Lehr- und Forschungsgebiet Informatik 5
RWTH Aachen

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Introduction to Artificial Intelligence (Winter 2016)

4. Assignment

Submit your solution electronically via the L2P until 20.12.2016.

Homework assignments are optional but strongly recommended.

Exercise 4.1 (25 points)

Prove each of the following statements.

(a) α is valid if and only if $\textsc{True} \models \alpha$.

(b) For any α, $\textsc{False} \models \alpha$.

(c) $\alpha \models \beta$ if and only if the sentence $\alpha \supset \beta$ is valid.

(d) α and β are equivalent[1] if and only if the sentence $\alpha \equiv \beta$ is valid.

(e) $\alpha \models \beta$ if and only if the sentence $\alpha \wedge \neg\beta$ is unsatisfiable.

Exercise 4.2 (25 points)

Formalize "*Norma Jeane Baker is a daughter of Marilyn Monroe's parents.*" and "*Norma Jeane is not a sister of Marilyn.*" together with the needed background knowledge on relationships as first-order sentences. Provide a semantical[2] proof that "*Norma Jeane is Marilyn.*" is an entailment thereof.

Exercise 4.3 (15 points)

Use resolution to prove the following logical consequences:

(a) Implication introduction: $\{\} \models (P \supset (Q \supset P))$

(b) Implication distribution: $\{(P \supset (Q \supset R))\} \models ((P \supset Q) \supset (P \supset R))$

(c) Contradiction realization: $\{(Q \supset P), (Q \supset \neg P)\} \models \neg Q$

[1] For any sentence ϕ, let $Mod(\phi) = \{I \mid I \text{ is an interpretation such that } I \models \phi\}$. Two sentences α and β are equivalent iff $Mod(\alpha) = Mod(\beta)$.

[2] i.e. *not* a resolution proof

Exercise 4.4 **(35 points)**

Are the following statements correct or not?

(a) $\{ \exists x P(x) \, , \, \exists x Q(x) \} \quad \models \quad \exists x [P(x) \land Q(x)]$

(b) $\{ \forall x P(x) \lor \forall x Q(x) \} \quad \models \quad \forall x [P(x) \lor Q(x)]$

(c) $\models \; (\forall x P(x) \land \forall x Q(x)) \equiv \forall x [P(x) \land Q(x)]$

(d) $\models \; \neg\phi$ where ϕ is $\forall x [P(x) \supset Q(x, g(x))] \land \exists x [P(g(x)) \land \neg Q(g(x), g(g(x)))]$

(e) $$\begin{aligned} \{ \forall x \exists y [\quad & (P(y) \supset P(f(x))) \\ & \land (P(f(x)) \supset Q(x, f(y))) \\ & \land (Q(x, f(x)) \lor P(y)) \quad] \} \quad \models \quad \forall x \exists y Q(x, y) \end{aligned}$$

(f) $\{ \forall x \exists y P(x, y) \, , \, \neg \exists z P(z, a) \} \; \models \; \exists y \forall x P(x, y)$

Prove each of your claims either by means of resolution (when the statement is correct) or by specifying a suitable interpretation as a counterexample (in case the statement is wrong).