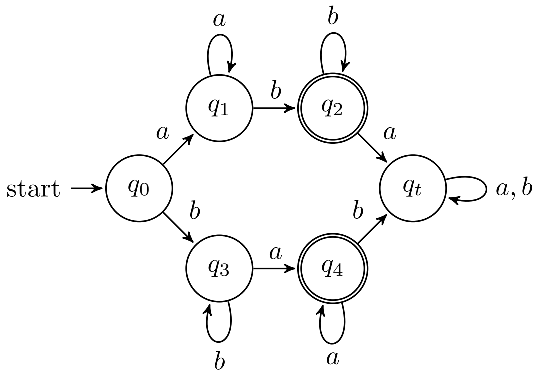
Exercise 27

a

$\mathfrak{A}$: The automaton $\mathfrak{A}$ does both, Büchi- and co-Büchi-recognize L.



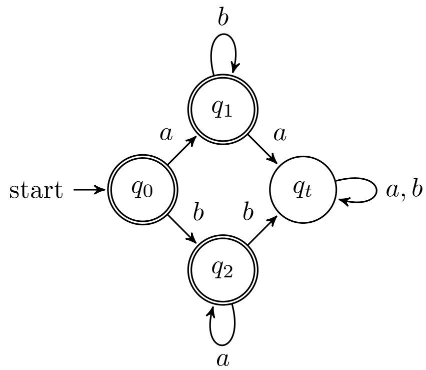
b

Assume $\mathfrak{A}'$ E-recognizes L. So for $\rho(i) = q_f$ and $q_f \in F$ for $i \in \mathbb{N}$. So the read letter before the accepting state is reached are final. Let the read word be $w = a^{1+n_1}b^{n_2}$. So $\mathfrak{A}'$ would recognize w but $w \notin L$. Contradiction $\mathfrak{A}'$ does not recognize L.

Assume $\mathfrak{A}''$ A-recognizes L. Let $w = a^u b^\omega$. So $\rho(i) = q_f$ where $q_f \in F$ and $i \leq u$. By repeating the letter a the automaton must allways reach a final state. So $w = a^\omega$ leads to a final state. This means $\mathfrak{A}''$ recognizes $w = a^\omega \notin L$. Contradiction $\mathfrak{A}''$ does not recognize L.

c

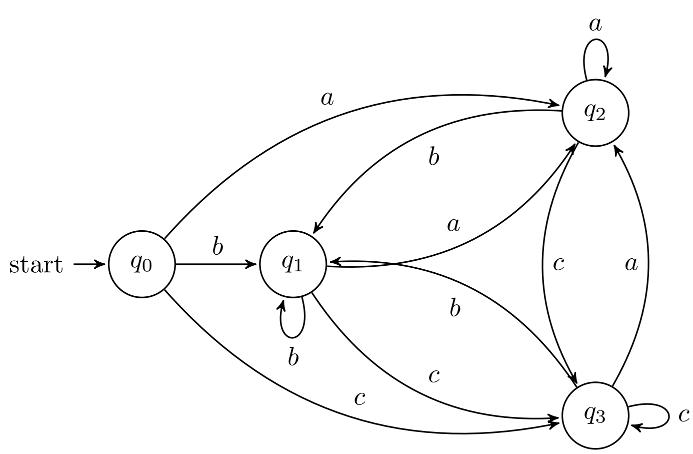
$\mathfrak{A}_A$:



Exercise 28

a

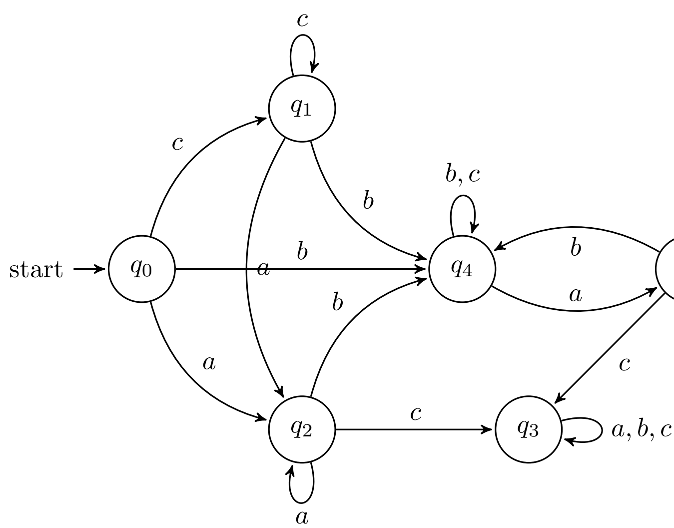
$\mathfrak{A}_{SW}$:



$\mathcal{F} = \{\{q_0\}, \{q_0, q_2\}, \{q_0, q_3\}, \{q_0, q_1, q_3\}, \{q_0, q_2, q_3\}, \{q_0, q_1, q_2, q_3\}\}$

b

$\mathfrak{A}'_{SW}$:



$\mathcal{F} = \{$
$\{q_0\}$,
$\{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_4\}$,
$\{q_0, q_1, q_2\}, \{q_0, q_1, q_4\}, \{q_0, q_2, q_4\}, \{q_0, q_4, q_5\}$,
$\{q_0, q_1, q_2, q_4\}, \{q_0, q_1, q_4, q_5\}, \{q_0, q_2, q_4, q_5\}$,
$\{q_0, q_1, q_2, q_4, q_5\}$
$\}$

Exercise 29

Let $F_i \in \mathcal{F}$ be the set of states that recognize the word $w = a_i^\omega$. It is obvious that $\mathcal{A}$ recognizes L_n.

Assume that $\mathcal{A}'$ L_n with $|\mathcal{F}| < n$ also recognize L_n. So there must be an i such that the set F_i recognizes both a_i^ω and $a_j^\omega, j \neq i$, otherwise some word $a_j^\omega \in L_n$ would not be recognized. Because $\mathcal{A}$ is Staiger-Wagner recognize L_n, recognizing a_i^ω means to find a $F \in \mathcal{F}$, with $Occ(\rho) = F$ for a deterministic run ρ on a_i^ω. Let be $F' = F_i \cup F_j$, $F' \in \mathcal{F}$ and $F_i, F_j \notin \mathcal{F}$. So $Occ(\rho) \stackrel{!}{=} F'$ for the word $w = a_i^\omega$. But $Occ(\rho) \neq F'$ for $w = a_i^\omega$, otherwise would words of the form $(a_i + a_j)^\omega$ also be recognized, which are obviously not in L_n. So $Occ(\rho) \notin \mathcal{F}$. Contradiction!

Exercise 30

Proof. Let $S_1, ..., S_n \subseteq Q$ be all possible non-accepting loops in $\mathcal{M}$ for the determinitsic Muller automaton. For the deterministic Büchi automaton this means that all states used during this loops cannot be part of F.

$$q \in \bigcup_{i=1}^{n} S_i \Leftrightarrow q \notin F$$

If this would not hold, there would exist a non-accepting run S_j on the word $\alpha \notin L(\mathcal{M})$ that visits an accepting state of the deterministic Büchi automaton infently often. So the Büchi automaton would accept it which is a contradiction because $a \notin L(\mathcal{M})$. It follows:

$$F = Q \backslash \bigcup_{i=1}^{n} S_i$$

Finding every non-accepting run can be done with the powerset $\mathcal{P}(Q)$ and the set of accepting loops $\mathcal{F}$ which are given by the Muller automaton.

$$\mathcal{P}(Q) \backslash \mathcal{F} = \bigcup_{i=1}^{n} \{S_i\}$$

□