Informed Search

Introduction to Artificial Intelligence

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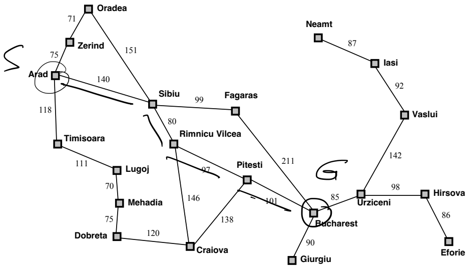
Best-First Search

Search methods differ in their strategies which node to expand next

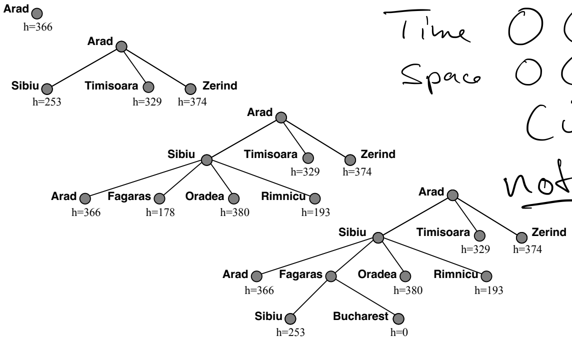
Uninformed search:	fixed strategies without information about the cost from a given node to a goal.
Informed search:	uses information about the cost from a given node to a goal in the form of an evaluation function f, assigning each node a real number.
Best-First Search:	always expand the node with the “best” f-value.
Greedy Search:	$h(n)$ = estimated cost from state at node n to a goal state. Expand node n where $h(n)$ is minimal. Use $f = h$.

Greedy Search Example



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374



Time $O(b^m)$

Space $O(b^m)$

(if $h = 0$)

not complete

A^*

combines uniform cost search with greedy search.

$g(n)$ = actual cost from the initial state to n.

$h(n)$ = estimated cost from n to the nearest goal.

$f(n) := g(n) + h(n)$.

$f(n)$ = is the estimated cost of the cheapest path which passes through n.

Let $h^*(n)$ be the actual cost of the optimal path from n to the nearest goal.

Admissible Heuristic

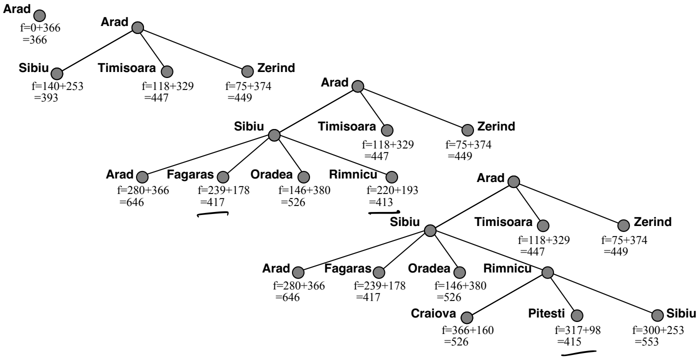
h is called admissible if we have for all n:

$$h(n) \leq h^*(n) \ .$$

$\Rightarrow f(n) \leq h^*(n)$

We require for A^* that h is admissible.

Example A*



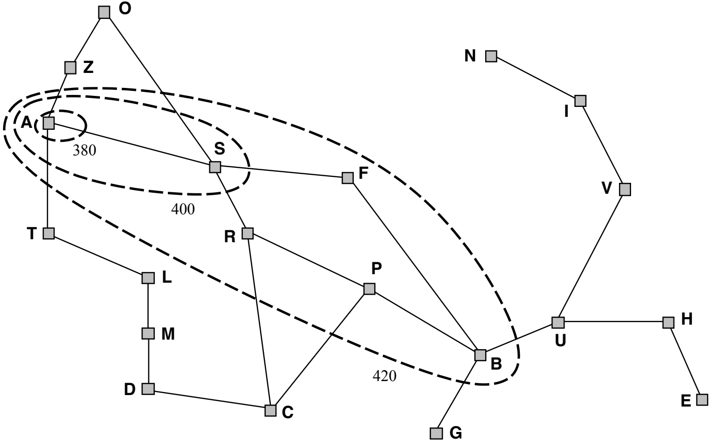
Note: in the example f is monotone nondecreasing.
The following can always be guaranteed:

Path-Max Equation

Let n, n' be nodes, where is n parent of n'. Then let

$$f(n') = max(f(n), g(n') + h(n')).$$

Contour Lines in A*



Heuristic Function 1

a typical solution has depth ≈ 20

s

5	4	
6	1	8
7	3	2

Start State

1	2	3
8		4
7	6	5

Goal State

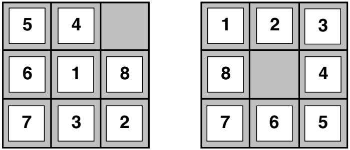
h_1 = number of tiles in the wrong position.

$h_1(s) = 7$

h_2 = sum of the distances to the goal location for all tiles (Manhattan Distance)

$h_2(s) = 2 + 3 + 3 + 2 + 4 + 2 + 0 + 2 = 18$

Heuristic Function 1



Start State   Goal State

h_1 = number of tiles in the wrong position.

h_2 = sum of the distances to the goal location for all tiles (Manhattan Distance)

Effective branching factor b^*: If A^* generates N nodes with solution depth d, then b^* is the branching factor of a uniform tree of depth d with $N+1$ nodes, i.e.

for the root

$$N + 1 = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d$$

b^* is a measure for the goodness of h: the closer b^* is to 1 the better.

Heuristic Function 2

iterative deepening

	Search Cost			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

How to Find a Heuristic

General Strategy:

- Simplify the problem
- Compute the exact solution for the simplified problem
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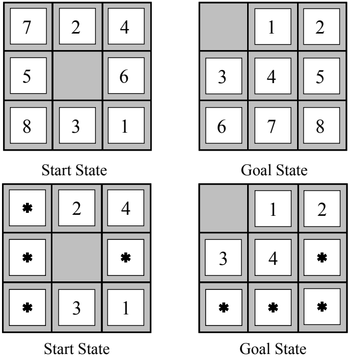
For example:

- h_1 is the solution cost for the simplified 8-puzzle where tiles can be placed at an arbitrary position with a single action.
- h_2 corresponds to the exact solution, if tiles can be moved to an arbitrary position but actions are restricted to moving a tile to a neighboring position.

Pattern Databases



Idea: Compute the exact solution for each *pattern* with four numbers and use that value as heuristic. When more than one pattern applies, use the maximum value.

Better than Manhattan!

can't just take the sum, as 2 patterns usually share moves, i.e. the sum is not admissible!

IDA*

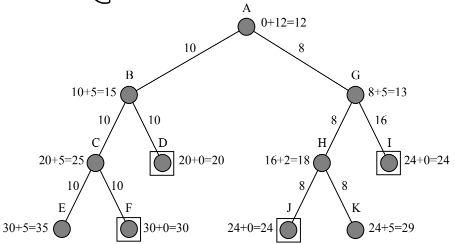
```
function IDA*(problem) returns a solution sequence
   inputs: problem, a problem
   static: f-limit, the current f- COST limit
           root, a node

   root ← MAKE-NODE(INITIAL-STATE[problem])
   f-limit ← f- COST(root)
   loop do
      solution, f-limit ← DFS-CONTOUR(root, f-limit)
      if solution is non-null then return solution
      if f-limit = ∞ then return failure; end
```

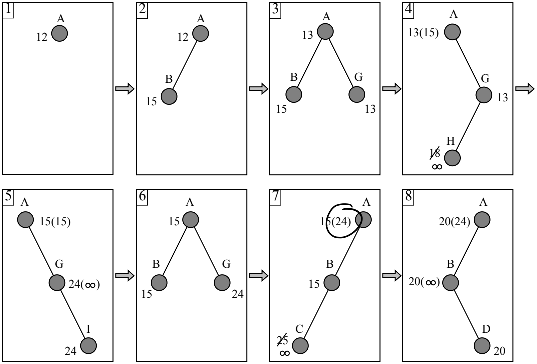
```
function DFS-CONTOUR(node, f-limit) returns a solution sequence and a new f- COST limit
   inputs: node, a node
           f-limit, the current f- COST limit
   static: next-f, the f- COST limit for the next contour, initially ∞

   if f- COST[node] > f-limit then return null, f- COST[node]
   if GOAL-TEST[problem](STATE[node]) then return node, f-limit
   for each node s in SUCCESSORS(node) do
      solution, new-f ← DFS-CONTOUR(s, f-limit)
      if solution is non-null then return solution, f-limit
      next-f ← MIN(next-f, new-f); end
   return null, next-f
```

eventually compute the minimum f-cost beyond the current f-limit.

SMA* 1

Simplified memory-bounded A*

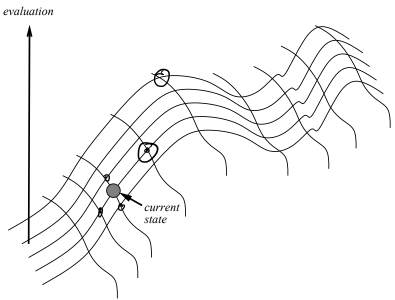




SMA* 2

```
function SMA*(problem) returns a solution sequence
  inputs: problem, a problem
  static: Queue, a queue of nodes ordered by f-cost

  Queue ← MAKE-QUEUE({MAKE-NODE(INITIAL-STATE[problem])})
  loop do
     if Queue is empty then return failure
     n ← deepest least-f-cost node in Queue
     if GOAL-TEST(n) then return success
     s ← NEXT-SUCCESSOR(n)
     if s is not a goal and is at maximum depth then
        f(s) ← ∞
     else
        f(s) ← MAX(f(n), g(s)+h(s))
     if all of n's successors have been generated then
        update n's f-cost and those of its ancestors if necessary
     if SUCCESSORS(n) all in memory then remove n from Queue
     if memory is full then
        delete shallowest, highest-f-cost node in Queue
        remove it from its parent's successor list
        insert its parent on Queue if necessary
     insert s on Queue
  end
```

Hill Climbing



function HILL-CLIMBING(*problem*) **returns** a solution state
inputs: *problem*, a problem
static: *current*, a node
next, a node

current ← MAKE-NODE(INITIAL-STATE[*problem*])
loop do
next ← a highest-valued successor of *current*
if VALUE[next] < VALUE[current] **then return** *current*
current ← *next*
end

Simulated Annealing

Thm: for every search space there is a schedule so that sim. ann. converges to global max.

function SIMULATED-ANNEALING(*problem, schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"
  static: *current*, a node
    next, a node
    T, a "temperature" controlling the probability of downward steps

  current ← MAKE-NODE(INITIAL-STATE[*problem*])
  for $t \leftarrow 1$ **to** ∞ **do**
    $T \leftarrow$ *schedule*[t]
    if T=0 **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$ $= \frac{1}{e^{\frac{|\Delta E|}{T}}}$

if $\Delta E < 0$

Ridges