Chair of Computer Science 7, RWTH Aachen University
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Infinite Computations – WS 2015

Exercises – Series 1 **October 23, 2015**

All exercises belong to one of the following three categories:

1. *Bachelor + Master* exercises, which count for both Bachelor and Master students.
2. *Bachelor* exercises, which count for Bachelor students.
3. *Master* exercises, which count for Master students.

Hand in your solutions in groups of preferably two (but at most three) students and state whether your group is going to solve Bachelor or Master exercises. Please note that this choice is binding. If possible, please do not form mixed groups of Bachelor and Master students (but if you do, state the type of exercises for each member of your group). Hand in this cover sheet together with your solutions for exercise series 1.

Type of exercises (Bachelor or Master): ____________________

1st student:

Name: ____________________ Matrikelnr.: ____________________

Signature: ____________________

2nd student:

Name: ____________________ Matrikelnr.: ____________________

Signature: ____________________

Exercise 1. ***(Bachelor)*** *2+2 points*

Let $U, V \subseteq \Sigma^*$ be languages (of finite words) over some alphabet Σ. Prove (by giving counterexamples) that the following equations for ω-languages do not hold in general:

(a) $(U \cup V)^\omega = U^\omega \cup V^\omega$

(b) $U^\omega = \lim(U)$

Exercise 2. ***(Bachelor + Master)*** *2+2 points*

Construct (nondeterministic or deterministic) Büchi automata recognizing the following ω-languages over the alphabet $\Sigma = \{a, b, c\}$:

(a) $L_1 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains both } a \text{ and } b \text{ infinitely often}\}$

(b) $L_2 = \{\alpha \in \Sigma^\omega \mid \alpha \text{ contains the infix } bc \text{ infinitely often, but } a \text{ only finitely often}\}$

Exercise 3. ***(Bachelor + Master)*** *3 points*

By adapting the corresponding proof presented in the lecture, show that the language L_2 from Exercise 2 is not recognizable by a deterministic Büchi automaton.

Exercise 4. ***(Bachelor + Master)*** *2+2 points*

Let $\mathcal{A}$ be a nondeterministic finite automaton over an alphabet Σ, let $U_\mathcal{A} \subseteq \Sigma^*$ be the language (of finite words) recognized by $\mathcal{A}$, and $L(\mathcal{A}) \subseteq \Sigma^\omega$ the ω-language recognized by $\mathcal{A}$ when interpreted as a Büchi automaton.

Prove, or disprove by counterexample, each of the following inclusions:

(a) $L(\mathcal{A}) \subseteq \lim(U_\mathcal{A})$

(b) $L(\mathcal{A}) \supseteq \lim(U_\mathcal{A})$

Exercise 5. ***(Master)*** *2+2 points*

Recall that we defined an ω-language $L \subseteq \Sigma^\omega$ to be regular if it is representable in the form $L = \bigcup_{i=1}^{n} U_i \cdot V_i^\omega$ for some $n \in \mathbb{N}$ and regular languages (of finite words) $U_i, V_i \subseteq \Sigma^*$.

(a) Show that for each $n \geq 2$, there exists an ω-language L_n such that L_n is representable as $L_n = \bigcup_{i=1}^{n} U_i \cdot V_i^\omega$, but not as $L_n = \bigcup_{i=1}^{n-1} U_i \cdot V_i^\omega$ with regular languages U_i, V_i.

Hint: Consider $L_n = a_1^\omega + \cdots + a_n^\omega$ over the alphabet $\Sigma = \{a_1, \ldots, a_n\}$.

(b) Show that the statement in (a) holds true even if we only consider languages L_n over the fixed alphabet $\Sigma = \{a, b\}$.

Please always consider announcements made in the L²P course room.

You can hand in your solutions until **12:00 noon on Friday, October 30, 2015** at the drop box at the Chair i7.