Introduction to Statistical Concepts

Session 3

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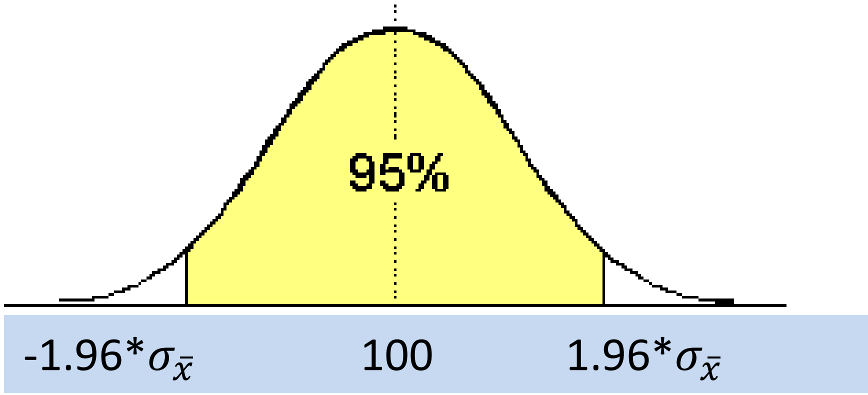
Basic Assumptions of the Models

- X $\sim N(\mu, \sigma^2)$
- X are i.i.d. (independent and identically distributed)

Inference When σ is Known – Z Test

- Sam, a 3rd grade boy, weights 100 pounds, where mean weight for 3rd grade boys is 90 and standard deviation is 15. How does Sam's weight compare to the weight of his peers?
  - Z=(100 – 90) / 15 = 0.67, p(z<0.67) = 0.747
  - 75% of 3rd grade boys weight less than Sam.
  - Sam would be overweight if he is within the top 5%.
  - α = 0.05, $z_{.95}$ = 1.64, upper limit =90+1.64*15 = 114.6 pounds
  - So, Sam is not over weight.
- 25 3rd grade boys are sampled randomly from the above population.
- $\bar{x}$ = 100 pounds.
- $\sigma_{\bar{x}} = \frac{15}{\sqrt{25}}$ = 3, z=(100 – 90) / 3 = 3.33, p(z<3.33) = 0.999.
- The sample mean of 100 (n=25) is significantly different from the population with μ=90, z=3.33, SEM = 3, p < .001.

Confidence Interval



Lower limit: 100 – 1.96 * 3 = 94.12
Upper limit = 100 + 1.96*3 = 105.88

In replications of the study, the interval [94, 105.88] will include the mean of 100 95% of the cases.

The sample mean of 100 is significantly different from the population mean of 90, since the confidence interval does not include 90.

Inference When σ is Unknown – One Sample T Test

- Degrees of Freedom

X	$\bar{x}$	X - $\bar{x}$
2	5	-3
4	5	-1
6	5	1
8	5	?

Df = N-1 = 4-1 = 3.

t is a sampling distribution and slightly leptokurtic

One Sample T Test

- Math scores with μ = 77.
- Random sample of N=30.
- $\bar{x}$ = 79.8 and $\sum_{i=1}^{N}(x - \bar{x})^2$ = 1562.8
- t = $\frac{\bar{x} - \mu}{SE_{\bar{x}}}$
- SD_x = $\sqrt{\frac{\sum_{i=1}^{N}(x-\bar{x})^2}{N-1}}$ = $\sqrt{\frac{1562.8}{29}}$ = 7.341
- $SE_{\bar{x}}$ = $\frac{SD_x}{\sqrt{N}}$ = $\frac{7.341}{\sqrt{30}}$ = 1.34
- $t = \frac{79.8 - 77}{1.34}$ = 2.09, df = N-1 = 39 -1 = 29
- P(t<2.09, 29) = 0.97 -> 0.023

H0: μ = 77
H1: μ ≠ 77 Two tailed test

- $t_{.025,29}$ = 2.045
- $t_{.05,29}$ = 1.67

H0: μ ≤ 77
H1: μ > 77 One tailed test

Confidence Interval

- CI : $\bar{x} \pm t_{\alpha/2,df}{}^{*}SE_{\bar{x}}$
- Lower limit: 79.8 – 2.045*1.34 = 77.06
- Upper limit: 79.8 + 2.045*1.34 = 82.54
- Sample mean is significantly different from the population mean of 77, t(29) = 2.09, p=0.023.

Two Sample T Test

Medication n=9

- $\bar{x}_M$ = 28.778
- SD_M= 10.45

Counselling n=9

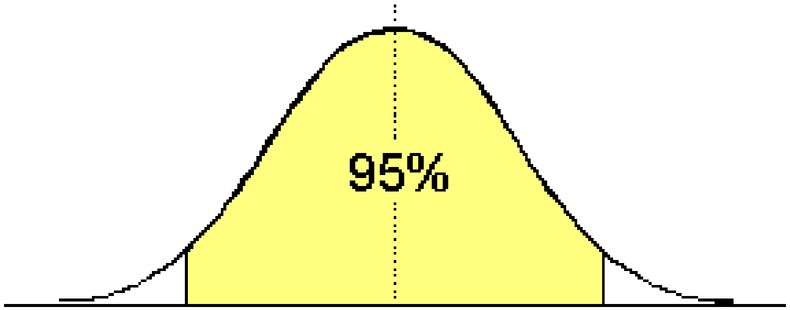
- $\bar{x}_C$ = 38.111
- SD_C= 5.302

H0: μ_C - μ_M = 0
H1: μ_C - μ_M ≠ 0

0 under H0

$$t = \frac{(\bar{x}_C - \bar{x}_M) - (\mu_C - \mu_M)}{SE_{(\bar{x}_C - \bar{x}_M)}}$$

$$t = \frac{(\bar{x}_C - \bar{x}_M)}{SE_{(\bar{x}_C - \bar{x}_M)}}$$



-2.045*$SE_{(\bar{x}_C - \bar{x}_M)}$ 0 2.045*$SE_{(\bar{x}_C - \bar{x}_M)}$

Two Sample T Test

Medication

- $SE_M = \frac{SD_M}{\sqrt{n_M}} = \frac{10.45}{3} = 3.483$

Counselling

- $SE_C = \frac{SD_C}{\sqrt{n_C}} = \frac{5.302}{3} = 1.767$

$$SE_{(\bar{x}_C - \bar{x}_M)} = \sqrt{\left(SE_C{}^2 + SE_M{}^2\right)} = \sqrt{3.483^2 + 1.767^2} = 3.91$$

$$SE_{(\bar{x}_C - \bar{x}_M)} = \sqrt{\frac{2*MSE}{n}} = \sqrt{\frac{2*(\frac{(SD_M{}^2 + SD_C{}^2)}{2}}{9}} = \sqrt{\frac{2*68.65}{9}} = 3.91$$

df = (n_C - 1) + (n_M - 1) = 8 + 8 = 16

Very Important Assumption - > Homogeneity of Variances

That is, $SE_C{}^2 \approx SE_M{}^2$

Two Sample T test

- t = $\frac{38.111 - 28.778}{3.91}$ = 2.39,   p(t(16) < 2.39) = 0.985
- p(t(16) > 2.39) = 0.015, which is smaller than 0.025
- The null hypothesis of no difference between the groups is rejected, t(16) = 2.39, p=0.015.
- CI: $(\bar{x}_C$ - $\bar{x}_M) \mp t_{\frac{\alpha}{2},df} SE_{(\bar{x}_C - \bar{x}_M)}$
- Lower limit: 9.33 – 2.12*3.91 = 1.125
- Upper Limit: 9.33 + 2.12*3.91 = 17.62
- CI does not include 0, so, H0 is rejected.

Multiple (Unplanned) Comparisons

Tukey HSD Test

Test Scores of Students in Courses with Different Modalities (n=15)

Traditional		**Hybrid**		**Online**	
Mean	Variance	Mean	Variance	Mean	Variance
89.93	24.60	88.57	59.60	84.10	43.30

H0: μ_T = μ_O
H1: μ_T ≠ μ_O

3 choose 2: 6 comparisons

MSE= $\frac{24.6+59.6+43.3}{3}$ = 42.5

$$Q = \frac{(\bar{x}_T - \bar{x}_O)}{\sqrt{\frac{MSE}{n}}} = \frac{(89.93 - 84.10)}{\sqrt{\frac{42.5}{15}}} = 3.35$$

df = N- r
r: # of groups
df = 45 – 3 = 42
P(Q(3, 42)=3.35) = 0.0573

T Test with Related Samples

Patient	Before	After	D	D^2
1	9	3	-6	36
2	4	1	-3	9
3	5	0	-5	25
4	4	3	-1	1
5	7	2	-5	25
Sum	29	9	-20	96

- t = $\frac{\bar{x}_1 - \bar{x}_2}{\sqrt{{\sigma_1}^2 + {\sigma_2}^2 - 2 * r * \sigma_1 * \sigma_2}}$
- D=$\bar{x}_1 - \bar{x}_2$
- t = $\frac{\bar{D}}{\sqrt{\frac{\sum_{i-1}^{N} D^2 - \frac{(\sum D)^2}{N}}{N * (N-1)}}}$

H0: $\mu_A = \mu_B$

H1: $\mu_A \neq \mu_B$

$$t = \frac{1.8 - 5.8}{\sqrt{\frac{96 - 400/5}{5 * 4}}}$$

t = -4.49

df = N -1 = 5-1 = 4.

P(t(4)=-4.49) = 0.005

Power

	True State	
Decision	H0	H1
H0	Confidence	Type II mistake β
H1	Type I mistake α	Power

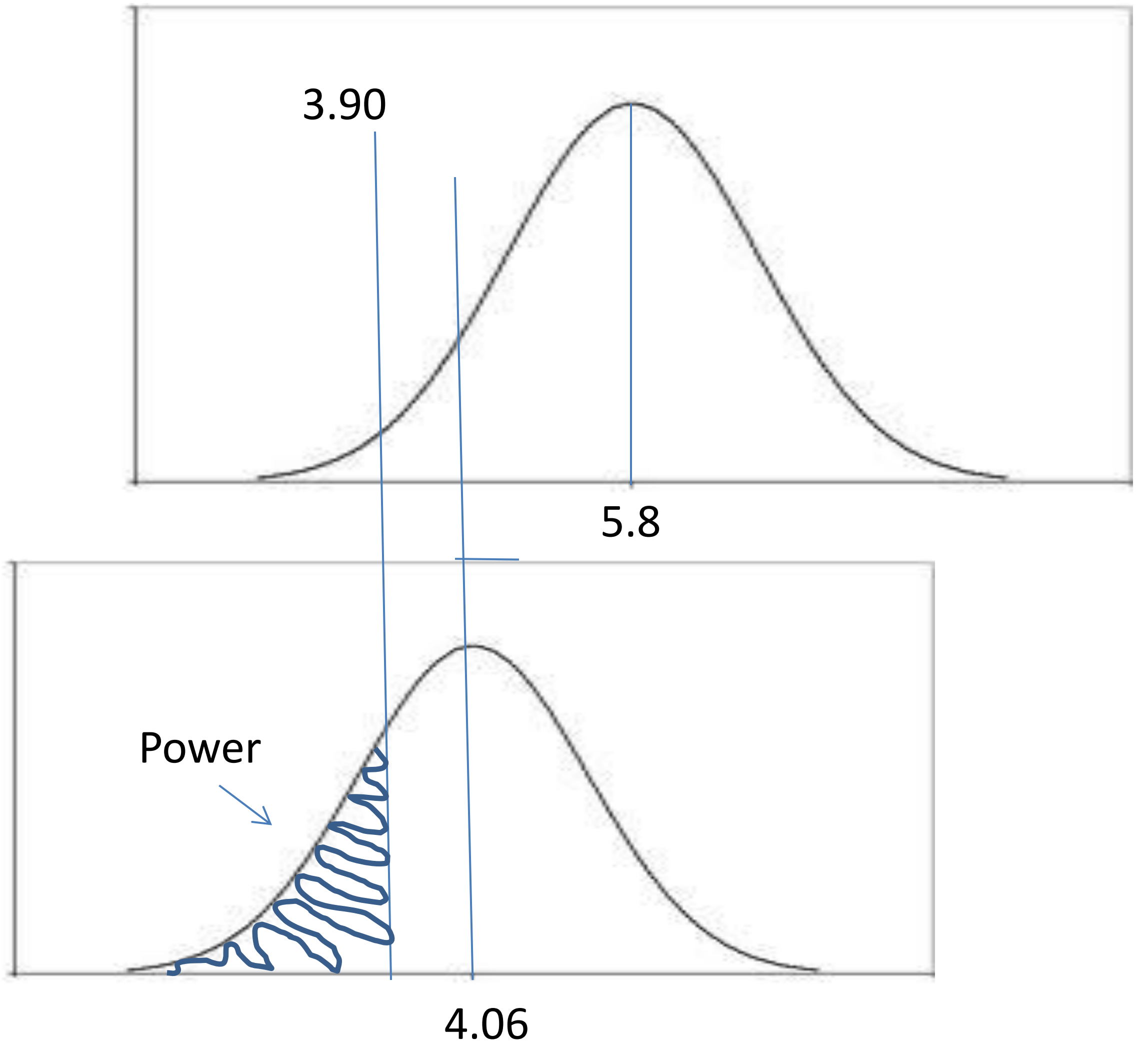
Power: Probability of rejecting the null hypothesis when it is false.
Power = 1- β

30% Decrease in Symptoms

Rejection region = 5.8 – 0.89 * 2.13 = 3.90

$t = \frac{3.90 - 4.06}{0.89} = -0.18$

P(t(4)<-0.18) = 0.43



Factors Affecting Power

- Sample size - large
- Effect size -large
- Standard deviation -small
- α - large
- One or two tailed test – one tailed
- Dependent or independent samples – dependent samples