## **Estimating Defocus Blur via Rank of Local Patches**

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## **Abstract**

This paper addresses the problem of defocus map estimation from a single image. We present a fast yet effective approach to estimate the spatially varying amounts of defocus blur at edge locations, which is based on the maximum ranks of the corresponding local patches with different orientations in gradient domain. Such an approach is motivated by the theoretical analysis which reveals the connection between the rank of a local patch blurred by a defocus-blur kernel and the blur amount by the kernel. After the amounts of defocus blur at edge locations are obtained, a complete defocus map is generated by a standard propagation procedure. The proposed method is extensively evaluated on real image datasets, and the experimental results show its superior performance to existing approaches.

## 1. Introduction

Conventional cameras produce images with best sharpness when the objects of a scene are exactly on the focal plane of focusing module. The further is an object away from the focal plane, the more blurred it appears in the image, as shown in Fig. 2 (a). Such a phenomenon is called *defocus* (or out-of-focus) whose blur amount is related to the translation of the object away from the focal plane along optical axis, as illustrated in Fig. 1. More specifically, when an object is placed at the focal distance  $d_f$ , all light beams from any point of the object will converge to a single sensor point, which leads to image pixels with best sharpness. In contrast, the light beams from the points with the distance  $d \neq d_f$  will arrive at a region with multiple sensor points, which leads to blurred image pixels. Such a region is called circle of confusion (CoC).

**Defocus amount and scene depth.** The *defocus amount* of a pixel, denoted by c, is defined as the diameter of CoC ([8]). The defocus amount c is related to the *scene depth*,

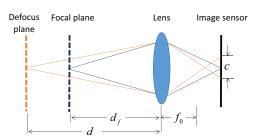


Figure 1: Illustration of focus and defocus [37].

denoted by d, as follows:

$$c = \frac{|d - d_f|}{d} \frac{f_0^2}{n_s(d_f - f_0)},\tag{1}$$

where  $n_s$  is the stop number and  $f_0$  is the focal length. Clearly, the defocus amount c monotonically increases when the scene depth f increases. Thus, for an image I captured for the scene with varying depth, the defocus amount is spatially varying. We define the *defocus map* of an image as the matrix c whose (i,j)-th entry c[i,j] is the defocus amount of the pixel at [i,j].

**Defocus amount and blur kernel**. Defocus map is also closely related to the image degradation caused by out-offocus, as it measures the blur amount of each pixel of an out-of-focus image. For example, as the blurring effect is often modeled as local averaging weighted by 2D isotropic Gaussian functions, local regions of defocused image can then be modeled by the convolution between sharp image regions and isotropic Gaussian kernels with spatially varying standard deviation (s.t.d.), denoted by  $\sigma[i,j]$ . The s.t.d.  $\sigma[i,j]$  is equivalent to the defocus map c[i,j] up to a constant, i.e.  $\sigma[i,j] = \kappa_0 c[i,j]$  for some global constant  $\kappa_0$ . See e.g. [7, 9, 25] for more details. In other words, defocus map is equivalent to the s.t.d. of spatially varying blur kernels of an out-of-focus image.

Applications. Since defocus map provides essential in-

