13讲

1、画向量函数图像、求向量函数极限

Example 1 Graph the vector function

$$r(t)=(\cos t,\sin t,t)=(\cos t)\mathbf{i}+(\sin t)\mathbf{j}+t\mathbf{k}$$

$$t\in(-\infty,+\infty)$$

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EXAMPLE 2 If $\mathbf{r}(t)=(\cos t)\mathbf{i}+(\sin t)\mathbf{j}+t\mathbf{k}$, then

$$\lim_{t\to\pi/4}\mathbf{r}(t)=\left(\lim_{t\to\pi/4}\cos t\right)\mathbf{i}+\left(\lim_{t\to\pi/4}\sin t\right)\mathbf{j}+\left(\lim_{t\to\pi/4}t\right)\mathbf{k}$$
$$=\frac{\sqrt{2}}{2}\mathbf{i}+\frac{\sqrt{2}}{2}\mathbf{j}+\frac{\pi}{4}\mathbf{k}.$$

2、求向量函数的速度、加速度

Exercises 13.1

Motion in the Plane

In Exercises 1–4, $\mathbf{r}(t)$ is the position of a particle in the xy-plane at time t. Find an equation in x and y whose graph is the path of the particle. Then find the particle's velocity and acceleration vectors at the given value of t.

1. $\mathbf{r}(t)=(t+1)\mathbf{i}+(t^2-1)\mathbf{j},\quad t=1$
2. $\mathbf{r}(t)=\frac{t}{t+1}\mathbf{i}+\frac{1}{t}\mathbf{j},\quad t=-\frac{1}{2}$
3. $\mathbf{r}(t)=e^t\mathbf{i}+\frac{2}{9}e^{2t}\mathbf{j},\quad t=\ln 3$
4. $\mathbf{r}(t)=(\cos 2t)\mathbf{i}+(3\sin 2t)\mathbf{j},\quad t=0$

Motion in Space P23

In Exercises 9–14, $\mathbf{r}(t)$ is the position of a particle in space at time t. Find the particle's velocity and acceleration vectors. Then find the particle's speed and direction of motion at the given value of t. Write the particle's velocity at that time as the product of its speed and direction.

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9. $\mathbf{r}(t)=(t+1)\mathbf{i}+(t^2-1)\mathbf{j}+2t\mathbf{k},\quad t=1$
10. $\mathbf{r}(t)=(1+t)\mathbf{i}+\frac{t^2}{\sqrt{2}}\mathbf{j}+\frac{t^3}{3}\mathbf{k},\quad t=1$
11. $\mathbf{r}(t)=(2\cos t)\mathbf{i}+(3\sin t)\mathbf{j}+4t\mathbf{k},\quad t=\pi/2$
12. $\mathbf{r}(t)=(\sec t)\mathbf{i}+(\tan t)\mathbf{j}+\frac{4}{3}t\mathbf{k},\quad t=\pi/6$
13. $\mathbf{r}(t)=(2\ln(t+1))\mathbf{i}+t^2\mathbf{j}+\frac{t^2}{2}\mathbf{k},\quad t=1$

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14. $\mathbf{r}(t)=(e^{-t})\mathbf{i}+(2\cos 3t)\mathbf{j}+(2\sin 3t)\mathbf{k},\quad t=0$

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Example 4 Find the velocity, speed, and acceleration of a particle whose motion in space is given by the position vector $\boldsymbol{r}(t)=2\cos t\,\mathbf{i}+2\sin t\,\mathbf{j}+5\cos^2 t\,\mathbf{k}$. Sketch the velocity vector $\boldsymbol{v}(7\pi/4)$.

3、求弧长

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EXAMPLE 1 A glider is soaring upward along the helix $\mathbf{r}(t)=(\cos t)\mathbf{i}+(\sin t)\mathbf{j}+t\mathbf{k}$. How long is the glider's path from $t=0$ to $t=2\pi$?

Example 2 Find the arc length parameter along the helix $\boldsymbol{r}(t)=(\cos t)\,\mathbf{i}+(\sin t)\,\mathbf{j}+t\,\mathbf{k}$. Set $t_0=0$. P61

4、求曲率

Example 2 Find the curvature of a circle $\boldsymbol{r}(t) = (a\cos t)\,\mathbf{i} + (a\sin t)\,\mathbf{j}$.

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5、求切向量与法向量

Example 3 Find $\boldsymbol{T}$ and $\boldsymbol{N}$ for the circular motion $\boldsymbol{r}(t) = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j}$.

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Exercises 13.3

Finding Tangent Vectors and Lengths

In Exercises 1–8, find the curve's unit tangent vector. Also, find the length of the indicated portion of the curve.

1. $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{j} + \sqrt{5}t\mathbf{k}, \quad 0 \le t \le \pi$
2. $\mathbf{r}(t) = (6\sin 2t)\mathbf{i} + (6\cos 2t)\mathbf{j} + 5t\mathbf{k}, \quad 0 \le t \le \pi$
3. $\mathbf{r}(t) = t\mathbf{i} + (2/3)t^{3/2}\mathbf{k}, \quad 0 \le t \le 8$
4. $\mathbf{r}(t) = (2+t)\mathbf{i} - (t+1)\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 3$
5. $\mathbf{r}(t) = (\cos^3 t)\mathbf{j} + (\sin^3 t)\mathbf{k}, \quad 0 \le t \le \pi/2$