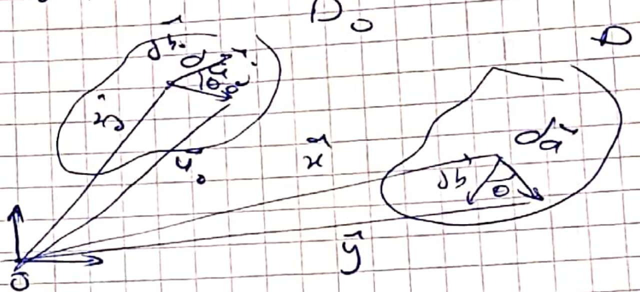
chapitre 1. mouvement de corps rigide :

$$\vec{x}(\vec{x}_0, t) = \underline{\underline{R}}(t)\, \vec{x}_0 + \vec{C}(t)$$



$$\|d\vec{a}_0\| \overset{?}{=} \|d\vec{a}\|$$

$$d\vec{a} = \vec{y} - \vec{x}$$

$$d\vec{a}_0 = \vec{y}_0 - \vec{x}_0$$

$$\|d\vec{a}_0\|^2 = (\vec{y}_0 - \vec{x}_0) \cdot (\vec{y}_0 - \vec{x}_0)$$

$$\|d\vec{a}\|^2 = (\vec{y} - \vec{x})\,(\vec{y} - \vec{x})$$

$$= \underline{\underline{R}}(\vec{y}_0 - \vec{x}_0)\; \underline{\underline{R}}(\vec{y}_0 - \vec{x}_0)$$

$$\|y - x\|^2 = (\vec{y}_0 - \vec{x}_0)\; \underline{\underline{R}}^T \underline{\underline{R}}\, (\vec{y}_0 - \vec{x}_0)$$

$$= \|\vec{y}_0 - \vec{x}_0\|^2 \quad \text{si } \boxed{\underline{\underline{R}}^T \cdot \underline{\underline{R}} = \underline{\underline{1}}}$$

<u>des angles</u> :

$$(d\vec{a} \cdot d\vec{b}) = \|d\vec{a}\|\, \|d\vec{b}\| \times \cos(\theta)$$

$$d\vec{a}_0 \cdot d\vec{b}_0 = \|d\vec{a}_0\|\, \|d\vec{b}_0\|\, \cos(\theta_0)$$

$$\Rightarrow \theta = \pm\, \theta_0 + 2k\pi$$

<u>orientation</u> ?

$$[d\vec{a}, d\vec{b}, d\vec{c}] = [\det \underline{\underline{R}}\, [d\vec{a}_0, d\vec{b}_0, d\vec{c}_0]$$

$$\rightarrow \boxed{\det(\underline{\underline{R}}) = 1} \hookrightarrow \boxed{\text{rotation}}$$

Scanné avec CamScanner

chapitre 2 : $\boxed{Ex\ 7}$ mvt de corps rigide.

$$\vec{x} = \underline{\underline{R}}(t)\,\vec{x}_0 + \vec{C}(t).$$

1/ la vitesse $\vec{v}$

$$\vec{x} = \underline{\underline{R}}(t)\,\vec{x}_0 + \vec{C}(t).$$

$$\vec{v}(x,t) = \frac{\partial x}{\partial t} = \underline{\underline{\mathring{R}}}(t)\,\vec{x}_0 + \vec{\dot{C}}(t).$$

$$= \underline{\underline{\mathring{R}}}\cdot\left(\underline{\underline{R}}^T(\vec{x} - \vec{C})\right) + \vec{\dot{C}}$$

$$= \underline{\underline{\mathring{R}}}\,\underline{\underline{R}}^T(\vec{x} - \vec{C}) + \vec{\dot{C}}$$

2/ $\underline{\underline{L}} = \underline{\underline{\nabla}}\,\vec{v} = \underline{\underline{\Omega}}$

$$\underline{\underline{\Omega}}^T = (\underline{\underline{\mathring{R}}}\,\underline{\underline{R}}^T)^T = \underline{\underline{R}}\cdot\underline{\underline{\mathring{R}}}^T$$

$$\left|\underline{\underline{R}}\,\underline{\underline{R}}^T = \underline{\underline{1}} \Rightarrow \mathring{R}\,R^T + = -R\,\dot{R}^T\right.$$

$$= -\underline{\underline{\mathring{R}}}\cdot\underline{\underline{R}}^T$$

$$\boxed{\underline{\underline{\Omega}}^T = -\underline{\underline{\Omega}}}$$

↳ mvt de corps rigide $\underline{\underline{L}} = \underline{\underline{\Omega}}$ $\quad \underline{\underline{D}} = \underline{\underline{L}}^S$

$$\Rightarrow \underline{\underline{D}} = 0$$

3/ 1/ mvt de corps rigide $\Rightarrow$ $\underline{\underline{L}} = \underline{\underline{\Omega}}$ $\quad \underline{\underline{D}} = \underline{\underline{L}}^S \Rightarrow \underline{\underline{D}} = 0$

2/ $\underline{\underline{D}} = 0 \Rightarrow -\underline{\underline{L}} = \underline{\underline{L}}^T \Rightarrow \underline{\underline{L}}^S = 0$

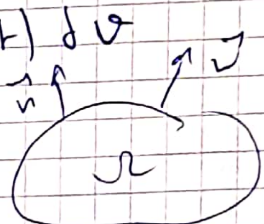
$$\Rightarrow d\vec{a}\cdot d\vec{b} = cte$$

$\rightarrow$ mvt de cort

4/ $\vec{a}(\vec{x},t) = \vec{\dot{v}} = \vec{\ddot{C}} + \vec{\mathring{\Omega}}\wedge(\vec{x} - \vec{C}) + \vec{\Omega}\wedge(\vec{v})$

chapiter 3 :

[Ex 1] $\int_D \varphi(\vec{x},t) = \int_D \phi(\vec{x},t)\, dv$



1)

Th Reynolds $\Rightarrow$

$$\frac{d}{dt}\int_\Omega \phi\, dV = \int_\Omega \partial_t \phi\, dV + \int_{\partial\Omega} \phi\,(\vec{v}-\vec{w})\,\vec{n}\, dA$$

$\Omega = D$

$\hookrightarrow \frac{d}{dt} \to \frac{D}{Dt}$;

Th Reynolds $\Rightarrow$

$$\boxed{\frac{D}{Dt}\int_D \phi\, dV = \int_D \partial_t \phi\, dV + \int_{\partial D} \phi\, \vec{v}\cdot\vec{n}\, dA}$$

Loi de conservation de la masse $\Rightarrow$

$$dM = \rho\, dV$$

donc

$$\phi \to \rho\varphi$$

$$\frac{D}{Dt}\int_D \rho\varphi\, dV = \int_D \partial_t \rho\varphi\, dV + \int_D \rho\varphi\,\vec{v}\cdot\vec{n}\, dA$$

$$\hookrightarrow \frac{D}{Dt}\int_D \rho\varphi\, dV = \int_D \partial_t \rho\varphi\, dV + \vec{\nabla}\cdot(\rho\varphi\vec{v})$$

$$\rho\,\partial_t\varphi + \varphi\,\partial_t\rho + \rho\,\vec{v}\cdot\vec{\nabla}\varphi + \varphi\,\vec{\nabla}\cdot(\rho\vec{v})$$

$$= \varphi\Big[\underbrace{\partial_t\rho + \vec{\nabla}\cdot(\rho\vec{v})}_{0}\Big] + \rho\Big[\partial_t\varphi + \vec{v}\cdot\vec{\nabla}\varphi\Big]$$

$$= \rho\,\frac{D\varphi}{Dt} = \rho\,\dot{\varphi}$$

$$\boxed{\frac{D}{Dt}\int_D \varphi \, dm = \int_D \frac{D\varphi}{Dt} \, dm}$$

2) volume conservé.

$$\frac{D}{Dt}\int_D \phi \, dV = \int_D \left(\partial_t \phi + \vec{\nabla}\cdot(\phi \vec{v})\right) dV$$

$$\phi \vec{\nabla}\vec{v} + \vec{v}\cdot\vec{\nabla}\phi$$

$$\dot{\phi} := \frac{D\phi}{Dt}$$

$$= \int_D \frac{D\phi}{Dt} \, dV.$$

<u>Conclusion</u> :

$$\boxed{\frac{D}{Dt}\int_D \varphi \, dX = \int_D \frac{D\varphi}{Dt} \, dX}$$

$$\boxed{X = cte}$$

Exercices 3 Lois de Bilan
Cercles de Mohr.

1/ $\sigma \quad \vec{T}\cdot\vec{n} = (\sigma\vec{n} + \tau\vec{t})\,\vec{n}$

$$= \sigma\,\underbrace{\vec{n}\cdot\vec{n}}_{1} + \tau\,\underbrace{\vec{t}\cdot\vec{n}}_{0}$$

$$\vec{T}\cdot\vec{n} = \sigma.$$

$$\sigma = \vec{T}\vec{n} = \underline{\underline{\sigma}}\,\vec{n}\cdot\vec{n}$$

$$\vec{\tau} = \vec{T} - \sigma\vec{n}$$

$$\tau^2 = (\vec{T} - \sigma\vec{n})(\vec{T} - \sigma\vec{n})$$

$$\vec{T}\cdot\vec{T} = (\sigma\vec{n} + \vec{\tau})(\sigma\vec{n} + \vec{\tau})$$

$$= \sigma^2\vec{n} + \tau^2$$

$$\boxed{\tau^2 = \|\vec{T}\|^2 - \sigma^2}$$

$$\vec{\tau} = \vec{T} - \sigma\vec{n}$$

$$= \vec{T} - (\underline{\underline{\sigma}}\,\vec{n}\cdot\vec{n})\,\vec{n}$$

$$= \vec{T} - (\vec{T}\cdot\vec{n})\,\vec{n}$$

$$= \vec{T} - (\vec{n}\otimes\vec{n})\,\vec{T}$$

$$\boxed{\vec{\tau} = (\underline{\underline{1}} - \vec{n}\otimes\vec{n})\,\vec{T}}$$

2) $\vec{n}$ unitaire $\Rightarrow$ $n_1^2 + n_2^2 + n_3^2 = 1 = \|n\|^2$ ①

* $\vec{T} = \underline{\sigma}\ \vec{n}$

$$= \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}_{\vec{c_i}} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}_{\vec{c_i}}$$

$$\vec{T} = \begin{pmatrix} \sigma_1 n_1 \\ \sigma_2 n_2 \\ \sigma_3 n_3 \end{pmatrix}_{\{\vec{c_i}\}}$$

$\sigma = \vec{T}\,\vec{n}$

$$\rightarrow \begin{cases} n_1^2 + n_2^2 + n_3^2 = 1 \qquad \|\vec{n}\| = 1 \\ \sigma_1 n_1^2 + \sigma_2 n_2^2 + \sigma_3 n_3^2 = \sigma \quad (\sigma) \\ \sigma_1^2 n_1^2 + \sigma_2^2 n_2^2 + \sigma_3^2 \sigma_3^2 = \sigma^2 + \tau^2 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ \sigma_1 & \sigma_2 & \sigma_3 \\ \sigma_1^2 & \sigma_2^2 & \sigma_3^2 \end{bmatrix} \begin{bmatrix} n_1^2 \\ n_2^2 \\ n_3^2 \end{bmatrix} = \begin{bmatrix} 1 \\ \sigma \\ \sigma^2 + \tau^2 \end{bmatrix}$$

$n_G^2 = \det(A) = (\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3)(\sigma_3 - \sigma_1) < 0$

$$\begin{cases} n_1^2 = \dfrac{\tau^2 + (\sigma - \sigma_2)(\sigma - \sigma_3)}{(\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3)} \\ n_2^2 = \dfrac{\tau^2 + (\sigma - \sigma_3)(\sigma - \sigma_1)}{(\sigma_2 - \sigma_3)(\sigma_2 - \sigma_1)} \\ n_3^2 = \dfrac{\tau^2 + (\sigma - \sigma_1)(\sigma - \sigma_2)}{(\sigma_3 - \sigma_1)(\sigma_3 - \sigma_2)} \end{cases}$$

$\hookrightarrow A^{-1}$

Scanné avec CamScanner

3) $n_i^2 \geq 0 \quad \tau^2 + (\sigma - \sigma_2)(\sigma - \sigma_3) \geq 0$

$\sigma^2 \cancel{- (\sigma_2 + \sigma_3)} -$

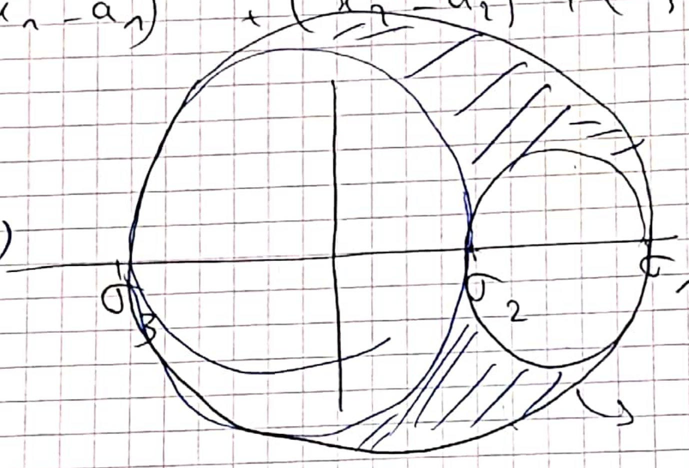
↳ on trouve les résult.

$$\sigma_2 \sigma_3 = \frac{1}{4}\left((\sigma_2 + \sigma_3)^2 - (\sigma_2 - \sigma_3)^2 \right)$$

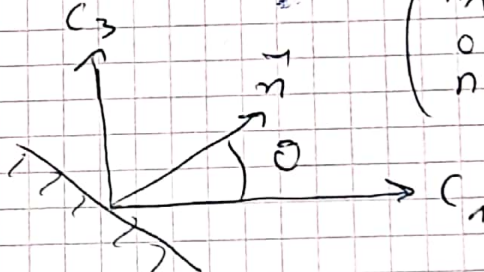
↳ equation du cercle $\|\vec{x} - \vec{a}\|^2 = R^2$

$$(x_1 - a_1)^2 + (x_2 - a_2)^2 + (x_3 - a_3)^2 = R^2$$

4) $\vec{T} = \begin{pmatrix} \sigma \\ \tau \end{pmatrix}_{(\sigma, \tau)}$



5) a)



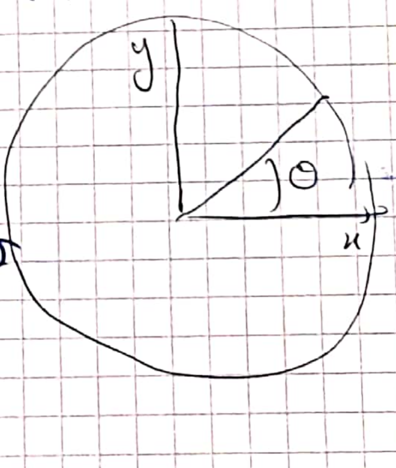
$\begin{pmatrix} n_1 \\ 0 \\ n_3 \end{pmatrix}$

$\vec{n} = \cos(\theta)\vec{C_1} + \sin(\theta)\vec{C_3}$

$\vec{t} = -\sin(\theta)\vec{C_1} + \cos(\theta)\vec{C_3}$

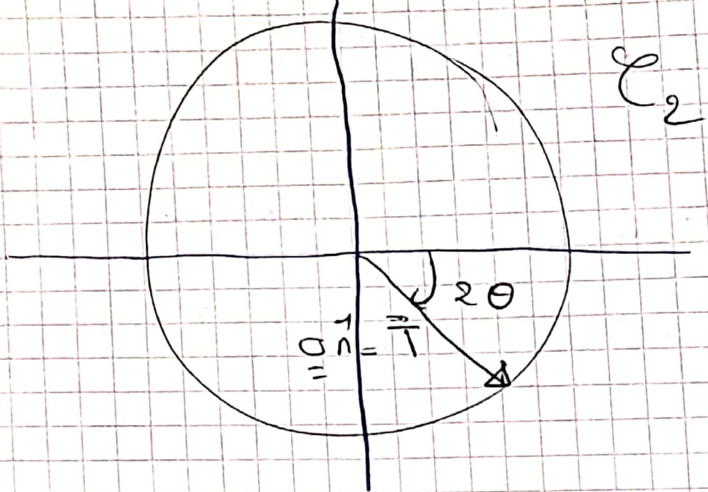
$\begin{cases} \sigma = \vec{T} \cdot \vec{n} \\ \tau = \vec{T} \cdot \vec{t} \end{cases} \qquad \frac{\cos(2\theta) + 1}{2} = \cos^2(\theta)$

$x = R$



$$\begin{cases} \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos(2\theta) = \sigma \\ -\frac{(\sigma_1 - \sigma_3)}{2}\sin(2\theta) = \tau \end{cases}$$

$\mathcal{C}_2$

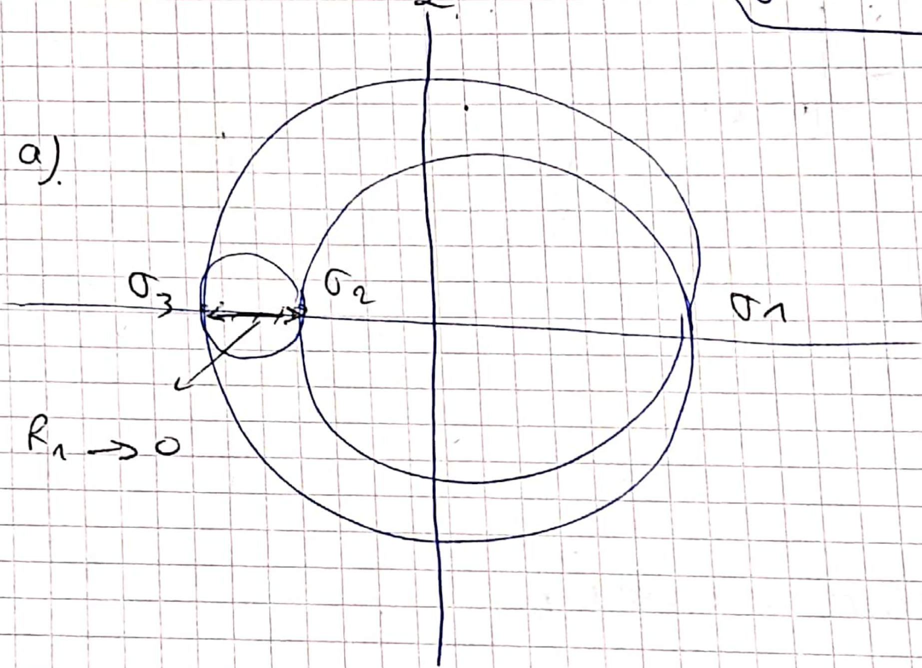


$$|\tau| = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \quad ; \quad 2\theta = \frac{\pi}{2}$$

$$\max |\tau| = \frac{\sigma_1 - \sigma_3}{2}$$

$$\boxed{\theta = \pm \frac{\pi}{4}}$$

6/ a)



$\sigma_1 > \sigma_2 = \sigma_3$

$\sigma_2 \longrightarrow \sigma_3$

$\sigma_1 \geq \sigma_2 \geq \sigma_3$

$$\underline{\underline{\sigma}} = \sigma \, \vec{a} \otimes \vec{a}$$

c) $\underline{\underline{\sigma}} = \tau (\vec{a} \otimes \vec{b} + \vec{b} \otimes \vec{a})$

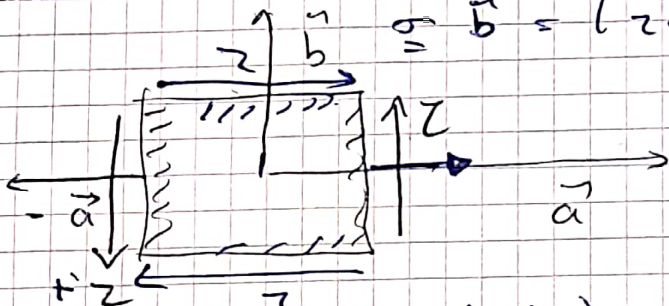
$$\begin{pmatrix} \sigma & \tau \\ \tau & 0 \end{pmatrix}_{(\vec{a},\vec{b})} \leftarrow \underline{\underline{\sigma}} = \begin{pmatrix} \tau & 0 & 0 \\ 0 & -\tau & 0 \\ 0 & 0 & 0 \end{pmatrix}_{\{\vec{C}\}} \quad \left(\text{chapitre 2 exo} \right)$$

$$\{\vec{C}\} \begin{cases} \vec{c}_1 & \dfrac{1}{\sqrt{2}} (\vec{a} + \vec{b}) \\ \vec{c}_2 & \dfrac{1}{\sqrt{2}} (\vec{a} - \vec{b}) \end{cases}$$

→ Diagonaliser la matrice

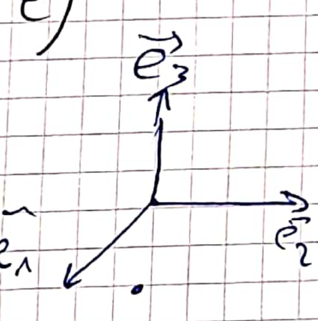
$$\underline{\underline{\sigma}} \, \vec{b} = \begin{pmatrix} 0 & \tau \\ \tau & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \tau \\ 0 \end{pmatrix}$$

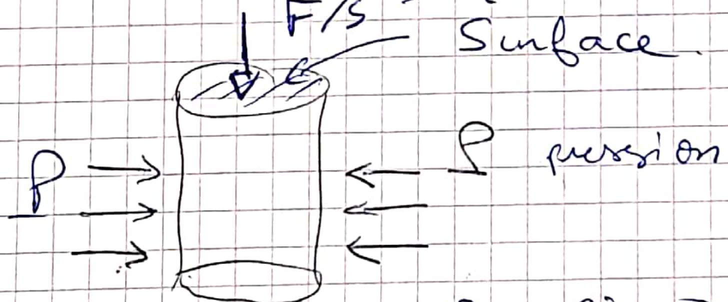
. d)



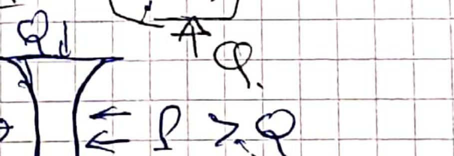
$$\underline{\underline{\sigma}} \, \vec{a} = \begin{pmatrix} 0 & \tau \\ \tau & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \tau \end{pmatrix} (\tau > 0)$$

e)



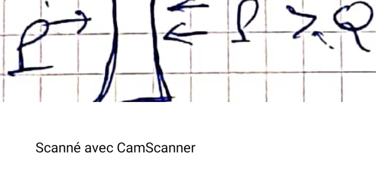


$$\underline{\underline{\sigma}} = \begin{pmatrix} -P & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & -Q \end{pmatrix}$$



Si $Q > P$ → compression

Si $Q < P$ → extension



chapitre 4 : Loi de comportement

Exercices 2 Lois d'état thermodynamique élastique

1)
$$\underline{\underline{\Pi}} = \rho_0 \, \partial_{\underline{\underline{\Delta}}} \, \psi$$
$$s = -\partial_T \psi$$
$$\psi = \psi(\underline{\underline{\Delta}}, T)$$
$$\dot{\underline{\underline{\Pi}}} = \partial_{\underline{\underline{\Delta}}} \Pi : \dot{\underline{\underline{\Delta}}} + \partial_T \underline{\underline{\Pi}} \, \dot{T}$$
$$\partial_{\underline{\Delta}} \Pi = \rho_0 \, \partial^2_{\underline{\underline{\Delta}}} \psi = \underset{\sim}{H}$$
$$\partial_T \Pi = -\underset{\sim}{H} \, \alpha$$
$$s = -\partial_T \psi = s(\underline{\underline{\Delta}}, T)$$
$$\dot{s} = \partial_{\underline{\underline{\Delta}}} s \, \dot{\underline{\underline{\Delta}}} + \partial_T s \, \dot{T}$$
$$\partial_{\Delta} s = -\partial_{\underline{\Delta}} (\partial_T \psi) = -\partial_{\underline{\underline{\Delta}}} (\partial_T \psi)$$
$\boxed{\text{Schwartz } C^2}$
$$= -\partial_T (\partial_{\underline{\underline{\Delta}}} \psi)$$

2)
$$\boxed{\underset{\sim}{H} = \partial_{\underline{\underline{\Delta}}} \Pi} \quad , \quad H_{ijkl} = \frac{\partial \Pi_{ij}}{\partial \Delta_{kl}} = \rho_0 \frac{\partial}{\partial \Delta_{kl}} \partial_{\Delta_{ij}} \psi$$
$$= \rho_0 \frac{\partial}{\partial \Delta_{ij}} \partial_{\Delta_{kl}} \psi$$
$$\Rightarrow H_{ijkl} = H_{klij}$$

↳ $\underset{\sim}{H}$ est symétrique