

MIMO multiple-input multiple-output Channel.

\downarrow multiple TX antenna \downarrow multiple RX antennas

TX 1 S_1 RX 1 $Y = HS + n$

TX 2 S_2 RX 2 \downarrow

Y: RX signal vector $\mathbb{C}^{N \times 1}$

TX M S_M RX N H: channel matrix $\mathbb{C}^{N \times M}$

M: # of TX antennas

N: " RX "

h_{ij} : channel gain between i -th RX antenna and j -th TX antenna

$$h_{ij} = h_{ij,I} + j h_{ij,Q}$$

\downarrow \downarrow
 $N(0, \frac{1}{2})$ $\sum_{j=1}^M$ \downarrow $N(0, \frac{1}{2})$

standard normal dist.

$$E(|h_{ij}|^2) = 1$$

S : transmitted signal vector $S \in \mathbb{C}^{M \times 1}$

n : noise vector. $n \in \mathbb{C}^{N \times 1}$

$$n_i = n_{i,I} + j n_{i,Q}$$

\downarrow \downarrow
 $N(0, \sigma^2)$ \downarrow $N(0, \sigma^2)$

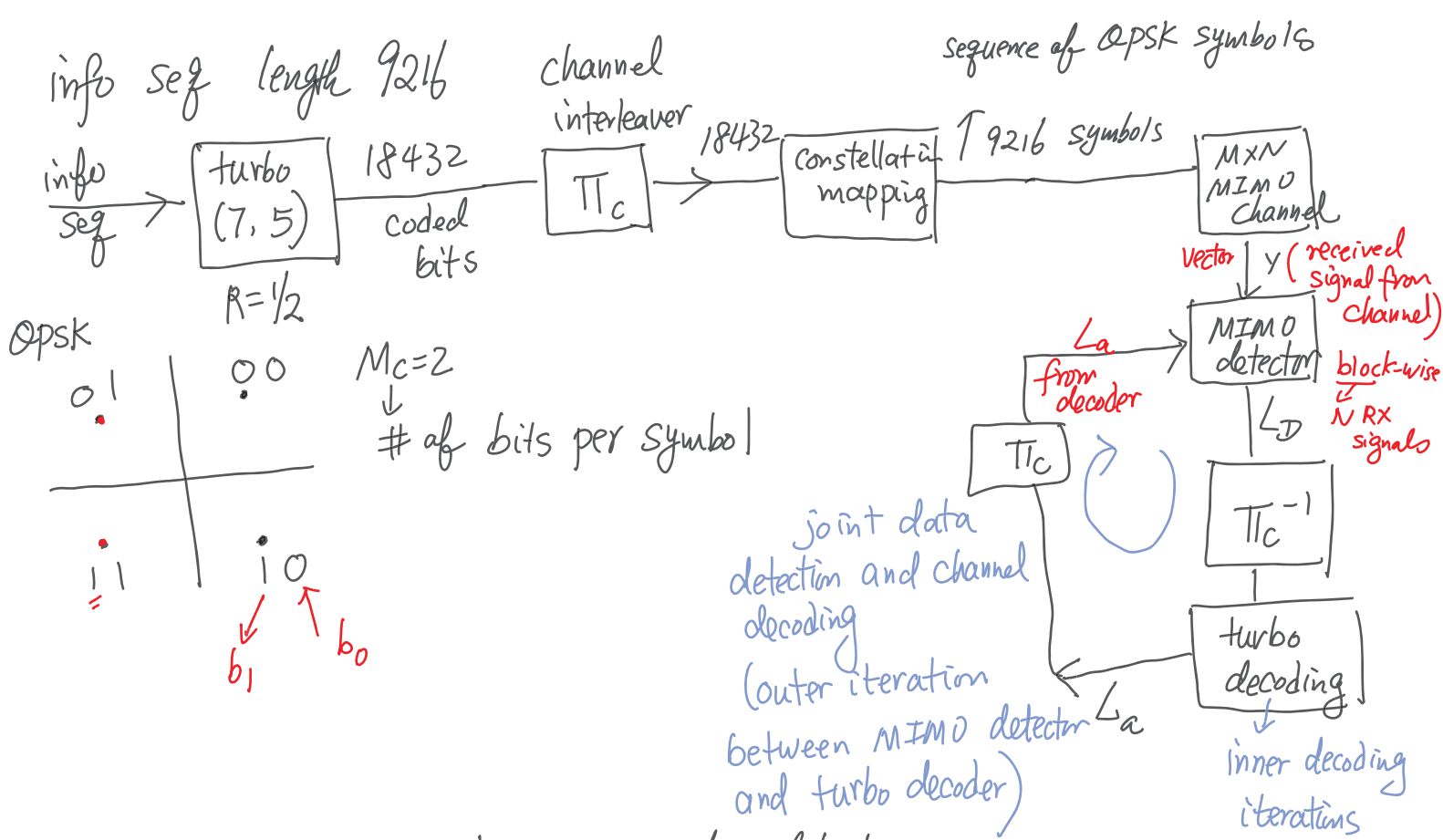
σ^2 related to $\frac{E_b}{N_0}$ value, needs to be adjusted!

Average power constraint: $E[|s_i|^2] = \frac{E_s}{M}$

$Y = HS + n$

$$y_i = \sum_{j=1}^M h_{ij} s_j + n_i$$

i -th RX antenna



Assume H is perfectly known at the detector.

Input to the MIMO detector

① $\underline{y} \in \mathbb{C}^{N \times 1}$ RX vector,

$$\underline{y} = \underline{H} \underline{s} + \underline{n}$$

② L_a from decoder: $M \times M_c$ real-valued

$M_c = 2 \Rightarrow$ QPSK

\underline{s} : $M \times 1$ transmitted vector

Each symbol consists of M_c bits

\underline{x} : bit vector $(x_0, x_1, x_2, \dots, x_{M \times M_c - 1})$

$$x_k = \pm 1$$

turbo code (same as in the paper)

$0 \rightarrow -1$ (reverse it for LDPC)

$$L_D(x_k | y, H, L_a) = \ln \frac{P(x_k = +1 | y, H, L_a)}{P(x_k = -1 | y, H, L_a)}$$

bit vector

" $x_k = +1$ " total $\frac{M \cdot M_c - 1}{2}$ many possible \underline{x} such that $x_k = +1$.

Bayes formula $f(y | x, H) \cdot P(x | L_a)$

$$P(x_k = +1 | y, H, L_a) = \sum_{\underline{x}: x_k = +1} P(\underline{x} | y, H, L_a) = \sum \frac{f_Y(y)}{f_Y(y)}$$

$\underline{x}: x_k = +1$

$$P(\underline{x} | L_a) \approx P(x_k | L_a) \cdot \prod_{j \neq k} P(x_j | L_a)$$

$$L_D(x_k | Y, H, L_A) = \underbrace{\ln \frac{p(x_k = +1 | L_A)}{p(x_k = -1 | L_A)}}_{L_A(x_k)} + \ln \frac{\sum_{x_i: x_k = +1} f(Y | x, H) \prod_{j \neq k} p(x_j | L_A)}{\sum_{x_i: x_k = -1} f(Y | x, H) \prod_{j \neq k} p(x_j | L_A)}$$

$$\sum_{x_i: x_k = +1} f(Y | x, H) \prod_{j \neq k} p(x_j | L_A) = \sum_{x_i: x_k = +1} e^{\ln f(Y | x, H) + \sum_{j \neq k} \ln p(x_j | L_A)}$$

$$\ln p(x_j | L_A) = \frac{x_j \cdot L_A(x_j)}{2} ; x_j = \pm 1 \quad \text{eqn (7) reference paper.}$$

$$f(Y | x, H) = \frac{1}{(\sqrt{2\pi\sigma^2})^N} e^{-\frac{1}{2\sigma^2} \|Y - HS\|^2}$$

$x \rightarrow s$

$$\ln f(Y | x, H) = \ln \frac{1}{(\sqrt{2\pi\sigma^2})^N} - \frac{1}{2\sigma^2} \|Y - HS\|^2$$

$$\left\{ \begin{aligned} L_D(x_k | Y, H, L_A) &= L_A(x_k) + \max_{x_i: x_k = +1}^* \left(-\frac{1}{2\sigma^2} \|Y - HS\|^2 + \frac{1}{2} \sum_{j \neq k} x_j L_A(x_j) \right) \\ &\quad - \max_{x_i: x_k = -1}^* \left(\text{same} \right) \end{aligned} \right.$$

$$L_D^e = L_D - L_A \quad \text{get extrinsic LLR going to decoder}$$