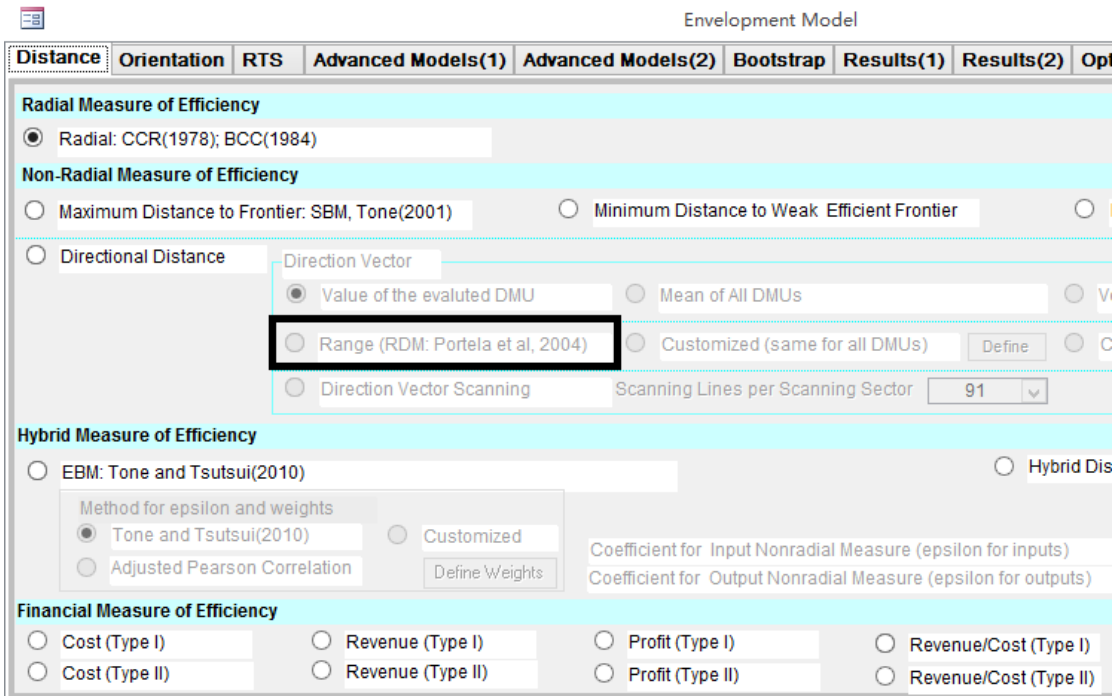
Super-efficiency model

The difference between Super-efficiency model and standard efficiency model is that in super models the DMU_0 (the DMU evaluated) is eliminated from the reference set (indicated by $j \neq 0$ in the LP).

Ref. to (Andersen & Petersen, 1993; Tone K., 2002).

The Super-efficiency score can be greater than 1. In some cases, the LP for some DMUs will be infeasible(Seiford & Zhu, 1999). In such cases, you can decide whether the program returns 1 as the score through the option “No optima”.

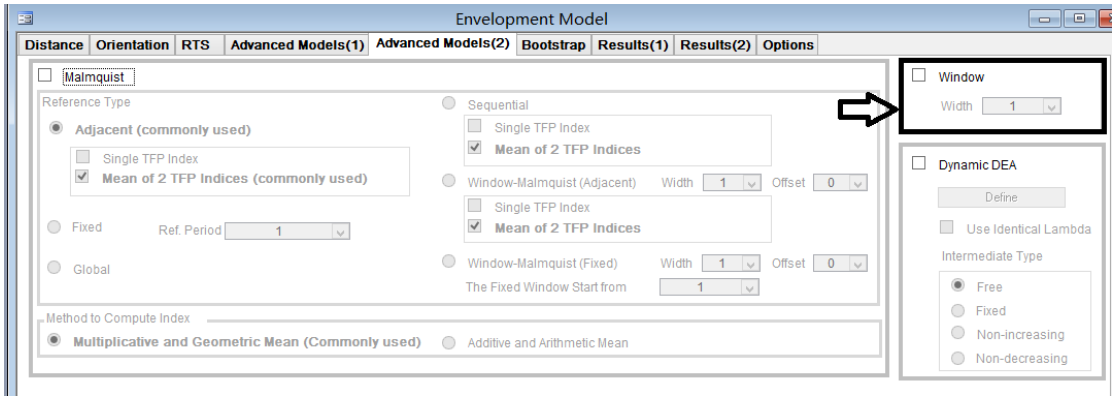
Range Directional model (RDM)



A type of directional distance function proposed by Portela, et al. (2004)

See Distance- Directional Distance Function

Window Model



The window model deals with panel data. Window width must be set for window models.

Panel data for Malmquist model and Window model are not necessary to be balanced, i.e., missing values at some periods are permitted.

If you want to get the DEA results for each year independently, you should set window width to 1. It is equivalent to dividing the panel data into many datasets (each dataset contains one year's data) and analyzing each dataset one by one.

Non-controllable Inputs/outputs Model

See Nondiscretionary Inputs/outputs Model

Measure Specific Model

See Nondiscretionary Inputs/outputs Model

Restricted Projection Model

The restricted projection model is to add constraints on the ratios of input or output projections (targets). The constraints are similar to the restricted multiplier model (type I).

See Restricted Multiplier Model

Nondiscretionary Inputs/outputs Model

MaxDEA Pro provides a generalized nondiscretionary inputs/outputs models. And the nondiscretionary model is actually a special case of a more generalized model - bounded model (see the next part), so we just provide the LPs for bounded models

To run the nondiscretionary model, you should first set the discretion status of inputs/outputs ("Full Discretion" means complete control, and "Non- or Part-discretion" means limited control), and set the discretion degree for nondiscretionary inputs/outputs. To do so, just click the "Define" button on the right side. The format of discretion degree is percent.

Let's talk about some special cases of nondiscretionary models.

1) The first special case is the nondiscretionary radial model with the discretion degrees of all nondiscretionary inputs/outputs being zero. Such a case is also called "**non-controllable**" radial model (Cooper, et al., 2007).
2) The second special case is the nondiscretionary radial model with the discretion degrees of all nondiscretionary inputs/outputs being 100%. Such a case is also called "**non-discretionary**" radial model (Cooper, et al., 2007), or "**measure specific**" model (Zhu, 2009).

Please note that setting the discretion degrees of nondiscretionary inputs/outputs to

be 100% is **not** equivalent to setting the inputs/outputs to be "full-discretion" in radial models, which is a little puzzling.

3) The third special case is the nondiscretionary SBM model with the discretion degrees of all nondiscretionary inputs/outputs being zero. Such a case is also called "**non-controllable**" SBM model (Cooper, et al., 2007).
4) The last special case is the nondiscretionary SBM model with the discretion degrees of all nondiscretionary inputs/outputs being 100%. Such a case is equivalent to a normal model. In other words, setting the discretion degrees of nondiscretionary inputs/outputs to be 100% is equivalent to setting the inputs/outputs to be "Full Discretion" in SBM models.

   If all the inputs/outputs are set to be "Full Discretion", it is equivalent to the normal model.

Bounded Inputs/outputs Model

To run a bounded model, you should first set the lower and upper bounds of the bounded inputs/outputs, by the "define" button on the right side. The fields indicating lower and upper bounds of inputs/outputs must be kept "Not define" at the stage of "Define Data". Lower bound must be less than or equal to the original value, and upper bound must be greater than or equal to the original value.

If all the inputs/outputs are set to be "Full Discretion", it is equivalent to the normal model.

The nondiscretionary model is equivalent to the bounded model by setting the lower and upper bounds as follows,

Lower bound = original value ×(1－discretion degree)

Upper bound = original value /(1－discretion degree)

Undesirable Outputs Model

Undesirable model deals with the circumstances that bad outputs exit, by setting the improvements of bad outputs in an opposite direction to the good outputs, which means that more good outputs and less bad outputs are desired.

In MaxDEA Pro, you can develop DEA models with undesirable outputs using

any types of distances, such as radial, directional distance function, SBM and hybrid model.

The radial measure with undesirable outputs is based on the directional distance function (Chung, et al., 1997), and can be seen as a special case of directional distance function.

The SBM model with undesirable outputs is expressed as

$$\min \quad \rho = \frac{1-\frac{1}{m}\sum_{i=1}^{m} s_i^- / x_{io}}{1+\frac{1}{s}\sum_{r=1}^{s} s_r^+ / y_{ro}}$$

$$st \quad x_0 - X\lambda - s^- = 0$$

$$Y^g\lambda - y_0^g - s^{g+} = 0$$

$$Y^b\lambda - y_0^b + s^{b+} = 0$$

$$\lambda, s^-, s^+ \geq 0$$

Ref. to Cooper, et al. (2007).

See Weak Disposability Model

Inseparable Good and Bad Outputs Model

Inseparable outputs model deals with the situation that certain bad outputs are inseparable from the corresponding good outputs (and certain inputs). Reducing bad outputs is inevitably accompanied by reduction in good outputs.

Refer to Cooper, et al. (2007) for details about this model. There are two types of inseparable models in this book, one is names as “SBM-NS”, and the other is named as “NS-Overall”.

Please note that MaxDEA provides a more flexible inseparable model, you should make the options in MaxDEA according to the linear programs in the literature. The relevant options include **orientation, weak disposability, discretion, and whether**

- ✓ Total amount of good outputs remains unchanged;

- ✓ Slacks of inseparable inputs are treated as inefficiency; and
- ✓ Slacks of inseparable bad outputs are treated as inefficiency.

Note that in MaxDEA the increasing upper bound for outputs is calculated with the following formula

Upper bound = original value /(1－discretion degree).

For example, if the discretion degree is set to be 20%, the actual percentage of increasing will be 1 / (1 – 20%) – 100% = 25%. If you want to set the upper bound of percentage of increasing to be 20%, the discretion degree should be 1 – 1 / (1 + 20%) = 16.6667%.

Weak Disposability Model

The weak disposability model restricts the slacks of input/outputs with weak disposability to being zero:

$$st\ \ \theta x_0^s - X^s\lambda - s^{s-} = 0$$

$$\theta x_0^w - X^w\lambda = 0$$

$$Y^s\lambda - \phi y_0^s - s^{s+} = 0$$

$$Y^w\lambda - \phi y_0^w = 0$$

s indicates strong disposability and w indicates weak disposability.

Click the button “Define” on the right side to set inputs/outputs with weak disposability.

Note: Weak Disposability is not applicable in non-radial (SBM) model.

See a note on weak disposability: Weak Disposability of Undesirable Outputs in Data Envelopment Analysis: Seemingly Reasonable but Actually Questionable http://www.maxdea.cn/Sharing/Weak_Disposability.pdf

Preference (weighted) Model

Weights can be assigned to inputs and outputs in SBM models according to their relative importance as follows:

$$\min \quad \rho = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} w_i s_i^- / x_{io}}{1 + \frac{1}{s}\sum_{r=1}^{s} w_r s_r^+ / y_{ro}}$$

$$\sum_{i=1}^{m} w_i = m$$

$$\sum_{r=1}^{s} w_r = s$$

Click the button “Define” on the right side to set weights.

The user can set the weights freely, for example, suppose there are 2 inputs and 2 outputs in the model, you can set

weight for input1 = 1,

weight for input1 = 2,

weight for output1 = 1,

weight for output2 = 2,

and MaxDEA will calculate the corresponding w_i and w_r automatically.

If you set the weights of all inputs/outputs to be 1, it is equivalent to the normal model.

Context–dependent model

Ref. to Seiford & Zhu (2003)

Cost, Revenue, Profit, and Revenue/Cost Ratio Model

See Distance

Cluster Model

Cluster Model deals with the situation that the DMUs are categorized according to their characteristics. There are 7 types of cluster models according to the relationship between the clusters evaluated and the clusters as benchmarks.

Data format for Cluster Model

For **cluster data**, there must be an additional column indicating the clusters. Each DMU belongs to only one cluster, and a DMU should not belong to different clusters.

The following are examples of cluster data:

Table 3-2(A) Example data with clusters

DMU	Cluster	Input1	Input2	Output1	Output2
A	1	4323	875	93608	187196
B	1	2295	469	225559	451099
C	1	6379	1286	327068	654116
D	1	6644	1339	201354	402688
E	1	1436	297	188926	377833
F	1	6281	1266	413738	827456
G	2	7459	1502	114022	228024
H	2	4464	903	212444	424867
I	2	4524	915	462677	925334

Cluster data needn't be balanced. The number of DMUs in each cluster can be different.

Table 3-2(B) Example panel data with clusters

Period	DMU	Cluster	Input1	Input2	Output1	Output2
1	A	1	4323	875	93608	187196
1	B	1	6644	1339	201354	402688
1	C	1	7459	1502	114022	228024
1	D	1	2295	469	225559	451099
1	E	1	1436	297	188926	377833
1	F	1	4464	903	212444	424867
1	G	2	6379	1286	327068	654116
1	H	2	6281	1266	413738	827456
1	I	2	4524	915	462677	925334
2	A	1	4341	944	93656	187279
2	B	1	6689	1374	201442	402767
2	C	1	7559	1511	114107	228101
2	D	1	2362	530	225604	451107
2	E	1	1513	381	188937	377886
2	F	1	4544	937	212446	424869
2	G	2	6406	1361	327102	654188
2	H	2	6369	1327	413797	827479
2	I	2	4563	942	462739	925393

Note: For panel data, the same DMU in different periods must belong to the same cluster. For example, DMU A belongs to cluster 1 in period 1, and it must belong to cluster 1 in period 2 and other periods.

The **cluster field** must be integer numbers, such as

1, 2, 3……

But they needn't to be continuous. The following time series are permitted:

1, 2, 5, 8……

2001, 2005, 2009……

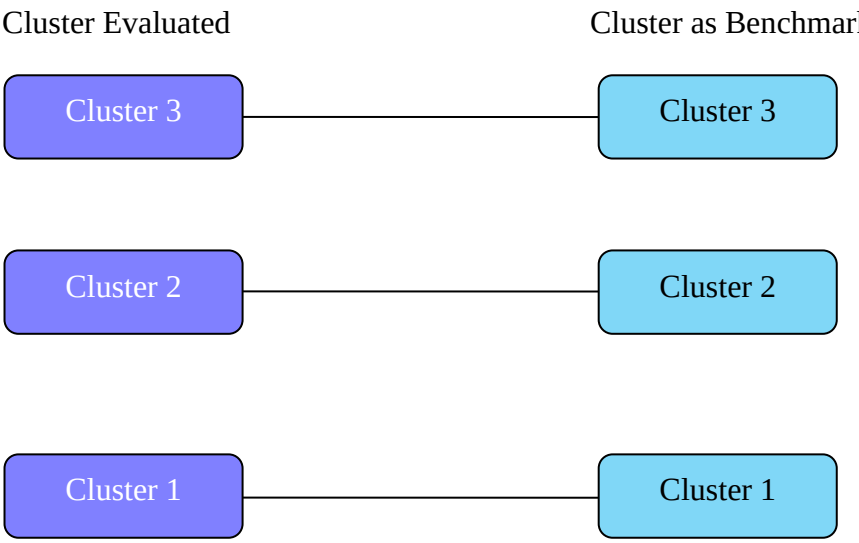
200101, 200102, 200302……

Self-benchmarking

In self-benchmarking model, each DMU is evaluated within the cluster it belongs to. In effect it is a batch mode of evaluating the DMUs many times, analyzing one cluster each time.

For example, if all the DMUs are categorized into 3 clusters, the results of self-benchmarking model are the same as the following models. Firstly run the model using the data of cluster 1 only, secondly cluster 2 only, and lastly cluster 3 only.

Self-benchmarking



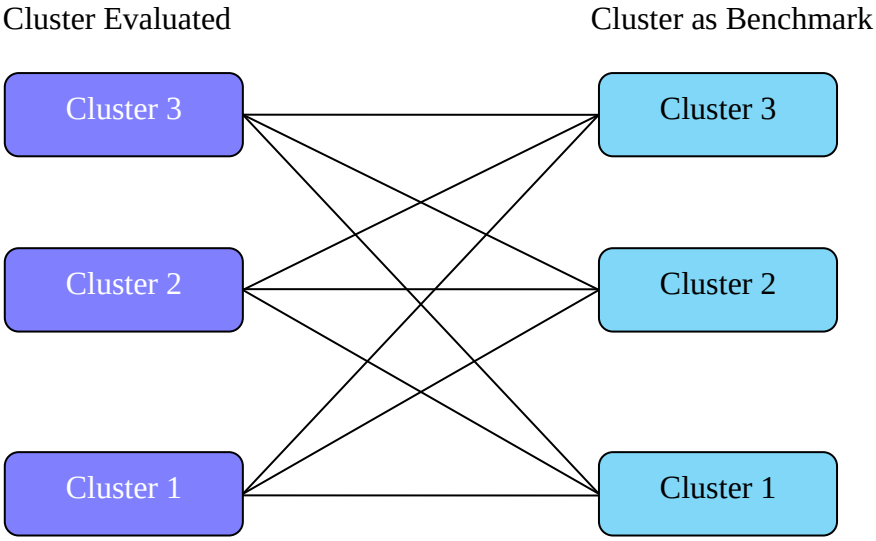
Cross-benchmarking

In cross-benchmarking model, each DMU is evaluated successively with every cluster as the reference set separately.

For example, if all the DMUs are categorized into 3 clusters,

1) Firstly all DMUs are evaluated using the DMUs in cluster 1 as reference set,
2) Secondly all DMUs are evaluated using the DMUs in cluster 2 as reference set, and
3) Lastly all DMUs are evaluated using the DMUs in cluster 3 as reference set.

Cross-benchmarking



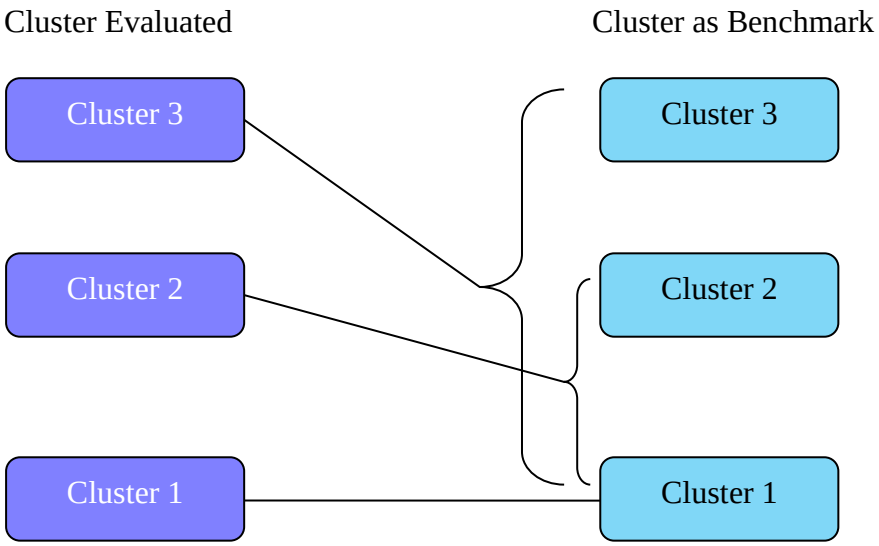
Downward-benchmarking

In downward-benchmarking, the DMUs in a cluster are evaluated with the DMUs in its own cluster and those in the cluster with a lower cluster number (cluster ID) as the reference set.

For example, if all the DMUs are categorized into 3 clusters: clusters 1, 2 and 3,

1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 1 only as the reference set;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 1 and cluster 2 as the reference set; and
3) the DMUs in cluster 3 are evaluated with the DMUs in cluster 1, cluster 2, and cluster 3 (all the DMUs) as the reference set.

Downward -benchmarking



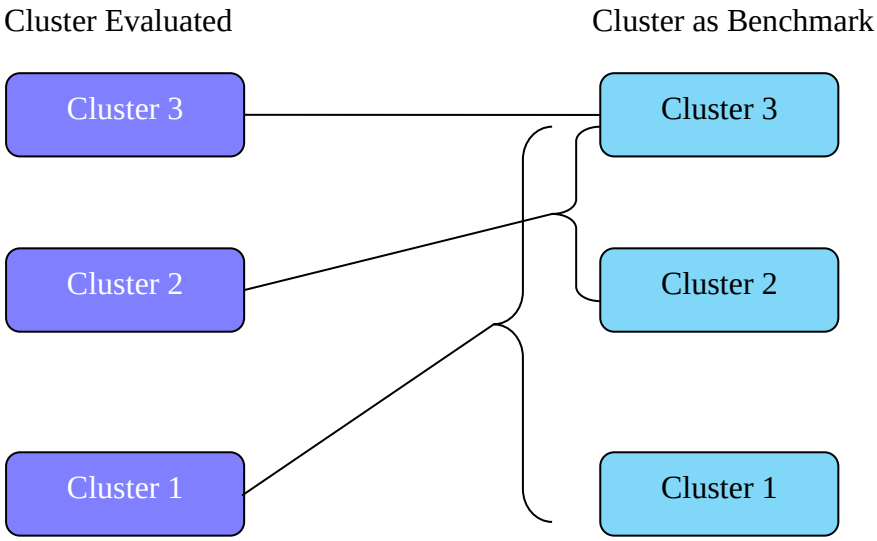
Upward-benchmarking

Contrary to downward-benchmarking, in upward-benchmarking, the DMUs in a cluster are evaluated with the DMUs in its own cluster and those in the cluster with a upper cluster number (cluster ID) as the reference set.

For example, if all the DMUs are categorized into 3 clusters: clusters 1, 2 and 3,

1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 1, cluster 2, and cluster 3 (all the DMUs) as the reference set;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 2 and cluster 3 as the reference set; and
3) the DMUs in cluster 3 are evaluated with the DMUs in cluster 3 only as the reference set.

Upward-benchmarking



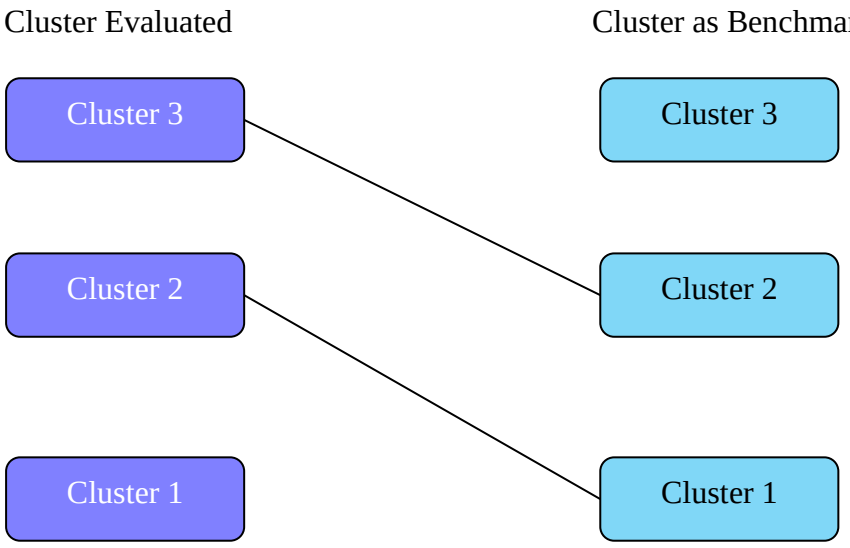
Lower-adjacent-benchmarking

In lower-adjacent-benchmarking, the DMUs in a cluster are evaluated with the DMUs in the cluster with a lower cluster number (cluster ID) as the reference set.

For example, if all the DMUs are categorized into 3 clusters: clusters 1, 2 and 3,

1) the DMUs in cluster 1 are not evaluated;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 1 as the reference set; and
3) the DMUs in cluster 3 are evaluated with the DMUs in and cluster 2 as the reference set.

Lower-adjacent-benchmarking



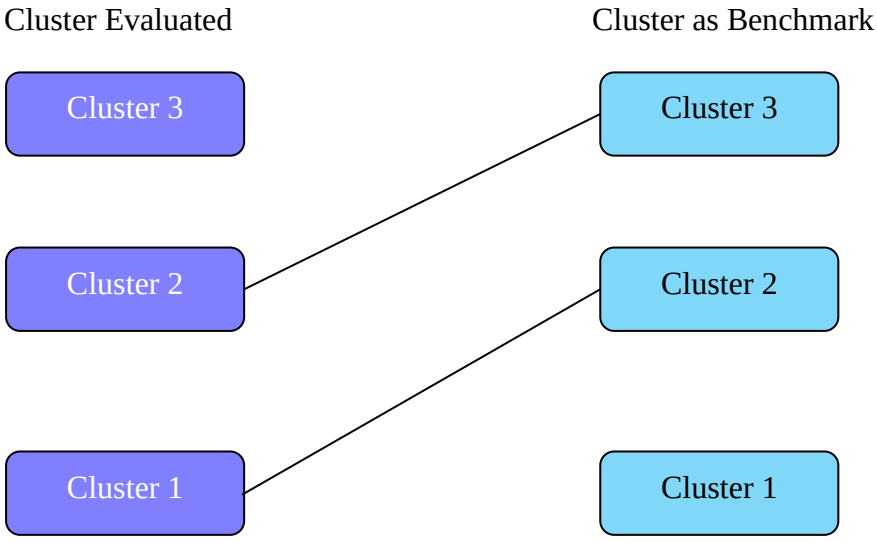
Upper-adjacent-benchmarking

Contrary to lower-adjacent-benchmarking, in upper-adjacent-benchmarking, the DMUs in a cluster are evaluated with the DMUs in the cluster with a upper cluster number (cluster ID) as the reference set.

For example, if all the DMUs are categorized into 3 clusters: clusters 1, 2 and 3,

1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 2 as the reference set;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 3 as the reference set; and
3) the DMUs in cluster 3 are not evaluated.

Upper-adjacent-benchmarking



Window-benchmarking

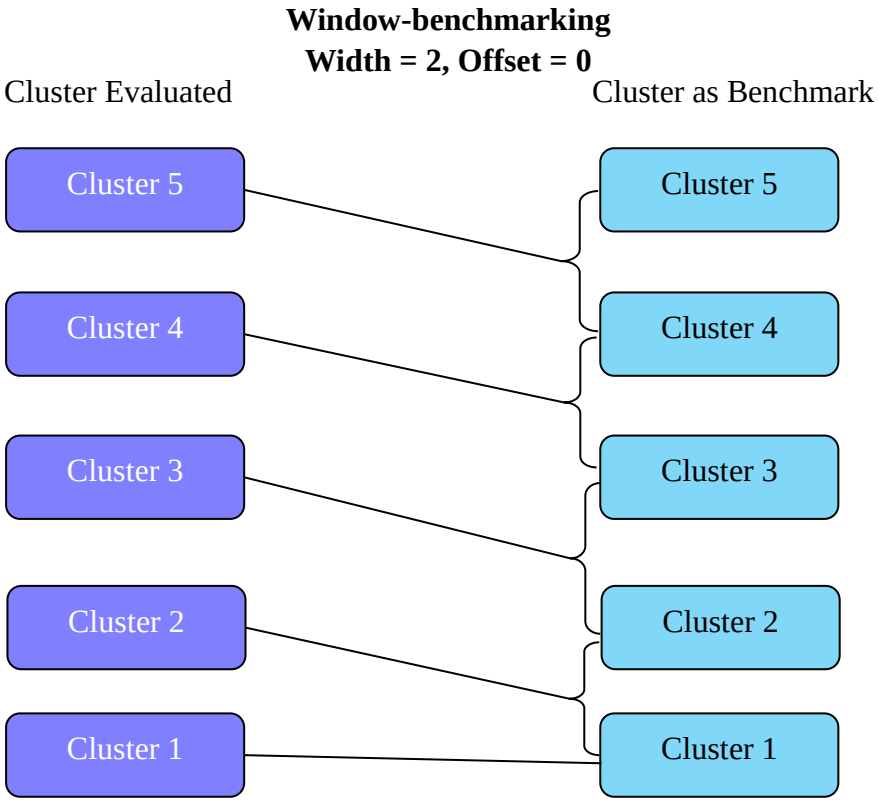
In window-benchmarking, the DMUs in a cluster are evaluated with the DMUs in a cluster-window including its own cluster as the reference set. By default (offset =0), the cluster-window is composed of its own cluster and a number of adjacent clusters. The number of the adjacent clusters included in the window is width – 1.

Suppose all the DMUs are categorized into 5 clusters: clusters 1, 2, 3, 4 and 5.

If window width = 2, offset = 0,

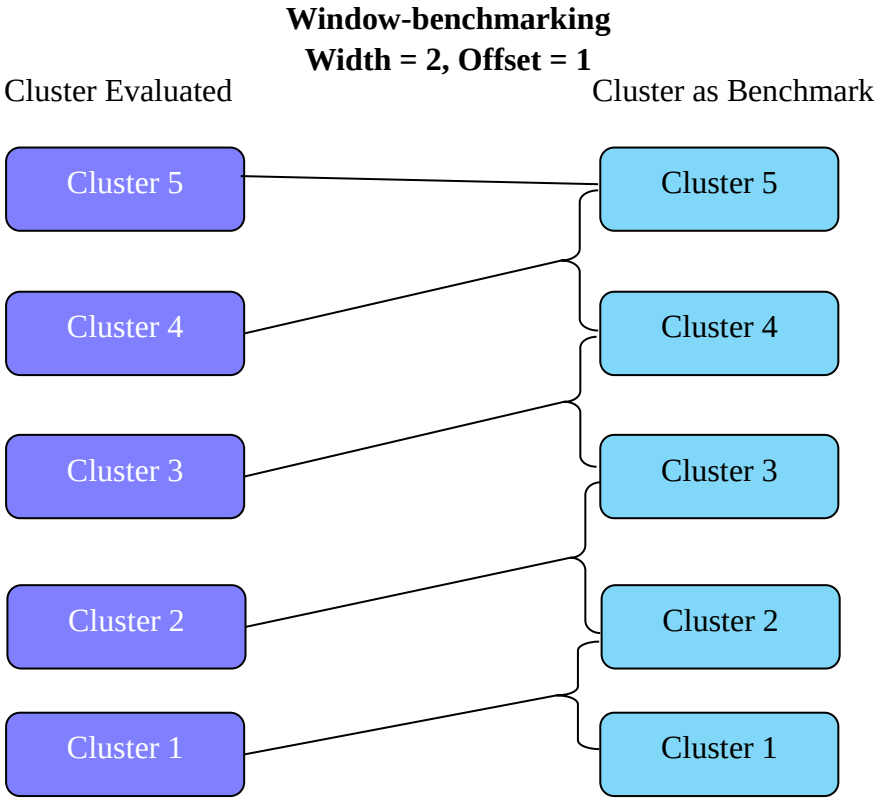
1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 1 only as the reference set, and its referred window is not full width;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 1 and cluster 2 as the reference set;

3) the DMUs in cluster 3 are evaluated with the DMUs in cluster 2 and cluster 3 as the reference set;
4) the DMUs in cluster 4 are evaluated with the DMUs in cluster 3 and cluster 4 as the reference set;
5) the DMUs in cluster 5 are evaluated with the DMUs in cluster 4 and cluster 5 as the reference set.



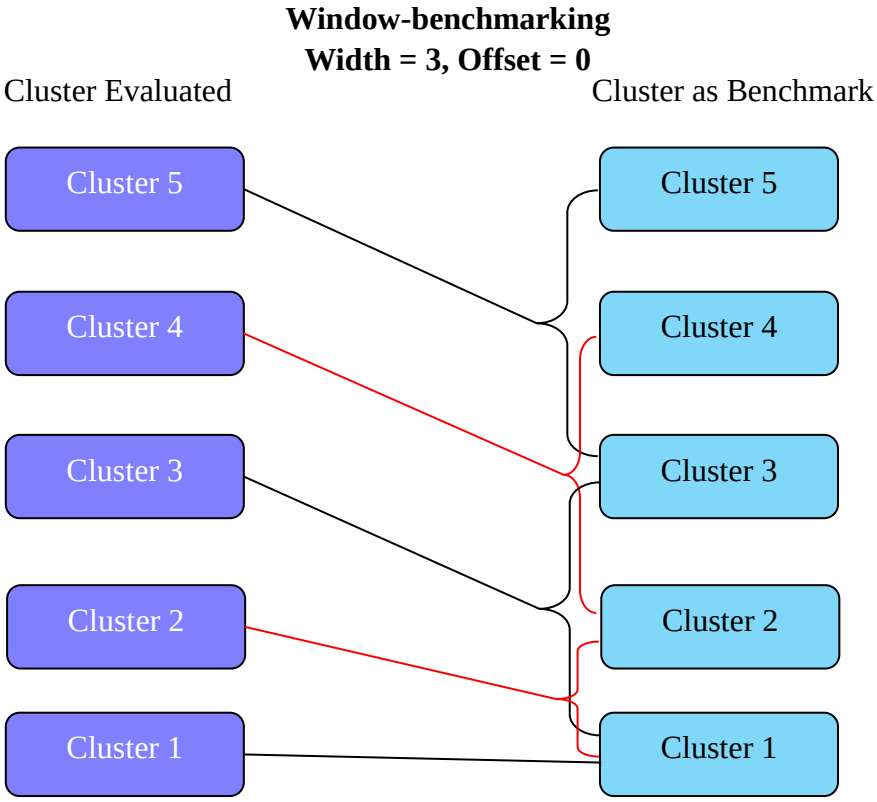
If window width = 2, offset = 1,

1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 1 and cluster 2 as the reference set;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 2 and cluster 3 as the reference set;
3) the DMUs in cluster 3 are evaluated with the DMUs in cluster 3 and cluster 4 as the reference set;
4) the DMUs in cluster 4 are evaluated with the DMUs in cluster 4 and cluster 5 as the reference set;
5) the DMUs in cluster 5 are evaluated with the DMUs in cluster 5 only as the reference set, and its referred window is not full width.



If window width = 3, offset = 0,

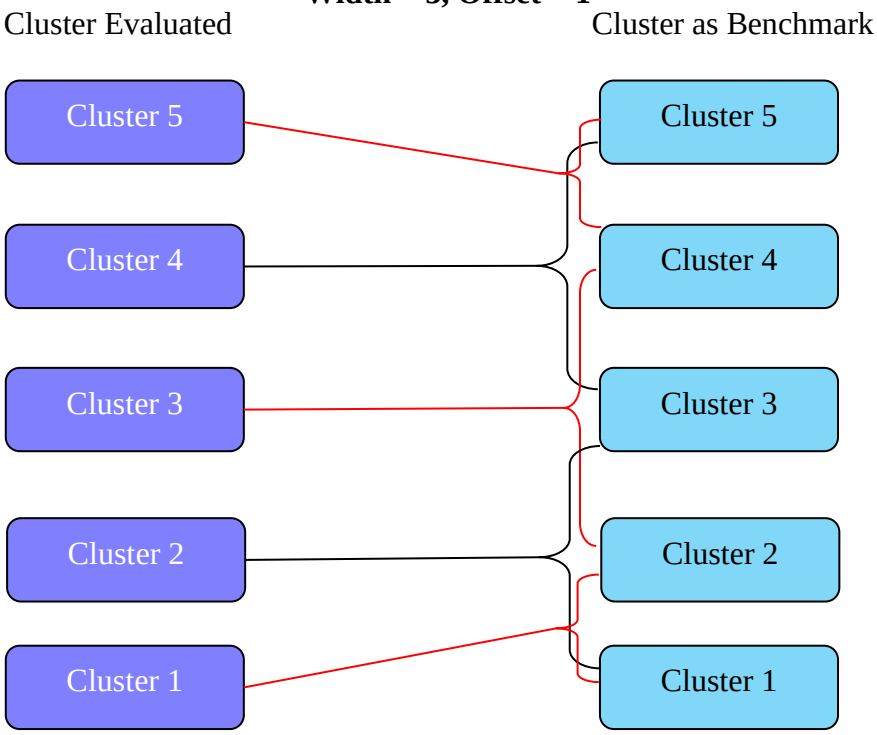
1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 1 only as the reference set, and its referred window is not full width;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 1 and cluster 2 as the reference set, and its referred window is not full width;
3) the DMUs in cluster 3 are evaluated with the DMUs in cluster 1, cluster 2 and cluster 3 as the reference set;
4) the DMUs in cluster 4 are evaluated with the DMUs in cluster 2, cluster 3 and cluster 4 as the reference set;
5) the DMUs in cluster 5 are evaluated with the DMUs in cluster 3, cluster 4 and cluster 5 as the reference set.



If window width = 3, offset = 1,

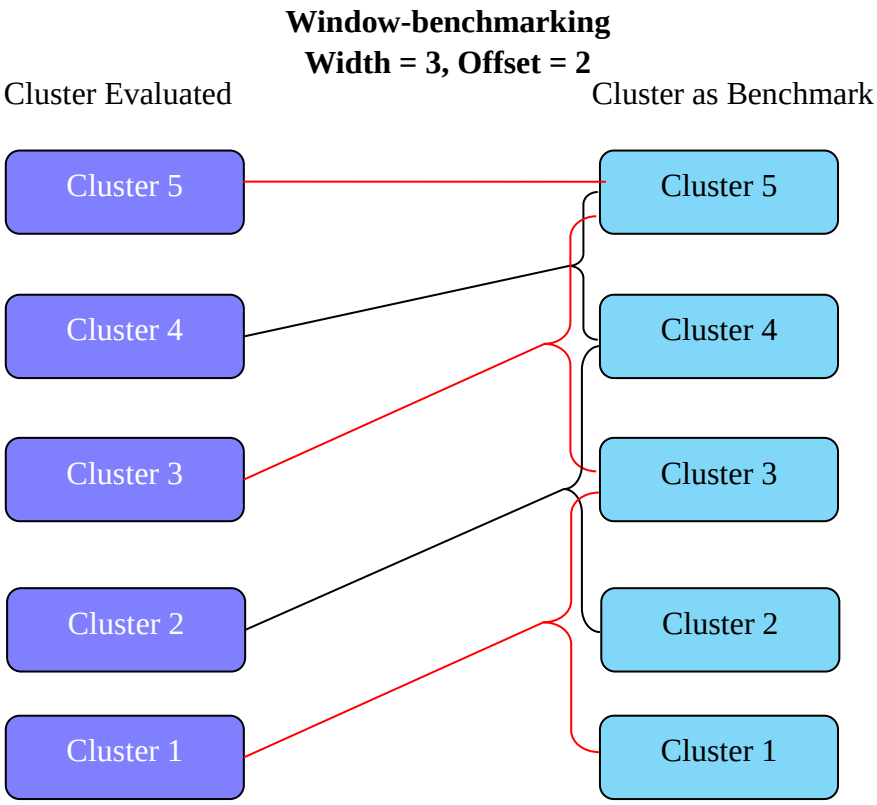
1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 1 and cluster 2 as the reference set, and its referred window is not full width;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 1, cluster 2 and cluster 3 as the reference set;
3) the DMUs in cluster 3 are evaluated with the DMUs in cluster 2, cluster 3 and cluster 4 as the reference set;
4) the DMUs in cluster 4 are evaluated with the DMUs in cluster 3, cluster 4 and cluster 5 as the reference set;
5) the DMUs in cluster 5 are evaluated with the DMUs in cluster 4 and cluster 5 as the reference set, and its referred window is not full width.

Window-benchmarking
Width = 3, Offset = 1



If window width = 3, offset = 2,

1) the DMUs in cluster 1 are evaluated with the DMUs in cluster 1, cluster 2 and cluster 3 as the reference set;
2) the DMUs in cluster 2 are evaluated with the DMUs in cluster 2, cluster 3 and cluster 4 as the reference set;
3) the DMUs in cluster 3 are evaluated with the DMUs in cluster 3, cluster 4 and cluster 5 as the reference set;
4) the DMUs in cluster 4 are evaluated with the DMUs in cluster 4 and cluster 5 as the reference set, and its referred window is not full width;
5) the DMUs in cluster 5 are evaluated with the DMUs in cluster 5 only as the reference set, and its referred window is not full width.



Note: in the stage of "Define Data", the field indicating cluster ID should be defined as "Cluster".

Variable-benchmark Model

See Customized Benchmarking

Customized Benchmarking

In customized benchmarking model, you can customize the reference DMU set for calculating efficiency scores. Take the input-oriented CCR model as an example,

$$\min \ \theta$$

$$st \ \ \theta x_0 - X^r \lambda - s^- = 0$$

$$Y^r \lambda - y_0 - s^+ = 0$$

$$\lambda, s^-, s^+ \geq 0$$

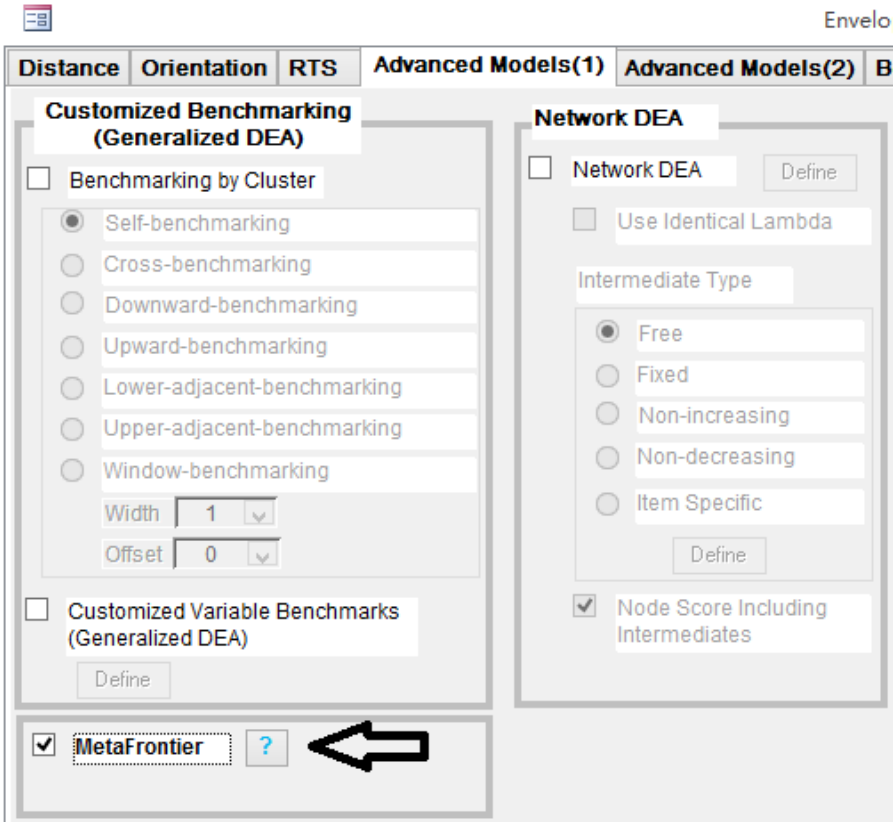
The superscript r indicates the DMUs included in the customized reference DMU set R.

The above model is also called **variable-benchmark model** by Cook, et al. (2004), because any of the DMUs in R might be a benchmark, but it is not necessary for all the DMUs in R to be the benchmarks.

To set a variable-benchmark model, open the menu "Run Envelopment Model", check the option "Variable-benchmark", and click the "Define" button to select variable benchmarks.

MetaFrontier Model

Click the "**?**" button in MaxDEA Pro to see details on MetaFrontier DEA and MetaFrontier Malmquist model.



Network DEA Model

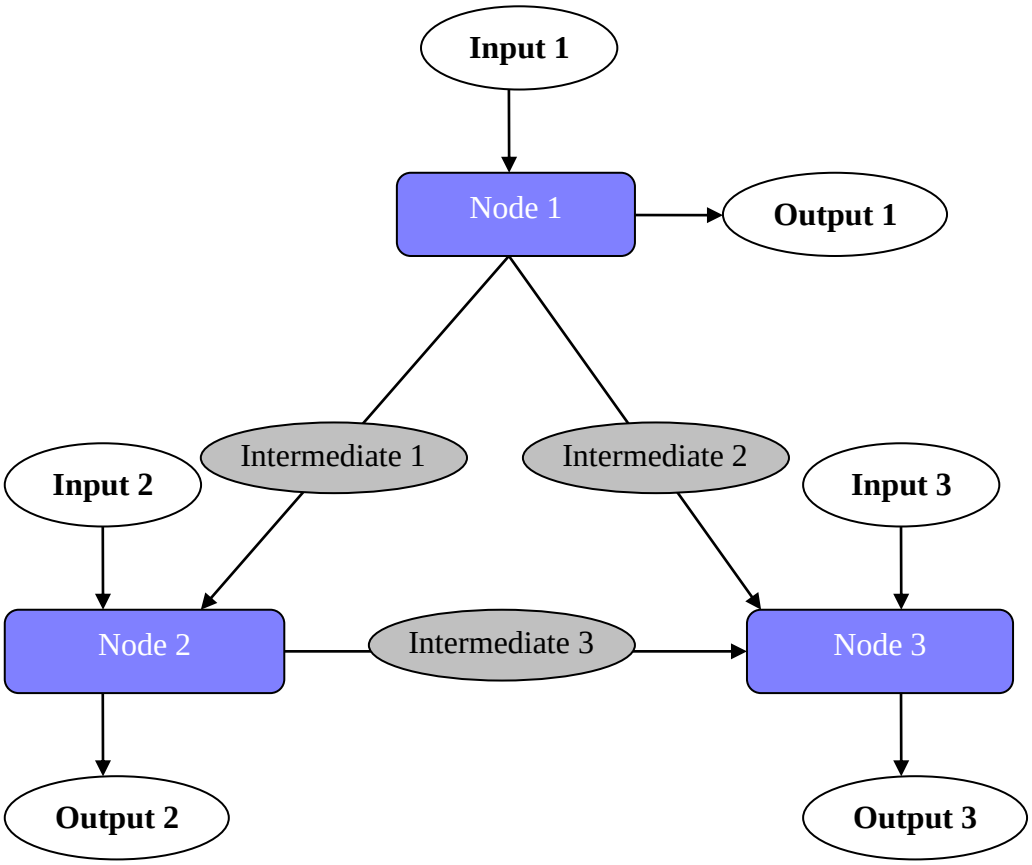
The Network DEA model deals with such circumstances in which many divisions (they are called as nodes in MaxDEA) are linked with each other(Tone K. & Tsutsui, 2009).

1) Each node **may and may not** have its own inputs and outputs, which are called as direct inputs/outputs in Network DEA models;

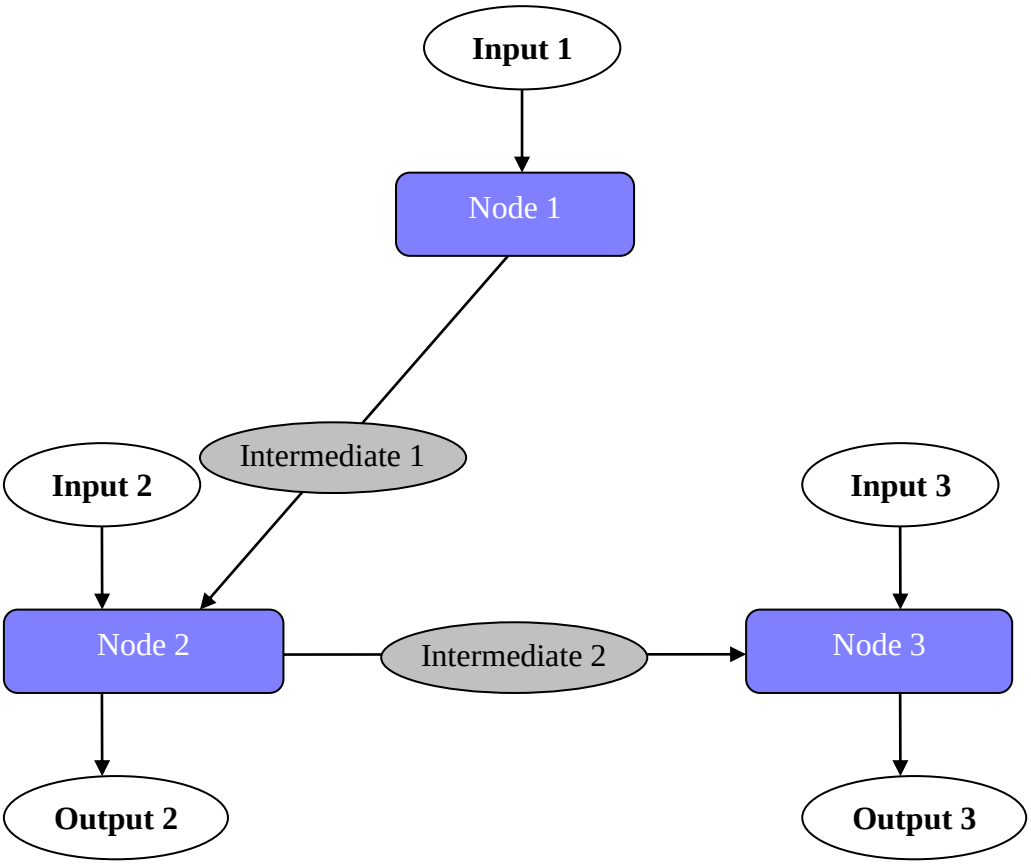
2) At least one node is linked with another node through intermediate inputs/outputs, which are outputs for one node, and meanwhile are outputs for the other node.

Network DEA models provides the overall efficiency and the efficiencies of the nodes in a systematic framework.

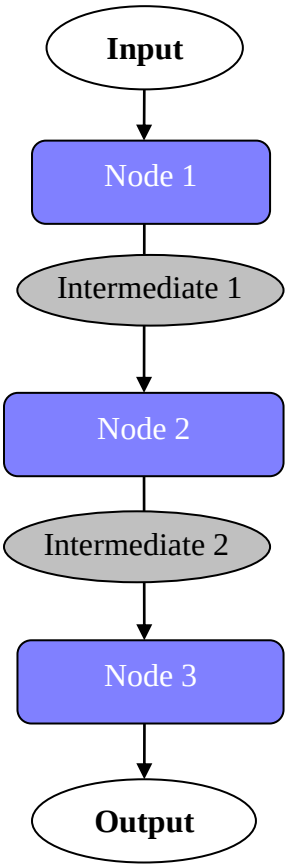
An example of Network DEA Model (net-shape)



An example of Network DEA Model (chain-shape)



An example of Network DEA Model (chain-shape)



MaxDEA Pro is a powerful tool for Network DEA. It provides the most extensive options for Network DEA models.

There are 2 options for Network DEA. One is the type of intermediates. There are 4 types of intermediates:

1) Free
2) Fixed
3) Non-increasing
4) Non-decreasing.

If you want the intermediates to be different types, you should choose the "Free" type and at the same time apply the "Bounded Inputs/Outputs" model. In the Bounded settings, you may set the lower and upper bounds for the intermediates.

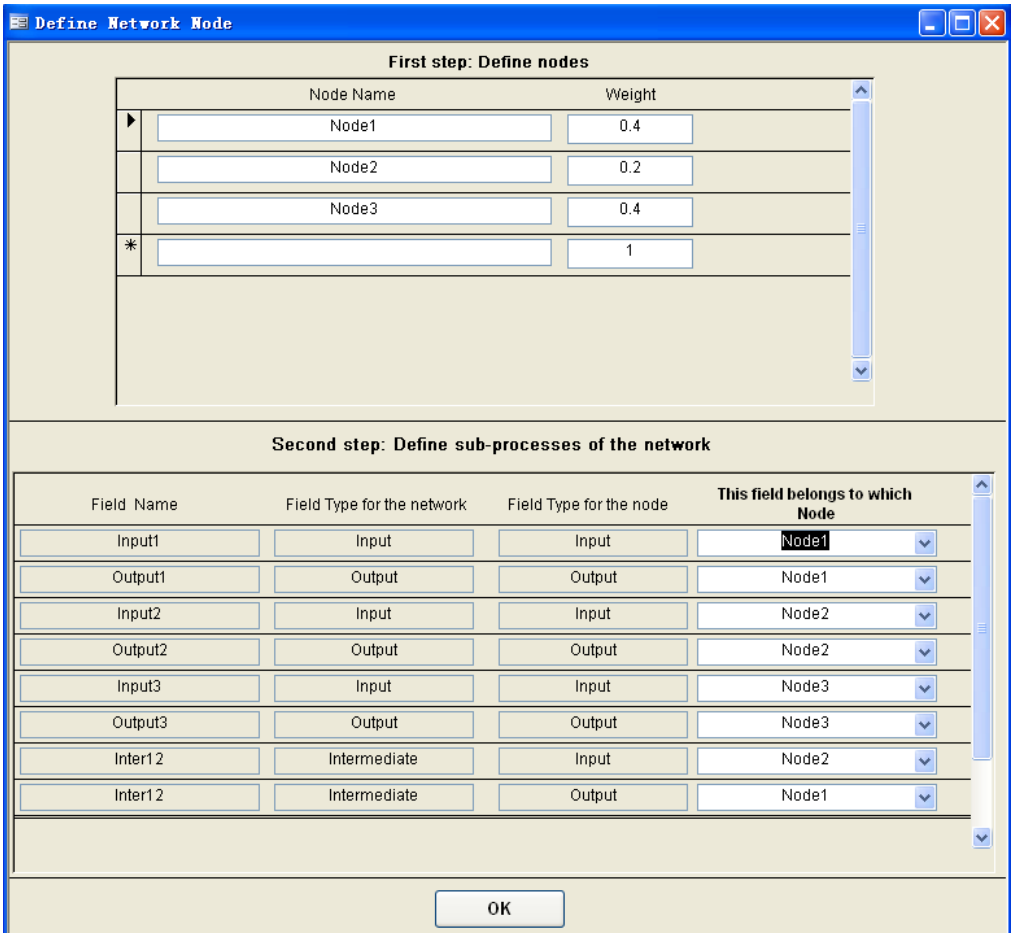
See Bounded Inputs/outputs Model.

The other option is "Use Identical Lambda". It means that the lambdas for all nodes are the same.

To run a Network DEA model, you should

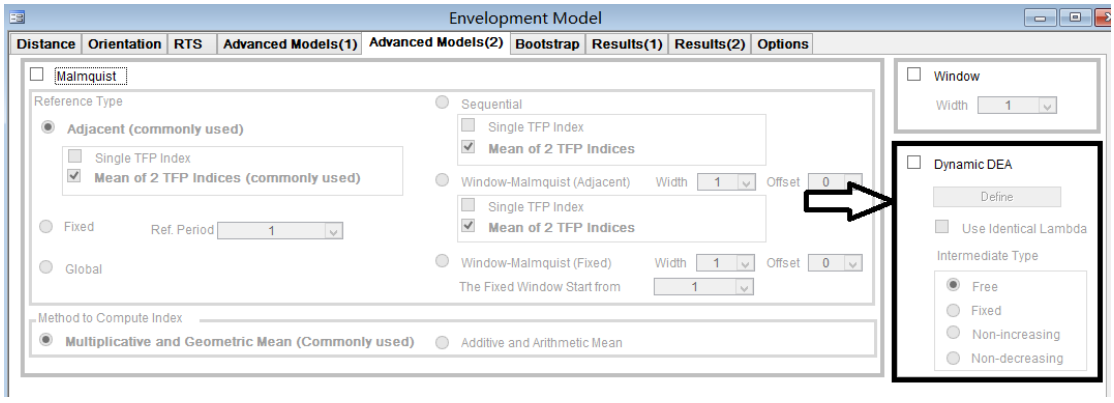
1) At the stage of "Define Data", define the direct inputs/outputs just as usual, but define indirect inputs/outputs (intermediate inputs/outputs) as "Intermediate";
2) At the stage of "Run Model", check the box "Network", and
    a) Decide whether different nodes use same or different intensity vector λ through the check box "Use Identical Lambda";
    b) Decide whether the intermediate value is free or fixed through the check box "Fixed Intermediate Value";
    c) Define the Sub-processes of the Network through the "Define" button;
        i. Firstly define the nodes for the network, including node name and node weight;
        ii. Secondly, assign the direct inputs, outputs and intermediate to corresponding nodes. Each input or output must be assigned to **one and only one** node, and each intermediate must be assigned to one node as input and another node as output.
    d) Run the model.

In MaxDEA Pro, it is very convenient to compare the results between the traditional model (so-called "Black box" model) and the Network model. Just uncheck "Network" with other options unchanged, and it will be the traditional model.



There are three tables for the results of the Network models: the first is a summary just as usual, the second is overall efficiency and slacks and projections, and the last is efficiency scores of the nodes and the intensity vectors.

Dynamic Model



Dynamic model deals with panel data, and take into consideration the links between periods. Panel data for Dynamic model must be balanced, i.e., each DMU must have one and only one observation in each period.

Table 3-3(A) is a panel dataset with one input, one output and one intermediate with three periods for dynamic model. Note that in MaxDEA Pro, the intermediate in period 1 is treated as the link between period 1 and 2, the intermediate in period 2 as

the link between period 2 and 3, and the intermediate in period 3 is not included in the dynamic model.

Essentially, dynamic model is a special case of network model, so all dynamic models can be equivalently achieved through network model. To do so, the panel data (long data) must be transformed into wide data for network model. Table 3-3(B) is the equivalent dataset to Table 3-3(A) for the same dynamic model achieved though network model, which treats the periods as the nodes.

With MaxDEA, more complex dynamic model , such as dynamic model with different intermediates, inputs and outputs in each period, can be developed through network model.

Refer to Tone K. & Tsutsui (2010a)

Table 3-3(A) Panel data for dynamic model (example)

Period	DMU	Input	Output	Intermediate
1	A	4323	93608	875
1	B	2295	225559	469
1	C	6379	327068	1286
2	A	6644	201354	1339
2	B	1436	188926	297
2	C	6281	413738	1266
3	A	7459	114022	1502
3	B	4464	212444	903
3	C	4524	462677	915

Structure of the dynamic model

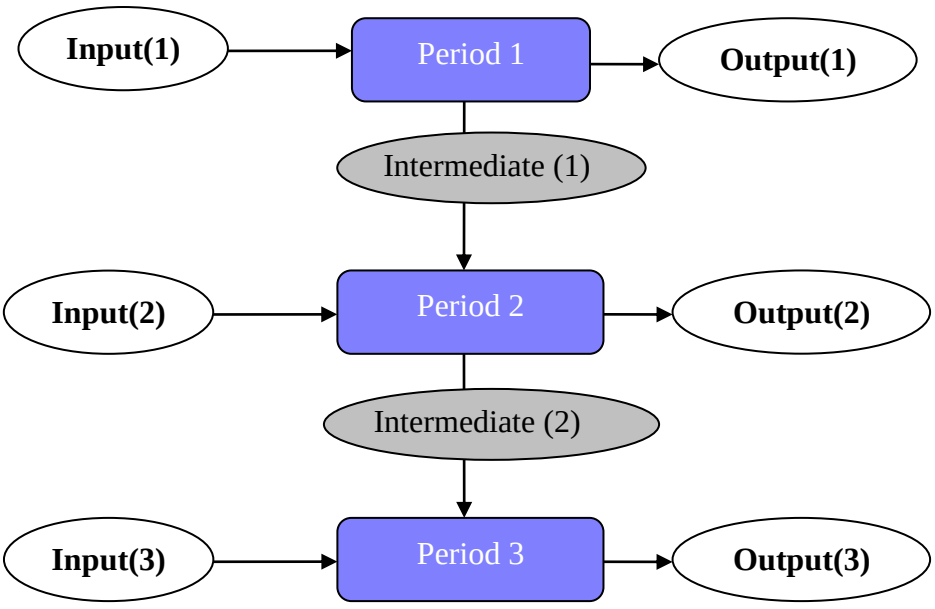
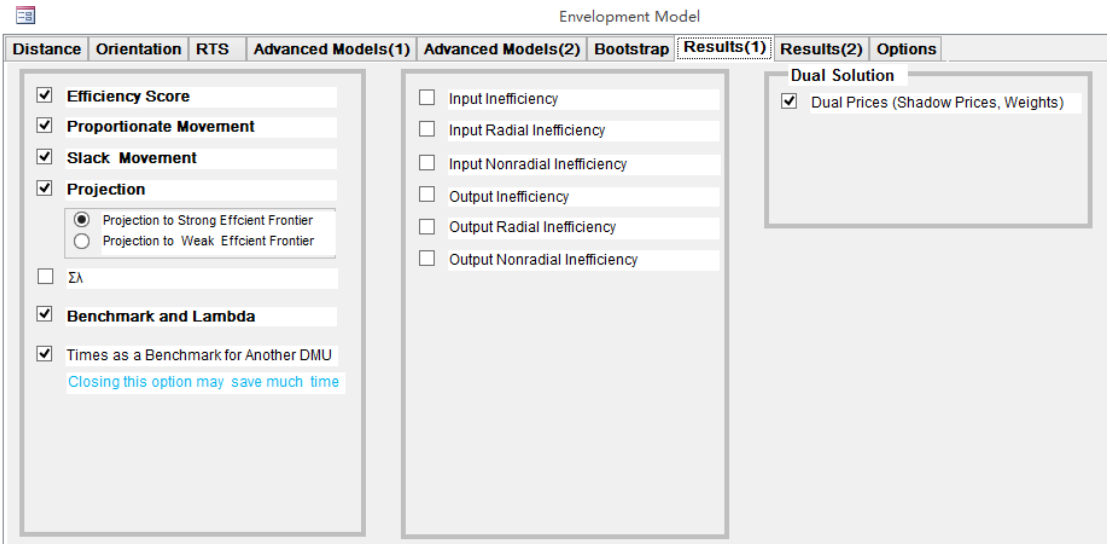


Table 3-3(B) The equivalent dataset to table 3-3(A) for the same dynamic model achieved though network model

DMU	Input(1)	Output(1)	Intermediate(1)	Input(2)	Output(2)	Intermediate(2)	Input(3)	Output(3)
A	4323	93608	875	6644	201354	1339	7459	114022
B	2295	225559	469	1436	188926	297	4464	212444
C	6379	327068	1286	6281	413738	1266	4524	462677

Results for Envelopment Model



Score

Please note that MaxDEA uses θ as efficiency score in input-oriented model, and uses 1/φ (the reciprocal of φ) as efficiency score in output-oriented model.

For non-oriented models,

$$\text{Efficiency Score} = \frac{1 - \text{Input Inefficiency}}{1 + \text{Output Inefficiency}}$$

The score may be technical efficiency, pure technical efficiency, cost, revenue, or profit efficiency. Its meaning depends on the specific model you develop.

Scale Efficiency

In Radial model, Scale Efficiency = CRS efficiency / VRS efficiency.

In non-radial models, Scale Effect = CRS efficiency / VRS efficiency.

Allocative Efficiency

Allocative Efficiency is not provide directly in MaxDEA. It can be calculated as follows

Allocative Efficiency = Cost (or Revenue or Profit) Efficiency / Technical Efficiency

Input Inefficiency

= Input Radial Inefficiency + Input Non-radial Inefficiency

Input Radial Inefficiency (for Hybrid model)

$$= \frac{m_1}{m}(1-\theta)$$

m: number of inputs; m_1: number of radial inputs

Input Non-radial Inefficiency(for Hybrid model)

$$= \frac{1}{m}\sum_{i=1}^{m_2} s_i^{NR-} / x_{io}^{NR}$$

m: number of inputs; m_2: number of non-radial inputs; NR indicates non-radial

Output Inefficiency

= Output Radial Inefficiency + Output Non-radial Inefficiency

Output Radial Inefficiency(for Hybrid model)

$$= \frac{s_1}{s}(\phi-1)$$

s: number of outputs; s_1: number of radial outputs

Output Non-radial Inefficiency(for Hybrid model)

$$= \frac{1}{s}\sum_{r=1}^{s_2} s_r^{NR+} / y_{ro}^{NR}$$

s: number of outputs; s_2: number of non-radial outputs; NR indicates non-radial

Benchmark (λ)

It contains the benchmark DMUs for the evaluated DMU with its λ. In other words, they are those DMUs in the reference set whose λ isn't zero.

Times as a benchmark for another DMU

It provides useful information for efficient DMUs (score = 1). If "Times as a benchmark for another DMU" is zero for an efficient DMU, it means that it is just efficient by default. There are no other DMUs take it as a benchmark, in other words, it is a lonely DMU with a special situation in terms of input and outputs. The more times of an efficient DMU as a benchmark for other DMUs, the more significant the benchmark is.

Σλ

It is the sum of the value of λ.

RTS Estimation

See Returns to scale (RTS)

Proportionate Movement (Radial Movement)

Only available for radial, directional distance and hybrid models. It is the radial part of improvement of inputs/outputs, the proportional decrease of inputs or the proportional increase of outputs. Positive values mean increase, and negative values mean decrease.

Slack Movement

It's absolute value is the s^- (input slack) or s^+ (output slack) in the LP equations. Positive values indicate increase, and negative values indicate decrease.

Projection

It is the efficient target.

For radial models,

Strong Efficient Projection =Original + Radial Movement + Slack Movement

Weak Efficient Projection = Original + Radial Movement

If strong efficient projection is wanted, it is suggested that "Two Stage" method be used to compute the slacks (in "Options – Slack Computation"), because the slacks from the first stage may be incomplete.

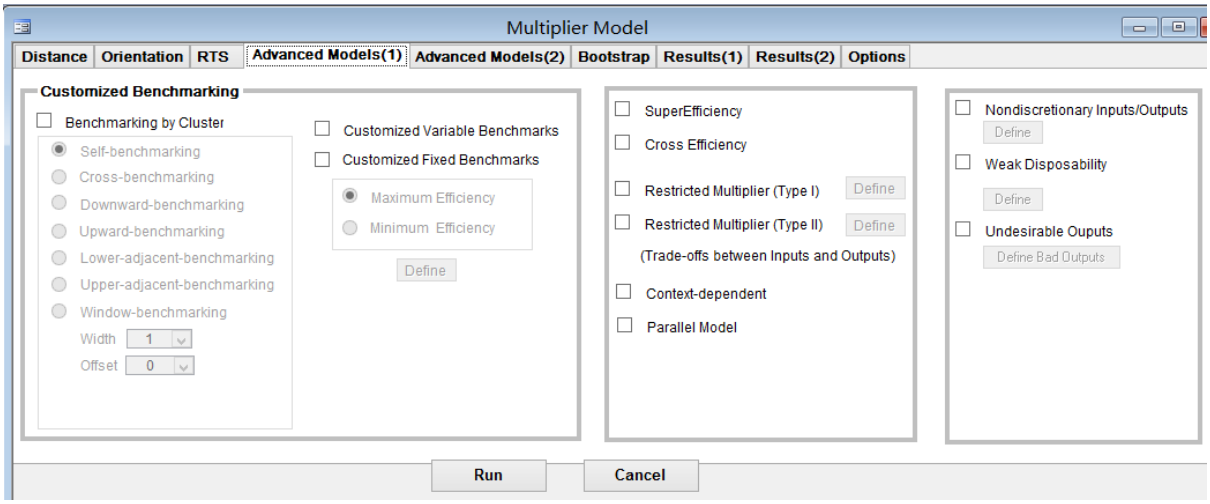
For SBM models, Projection =Original + Slack Movement

Dual Prices

The dual price (also named as shadow price, dual value) of an input (output) indicate that the objective function will change with the value of the dual price if the value of the right-hand side (RHS) for the input (output) in the LP is changed with 1 unit. Note that the sign indicates whether the objective function will increase or decrease.

The absolute values of the dual prices correspond to the values of the weights of inputs (outputs) in the dual model (Multiplier Model). Note that there often exist multiple optimal solutions for multiplier model, so the absolute values of the dual prices got from envelopment model might not be equal to the values of the weights got from corresponding multiplier model.

4 Multiplier DEA Model



Scale Elasticity

A quantitative estimate of RTS.

Ref. to Banker & Thrall (1992).

Cross Efficiency Model

Mean cross efficiency is computed in addition to routine efficiency. And if the number of DMUs is not more than 252, the cross efficiency matrix will be shown in the results.

Note: there usually exist multiple optimal solutions for multiplier models, as a result, the cross efficiency score of a DMU is usually not unique. Using the "Two Stage" method to computing input and output weights can reduce the probability of multiple solutions dramatically.

Cross Efficiency model can be combined with Global Malmquist model.

Game Cross Efficiency Model

Ref. to (Liang, et al., 2008; Wu, et al., 2009)

Game Cross Efficiency model can be combined with Global Malmquist model.

Assurance Region Model

See Restricted Multiplier Model

Trade-offs between Inputs and Outputs

See Restricted Multiplier Model

Ref. to Podinovski (2004)

Restricted Multiplier Model

There are two types of restricted multiplier models available in MaxDEA, named as type I and type II respectively. The restricted multiplier model is to add constraints on the ratios or proportions of weights for inputs/outputs. It is also called Assurance Region model in literature. Ref. to Cooper, et al. (2007)

Type I restricted multiplier model is to add constraints on the ratios of weights for inputs/outputs, such as

$$L_1 \le \frac{\mu_2}{\mu_1} \le U_1$$

$$L_2 \le \frac{\mu_3}{\mu_1} \le U_2$$

while Type II restricted multiplier model is to add constraints on the proportion of a (some or all) weighted (virtual) input/output over the total weighted (virtual) inputs/outputs, for example, suppose there are two inputs and two outputs, we can restrict their virtual proportions as follows,

$$0.1 \le \frac{v_1 x_1}{v_1 x_1 + v_2 x_2} \le 0.3,$$

$$0.1 \le \frac{\mu_1 y_1}{\mu_1 y_1 + \mu_2 y_2} \le 0.3,$$

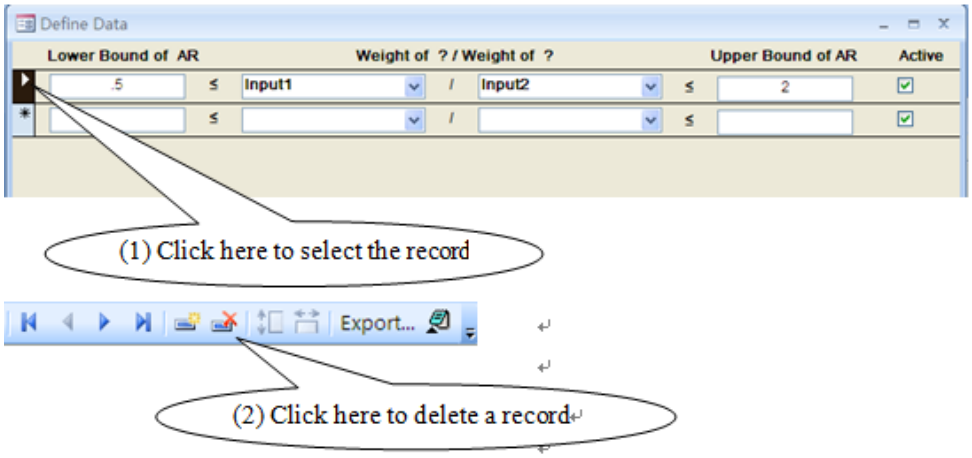
$$0.5 \le \frac{\mu_2 y_2}{\mu_1 y_1 + \mu_2 y_2} \le 0.8.$$

Please note that in type II restricted multiplier model, (1) the sum of the lower bounds of weight proportions must be less than or equal to 1, and (2) the sum of the upper bounds must be greater than or equal to 1 in case that all the weight proportions of inputs/outputs

are restricted such as the above example, in which all the weight proportions of outputs are restricted.

To run the restricted multiplier model, set the restrictions first by a click on the “Define” button on the right side.

If you want to eliminate one piece of restriction, (1)click the arrow on the left, and (2) press the “Delete” key or use the “delete record button” on the toolbar. You can also disable the restriction through the active check box.



Parallel model

The parallel DEA model takes the operation of individual components of the system into account in calculating the efficiency.

The data for parallel system must have two columns indicating the DMU and Sub-DMU respectively.

Ref. to Kao (2009)

Table 4-1 An example for parallel data format

DMU	Sub-DMU	Capital	Labor	Product
A	a	4323	875	93608
A	b	2295	469	225559
A	c	6379	1286	327068
B	d	6644	1339	201354
B	e	1436	297	188926
C	f	6281	1266	413738

C	g	7459	1502	114022

Variable-benchmark Model

See Customized Benchmarking

Fixed-benchmark Model

See Customized Benchmarking

Minimum Efficiency Model

See Customized Benchmarking

Customized Benchmarking

In customized benchmarking model, you can customize the reference DMU set for calculating efficiency scores. Take the input-oriented CCR model as an example,

$$\max \theta = \mu' y_0$$

$$st \quad \nu' x_0 = 1$$

$$\mu' \mathrm{Y}^{\mathrm{r}} - \nu' \mathrm{X}^{\mathrm{r}} \le 0$$

$$\mu, \nu \ge 0 \text{ (or } \mu, \nu \ge \varepsilon)$$

The superscript r indicates the DMUs included in the customized reference DMU set. The above model is also called **variable-benchmark model** (Zhu, 2009), because any of the DMUs in R might be the benchmark for a DMU evaluation, but it is not necessary for all the DMUs in R to be the benchmarks.

If you want one or some DMUs in R to be fixed benchmarks, you can develop a **fixed-benchmark model** (Zhu, 2009) as follows, also take the input-oriented CCR model as an example,

$$\max \theta = \mu' y_0$$

$$st \quad \nu' x_0 = 1$$

$$\mu' \mathrm{Y}^{\mathrm{f}} - \nu' \mathrm{X}^{\mathrm{f}} = 0$$

$$\mu' Y^n - \nu' X^n \le 0$$

$$\mu, \nu \ge 0 \text{ (or } \mu, \nu \ge \varepsilon)$$

f indicats the fixed benchmarks, and n indicates the non-fixed (variable) benchmarks.

If change the objective function of the fixed-benchmark model

$$\max \theta = \mu' y_0,$$

to

$$\min \theta = \mu' y_0,$$

The model will change from a maximum efficiency fixed-benchmark model to a **minimum efficiency** fixed-benchmark model (Zhu, 2009). Such a change for output-oriented models is vice versa.

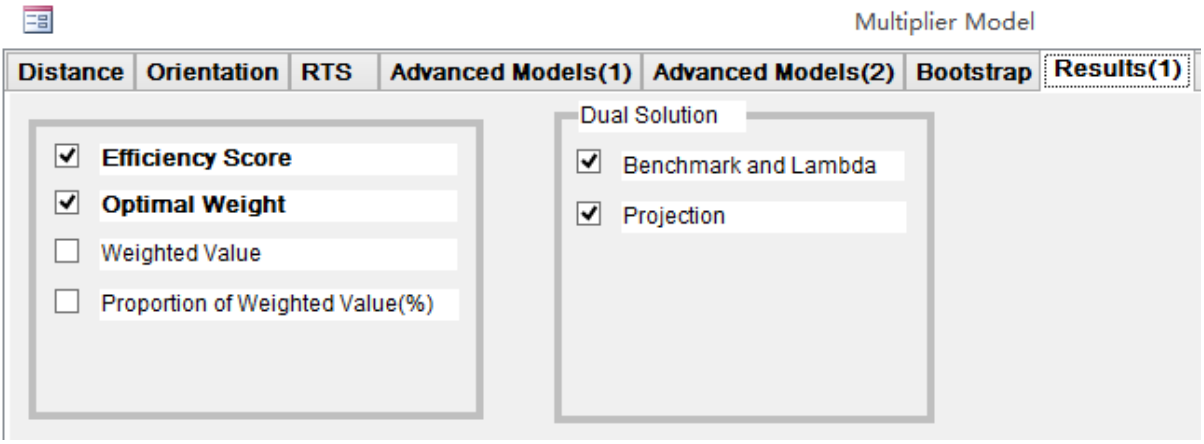
Variable-benchmarking can be applied to both envelopment models and multiplier models, but fixed-benchmarking can be applied to multiplier models only.

To set a variable-benchmark model, open the menu “Run Multiplier Model”, check the option “Variable-benchmark”, and click the “Define” button to select variable benchmarks.

To set a fixed-benchmark model, check the option “Fixed-benchmark”, choose maximum efficiency or minimum efficiency, and click the “Define” button to select both variable benchmarks and fixed benchmarks. The selection of variable benchmarks is optional.

Note: Be cautious about setting the fixed benchmarks. If the fixed benchmarks are not properly chosen, it will result in infeasible LP.

Results for Multiplier Model



Weight

The optimal weight (multiplier) of the inputs (outputs), i.e., the optimal solution of v and μ.

Weighted Value

The product of an input (output) and its multiplier, i.e., $v^{'}x_0$ or $\mu^{'}y0$.

Dual Solution

It provides the benchmarks with the values of lambda and the values of projection.

5 Productivity Analysis (Malmquist Model)

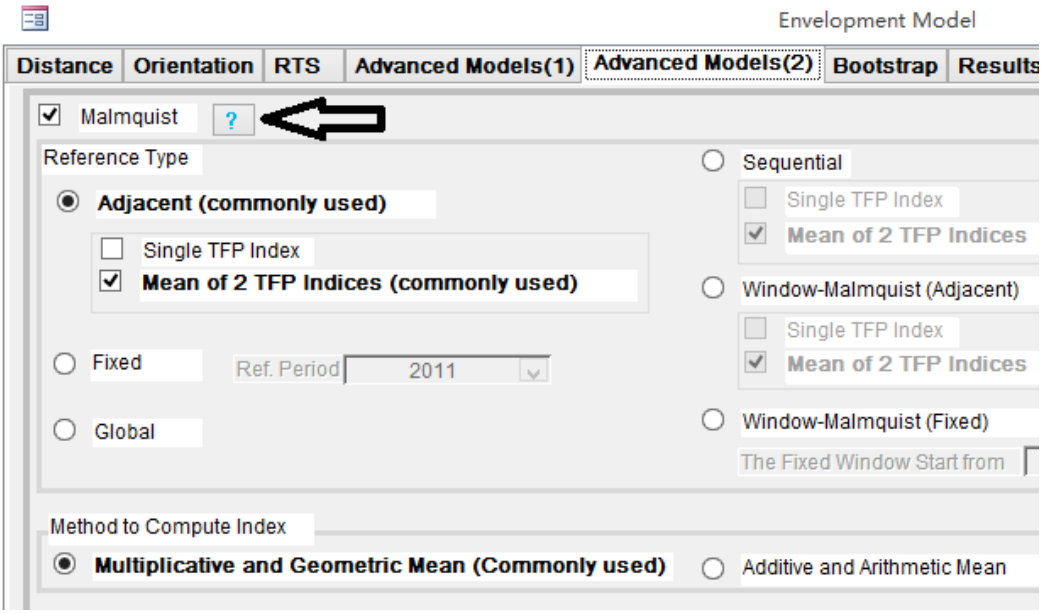
The Malmquist productivity index evaluates the total factor productivity change of a DMU between two periods. It is defined as the product of efficiency change (catch-up) and technological change (frontier-shift). The efficiency change reflects to what extent a DMU improves or worsens its efficiency, while technological change reflects the change of the efficiency frontiers between two periods.

The Malmquist model deals with panel data. Panel data for Malmquist model and Window model are not necessary to be balanced, i.e., missing values at some periods are permitted. If the values for some DMUs in one or more periods are missing, the Malmquist indices for the relevant periods will not be computed, but the Malmquist indices for the periods that have complete data will be computed. In addition, the DMUs with missing values may serve as benchmarks for other DMUs at the periods that they have data.

MaxDEA Pro provides 6 types of Malmquist Indices.

1) Adjacent Malmquist with Mean of 2 TFP Indices
2) Adjacent Malmquist with Single TFP Index (Biennial)
3) Fixed Malmquist
4) Global Malmquist
5) Adjacent Sequential Malmquist with Mean of 2 TFP Indices
6) Adjacent Sequential Malmquist with Single TFP Index
7) Adjacent Window Malmquist with Mean of 2 TFP Indices
8) Adjacent Window Malmquist with Single TFP Index
9) Fixed Window Malmquist

Click the "**?**" button in MaxDEA Pro to see a detailed description of Malmquist model.

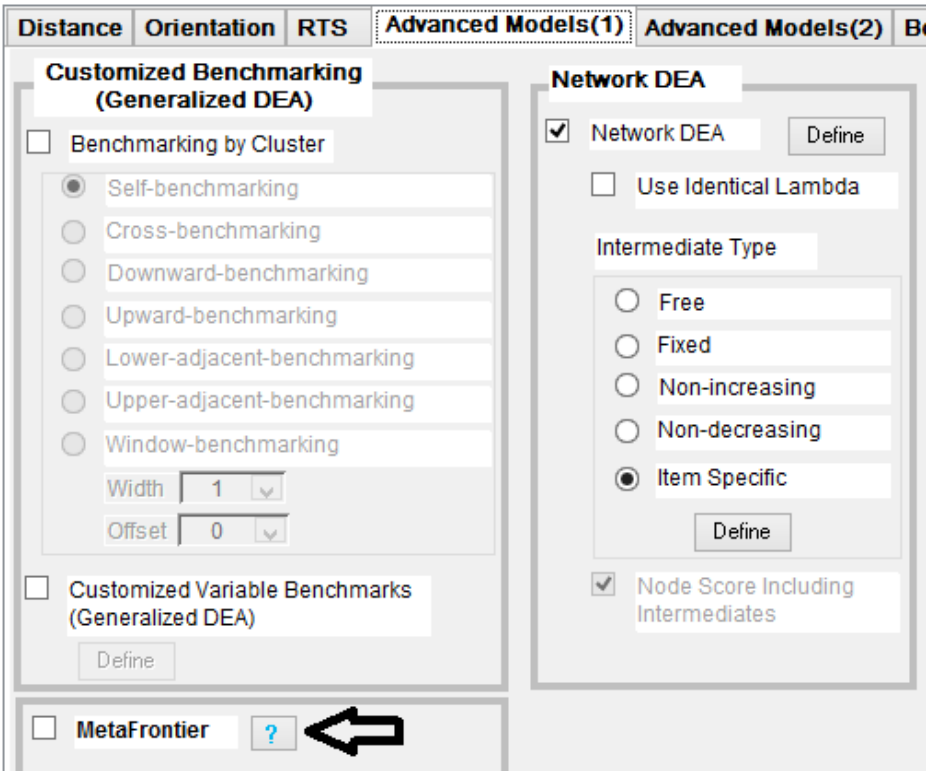


Malmquist-Luenberger index is also seen in the details in "**?**" button.

Cost Malmquist model and similar **Revenue Malmquist**, **Profit Malmquist** and **Revenue Cost Ratio Malmquist** models are also available in MaxDEA Pro.

See Cluster Model, window-benchmarking for the meaning of Width and Offset in Window-Malmquist model.

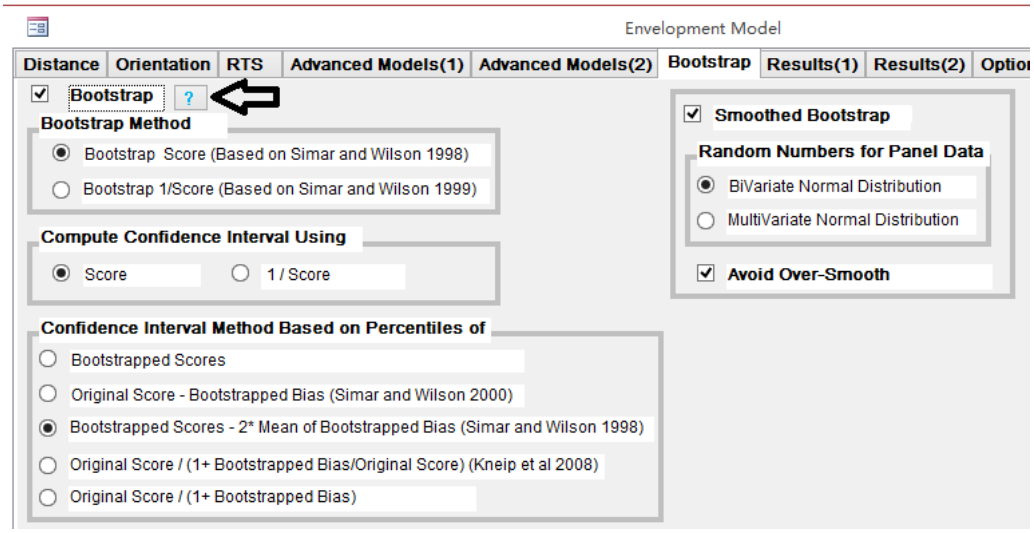
Click the "**?**" button in MaxDEA Pro to see details on MetaFrontier DEA and **MetaFrontier Malmquist** model.



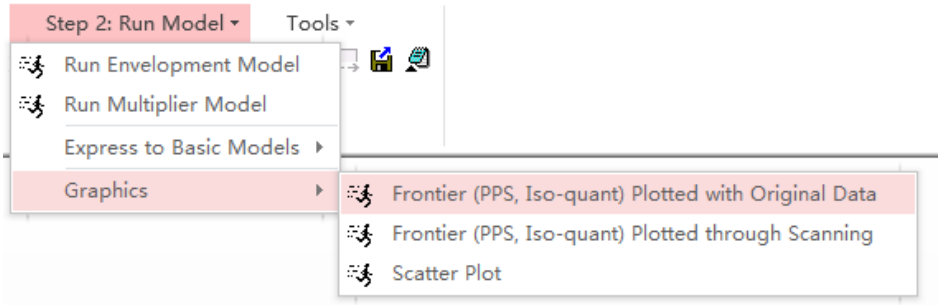
6 Bootstrapping DEA Scores and Malmquist Indices

MaxDEA Pro provides two types of bootstrap method: one is that proposed by Simar & Wilson (1998), and the other is by Simar & Wilson (1999).

Click the "**?**" button in MaxDEA Pro to see a detailed description.



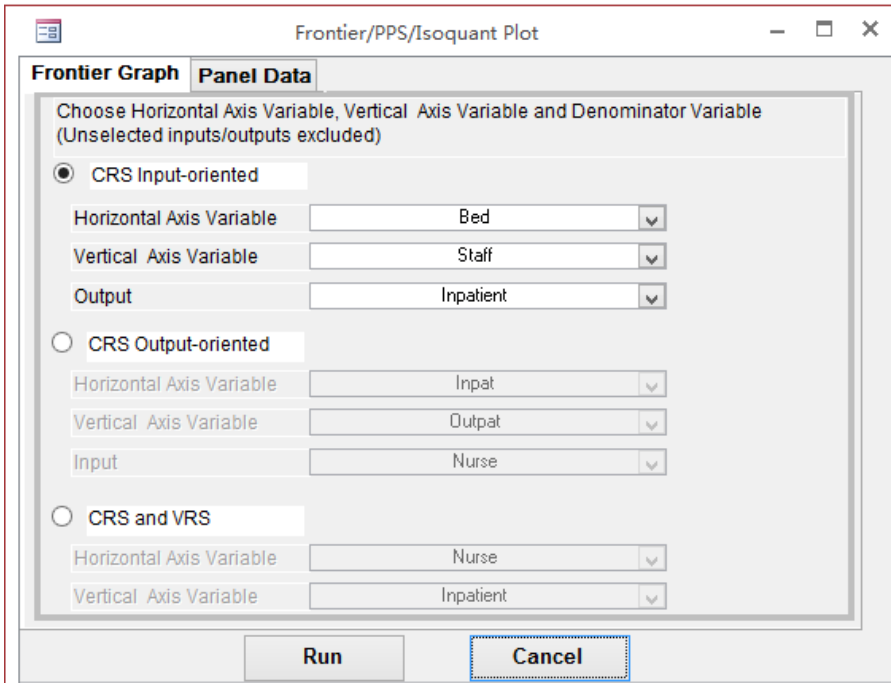
7 Graphics



7.1 Frontier Plot

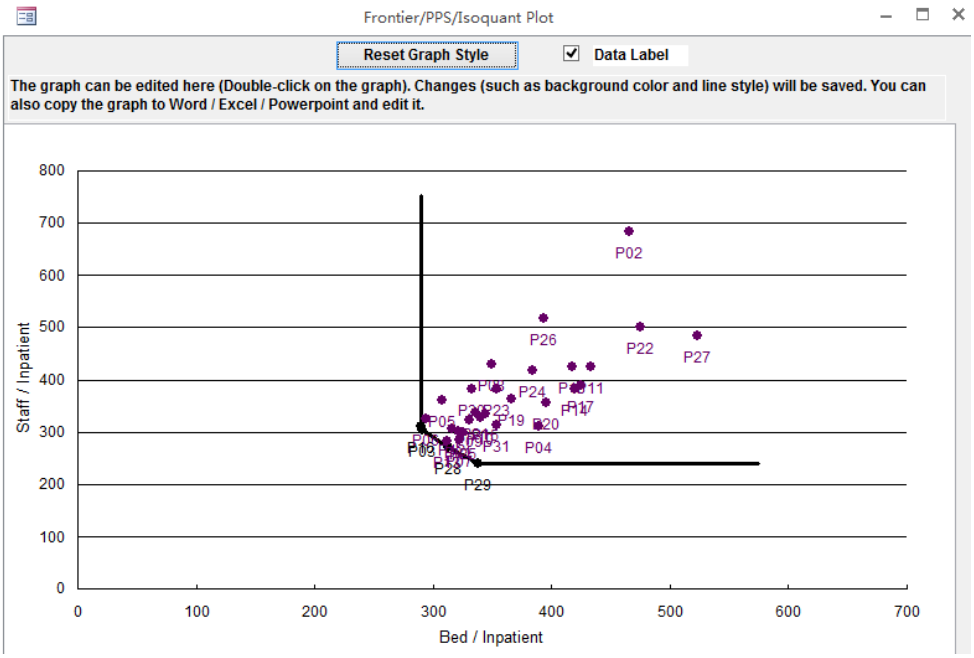
There are two methods of plotting frontiers.

- **One method is plotting the frontier using the original data.** Two or three inputs/outputs are used to plot the frontier, and all the other defined inputs and outputs are excluded from the model.

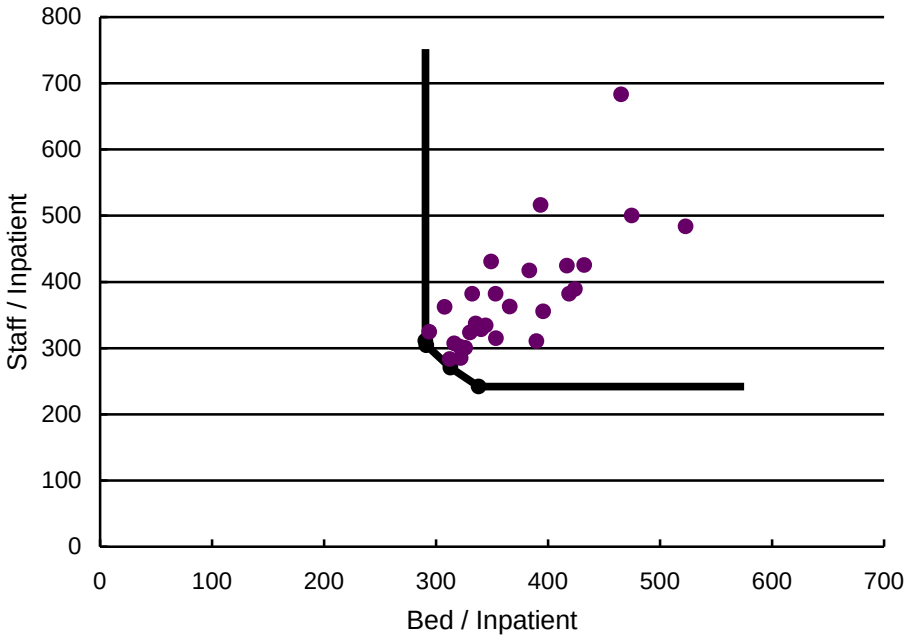


There are 3 types of frontiers.

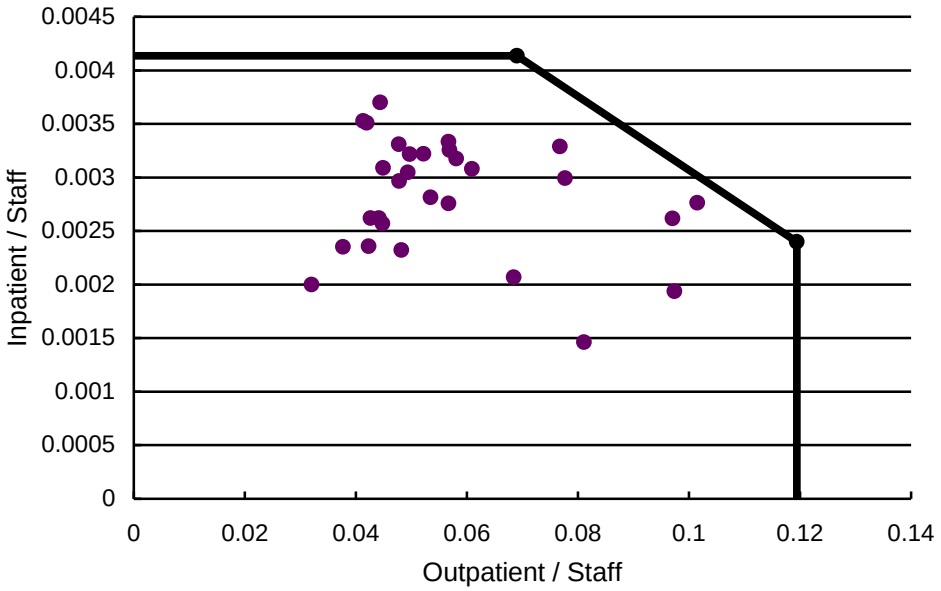
1) CRS input-oriented frontier: two inputs and one output are used to plot the frontier.



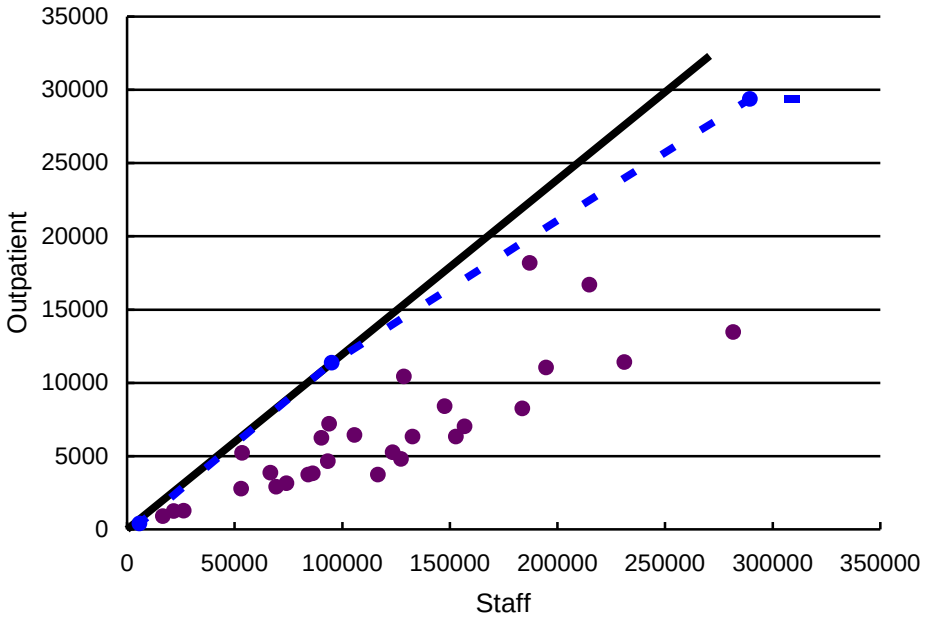
You can copy the plot to Word, Excel or PowerPoint, and edit it just like editing an Excel graph.



2) CRS output-oriented frontier: two outputs and one input are used to plot the frontier.

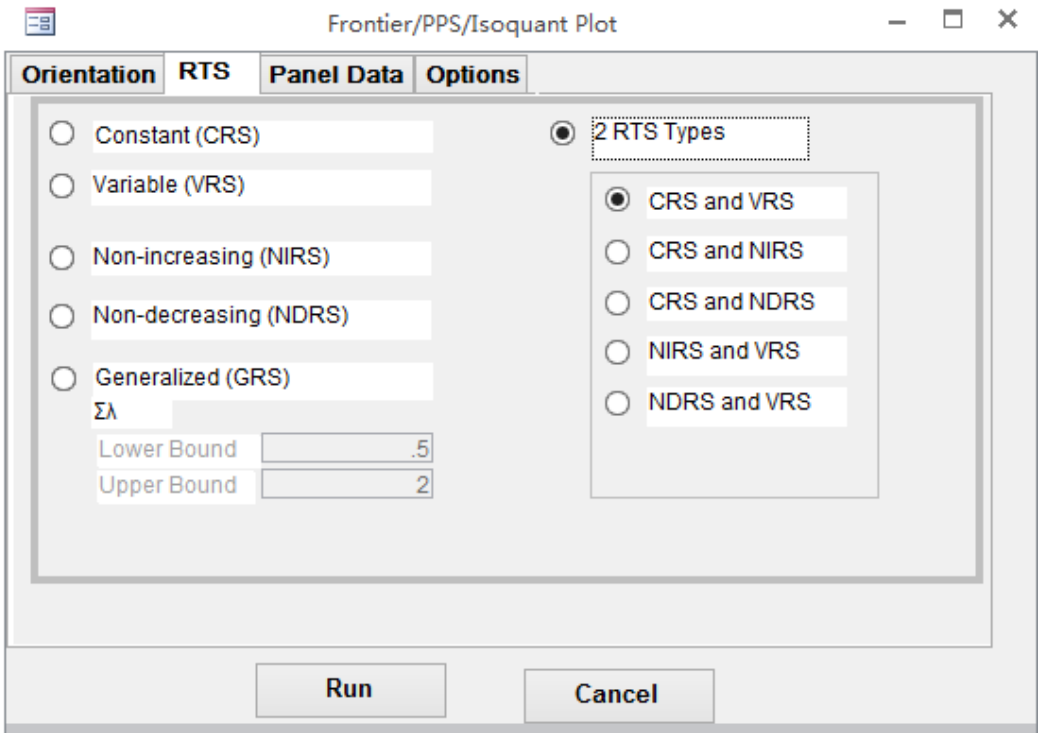


3) CRS and VRS frontiers: one input and one output are used to plot the CRS and VRS frontiers in the same graph.

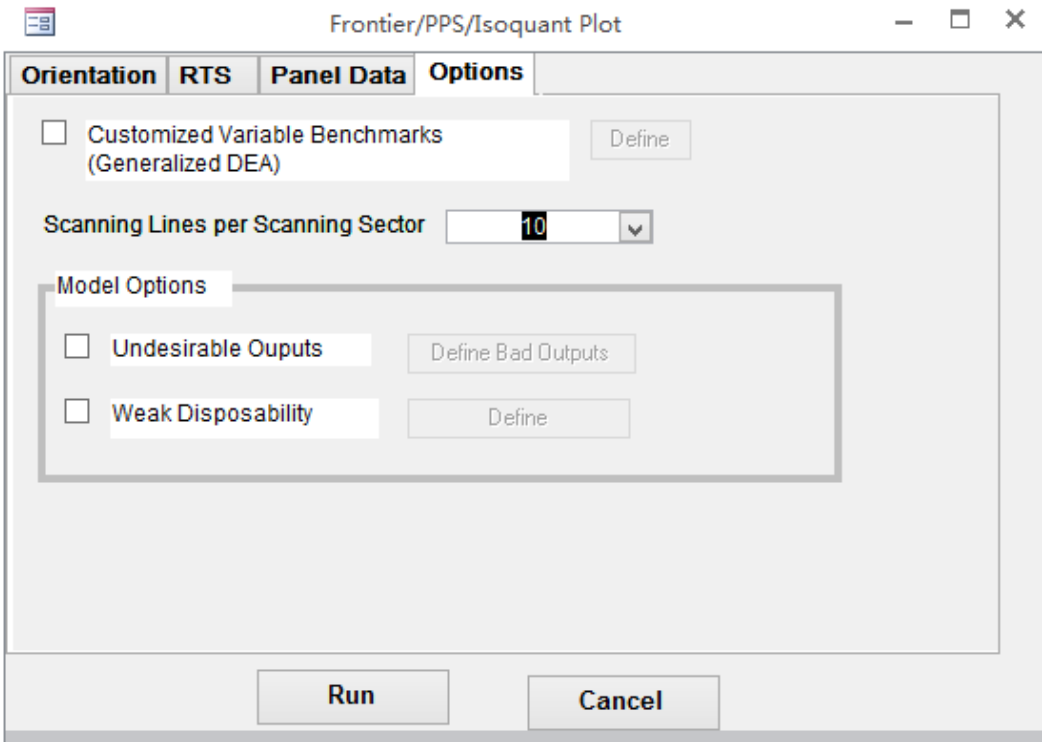


- **The other method is plotting the frontier for the inefficient DMUs through scanning.** Two inputs/outputs are used to plot the frontier, and all the other defined inputs and outputs are also included in the model, but kept fixed, more exactly, inputs non-increasing and outputs non-decreasing.

  The plot can be frontier of any types of RTS, or 2 frontiers of different types of RTS.

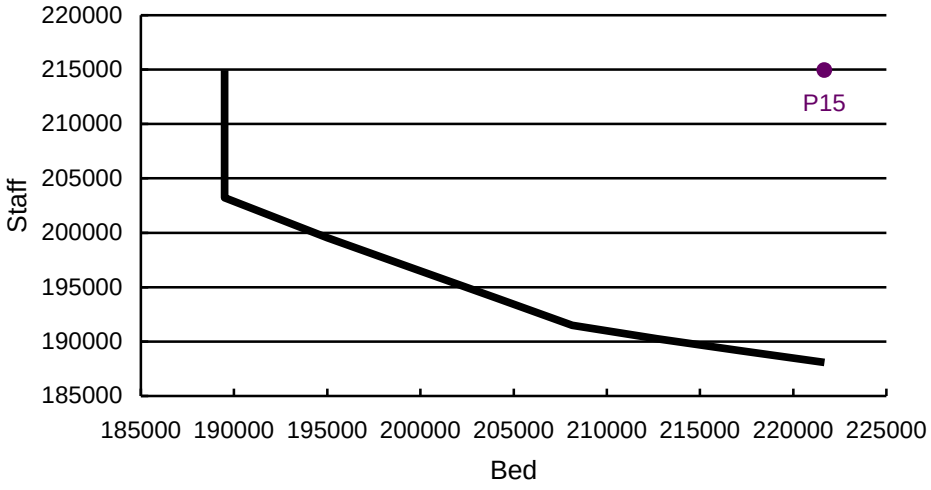


The number of scanning lines can be set in "Options". More scanning lines mean more precision of the plotted frontier. Undesirable outputs and weak disposability can be defined.

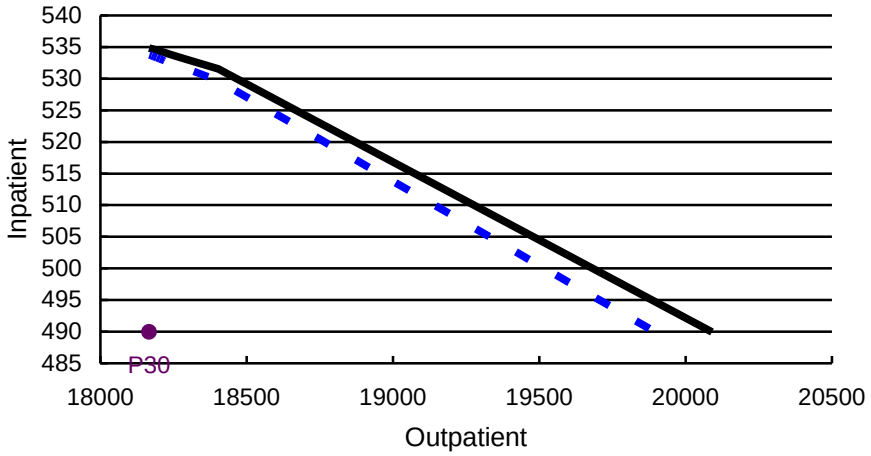


There are 3 types of frontiers.

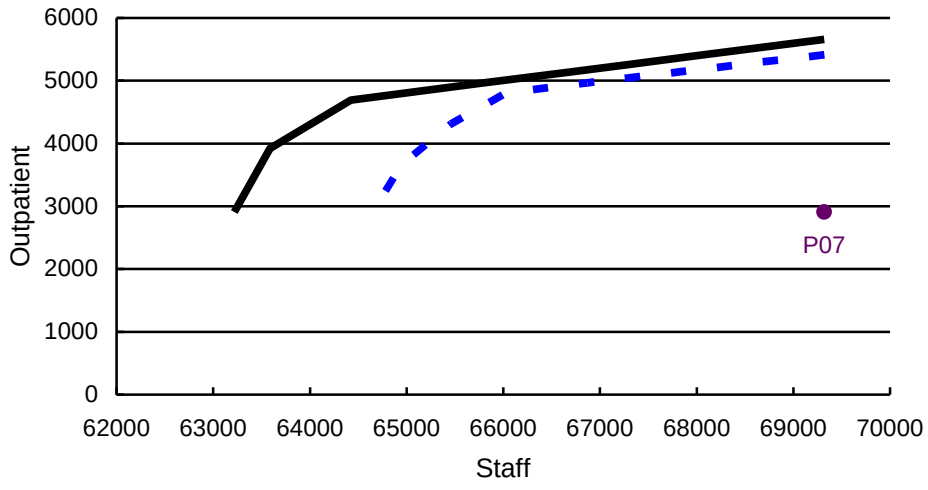
1) Input-oriented frontier: two inputs are used to plot the frontier.



2) Output-oriented frontier: two outputs and are used to plot the frontier.

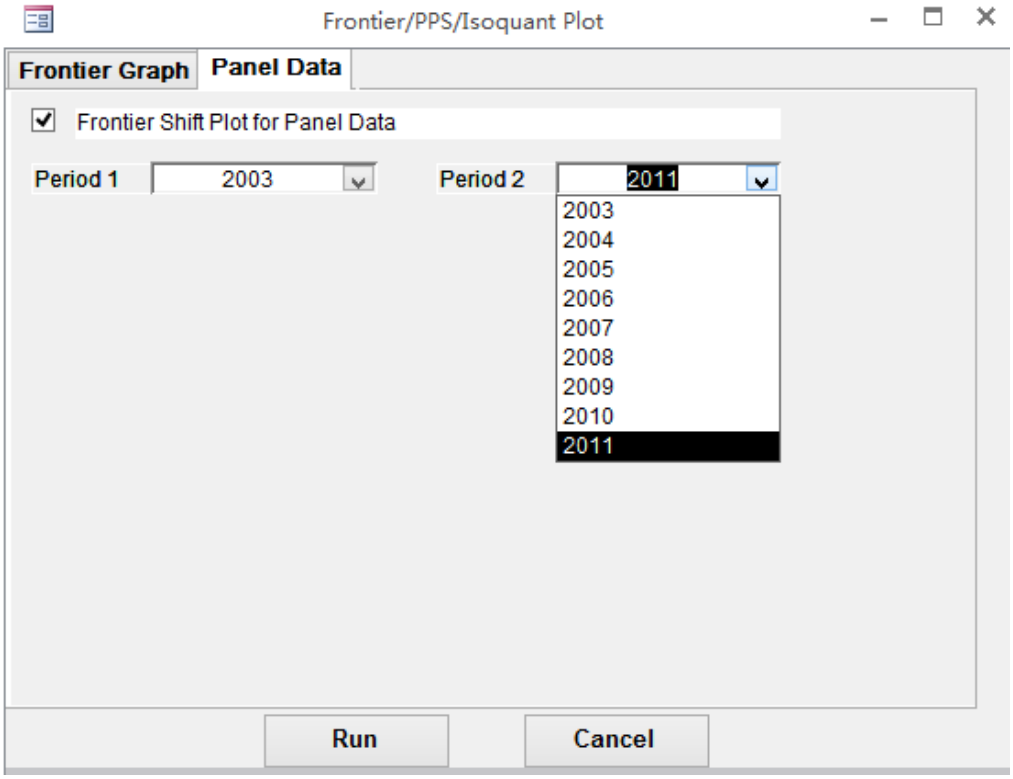


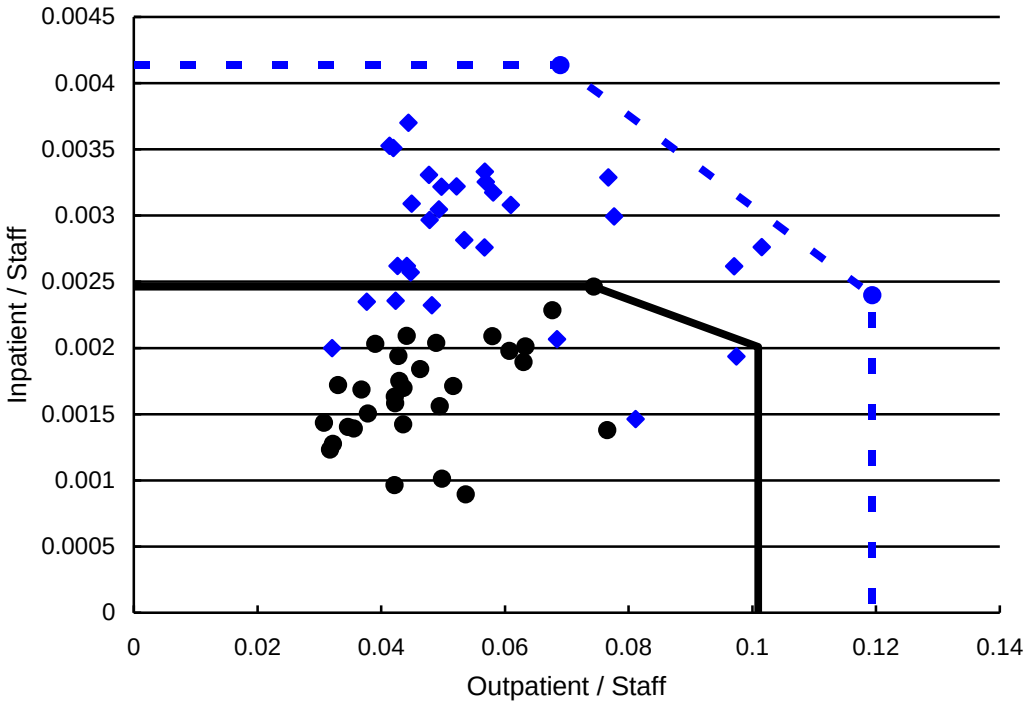
3) Non-oriented frontier: one input and one output are used to plot the frontier.

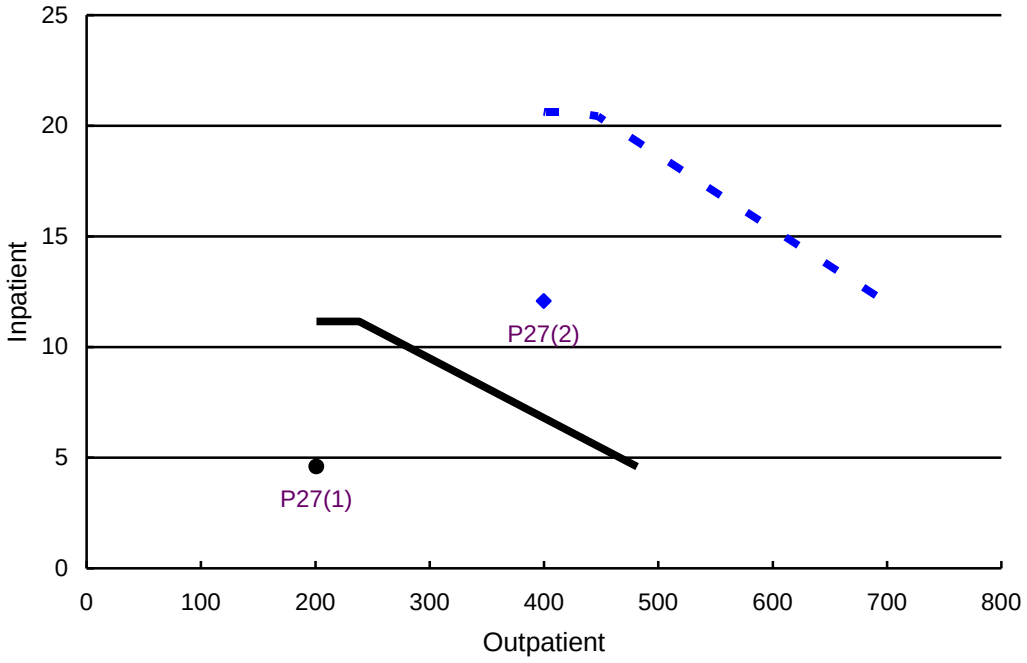


7.2 Frontier Shift Plot

The frontier shit plot can show frontier movement between 2 periods.
The methods are similar to frontier plot, except that two periods must be set.

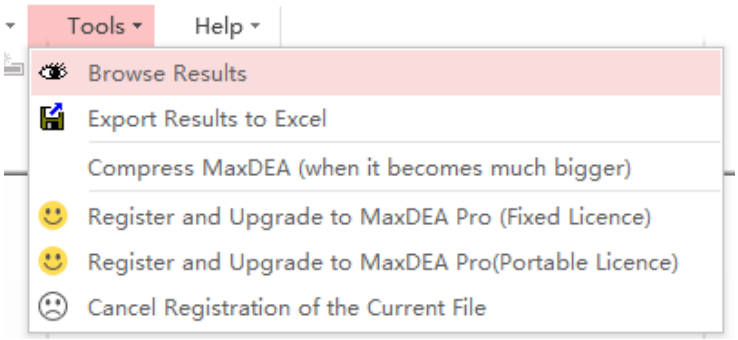





25
20
15
10
5
0
Inpatient
P27(2)
P27(1)
0
100
200
300
400
500
600
700
800
Outpatient

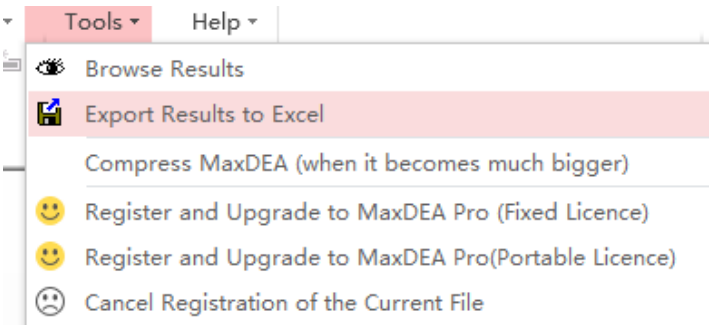
8 Browse and Export Results

Browse results

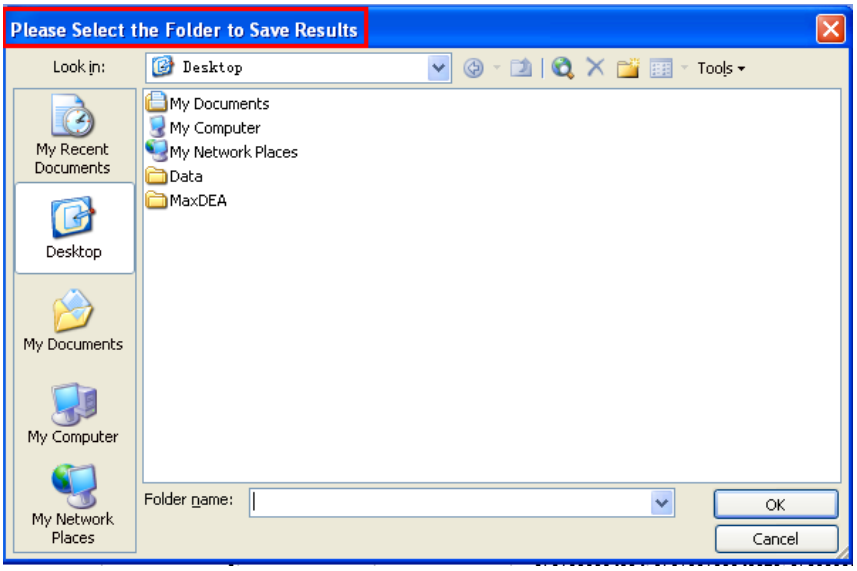


After running your model, the results will show automatically in the result form. You can also browse the results in the result tables saved in MaxDEA Pro. After MaxDEA is closed, the results will not be lost.

Export results to Excel



The results can be exported to Excel. Each result table is exported as a single Excel file. You need to choose in which folder the exported files are saved.



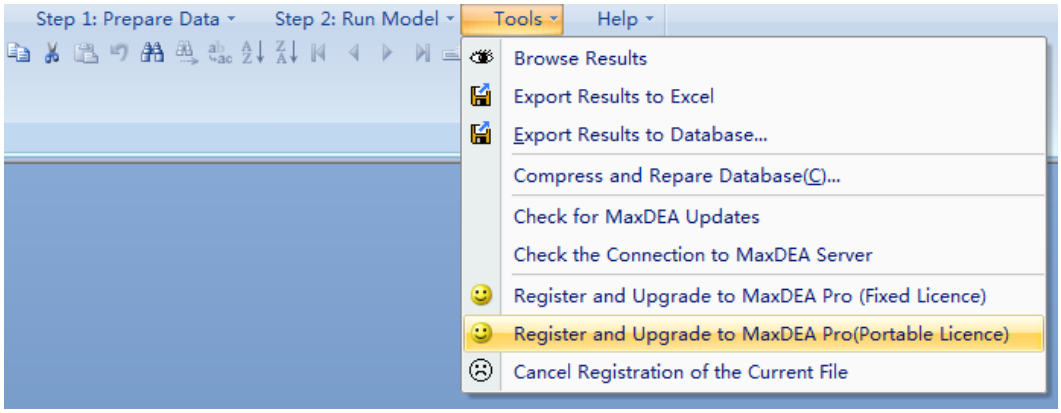
9 Buy a licence and register MaxDEA Pro

MaxDEA Basic can be downloaded at **http://www.MaxDEA.cn** **free**. It is limited to basic DEA models, but it has no limitation on the number of DMUs.

After registration, MaxDEA Basic will be upgraded to MaxDEA Pro, and all the disabled functions (in grey color) will be activated. (Tutorial Video)

Upgrade to MaxDEA Pro

Click "Tools – Register and Upgrade to MaxDEA Pro (…)", and follow the guide.



Steps for portable licence:

1) Buy a licence at www.maxdea.cn;

2) Insert the flash disk you want to use as the USB key for MaxDEA into your computer (Don't use USB Hub);

3) Open MaxDEA Basic (MaxDEA can be run anywhere, not necessarily in the USB disk), Click the Menu "Tools - Register and Upgrade to MaxDEA Pro (Portable Licence)";

4) Send the "USB Disk Code" to MaxDEA@qq.com to get the "Registration Code";

5) Fill in the "Registration Code", and click the button "Register".

Notes:

1) The portable licence holder can use MaxDEA Pro on any computer the USB key is plugged into.

2) The flash disk is prepared by the user. Please ensure that the flash disk is of high quality;

3) Every time MaxDEA Pro is used, insert the USB disk first, and do not remove it until MaxDEA Pro is closed.

Update to a newer version

Check for and download a newer version through the menu “Help – Check for Updates”.

The newly downloaded MaxDEA Basic can automatically upgrade to MaxDEA Pro, i.e., you needn’t register it manually.

Any problem, please contact MaxDEA@qq.com

Home page: http://www.MaxDEA.cn

10 Frequently asked questions （FAQ）

1) **I cannot find the "setup" file in the downloaded zip file, why?**

MaxDEA doesn't need installation. The only file needed to run MaxDEA is the mdb file (such as MaxDEA 4.mdb). All the data needed to develop a DEA model, including dataset and model options, are permanently stored in this file, which means that after closing and reopening the file, the database and model options are still there unchanged. MaxDEA provides extreme convenience to develop DEA models and keep backups of the models. If you want to keep a backup of a DEA model, just rename it as you want, such as Model_CCR.mdb, it is a full backup of the DEA model.

2) **Why I cannot open the program file (mdb file), why?**

MaxDEA is developed with VBA for Access, so Microsoft **Access** is required for running MaxDEA.

If you cannot open the program file (the file with extension "mdb" cannot be recognized), it indicates that Microsoft Office Access is not installed in your computer, and you must install MS Access 2003, 2007 or 2010 (Professional **OR** Runtime) first. Access **Runtime** can be downloaded **free** at Microsoft website.

3) **After registration, Can I use MaxDEA on multiple computers?**

MaxDEA has two types of licences.

One is fixed licence, which is bonded to a computer and can be used in this computer only. The licence will not be affected by reinstalling system or formatting hard disk.

The other is portable licence. This licence type uses a flash disk as the USB key. The license holder may work with MaxDEA on any computer the USB key is plugged into. The flash disk is prepared by the user.

4) **How to update MaxDEA to a higher version? Is update free?**

Update to a minor higher version within the same major version is free of charge. For instance, MaxDEA Pro 5.0 can be updated to 5.x free.

Download an updated version of MaxDEA Basic from http://www.MaxDEA.cn,

and register it with your registration code.

You can check for and download MaxDEA updates through menu "**Tools – Check for MaxDEA Updates**".

5) **How to backup my DEA models?**

MaxDEA provides extreme convenience to develop DEA models and keep backups of the models. If you want to keep a backup of a DEA model, just rename it as you want, such as Model_CCR.mdb, it is a full backup of the DEA model.

6) **When I reopen MaxDEA, the data and model options are missing, why?**

If you open MaxDEA directly in the zip file, the data may be lost after the program file is closed. It is strongly suggested that you unzip the file (MaxDEA.zip) first.

7) **Can MaxDEA run multiple models at the same time? If yes, how to do it?**

Multiple models can be run at the same time. Because the only files needed for running the program is MaxDEA.mdb, you can rename or copy this file freely. Each copy of this file contains one DEA model with all its data and model options saved in the file. And you can open and run multiple files simultaneously.

8) **How to deal with zeros in inputs or outputs in MaxDEA?**

There are many methods to deal with zero data in the literature. One easy approach is replacing zeros with a small number, such as 0.01. Please note that a too small number might cause a numerical problem in linear programming. Suppose, the value of input1 of DMU1 is 100000000, and the value of input1 of DMU2 is 0. If you replace the zero with 0.0000001, it may cause numerical failure in solving the linear program.

9) **What is the largest number of DMUs that MaxDEA can deal with?**

Theoretically the number of DMUs that MaxDEA can deal with is unlimited. We have tested with a dataset with 100000 DMUs. Please note that the computing time will increase dramatically when the number of DMUs increases.

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