15212021106 - Doğukan Kayıkçık

Q1

$$\begin{aligned} x_1 - x_2 + 3x_3 &= 1 \\ 5x_1 - x_2 - x_3 &= 5 \\ x_1 - x_2 + x_3 &= -3 \end{aligned} \Bigg\} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 5 & -1 & -1 & 5 \\ 1 & -1 & 1 & -3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 5 & -1 & -1 & 5 \\ 1 & -1 & 1 & -3 \end{array}\right] \begin{array}{l} S_2 = S_2 - 5S_1 \\ S_3 = S_3 - S_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 0 & 4 & -16 & 0 \\ 0 & 0 & -2 & -4 \end{array}\right]$$

$-2x_3 = -4 \Rightarrow x_3 = 2$

$4x_2 - 16x_3 = 0$

$4x_2 = 32 \Rightarrow x_2 = 8$

$x_1 - x_2 + 3x_3 = 1$

$x_1 - 8 + 6 = 1 \Rightarrow x_1 = 3$

Cevap $\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 2 \end{pmatrix}$

Q2

$$\begin{aligned} x_1 - x_2 + 3x_3 &= 1 \\ 5x_1 - x_2 - x_3 &= 5 \\ x_1 - x_2 + x_3 &= -3 \end{aligned} \Bigg\} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 5 & -1 & -1 & 5 \\ 1 & -1 & 1 & -3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 5 & -1 & -1 & 5 \\ 1 & -1 & 1 & -3 \end{array}\right] \begin{array}{l} S_2 = S_2 - 5S_1 \\ S_3 = S_3 - S_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 0 & 4 & -16 & 0 \\ 0 & 0 & -2 & -4 \end{array}\right] \begin{array}{l} S_2 = S_2 - 8S_3 \\ S_1 = S_1 + \frac{3}{2} S_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & -5 \\ 0 & 4 & 0 & 32 \\ 0 & 0 & -2 & -4 \end{array}\right] \begin{array}{l} S_1 = \frac{1}{4} S_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 4 & 0 & 32 \\ 0 & 0 & -2 & -4 \end{array}\right]$$

$-2x_3 = -4 \Rightarrow x_3 = 2$

$4x_2 = 32 \Rightarrow x_2 = 8$

$x_1 = 3$

Cevap $\Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 2 \end{pmatrix}$

152120211104 - Doğukan Kayıkçık

Q3

$$\begin{aligned} x_1 - x_2 + 3x_3 &= 1 \\ 5x_1 - x_2 - x_3 &= 5 \\ x_1 - x_2 + x_3 &= -3 \end{aligned} \Bigg\} \underbrace{\begin{bmatrix} 1 & -1 & 3 \\ 5 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$$

$$L \Rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 5 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{matrix} S_2 = S_2 - 5S_1 \\ \\ S_3 = S_3 - S_1 \end{matrix} \rightarrow \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & -16 \\ 0 & 0 & -2 \end{bmatrix}$$

$E_1 \Rightarrow S_2 = S_2 - 5S_1$

$E_2 \Rightarrow S_3 = S_3 - S_1$

$$E_2 \cdot E_1 \cdot A = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & -16 \\ 0 & 0 & -2 \end{bmatrix} \Rightarrow E_1^{-1} \cdot E_2^{-1} \cdot \begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & -16 \\ 0 & 0 & -2 \end{bmatrix} = A$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{S_2 = S_2 - 5S_1} \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{S_3 = S_3 - S_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \Rightarrow E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \rightarrow L$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & -16 \\ 0 & 0 & -2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{olsun}} = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix} \Rightarrow \begin{matrix} a_1 = 1 \\ a_2 = 0 \\ a_3 = -4 \end{matrix}$$

$$\begin{bmatrix} 1 & -1 & 3 \\ 0 & 4 & -16 \\ 0 & 0 & -2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} \Rightarrow \begin{matrix} x_3 = 2 \\ x_2 = 8 \\ x_1 = 3 \end{matrix}$$

$$\text{Cevap} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 8 \\ 2 \end{pmatrix}$$

152120211104 - Doğukan Kıyıklık

Q4

$$5x_1 - x_2 - x_3 = 5$$
$$x_1 - x_2 + x_3 = -3$$
$$x_1 - x_2 + 3x_3 = 1$$

$$x_1 = \frac{5 + x_2 + x_3}{5} \qquad x_3 = \frac{1 - x_1 + x_2}{3}$$

$$x_2 = x_1 + x_3 + 3$$

1. İterasyon

$$\begin{pmatrix} 0,5 \\ 0,5 \\ 0,5 \end{pmatrix} \Rightarrow x_1 = \frac{5 + 0,5 + 0,5}{5} \rightarrow \frac{6}{5} \rightarrow 1,2 \qquad x_3 = \frac{1 - 0,5 + 0,5}{3} = \frac{1}{3} = 0,3\overline{3}$$

$$x_2 = 0,5 + 0,5 + 3 = 4$$

2. İterasyon

$$\begin{pmatrix} 1,2 \\ 4 \\ 0,\overline{3} \end{pmatrix} \Rightarrow x_1 = \frac{5 + 4 + 0,3}{5} = 1,86 \qquad x_3 = \frac{1 - 1,2 + 4}{3} = 1,2\overline{6}$$

$$x_2 = 1,2 + \frac{1}{3} + 3 \Rightarrow 4,5$$

3. İterasyon

$$\begin{pmatrix} 1,86 \\ 4,5 \\ 1,26 \end{pmatrix} \Rightarrow x_1 = \frac{5 + 4,5 + 1,26}{5} = 2,152 \qquad x_3 = \frac{1 - 1,86 + 4,5}{3} = 1,213$$

$$x_2 = 1,86 + 1,26 + 3 = 6,12$$

Q5

$$f(x) = x^3 - x + 5$$
$$f'(x) = 3x^2 - 1$$
$$f''(x) = 6x$$
$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$$

1. iterasyon $x_0 = 1$

$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)} \Rightarrow 1 - \frac{2}{6} = 0,66666$$

2. iterasyon $x_1 = 0,66666$

$$x_2 = x_1 - \frac{f'(x_1)}{f''(x_1)} \Rightarrow 0,66666 - \frac{0,3333}{3,9999} \Rightarrow 0,58366$$

3. iterasyon $x_2 = 0,58366$

$$x_3 = x_2 - \frac{f'(x_2)}{f''(x_2)} \Rightarrow 0,58366 - \frac{0,02197}{3,5019} = 0,57746$$