

西南大学计算机与信息科学学院

《高等数学 IB》课程试题【B】卷参考答案和评分标准

2020～2021 学 年 第 2 学 期								期末考试		
考试时间	120 分钟		考核方式	闭卷笔试		学生类别		本科	人数	
适用专业或科类			计信院计科、软件工程、自动化专业					年级	2020 级	
题号	一	二	三	四	五	六	七	八	九	合计
得分										
签名										

阅卷须知：阅卷用红色墨水笔书写，得分用阿拉伯数字写在每小题题号前，用正分表示，不得分则在题号前写 0；大题得分登录在对应的分数框内；统一命题的课程应集体阅卷，流水作业；阅卷后要进行复核，发现漏评、漏记或总分统计错误应及时更正；对评定分数或统分记录进行修改时，修改人必须签名。

特别提醒：学生必须遵守课程考核纪律，违规者将受到严肃处理

PLEASE ANSWER IN CHINESE OR IN ENGLISH OR BILINGUALISM!!

1. Fill the correct answer in the blanks (3 points each, 15 points in all)

(1) $y = \frac{3}{2} + ce^{-2x}$

(2) $3x + y + z = 5$

(3) $e^y + ye^x + \frac{2}{x}$

(4) $3\sqrt{3}$

(5) $\frac{4}{(1+x)(3-x)} = \frac{1}{1 - \left(\frac{1-x}{2}\right)^2}$

2. Choose the corresponding letter of the best answer that best completes the statement or answers the question among A, B, C, and D, and fill in the blanks (3 points each, 15 points in all).

(1)C (2)A (3)B (4)D (5)A

3. Find the solutions for following problems by computing (8 points each, 40 points in all)

(1) Show that the limit $\lim_{(x,y) \rightarrow (1,1)} \frac{xy^2 - 1}{y - 1}$ do not exists.

命题教师：

教研室或系负责人：

主管院长：

年 月 日

[Solution]

$$\text{Let } x=1, \quad \lim_{(x,y) \rightarrow (1,1)} \frac{xy^2-1}{y-1} = \lim_{y \rightarrow 1} \frac{y^2-1}{y-1} = 2 \quad \text{-----3points}$$

$$\text{Let } x=y^2, \quad \lim_{(x,y) \rightarrow (1,1)} \frac{xy^2-1}{y-1} = \lim_{y \rightarrow 1} \frac{y^4-1}{y-1} = 4 \quad \text{-----3points}$$

So the limit is not exist. -----2points

(2) Find the total differential of the function $f(x, y, z) = x^2 \sin y + y^z + e^{xz}$.

[Solution]

$$\begin{aligned} df(x, y, z) &= d(x^2 \sin y + y^z + e^{xz}) \\ &= d(x^2 \sin y) + d(y^z) + d(e^{xz}) \\ &= 2x \sin y dx + x^2 \cos y dy + zy^{z-1} dy + y^z \ln y dz + ze^{xz} dx + xe^{xz} dz \\ &= (2x \sin y + ze^{xz}) dx + (x^2 \cos y + zy^{z-1}) dy + (y^z \ln y + xe^{xz}) dz \end{aligned}$$

-----8points

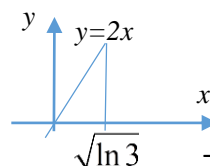
(3) Sketch the region of integration $\int_0^{2\sqrt{\ln 3}} \int_{\frac{y}{2}}^{\sqrt{\ln 3}} e^{x^2} dx dy$, reverse the order of integration, and

evaluate the integral.

[Solution]

$$\int_0^{2\sqrt{\ln 3}} \int_{\frac{y}{2}}^{\sqrt{\ln 3}} e^{x^2} dx dy$$

-----4points



-----2points

$$= \int_0^{\sqrt{\ln 3}} \int_0^{2x} e^{x^2} dy dx$$

$$= \int_0^{\sqrt{\ln 3}} 2xe^{x^2} dx$$

$$= e^{\ln 3} - 1$$

-----2points

$$= 2$$

(4) Calculate $\int_C (2xy + z) ds$, where $C: \vec{r}(t) = t\vec{i} + t\vec{j} + t\vec{k}, 1 \leq t \leq 3$.

[Solution]

$$\int_C (2xy + z) ds$$

$$= \int_1^3 (2t^2 + t)\sqrt{3} dt$$

-----4points

$$= \sqrt{3} \left(\frac{16}{3} + 2 \right) \quad \text{-----4points}$$

$$= \frac{22}{3} \sqrt{3}$$

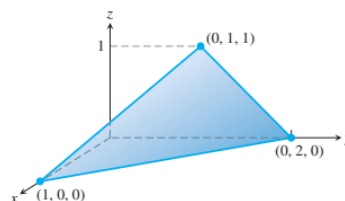
(5) Evaluate $\iint_S z d\sigma$ over the triangular surface with vertices $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 1, 1)$.

[Solution]

The surface equation is $2x+y+z-2=0$ -----2points

$$\iint_S y d\sigma$$

$$= \iint_{R_{xz}} (2-2x-z) \frac{|\nabla(2x+y+z-2)|}{|\nabla(2x+y+z-2) \cdot \vec{j}|} dx dz$$



-----4points

$$= \int_0^1 \int_0^{1-x} (2-2x-z) \sqrt{6} dz dx$$

$$= \sqrt{6} \int_0^1 \frac{3}{2} (1-x)^2 dx \quad \text{-----2points}$$

$$= \frac{\sqrt{6}}{2}$$

4. Solve the following comprehensive problems (10 points each, 30 points in all)

(1) You are to construct an open rectangular box from 12 m^2 of material. What dimensions will result in a box of maximum volume?

[Solution 1]

$$V = xyz, \quad xy + 2xz + 2yz = 12$$

$$V = xyz = xy \left(\frac{12-xy}{2(x+y)} \right) = \frac{12xy - x^2 y^2}{2(x+y)} \quad \text{-----2points}$$

$$\frac{\partial V}{\partial x} = \frac{(12y - 2xy^2)(x+y) - (12xy - x^2 y^2)}{2(x+y)^2} = 0$$

$$\frac{\partial V}{\partial y} = \frac{(12x - 2x^2 y)(x+y) - (12xy - x^2 y^2)}{2(x+y)^2} = 0 \quad \text{-----4points}$$

$$\begin{cases} x^2 + 2xy = 12 \\ y^2 + 2xy = 12 \\ xy + 2xz + 2yz = 12 \end{cases} \quad \text{-----4points}$$

$$\Rightarrow x = y = 2, z = 1$$

[Solution 2]

$$\nabla V = (yz, xz, xy) \quad \nabla f = (y + 2z, x + 2z, 2x + 2y)$$

$$\begin{cases} \nabla V = (yz, xz, xy) = \lambda(y + 2z, x + 2z, 2x + 2y) \\ xy + 2xz + 2yz = 12 \end{cases}$$

$$\begin{cases} yz = \lambda(y + 2z) \\ xz = \lambda(x + 2z) \\ xy = \lambda(2x + 2y) \\ xy + 2xz + 2yz = 12 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 2 \\ z = 1 \end{cases}$$

(2) Find the sum for $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{2n+1}}{2n+1}, 0 < x < 2$

[Solution]

$$\text{Let } f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{2n+1}}{2n+1}$$

$$\begin{aligned} \frac{df(x)}{dx} &= \sum_{n=0}^{\infty} (-1)^n (x-1)^{2n} \\ &= \sum_{n=0}^{\infty} [-(x-1)^2]^n \end{aligned} \quad \text{-----6points}$$

$$= \frac{1}{1 + (x-1)^2} \quad |x-1| < 1$$

$$\begin{aligned} &\int \frac{1}{1 + (x-1)^2} dx \\ &= \tan^{-1}(x-1) + C \end{aligned} \quad \text{-----2points}$$

$$f(1) = 0, C=0, f(x) = \tan^{-1}(x-1) \quad \text{-----2points}$$

- (3) Find the counterclockwise circulation and the outward flux of the field $\vec{F} = xy\vec{i} + y^2\vec{j}$ around and over the boundary of the region enclosed by the curves $y = x^2$, $y = x$ in the first quadrant.

[Solution 1]

$$\begin{aligned} \text{circulation} &= \oint_C \vec{F} \cdot \vec{T} ds \\ &= \oint_C \vec{F} \cdot d\vec{r} \\ &= \int_0^1 (x^3, x^4) \cdot (1, 2x) dx + \int_1^0 (x^2, x^2) \cdot (1, 1) dx \quad \text{-----5points} \\ &= -\frac{1}{12} \end{aligned}$$

$$\begin{aligned} \text{Flux} &= \oint_C \vec{F} \cdot \vec{n} ds \\ &= \oint_C xy dy - y^2 dx \\ &= \int_0^1 x^3 dx^2 - x^4 dx + \int_1^0 x^2 dx - x^2 dx \quad \text{-----5points} \\ &= \frac{1}{5} \end{aligned}$$

[Solution 2]

$$\begin{aligned} \text{circulation} &= \oint_C \vec{F} \cdot \vec{T} ds \\ &= \oint_C \vec{F} \cdot d\vec{r} = \oint_C xy dx + y^2 dy \\ &= \iint_R -x dx dy \\ &= -\frac{1}{12} \end{aligned}$$

$$\begin{aligned} \text{Flux} &= \oint_C \vec{F} \cdot \vec{n} ds \\ &= \oint_C xy dy - y^2 dx \\ &= \iint_R 3y dx dy \\ &= \frac{1}{5} \end{aligned}$$