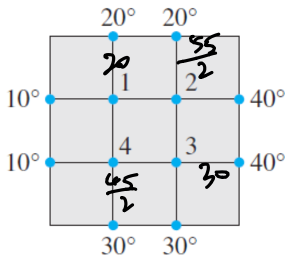
Linear Algebra Assignment #1

2020/09/07

2018112749 전현승

Answer the following questions.



An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume the plate shown in the figure represents a cross section of metal beam, with negligible heat flow in the direction perpendicular to the plate. Let $T_1, \dots, T_4$ denote the temperatures at the four interior nodes of the mesh in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes to the left, above, to the right, and blow. For instance, $T_1 = \frac{10+20+T_2+T_4}{4}$, or $4T_1 - T_2 - T_4 = 30$.

Q1. (5pt) Write a system of four equations whose solution gives estimates for the temperatures $T_1, \dots, T_4$.

Q2. (10pt) Solve the system of equations from Q1.

Q3. (10pt) In a wind tunnel experiment, the force on a projectile due to air resistance was measured at different velocities as following:

t	Velocity(100ft/sec)	0	2	4	6	8	10
$P(t)$	Force(100lb)	0	2.9	14.8	39.6	74.3	119

Find an interpolating polynomial for these data and estimate the force on the projectile when the projectile is traveling at 750 ft/sec. Use $p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$. What happens if you try to use a polynomial of degree less than 5? (Try a cubic polynomial, for instance)

Q1. $\begin{cases} T_1 = \frac{1}{4}T_2 + \frac{1}{4}T_4 + \frac{30}{4} \\ T_2 = \frac{1}{4}T_1 + \frac{1}{4}T_3 + 15 \\ T_3 = \frac{1}{4}T_2 + \frac{1}{4}T_4 + \frac{70}{4} \\ T_4 = \frac{1}{4}T_1 + \frac{1}{4}T_3 + 10. \end{cases} \Leftrightarrow \begin{array}{l} 4T_1 - T_2 - T_4 = 30 \\ -T_1 + 4T_2 - T_3 = 60 \\ -T_2 + 4T_3 - T_4 = 70 \\ -T_1 - T_3 + 4T_4 = 40 \end{array}$

Q2. $\begin{bmatrix} 4 & -1 & 0 & -1 & 30 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40 \end{bmatrix} \overset{①}{=} \begin{bmatrix} 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & \frac{15}{2} \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40 \end{bmatrix} \overset{②}{=} \begin{bmatrix} 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & \frac{15}{2} \\ 0 & \frac{15}{4} & -1 & -\frac{1}{4} & \frac{135}{2} \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -\frac{1}{4} & -1 & \frac{15}{4} & \frac{95}{2} \end{bmatrix}$

$\overset{③}{=} \begin{bmatrix} 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & \frac{15}{2} \\ 0 & 1 & -\frac{4}{15} & -\frac{1}{15} & 18 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -\frac{1}{4} & -1 & \frac{15}{4} & \frac{95}{2} \end{bmatrix} \overset{④}{=} \begin{bmatrix} 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & \frac{15}{2} \\ 0 & 1 & -\frac{4}{15} & -\frac{1}{15} & 18 \\ 0 & 0 & \frac{56}{15} & -\frac{16}{15} & 88 \\ 0 & 0 & -\frac{16}{15} & \frac{56}{15} & 52 \end{bmatrix}$

$$\overset{5}{=}\begin{bmatrix} 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & \frac{15}{2} \\ 0 & 1 & -\frac{4}{15} & -\frac{1}{15} & 18 \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{165}{7} \\ 0 & 0 & -\frac{16}{15} & \frac{56}{15} & 52 \end{bmatrix} \overset{6}{=}\begin{bmatrix} 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & \frac{15}{2} \\ 0 & 1 & -\frac{4}{15} & -\frac{1}{15} & 18 \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{165}{7} \\ 0 & 0 & 0 & \frac{24}{7} & \frac{540}{7} \end{bmatrix}$$

$$\overset{7}{=}\begin{bmatrix} 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & \frac{15}{2} \\ 0 & 1 & -\frac{4}{15} & -\frac{1}{15} & 18 \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{165}{7} \\ 0 & 0 & 0 & 1 & \frac{45}{2} \end{bmatrix} \overset{8}{=}\begin{bmatrix} 1 & 0 & -\frac{1}{15} & -\frac{4}{15} & 12 \\ 0 & 1 & -\frac{4}{15} & -\frac{1}{15} & 18 \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{165}{7} \\ 0 & 0 & 0 & 1 & \frac{45}{2} \end{bmatrix}$$

$$\overset{9}{=}\begin{bmatrix} 1 & 0 & 0 & -\frac{2}{7} & \frac{95}{7} \\ 0 & 1 & 0 & -\frac{1}{7} & \frac{170}{7} \\ 0 & 0 & 1 & -\frac{2}{7} & \frac{165}{7} \\ 0 & 0 & 0 & 1 & \frac{45}{2} \end{bmatrix} \overset{10}{=}\begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & \frac{55}{2} \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & \frac{45}{2} \end{bmatrix}$$

$\therefore T_1=20,\ T_2=\frac{55}{2},\ T_3=30,\ T_4=\frac{45}{2}$ $\blacksquare$

Q3. $P(0)=a_0=0.$

$P(2)=2.9,\ P(4)=14.8,\ P(6)=39.6,\ P(8)=74.3,\ P(10)=119.$

$$\begin{bmatrix} 2 & 4 & 8 & 16 & 32 & 2.9 \\ 4 & 4^2 & 4^3 & 4^4 & 4^5 & 14.8 \\ 6 & 6^2 & 6^3 & 6^4 & 6^5 & 39.6 \\ 8 & 8^2 & 8^3 & 8^4 & 8^5 & 74.3 \\ 10 & 10^2 & 10^3 & 10^4 & 10^5 & 119 \end{bmatrix} = \cdots = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \frac{137}{80} \\ 0 & 1 & 0 & 0 & 0 & -\frac{1147}{960} \\ 0 & 0 & 1 & 0 & 0 & \frac{127}{192} \\ 0 & 0 & 0 & 1 & 0 & -\frac{269}{3840} \\ 0 & 0 & 0 & 0 & 1 & \frac{1}{384} \end{bmatrix}$$

$\therefore P(t)=\frac{137}{80}t-\frac{1147}{960}t^2+\frac{127}{192}t^3-\frac{269}{3840}t^4+\frac{1}{384}t^5.$

$P(7.5)\approx 64.838$ $\blacksquare$

if $P(t)=a_0+a_1t+a_2t^2+a_3t^3$. $(P(2)=2.9,\ P(4)=14.8,\ P(6)=39.6)$

$$\begin{bmatrix} 2 & 4 & 8 & 2.9 \\ 4 & 16 & 64 & 14.8 \\ 6 & 6^2 & 6^3 & 39.6 \end{bmatrix} = \cdots = \begin{bmatrix} 1 & 0 & 0 & -\frac{3}{20} \\ 0 & 1 & 0 & \frac{51}{80} \\ 0 & 0 & 1 & \frac{13}{160} \end{bmatrix}.$$

$\therefore P(t)=-\frac{3}{20}t+\frac{51}{80}t^2+\frac{13}{160}t^3.$

$P(7.5)\approx 69.012$ $\blacksquare$