Report on Task 19 – Adam Jaskierski

## Problem description

## A diagram of a diagram of a beam Description automatically generatedTwo masses and of negligible dimensions and charges and are fastened to two springs of free lengths and and elasticity coefficients and . The top ends of the springs are fastened at the same height at a distance one from the other. Find the equilibrium positions of both masses

## Equations

For the electric force the following formula have been used:

Where:

And are the coordinates of masses.

Components of the force were calculated using the following formula:

For the spring force the following formula have been used:

Where :

And , for each spring

Components of this force have been calculated using the following formula:

Additionally:

Net forces have been calculated using the following formulas:

To achieve equilibrium, all forces have to cancel each other out in and direction

These equations were implemented in Python in a form of a function which was then passed to

def equilibrium(vars):  
 x1, y1, x2, y2 = vars  
  
 r1 = np.sqrt(x1 \*\* 2 + y1 \*\* 2) # Length of spring 1  
 r2 = np.sqrt((x2 - D) \*\* 2 + y2 \*\* 2) # Length of spring 2  
 r12 = np.sqrt((x2 - x1) \*\* 2 + (y2 - y1) \*\* 2) # Distance between charges  
  
 # Spring forces  
 F\_spring1 = K1 \* (r1 - L1)  
 F\_spring2 = K2 \* (r2 - L2)  
  
 # Components of spring forces  
 F\_spring1\_x = F\_spring1 \* (x1 / r1)  
 F\_spring1\_y = F\_spring1 \* (y1 / r1)  
 F\_spring2\_x = F\_spring2 \* ((x2 - D) / r2)  
 F\_spring2\_y = F\_spring2 \* (y2 / r2)  
  
 # Electrostatic force  
 F\_electro = K\_E \* Q1 \* Q2 / r12 \*\* 2  
 F\_electro\_x = F\_electro \* ((x2 - x1) / r12)  
 F\_electro\_y = F\_electro \* ((y2 - y1) / r12)  
  
 # Net forces  
 eq1 = F\_spring1\_x + F\_electro\_x  
 eq2 = F\_spring1\_y - M1 \* G + F\_electro\_y  
 eq3 = F\_spring2\_x - F\_electro\_x  
 eq4 = F\_spring2\_y - M2 \* G - F\_electro\_y  
  
 return [eq1, eq2, eq3, eq4]

## Typical Parameters

Parameters have been picked to achieve the best visibility of correctness on the graphs:

Q1 = 5e-6 # Charge on m1 (C)  
Q2 = 5e-6 # Charge on m2 (C)  
M1 = 0.5 # Mass of m1 (kg)  
M2 = 0.5 # Mass of m2 (kg)  
L1 = 1.5 # Free length of spring 1 (m)  
L2 = 1.5 # Free length of spring 2 (m)  
K1 = 50.0 # Spring constant for spring 1 (N/m)  
K2 = 50.0 # Spring constant for spring 2 (N/m)  
D = 1.0 # Distance between the tops of the springs (m)  
G = 9.81 # Acceleration due to gravity (m/s^2)  
K\_E = 8.9875517923e9 # Coulomb constant (N·m²/C²)

## Results

Results of the model were presented in form of a graphs showing the masses attached to springs

Fig.1 Masses of same sign chargesA white sheet with black lines

Description automatically generated

Fig.2 Masses of opposite sign charges

A graph with lines in it

Description automatically generated

Fig.3 Masses of opposite sign charges and different masses and spring free lengths A graph with a blue line

Description automatically generated

## Correctness

## The results are consistent with expectations for all configurations which were expected. The graphs clearly demonstrate the behavior predicted by the model, showcasing the accuracy of the calculations and the validity of the model's predictions. Each graphical representation aligns with the theoretical framework and confirms the correctness of the model's results