Linear Algebra Assignment #4

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~~Q1~~. (5pt) Determine the dimension of the subspace H of $\mathbb{R}^3$ spanned by the vectors $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$.(First, find a basis for H.)

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 3 \\ -7 \\ -1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -1 \\ 6 \\ 7 \end{bmatrix}$$

~~Q2~~. (5pt) Consider the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0.2 \end{bmatrix}, \begin{bmatrix} 0.2 \\ 1 \end{bmatrix} \right\}$$

For $\mathbb{R}^2$. If $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, what is $\mathbf{x}$?

Q3. (10pt) Given $\mathbf{u}$ in $\mathbb{R}^n$ with $\mathbf{u}^T\mathbf{u} = 1$, let $P = \mathbf{u}\mathbf{u}^T$(and outer product) and $Q = I - 2P$. Justify statements (a), (b), and (c).

(a) $P^2 = P$ (b) $P^T = P$ (c) $Q^2 = I$

(a) $P^2 = (u \cdot u^T)(u \cdot u^T) = u \cdot u^T u \cdot u^T = uu^T = P$.

(b) $P^T = (uu^T)^T = u^T \cdot u = uu^T = P$.

(c) $Q^2 = (I-2P)(I-2P) = I^2 - 4P + 4P^2 = I - 4P + 4P = I$. ∎

Q4. (3pt) Let $\mathbf{u} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$. Determine P and Q as in Q3, and compute $P\mathbf{x}$ and $Q\mathbf{x}$.

$P = uu^T = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$Q = I - 2P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$P\mathbf{x} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$.

$Q\mathbf{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}$. ∎

★ The transformation $\mathbf{x} \mapsto P\mathbf{x}$ is called a projection, and $\mathbf{x} \mapsto Q\mathbf{x}$ is called a Householder reflection. Such reflections are used in computer programs to create multiple zeros in a vector(usually a column of a matrix)