Data Structure and Algorithm

Class 7

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Tree

Introduction

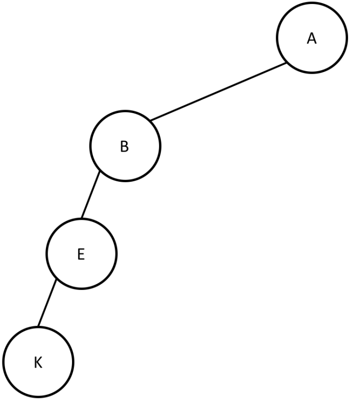
Tree It is finite set of one or more nodes such that

1. there is a special node called root
2. remaining nodes are partitioned into ngeq0 disjoint trees $T_1, T_2, \cdots, T_n$ where each of these is a tree; we call each T_i subtree of the root

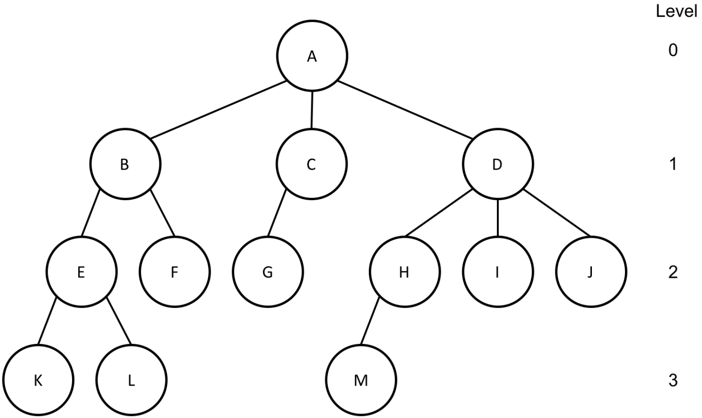
Acyclic graph A tree that contains no cycle

It has a hierarchical structure

Tree

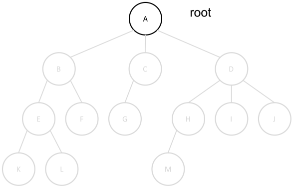


Tree

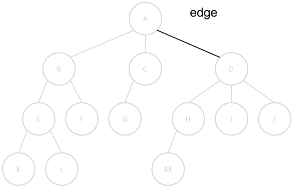


Terminology

Root A node with no parent (e.g., A is the root)

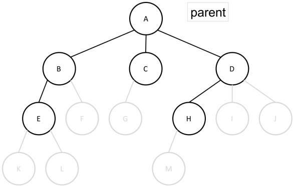


edge The connecting link between any two nodes (e.g., link between A and B is edge)

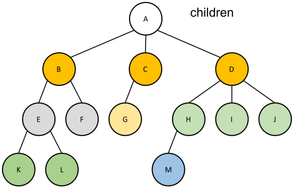


Terminology II

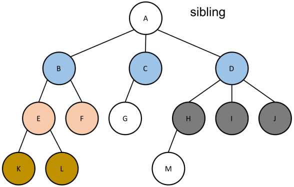
parent a node that has subtrees (e.g., A is parent of B, C, and D. E is parent of K and L)



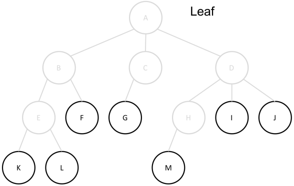
child a root of the subtrees (e.g., E is child of B, C is child of A)



sibling child nodes of the same parent (e.g., B, C, and D are siblings, K, L, and M are siblings)



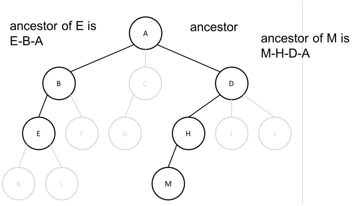
Leaf (terminal, external) node A node with degree zero (e.g., K, L, F, G, M, I, and J)



Terminology IV

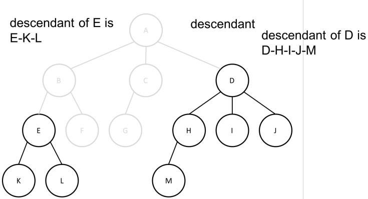
Internal (non-terminal, internal) node node with degree one or more (e.g., A, B, C, D, E, and H)

ancestor all the nodes along the path from the root to the node (e.g., ancestor of K is K, E, B, and A. Ancestor of H is H, D, and A)



Terminology V

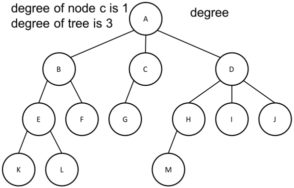
descendant all the nodes that are in its subtrees (e.g., Descendants of E is E, K, and L. Descendants of D is D, H, M, I, and J)



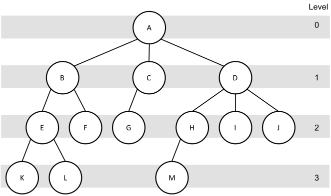
Degree of a node The number of subtrees of node, in other words the total number of children of a node (e.g., Degree of A is 3. Degree of C is 1)

Terminology VI

Degree of a tree The maximum degree of the nodes in the tree (e.g., the degree of the tree is 3)



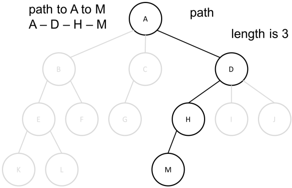
level each step from top to bottom, the level of the root node is 0 (e.g., level of F is 2. Level of M is 3)



Terminology VII

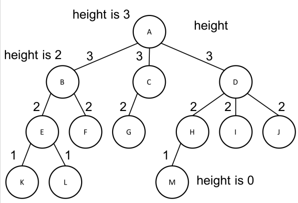
path the set of edges from the root to a node (e.g., the path to M from A is (A, D), (D, H), (H, M))

path length The number of edges in a path (e.g., path length from A to M is 3)



Terminology VIII

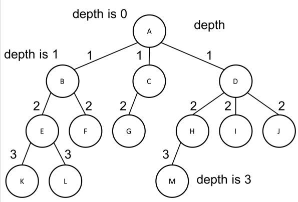
Height of a tree The longest path length from the root to a leaf (e.g., the height is 3)



Terminology IX

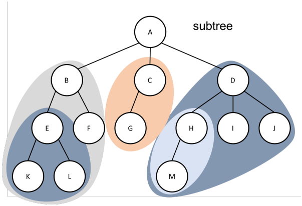
depth the total number of edges from root node to a particular node (e.g., depth of F is 2. Depth of M is 3)

depth of the tree the total number of edges from root node to a leaf node in the longest path (e.g., depth of the tree is 3)

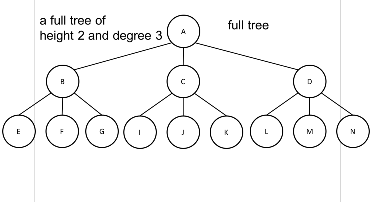


Terminology X

subtree each child from a node forms a subtree recursively. Every child node will form a subtree on its parent node

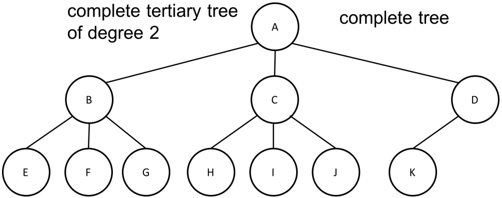


proper (or full) tree every node other than the leaves has non-void children



Terminology XI

complete tree All levels are full except for the deepest level, which is partially filled from the left



Tree Representation

Tree Data Structure

Two types of representation of tree

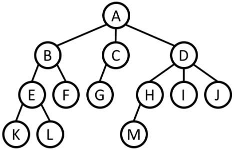
1. List representation
2. left child - right sibling representation

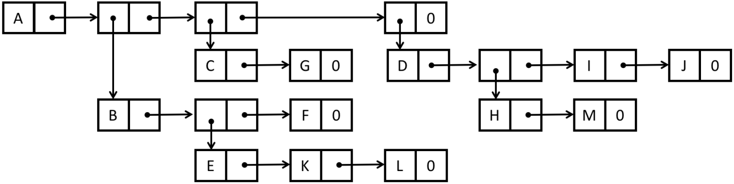
List Representation

Two types of nodes

1. Node with data
2. Node with the reference

The information in the root node comes first and it is followed by a list of the subtrees of that node

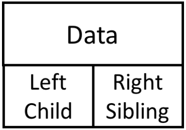




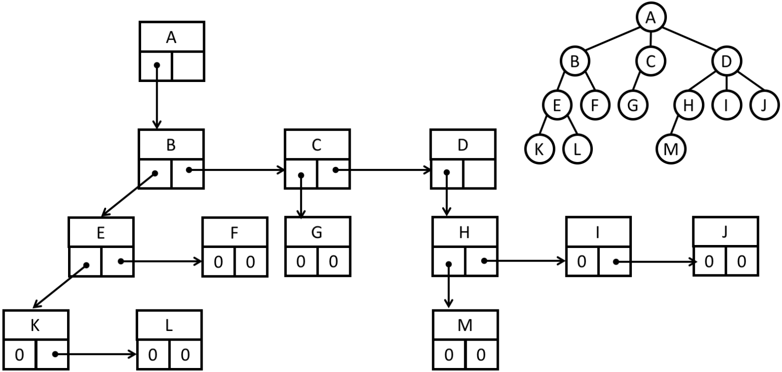
Left-child Right-Sibling Representation

nodes of a fixed size

- easier to work
- two link / pointer fields and a data field per node
  - left reference field stores the address of the left child
  - right reference field stores the address of the right sibling node



Left-child Right-sibling Representation

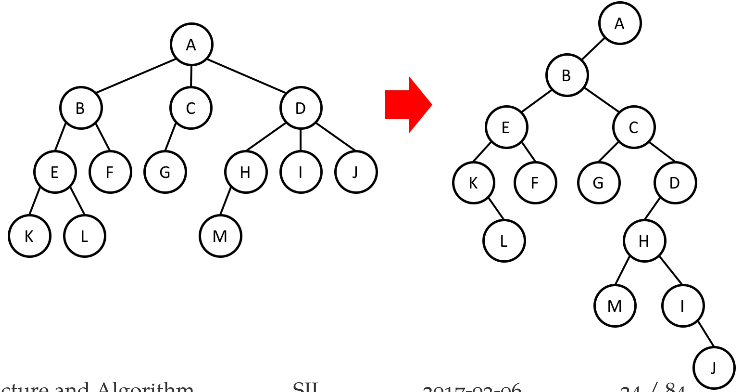


Representation of trees

Changing a representation of a tree as a degree two tree

- simply rotate the left-child right-sibling tree clockwise by 45 degrees

A tree with degree 2 (two children, left and right child) is called a binary tree



Binary Tree

Definition

A binary Tree is a finite set of nodes such that

1. empty or
2. consists of root note and two disjoint binary trees, called left subtree and right subtree

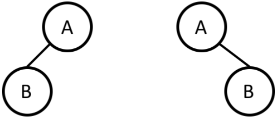


Figure: two different types of binary tree

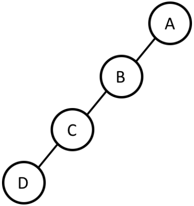
Properties

Difference between a binary tree and a tree

- may have empty node
- the order of subtree are important
- a binary tree is not a subset of a tree
- maximum number of nodes in a Binary Tree is $2^k - 1$ where k is depth of the tree
- relationship between the number of leaf nodes ($n0$) and the number of nodes with degree 2 ($n2$) $n0 = n2 + 1$

Special Types of Binary Trees I

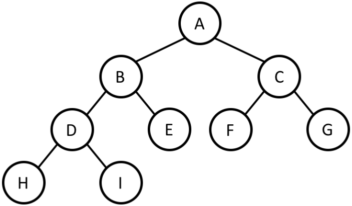
- skewed binary tree



Special Types of Binary Trees II

- full binary tree (of depth k)

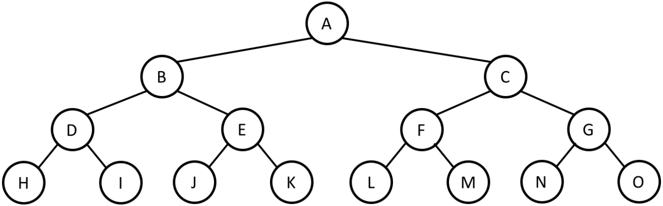
  full (or proper or strictly binary tree every node has either two or zero number of children



Special Types of Binary Trees III

- complete binary tree

complete (or perfect) binary tree A binary tree in which every internal node has exactly two children and all leaf nodes are at same level a binary tree with n nodes that correspond to the nodes numbered from 1 to n in the full binary tree of depth k



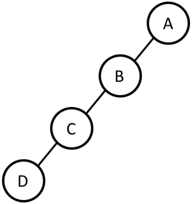
Binary Tree Representation

There are two methods to represent the binary tree

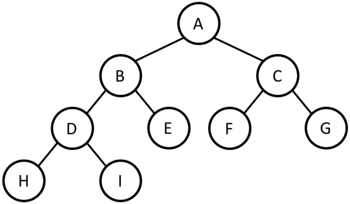
1. Array Representation
2. Linked List Representation

Array Representation

- sequential representations
- determine the locations of the parent, left child, and right child of any node i in the binary tree
  1. parent (i) is at $\lfloor i/2 \rfloor$ if $i \neq 1$, if $i = 1$, no parent
  2. left_child (i) is at $2 \cdot i$ if $2i \leq n$
  3. right_child (i) is at $2 \cdot i + 1$ if $2 \cdot i + 1 \leq n$



1	A
2	B
3	-
4	C
5	-
6	-
7	-
8	D
9	-
10	-
11	-
12	-
13	-
14	-
15	-
16	E



1	A
2	B
3	C
4	D
5	E
6	F
7	G
8	H
9	I

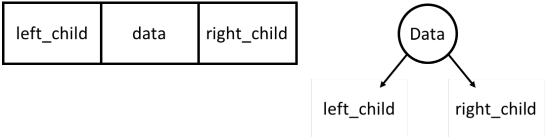
The Problem of Array Representation of Tree

- inefficient storage utilization
  $S(n) = 2 - 1$ where k is depth of binary tree
  ideal for complete binary trees
- hard to insert/delete

Representing tree with linked list

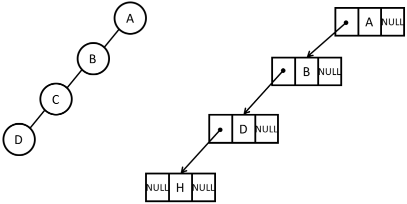
- each node has three fields
  1. left_child
  2. data
  3. right_child

```
typedef struct BinaryTreeNode {
    int data;
    struct BinaryTreeNode* left_child;
    struct BinaryTreeNode* right_child;
} node;
```

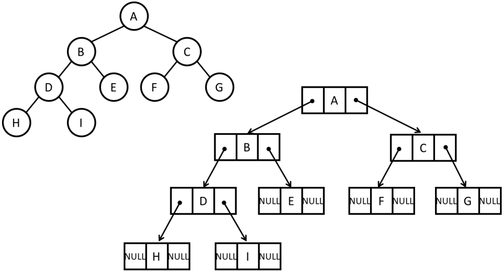


example

Skewed



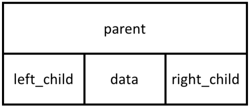
complete

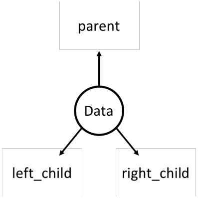


Linked List Representation cont'd

leaf node's link field contains NULL pointer

Add a fourth field, called parent, to know the parent of a random nodes





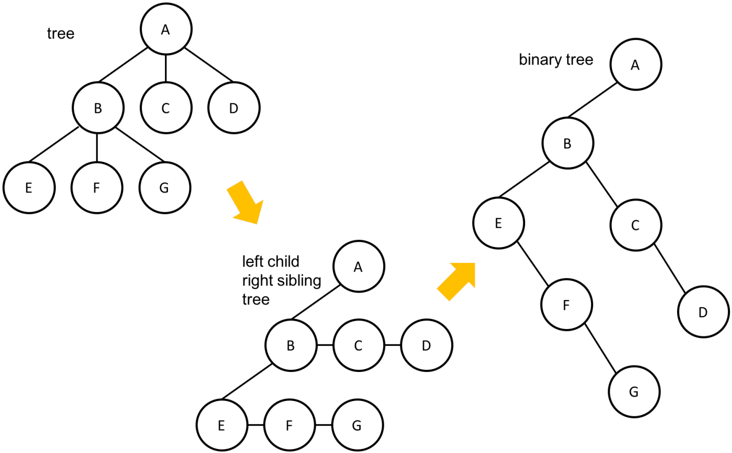
Tree Representation

- each node in a tree has a variable sized nodes
- hard to represent it by using array
- use linked list to represent a tree needs k link fields per node
  - k is the degree of tree
- There are two types of links
  - non-null links
  - null links
- if the number of non-null links are $n - 1$
  - the number of null links are $n \cdot k - (n - 1)$

Converting a tree into a binary Tree

1. Use left-child right sibling representation
   - (parent, $child_1$, $child_2$, ..., $child_x$) → (parent, leftmost-child, next-right-sibling)
2. simply rotate the left-child right-sibling tree clockwise by 45 degrees
   - right field of root node always have null link
   - null links: approximately 50%
   - depth increased

Converting a tree into a binary Tree



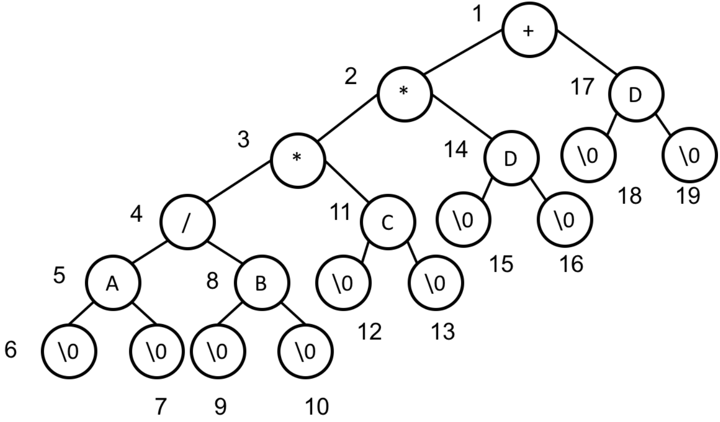
Binary Tree Traversal and Other Operations

Binary Tree Traversals

visit each node in the tree exactly once

- produce a linear order for the information in a tree
- what order?
  - inorder: LVR (Left Visit Right)
  - preorder: VLR (Visit Left Right)
  - postorder: LRV (Left Right Visit)

$A / B * C * D + E$ (infix form)



Binary Tree Traversals

Inorder Traversal

```
void inorder(TreeNode *ptr) {
    if(ptr) {
        inorder(ptr->left_child);
        printf("%d",ptr->data);
        inorder(ptr->right_child);
    }
}
```

inorder is invoked 19 times for the complete traversal: 19 nodes

output: $A / B * C * D + E$

- corresponds to the infix form

Binary Tree Traversal

call of in-order	value in root	action	call of in-order	value in root	action
1	+		11	C	
2	*		12	NULL	
3	*		11	C	printf
4	/		13	NULL	
5	A		2	*	printf
6	NULL		14	D	
5	A	printf	15	NULL	
7	NULL		14	D	printf
4	/	printf	16	NULL	
8	B		1	+	printf
9	NULL		17	E	
8	B	printf	18	NULL	
10	NULL		17	E	printf
3	*	printf	19	NULL	

Preorder Traversal

```
void preorder(TreeNode *ptr) {
    if(ptr) {
        printf("%d",ptr->data);
        preorder(ptr->left_child);
        preorder(ptr->right_child);
    }
}
```

output in the order $+ * * / ABCDE$

Postorder Traversal

```
void postorder(TreeNode *ptr) {
    if(ptr) {
        postorder(ptr->left_child);
        postorder(ptr->right_child);
        printf("%d",ptr->data);
    }
}
```

output in the order $AB/C*D*E+$

Iterative Inorder Traversal

Recursion

- call itself directly or indirectly
- simple, compact expression: good readability
- don't need to know implementation details
- much storage: multiple activations exist internally
- slow execution speed
- application: factorial, Fibonacci number, tree traversal, binary search, tower of Hanoi, quick sort, LISP structure

Iterative Inorder traversal

```c
void iter_inorder(tree_ptr node) {
    int top = -1;
    tree_ptr stack[MAX_STACK_SIZE];
    while (1) {
        while (node) {
            push(&top, node);
            node = node->left_child;
        }
        node = pop(&top);
        if (!node) break;
        printf("%d", node->data);
        node = node->right_child;
    }
}
```

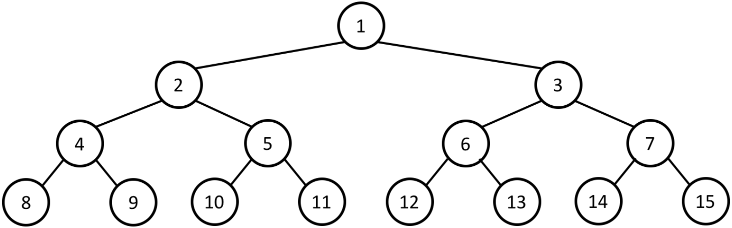
Iterative Inorder Traversal

every node of the tree is placed on and removed from the stack exactly once

- time complexity: O(n) where n is the number of nodes in the tree
- space complexity: stack size O(n) where n is worst case depth of the tree (case of skewed binary tree)

Level Order Traversal

Traversal by using queue (FIFO)



Output in the order: 1, 2, 3, 4, . . ., 14, 15

Level Order Traversal

```
void level_order(tree_ptr ptr) {
    int front = rear = 0;
    tree_ptr queue[MAX_QUEUE_SIZE];

    if (!ptr) return;

    addq(front,&rear,ptr);

    while (1) {
        ptr = deleteq(&front, rear);

        if (ptr) {
            printf("%d", ptr->data);
            if (ptr->left_child)
                addq(front,&rear,ptr->left_child);
            if (ptr->right_child)
                addq(front,&rear,ptr->right_child);
            else break;
        }
    }
}
```

Copying Binary Tree

Modified version of postorder

```
tree_ptr copy(tree_ptr original) {
    tree_ptr temp;
    if (original) {
        temp = (tree_ptr)malloc(sizeof(node));

        if (IS_FULL(temp)) exit(1);
            temp->left_child = copy(original->left_child);
            temp->right_child = copy(original->right_child);
            temp->data = original->data;
            return temp;
         }
    return NULL;
}
```

Testing for equality of binary trees

Modified version of preorder

```
int equal(tree_ptr first, tree_ptr second) {
return ((!first && !second) ||
      (first && second
      && (first->data == second->data)
      && equal(first->left_child,
              second->left_child)
      && equal(first->right_child,
              second->right_child));
}
```

Heaps

Heaps: Definition

MAX (or MIN) Tree a tree in which the key value in each node is no smaller (larger) than the key value in its children (if any)

MAX (or MIN) Heap a complete binary tree that is also a max (or min) tree

- the root of a max (or min) tree contains the largest (smallest) key in the tree

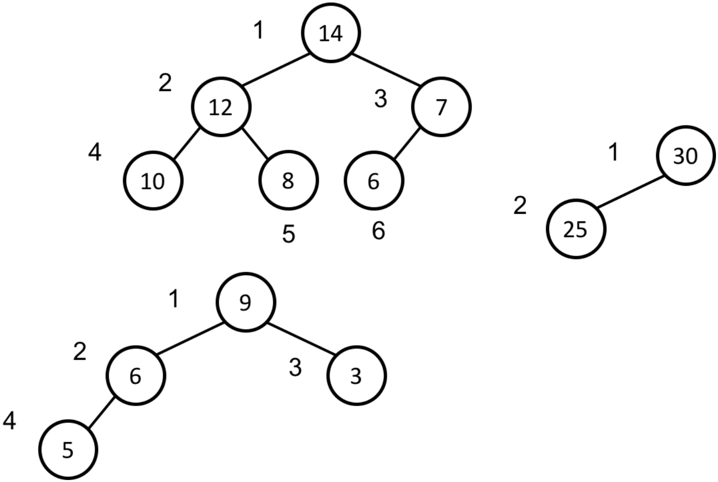
Representation of MAX (or MIN) heaps

- array representation because heap is a complete binary tree
- simple addressing scheme for parent, left(right) child

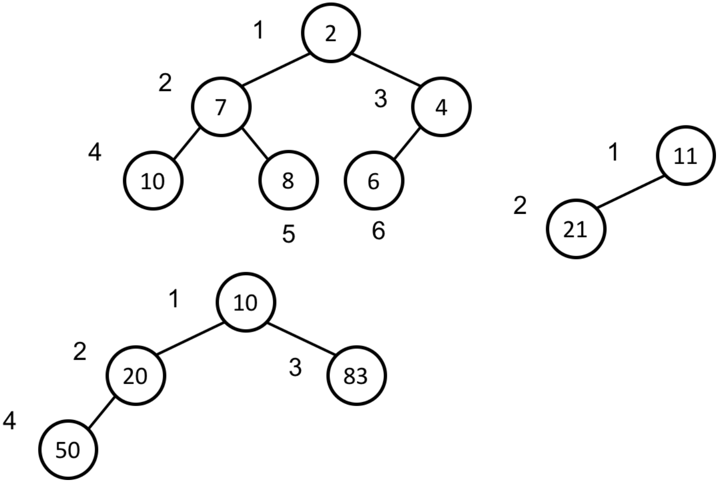
Heap Structure

```
#define MAX_ELEMENTS 200
#define HEAP_FULL(n) (n == MAX_ELEMENTS - 1)
#define HEAP_EMPTY(n) (!n)

typedef struct {
    int key;
    /* other field */
} element;

element heap[MAX_ELEMENTS];
int n = 0;
```

Sample Max Heaps



Sample Min Heaps



Priority Queues

deletion deletes the element with the highest(or the lowest) priority

insertion insert an element with arbitrary priority into a priority queue at any time

Ex. Job scheduling of OS

Priority Queues

We use a max (or Min) Heap to implement the Priority Queues

Possible priority queue representations

Representation	insertion	deletion
unordered array	O(1)	O(n)
unordered linked list	O(1)	O(n)
sorted array	O(n)	O(1)
sorted linked list	O(n)	O(1)
max heap	O($log_2 n$)	O($log_2 n$)

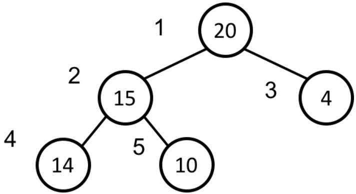
Insertion into a max heap

Need to go from a node to its parent

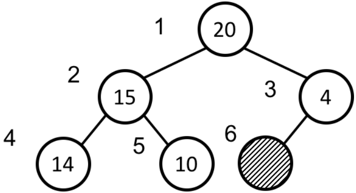
- linked representation add a parent field to each node
- array representation a heap is a complete binary tree simple addressing scheme

Insertion into a max heap

Heap before Insertion

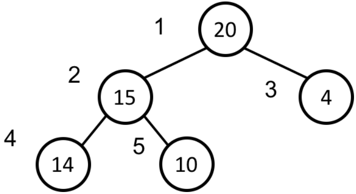


Initial location of new node

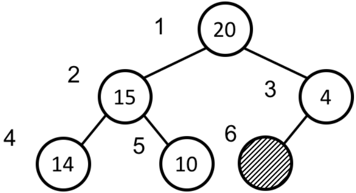


Insertion into a max heap

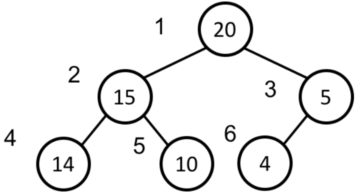
Heap before Insertion



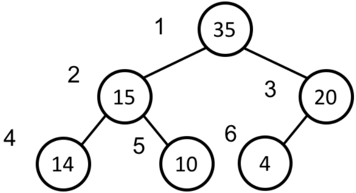
Initial location of new node



Insert 5 into heap



Insert 35 into heap



Insertion into a max heap

- select the initial location for the node to be inserted $\rightarrow$ bottommost-rightmost leaf node
- insert a new key value adjust key value from leaf to root parent position: $\lfloor i/2 \rfloor$
- time complexity : $O(depthoftree) \rightarrow O(log_2 n)$

Insertion into a max heap

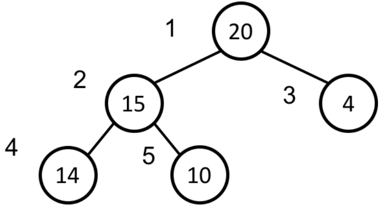
```
void insert_max_heap(element item, int *n) {
    int i;
    if (HEAP_FULL(*n)) {
        fprintf(stderr,"The heap is full. \n");
        exit(1);
    }
    i = ++(*n);
    while ((i != 1 ) && (item.key > heap[i/2].key)) {
        heap[i] = heap[i/2];
        i /= 2;
    }
    heap[i] = item;
}
```

Deletion from a max heap

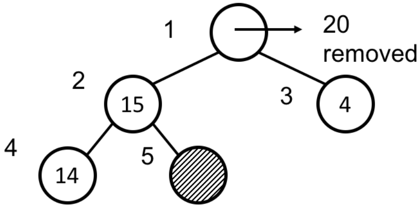
- always delete an element from the root of the heap
- restructure the tree so that it corresponds to a complete binary tree
- place the last node to the root and from the root compare the parent node with its children and exchanging out-of-order elements until the heap is reestablished

Deletion from a max heap

Heap before deletion

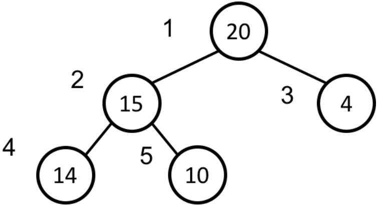


Heap Structure

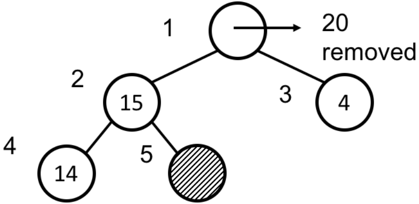


Deletion from a max heap

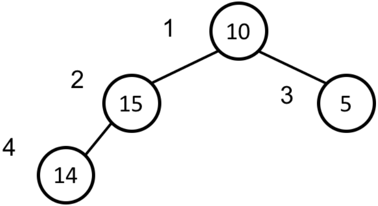
Heap before deletion



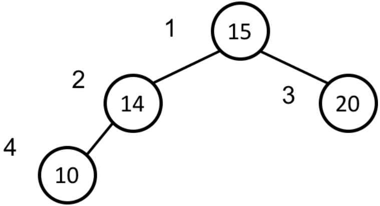
Heap Structure



10 inserted at the root



Final heap



Deletion from a max heap

- select the removed node bottommost-rightmost leaf node
- place the node's element in the root node
- adjust key value from root to leaf compare the parent node with its children and exchange out-of-order elements until the heap is reestablished -
- time complexity : $O(depth\ of\ tree) \rightarrow\ O(log_2 n)$

Deletion from a max heap

```
element delete_max_heap(int *n) {
    element item, temp;
    if (HEAP_EMPTY(*n)) {
        fprintf(stderr,"The heap is empty\n");
        exit(1);
    }
    item = heap[1];
    temp = heap[(*n)--];
    parent = 1;
    child = 2;
    while (child <= *n) {
    /* compare left and right child's key values */
        if ((child < *n) && (heap[child].key <
                             heap[child+1].key))
            child++;
        if (temp.key >= heap[child].key) break;
        /* move to the next lower level */
        heap[parent] = heap[child];
        parent = child;
        child *= 2;
    }
    heap[parent] = temp; return item;
}
```

Binary Search Tree

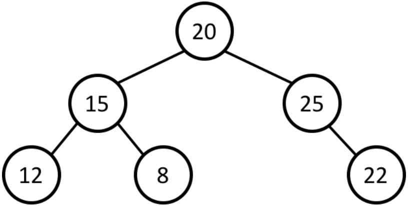
Binary Search Tree (BST)

Binary search tree(BST) is a binary tree that is empty or each node satisfies the following properties:

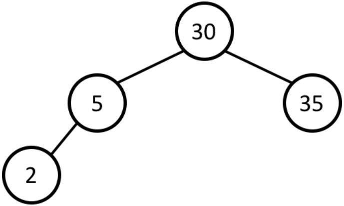
1. every element has a key, and no two elements have the same key
2. the keys in a nonempty left subtree must be smaller than the key in the root of the subtree
3. the keys in a nonempty right subtree must be larger than the key in the root of the subtree
4. the left and right subtrees are also BST

Binary Search Tree

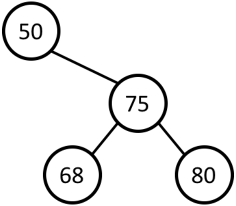
Not a BST



BST



BST



Operations and their Complexity

Searching, Insertion, Deletion is bounded by $O(h)$ where h is the height of the BST

can perform these operations both

- by key value and
  e.g., delete the element with key x
- by rank
  e.g., delete the fifth smallest element

Recursive search of a BST

```
tree_ptr search(tree_ptr root, int key) {
    /* return a pointer to the node that contains
     * key. If there is no such node, return NULL
     */
    if (!root) return NULL;
    if (key == root->data) return root;
    if (key < root->data)
       return search(root->left_child, key);
    return search(root->right_child, key);
}
```

Iterative Search of a BST

```
tree_ptr iter_search(tree_ptr tree, int key) {
    while (tree) {
        if (key == tree->data) return tree;
        if (key < tree->data)
            tree = tree->left_child;
        else
            tree = tree->right_child;
    }
    return NULL;
}
```

Time complexity for searching

- Average case
  - $O(h)$ where h is the height of BST
- Worst case
  - $O(n)$ for skewed binary tree

Inserting into a BST

```
void insert_node(tree_ptr *node, int num) {
    tree_ptr ptr, temp = modified_search(*node, num);
    if (temp || !(*node)) {
        ptr = (tree_ptr)malloc(sizeof(node));
        if (IS_FULL(ptr)) {
            fprintf(stderr,"The memory is full\n");
            exit(1);
        }
        ptr->data = num;
        ptr->left_child = ptr->right_child = NULL;
        if (*node)
            if (num < temp->data)
                temp->left_child = ptr;
            else
                temp->right_child = ptr;
        else *node=ptr;
    }
}
```

Inserting into a BST

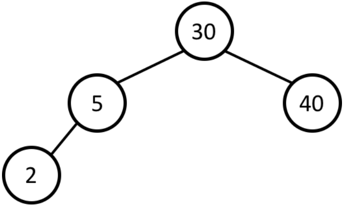
FIGURE-CODE?? **modified_search** is slightly modified version of function **iter_search**

- return NULL, if the tree is empty or num is present
- otherwise, return a pointer to the last node of the tree that was encountered during the search

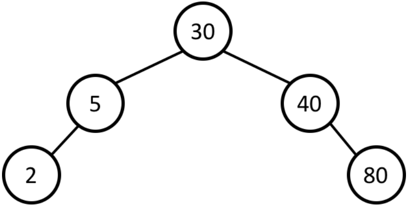
time complexity for inserting

- $O(h)$, where h is the height of the tree

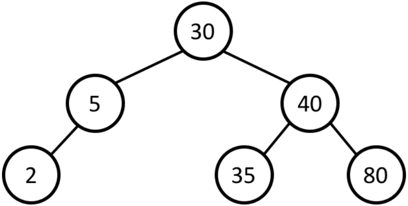
Initial BST



Insert 80



Insert 35



Deleting from BST

deletion of a leaf node

- deletion of a node with 1 child
- deletion of a node with 2 children

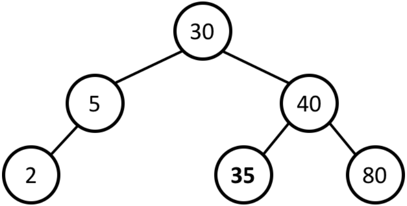
FIGURE- Deleting Algorithm is discussed in Chapter 10

time complexity for deleting

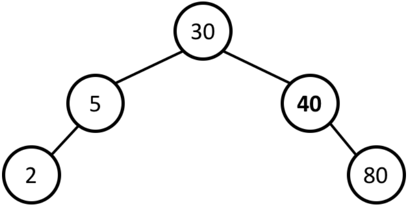
- $O(h)$ where h is the height of the tree

Deleting a leaf or a node with a child

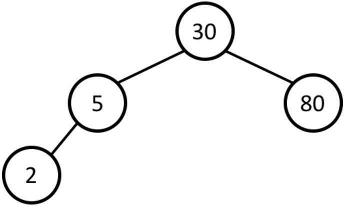
Initial BST



Delete 35 (leaf)

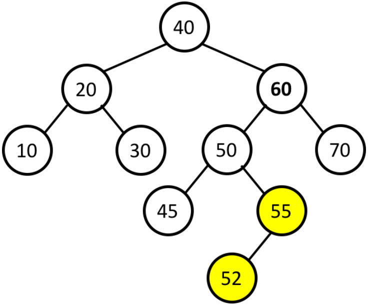


Delete 40 (node with single child)

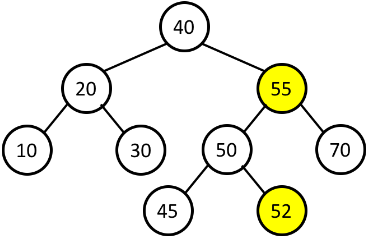


Deletion of a node with two children

Tree before deletion of 60



Tree after deletion of 60



Height of a BST

the height of a BST with n elements

- average case: $O(log_2 n)$
- worst case: $O(n)$
  - e.g., use `insert_node` to insert the keys $1, 2, 3, \ldots, n$ into an initially empty BST

Balanced (binary) Search Tree

- worst case height: $O(log_2 n)$
- searching, insertion, deletion is bounded by $O(h)$ where h is the height of a binary tree
- *AVL tree, 2-3 tree, red-black tree are all introduced in Chapter 10*