Chair of Computer Science 7, RWTH Aachen University
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Infinite Computations – WS 2015/2016

Exercises – Series 5 November 20, 2015

Exercise 21. ***(Bachelor)*** *4 points*

Show that the Union Lemma for Rabin automata presented in the lecture does not hold for *accepting* runs. That means: There exists a Rabin automaton that has runs ρ_1, ρ_2, ρ with $Inf(\rho_1) \cup Inf(\rho_2) = Inf(\rho)$ such that ρ_1 and ρ_2 are accepting, but ρ is not accepting.

Prove this statement by giving an appropriate example.

Exercise 22. ***(Bachelor + Master)*** *2 points*

Let $\Sigma = \{a, b\}$ and let $U = \{ab, bb, bbb\}$. Show that $L = U \cdot \Sigma^\omega$ is both E- and A-recognizable.

Exercise 23. ***(Bachelor + Master)*** *2 points*

Let $L = \{\alpha \in \{a, b, c\}^\omega \mid \alpha \text{ contains exactly one } c \text{ and after that only } b\text{'s}\}$. Show that L is both deterministically Büchi recognizable and deterministically co-Büchi recognizable.

Exercise 24. ***(Bachelor + Master)*** *4 points*

Reconsider the ω-languages $L(\mathcal{A}_n)$ over the alphabet $\Sigma_n = \{1, \ldots, n, \#\}$ defined in the lower bound theorem on the size of deterministic Rabin automata. Provide a general construction (for $n \geq 1$) of a deterministic Muller automaton $\mathcal{M}_n$ with only $\mathcal{O}(n^2)$ states that recognizes $L(\mathcal{A}_n)$. Give a brief justification for the correctness of your construction.

Hint: Use the "cycle characterization" of $L(\mathcal{A}_n)$ presented in the lecture.

Exercise 25. ***(Master)*** *4 points*

Again, consider the ω-languages $L(\mathcal{A}_n)$ as in exercise 24. Show that each nondeterministic Büchi automaton for the complement of $L(\mathcal{A}_n)$ (i.e., $\Sigma_n^\omega \setminus L(\mathcal{A}_n)$) needs at least $n!$ states.

Please always consider announcements made in the L²P course room.

You can hand in your solutions until **12:15 on Friday, November 27, 2015,** before the beginning of the lecture or at the drop box at the Chair i7.