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Calculus I, Section 4.1 Classwork   Due_________________

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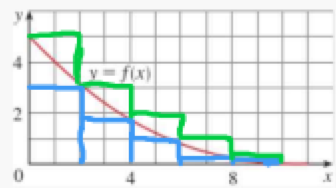
4.1 Exercises

1. a. By reading values from the given graph of f, use five rectangles to find a lower estimate and an upper estimate for the area under the given graph of f from $x = 0$ to $x = 10$. In each case sketch the rectangles that you use.



$[0,10] = [0,2],[2,4],[4,6],[6,8],[8,10]$

$R_5 = 2[f(2)+f(4)+f(6)+f(8)+f(10)]$

$2(3+1.75+.8+.2+0]$

$R_5 \approx 11.5$

$L_5 = 2(f(0)+f(2)+f(4)+f(6)+f(8))$

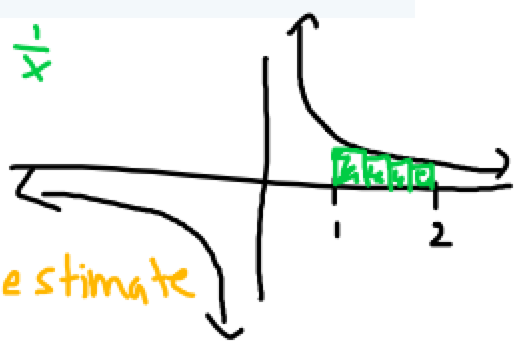
$2(5+3+1.75+.8+.2)$

$L_5 \approx 21.5$

3. a. Estimate the area under the graph of $f(x) = 1/x$ from $x = 1$ to $x = 2$ from to using four approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

$n=4 \quad [1,2) \quad \text{int} = \frac{2-1}{4} = \frac{1}{4} \qquad f(x) = \frac{1}{x}$

$[1,1.25],[1.25,1.5],[1.5,1.75],[1.75,2]$



$R_5 = \frac{1}{4}(f(1.25)+f(1.5)+f(1.75)+f(2))$

$\frac{1}{4}\left(\frac{1}{1.25}+\frac{1}{1.5}+\frac{1}{1.75}+\frac{1}{2}\right) = 6345$ underestimate

b) Estimate the area using left endpoints.

$1, 1.25, 1.5, 1.75$

$L_5 = \frac{1}{4}(f(1)+f(1.25)+f(1.5)+f(1.75))$

$\frac{1}{4}\left(1+\frac{1}{1.25}+\frac{1}{1.5}+\frac{1}{1.75}\right)$

$L_5 \approx .7595$

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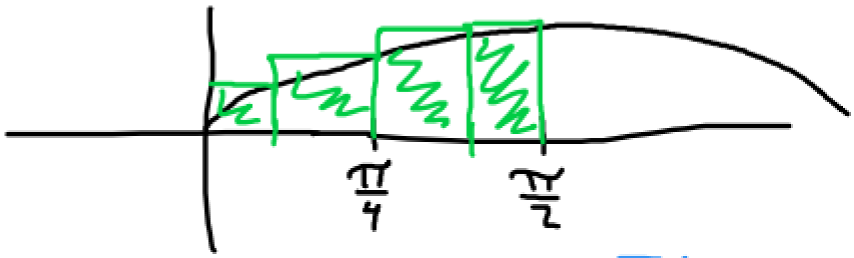
4. a. Estimate the area under the graph of $f(x) = \sin x$ from $x = 0$ to $x = \pi/2$ from to using four approximating rectangles and right endpoints. Sketch the graph and the rectangles. Is your estimate an underestimate or an overestimate?

b. Repeat part (a) using left endpoints.

$[0, \frac{\pi}{2}] = [0, \frac{\pi}{8}][\frac{\pi}{8}, \frac{\pi}{4}][\frac{\pi}{4}, \frac{3\pi}{8}][\frac{3\pi}{8}, \frac{\pi}{2}]$



$\frac{0 - \frac{\pi}{2}}{4} = \frac{\pi}{8}$

$\frac{\pi}{8}\left(f(\frac{\pi}{8}) + f(\frac{\pi}{4}) + f(\frac{3\pi}{8}) + f(\frac{\pi}{2})\right)$

$R_4 = 1.18346$ overestimate

$\frac{\pi}{8}\left(f(0) + f(\frac{\pi}{8}) + f(\frac{\pi}{4}) + f(\frac{3\pi}{8})\right)$

$L_4 = .7908$ underestimate
left points

5. a. Estimate the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ from to using three rectangles and right endpoints. Then improve your estimate by using six rectangles. Sketch the curve and the approximating rectangles.

$[-1, 2] = [-1, 0][0, 1][1, 2]$

$1(f(0) + f(1) + f(2))$
$1 + 2 + 5$
$R_3 = 8$

$\frac{2+1}{3} = 1 \quad \frac{2+1}{6} = \frac{1}{2}$

$[-1, 2] = [-1, -\frac{1}{2}][-\frac{1}{2}, 0][0, \frac{1}{2}][\frac{1}{2}, 1][1, 1.5][1.5, 2]$

$\frac{1}{2}(f(-\frac{1}{2}) + f(0) + f(\frac{1}{2}) + f(1) + f(1.5) + f(2))$

$\frac{1}{2}(1.25 + 1 + 1.25 + 2 + 3.25 + 5)$

$R_6 = 6.875$

b) Repeat using left endpoints and midpoints.

$1(f(-1) + f(0) + f(1))$
$2 + 1 + 2$
$L_3 = 5$

$1(f(-.5) + f(.5) + f(1.5))$
$1.25 + 1.25 + 3.25$
$=$
$M_3 = 5.75$ midpoint

$\frac{1}{2}(f(-1) + f(-\frac{1}{2}) + f(0) + f(\frac{1}{2}) + f(1) + f(1.5))$
$\frac{1}{2}(2 + 1.25 + 1 + 1.25 + 2 + 3.25)$

$L_6 = 5.375$

$\frac{1}{2}(f(-.75) + f(-\frac{1}{4}) + f(\frac{1}{4}) + f(\frac{3}{4}) + f(1.25) + f(1.75))$

$M_6 = 5.9375$

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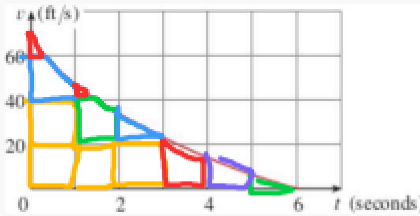
13. The velocity graph of a braking car is shown. Use it to estimate the distance traveled by the car while the brakes are applied.



$7(20) + 10$

150ft

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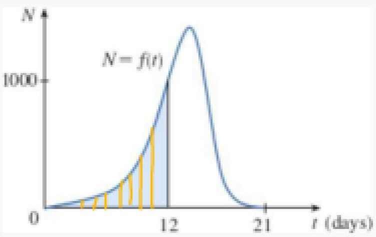
15. In a person infected with measles, the virus level N (measured in number of infected cells per mL of blood plasma) reaches a peak density at about $t = 12$ days (when a rash appears) and then decreases fairly rapidly as a result of immune response. The area under the graph of $N(t)$ from $t = 0$ to $t = 12$ (as shown in the figure) is equal to the total amount of infection needed to develop symptoms (measured in density of infected cells $\times$ time). The function N has been modeled by the function

$$f(t) = -t(t-21)(t+1)$$

Use this model with six subintervals and their midpoints to estimate the total amount of infection needed to develop symptoms of measles.

$\frac{12-0}{6} = 2$



Source: J. M. Heffernan et al., "An In-Host Model of Acute Infection: Measles as a Case Study," *Theoretical Population Biology* 73 (2006): 134– 47.

$[0,12] = [0,2][2,4][4,6][6,8][8,10][10,12]$

$2(f(0)+f(2)+f(4)+f(6)+f(8)+f(10))$

$2(0+114+340+630+936+1210)$

$2(f(2)+f(4)+f(6)+f(8)+f(10)+f(12))$

$2(114+340+630+936+1210+1404)$

$L_6 = 6460$

$R_6 = 9268$

$2(f(1)+f(3)+f(5)+f(7)+f(9)+f(11))$

$2(40+216+480+784+1080+1320)$

$M_6 = 7840$