HW2 Solutions

Sunday, April 21, 2024 10:16 AM

Q1) $i_L(t) = \frac{1}{L}\int_0^t v_L(t)\,dt + i_L(0)$

$$= \frac{1}{10\times10^{-6}}\int_0^t 20\,dt - 200\times10^{-3}$$

$$i_L(t) = 20t\,10^5 - 200\times10^{-3}$$

$$200\times10^{-3} = 20t_x 10^5 - 200\times10^{-3}$$

$$400\times10^{-3} = 20t_x 10^5$$

$$t_x = \frac{400\times10^{-3}}{20\times10^5} = 20\times10^{-8}\ A$$

$$= \boxed{0{,}2\,\mu s}$$

Q2) the switch is closed at $t=0^-$, therefore initial voltage is zero

$$V(t) = \frac{1}{C}\int i(t)\,dt + V(0) \Rightarrow 10^5\int_0^t (2\times10^{-3})\,dt + 0$$

$$= 200t$$

$@t = 10\text{ ms}$ $\quad V(10\times10^{-3}) = \boxed{2V}$

$$P = V\cdot i = 0.4t$$

$$P(10\times10^{-3}) = \boxed{4\text{ mW}}$$

$$w(t) = \frac{1}{2}C\cdot V^2(t) = 0.2t^2$$

$$w(10\times10^{-3}) = \boxed{20\text{ mJ}}$$

Q3) $2e^{-3t} = \frac{V_c(t)}{R} + C \cdot \frac{dV_c(t)}{dt}$

particular solution $V_{cp}(t) = A \cdot e^{-3t}$

$$2e^{-3t} = \frac{A}{R} e^{-3t} - 3ACe^{-3t}$$

$R = 1\,M\Omega = 10^6\,\Omega$

$C = 1\times10^{-6}\,F$

$$2e^{-3t} = \frac{A}{10^6} \cdot e^{-3t} - 3 \cdot A \cdot 10^{-6} \cdot e^{-3t}$$

$$2e^{-3t} = e^{-3t}\left(\frac{A}{10^6} - 3A \cdot 10^{-6}\right)$$

$$= e^{-3t}\left(A \cdot 10^{-6} - 3A \cdot 10^{-6}\right)$$

$$= e^{-3t}\left(-2A\, 10^{-6}\right)$$

$$A = -10^6$$

$V_c(t) = V_{particular} + V_{comp}$

$= -10^6 e^{-3t} + K_1 e^{-t/\tau}$ $\quad V_c(0+) = 0$ & $\tau = RC = 1$

$V_c(0^+) = -10^6 + K_1 = 0 \Rightarrow K_1 = 10^6$

General solution $V_c(t) = 10^6 e^{-t} - 10^6 e^{-3t}$

Q4) $\dfrac{d\left(5\cdot\cos(10t)\right)}{dt} = \dfrac{d\left(\dfrac{V(t)}{10}\right)}{dt} + \dfrac{d\left(\int_0^t v(t)\,dt + i_L(0)\right)}{dt}$

→ After writing $\underline{\underline{KCL}}$, we differentiate each term wrt time

$$-50\cdot\sin(10t) = \frac{1}{10}\frac{dV(t)}{dt} + V(t)$$

$$V_p = A.\cos(10t) + B.\sin(10t)$$ → substitute into solution.

$$-50\cdot\sin(10t) = \frac{1}{10}\left[-10A\sin(10t) + 10B\cos(10t)\right] + \left[A.\cos(10t) + B.\sin(10t)\right]$$

the cosine coefficients should be zero on the right hand side and sine coefficients should be -50

$$\left.\begin{aligned} -50 &= -A + B \\ 0 &= B + A \end{aligned}\right\} \quad \begin{aligned} A &= 25 \\ B &= -25 \end{aligned}$$

$$V(t) = V_{particular} + V_{complementary}$$

$$= V_p(t) + K_1.e^{-t/\tau}$$

τ = time constant for RL circuit

$\tau = \dfrac{L}{R} = 0.1$ sec.

$$= 25.\cos(10t) - 25.\sin(10t) + K_1.e^{-10t}$$

Current of inductor at $t=0^+$ is zero $i_L(0^+)=0$

at $t=0^+$ all current goes through Resistor

at $t=0^+$ $\quad i_R(0^+) = 5A \Rightarrow v_R(0^+) = 5\times 10 = 50V$

therefore $\quad 50 = 25.\underbrace{\cos(0)}_{1} - 25.\underbrace{\sin(0)}_{0} + K_1.\underbrace{e^{-0}}_{1}$

therefore $50 = 25\cdot\underbrace{\cos(0)}_{1} - 25\cdot\underbrace{\sin(0)}_{0} + K_1\cdot \underbrace{e^{0}}_{1}$

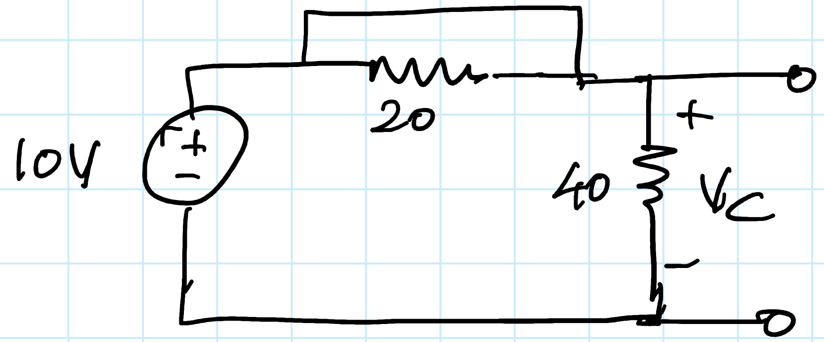
$K_1 = 25$

$$\boxed{v(t) = 25e^{-10t} + 25\cdot\cos(10t) - 25\sin(10t)}$$

Q5) Steady state representations for L & C

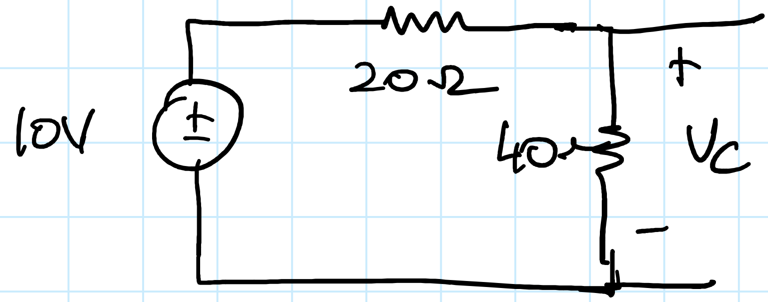
L → short circuit
C → open circuit

prior to t=0 (before switch opens) the circuit can be represented as:



$$\boxed{V_C = 10\,V}$$

after t>0 the circuit becomes



$$V_C = 10\cdot\frac{40}{40+20}$$

$$\boxed{V_C = 6.66}$$

Q6) using KVL we obtain:

$$L \cdot \frac{d i_s(t)}{dt} + R \cdot i_s(t) = 15 \cdot \cos(300t)$$

$$2 \cdot \frac{d i_s(t)}{dt} + 300\, i_s(t) = 15 \cdot \cos(300t)$$

particular solution given in the hint:

$$i_{sp} = A \cdot \cos(300t) + B \cdot \sin(300t)$$

substitute i_{sp} into eqn:

$$\frac{d i_{sp}(t)}{dt} = -300A\sin(300t) + 300B\cos(300t)$$

$$-600A\sin(300t) + 600B \cdot \cos(300t) +$$
$$300A\cos(300t) + 300B\sin(300t) = 15 \cdot \cos(300t)$$

coefficients of sines $\rightarrow 0$

of cosines $\rightarrow 15$

$$-600A + 300B = 0 \rightarrow 300B = 600A$$
$$B = 2A$$
$$600B + 300A = 15$$

$$1200A + 300A = 15$$

$$1500A = 15 \rightarrow A = 10^{-2}$$

$$B = 2 \times 10^{-2}$$

$\hat{i}_{sc}(t) = \hat{i}_{s_{complementary}} + \hat{i}_{s_{particular}}$

$\hat{i}_{sc} = k_1 \cdot e^{-t/\tau} \rightarrow k_1 \cdot e^{-Rt/L} = k_1 \cdot e^{-150t}$

$\hat{i}_{sc}(t) = k_1 \cdot e^{-150t} + 10^{-2} \cos(300t) + 2 \cdot 10^{-2} \sin(300t)$

current accross L cannot change instantaneously

therefore $\hat{i}_s(0^+) = 0 \rightarrow$ substitute into eqn to find K_1

$\hat{i}_s(0) = 0 = k_1 + 10^{-2} \Rightarrow k_1 = -10^{-2}$

the solution is

$$\boxed{\hat{i}_s(t) = -10^{-2} e^{-150t} + 10^{-2} \cos(300t) + 2 \cdot 10^{-2} \sin(300t)}$$