西南大学计算机与信息科学学院

《 高等数学 IB 》课程试题 【A】卷

2020～2021 学 年 第 2 学 期						期末考试			
考试时间	120 分钟	考核方式	闭卷笔试	学生类别		本科	人数		
适用专业或科类		计信院计科、软件工程、自动化专业				年级	2020 级		
题号	一	二	三	四	五	六	七	八	九	合计
得分										
签名										

阅卷须知：阅卷用红色墨水笔书写，得分用阿拉伯数字写在每小题题号前，用正分表示，不得分则在题号前写0；大题得分登录在对应的分数框内；统一命题的课程应集体阅卷，流水作业；阅卷后要进行复核，发现漏评、漏记或总分统计错误应及时更正；对评定分数或统分记录进行修改时，修改人必须签名。

特别提醒：学生必须遵守课程考核纪律，违规者将受到严肃处

PLEASE ANSWER IN CHINESE OR IN ENGLISH OR IN BILINGUALISM!!

1. Fill the correct answer in the blanks (3 points each，15 points in all)

(1) The solution to the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} + xy = x$ is y=_____..

(2) The distance from the point (0,0,0) to the line $x = 1+t, y = 2+t, z = -1-t$ is _____.

(3) The derivative of the function $f(x, y, z) = xy + yz + zx$ at the point (1,-1,2) in the direction of $\vec{u} = 3\vec{i} + 6\vec{j} - 2\vec{k}$ is _____.

(4) The velocity vectors of the motion $\vec{r}(t) = \vec{i} + (5\cos t)\vec{j} + (3\sin t)\vec{k}$ is_____.

(5) The convergence set of the series $\displaystyle\sum_{n=0}^{\infty}\left(\dfrac{3-x}{2}\right)^{n}$ is_____.

2. Choose the corresponding letter of the best answer that best completes the statement or answers the question among A, B, C, and D, and fill in the blanks (3 points each，15 points in all).

(1) Which of the series conditionally converges (　　　)

(A) $\sum_{n=1}^{\infty}\left(\frac{(-1)^n}{n}-\frac{1}{n+1}\right)$

(B) $\sum_{n=1}^{\infty}\left(\frac{\ln(n)}{n}\right)$

(C) $\sum_{n=1}^{\infty}\left((-1)^n\frac{n+2}{n^2\sqrt{n}}\right)$

(D) $\sum_{n=1}^{\infty}\left(\frac{\cos(n\pi)}{n}\right)$

(2) Which of the following is the equation of the plane through the points (1, 1, -1), (2, 0, 2), and (0, -2, 1)? (　　　)

(A) $7x-5y-4z=0$

(B) $7x-5y-4z=6$

(C) $\frac{x}{7}=\frac{y}{-5}=\frac{z}{-4}$

(D) $\frac{x}{7}=\frac{y+2}{-5}=\frac{z-1}{-4}$

(3) For the function $f(x,y)=\begin{cases}\dfrac{y^2}{(x-1)^2+y^2} & (x,y)\neq(1,0)\\ 0 & (x,y)=(1,0)\end{cases}$, which of the following

conclusions is right? (　　　)

(A) The function is continuous and has partial derivatives at the point (1, 0).

(B) The function is continuous but has no partial derivatives at the point (1, 0).

(C) The function is not continuous but the partial derivatives exist at the point (1, 0).

(D) The function is not continuous and the partial derivatives do not exist at the point (1, 0).

(4) The plane $x-y+2z=1$ is tangent to some surface at the point (1,2,1). Which of the following

surfaces is right? (　　　)

(A) $z=\sqrt{y-x}$

(B) $x^2-y+2z=1$

(C) $z=\sqrt{2y-3x}$

(D) $x^2-y+2z^2=1$

(5) Which of the following is an equivalent double integral to $\int_0^3\int_1^{e^y}(x+y)dxdy$ with the order of

integration reversed? (　　　)

(A) $\int_1^{e^y}\int_0^3(x+y)dydx$

(B) $\int_1^{e^3}\int_0^{\ln x}(x+y)dydx$

(C) $\int_1^{e^3}\int_{\ln x}^3(x+y)dydx$

(D) $\int_1^{e^3}\int_0^{e^x}(x+y)dydx$

3. **Find the solutions for following problems by computing (8 points each, 40 points in all)**

(1) Is the function $f(x, y) = \begin{cases} \dfrac{2 - 2\cos(x + y)}{x^2 + 2xy + y^2} & x + y \neq 0 \\ 1 & x + y = 0 \end{cases}$ continuous at (0,0)?

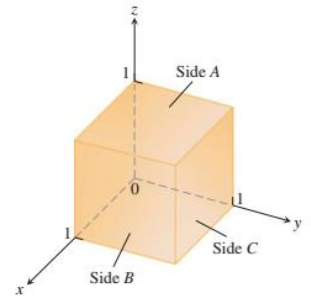(2) Assuming that the equation $x^2 + 2xy - z^2 + 2xz = 3$ defines z as a differentiable function of

x and y, then find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z^2}{\partial y \partial x}$.

(3) Calculate $\iint\limits_{R} (x^2 + y^2)dA$ $\quad R : (x-1)^2 + y^2 \leq 1$.

(4) A fluid's velocity field is $\vec{F} = x\vec{i} + z\vec{j} + y\vec{k}$. Find the flow along the helix $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + t\vec{k}, 0 \leq t \leq \frac{\pi}{2}$.

(5) Calculate $\iint_S xyz d\sigma$, S: The surface of the cube is cut from the first octant by the planes $x = 1$, $y = 1$, and $z = 1$.

4. Solve the following comprehensive problems (10 points each，30 points in all)

（1）Find the point on the sphere $x^2 + y^2 + z^2 = 27$ farthest from the point (1, -1, 1).

（2）Find the Maclaurin series for the function $x\tan^{-1} x$, and find the sum of series $\sum\limits_{n=0}^{\infty} \dfrac{(-1)^n}{2n+1}$.

(3) Find the net outward flux of the field $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ across the boundary of the region

$D : 1 \le x^2 + y^2 + z^2 \le 4$.