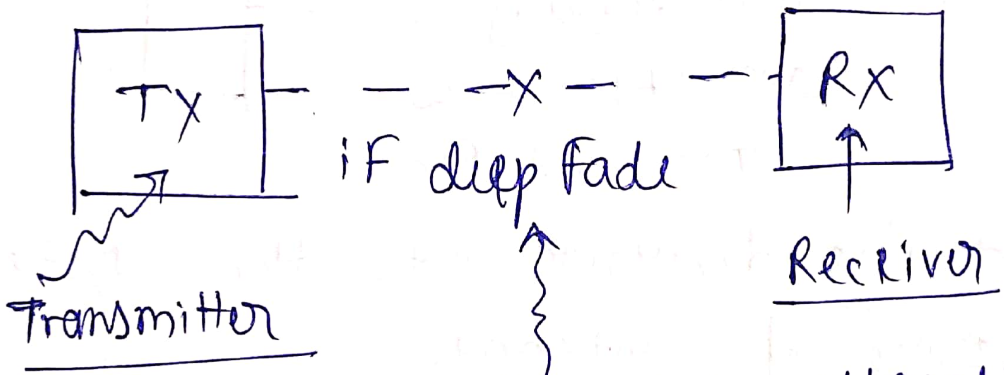
* Unit - 6 *

* MIMO :-

* Principal of Diversity :-

→ Can be employed to over come the effect of deep fade

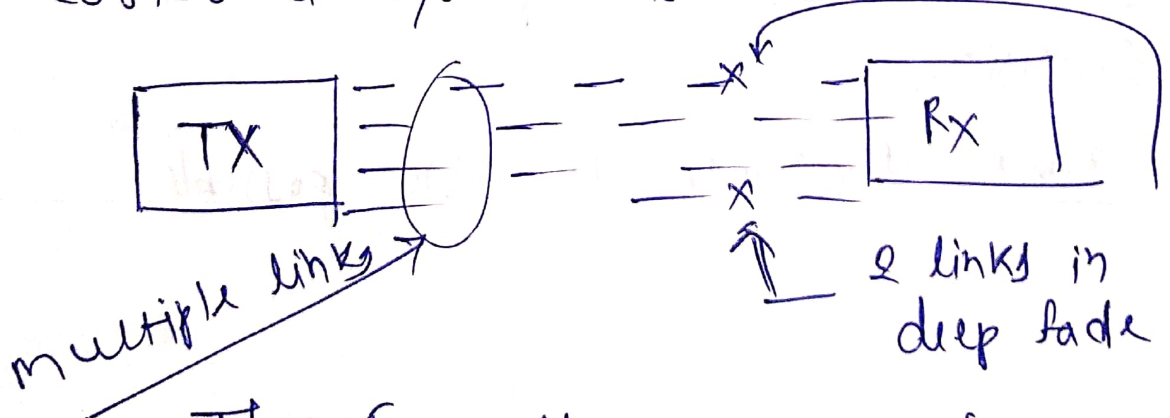
→ To combate the impact of fading.



There is no alternative link

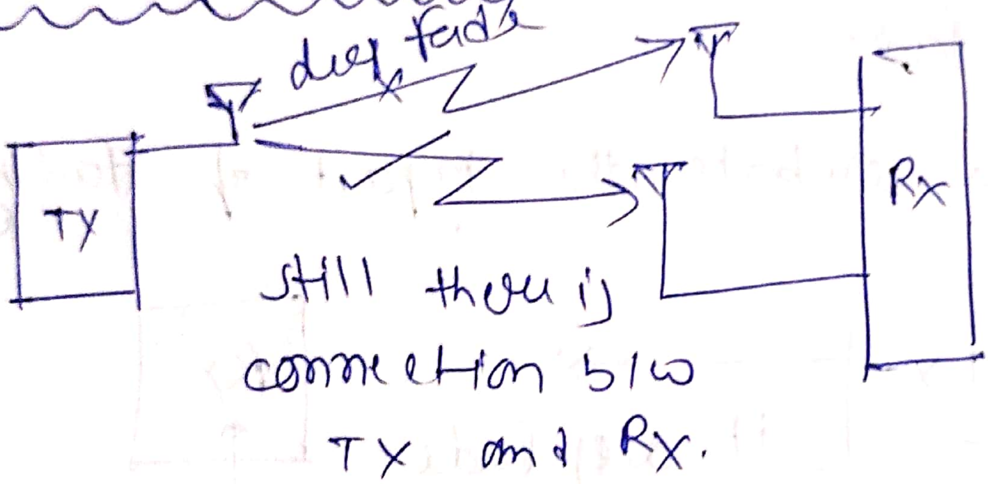
How we do improve it ?

* consider a system with multiple links



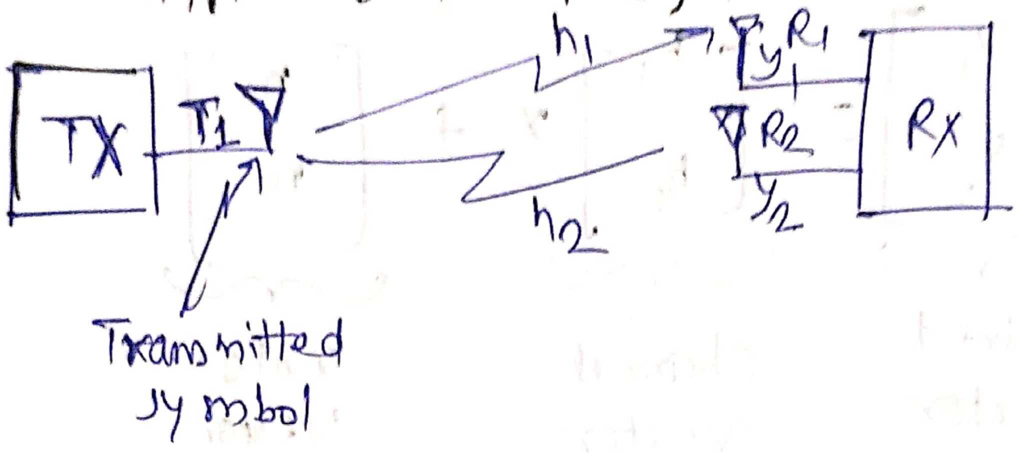
Therefore, the comm. will not be disrupted.

* There is diversity into the system
* Signals can be reached with multiple path
* This is the principle of Diversity.
* Antenna Diversity (Example of Diversity



* Introducing diversity into the system. In terms of antenna.
* It is called Rx diversity
* we can also have multiple TX diversity
* It can be multiple TX and RX diversity.
* Time diversity is also possible.

* Multiple Antenna system



h_1 = Fadding coefficient between $T_1 \to R_1$

h_2 = fading coefficient betwon $T_1 \Rightarrow R_2$

y_1 = Rx signal at R_1

y_2 = Rx signal at R_2

__System model:-__

$y_1 = h_1 x + n_1$ ~ noise at R_1

$y_2 = h_2 x + n_2$ ~ noise at R_2

{ n_1 and n_2 are Gaussian, zero mean r.v.

n_1 and n_2 are un-correlated }

↳ Each has power σ^2

$$E\{|n_1|^2\} = E\{|n_2|^2\} = \sigma^2$$

$$E\{n_1, n_2\} = 0$$ n_1, n_2 are uncorrelated

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{\text{Received vector}} = \underbrace{\begin{bmatrix} h_1 \\ h_2 \end{bmatrix}}_{\substack{\text{channel vector} \\ h}} x + \underbrace{\begin{bmatrix} n_1 \\ n_2 \end{bmatrix}}_{\text{noise vector}}$$

$$\boxed{\hat{y} = \hat{h} x + \hat{n}}$$

Vector of Rx signal

↳ <u>Combining</u> :—

combining weights

$$\hat{y} = \underbrace{w_1 y_1 + w_2 y_2}$$

$$= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$\hat{y} = \hat{w}^T \hat{y}$$

$$= \hat{w}^T (\hat{h} x + \hat{n})$$

$$= \underbrace{\hat{w}^T \hat{h} x}_{\text{signal}} + \underbrace{\hat{w}^T \hat{n}}_{\text{noise}}$$

* SNR at the system:-

$$SNR = \frac{|\hat{w}^T \hat{h}|^2 P}{E\{|\hat{w}^T \hat{n}|^2\}}$$

* Noise:-

$$\hat{w}^T \hat{n} = [w_1 \;\; w_2] \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\Rightarrow \hat{w}^T \hat{n} = w_1 n_1 + w_2 n_2$$

$$\Rightarrow E\left[(w_1 n_1 + w_2 n_2)^2\right] =$$

$$\Rightarrow E\{w_1^2 n_1^2 + w_2^2 n_2^2 + 2 w_1 w_2 n_1 n_2\}$$

$$\Rightarrow w_1^2 E[n_1^2] + w_2^2 E[n_2^2] + \underbrace{\cancel{2 w_1 w_2 E[n_1 n_2]}}_{\text{uncorrelated}} \rightarrow 0$$

$$\Rightarrow w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2$$

$$E\{|\hat{w}^T \hat{n}|^2\} = w_1^2 \sigma^2 + w_2^2 \sigma^2$$

$$= \sigma^2 (w_1^2 + w_2^2)$$

$$= \sigma^2 \|\hat{w}\|^2$$

since

$$\Rightarrow \quad \hat{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$\Rightarrow \quad \|\hat{w}\| = \sqrt{w_1^2 + w_2^2}$$

Then $SNR = \dfrac{P\,|\hat{w}^T \hat{h}|^2}{\sigma^2 \|\hat{w}\|^2}$

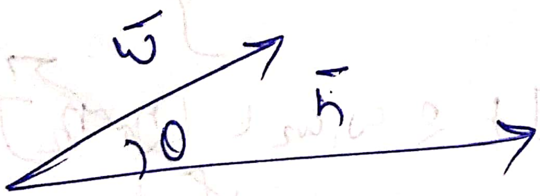
$$\hat{w}^T \hat{h} = [w_1 \; w_2] \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$= w_1 h_1 + w_2 h_2$$

$$= \underbrace{\bar{w} \cdot \bar{h}}$$

From vector calculus

Dot product b/w $\bar{w}$, $\bar{h}$



$$|\bar{w} \cdot \bar{h}| = \|\bar{w}\| . \|\bar{h}\| \cos\theta$$

$$SNR = \frac{P \cdot \cancel{\|\bar{w}\|^2} \|\bar{h}\|^2 \cos^2\theta}{\sigma^2 \cancel{\|\bar{w}\|^2}}$$

$$= \frac{P/\sigma^2 \cdot \|h\|^2 \cos^2\theta}{}$$

This is maximum when $\cos^2\theta = 1$

$$\theta = 0^\circ$$

* For SNR to be maximized $\bar{w}$ has to be along $\hat{h}$

$$\hat{w} \propto \hat{h}$$

$$SNR = \frac{P \cdot \|h\|^2}{\sigma^2} = \text{For two antennas}$$

$$= \frac{P \cdot (|h_1|^2 + |h_2|^2)}{\sigma^2}$$

$$\boxed{\hat{w} = \frac{1}{\|\hat{h}\|} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}}$$

$$\boxed{\hat{w} = \frac{1}{\sqrt{|h_1|^2 + |h_2|^2}} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}}$$

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$$W^H \bar{y} = [W_1^* \quad W_2^*] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= [W_1^* \quad W_2^*] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

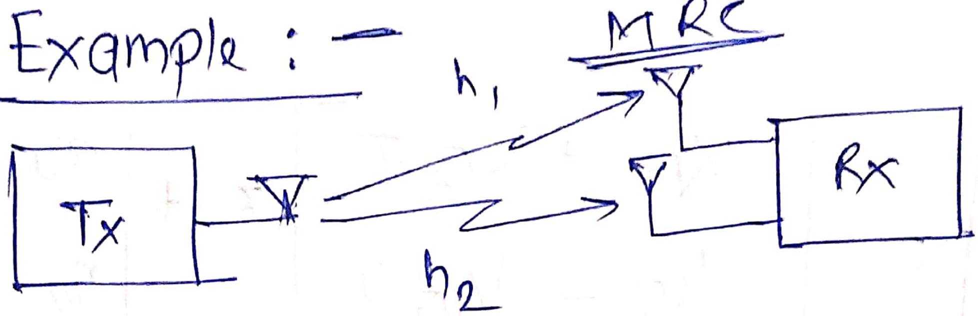
$$= W_1^* y_1 + W_2^* y_2$$

$$\bar{W} = \frac{\bar{h}}{\|h\|}$$

maximal ratio combiner

$$= \frac{\bar{h}}{\sqrt{h_1^2 + h_2^2}}$$

* Example :-



$$h_1 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} J$$

$$h_2 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} J$$

$$y_1 = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} J \right) x + n_1$$

$$y_2 = \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} J \right) x + n_2$$

$$E\left\{ |h_1|^2 \right\} = E\left\{ |h_2|^2 \right\} = \sigma^2 = \frac{1}{2}$$

Therefore, $10 \log_{10} \frac{1}{2} = -3\,dB$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} J \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} J \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$\bar{y} = \bar{h} x + \bar{n}$$

* optimal MRC vector $= \dfrac{\bar{h}}{\|\bar{h}\|}$

$$\|\bar{h}\| = \sqrt{|h_1|^2 + |h_2|^2}$$

$$\bar{h} = \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} J \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} J \end{bmatrix}$$

$$\|\bar{h}\|^2 = \sqrt{|h_1|^2 + |h_2|^2}$$

$$= \sqrt{\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}}$$

$$= \sqrt{2}$$

$$\bar{w} = \frac{\bar{h}}{\|\bar{h}\|} = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} J \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} J \end{bmatrix}$$

this is the MRC vector

* $SNR = \dfrac{|\bar{w}^H \bar{h}|^2 P}{\sigma^2 \|\bar{w}\|^2}$

$\bar{w}^H \bar{h} = \dfrac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j & \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j \\ \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j \end{bmatrix}$

$= \dfrac{1}{\sqrt{2}} \left[\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right]$

$= \dfrac{2}{\sqrt{2}} = \sqrt{2}$

$\boxed{\bar{w}^H \bar{h} = \sqrt{2}}$

$|\bar{w}^H \bar{h}|^2 = 2$

* $\|\bar{w}\|^2 = |w_1|^2 + |w_2|^2$

$= \frac{1}{2} \left(1/2 + 1/2 + 1/2 + 1/2 \right)$

$= \frac{1}{2} \cdot 2 = 1$

$SNR = \dfrac{P \cdot |\bar{w}^H \bar{h}|^2}{\sigma^2 \|\bar{w}\|^2}$

$$= \frac{P \cdot 2}{\frac{1}{2} \cdot 1}$$

$$\boxed{SNR = 4P}$$

System with maximal ratio combining

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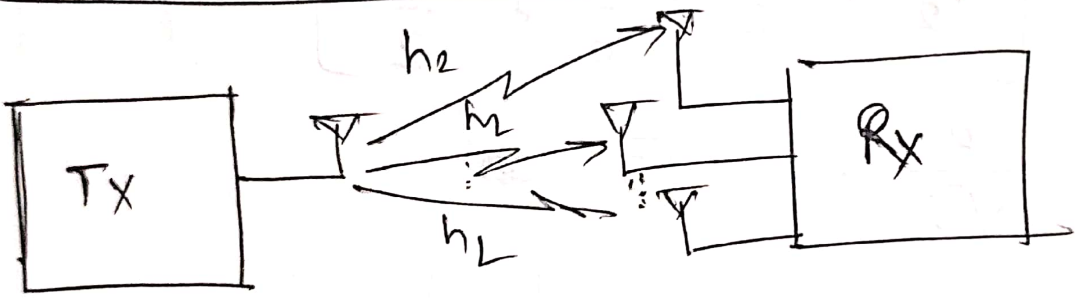
* Generalized it for ~~Two~~ ~~Three~~ (L) antennas:-



* Channel coefficients = $h_1, h_2, \ldots h_L$

* $$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

$\Rightarrow \quad \bar{y} = \bar{h} x + \bar{n}$

$\Rightarrow \quad E\{|h_i|^2\} = \sigma^2$

$\Rightarrow \quad E\{n_i n_j^*\} = 0 \quad \forall \; i \neq j$

noise on any pair of antenna are uncorrelated

$$\Rightarrow \quad \bar{w}^H \bar{y} = \bar{w}^H (\bar{h}x + \bar{n})$$

$$= \underbrace{\bar{w}^H \bar{h} x}_{\text{signal part}} + \underbrace{\bar{w}^H \bar{n}}_{\text{noise part}}$$

$\Rightarrow$ For maximum SNR, choose MRC (maximal ratio combining)

$$\bar{w} = \frac{\bar{h}}{\|\bar{h}\|} = \frac{1}{\|\bar{h}\|}$$

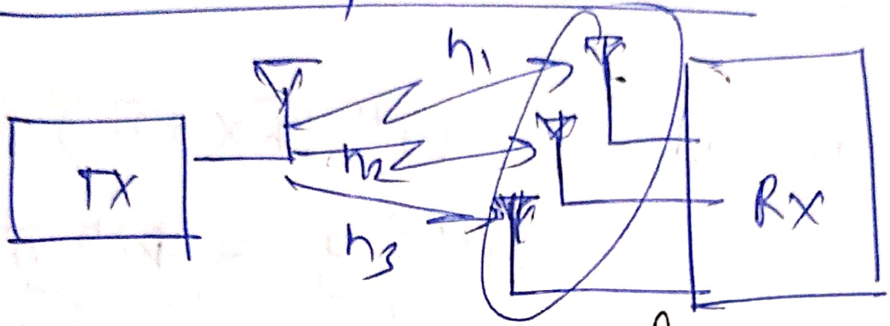
$$\Rightarrow \|\bar{h}\| = \sqrt{|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2}$$

$$\boxed{SNR = \frac{\|\bar{h}\|^2 p}{\sigma^2}}$$

$$= \frac{p(|h_1|^2 + |h_2|^2 + \cdots |h_L|^2)}{\sigma^2}$$

* Diversity :- (MRC)

BER of multiple antenna tx antenna system :-



$\{h_1, h_2, ---, h_L\}$ channel coefficient

$$SNR = \frac{P\|\bar{h}\|^2}{\cancel{\|h\|^2}\,\sigma^2}$$

$$= \frac{P}{\sigma^2}\left\{\|\bar{h}\|^2\right\}$$

$$SNR = \frac{P}{\sigma^2}\left\{|h_1|^2 + |h_2|^2 + \text{---} + |h_L|^2\right\}$$

$$g = \|\bar{h}\|^2$$

$$g = |h_1|^2 + |h_2|^2 + \text{---} + |h_L|^2$$

BER of multiple antenna system

$$g = |h_1|^2 + |h_2|^2 + \cdots + |h_L|^2$$

i.i.d. Rayleigh fadded coefficient of variance

$$\boxed{E\{|h_i|^2\} = 1}$$

* $g = \|\bar{h}\|^2$

chi - squared Random variable

$$F_g(g) = \frac{1}{\lfloor(L-1)}\, g^{L-1}\, e^{-g}$$

here $\boxed{g = \|\bar{h}\|^2}$

* Received SNR

$$SNR_m = \frac{\|\bar{h}\|^2 P}{\sigma^2}$$

$$\boxed{SNR_m = g \cdot SNR}$$

From previous BER Analysis

$$\boxed{BER = Q(\sqrt{SNR})}$$

$$BER = Q\left(\sqrt{g\,SNR}\right)$$

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g is random variable

& I have to average

$Q(\sqrt{g\,SNR})$

for multiple antenna system

$$\text{Average BER} = \int_{-\infty}^{\infty} Q(\sqrt{g\,SNR})\, f_g(g)\, dg$$

$$= \underbrace{\int_{0}^{\infty} Q(\sqrt{g\,SNR})\, \frac{g^{L-1} e^{-g}}{(\lfloor L-1)}\, dg}$$

$$= \frac{(1-\lambda)^L}{2} \sum_{l=0}^{L-1} {}^{L+l-1}C_l \times$$

$$\times \left(\frac{1+\lambda}{2}\right)^l ,$$

Here $\boxed{\lambda = \sqrt{\frac{SNR}{2+SNR}}}$

For $L = 2$

$$BER = \left(\frac{1-\lambda}{2}\right)^2 \left\{ {}^1C_0 \left(\frac{1+\lambda}{2}\right)^0 + {}^2C_1 \left(\frac{1+\lambda}{2}\right)^2 \right\}$$

$${}^1C_0 = \frac{\lfloor 1}{\lfloor 0 \lfloor 1} = 1$$

$${}^2C_1 = \frac{\lfloor 2}{\lfloor 1 \lfloor 1} = 2$$

$$= \left(\frac{1-\lambda}{2}\right)^2 \left\{ 1 + 2\left(\frac{1+\lambda}{2}\right) \right\}$$

$$= \left(\frac{1-\lambda}{2}\right)^2 \left\{ 2+\lambda \right\}$$

Average BER = with $L=2$