Chair of Computer Science 7, RWTH Aachen University
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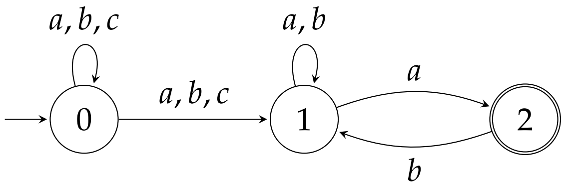
Infinite Computations – WS 2015/2016

Exercises – Series 2 October 30, 2015

Exercise 6. ***(Bachelor)*** *1+2 points*

(a) Let Σ be a finite alphabet and let UP be the set of all ultimately periodic ω-words over Σ. Show that UP is not regular. (You may use the closure properties of the class of regular ω-languages.)

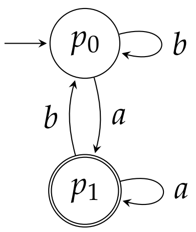
(b) Consider the following Büchi automaton $\mathcal{A}$. Give a Büchi automaton recognizing the complement of $L(\mathcal{A})$. (You can construct the automaton directly, without using the complementation construction from the lecture.)



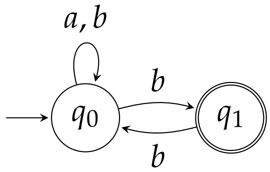
Exercise 7. ***(Bachelor + Master)*** *3+3 points*

Consider the following Büchi automata over the alphabet $\Sigma = \{a, b\}$:

$\mathcal{A}_1$



$\mathcal{A}_2$



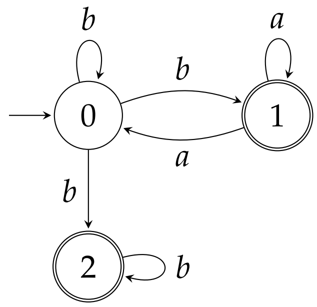
Apply the following steps to obtain a nondeterministic Büchi automaton (NBA) recognizing $L(\mathcal{A}_1) \cap L(\mathcal{A}_2)$:

(a) Construct a generalized Büchi automaton (GNBA) $\mathcal{A}$ recognizing $L(\mathcal{A}_1) \cap L(\mathcal{A}_2)$ as shown in the lecture.

(b) Construct an NBA $\mathcal{A}'$ that is equivalent to the GNBA $\mathcal{A}$, using the construction presented in the lecture.

Exercise 8. ***(Bachelor + Master)*** *3+1 points*

Consider the following Büchi automaton $\mathcal{A}$ over the alphabet $\Sigma = \{a, b\}$.



(a) Give graphical representations of the six transition profiles of $\mathcal{A}$.

(b) Draw the transition profile automaton $TP(\mathcal{A})$.

Exercise 9. ***(Bachelor + Master)*** *3 points*

We show that complementation of Büchi automata is easy if we start with a *deterministic* Büchi automaton:

Let $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ be a deterministic Büchi automaton. Give a formal construction of a nondeterministic Büchi automaton $\mathcal{B}$ recognizing the complement of $L(\mathcal{A})$ such that the number of states of $\mathcal{B}$ is polynomial in $|Q|$. Show the correctness of your construction.

Exercise 10. ***(Master)*** *3 points*

For a nondeterministic Büchi automaton $\mathcal{A} = (Q, \Sigma, q_0, \Delta, F)$, let $\Delta(P, a)$ be the set of states that are reachable from a state in P via input a, i.e., $\Delta(P, a) = \{q \in Q \mid \exists p \in P\colon (p, a, q) \in \Delta\}$.

We define *simple Büchi automata* as nondeterministic Büchi automata with the property that $\Delta(\{q\}, a) \cap F \neq \emptyset$ for all $q \in F, a \in \Sigma$. In other words, each final state allows for each letter a transition again to a final state.

Let $\mathcal{A} = (Q, \Sigma, q_0, \Delta, F)$ be a simple Büchi automaton. Give a formal construction of a *deterministic* Büchi automaton $\mathcal{B}$ with $L(\mathcal{B}) = L(\mathcal{A})$. Show the correctness of your construction.

Please always consider announcements made in the L²P course room.

You can hand in your solutions until **12:15 on Friday, November 6, 2015,** before the beginning of the lecture or at the drop box at the Chair i7.