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SS 16

Exercises for Computational Complexity Theory

Assignment 8

Deadline: Thursday, June 23rd, 2016

Exercise 29 (*Sparse languages and the polynomial hierarchy*)

A language $L \subseteq \{0, 1\}^*$ is called *sparse* if $f_L(n) \in n^{O(1)}$ holds, where $f_L(n) := |L \cap \Sigma^n|$.

Prove that if there exists a sparse NP-complete language, then $\text{PH} = \Sigma_2^P$, i.e., the polynomial hierarchy collapses to the second level.

Exercise 30 (*The class P_{poly}*) [Exercise 6.3 in AB]

Describe a *decidable* language in P_{poly} that is not in P .

Exercise 31 (*The class NC^0*)

Show that

- a) NC^0 does not contain any infinite unary language.
- b) NC^0 contains an undecidable language.
- c) $\text{PARITY} \notin \text{NC}^0$.

Exercise 32 (*Circuits and the polynomial hierarchy I*)

Show that for every $k \in \mathbb{N}$ it holds that $\text{PH} \not\subseteq \text{SIZE}(n^k)$.

Exercise 32' (*Circuits and the polynomial hierarchy II*) [Exercise 6.5 in AB]

Show that for every $k \in \mathbb{N}$ there is a language in PH that has a circuit complexity of $\Omega(n^k)$.