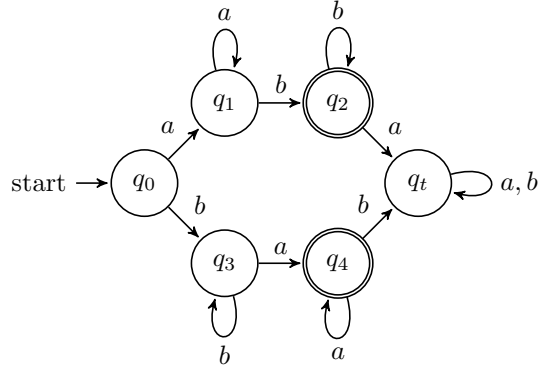


## Exercise 27

**a**

$\mathfrak{A}$  : The automaton  $\mathfrak{A}$  does both, Büchi- and co-Büchi-recognize  $L$ .



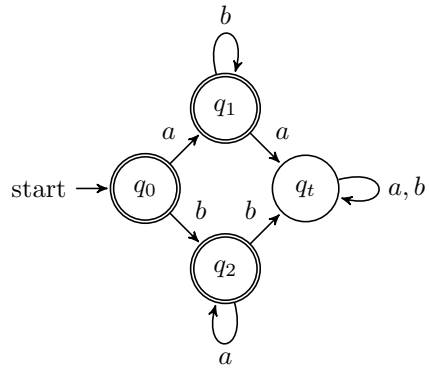
**b**

Assume  $\mathfrak{A}'$  E-recognizes  $L$ . So for  $\rho(i) = q_f$  and  $q_f \in F$  for  $i \in \mathbb{N}$ . So the read letter before the accepting state is reached are final. Let the read word be  $w = a^{1+n_1}b^{n_2}$ . So  $\mathfrak{A}'$  would recognize  $w$  but  $w \notin L$ . Contradiction  $\mathfrak{A}'$  does not recognize  $L$ .

Assume  $\mathfrak{A}''$  A-recognizes  $L$ . Let  $w = a^u b^\omega$ . So  $\rho(i) = q_f$  where  $q_f \in F$  and  $i \leq u$ . By repeating the letter  $a$  the automaton must always reach a final state. So  $w = a^\omega$  leads to a final state. This means  $\mathfrak{A}''$  recognizes  $w = a^\omega \notin L$ . Contradiction  $\mathfrak{A}''$  does not recognize  $L$ .

**c**

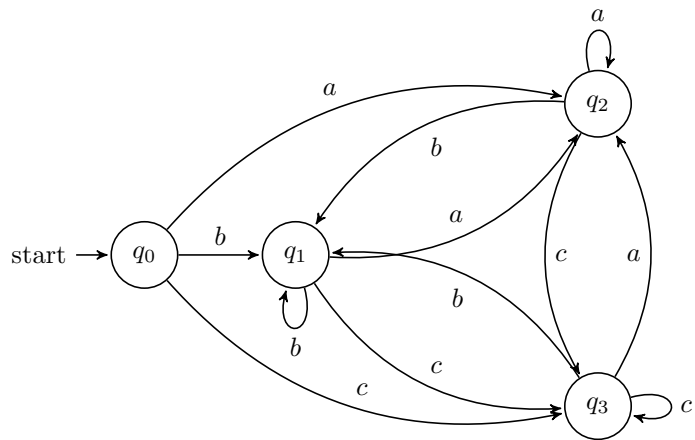
$\mathfrak{A}_A :$



## Exercise 28

**a**

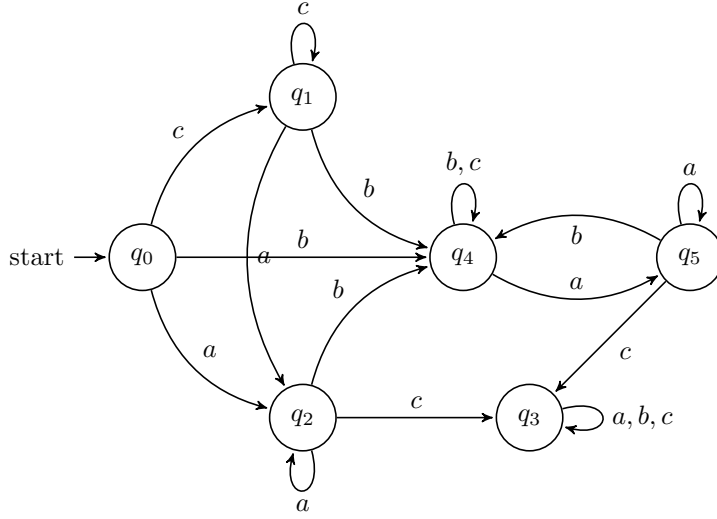
$\mathfrak{A}_{SW} :$



$$\mathcal{F} = \{\{q_0\}, \{q_0, q_2\}, \{q_0, q_3\}, \{q_0, q_1, q_3\}, \{q_0, q_2, q_3\}, \{q_0, q_1, q_2, q_3\}\}$$

**b**

$\mathcal{A}'_{SW} :$



$\mathcal{F} = \{$   
 $\{q_0\},$   
 $\{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_4\},$   
 $\{q_0, q_1, q_2\}, \{q_0, q_1, q_4\}, \{q_0, q_2, q_4\}, \{q_0, q_4, q_5\},$   
 $\{q_0, q_1, q_2, q_4\}, \{q_0, q_1, q_4, q_5\}, \{q_0, q_2, q_4, q_5\},$   
 $\{q_0, q_1, q_2, q_4, q_5\}$   
 $\}$

## Exercise 29

Let  $F_i \in \mathcal{F}$  be the set of states that recognize the word  $w = a_i^\omega$ . It is obvious that  $\mathcal{A}$  recognizes  $L_n$ .

Assume that  $\mathcal{A}'$   $L_n$  with  $|\mathcal{F}| < n$  also recognize  $L_n$ . So there must be an  $i$  such that the set  $F_i$  recognizes both  $a_i^\omega$  and  $a_j^\omega, j \neq i$ , otherwise some word  $a_j^\omega \in L_n$  would not be recognized. Because  $\mathcal{A}$  is Staiger-Wagner recognize  $L_n$ , recognizing  $a_i^\omega$  means to find a  $F \in \mathcal{F}$ , with  $Occ(\rho) = F$  for a deterministic run  $\rho$  on  $a_i^\omega$ . Let be  $F' = F_i \cup F_j, F' \in \mathcal{F}$  and  $F_i, F_j \notin \mathcal{F}$ . So  $Occ(\rho) \stackrel{!}{=} F'$  for the word  $w = a_i^\omega$ . But  $Occ(\rho) \neq F'$  for  $w = a_i^\omega$ , otherwise would words of the form  $(a_i + a_j)^\omega$  also be recognized, which are obviously not in  $L_n$ . So  $Occ(\rho) \notin \mathcal{F}$ . Contradiction!

## Exercise 30

*Proof.* Let  $S_1, \dots, S_n \subseteq Q$  be all possible non-accepting loops in  $\mathcal{M}$  for the deterministic Muller automaton. For the deterministic Büchi automaton this means that all states used during this loops cannot be part of  $F$ .

$$q \in \bigcup_{i=1}^n S_i \Leftrightarrow q \notin F$$

If this would not hold, there would exist a non-accepting run  $S_j$  on the word  $\alpha \notin L(\mathcal{M})$  that visits an accepting state of the deterministic Büchi automaton infinitely often. So the Büchi automaton would accept it which is a contradiction because  $\alpha \notin L(\mathcal{M})$ . It follows:

$$F = Q \setminus \bigcup_{i=1}^n S_i$$

Finding every non-accepting run can be done with the powerset  $\mathcal{P}(Q)$  and the set of accepting loops  $\mathcal{F}$  which are given by the Muller automaton.

$$\mathcal{P}(Q) \setminus \mathcal{F} = \bigcup_{i=1}^n \{S_i\}$$

□