Egyptian Multiplication

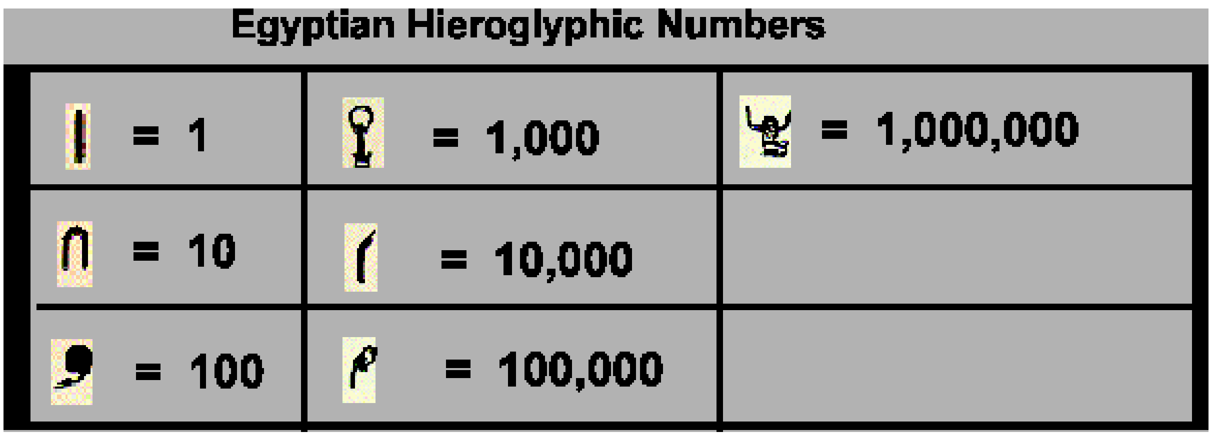
In 1858, A. Henry Rhind, a Scottish antiquary, came into possession of a document which is now called the Rhind Papyrus. Titled "Directions for Attaining Knowledge into All Obscure Secrets", the document provides important clues as to how the ancient Egyptians performed arithmetic.

There is no zero in the number system. There are separate characters denoting ones, tens, hundreds, thousands, ten-thousands, hundred-thousands, millions and ten-millions. For the purposes of this problem, we use near ASCII equivalents for the symbols:

- `|` for one (careful, it's a vertical line, not 1)
- `n` for ten
- `9` for hundred
- `8` for thousand
- `r` for ten-thousand

(The actual Egyptian hieroglyphs were more picturesque but followed the general shape of these modern symbols. For the purpose of this problem, we will not consider numbers greater than 99,999.)

Numbers were written as a group of ones followed by groups of tens, hundreds, thousands and ten-thousands. Thus our number 4,023 would be rendered: `|||nn8888`. Notice that a zero digit is indicated by a group consisting of none of the corresponding symbol. The number 40,230 would be rendered: `nnn99rrrr`. (In the Rhind Papyrus, the groups are drawn more picturesquely, often spread across more than one horizontal line; but for the purposes of this problem, you should write numbers all on a single line.)



To multiply two numbers a and b, the Egyptians would work with two columns of numbers. They would begin by writing the number | in the left column beside the number a in the right column. They would proceed to form new rows by doubling the numbers in both columns. Doubling continues as long as the number in the left column does not exceed the other multiplicand b. The numbers in the first column that summed to the multiplicand b were marked with an asterisk. The numbers in the right column alongside the asterisks were then added to produce the result. Below, we show the steps corresponding to the multiplication of 19 and 83:

```
| *                          |||nnnnnnnn (83)
|| *                         ||||||nnnnnn9 (166)
||||                         ||nnn999 (332)
||||||||                     ||||nnnnnn999999 (664)
||||||n *                    ||||||||nn9998 (1328)
The solution is: |n99998
```

(The solution came from adding together:

```
|||nnnnnnnn
||||||nnnnnn9
||||||||nn9998
```

For the purpose of this problem, we will not consider numbers greater than 99,999. In this problem you are to complete three methods in the `EgyptianMultiplication` class. The three methods are `toDecimal`, `toEgyptianNumber` and `multiply`.

`toDecimal(eNum)` returns the equivalent `int` of its `String` parameter `eNum`.

The following tables show sample results of the `toDecimal` method.
You may assume `eNum.length() > 0, eNum` only contain `|`, `n`, `9`, `8`, and/or `r` and will not contain any other symbols including spaces(" ") and the Egyptian number will be properly formed.

<table>
<tr><th>The following code</th><th>Returns</th></tr>
<tr><td>EgyptianMultiplication.toDecimal("|")</td><td>1</td></tr>
<tr><td>EgyptianMultiplication.toDecimal("|||||||nnn")</td><td>37</td></tr>
<tr><td>EgyptianMultiplication.toDecimal("|||9888")</td><td>3103</td></tr>
<tr><td>EgyptianMultiplication.toDecimal("nnnnnnnnn99rrrrr")</td><td>50290</td></tr>
</table>

The `toEgyptianNumber(value)` method returns the Egyptian hieroglyphs rendering (as a `String`) of its `int` parameter `value`. The returned rendering should only contain `|`, `n`, `9`, `8`, and/or `r` and shall not contain any other symbols including spaces(" ").

The following tables show sample results of the `toEgyptianNumber` method.
Remember, we will not consider numbers greater than 99,999

<table>
<tr><th>The following code</th><th>Returns</th></tr>
<tr><td>EgyptianMultiplication.toEgyptianNumber(1)</td><td>"|"</td></tr>
<tr><td>EgyptianMultiplication.toEgyptianNumber(37)</td><td>"|||||||nnn"</td></tr>
<tr><td>EgyptianMultiplication.toEgyptianNumber(3103)</td><td>"|||9888"</td></tr>
<tr><td>EgyptianMultiplication.toEgyptianNumber(50290)</td><td>"nnnnnnnnn99rrrrr"</td></tr>
</table>

The `multiply(n1, n2)` method returns a `String[]` representation of the steps described above in Egyptian multiplication. The array should only contain the rows which are summed and the last index contains the final answer. Each entry in the array representing a row from the multiplication process must be formatted as follows:

- The array element begins (with no leading spaces) with the rendering for the first parameter (`n1` - which starts at one).
- Exactly five spaces are added. This provides exactly five spaces between the rendering of the two Egyptian numbers
- The array element ends with the rendering for the second parameter (`n2` - which starts at `n2`).

  Note: the length of each entry in the array representing a row from the multiplication process is exactly the length of each Egyptian number plus 5 (for the 5 spaces).
- The last element of the array contains the `String` rendering of the product of n1 `and` `n2`. The String shall not contain any spaces.

The following tables show sample results of the `multiply` method.

<table>
<tr><th>The following code</th><th>Returns</th></tr>
<tr><td>EgyptianMultiplication.multiply("|||", "|n ")</td><td>"| |n"
"|| ||nn"
"|||nnn"</td></tr>
<tr><td>EgyptianMultiplication.multiply("|||||||||n",
"|||nnnnnnnn")</td><td>"| |||nnnnnnnn",
"|| ||||||nnnnnn9"
||||||n ||||||||nn9998"
"|||||||nnnnnnn999998"</td></tr>
</table>

If you still need further clarification, I found this description somewhere on the internet.

Note, this is an example of 29 x 59, not 29 x 58 as stated on line 3

The great Egyptian civilisation used a very different method to work out multiplication calculations. Rather than learning tables, they just got very good at adding up (or doubling as we know it today).

This is how they did it. Let's look at 29 x 58 which is quite a hard example to show you.

29	59
1	59
2	118
4	236
8	472
16	944

Start by writing 1 in the left hand column and 59 in the right hand column.

Then add 1 to itself (2) and 59 to itself, which is 118, and write these underneath.

Then add 2 to itself (4) and 118 to itself (236).

Then add 4 to itself (8) and 236 to itself (472).

Then add 8 to itself (16) and 472 to itself (944).

Since 16 + 16 = 32 which is larger than 29 we do not need to go any further.

29	59
1	59
2	118
4	236
8	472
16	944

$$\begin{array}{r} 944 \\ 472 \\ 236 \\ +\ 59 \\ \hline 1711 \end{array}$$

Now look to see how 29 can be made up from the numbers in the left hand column.

Start by adding 16 and 8 which is 24. Then add 4 which is 28 and then add 1, making 29.

29 = 16 + 8 + 4 + 1

Check the numbers on the right hand side which correspond to 16, 8, 4 and 1.

These numbers are 944, 472, 236 and 59.

Just add them together!

29 x 59 is 1711.