Chair of Computer Science 7, RWTH Aachen University
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Infinite Computations – WS 2015/2016

Exercises – Series 7 December 4, 2015

Exercise 32. ***(Bachelor)*** *2+2 points*

Consider the following ω-language:

$$L = \{\alpha \in \{a, b, c\}^\omega \mid \text{in } \alpha \text{, between every two occurrences of } b, \text{ the letter } c \text{ occurs at least once}\}$$

(a) Give a deterministic A-automaton recognizing L.

(b) Give a deterministic Staiger-Wagner automaton recognizing L.

Exercise 33. ***(Bachelor + Master)*** *3 points*

A *deterministic weak Büchi automaton (DWBA)* is a deterministic Büchi automaton with the property that each strongly connected component (SCC) contains either only accepting states or only non-accepting states.

Show that every ω-language L that is recognized by a DWBA is also recognized by a deterministic Staiger-Wagner automaton.

Exercise 34. ***(Bachelor + Master)*** *3 points*

We say that the acceptance component $\mathcal{F}$ of a Muller automaton is *closed under co-reachable loops* if for every two loops S, S' with $S \rightsquigarrow S'$ (that means, S' is reachable from S), the following holds: if $S' \in \mathcal{F}$ then also $S \in \mathcal{F}$.

Let $\mathcal{M} = (Q, \Sigma, q_0, \delta, \mathcal{F})$ be a deterministic Muller automaton. Show that $L(\mathcal{M})$ is A-recognizable iff $\mathcal{F}$ is closed under co-reachabe loops.

Hint: There is an analogy to a proof presented in the lecture (characterization of co-Büchi recognizability).

Exercise 35. ***(Master)*** *5 points*

We consider the class of *$\star$-regular ω-languages* over an alphabet Σ, which is inductively defined as follows (as in the lecture):

- Σ^ω and $\emptyset$ are $\star$-regular,
- if L is $\star$-regular, then also $\overline{L}$ is $\star$-regular,
- if L_1 and L_2 are $\star$-regular, then also $L_1 \cap L_2$ and $L_1 \cup L_2$ are $\star$-regular,
- if L is $\star$-regular and $W \subseteq \Sigma^*$ is a regular language of finite words, then $W \cdot L$ is $\star$-regular.

Show that for every ω-language $L \subseteq \Sigma^\omega$, it holds that

$$L \text{ is regular } \Leftrightarrow \ L \text{ is } \star\text{-regular.}$$

Hints:

- Recall that L is Muller recognizable (that means, regular) iff L is a Boolean combination of ω-languages of the form $\lim(U)$ where $U \subseteq \Sigma^*$ is a regular language.
- At some point it is useful to consider $\overline{\lim(U)}$ rather than $\lim(U)$.

Please always consider announcements made in the L2P course room.

You can hand in your solutions until **12:15 on Friday, December 11, 2015,** before the beginning of the lecture or at the drop box at the Chair i7.