Lehrstuhl für Informatik 7
Logik und Theorie diskreter Systeme

RWTH AACHEN UNIVERSITY

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Exercises for Computational Complexity Theory

Assignment 5 Deadline: Thursday, June 2nd, 2016

Exercise 17 (*Universal exploration sequences*)

A graph with cyclically ordered adjacency lists is a graph such that for each vertex v there is a cyclic ordering $(u_1, \dots, u_t)$ of the neighborhood $N(v)$. Given an infinite sequence of natural numbers $n_3, n_4, n_5 \dots \in \mathbb{N}$ and a starting edge (v_1, v_2) of G we explore G using the sequence by defining iteratively v_{i+1} to be the (n_{i+1})-st successor of v_{i-1} in the cyclic ordering of $N(v_i)$ (i.e., if $v_{i-1} = u_j$ then $v_{i+1} = u_{j+n_{i+1}}$ where the index is taken modulo t).

Show that there exists a *universal exploration sequence*, that is, a fixed sequence of natural numbers $n_3, n_4, n_5 \dots$ such that for every finite connected graph G and every starting edge (v_1, v_2), when exploring the graph G using the sequence, we will visit every vertex of G.

Exercise 18 (NL-*completeness of* 2SAT)

Show that 2SAT is NL-complete.

Exercise 19 (*The class* polyL) [Exercise 4.12 in AB, modified]

Define polyL to be $\cup_{c>0}\text{SPACE}(\log^c n)$. Steve's Class SC (named in honor of Steve Cook) is defined to be the set of languages that can be decided by deterministic machines that run in polynomial time and $\log^c n$ space for some $c > 0$.

a) It is an open problem whether PATH $\in$ SC. Why does Savitch's Theorem not resolve this question?

b) Is SC $=$ polyL $\cap\, P$?

c) A language $L \subseteq \{0,1\}^*$ is called polyL-*complete* if both $L \in$ polyL and $L' \leq_l L$ for every $L' \in$ polyL hold. Show that there is no polyL-complete problem.

Exercise 20 (*Zermelo's theorem*) [Exercise 4.10 in AB]

Show that in every deterministic finite two-person game with perfect information where there are no draws, one of the two players has a winning strategy.