

# Domain Adaptation for Regression under Monotonic Causal Shift

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**InfoSci**

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2-1. Introduction

2-2. Proposed Method

2-3. Experiments

## 3. Conclusion

3-1. Conclusion

3-2. Future Works

# Domain **A**daptation under monotonic causal shift via **S**urrogate **D**omain **E**stimation (**DASDE**)

## [Keywords]

Monotonic Causal Shift, Data Augmentation, Multi-Domain Adaptation, Regression for Tabular Dataset

# INTRODUCTION

## [Index]

1. Monotonic Covariate Shift
2. Data Augmentation

# Monotonic Causal Shift

## Traditional Domain Shift

### - Covariate Shift

$$P(Y_S|X_S) = P(Y_T|X_T) \text{ and } P(X_S) \neq P(X_T)$$

“Unseen/Biased Data Problem”

### - Concept Shift

$$P(Y_S|X_S) \neq P(Y_T|X_T) \text{ and } P(X_S) = P(X_T) \quad [X \rightarrow Y]$$

$$P(X_S|Y_S) \neq P(X_T|Y_T) \text{ and } P(Y_S) = P(Y_T) \quad [Y \rightarrow X]$$

“Fixed Input & Shift Output”

$\{X_S, Y_S\}$  : Source Domain,  $\{X_T, Y_T\}$  : Target Domain

## Monotonic Causal Shift

	Sensor A	Sensor B
CJ	190	183
DH	180	?
YJ	170	177

100% CI : (177, 183)

$$P(X_T) = P(\Phi(X_S)) \text{ and } P(X_T|Y_T) = P(\Phi(X_S)|Y_S)$$

$$\text{and if } X_i > X_k \text{ then } \Phi(X_i) > \Phi(X_k)$$

$\Phi(\cdot)$  : Shift Function

“Sensor : Replacement Effect”

“Image : Light Reflection/Shadow Effect”

# Data Augmentation

## Traditional Data Augmentation

- **Image Data** Bi-Direction Graph
  - Mirroring / Random Cropping
  - Rotation / Local Wrapping
  - Shearing / Color Shifting
- **Noise Injection** Robustness
  - Random Masking
- **Generative Model**

## Monotonic Transformation

“Monotonic Causal Shift Characteristic”



**“Monotonic Feature Augmentation”**



1. Robust to Shift Situation
2. Fast Adaptation to Shifted Domain

# RELATED WORKS

## [Index]

1. Monotonic Neural Network
2. Multi-Domain Adaptation

# Monotonic Neural Networks

## Monotonic NN Structure

### [Paper Info]

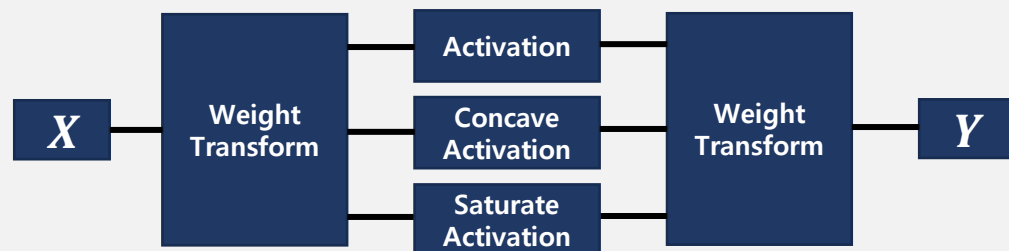
- Constrained Monotonic Neural Networks, 2023, ICML

### [MAIN IDEA]

- **Weight Transformation Function** : Embedding to  $[0, \infty)$
- **Concave Activation Function** :  $-f(-x)$  ( $f(x)$  : activation function)
- **Saturate Activation Function** :

$$\begin{cases} f(x+1) - f(1) & (\text{if } x < 0) \\ -f(-x+1) + f(1) & (\text{otherwise}) \end{cases}$$

- **Monotonic NN Architecture**



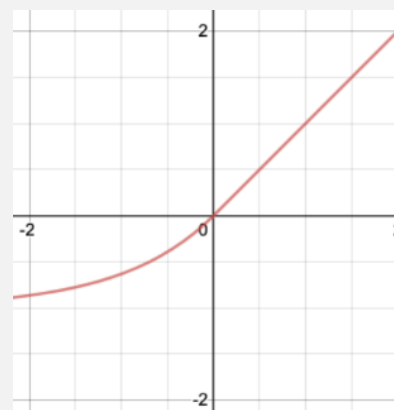
## Monotonic Function

### [Paper Info]

- Fast And Accurate Deep Network Learning by Exponential Linear Units (ELUs), 2016, ICLR

### [MAIN IDEA]

- **No Dead Neuron** :  $\forall x \quad f'(x) > 0$
- **Smooth and Strong Monotonic Increasing Function**
- **Fast Convergence**



$$ELU(x) = \begin{cases} x & (\text{if } x > 0) \\ e^x - 1 & (\text{otherwise}) \end{cases}$$



# Multi-Domain Adaptation

## DARC

### [Paper Info]

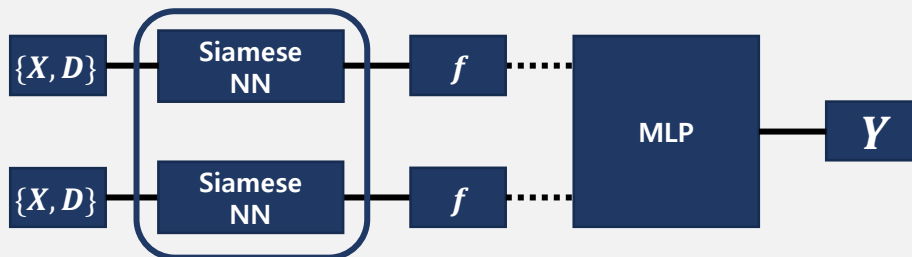
- Multi-domain adaptation for regression under conditional distribution shift, 2023, Expert Systems With Applications

### [MAIN IDEA]

- **Domain Labeling** : One-Hot Encoding
- **Shame Network** : Feature & Domain  $\rightarrow$  Shared Feature
- **Conditional Distribution Matching** :

$$\text{minimize } ||f_i - f_k| - |y_i - y_k|| \quad f : \text{shared feature}$$

### - DARC Architecture



## MAML

### [Paper Info]

- Model-Agnostic Meta-Learning for Fast Adaptation of Deep Networks, 2017, ICML
- Online Meta-Learning, 2019, ICML

### [MAIN IDEA]

- **Meta-Parent Model** : Shared Feature
- **Domain-Specific Child Model** : Domain Specific Feature
- **Fast Few-Shot Learning** :

1. Child Model : Initial weight = Parent Model
2. Child Model update weight using their Domain
3. Parent Model update weight

$$\rightarrow \text{minimize } \sum_{n_D} \text{Child\_Error}_k \quad n_D : \# \text{ of Domain}$$

Loop (1 ~ 3)

4. Use Parent Model as Initial Weight

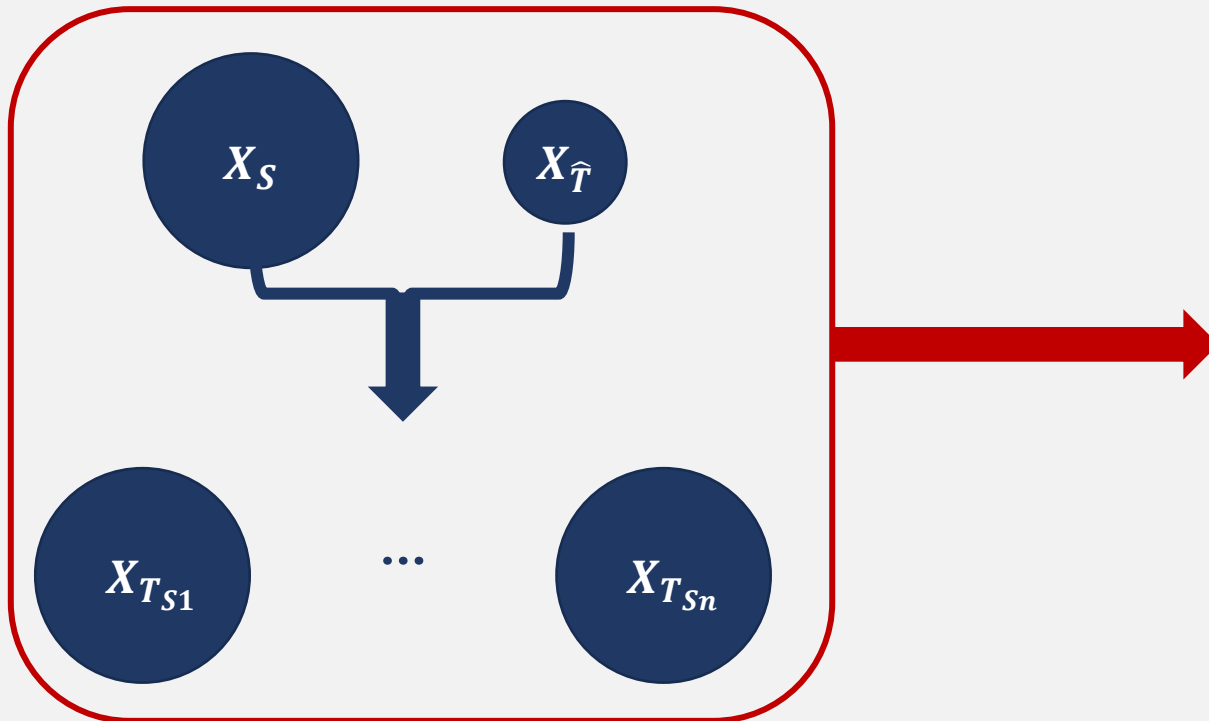
# PROPOSED METHOD

## [Index]

1. Full Architecture
2. Surrogate Domain Estimation

# Full Architecture

## ① SDE Surrogate Domain Estimation



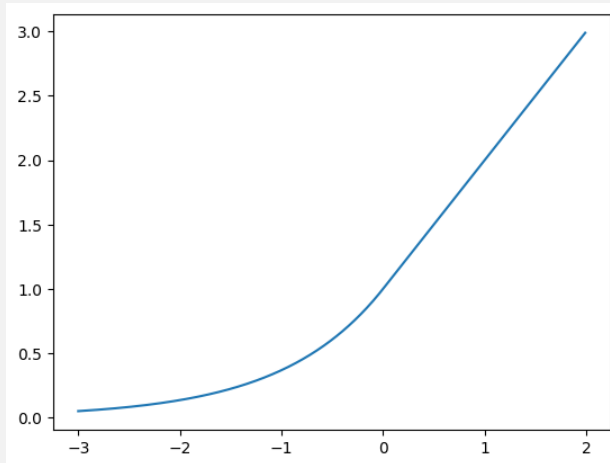
$X_S$  : Source Domain Input  
 $X_{\hat{T}}$  : Known Target Domain Input  
 $X_{T_s}$  : Surrogate Domain Input

## ② apply DARC/MAML **Multi-Domain Adaptation**

# Surrogate Domain Estimation (SDE)

## Modified Exponential Linear Units (MELU)

$$MELU(x) = \begin{cases} e^x & (\text{if } x < 0) \\ x + 1 & (\text{otherwise}) \end{cases}$$



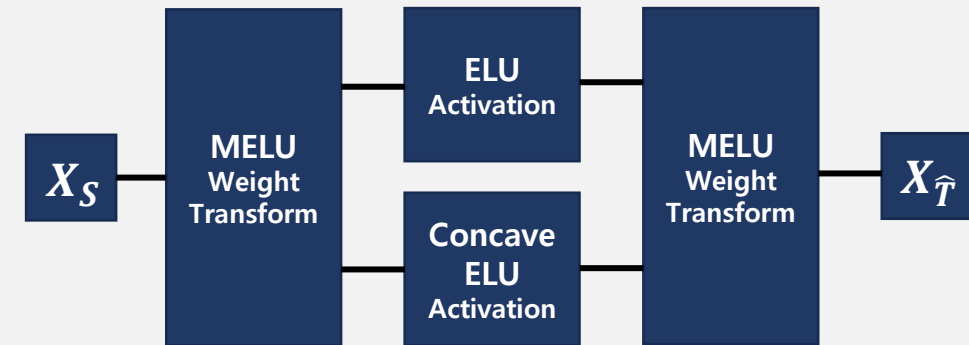
- **Fast Convergence** : ELU Characteristic

- **Computation Cost**

: high when  $x < 0$ , but low when  $x > 0$

## Surrogate Domain Estimation (SDE)

“Estimate Monotonic Causal Shift Function”



$X_S$  : Source Domain Input  
 $X_{\hat{T}}$  : Known Target Domain Input

- MELU Weight & ELU based Activation Function
- Shift Function : Source Domain  $\rightarrow$  Target Domain
- Strong Monotonicity :  $\forall x \ f'(x) > 0$
- Available to Explain Convex/Concave Function

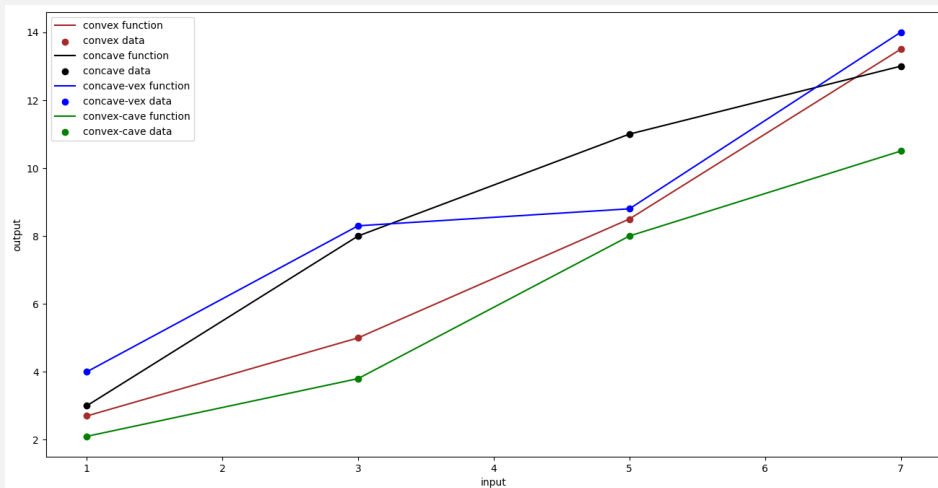
# EXPERIMENTS

## [Index]

1. Monotonic Structure Experiment
2. DA Experiment Setting
3. Result & Conclusion

# Monotonic Structure Experiment - Setting

## [Dataset]



- 4 kinds of Function Setting(4 Shot Learning)

: Convex, Concave, Concave-Convex, Convex-Concave

- Learning Rate : 0.01, NN Structure : 1-10-10-1

## [Comparison]

### Weight Transformation Function

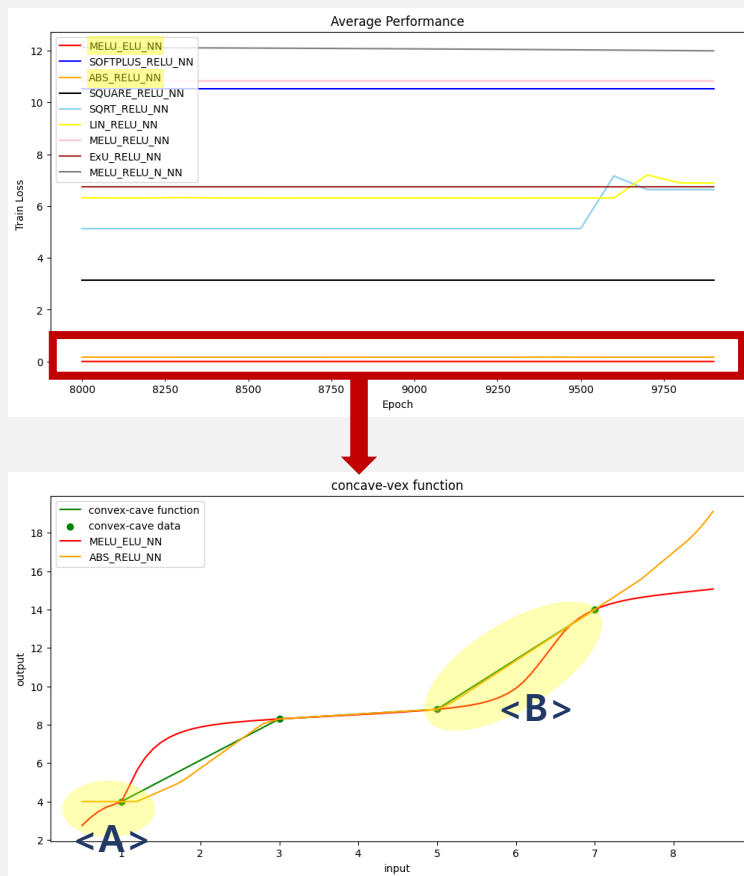
- MELU  $\min(1, e^w) + \max(1, w + 1)$
- SOFTPLUS  $\log(1 + e^w)$
- ABS  $|w|$
- SQUARE  $w^2$
- SQRT  $\sqrt{|w|}$

### Activation Function

- ELU & Concaved  $\text{ELU}(-\text{ELU}(-x))$
- ReLU & Concaved  $\text{ReLU}(-\text{ReLU}(-x))$
- Saturated ReLU
$$\begin{cases} \text{ReLU}(x + 1) - 1 & (\text{if } x < 0) \\ -\text{ReLU}(-x + 1) + 1 & (\text{otherwise}) \end{cases}$$

# Monotonic Structure Experiment - Result

**[Result]** 4 Experiments (4 dataset x 1)



**[Conclusion]**

## 1. Fast & Stable Train Structure

: MELU with ELU & Concave ELU

: ABS with ReLU & Concave ReLU

## 2. Strong Monotonicity

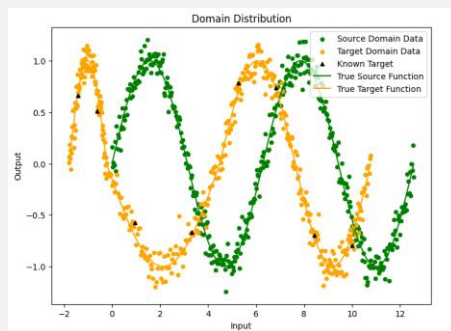
: MELU with ELU & Concave ELU <A>

## 3. Smooth Estimation

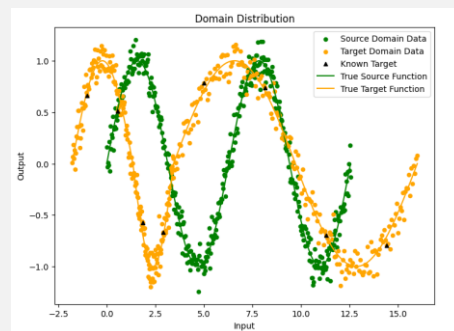
: MELU with ELU & Concave ELU <B>

# DA Experiment - Setting

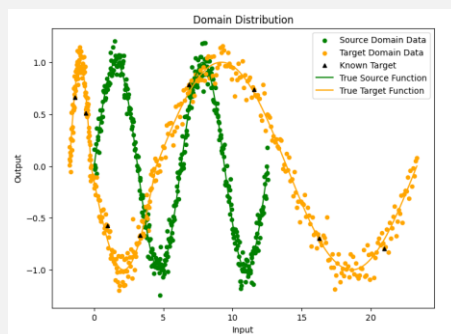
## [Dataset]



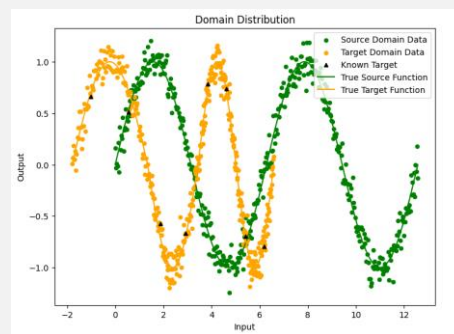
(a)



(c)



(b)



(d)

### <Source Dataset>

$$P(X_S) \sim \text{Unif}(0, 4\pi)$$

$$P(Y_S|X_S) \sim N(\sin(X_S), 0.1)$$

$$n_S = 400$$

### <Target Dataset>

$$P(X_T) = P(\Phi(X_S))$$

$$P(Y_T) = P(Y_T)$$

$$P(X_T|Y_T) = P(\Phi(X_S)|Y_S)$$

$$n_T = 400, \quad n_{\hat{T}} = 8$$

$n_S$  : # of Source Domain Data,  $\{X_S, Y_S\}$  : Source Domain

$n_T$  : # of Target Domain Data,  $\{X_T, Y_T\}$  : Target Domain

$n_{\hat{T}}$  : # of Known Target Domain Data,  $\Phi(\cdot)$  : Shift Function



# DA Experiment – Surrogate Domain Estimation

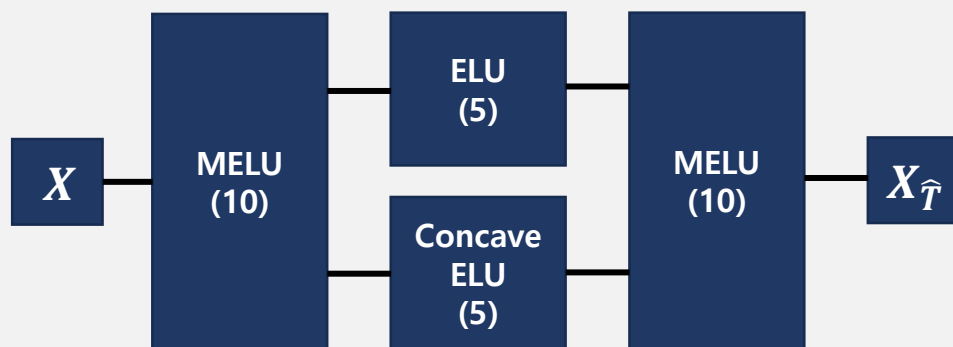
## [Surrogate Domain Estimator]

### Architecture

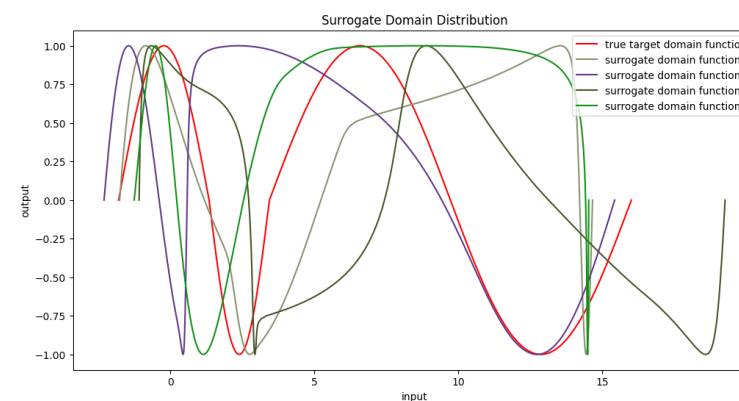
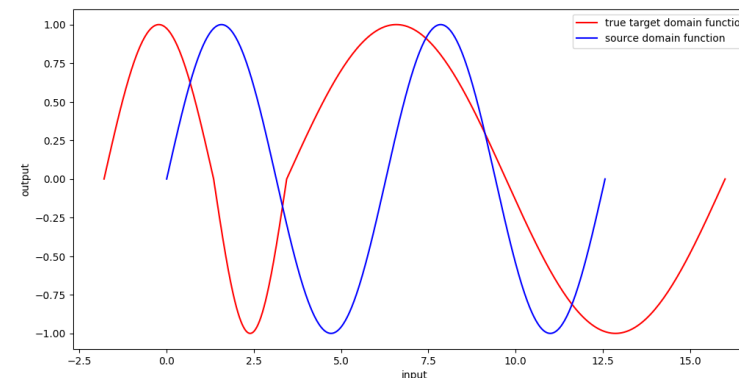
- Input :  $X \sim P(X_S)$  [Random Sampling & Sorting]
- Output :  $X_{\hat{T}}$  [Known Target Domain Input & Sorting]
- Structure

weight : MELU

activation function : ELU & Concave ELU



## [Result] 4 Surrogate Domain Estimation



# DA Experiment – Multi-Domain Adaptation

## MAML-SDE

### Meta-Learning Architecture

- Input :  $\{X_S, X_{T_{s1}}, X_{T_{s2}}, X_{T_{s3}}, X_{T_{s4}}\}$  [ $X_{T_s}$  : Surrogate Domain]
- Output :  $\{Y_S, Y_S, Y_S, Y_S, Y_S\}$
- Structure      iterative update Child & Meta-Model  
                         (Child-Model) initial weight : Meta-Model  
                         (Meta-Model)  $Val\_Error_{child} \rightarrow$  update

### Inference Architecture

- Input :  $X_{\hat{T}}$
- Output :  $Y_{\hat{T}}$
- Structure  
                         initial weight : Meta-Model

## DARC-SDE

### Latent Transformation Architecture

- Input :  $\{X_S, X_{T_{s1}}, X_{T_{s2}}, X_{T_{s3}}, X_{T_{s4}}, X_{\hat{T}}\} + \text{Domain Label}$
- Output :  $\{F_S, F_{T_{s1}}, F_{T_{s2}}, F_{T_{s3}}, F_{T_{s4}}, F_{\hat{T}}\}$  [ $F$ : Shared Feature]
- Structure  
                         Siamese Neural Network

### Inference Architecture

- Input :  $\{F_S, F_{T_{s1}}, F_{T_{s2}}, F_{T_{s3}}, F_{T_{s4}}, F_{\hat{T}}\}$
- Output :  $\{Y_S, Y_S, Y_S, Y_S, Y_S, Y_{\hat{T}}\}$
- Structure  
                         MLP Structure

# DA Experiment – Comparison Method

## DEFAULT SETTING

### Few-Shot Learning Architecture

- Input :  $X_{\hat{T}}$       - Output :  $Y_{\hat{T}}$
- Structure  
initial weight : MLP Training on  $\{X_S, Y_S\}$

### Only Target Domain Architecture

- Input :  $X_{\hat{T}}$       - Output :  $Y_{\hat{T}}$
- Structure  
MLP Structure

## DARC

### Latent Transformation Architecture

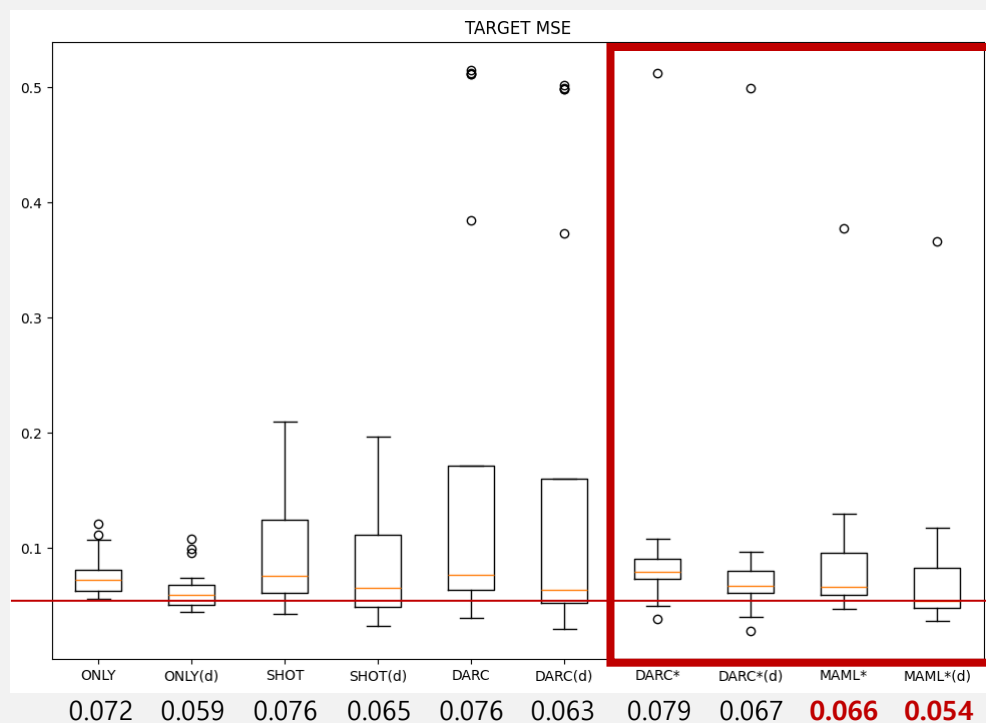
- Input :  $\{X_S, X_{\hat{T}}\}$  + **Domain Label**
- Output :  $\{F_S, F_{\hat{T}}\}$  [ $F$ : Shared Feature]
- Structure  
Siamese Neural Networks

### Inference Architecture

- Input :  $\{F_S, F_{\hat{T}}\}$
- Output :  $\{Y_S, Y_{\hat{T}}\}$
- Structure  
MLP Structure

# Result & Conclusion

**[Result]** 20 Experiments (4 dataset x 5)



ONLY : Only Target Domain, SHOT : Few-Shot Learning  
\* : applying SDE method, (d) : denoised MSE

**[Conclusion]**

## 1. MAML-SDE show high Performance

: Median Performance (MSE 0.066, MSE(d) 0.054)

## 2. DARC → DARC-SDE

: Lower Variance (Std(d) 0.186 → 0.098)

: Lower Median Performance (MSE(d) 0.063 → 0.067)

## 3. MAML/DARC is not suitable under Monotonic Causal Shift

[vs Only Target Domain Performance]

: Low Performance & Stable (DARC, DARC-SDE)

: High Performance & Unstable (MAML-SDE)

∴ DARC & MAML : Concept Shift(+Covariate Shift)

# Domain **A**daptation under monotonic causal shift via **C**ausal **D**istribution **T**ransformation (**DACDT**)

## [Keywords]

Monotonic Causal Shift, Restore Function, Domain Transformation, Regression for Tabular Dataset

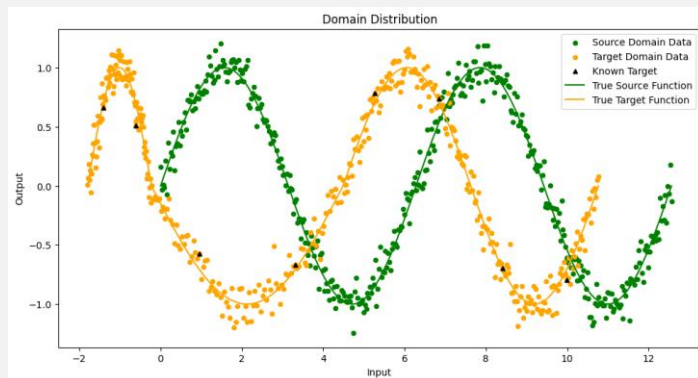
# INTRODUCTION

## [Index]

1. Monotonic Causal Shift Function
2. Restore Function

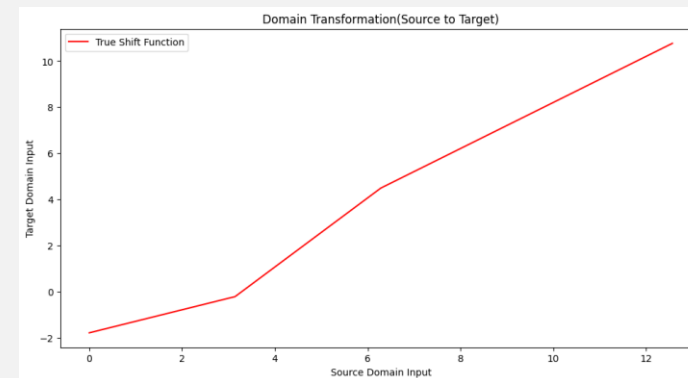
# Monotonic Causal Shift Function

## Data Perspective

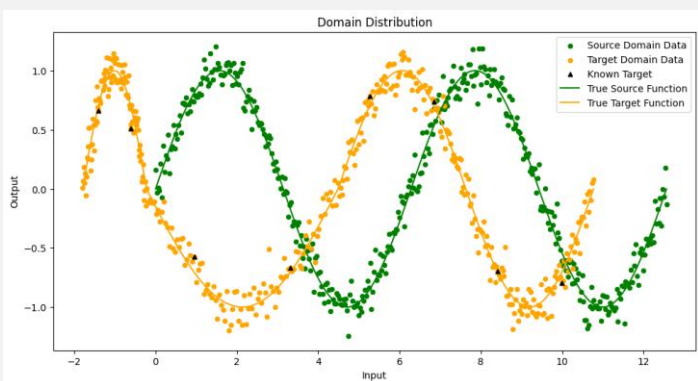


(a)

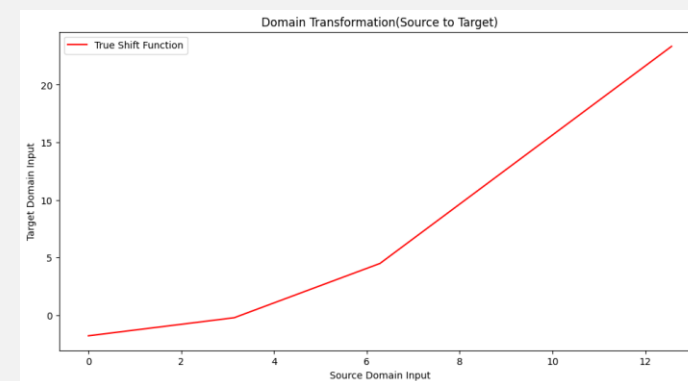
## Shift Function Perspective



(a)



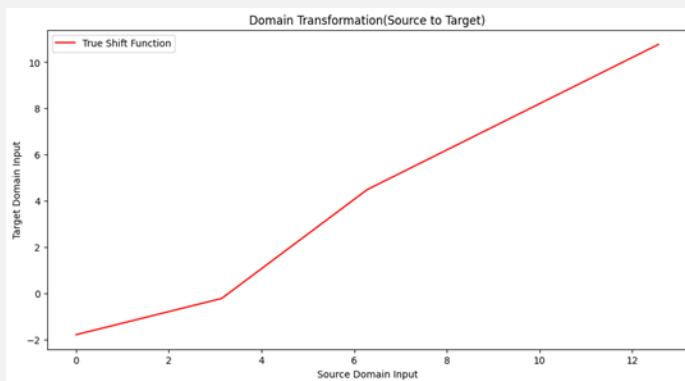
(c)



(c)

# Restore Function

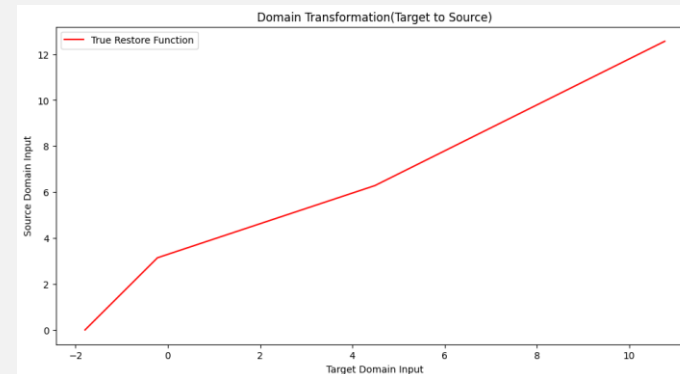
## Shift Function



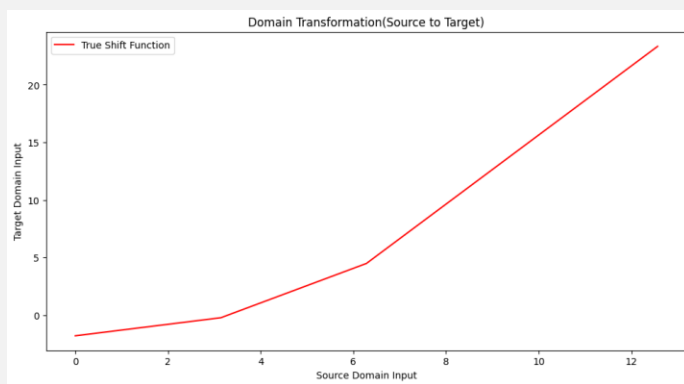
(a)

**Inverse Function!!**

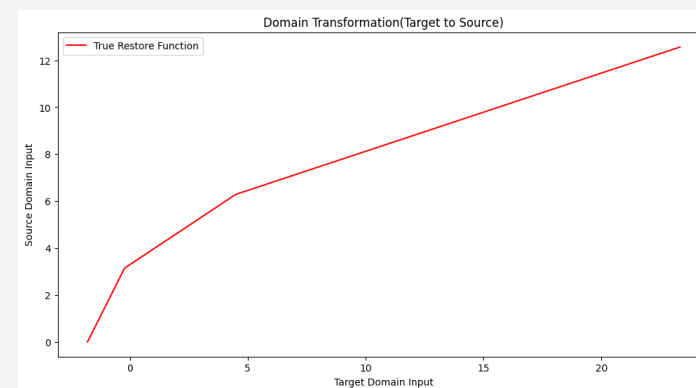
## Restore Function



(a)



(c)



(c)



# PROPOSED METHOD

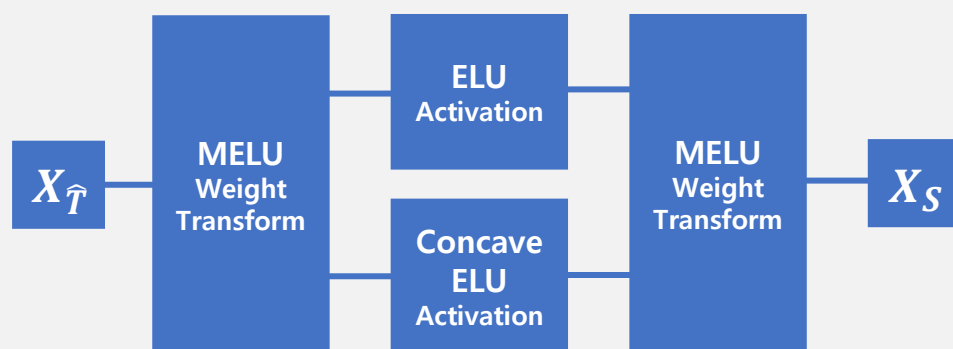
## [Index]

1. Full Architecture
2. Causal Distribution Transformation

# Full Architecture

## [Restore Model Architecture]

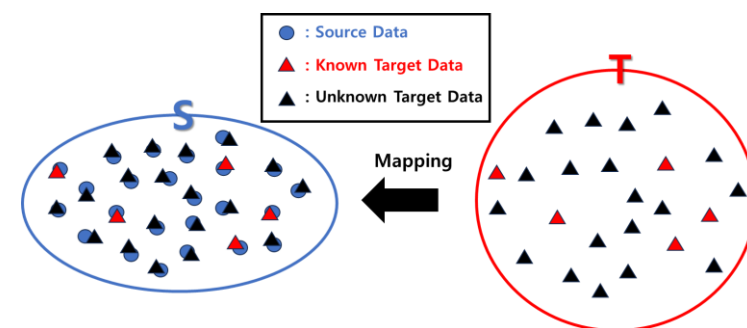
### “Monotonic Neural Network”



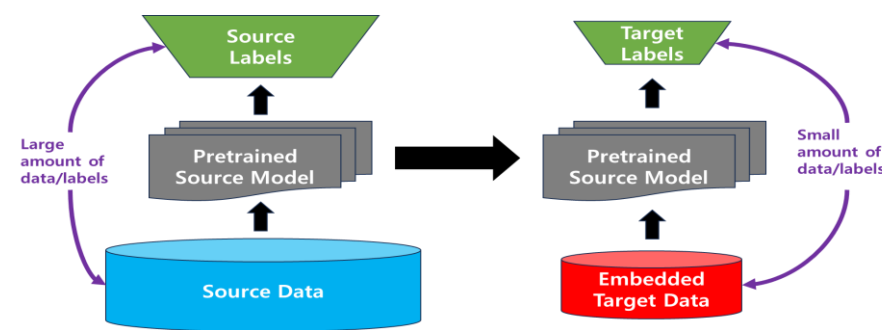
$X_{\hat{T}}$  : Known Target Domain Input  
 $X_S$  : Source Domain Input

## [Use Cases]

### ① Restore Function Estimation

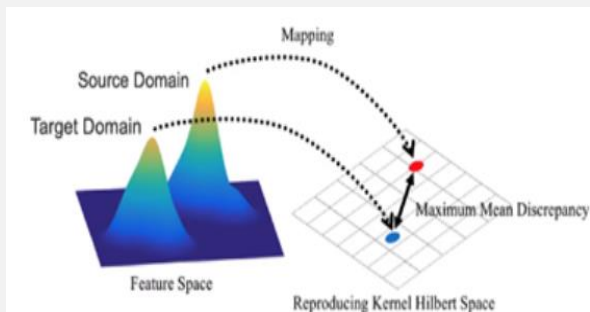


### ② Use Source Function to Inference



# Causal Distribution Transformation

## Maximum Mean Discrepancy (MMD)



“Marginal Distribution”

$$\|P(X_S) - P(\Phi^{-1}(X_{\hat{T}}))\|$$

$$L_{MMD} = \max_m \left( \left\| \frac{1}{n^S} \sum_{j=1}^{n^S} \varphi_m(x_i^S) - \frac{1}{n^{\hat{T}}} \sum_{j=1}^{n^{\hat{T}}} \varphi_m(x_j^{\hat{T}}) \right\|^2 \right)$$

$n^S$  : # of Source Domain Data,

$x^S$  : Source Domain Input

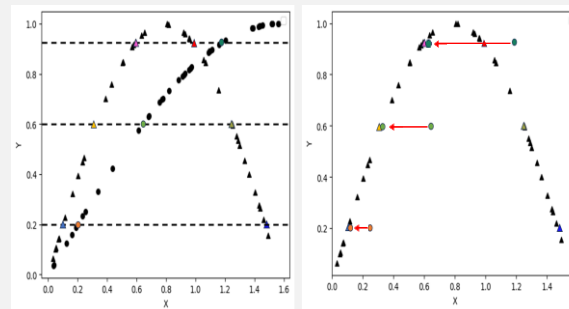
$n^{\hat{T}}$  : # of Known Target Domain Data,

$x^{\hat{T}}$  : Known Target Domain Input

$\varphi(\cdot)$  : RKHS embedding function,

$\Phi^{-1}(\cdot)$  : Restore Function

## Neighborhood Similarity (NS)



“Causal Distribution”

$$\|P(X_S|Y_S) - P(\Phi^{-1}(X_{\hat{T}})|Y_{\hat{T}})\|$$

$$L_{ns} = \frac{1}{n^{\hat{T}}} \sum_{i=1}^{n^{\hat{T}}} |x_j^S - \Phi^{-1}(x_i^{\hat{T}})|_2$$

$$(\hat{j} = \arg \min_j |x_{(j)}^S - \Phi^{-1}(x_i^{\hat{T}})|_2 + |y_{(j)}^S - y_i^{\hat{T}}|)$$

$$\{x_{(1)}^S, x_{(2)}^S, \dots, x_{(k)}^S \mid |y_{(1)}^S - y_i^{\hat{T}}| < \dots < |y_{(k)}^S - y_i^{\hat{T}}|\}$$

$n^S$  : # of Source Domain Data,

$\{x^S, y^S\}$  : Source Domain

$n^{\hat{T}}$  : # of Known Target Domain Data,

$\{x^S, y^S\}$  : Known Target Domain

$k$  : # of neighbor candidates,

$\Phi^{-1}(\cdot)$  : Restore Function

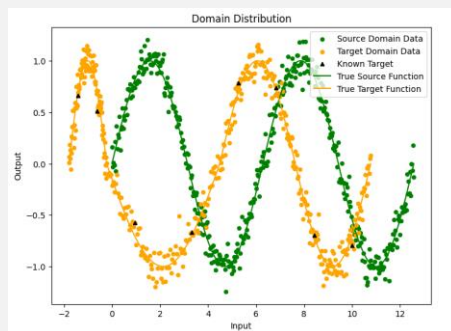
# EXPERIMENTS

## [Index]

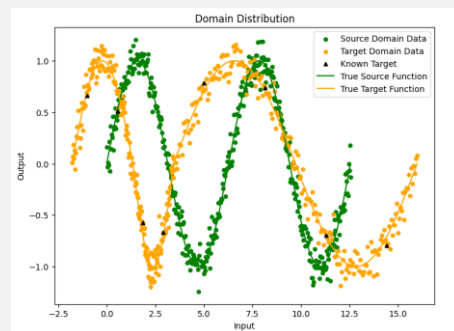
1. DA Experiment Setting
2. Result

# DA Experiment - Setting

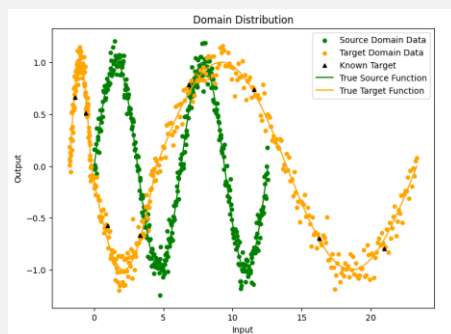
## [Dataset]



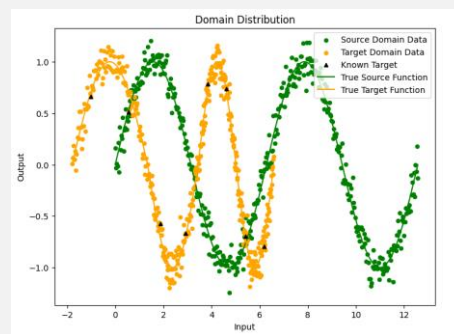
(a)



(c)



(b)



(d)

### <Source Dataset>

$$P(X_S) \sim \text{Unif}(0, 4\pi)$$

$$P(Y_S|X_S) \sim N(\sin(X_S), 0.1)$$

$$n_S = 400$$

### <Target Dataset>

$$P(X_T) = P(\Phi(X_T))$$

$$P(Y_T) = P(Y_T)$$

$$P(X_T|Y_T) = P(\Phi(X_S)|Y_S)$$

$$n_T = 400, \quad n_{\hat{T}} = 8$$

$n_S$  : # of Source Domain Data,  $\{X_S, Y_S\}$  : Source Domain

$n_T$  : # of Target Domain Data,  $\{X_T, Y_T\}$  : Target Domain

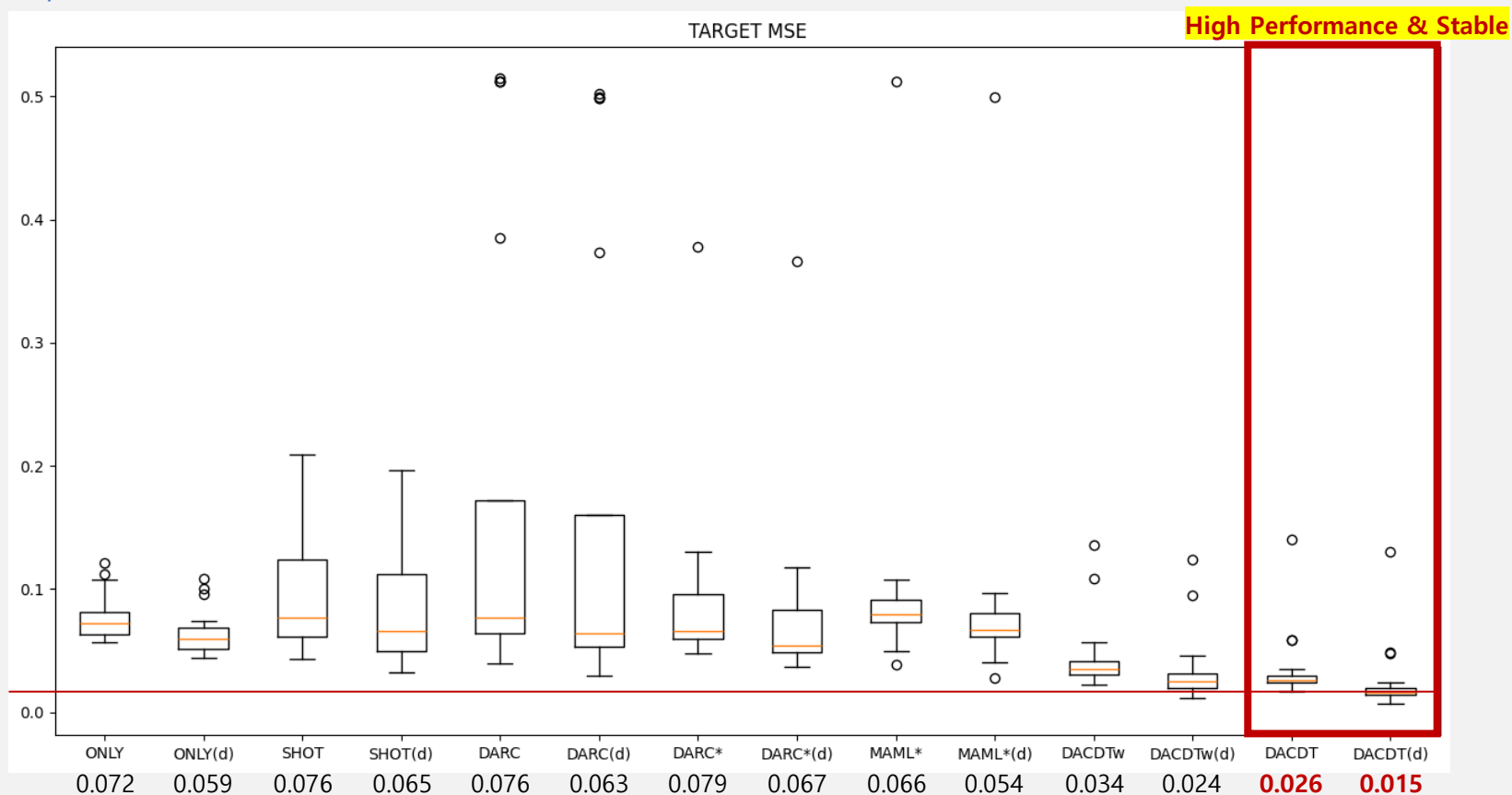
$n_{\hat{T}}$  : # of Known Target Domain Data,  $\Phi(\cdot)$  : Shift Function

# DA Experiment – DA Method

Proposed Method (DACDT)	Comparison Method
<b>Restore Model Architecture</b> <ul style="list-style-type: none"><li>- Input : <math>X_{\hat{T}}</math></li><li>- Output : <math>X_S</math></li><li>- Structure<ul style="list-style-type: none"><li>weight : MELU</li><li>activation function : ELU &amp; Concave ELU</li><li>Loss function : <math>L_{MMD} + L_{NS}</math></li></ul></li></ul>	<ul style="list-style-type: none"><li>- Only Target Domain</li><li>- Few-Shot Learning</li><li>- DARC</li><li>- DARC-SDE</li><li>- MAML-SDE</li><li>- DACDT with MLP Restore Model</li></ul>
<b>Inference Model Architecture</b> <ul style="list-style-type: none"><li>- Input : <math>X_S</math></li><li>- Output : <math>Y_S</math></li><li>- Structure<ul style="list-style-type: none"><li>MLP Structure</li></ul></li></ul>	

# Result

**[Result]** 20 Experiments (4 dataset x 5)



ONLY : Only Target Domain, SHOT : Few-Shot Learning, \* : applying SDE method, (d) : denoised MSE, DACDTw : DACDT with MLP Restore Model

# CONCLUSION



# Conclusion

## [Contribution]

- **Novel Problem Definition**  
: Monotonic Causal Shift
- **Tabular Data Augmentation**  
: Surrogate Domain Estimation
- **Efficient Monotonic Unit**  
: MELU
- **Achieve High Performance**  
: DACDT

## [Future Works]

- **More Complex Experiments**  
: n-dim Toy Data Experiments  
with non-shift and shift feature  
: Real Dataset Experiments
- **Meta-Restore Model**  
: SDE + MAML  $\rightarrow$  Estimate Initial Weight
- **Explainable Causal Shift Detection**  
: Identity Function Similarity

# Thank You

2024. 01. 11.

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**InfoSci**