

§9.1.

# 1. 一阶方程求解 (求解)

## Example 1

$$y = \frac{C}{x} + 2, 0 < x < +\infty, \forall C.$$

$$\frac{dy}{dx} = \frac{1}{x}(2 - y)$$

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## Example 2

$$y = (x + 1) - \frac{1}{3}e^x$$

$$\frac{dy}{dx} = y - x, y(0) = \frac{2}{3}$$

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# 2. 一阶线性方程求解

## ① 常数变易法

Example 3  $3xy' - y = \ln x + 1$

Note The requirement  $x > 0$  is superfluous.

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Example 2  $x \frac{dy}{dx} = x^2 + 3y, x > 0.$

Solution  $\frac{dy}{dx} - \frac{3}{x}y = x$

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## Class Work

### 1. Solve the equation

$$x \frac{dy}{dx} + 2y = x^2 + 1, x > 0$$

satisfying  $y(1) = 1.$

Answer:  $y = \frac{1}{4}x^4 + \frac{1}{4x^2} + \frac{1}{2}$

Step 1  $x \frac{dy}{dx} + 2y = 0 \Rightarrow y = Cx^{-2}$

Step 2

$y = u(x)x^{-2} \Rightarrow u(x) = \frac{1}{4}x^4 + \frac{1}{2}x^2 + C$

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### 2. Solve the equation

$$\frac{dy}{dx} + 3x^2y = x^2$$

satisfying  $y(0) = -1.$

Answer  $y = \frac{1}{3}(1 - 4e^{-x^3})$

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## Exercises 9.2

### First-Order Linear Equations

Solve the differential equations in Exercises 1-14.

1.  $\frac{dy}{dx} + y = e^x, x > 0$

2.  $e^x \frac{dy}{dx} + 2e^x y = 1$

5.  $\frac{dy}{dx} + 2y = 1 - \frac{1}{x}, x > 0$

6.  $(1 + x)y' + y = \sqrt{x}$

8.  $e^{2x}y' + 2e^{2x}y = 2x$

10.  $\frac{dy}{dx} = \frac{\cos x}{x} - 2y, x > 0$

3.  $xy' + 3y = \frac{\sin x}{x^2}, x > 0$

4.  $y' + (\tan xy) = \cos^2 x, -\pi/2 < x < \pi/2$

7.  $2y' = e^{x^2} + y$

9.  $xy' - y = 2x \ln x$

### 3. Solve the equation

$$x \frac{dy}{dx} + 2y = 1 - \frac{1}{x}, x > 0$$

Answer  $y = \frac{1}{2} - \frac{1}{x} + \frac{C}{x^2}$

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## ② 齐次方程

### □ 3. Homogeneous Equations

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### □ Chapter 9 Additional and Advanced Exercises: 5

$$(x^2 + y^2)dx + xydy = 0$$

$$\frac{dy}{dx} = -\frac{x^2 + y^2}{xy} = -\frac{1 + \left(\frac{y}{x}\right)^2}{\frac{y}{x}}, xy \neq 0$$