Knowledge Representation

Introduction to Artificial Intelligence

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Rationally Thinking Agents

- So far the focus was on rationally acting agents. (Decision making is done implicitly via the evaluation function, i.e. the designer "thinks.")
- Often rational action requires rational thought by the agent itself.
- Part of the world must be represented explicitly in a Knowledge Base (KB):
  - KB contains sentences in a language with a truth theory (Logic) which we can interpret as propositions about the world.
  - The sentences, through their form alone, have a causal effect on the agent's behavior in correlation with the contents of the sentences.
- Ideally, interaction with a KB through ASK and TELL:

  ASK(KB,α) = YES iff α follows from KB
  TELL(KB,α) = KB$'$, so that α follows from KB$'$

3 Levels

In knowledge representation one distinguishes 3 levels [Newell 1990]:

Knowledge Level: the most abstract level; addresses what is known by the KB. E.g.: automatic taxi driver knows that *Vaalser St* connects *Aachen* and *Vaals*.

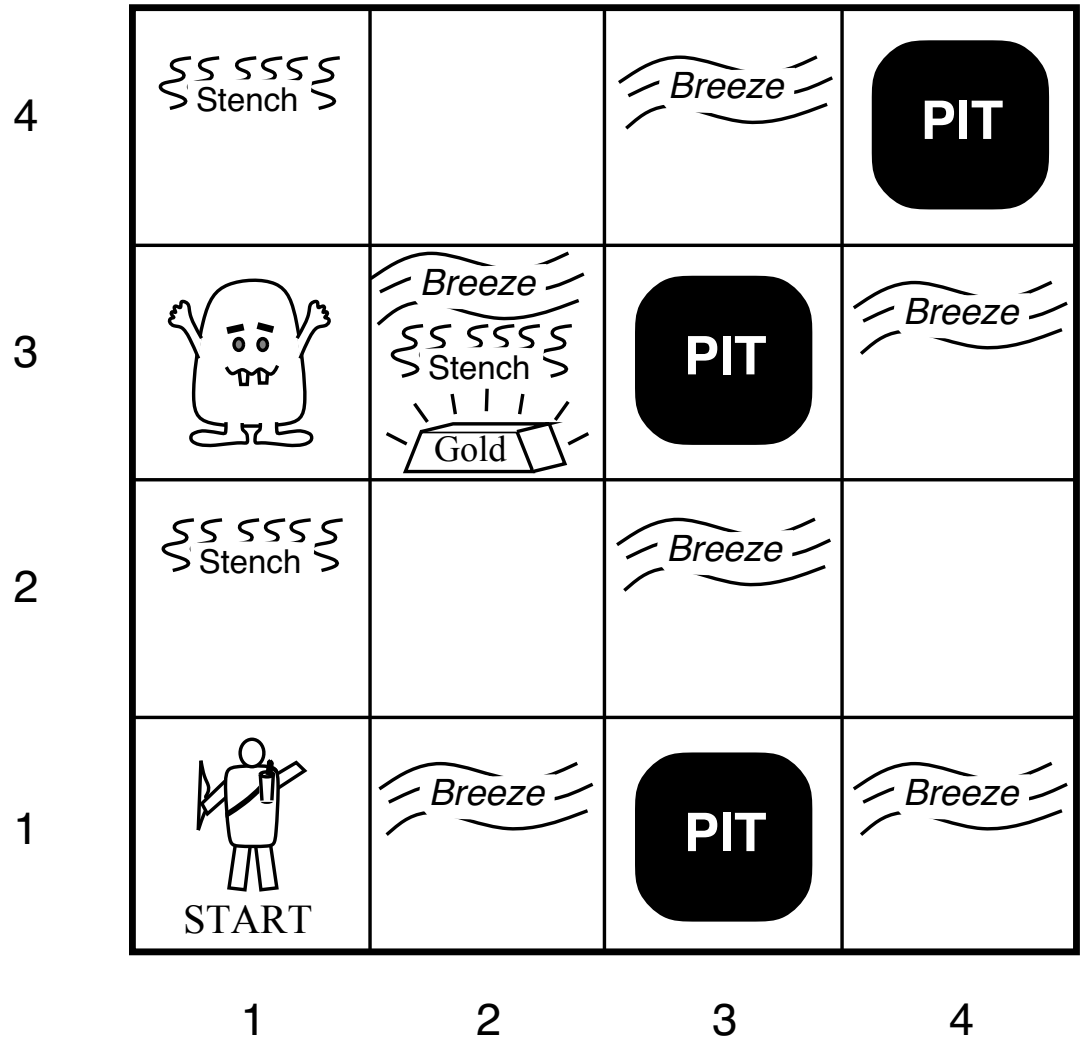
Symbolic Level: Encoding of the KB as sentences in a formal language: *Connects*(*Vaalser_St*, *Aachen*, *Vaals*)

Implementation Level: The internal representation of sentences. Taxi-example:

- a string "`Connects(Vaalser_St, Aachen, Vaals)`"
- a bit in in a 3-D-matrix representing connections between places.

When ASK and TELL work correctly, it suffices to stay at the knowledge level. Advantage: very nice user interface. A user has her own mental world model (propositions about the world) and simply tells that to the agent.

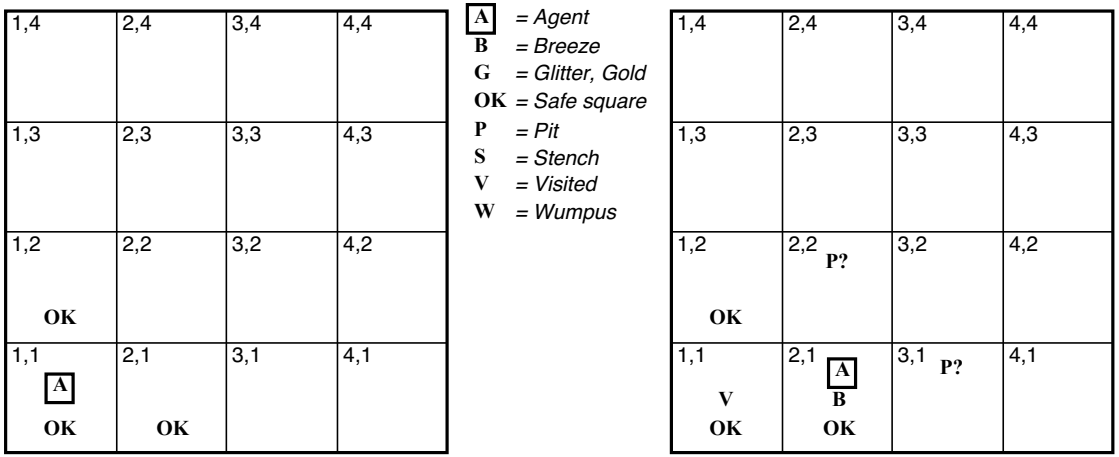
The Wumpus World 1



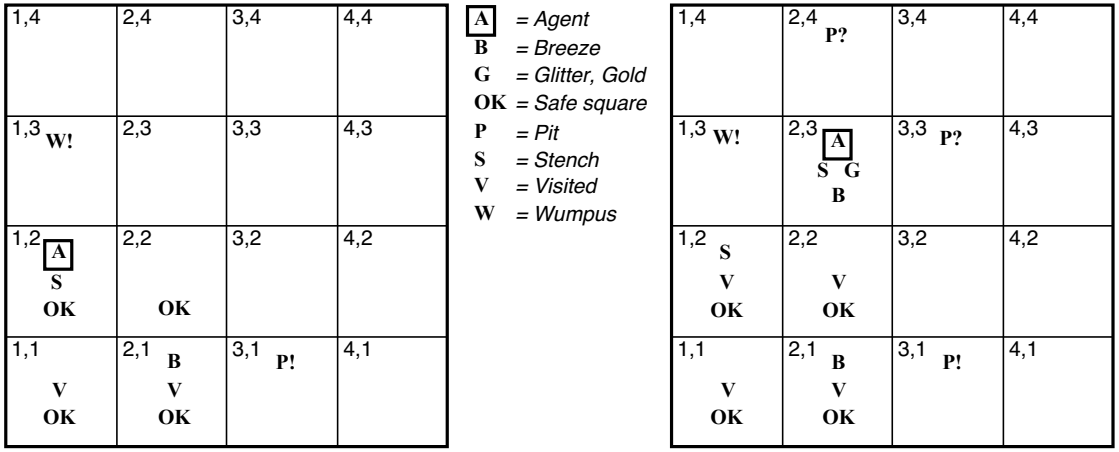
The Wumpus World 2

- In the square where the Wumpus is and those next to it there is stench.
- In the square next to a pit there is a breeze.
- In the square with the gold there is glitter.
- When hitting a wall, the agent receives a bump.
- When the Wumpus dies, its howl is heard everywhere.
- Perceptions are quintuples. [Stench, Breeze, Glitter, None, None] means that there is stench, breeze, and glitter, but there is no bump nor a howl. There is no location sensor.
- Actions: forward, turn right (90^o), turn left (90^o), grab an object, shoot (only one arrow), leave the cave (only from square (1,1)).
- The agent dies, if he falls into a pit or meets the live Wumpus.
- Goal: fetch the gold and leave the cave.
- initial state: agent in (1,1), 1 Wumpus, 1 pile of gold and 3 pits, randomly distributed.

The Wumpus World 3



(a) (b)



(a) (b)

A Declarative Language

We need a precise declarative language for representation and reasoning.

- declarative: System believes P iff P is thought to be true.

  (One cannot believe P without having an idea what it means for P to be true (satisfied) in the world.)
- precise: need to know
  - which strings are sentences;
  - what it means for a sentence to be true (without having to explicitly specify for every sentence whether or not it is true).

Here: the language of first-order logic (FOL)

Note: this is only one of many languages which satisfy the above criteria.

Alphabet

Logical symbols:

- Delimiter : ( ) ,
- Operators: $\neg, \wedge, \vee, \forall, \exists, =$
- Variables: $x, x_1, x_2, \ldots, x', x'', \ldots, y, \ldots, z, \ldots$

  fixed meaning and use (like keywords in a programming language).

Nonlogical symbols

- Predicate symbols (like Friend)
- Function symbols (like bestFriendOf)

  0-ary predicates: often called propositional variables
  0-ary function symbols: constants

  the meaning is application dependent (like identifiers in a PL)

  Note: $=$ is not considered a predicate!

Grammar

well-formed formulas

Expressions: terms and formulas (wffs).

Terms:

1. Every variable is a term.
2. If $t_1, t_2, \ldots, t_n$ are terms and f is an n-ary function symbol, then $f(t_1, t_2, \ldots, t_n)$ is a term.

Atomic wffs:

1. If $t_1, t_2, \ldots, t_n$ are terms and P is an n-ary predicate symbol, then $P(t_1, t_2, \ldots, t_n)$ is an atomic formula.
2. If t_1 and t_2 are terms, then $t_1 = t_2$ is an atomic formula.

Formulas:

1. Every atomic wff is a wff.
2. For wffs α, β and variable x, $\neg\alpha, (\alpha \wedge \beta), (\alpha \vee \beta), \exists x\alpha, \forall x\alpha$ are wffs.

Special Case: Propositional Logic

Propositional logic as a sublanguage of FOL:

- No terms;
- atomic wffs: only 0-ary predicate symbols (propositional variables);
- neither variables nor quantifiers;
- Example: $(p \wedge \neg(q \vee r))$.

Notation

We often omit parentheses or use other kinds ($\{, \}, [,]$) for better readability.

Abbreviations: $(\alpha \overrightarrow{\supset} \beta)$ instead of $(\neg\alpha \vee \beta)$

$(\alpha \equiv \beta)$ instead of $(\alpha \supset \beta) \wedge (\beta \supset \alpha)$

Nonlogical symbols:

Predicates: Person, Nice, OlderThan

Functions: fatherOf, successor, janeDoe

Lexical binding of variables:

$P(x) \quad \wedge \quad \exists x[P(x) \vee Q(x)]$

free bound occurrence of x

Sentences $=$ wffs without free variables.

Substitution: $\alpha[x/t]$ means α with all free occ. of x replaced by t.

Semantics

How are sentences interpreted?

- What do sentences tell us about what is true in the world?
- What does it mean to know/believe a sentence?

Without answers to these questions knowledge representation is impossible!

Problem: The semantics of sentences takes us outside the language because of the nonlogical symbols.

Therefore it is important to make precise the dependence of interpretations from the nonlogical symbols.

Logical interpretations: Specification of how to understand predicates and functions. Can be very complex!

Examples: *FavoriteMovie*, *SoccerCoach*, *DemocraticCountry*.

Interpretations (informal)

There are objects, some of which satisfy a predicate P, others don't. Each interpretation determines the extension of P.

Each interpretation determines a mapping from objects to objects for each function symbol.

Functions are single-valued and defined everywhere.

Note:

That's all one needs to know about the nonlogical symbols to determine which sentences are true and which are false.
In other words, after specifying

- which objects there are,
- which objects satisfy P and
- which mapping corresponds to f,

it is possible to determine the truth value of all sentences.

Interpretations (formal)

$I = \langle D, \Phi \rangle$ is an interpretation where

- D is the universe of discourse or domain
  can be any non-empty set (mathematical objects, but also students, tables, sentences, cars, etc.)
- Φ is an interpretation function

  where P is an n-ary predicate symbol,

  $\Phi(P) \subseteq D \times D \times \ldots \times D$ an n-ary relation over D

  where f is an n-ary function symbol,

  $\Phi(f) \in [D \times D \times \ldots \times D \longrightarrow D]$ an n-ary function over D

  In propositional logic::

  $\Phi(P) = \{\}$ (FALSE) or $\Phi(P) = \{\langle\rangle\}$ (TRUE)
  Simplification: $I = \Phi \in$ [prop. variables $\longrightarrow \{\text{TRUE}, \text{FALSE}\}$]

Denotation

The denotation of a term is the element of D assigned by $I = \langle D, \Phi \rangle$.

Notation: $I||t||$

For terms with free variables the denotation depends on the assignment to the variables as well:

Notation: $I, \nu||t||$, where $\nu \in [\mathrm{Variables} \longrightarrow D]$ defines a variable map.

Rules:

1. $I, \nu||x|| = \nu(x)$ (for every x)
2. $I, \nu||f(t_1, \ldots, t_n)|| = H(d_1, \ldots, d_n)$,
   where $H = \Phi(f)$ and $d_i = I, \nu||t_i||$ (recursively)

Satisfaction

For a given I the truth value of a wff depends also on the assignment to the variables.

$I, \nu \models \alpha$ stands for "α is satisfied by I and ν."
We write $I \models \alpha$ if α is a sentence and $I \models S$ if S is a set of sentences.

Rules:

1. $I, \nu \models P(t_1, \ldots, t_n)$ iff $\langle d_1, \ldots, d_n \rangle \in R$, where $R = \Phi(P)$ and $d_i = I, \nu || t_i ||$;
2. $I, \nu \models (t_1 = t_2)$ iff $I, \nu || t_1 ||$ equals $I, \nu || t_2 ||$;
3. $I, \nu \models \neg\alpha$ iff $I, \nu \not\models \alpha$;
4. $I, \nu \models (\alpha \wedge \beta)$ iff $I, \nu \models \alpha$ and $I, \nu \models \beta$;
5. $I, \nu \models (\alpha \vee \beta)$ iff $I, \nu \models \alpha$ or $I, \nu \models \beta$;
6. $I, \nu \models \exists x \alpha$ iff for some $d \in D$, $I, \nu^x_d \models \alpha$, where ν^x_d is like ν except that $\nu^x_d(x) = d$;
7. $I, \nu \models \forall x \alpha$ iff for all $d \in D$, $I, \nu^x_d \models \alpha$;

In propositional logic: $I \models p$ iff $\Phi(p) \neq \{\}$ and else as above.

Logical Consequence

Rules tell us how the truth value of a sentence depends on the meaning of the nonlogical symbols.

But not all connections between sentences depend on them.

Example: if α is satisfied by I, then $\neg(\beta \wedge \neg\alpha)$ is true independent of why α is true and what β is!

Logical Consequence:

S implies α or α is a logical consequence of S: $\{\alpha\} \models \neg(\beta \wedge \neg\alpha)$

$$S \models \alpha \text{ iff for all } I, \text{ if } I \models S \text{ then } I \models \alpha$$

In other words: for all I, $I \not\models S \cup \{\neg\alpha\}$, or, $S \cup \{\neg\alpha\}$ is unsatisfiable.

Special case: S is empty: $\models \alpha$ iff for all I, $I \models \alpha$. (α is valid.)

Note: $\{\alpha_1, \ldots, \alpha_n\} \models \alpha$ iff $\models (\alpha_1 \wedge \ldots \wedge \alpha_n) \supset \alpha$.
(reduces finite implication to validity)

Why Implication?

System does not have access to the interpretation of the nonlogical symbols as intended by the user!

With implication we know that α is true as long S is true in the intended interpretation.

> *If the world which the user envisions satisfies S, then it must also satisfy α. Other sentences may be true in the user's model of the world, but α need not be satisfied necessarily.*

How about $Dog(fido) \models Mammal(fido)$??

No! Not a logical consequence. $\Phi(Dog) \not\subset \Phi(Mammal)$for some Φ.

Central idea of KR:

such connections are explicitly represented in S:

$$\forall x[Dog(x) \supset Mammal(x)]$$

Then: $S \cup \{Dog(fido)\} \models Mammal(fido)$

Knowledge Bases

A knowledge base (KB) is a set of sentences

explicit representation of the believed sentences
(includes the assumed connections between the nonlogical symbols.)

$\mathrm{KB} \models \alpha$: α is a consequence of one's beliefs.

- explicit knowledge: KB
- implicit knowledge: $\{\alpha \mid \mathrm{KB} \models \alpha\}$

An Example

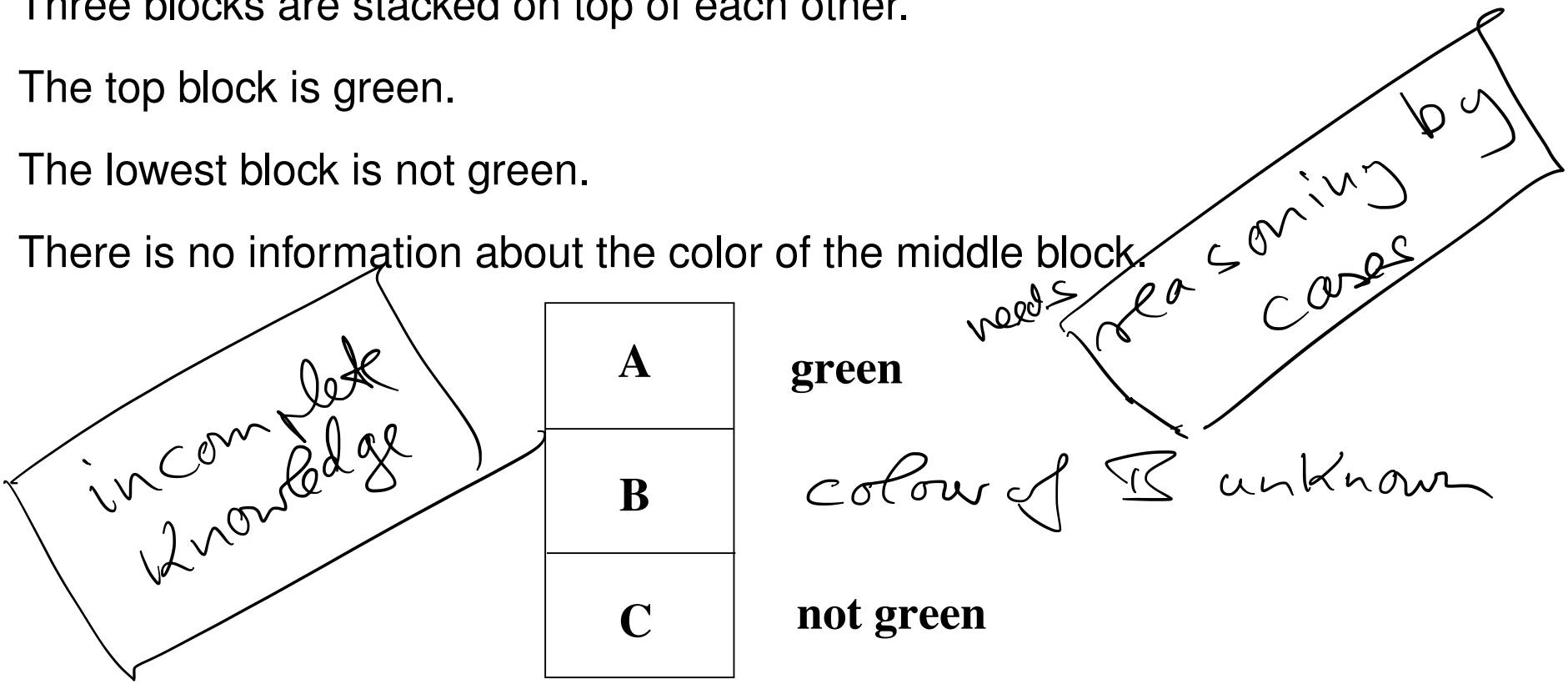
Often not trivial to extract implicit from explicit knowledge:

Three blocks are stacked on top of each other.

The top block is green.

The lowest block is not green.

There is no information about the color of the middle block.



Question: Is there a green block on top of a non-green block? Yes

A Formalization

$$
\begin{aligned}
S &= \{On(a,b), On(b,c), Green(a), \neg Green(c)\} \\
\alpha &= \exists x \exists y [Green(x) \wedge \neg Green(y) \wedge On(x,y)]
\end{aligned}
$$

Claim: $S \models \alpha$

Proof: Let I be an arbitrary interpretation such that $I \models S$.

Case 1: $I \models Green(b)$.
Then $I \models Green(b) \wedge \neg Green(c) \wedge On(b,c)$ and thus $I \models \alpha$.

Case 2: $I \not\models Green(b)$.
Then $I \models \neg Green(b)$.
Hence $I \models Green(a) \wedge \neg Green(b) \wedge On(a,b)$ and thus $I \models \alpha$.

In any case: if $I \models S$, then $I \models \alpha$.

Therefore $S \models \alpha$ **QED**

Where is the Wumpus?

1,4	2,4	3,4	4,4
1,3 **W!**	2,3	3,3	4,3
1,2 **A** **S** **OK**	2,2 **OK**	3,2	4,2
1,1 **V** **OK**	2,1 **B** **V** **OK**	3,1 **P!**	4,1

A = *Agent*
B = *Breeze*
G = *Glitter, Gold*
OK = *Safe square*
P = *Pit*
S = *Stench*
V = *Visited*
W = *Wumpus*

Knowledge about the current situation:
[B = breeze, S = stench, $B_{i,j}$ = there is a breeze in (i, j)]

$\neg S_{1,1}$ $\quad$ $\neg B_{1,1}$
$\neg S_{2,1}$ $\quad$ $B_{2,1}$
$S_{1,2}$ $\quad$ $\neg B_{1,2}$

Invariant knowledge about the Wumpus and stench:

R_1 : $\neg S_{1,1} \supset \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,1}$
R_2 : $\neg S_{2,1} \supset \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$
R_3 : $\neg S_{1,2} \supset \neg W_{1,1} \wedge \neg W_{1,2} \wedge \neg W_{2,2} \wedge \neg W_{1,3}$
R_4 : $S_{1,2} \supset W_{1,3} \vee W_{1,2} \vee W_{2,2} \vee W_{1,1}$

$\models W_{1,3}$

. . .

Knowledge-Based Systems

Start with a large KB representing explicit knowledge (e.g. what it has been told explicitly.)

The behavior of the system should then depend on the implicit knowledge. This requires inference methods:

Deductive inference:

Process to compute the logical consequences of a KB.

given a KB and a query α, computes whether $\mathrm{KB} \models \alpha$.

Process is correct if for every derivable α we have that $\mathrm{KB} \models \alpha$.

does not allow plausible inferences, which hold only in intended interpretations.

Process is complete if every α for which $\mathrm{KB} \models \alpha$ holds is derivable.

usually difficult

does not allow the process to abort computing difficult inferences!