Chapter 13

Metallic Conductor and Dielectric in Electrostatic Field

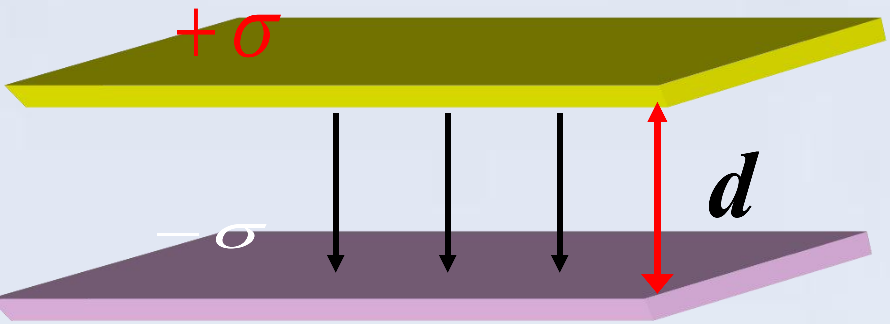
Main Contents

- ➢Conditions and Properties of a Conductor in Electrostatic Equilibrium. ▲
- ➢Properties of a Dielectric in Electrostatic Equilibrium
- ➢Gauss's Law in Dielectric ▲
- ➢Calculation of Capacitor ▲

Important Contents

Properties of a Dielectric in Electrostatic Equilibrium

Effect of Dielectric to Voltage and Electric Field



- **Experiment shows that, when the electric quantity of a parallel plate capacitor is given, if a kind of isotropic(各向同性) uniform dielectric is filled between two plates, then the voltage between two plates decreases to $1/\varepsilon_r$ times of the voltage U_0 without dielectric:**

$$U = \frac{U_0}{\varepsilon_r} \qquad \varepsilon_r \geq 1$$

➢Properties of a Dielectric in Electrostatic Equilibrium

- where ε_r is a non-dimensional(无量纲) constant and is called the relative dielectric constant or **the relative permittivity**.
- The relative dielectric constant of different dielectric is different.
- There is $\varepsilon_r = 1$ in the vacuum and $\varepsilon_r \approx 1$ in air as dielectric.
- The capacity of capacitor filled by dielectric increases.

$$C = \frac{Q}{U} = \frac{Q}{U_0}\varepsilon_r = \varepsilon_r C_0 \qquad \varepsilon_r = \frac{C}{C_0}$$

Properties of a Dielectric in Electrostatic Equilibrium

- The **absolute dielectric constant** or the absolute permittivity（shortly the dielectric constant or the permittivity）

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

- **Electric polarizability** χ_e

$$1 + \chi_e = \varepsilon_r$$

$$\boxed{\vec{P} = \varepsilon_0 \chi_e \vec{E}}$$

Properties of a Dielectric in Electrostatic Equilibrium

$$U = \frac{U_0}{\varepsilon_r} \quad \Rightarrow \quad E = \frac{E_0}{\varepsilon_r}$$

- **The field intensity between two plates decreases to $\frac{1}{\varepsilon_r}$ times of the intensity E_0 in the vacuum.**

§ 2. Effects of Dielectric in Electrostatic Field

(1) The called dielectric is insulating（绝缘） medium.

resistivity $\rho = 10^{8} \sim 10^{18}\,\Omega \cdot \mathrm{m}$

In practice, most dielectric materials are solid. Examples include ceramic, mica(云母), glass, plastics, and the oxides（氧化物） of various metals.

Some liquids and gases can serve as good dielectric materials. Dry air is an excellent dielectric, and is used in variable capacitors(电容器). Distilled(蒸馏) water is a fair dielectric.

A vacuum is an exceptionally(特殊的) efficient dielectric.

Properties of a Dielectric in Electrostatic Equilibrium

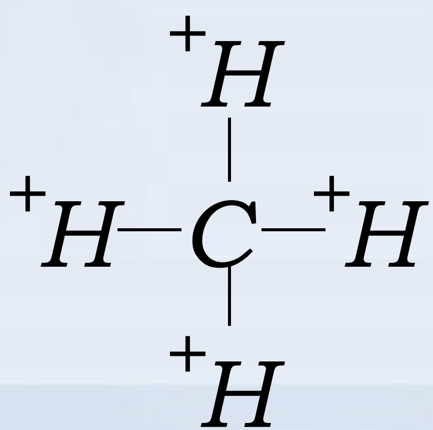
the Micro Mechanism of Dielectric Polarization (电介质极化)

① classification

<u>**Polar molecule:**</u> **the geometric(几何) center of positive charge and negative charge doesn't coincide(一致).**

<u>**Non-polar molecule:**</u> **the geometric center of positive charge and negative charge in non-polar molecule coincides.**

CH_4

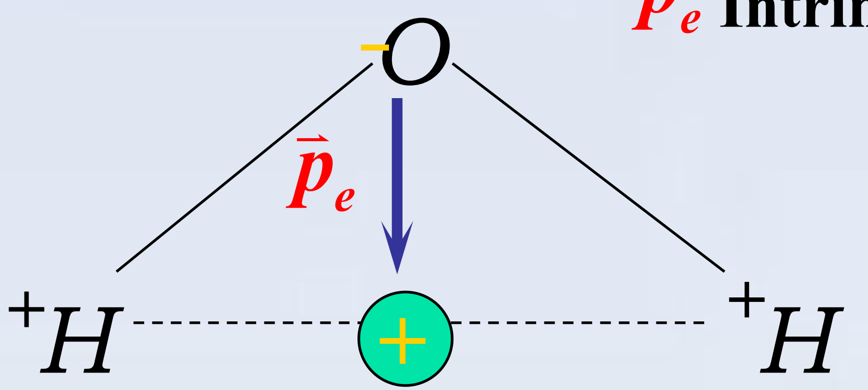


$$\vec{p}_e = 0$$

Properties of a Dielectric in Electrostatic Equilibrium

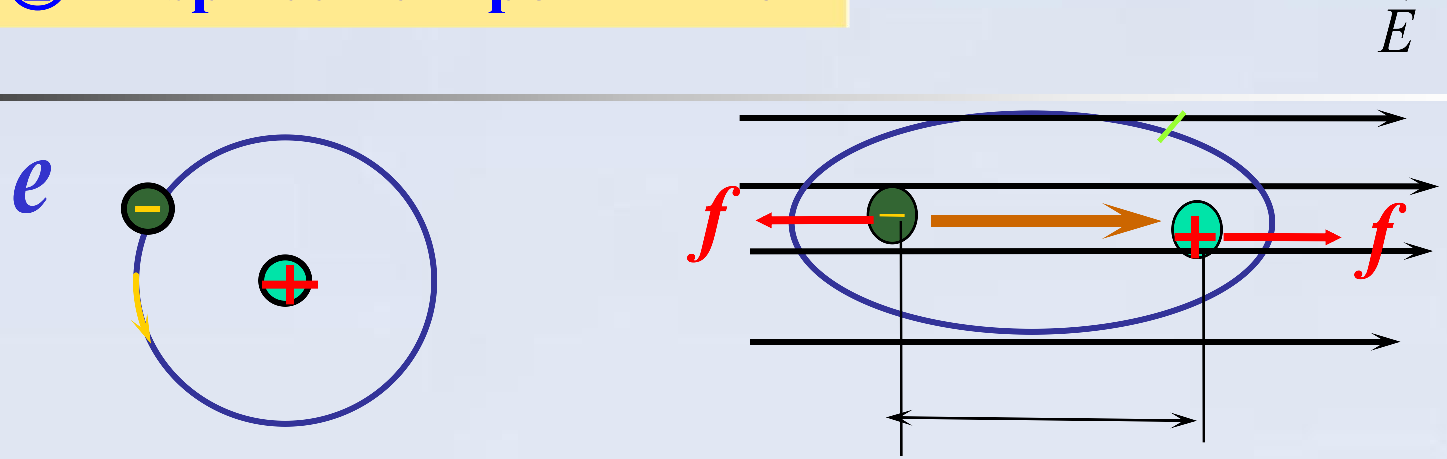
H_2O

$\vec{p}_e$ Intrinsic(固有) electric dipole moment



➢Properties of a Dielectric in Electrostatic Equilibrium

② Displacement polarization



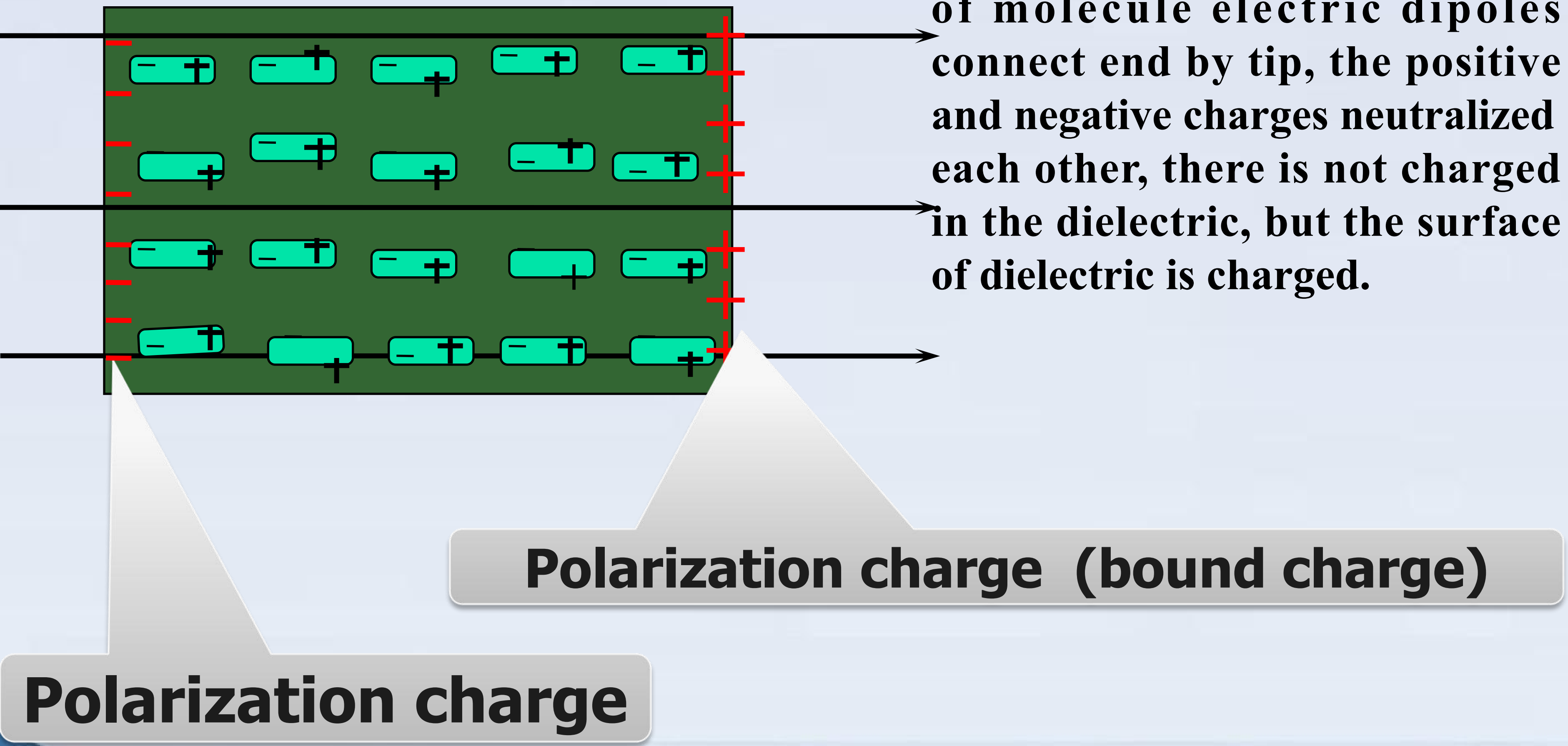
$\vec{p}_e = 0$ $\qquad$ $\vec{p}\,'_e \neq 0$

◆**Under action of out electric field the geometric center of positive and negative charge in non-polar molecule moved away each other and forms a lot of molecule electric dipoles.**

◆ $\vec{p}\,'_e$: **induced electric dipole moment**

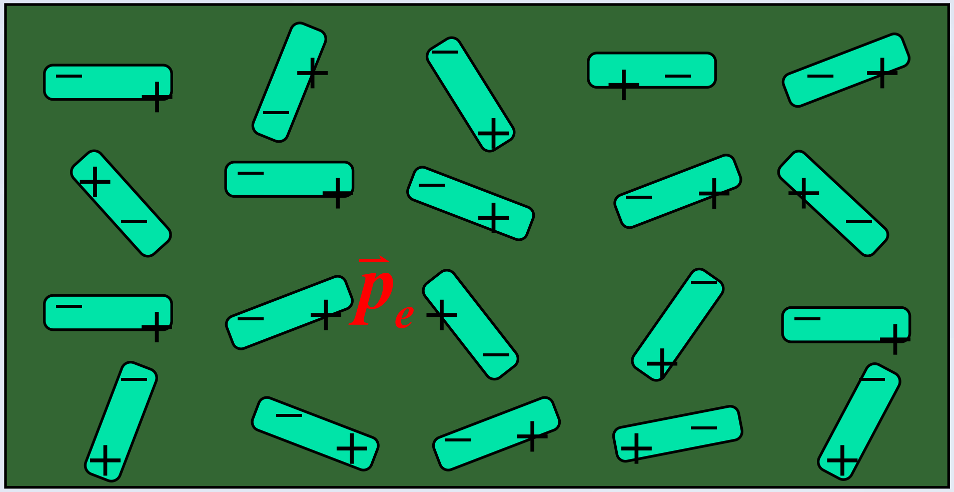
➢Properties of a Dielectric in Electrostatic Equilibrium

◆The displacement polarization in uniform dielectric makes a lot of molecule electric dipoles connect end by tip, the positive and negative charges neutralized each other, there is not charged in the dielectric, but the surface of dielectric is charged.



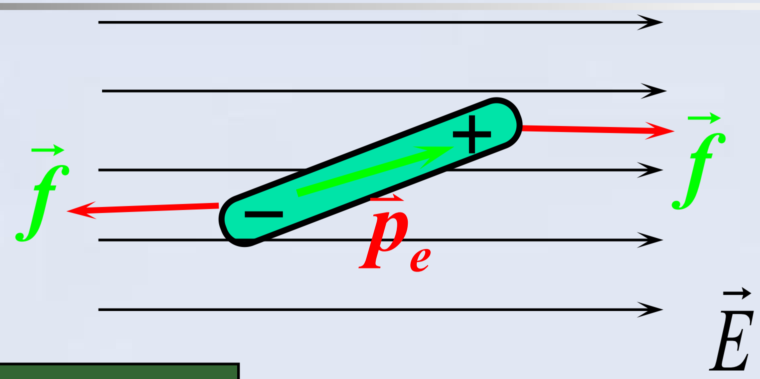
Properties of a Dielectric in Electrostatic Equilibrium

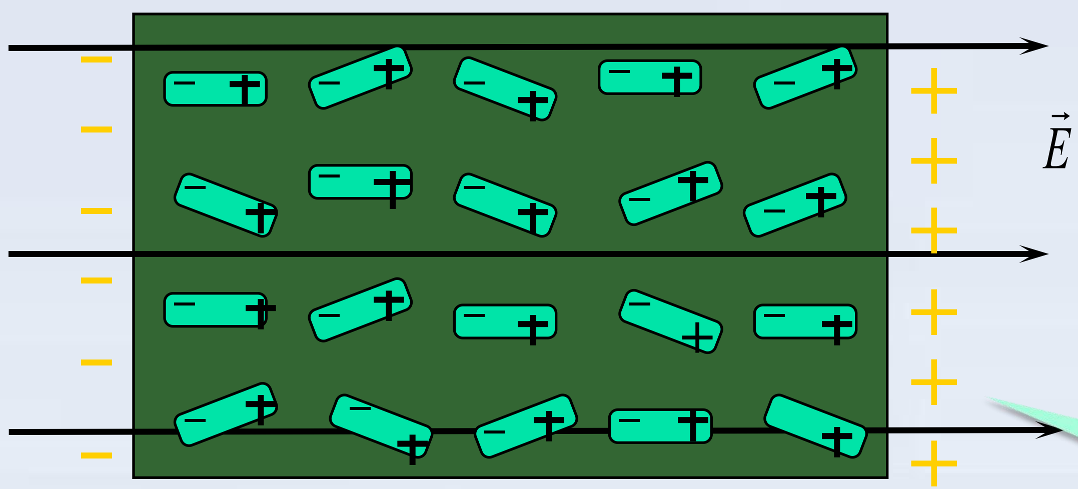
② Rotation Polarization



when $\vec{E} = 0$, there is $\sum \vec{p}_{\mathrm{i}} = 0$.

Electric polarization $\boxed{\vec{P} = \dfrac{\sum \vec{p}_i}{\Delta V} = 0}$ **order or disorder**





$$\boxed{\vec{P} = \frac{\sum \vec{p}_i}{\Delta V} \neq 0}$$

$$\boxed{\vec{P} \propto \vec{E}}$$

Polarization charge (bound charge)

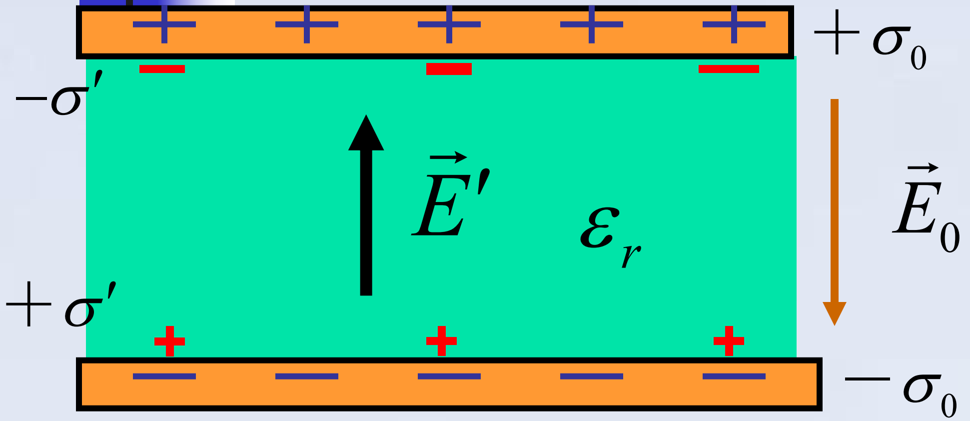
➢Properties of a Dielectric in Electrostatic Equilibrium

③ Distinguish Between Polarization Charge and Free Charge

◆**Relationship Between σ' and σ_0**

σ' , the bound charge density

σ_0 , the free charge density



$$\vec{E} = \vec{E}_0 + \vec{E}' = \frac{1}{\varepsilon_r}\vec{E}_0$$

$$E_0 = \frac{\sigma_0}{\varepsilon_0} \qquad E' = \frac{\sigma'}{\varepsilon_0}$$

$$\Rightarrow \quad \frac{\sigma_0}{\varepsilon_0} - \frac{\sigma'}{\varepsilon_0} = \frac{1}{\varepsilon_r}\frac{\sigma_0}{\varepsilon_0}$$

$$\boxed{\sigma' = \left(1 - \frac{1}{\varepsilon_r}\right)\sigma_0}$$

Discussion

$$\sigma' = \left(1 - \frac{1}{\varepsilon_r}\right)\sigma_0$$

- $\sigma' \prec \sigma_0$, then in dielectric $\vec{E} \neq 0$.

  It is different to the conductor .

- $$\frac{\sigma_0}{\varepsilon_0} - \frac{\sigma'}{\varepsilon_0} = \frac{1}{\varepsilon_r}\frac{\sigma_0}{\varepsilon_0} = \frac{\sigma_0/\varepsilon_r}{\varepsilon_0}$$

  The net charge decreases, therefore the field intensity decreases too.

- . $$E = \frac{1}{\varepsilon_r} E_0 = \frac{\sigma_0}{\varepsilon_r \varepsilon_0} = \frac{\sigma_0}{\varepsilon}$$

Is E irrelevant to(无关) the bound charge ?

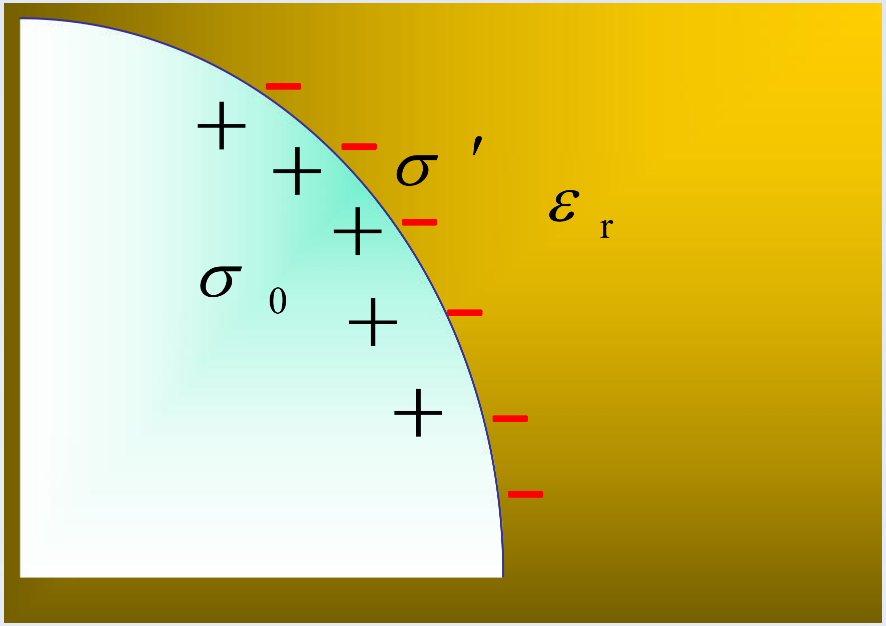
EXAMPLE

◆As is shown in the figure, there is a part of a charged conductor surrounded by dielectric with relative permittivity ε_r.

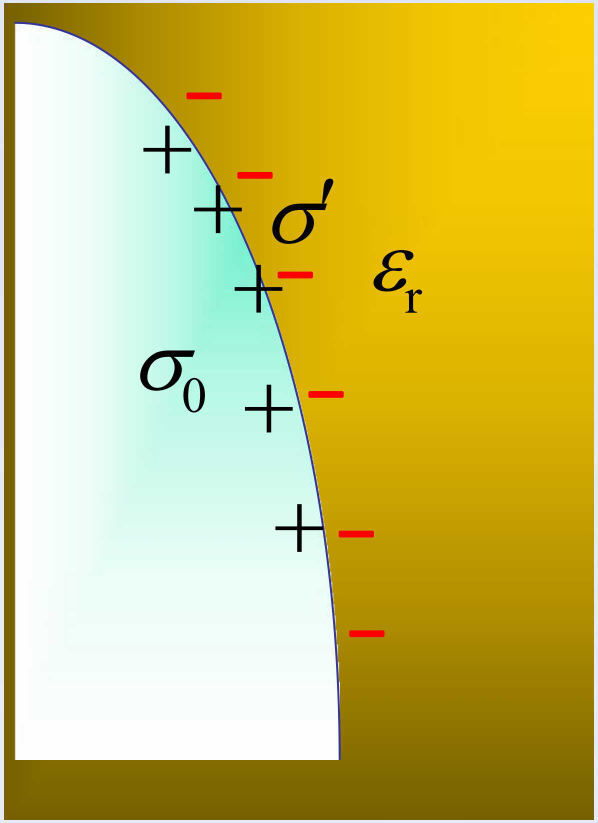
σ_0 : the free charge density on the surface of the conductor.

σ' : the polarization charge density

σ : the net charge density



◆Distinguish between right and wrong.

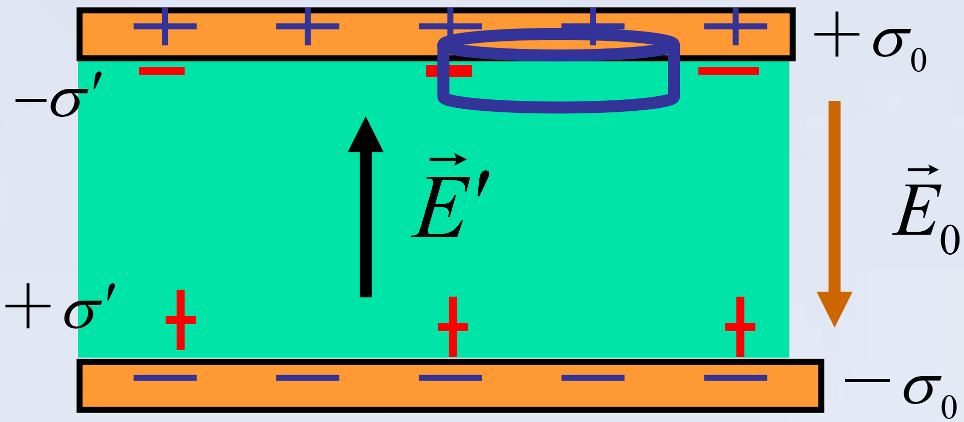


(1) $E = \dfrac{\sigma}{\varepsilon_0}$ (2) $E = \dfrac{\sigma_0}{\varepsilon_r \varepsilon_0}$

(3) $E = \dfrac{\sigma_0}{\varepsilon_0} - \dfrac{\sigma'}{\varepsilon_0}$ (4) $E = \dfrac{\sigma_0}{\varepsilon_0}$

(5) $E = \dfrac{\sigma}{\varepsilon_r \varepsilon_0}$ (6) $E = \dfrac{\sigma_0}{\varepsilon_0} - \dfrac{\sigma'}{\varepsilon_r \varepsilon_0}$

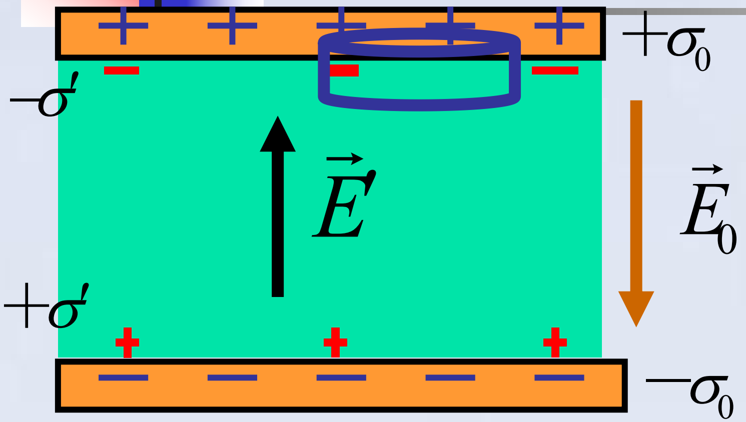
§ 3. Gauss Theorem of Electrostatic Field in Dielectric **P47**



◆**Draw a gaussian surface as shown in figure.**

◆**Apply Gauss theorem (contain bound charges into the sum of all charges) .**

$$\oiint_S \vec{E} \cdot d\vec{S} = \frac{\sum Q}{\varepsilon_0} = \frac{\sigma_0 - \sigma'}{\varepsilon_0} S = \frac{\sigma_0}{\varepsilon_0 \varepsilon_r} S$$



$$\oint\!\!\!\oint_S \vec{E}\cdot d\vec{S} = \frac{\sigma_0 S}{\varepsilon_0\varepsilon_r}$$

$$\oint\!\!\!\oint_S \varepsilon_r\varepsilon_0\vec{E}\cdot d\vec{S} = \sigma_0 S = Q_0$$

- **Electric displacement**

$$\vec{D} \equiv \varepsilon_0\varepsilon_r\vec{E}$$

- **(isotropic uniform dielectric)**

$$\oint\!\!\!\oint_S \vec{D}\cdot d\vec{S} = Q_0$$

- **Guass theorem in dielectric**

Discussion

$$\oiint_S \vec{D} \cdot d\vec{S} = Q_0$$

➢ Physical meaning: **the electric displacement flux of a closed surface equals the algebraic sum of all free charges in the closed surface.**

➢ .

$$\oiint_S \varepsilon_r \varepsilon_0 \vec{E} \cdot d\vec{S} = Q_0$$

The effect of polarization charge has been embodied by ε_r .

◆ $\vec{D}$ **is decided by the polarization charge and free charge, however, the flux of $\vec{D}$ is only related to free charge.**

- **Generally,** $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

- **In isotropic uniform dielectric,**

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_{\mathrm{e}} \vec{E} = \varepsilon_0 \varepsilon_{\mathrm{r}} \vec{E} = \varepsilon \vec{E}$$

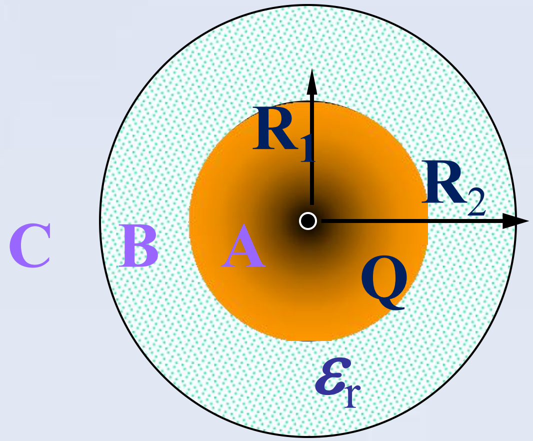
Example

◆**Proving :**

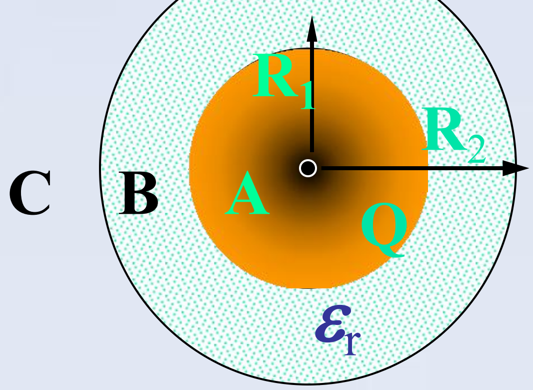
Coulomb law in isotropic uniform dielectric is

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r r^2}$$

Example



◆**Out of a metal sphere shell of radius R_1 and electric quantity Q is covered closely with a medium shell of radius R_2 and relative dielectric constant ε_r . Find the electric displacement, the field intensity and the electric potential in the medium.**



- $D_A = 0 \qquad D_B = D_c = \dfrac{Q}{4\pi r^2}$
- $\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

$$E_A = 0 \qquad E_B = \frac{Q}{4\pi \varepsilon_0 \varepsilon_r r^2}$$

$$E_C = \frac{Q}{4\pi \varepsilon_0 r^2}$$

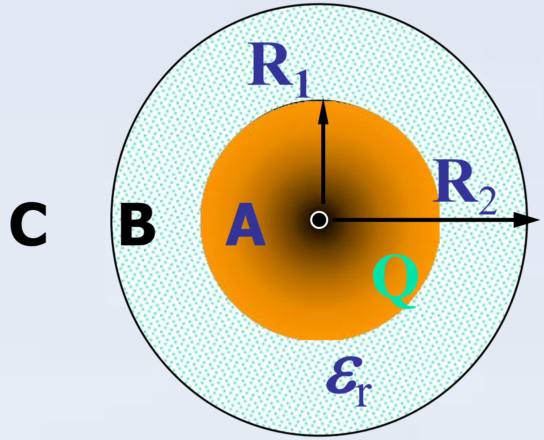
$$U_A = \int_r^{\infty} \vec{E} \cdot d\vec{r}$$

$$= \int_r^{R_1} \vec{E}_A \cdot d\vec{r} + \int_{R_1}^{R_2} \vec{E}_B \cdot d\vec{r} + \int_{R_2}^{\infty} \vec{E}_C \cdot d\vec{r}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_r R_1} - \frac{1}{\varepsilon_r R_2} + \frac{1}{R_2}\right)$$

$$U_B = \int_r^{R_2} \vec{E}_B \cdot d\vec{r} + \int_{R_2}^{\infty} \vec{E}_C \cdot d\vec{r}$$

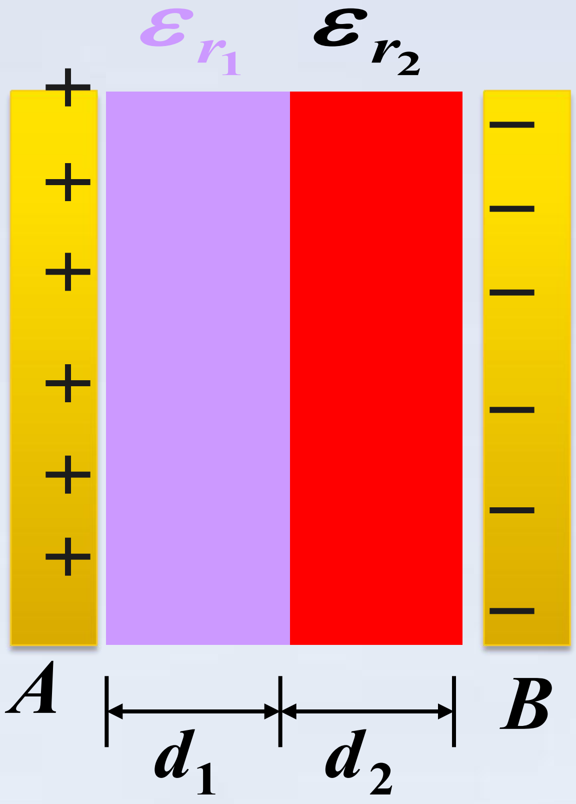
$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{\varepsilon_r r} - \frac{1}{\varepsilon_r R_2} + \frac{1}{R_2}\right)$$

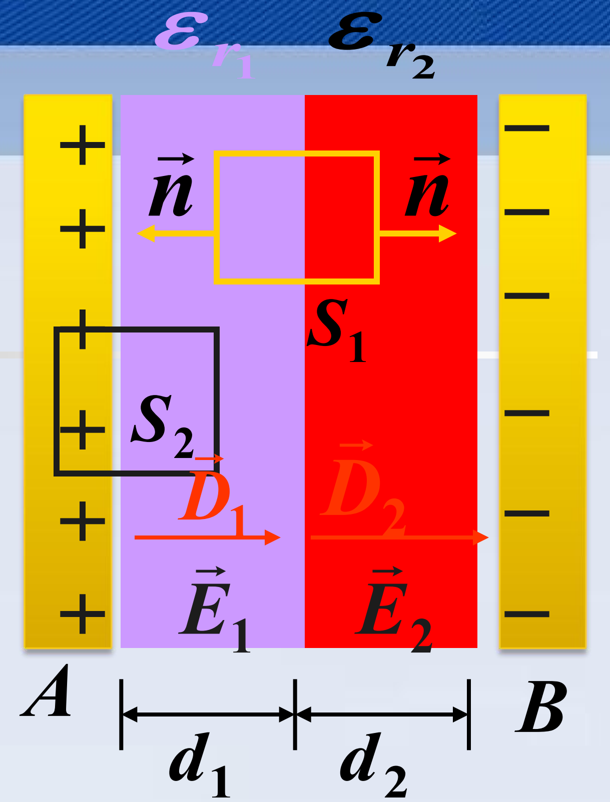


$$U_C = \int_r^{\infty} \vec{E}_C \cdot d\vec{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

Example

◆There are two same parallel metal plates *A* and *B* of area *S*, their surface charge density is +σ and -σ respectively. Suppose fill the space between A and B with the isotropic uniform dielectric of the relative dielectric constant ε_{r1} and ε_{r2}





◆Select gaussian surface as shown in figure.

◆According to Gauss theorem, there is

$$\oint_{S_1} \vec{D} \bullet d\vec{S} = -D_1 \Delta S + D_2 \Delta S = 0$$

$$\therefore D_1 = D_2$$

$$\oint_{S_2} \vec{D} \bullet d\vec{S} = D_1 \Delta S + 0 = \sigma \Delta S$$

$$\therefore D_1 = \sigma \qquad D_2 = \sigma$$

◆From equation $D_1 = \varepsilon_0 \varepsilon_{r_1} E_1$ we have

$$E_1 = \frac{\sigma}{\varepsilon_0 \varepsilon_{r_1}} \qquad E_2 = \frac{\sigma}{\varepsilon_0 \varepsilon_{r_2}}$$

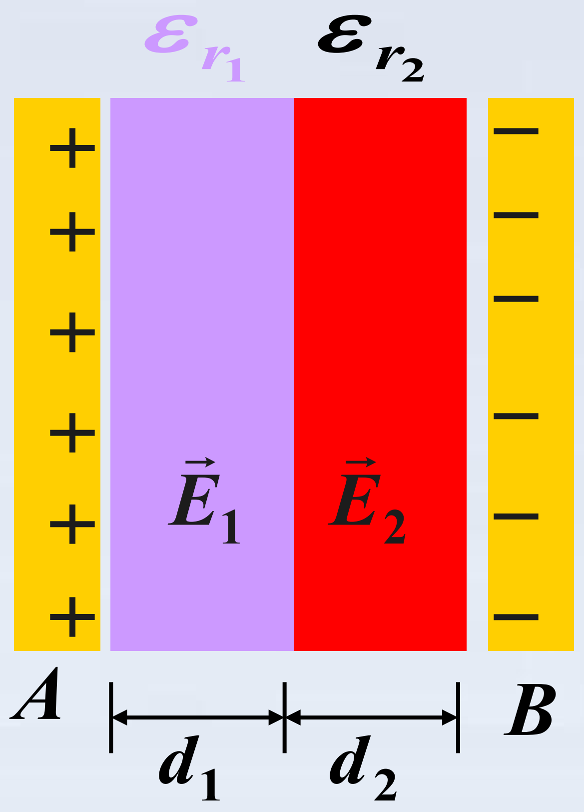
EXAMPLE

d_1, ε_{r1}, d_2, ε_{r2} *and S is given.*

Find : C

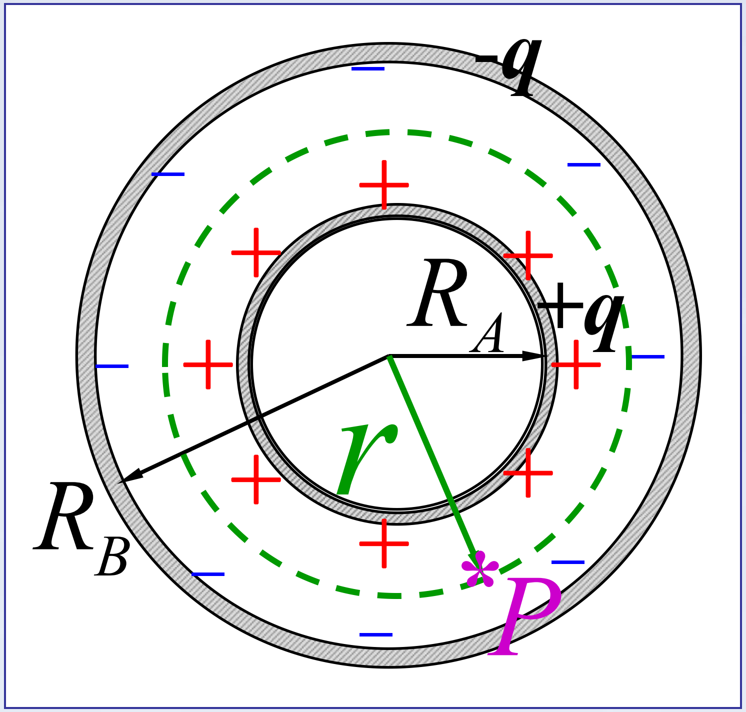
$$C=\frac{q}{u_A-u_B}=\frac{\sigma S}{\frac{\sigma}{\varepsilon_0}\left(\frac{d_1}{\varepsilon_{r_1}}+\frac{d_2}{\varepsilon_{r_2}}\right)}$$

$$=\frac{\varepsilon_0\varepsilon_{r_1}\varepsilon_{r_2}S}{d_1\varepsilon_{r_2}+d_2\varepsilon_{r_1}}$$



•球形电容器

（1）设内球带正电（+*q*），外球带负电（-*q*）



（2） $$E = \frac{q}{4\pi\varepsilon_0 r^2} \quad (R_A < r < R_B)$$

（3） $$U_{AB} = \int_{R_A}^{R_B} \vec{E}\cdot d\vec{l} = \int_{R_A}^{R_B} E dr$$

$$= \frac{q}{4\pi\varepsilon_0}\int_{R_A}^{R_B}\frac{\mathrm{d}r}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_A} - \frac{1}{R_B}\right)$$

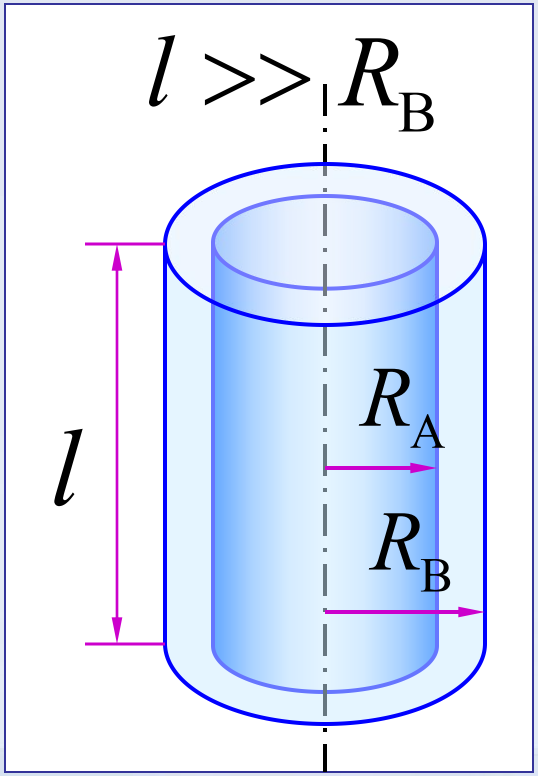
（4） $$C = \frac{q}{U_{AB}} = 4\pi\varepsilon_0 \frac{R_A R_B}{R_B - R_A}$$

两球壳间充满相对介电常量为ε_r的介电质时：

$$C = \varepsilon_r C_0 = \frac{4\pi\varepsilon_0\varepsilon_r R_A R_B}{R_B - R_A}$$

采用类似的方法可以求出，圆柱形电容器的电容

$$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{\ln\dfrac{R_B}{R_A}}$$



SUMMARY

- 1.

$$\vec{P} = \frac{\sum \vec{p}_i}{\Delta V} \qquad \varepsilon = \varepsilon_0 \varepsilon_r$$

- 2. $\boxed{\sigma' = \left(1 - \frac{1}{\varepsilon_r}\right) \sigma_0}$ $\boxed{E = \frac{1}{\varepsilon_r} E_0}$

- 3.in isotropic uniform dielectric： $\boxed{\vec{D} \equiv \varepsilon_0 \varepsilon_r \vec{E}}$

- 4.Gauss theorem in dielectric:

$$\boxed{\oiint_S \vec{D} \cdot d\vec{S} = Q_0}$$