

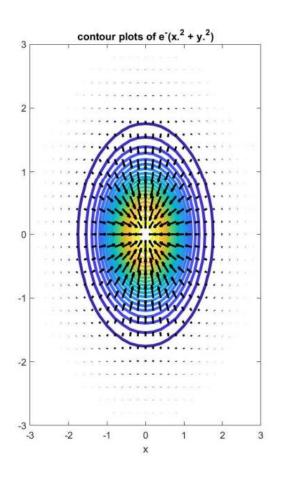
EE323 Electromagnetic Waves and Transmission Lines

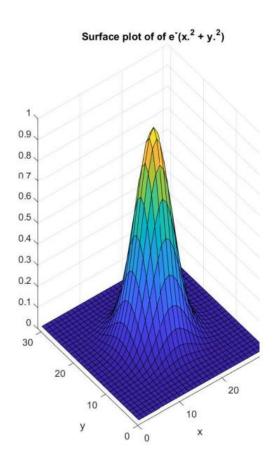
Experiment 1

Doğukan Köseoğlu 20190701027

GRADIENT of $z = e^{-(x^2+y^2)}$

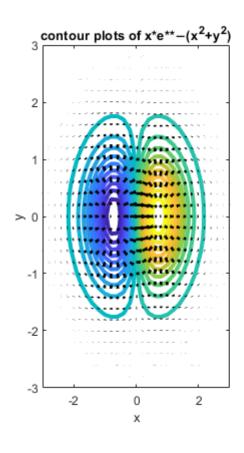
```
clear all
close all
clc
v = -3:0.2:3;
[x,y] = meshgrid(v);
z = exp(-(x.^2 + y.^2));
[px,py] = gradient(z,0.2,0.2);
subplot(1,2,1)
contour(v,v,z,20,'Linewidth',3);
hold on;
quiver(v,v,px,py,'Linewidth',2,'color','black');
hold off
xlabel('x');
ylabel('y');
title('contour plots of exp(-(x.^2 + y.^2))');
subplot(1,2,2)
surf(z)
xlabel('x');
ylabel('y');
title('Surface plot of of exp(-(x.^2 + y.^2))');
```

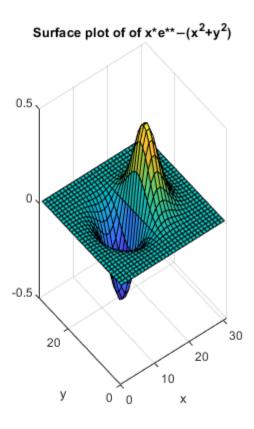




GRADIENT of $z = xe^{-(x^2+y^2)}$

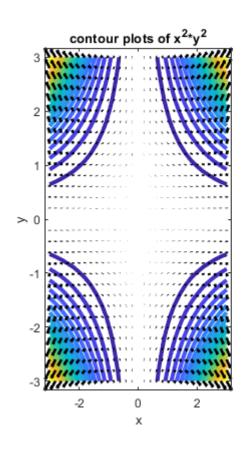
```
clear all
close all
clc
v = -3:0.2:3;
[x,y] = meshgrid(v);
z = x.*exp(-(x.^2+y.^2));
[px,py] = gradient(z,0.2,0.2);
subplot(1,2,1)
contour(v,v,z,20,'Linewidth',3);
hold on;
quiver(v,v,px,py,'Linewidth',2,'color','black');
hold off
xlabel('x');
ylabel('y');
title('contour plots of x*e**-(x^2+y^2)');
subplot(1,2,2)
surf(z)
xlabel('x');
ylabel('y');
title('Surface plot of of x*e**-(x^2+y^2)');
```

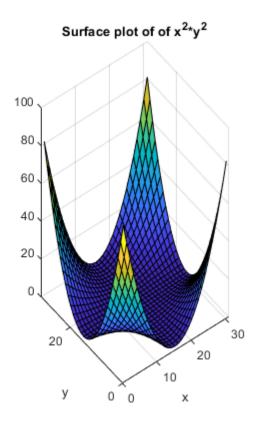




GRADIENT of $z = x^2 * y^2$

```
clear all
close all
clc
v = -3:0.2:3;
[x,y] = meshgrid(v);
z = ((x.*x).*(y.*y));
[px,py] = gradient(z,0.2,0.2);
subplot(1,2,1)
contour(v,v,z,20,'Linewidth',3);
hold on;
quiver(v,v,px,py,'Linewidth',2,'color','black');
hold off
xlabel('x');
ylabel('y');
title('contour plots of x^2*y^2');
subplot(1,2,2)
surf(z)
xlabel('x');
ylabel('y');
title('Surface plot of of x^2*y^2');
```

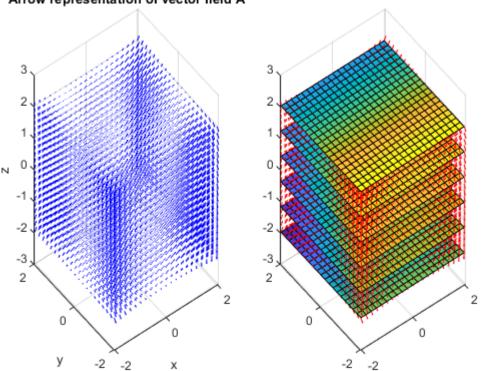




DIVERGENCE of $A = xza_x - y^2 a_y + 2x^2ya_z$

```
clear all
close all
clc
v=-2:0.2:2;
[x,y,z]=meshgrid(v);
fx= x.*z;
fy= -y.^2;
fz = 2*x.^2.*y;
div=divergence(x,y,z,fx,fy,fz);
subplot(1,2,1)
quiver3(x,y,z,fx,fy,fz,'color','blue');
title('Arrow representation of vector field A')
xlabel('x')
ylabel('y')
zlabel('z')
subplot(1,2,2);
quiver3(x,y,z,fx,fy,fz,'color','red');
hold on
for i=1:round(length(x)/5):length(x)
surf(x(:,:,i),y(:,:,i),z(:,:,i),div(:,:,i))
```

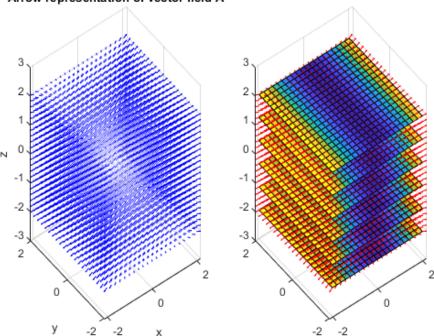
Arrow representation of vector field A



DIVERGENCE of $A = x^3 a_x + y a_y + z a_z$

```
clear all
close all
v=-2:0.2:2;
[x,y,z]=meshgrid(v);
fx= x.^3;
fy= y;
fz= z;
div=divergence(x,y,z,fx,fy,fz);
subplot(1,2,1)
quiver3(x,y,z,fx,fy,fz,'color','blue');
title('Arrow representation of vector field A')
xlabel('x')
ylabel('y')
zlabel('z')
subplot(1,2,2);
quiver3(x,y,z,fx,fy,fz,'color','red');
for i=1:round(length(x)/5):length(x)
surf(x(:,:,i),y(:,:,i),z(:,:,i),div(:,:,i))
```

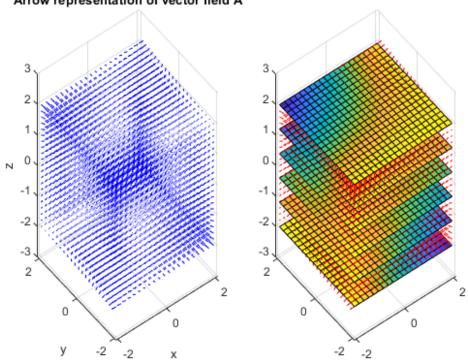
Arrow representation of vector field A



DIVERGENCE of $A = x^2ya_x + y^2za_y + z^2xa_z$

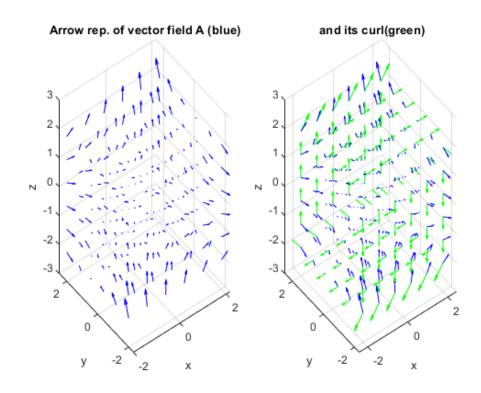
```
clear all
close all
clc
v=-2:0.2:2;
[x,y,z]=meshgrid(v);
fx = x.^2.*y;
fy= -y.^2.*z;
fz= z.^2.*x;
div=divergence(x,y,z,fx,fy,fz);
subplot(1,2,1)
quiver3(x,y,z,fx,fy,fz,'color','blue');
title('Arrow representation of vector field A')
xlabel('x')
ylabel('y')
zlabel('z')
subplot(1,2,2);
quiver3(x,y,z,fx,fy,fz,'color','red');
hold on
for i=1:round(length(x)/5):length(x)
surf(x(:,:,i),y(:,:,i),z(:,:,i),div(:,:,i))
```

Arrow representation of vector field A



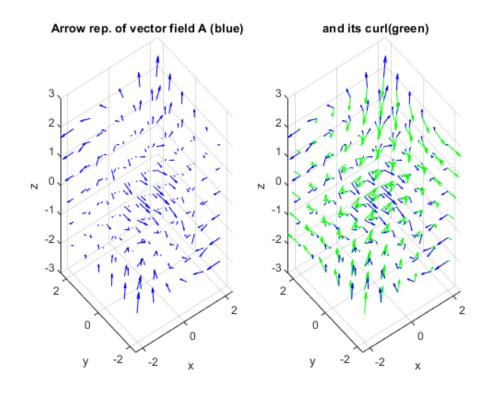
CURL of $A = y^{2}a_{x} + (2xy + z^{2})a_{y} + 2yza_{z}$

```
clear all
close all
clc
v=-2:0.8:2;
[x,y,z]=meshgrid(v);
fx=y.*y;
fy=2* (x.*y)+z.^2;
fz=2* (y.*z);
[curlx,curly,curlz]=curl(x,y,z,fx,fy,fz);
subplot(1,2,1)
quiver3(x,y,z,fx,fy,fz,'color','blue');
title('Arrow rep. of vector field A (blue)')
xlabel('x')
ylabel('y')
zlabel('z')
subplot(1,2,2)
quiver3(x,y,z,fx,fy,fz,'color','blue');
hold on;
quiver3(x,y,z,curlx,curly,curlz,'color','green')
title(' and its curl(green)')
xlabel('x')
ylabel('y')
zlabel('z')
```



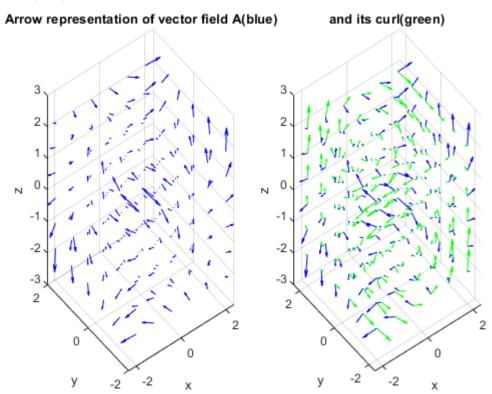
CURL of $A = xya_x + yza_y + zxa_z$

```
clear all
close all
clc
v=-2:0.8:2;
[x,y,z]=meshgrid(v);
fx= x.*y;
fy= y.*z;
fz=z.*x;
[curlx,curly,curlz]=curl(x,y,z,fx,fy,fz);
subplot(1,2,1)
quiver3(x,y,z,fx,fy,fz,'color','blue');
title('Arrow rep. of vector field A (blue)')
xlabel('x')
ylabel('y')
zlabel('z')
subplot(1,2,2)
quiver3(x,y,z,fx,fy,fz,'color','blue');
hold on;
quiver3(x,y,z,curlx,curly,curlz,'color','green')
title(' and its curl(green)')
xlabel('x')
ylabel('y')
zlabel('z')
```



CURL of $A = xy^2a_x - yz^2a_y + zx^2a_z$

```
clear all
close all
clc
v=-2:0.8:2;
[x,y,z]=meshgrid(v);
fx= x.*y.^2;
fy= -y.*z.^2;
fz= z.*x.^2;
[curlx,curly,curlz]=curl(x,y,z,fx,fy,fz);
subplot(1,2,1)
quiver3(x,y,z,fx,fy,fz,'color','blue');
title('Arrow representation of vector field A(blue)')
xlabel('x')
ylabel('y')
zlabel('z')
subplot(1,2,2)
quiver3(x,y,z,fx,fy,fz,'color','blue');
hold on;
quiver3(x,y,z,curlx,curly,curlz,'color','green')
title('and its curl(green)')
xlabel('x')
ylabel('y')
zlabel('z')
```



- 2. Using the definitions of vector operators in Cartesian system above, prove the following 2 null-identities (analytically) for any vector field "A":
 - $\nabla \times (\nabla A) = 0$ curl of the gradient of any scalar field is zero.
 - ∇·(∇×A)=0 divergence of the curl of any vector field is zero.

```
gradA = (\partial A/\partial x, \partial A/\partial y, \partial A/\partial z)
curlA = \nabla x A = ((\partial/\partial x, \partial/\partial y, \partial/\partial z) x (Ax, Ay, Az)=
curlA = ((\partial Az/\partial y - \partial Ay/\partial z), (\partial Ax/\partial z - \partial Az/\partial x), (\partial Ay/\partial x - \partial Ax/\partial y))
divA = \nabla \cdot A = (\partial Ax/\partial x + \partial Ay/\partial y + \partial Az/\partial z)
2.1 curl of gradient of a scalar field = 0
curl(gradA) = curl(\partial A/\partial x, \partial A/\partial y, \partial A/\partial z)
((\partial^2 A/\partial y \partial z - \partial^2 A/\partial z \partial y), (\partial^2 A/\partial z \partial x - \partial^2 A/\partial x \partial z), (\partial^2 A/\partial x \partial y - \partial^2 A/\partial x \partial z)
\partial^2 A/\partial y \partial x)
\partial^2 A / \partial y \partial z = \partial^2 A / \partial z \partial y
\partial^2 A/\partial z \partial x = \partial^2 A/\partial x \partial z
\partial^2 A/\partial x \partial y = \partial^2 A/\partial y \partial x
Therefore, curl(gradA) = (0, 0, 0), which means curl(gradA) = 0.
2.2 divergence of curl of a vector field = 0
div(curlA) = \frac{\partial(\partial Az}{\partial y} - \frac{\partial Ay}{\partial z})/\partial x + \frac{\partial(\partial Ax}{\partial z} - \frac{\partial Az}{\partial x})/\partial y +
\partial(\partial Ay/\partial x - \partial Ax/\partial y)/\partial z
\partial^2 Az / \partial x \partial y = \partial^2 Az / \partial y \partial x
\partial^2 Ay / \partial z \partial x = \partial^2 Ay / \partial x \partial z
\partial^2 Ax/\partial y \partial z = \partial^2 Ax/\partial z \partial y
div(curlA) = \nabla \cdot (curlA) = (\partial/\partial x, \partial/\partial y, \partial/\partial z) \cdot ((\partial Az/\partial y - \partial Ay/\partial z),
(\partial Ax/\partial z - \partial Az/\partial x), (\partial Ay/\partial x - \partial Ax/\partial y))
div(curlA) = (\partial^2 Az/\partial x \partial y - \partial^2 Ay/\partial x \partial z) + (\partial^2 Ax/\partial y \partial z - \partial^2 Az/\partial y \partial x) +
(\partial^2 Ay/\partial z \partial x - \partial^2 Ax/\partial z \partial y)
\partial^2 Az/\partial x \partial y = \partial^2 Az/\partial y \partial x
\partial^2 Ay / \partial z \partial x = \partial^2 Ay / \partial x \partial z
\partial^2 Ax/\partial y \partial z = \partial^2 Ax/\partial z \partial y
Therefore, div(curl) = 0,
```

3. Prove the 2 null identities using MATLAB. You can select the field functions used in the examples.

```
3.1 curl of gradient of a scalar field = 0
clear all
close all
v = -3:0.2:3;
[x,y,z] = meshgrid(v);
k=x.*z.^3+z.^3+2.*x;
[px,py,pz] = gradient(k,0.2,0.2,0.2);
[curlx,curly,curlz]=curl(x,y,z,px,py,pz);
curlx;
curly;
curlz;
3.2 divergence of curl of a vector field = 0
clear all
close all
clc
v=-2:0.8:2;
[x,y,z]=meshgrid(v);
fx= x.*(z.^2);
fy= -y.*(y.*x);
fz= -y.*(x.^2);
[curlx,curly,curlz]=curl(x,y,z,fx,fy,fz);
div=divergence(x,y,z,curlx,curly,curlz);
```

4. The characteristic impedance of a medium is defined as the ratio of Electric field and Magnetic Field propagating in that media as shown in the equation below.

$$E_0(z)$$

 $\eta = \frac{1}{H^0(z)}$. Now suppose that you have the data of measured values of Electric Field and Magnetic field in 15 different measurement setups. The resulting values are in the following table:

Number of measurements	Electric Field (V/m)	Magnetic Field(A/m)
1	0.58	0.0023
2	0.58	0.0024
3	0.53	0.0021
4	0.59	0.0025
5	0.54	0.0022
6	0.61	0.0023
7	0.59	0.0024
8	0.55	0.0022
9	0.51	0.0020
10	0.52	0.0020
11	0.56	0.0022
12	0.55	0.0021
13	0.55	0.0022
14	0.57	0.0023
15	0.55	0.0022

Find the average value of impedance.

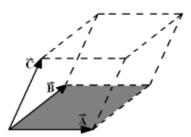
```
clear all
close all
clc
E = [0.58 0.58 0.53 0.59 0.54 0.61 0.59 0.55 0.51 0.52 0.56 0.55 0.55 0.57 0.55];
H = [0.0023 0.0024 0.0021 0.0025 0.0022 0.0023 0.0024 0.0022 0.0020 0.0020 0.0022
0.0021 0.0022 0.0023 0.0022];
n= E./H;
result = mean(n);
```

5. You are given three vectors;

$$A = i + 2 j + k$$

$$B = 2i + j + 2k$$

$$C = i + 3j + 4k$$



Please find the volume of the parallelepiped, formed by three vectors A, B and C by using MATLAB. Write the corresponding code.

```
clear all
close all
clc
A = [1 2 1];
B = [2 1 2];
C = [1 3 4];
v= cross(A,B);
volume = abs(dot(v,C));
```