Algorithmic Game Theory - Assignment 4

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June 16, 2016

Exercise 12

The total amount of seats in the Bundestag is 631. So in case a coalition wants to form a regime it needs 316 seats. With the given table the statements below follow:

1. In case CDU/CSU is not part of the Bundestag all other parties are part of the Bundestag beacuse the CDU/CSU has 311 seats. For the permutation table this means that all rows where CDU/CSU is in the beginning or at the end lead to 0 entry. Because there are 4 parties in total there are 3! possible permutations with CDU/CSU in the beginning and 3! permutations with CDU/CSU in the ending. So there are $2 \cdot 3!$ permutation that lead tu a 0 entry for the CDU/CSU.
2. For each party of the set {SPD,Linke,Grüne} there exists four possible combinations so they are part of the regime, three where CDU/CSU is part of the regime and one without CDU/CSU.

The calculation could be done with a permutation table, but that would be a waste of space.

$$\Phi_i = \frac{1}{n!}\Sigma\{x_i^\pi | \pi \text{ permutation of } N\} \;\forall i \in N$$

With the given information above this leads to

$$\Phi = \frac{1}{24}(2 \cdot 3!, 4, 4, 4)^T = \frac{1}{24}(12, 4, 4, 4)^T$$

as Shapley value.

Exercise 13

1)

$$\begin{aligned}
& f(S \cup \{p_1, p_2\}) - f(S) \\
= \;& f(S \cup \{p_1, p_2\} + f(S \cup \{p_1\}) - f(S \cup \{p_1\} f(S) \\
\overset{a)}{\geq} \;& f(T \cup \{p_1, p_2\} - f(T \cup \{p_1\}) + f(T \cup \{p_1\}) - f(T)
\end{aligned}$$

$b) \to a)$: Let be $S' = S \cup \{p\}$.

$$\begin{array}{cl} & f(S') + f(T) \\ = & f(S \cup \{p\}) + f(T) \\ \overset{(b)}{\geq} & f((S \cup \{p\}) \cap T) + f((S \cup \{p\}) \cup T) \\ = & f(S) + f(T \cup \{p\}) \end{array}$$

$a) \to b)$ Let $S' = S \cap T, P = S \setminus T$.

$$\begin{array}{cl} & f(S) - f(S \cup T) \\ = & f(P \cup S') - f(S') \\ \overset{1), S' \subseteq T'}{\geq} & f(T' \cup P) - f(T') \\ = & f(T \cup S) - f(T) \end{array}$$

Exercise 14

The dual linear program

$$\begin{array}{lll} \min_{y \geq 0} & (10 \cdot x_1' + 4 \cdot x_3' + 2 \cdot x_4' + 3 \cdot x_5') \cdot y_1 & \\ & +(1 \cdot x_1' + 7 \cdot x_3' + 2 \cdot x_5') y_2 & \\ & +(2 \cdot x_1' + 6 \cdot x_2' + 5 \cdot x_4' + 1 \cdot x_5') \cdot y_3 & \\ & +(6 \cdot x_1' + 4 \cdot x_2' + 8 \cdot x_3' + 1 \cdot x_4') \cdot y_4 & \\ s.t. & & \\ & x_1' y_1 + 2 \cdot x_3' y_3 & \geq \ 5 \\ & x_1' y_1 + x_2' y_2 + x_3' y_3 & \geq \ 6 \\ & x_1' y_1 + 2 \cdot x_2' y_2 & \geq \ 3 \\ & x_1' y_1 & \geq \ 1 \\ & x_1' y_1 + x_2' y_2 & \geq \ 5 \\ & x_1' y_1 + x_4' y_4 & \geq \ 4 \\ & x_i' \in \{0,1\}, i \in \{1, ..., 5\} & \end{array}$$