Exercise 20

Given Information:

1. no draws
   (a) no loops where the loosing player can go in, in order to prevent the loose and loops the winning player can go in can be ignored (why go in a loop in case you can win the game?).
   (b) the game structure can be represented by a graph which has the form of a tree
2. finite
   (a) the graph structure is finite
3. perfect information
   (a) both player know the graph

Let V be the set of different game position in the graph $G = (V, E)$, $V_0 \subseteq V$ the positions where player 0 has to move, analogous V_1 and $V_1 = V \setminus V_0$. Let T_σ be all the leaves of the tree where $T_{\sigma_0} \subseteq T_\sigma$ are the game positions player 0 wins and $T_{\sigma_1} = T_\sigma \setminus T_{\sigma_0}$ are the game positions player 1 wins,

Base Case:

$\forall v \in T_\sigma$ a player has a winning strategy. So for every leave there exists a winning strategy for a player.

Induction Step:

$\circ, \star \in \{\square, \bigcirc\}$, where $\square$ means that player 0 has a winning strategy and $\bigcirc$ means that player 1 has a winning strategy. If $\circ = \square$, player 0 has a winning strategy in $\star$, if $\circ = \bigcirc$ the choosing player has a winning strategy in $\star$.

By induction every node $v \in V$ has a winning strategy for one player.