Linear Algebra Assignment #2

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2018112749 전현승.

1. (3pt) Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is $\mathbf{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why or why not?

2. (3pt) Let $\mathbf{u} = \begin{bmatrix} 7 \\ 2 \\ 5 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix}$. Show that these three vectors are linearly dependent.

3. (5pt) Solve the following linear equation. Write the solution set of the given homogeneous system in parametric vector form.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 0 \\ -4x_1 - 9x_2 + 2x_3 &= 0 \\ -3x_2 - 6x_3 &= 0 \end{aligned}$$

1. $A\mathbf{x} = \mathbf{u}$의 해가 존재하는가?

$$\begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 5 & 8 & 7 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & -7 & 7 & -8 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -29 \end{bmatrix}$$

$0 = -29$ $\therefore$ 해가 없다.

$\therefore \mathbf{u} \notin \mathrm{Span}\{A\}$ $\blacksquare$

2. $A\mathbf{x} = 0$이 자명해 이외의 해를 가진다.

$$\begin{bmatrix} 7 & 3 & 6 \\ 2 & 1 & 1 \\ 5 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 1 \\ 7 & 3 & 6 \\ 5 & 3 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 \\ 0 & -\frac{1}{2} & \frac{5}{2} \\ 0 & \frac{1}{2} & -\frac{5}{2} \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 \\ 0 & -\frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3x_3 \\ 5x_3 \\ x_3 \end{bmatrix} = x_3 \cdot \begin{bmatrix} -3 \\ 5 \\ 1 \end{bmatrix}.$$

해가 무수히 많다 $\blacksquare$

3. $$\begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & -3 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 \\ 0 & 3 & 6 \\ 0 & -3 & -6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \cdot \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} \blacksquare$$