Exercise 36

$$\varphi'(X_1) = 0 \subseteq X_1 \wedge \forall X \exists Y (Sing(X) \wedge Sing(Y) \wedge X \subseteq X_1 \wedge Succ(X,Y) \rightarrow \neg(Y \subseteq X_1))$$

Exercise 37

a

$\forall s \exists x (X_3(x) \wedge s < x \rightarrow ((X_0(s) \rightarrow X_1(s+1)) \vee (X_1(s) \rightarrow X_2(s+1)) \vee (s+1 < x \wedge X_2(s) \rightarrow X_0(s+1)))) \wedge \forall x \exists y (x+1 = y)$

b

$$\begin{pmatrix} * \\ 0 \end{pmatrix}^* \cdot \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} * \\ 0 \end{pmatrix}^* \cdot \begin{pmatrix} * \\ 1 \end{pmatrix}^* \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} * \\ 1 \end{pmatrix}^*$$

The first row of the formula says that if there is an element in X_2 all successors are in X_2 as well.
The second row of the formula name the two elements that must occur ($\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}$).

Exercise 38

a

$$\begin{aligned} \varphi(X_1) \quad = \quad & \exists Y_1 Y_2 Y_3 (Partition(Y_1, Y_2, Y_3) \wedge Y_1(0) \forall t (\\ & (Y_1(t) \wedge X_1(t) \wedge Y_2(t')) \vee \\ & (Y_2(t) \wedge \neg X_1(t) \wedge Y_2(t')) \vee \\ & (Y_2(t) \wedge X_1(t) \wedge Y_3(t')) \vee \\ & (Y_3(t) \wedge \neg X_1(t) \wedge Y_2(t')) \\ & \forall s \exists t (s < t \wedge Y_3(t))) \end{aligned}$$