Exercise 3

a)

$X_i(f)$	:	Input signal
$Y_i(f)$	:	Electric signal for loudspeaker
$L(f), R(f)$	:	Acoustic signal produced by loudspeaker
$Z_i(f)$	:	Signal arriving at the ear
O_{xx}	:	head related transfer function

Goal: Find $Y_i(f)$ such that $Z_i(f) \overset{!}{=} X_i(f)$.

At each ear a acoustic signal of each loudspeaker arrives. The signal arriving from each loudspeaker is of the form $Y_L \cdot L \cdot O_{uL}$ for each ear u and the left loudspeaker, analogous for the right loudspeaker. Because the frequencies of the two loudspeaker (left and right) the signal arriving at the ear is the sum of signal of each loudspeaker. So Z_i is the following.

$$Z_L = Y_R \cdot L \cdot O_{LL} + Y_L \cdot R \cdot O_{LR}$$

The formula for Z_R is analogous. With both Z_R and Z_L

$$Y_R = \frac{Z_R - Y_L L \cdot O_{RL}}{R \cdot O_{RR}} \tag{1}$$

$$Y_L = \frac{Z_L - Y_R R \cdot O_{LR}}{L \cdot O_{LL}} \tag{2}$$

$$= \frac{Z_L}{L \cdot O_{LL}} - \frac{Z_R - Y_L L \cdot O_{RL}}{\cancel{R} \cdot O_{RR}} \frac{\cancel{R} \cdot O_{LR}}{L \cdot O_{LL}} \tag{3}$$

$$= \frac{Z_L}{L \cdot O_{LL}} - \frac{Z_R O_{LR}}{L \cdot O_{RR} O_{LL}} + \frac{Y_L \cancel{L} \cdot O_{RL} O_{LR}}{\cancel{L} \cdot O_{RR} O_{LL}} \tag{4}$$

$$Y_L \left(1 - \frac{O_{LR} O_{RL}}{O_{RR} O_{LL}}\right) = L^{-1} \left(\frac{Z_L}{O_{LL}} - \frac{Z_R O_{LR}}{O_{RR} O_{LL}}\right) \tag{5}$$

$$Y_L = L^{-1} \left(\frac{Z_L}{O_{LL}} - \frac{Z_R O_{LR}}{O_{RR} O_{LL}}\right) \left(1 - \frac{O_{LR} O_{RL}}{O_{RR} O_{LL}}\right)^{-1} \tag{6}$$

$$= L^{-1} \left(\frac{Z_L}{O_{LL}} \left(\frac{1}{1 - \frac{O_{LR} O_{RL}}{O_{LL} O_{RR}}}\right) - \frac{Z_R O_{LR}}{O_{RR} O_{LL}} \left(\frac{1}{1 - \frac{O_{LR} O_{RL}}{O_{LL} O_{RR}}}\right)\right) \tag{7}$$

$$= L^{-1} \left(\frac{Z_L}{O_{LL} - \frac{O_{LR} O_{RL}}{\cancel{O_{LL}} O_{RR}} \cancel{O_{LL}}} - \frac{Z_R O_{LR}}{O_{LL} O_{RR} - \frac{O_{RL} O_{LR}}{O_{LL} \cancel{O_{RR}}} \cancel{O_{RR}}}\right) \tag{8}$$

$$= L^{-1} \left(\frac{O_{RR}}{O_{RR} O_{LL} - O_{LR} O_{RL}} Z_L - \frac{O_{LR}}{O_{LL}} \frac{O_{LL}}{O_{LL} O_{RR} - O_{RL} O_{LR}} Z_R\right) \tag{9}$$

b)

The equation can be split into two parts. The part for the right ear and the part for the left ear. Because the sound you can hear with your right ear from the left loudspeaker should be reduced, the part for the left ear has a negative sign. Because the frequencies are manipulated during the time between loudspeaker and ear, the made sound is not equal to the sound the ear hears. In order to reverse the manipulation the fractions of considering O are inserted.

c)

The analytical solution is not applicable in praxis because it is numerical instable for rather low low frequencies with $O_{LL} \sim O_{LR}$ and $O_{RR} \sim O_{RL}$. This leads to a denominator close to 0.