Chair of Computer Science 7, RWTH Aachen University
Prof. Dr. Dr. h.c. W. Thomas • Dipl.-Inform. B. Brütsch

Infinite Computations – WS 2015/2016

Exercises – Series 8 December 11, 2015

Exercise 36. ***(Bachelor + Master)*** *3 points*

Consider the following S1S formula:

$$\varphi(X_1) = X_1(0) \wedge \forall y\big(X_1(y) \to \neg X_1(y+1)\big)$$

Transform φ into an equivalent $\mathrm{S1S}_0$ formula φ' as described in the lecture.

Exercise 37. ***(Bachelor + Master)*** *2+2 points*

We consider the alphabet $\mathbb{B} = \{0, 1\}$.

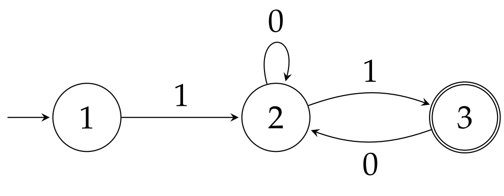
(a) Give an S1S formula for the ω-language $(000)^*1\mathbb{B}^\omega$. Briefly explain your formula.

(b) Give a short colloquial description and an ω-regular expression for the ω-language defined by the S1S formula

$$\begin{aligned}\varphi(X_1) = \exists X_2\Big(&\forall z\big(X_2(z) \to X_2(z+1)\big)\\ &\wedge \exists x \exists y\big(\neg X_1(x) \wedge \neg X_2(x) \wedge X_1(y) \wedge X_2(y)\big)\Big).\end{aligned}$$

Exercise 38. ***(Bachelor + Master)*** *2+2+2 points*

Consider the following Büchi automaton $\mathcal{A}$:



(a) Using the method from the lecture, construct an S1S formula $\varphi_1(X_1)$ with auxiliary set variables Y_1, Y_2, Y_3 such that $L(\varphi_1) = L(\mathcal{A})$.

(b) Do the same with only two auxiliary set variables Y_1, Y_2.

(c) Construct an FO formula $\varphi_2(X_1)$ such that $L(\varphi_2) = L(\mathcal{A})$.

(In an FO formula, no set quantifiers occur.)

Exercise 39. ***(Bachelor + Master)**** *4 points**

We call a relation $R \subseteq \mathbb{N} \times \mathbb{N}$ *S1S-definable* if there exists an S1S-formula $\varphi(x, y)$ such that $(m, n) \in R \Leftrightarrow \varphi[x/m, y/n]$ is satisfied (i.e., the formula $\varphi(x, y)$ evaluates to `true` when the variables x and y are substituted with the values m and n).

For instance, the successor relation *Succ* $\subseteq \mathbb{N} \times \mathbb{N}$ can be defined by the S1S-formula $\varphi_{\text{Succ}}(x, y) := \; y = x + 1$.

Show that the relation *Double* $= \{(m, n) \mid n = 2 \cdot m\} \subseteq \mathbb{N} \times \mathbb{N}$ is not S1S-definable.

Hint: It is useful to consider the ω-language $L = \left\{ \binom{0}{0}^n \binom{0}{1}^n \binom{1}{1}^\omega \mid n \geq 1 \right\}$ and to show that L is not regular.

Please always consider announcements made in the $\mathrm{L^2P}$ course room.

For solutions to the exercise marked with an asterisk (*), bonus points will be awarded.

You can hand in your solutions until **12:15 on Friday, December 18, 2015** at the drop box at the Chair i7.