Chair of Computer Science 7, RWTH Aachen University
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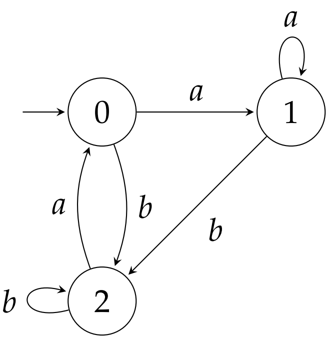
Infinite Computations – WS 2015/2016

Exercises – Series 6 | November 27, 2015

Exercise 26. ***(Bachelor)*** *2+2 points*

Consider the following Muller automaton $\mathcal{M}$ with the acceptance component $\mathcal{F} = \{\{2\}, \{0, 2\}\}$.



(a) Present a deterministic Büchi automaton recognizing $L(\mathcal{M})$ or show that no such automaton exists.

(b) Present a deterministic co-Büchi automaton recognizing $L(\mathcal{M})$ or show that no such automaton exists.

Exercise 27. ***(Bachelor + Master)*** *1+2+1 points*

Consider the ω-language $L = aa^*b^\omega + bb^*a^\omega$.

(a) Show that L is deterministically Büchi and co-Büchi recognizable.

(b) Show that L is not E-recognizable and not A-recognizable.

(c) Show that $L' = ab^\omega + ba^\omega$ is A-recognizable.

Exercise 28. ***(Bachelor + Master)*** *2+2 points*

Construct deterministic Staiger-Wagner automata for the following ω-languages:

(a) $L_1 = \{\alpha \in \{a, b, c\}^\omega \mid$ if b occurs in α, then also c occurs in $\alpha\}$

(b) $L_2 = \{\alpha \in \{a, b, c\}^\omega \mid \alpha$ contains b at least once, but does not contain the infix $ac\}$

Exercise 29. ***(Bachelor + Master)*** *3 points*

Show that for every $n \geq 1$, the ω-language $L_n = a_1^\omega + \ldots + a_n^\omega$ over the alphabet $\Sigma_n = \{a_1, \ldots, a_n\}$ can be recognized by a deterministic Staiger-Wagner automaton $\mathcal{A} = (Q, \Sigma_n, q_0, \delta, \mathcal{F})$ with $|\mathcal{F}| = n$, but not by one with $|\mathcal{F}| < n$.

Exercise 30. ***(Master)*** *4 points*

Let $\mathcal{M} = (Q, \Sigma, q_0, \delta, \mathcal{F})$ be a deterministic Muller automaton with the following properties:

1. $\mathcal{F}$ is closed under superloops and
2. for any k loops $S_1, \ldots, S_k \subseteq Q$ with $S_1, \ldots, S_k \notin \mathcal{F}$, the union $\bigcup_{i=1}^{k} S_i$ is also not in $\mathcal{F}$.

Show that there exists a set $F \subseteq Q$ such that the Büchi automaton $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ recognizes $L(\mathcal{M})$.

Hint: Show that in each accepting loop, there exists a state that does not appear in any non-accepting loop.

Exercise 31. ***(Bachelor + Master)****** *1+1 points**

Recall that an elementary Muller automaton as defined in exercise 20 (series 4) has an acceptance component $\mathcal{F} = \{F_1, \ldots, F_k\}$ where each F_i is a singleton (that means, each F_i contains exactly one state).

(a) When we discussed exercise 20 in the tutorial, the following question was raised: Is every ω-language that is recognized by a (deterministic) elementary Muller automaton also recognized by a deterministic Büchi automaton? Prove or disprove this.

(b) Prove or disprove the following statement: Every ω-language that is recognized by a (deterministic) elementary Muller automaton is also recognized by a deterministic co-Büchi automaton.

Please always consider announcements made in the L^{2}P course room.

For solutions to the exercise marked with an asterisk (*), bonus points will be awarded.

You can hand in your solutions until **12:15 on Friday, December 4, 2015** at the drop box at the Chair i7.