2019-1, 2019/5/31

오토마타 및 형식언어(COMP315)

Homework 4

Due date: 2019년 6월 7일 (금)

Late submission not allowed

How to submit: Upload the answers as one PDF file to LMS

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Q1, 5점) Chapter 7.1, Exercise 1 (pg. 189)

a) 언어 $L = \{a^n b^{2n} : n \geq 0\}$을 나타내는 Pushdown Automata를 구하시오

답: $M = (\{q_0, q_1, q_f\}, \{a, b\}, \{a, b, z\}, \delta, q_0, z, F)$

Transition functions (δ): $\delta(q_0, a, z) = \{(q_0, aaz)\}$. $\delta(q_0, a, a) = \{(q_0, aaa)\}$.

$\delta(q_0, \lambda, z) = \{(q_1, z)\}$. $\delta(q_0, \lambda, a) = \{(q_1, a)\}$.

$\delta(q_1, b, a) = \{(q_1, \lambda)\}$.

$\delta(q_1, \lambda, z) = \{(q_f, z)\}$.

b) 위의 언어에 속하는 *w=aabbbb*를 accept하는 과정을 (instantaneous description) 보이시오

답: $(q_0, aabbbb, z) \vdash (q_0, abbbb, aaz)$

$\vdash (q_0, bbbb, aaaaz)$

$\vdash (q_1, bbbb, aaaaz)$

$\vdash (q_1, bbb, aaaz)$

$\vdash (q_1, bb, aaz)$

$\vdash (q_1, b, az)$

$\vdash (q_1, \lambda, z)$

$\vdash (q_f, \lambda, z)$

Q2, 5점) Chapter 7.1, Exercise 12 (pg. 190)

다음 오토마타는 어떠한 언어 L을 나타내는가?

$M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b\}, \{0, 1, z\}, \delta, q_0, z, \{q_5\})$

Transition fuctions (δ):

$\delta(q_0, b, z) = \{(q_1, 1z)\}$,
$\delta(q_1, b, 1) = \{(q_2, 11)\}$,
$\delta(q_2, a, 1) = \{(q_3, \lambda)\}$,
$\delta(q_3, a, 1) = \{(q_4, \lambda)\}$,
$\delta(q_4, a, z) = \{(q_4, z), (q_5, z)\}$.

답: $L = \{ \quad \}$ $L = \{bbaaaa^n : n \geq 0.\}$.

Q3, 5점) Chapter 7.2, Exercise 2 (pg. 201)

다음 grammar를 PDA로 나타내시오

$S \to aSSSab|\lambda$

Hint: Greibach form으로 우선 변환 후 transition function(δ)들을 정의

답: $S \to aSSSab \mid aab$

$\Downarrow$

GNF: $S \to aSSSV_aV_b \mid aV_aV_b$.
$V_a \to a$,
$V_b \to b$.

$M = (\{q_0, q_1, q_f\}, \{a, b\}, \{z, S, V_a, V_b\}, \delta, q_0, z, \{q_f\})$.

$\delta(q_0, \lambda, z) = \{(q_1, Sz)\}$,
$\delta(q_1, a, S) = \{(q_1, SSSV_aV_b), (q_1, V_aV_b)\}$,
$\delta(q_1, a, V_a) = \{(q_1, \lambda)\}$,
$\delta(q_1, b, V_b) = \{(q_1, \lambda)\}$,
$\delta(q_1, \lambda, z) = \{(q_f, z)\}$. ■

Q4, 5점) Chapter 8.1, Exercise 1 (pg. 221)

언어 $L = \{w: n_a(w) < n_b(w) < n_c(w)\}$이 context-free 하지 않다는 것을 pumping lemmar를 사용하여 보이시오

답: 충분히 큰 m에 대해 $w = a^m b^{m+1} c^{m+2}$라 하자. $w \in L$이고 $|w| \geq m$이다.
Decomposition $w = uvxyz$, $|vxy| \leq m$, $|vy| \geq 1$에 대해
① $v = a^k$, $y = a^l \to w_2 = uv^2xy^2z$
$= a^{m+k+l} b^{m+1} c^{m+2}$. $|vy| \geq 1$이므로 $k+l \geq 1$, $\therefore n_a(w) \geq n_b(w)$.
② $v = a^k$, $y = b^l \to w_2 = a^{m+k} b^{m+1+l} c^{m+2}$.
$l \geq 1$ 이면 $n_b(w) \geq n_c(w)$.
$k \geq 1$ 이면 $n_a(w) \geq n_b(w)$
③ $v = b^k$, $y = b^l \to w_2 = a^m b^{m+1+k+l} c^{m+2}$. $k+l \geq 1$ $\therefore n_b(w) \geq n_c(w)$.
④ $v = b^k$, $y = c^l \to w_0 = uxz = a^m b^{m+1-k} c^{m+2-l}$
$k \geq 1$ 이면 $n_a(w) \geq n_b(w)$,
$l \geq 1$ 이면 $n_b(w) \geq n_c(w)$.
⑤ $v = c^k$, $y = c^l \to w_0 = a^m b^{m+1} c^{m+2-(k+l)}$.
$k+l \geq 1$ $\therefore n_b(w) \geq n_c(w)$.
$\therefore L$ is not Context-free. ■