学号＿＿＿＿ 姓名＿＿＿＿ 年级＿＿＿＿ 专业＿＿＿＿ 学院＿＿＿＿

密 封 线

西南大学计算机与信息科学学院

《 高等数学 IB 》课程试题 【A】卷参考答案和评分标准

2020～2021 学年 第 2 学期								期末考试		
考试时间	120 分钟		考核方式	闭卷笔试		学生类别		本科	人数	
适用专业或科类		计信院计科、软件工程、自动化专业						年级	2020 级	
题号	一	二	三	四	五	六	七	八	九	合计
得分										
签名										

阅卷须知：阅卷用红色墨水笔书写，得分用阿拉伯数字写在每小题题号前，用正分表示，不得分则在题号前写 0；大题得分登录在对应的分数框内；统一命题的课程应集体阅卷，流水作业；阅卷后要进行复核，发现漏评、漏记或总分统计错误应及时更正；对评定分数或统分记录进行修改时，修改人必须签名。

特别提醒：学生必须遵守课程考核纪律，违规者将受到严肃处

PLEASE ANSWER IN CHINESE OR IN ENGLISH OR BILINGUALISM!!

1. Fill the correct answer in the blanks (3 points each，15 points in all)

(1) $y=1+ce^{-\frac{x^2}{2}}$ (2) $\dfrac{\sqrt{6}}{3}$

(3) 3 (4) $(-5\sin t)\vec{j}+(3\cos t)\vec{k}$ (5) $1<x<5$

2. Choose the corresponding letter of the best answer that best completes the statement or answers the question among A, B, C, and D, and fill in the blanks (3 points each，15 points in all).

(1)D (2) B (3)D (4)A (5)C

3. Find the solutions for following problems by computing (8 points each，40 points in all)

(1) Is the function $f(x,y)=\begin{cases}\dfrac{2-2\cos(x+y)}{x^2+2xy+y^2} & x+y\neq 0\\ 1 & x+y=0\end{cases}$ continuous at (0,0)?

命题教师： 教研室或系负责人： 主管院长： 年 月 日

[Solution]

$$\lim_{(x,y)\to(0,0)} \frac{2-2\cos(x+y)}{x^2+2xy+y^2}$$

$$=\lim_{(x,y)\to(0,0)} \frac{2-2\cos(x+y)}{(x+y)^2}$$ -----------6points

$$\overset{t=x+y}{=} \lim_{t\to 0} \frac{2-2\cos t}{t^2}$$

$$=\lim_{t\to 0} \frac{\sin t}{t}$$ -----------------2points

$$=1$$

(2) Assuming that the equation $x^2+2xy-z^2+2xz=3$ defines z as a differentiable function of x and y, then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z^2}{\partial y \partial x}$.

[Solution]

$$\frac{\partial z}{\partial x}=-\frac{2x+2y+2z}{2x-2z}$$ -----------------2points

$$=\frac{x+y+z}{z-x}$$

$$\frac{\partial z}{\partial y}=-\frac{2x}{2x-2z}$$ -----------------2points

$$=\frac{x}{z-x}$$

$$\frac{\partial z^2}{\partial y \partial x}=\frac{\left(1+\frac{\partial z}{\partial y}\right)(z-x)-(x+y+z)\frac{\partial z}{\partial y}}{(z-x)^2}$$

$$=\frac{z^2-2xz-x^2-xy}{(z-x)^3}$$

$$=\frac{3-xy}{(z-x)^3}$$

-----------------4points

(3) Calculate $\iint\limits_R (x^2+y^2)dA \qquad R:(x-1)^2+y^2\le 1$

[Solution]

$$\iint_R (x^2+y^2)dA$$

-----------------6points

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\cos\theta} r^2 r dr d\theta$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\cos^4\theta d\theta$$

-----------------2points

$$=\frac{3\pi}{2}$$

(4) A fluid's velocity field is $\vec{F}=x\vec{i}+z\vec{j}+y\vec{k}$. Find the flow along the helix $\vec{r}(t)=\cos t\vec{i}+\sin t\vec{j}+t\vec{k}, 0\le t\le\frac{\pi}{2}$.

[Solution]

$$Flow=\int_C \vec{F}\bullet d\vec{r}$$ -----------------2points

$$=\int_0^{\frac{\pi}{2}}(\cos t, t, \sin t)\bullet(-\sin t, \cos t, 1)dt$$

-----------------4points

$$=\int_0^{\frac{\pi}{2}}(-\sin t\cos t+t\cos t+\sin t)dt$$

$$=\frac{\pi}{2}-\frac{1}{2}$$ -----------------2points

(5) Integrate G(x, y, z) = xyz over the surface of the cube cut from the first octant by the planes x = 1, y = 1, and z = 1.

[Solution]

$$\iint_S xyz d\sigma=\iint_R xyz\frac{|\nabla F|}{|\nabla F\bullet\vec{p}|}dA$$

-----------------4points

$$=\iint_{R_{xy}} xydxdy+\iint_{R_{xz}} xzdxdz+\iint_{R_{yz}} yzdydz$$

$$=\int_0^1\int_0^1 xydxdy+\int_0^1\int_0^1 xzdxdz+\int_0^1\int_0^1 yzdydz$$

$$=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$$ -----------------4points

$$=\frac{3}{4}$$

4. Solve the following comprehensive problems (10 points each，30 points in all)

(1) Find the point on the sphere $x^2+y^2+z^2=27$ farthest from the point (1, -1, 1).

[Solution]

$$d=\sqrt{(x-1)^2+(y+1)^2+(z-1)^2}$$

$$f(x,y,z)=(x-1)^2+(y+1)^2+(z-1)^2$$

$$g(x,y,z)=x^2+y^2+z^2-27$$

$$\nabla f=2(x-1)+2(y+1)+2(z-1)$$

$$\nabla g=2x+2y+2z$$ -----------------2points

$$\begin{cases}\left(2(x-1),2(y+1),2(z-1)\right)=\lambda(2x,2y,2z)\\ x^2+y^2+z^2-27=0\end{cases}$$ -----------------4points

$$\begin{cases}x-1=\lambda x\\ y+1=\lambda y\\ z-1=\lambda z\\ x^2+y^2+z^2-27=0\end{cases}$$

$$\frac{1}{1-\lambda}=\pm 3$$

$$\begin{cases}x=\pm 3\\ y=\mp 3\\ z=\pm 3\end{cases}$$ -----------------2points

$$f(3,-3,3)=24$$

$$f(-3,3,-3)=36$$

So the farthest point is (-3,3,-3), the distance is 6. -----------------2points

(2) Find the Maclaurin series for the function $x\tan^{-1}x$, and find the sum of series $\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$.

[Solution]

$$\frac{d\tan^{-1}x}{dx} = \frac{1}{1+x^2}$$

$$= \sum_{n=0}^{\infty}(-x^2)^n$$ -----------------2points

$$= \sum_{n=0}^{\infty}(-1)^n x^{2n} \qquad |x|<1$$

$$\tan^{-1}x = \int_0^x \sum_{n=0}^{\infty}(-1)^n x^{2n}dx$$

-----------------4points

$$= \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1} \qquad |x|\le 1$$

$$x\tan^{-1}x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+2}}{2n+1} \qquad |x|\le 1$$ -----------------2points

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = \tan^{-1}1 = \frac{\pi}{4}$$ -----------------2points

(3) Find the net outward flux of the field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ across the boundary of the region $D: 1 \le x^2 + y^2 + z^2 \le 4$.

[Solution 1]

$$Flux = \oiint_S \vec{F}\bullet\vec{n}d\sigma$$

-----------------6points

$$= \iiint_D \nabla\bullet\vec{F}dV$$

$$= \iiint_D 3dV$$

-----------------4points

$$= 28\pi$$

[Solution 2]

$$
\begin{aligned}
Flux &= 2\iint_{S_1} \vec{F}\bullet\vec{n}d\sigma + 2\iint_{S_2} \vec{F}\bullet\vec{n}d\sigma \\
&= 2\iint\limits_{x^2+y^2\le 4} (x,y,z)\bullet\frac{(x,y,z)}{\sqrt{4-x^2-y^2}}dxdy + 2\iint\limits_{x^2+y^2\le 1} (x,y,z)\bullet\frac{-(x,y,z)}{\sqrt{1-x^2-y^2}}dxdy \\
&= 2\iint\limits_{x^2+y^2\le 4} \frac{4}{\sqrt{4-x^2-y^2}}dxdy - 2\iint\limits_{x^2+y^2\le 4} \frac{1}{\sqrt{1-x^2-y^2}}dxdy \\
&= 2\int_0^{2\pi}\int_0^2 \frac{4}{\sqrt{4-r^2}}rdrd\theta - 2\int_0^{2\pi}\int_0^1 \frac{1}{\sqrt{1-r^2}}rdrd\theta \\
&= 32\pi - 4\pi = 28\pi
\end{aligned}
$$