# Linear Algebra Assignment #3

2020/09/14

2018112749 전현승

1. (5pt) Let a rotation transformation $T$ be a transformation from $\mathbb{R}^2$ onto $\mathbb{R}^2$. The standard matrix $A$ can be defined as $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ for a given angle $\theta$. Show that this transformation is a linear transformation.

2. (5pt) Let a scaling transformation $T$ be a transformation from $\mathbb{R}^2$ onto $\mathbb{R}^2$. The standard matrix $A$ can be defined as $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$ for a given angle $\theta$. Show that this transformation is a linear transformation.

1. $T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \cdot \cos\theta - x_2 \cdot \sin\theta \\ x_1 \cdot \sin\theta + x_2 \cdot \cos\theta \end{bmatrix}.$

$$T(\mathbf{u}+\mathbf{v}) = T\left(\begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix}\right) = \begin{bmatrix} (u_1+v_1)\cos\theta - (u_2+v_2)\sin\theta \\ (u_1+v_1)\sin\theta + (u_2+v_2)\cos\theta \end{bmatrix}.$$

$$= \begin{bmatrix} u_1\cos\theta - u_2\sin\theta \\ u_1\sin\theta + u_2\cos\theta \end{bmatrix} + \begin{bmatrix} v_1\cos\theta - v_2\sin\theta \\ v_1\sin\theta + v_2\cos\theta \end{bmatrix}$$

$$= T(\mathbf{u}) + T(\mathbf{v}).$$

$$T(c \cdot \mathbf{u}) = T\left(\begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}\right) = \begin{bmatrix} cu_1 \cdot \cos\theta - cu_2 \cdot \sin\theta \\ cu_1 \cdot \sin\theta + cu_2 \cdot \cos\theta \end{bmatrix}$$

$$= c \cdot \begin{bmatrix} u_1 \cdot \cos\theta - u_2 \cdot \sin\theta \\ u_1 \cdot \sin\theta + u_2 \cdot \cos\theta \end{bmatrix}$$

$$= c \cdot T(\mathbf{u}). \blacksquare$$

2. $T(\mathbf{x}) = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} s_x \cdot x_1 \\ s_y \cdot x_2 \end{bmatrix}.$

$$T(\mathbf{u}+\mathbf{v}) = T\left(\begin{bmatrix} u_1+v_1 \\ u_2+v_2 \end{bmatrix}\right) = \begin{bmatrix} s_x(u_1+v_1) \\ s_y(u_2+v_2) \end{bmatrix} = \begin{bmatrix} s_x \cdot u_1 \\ s_y \cdot u_2 \end{bmatrix} + \begin{bmatrix} s_x \cdot v_1 \\ s_y \cdot v_2 \end{bmatrix} = T(\mathbf{u}) + T(\mathbf{v}).$$

$$T(c\mathbf{u}) = T\left(\begin{bmatrix} cu_1 \\ cu_2 \end{bmatrix}\right) = \begin{bmatrix} s_x \cdot cu_1 \\ s_y \cdot cu_2 \end{bmatrix} = c \cdot \begin{bmatrix} s_x \cdot c_1 \\ s_y \cdot c_2 \end{bmatrix} = c \cdot T(\mathbf{u}). \blacksquare$$