Lect 8

* Fading and multipath propagation model :—

* Wireless communication (WC) represented as an LTI system

*

$$s(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t)$$

Wireless channel

Multipath

$h(t)$

$s(t)$ — Base band signal

$$s(t) = Re\{ s_b(t)\, e^{j2\pi f_c t} \}$$

** complex base band Tx signal **

$$h(t) = \sum_{i=0}^{L-1} a_i\, \delta(t - \tau_i)$$

L = no. of paths
τ_i = delay of i^{th} path component
a_i = * attenuation * magnitude of i^{th} impulse
f_c = carrier freq.

$$y(t) = s(t) * h(t)$$

For first path :—

$$y_0(t) = Re\{ a_0\, s_b(t - \tau_0)\, e^{j2\pi f_c(t - \tau_0)} \}$$

$$y_1(t) = Re\{ a_1\, s_b(t - \tau_1)\, e^{j2\pi f_c(t - \tau_1)} \}$$

$\vdots$

$$y_{L-1}(t) = Re\{ a_{L-1}\, s_b(t - \tau_{L-1})\, e^{j2\pi f_c(t - \tau_{L-1})} \}$$

ith path amplitude (classmate) attenuation of; delayed version of; delay of ith component; the carrier

NET Signal :-

$$y(t) = Re\left\{ \sum_{i=0}^{L-1} a_i \, S_b(t-\tau_i) \, e^{j2\pi f_c (t-\tau_i)} \right\}$$

$$\Rightarrow \quad = Re\left\{ \underbrace{\left[\sum_{i=0}^{L-1} a_i \, S_b(t-\tau_i) \, e^{-j2\pi f_c \tau_i} \right]}_{\text{is a complex signal}} \times e^{j2\pi f_c t} \right\}$$

[Complex Baseband Rx signal]

—— x —— x —— x —— x ——

Complex baseband signal Rx =

$$\boxed{y_b(t) = \sum_{i=0}^{L-1} a_i \, S_b(t-\tau_i) \, \underbrace{e^{-j2\pi f_c \tau_i}}_{\text{complex phase factor}}}$$

τ_i = delay Factor

a_i = attenuation Factor

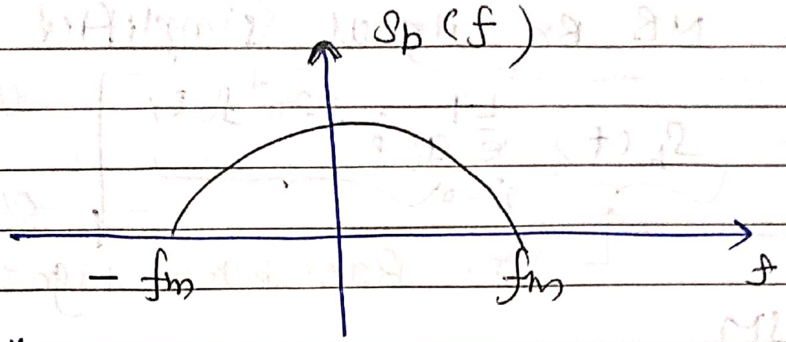
* complex phase factor

$$y_b(t) = \sum_{i=0}^{L-1} a_i \, S_b(t-\tau_i) \, e^{-j2\pi f_c \tau_i}$$

—— x —— x ——

[Narrow band signal assumption]

Let f_m be the max. freq. component of $S_b(t)$.



Ex: * GSM $\rightarrow$ $f_m = 100KHz$ ✓

if $f_m < \frac{1}{\tau_i}$ $\forall i$

then signal is called as NB signal

typically $\tau_i \simeq 1\mu s$

$1/\tau_i = 1MHz$

GSM is N.B. signal

** [For CDMA it is not valid assumption since CDMA is W.B.]

For a N.B. Signal :-

$$S_b(t - \tau_i) \simeq S_b(t)$$

i.e., delay does not cause significant changes due to $f_m < 1/\tau_i$

complex phase factor

So the MB Rx signal simplified to

$$\boxed{y_b(t) = \underbrace{S_b(t)}_{\text{Tx Base band signal}} \underbrace{\sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}}_{}}$$

complex coefficient or complex factor

h

also named as fading coefficient

For GSM

This sum leads to either constructive or distructive interference

① Ex: — Two paths

$a_0 = 1 \qquad \tau_0 = 0$

$a_1 = 1 \qquad \tau_1 = 1/2f_c$

$$h = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$$

$$= 1 + (-1) = 0$$

$$\boxed{Rx\ signal = S_b(t) \times h = 0}$$

② Ex: Two paths

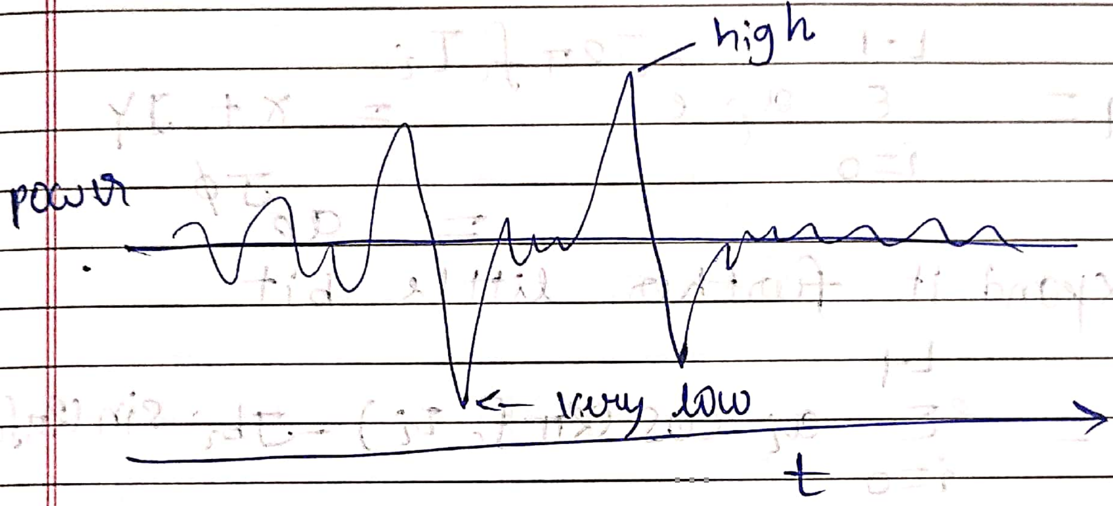
$a_0 = 1 \qquad \tau_0 = 0$

$a_1 = -1 \qquad \tau_1 = 1/f_c$

$$|h = 1 + 1 = 2|$$

$$\boxed{Rx\ signal = S_b(t) \times h = 2S_b(t)}$$

* Because of the fading, we receive the range of the Rx power



variation in signal power is known as fading in wireless communication.

* Analytical models of wireline and wireless

$$Y_b(t) = h\, S_b(t)$$ } wireless system

complex fading coefficient

$$Y_b(t) = S_b(t)$$ } wireline system

complex
* Properties of ^ fading coefficient

Statistics of Fading coefficient

$$h = \sum_{i=0}^{L-1} a_i e^{-J2\pi f_c \tau_i} = x + JY$$

$$= a e^{J\phi}$$

expand it further little bit

$$= \sum_{i=0}^{L-1} a_i \cos(2\pi f_c \tau_i) - J b_i \sin(2\pi f_c \tau_i)$$

$$x = \sum_{i=0}^{L-1} a_i \cos(2\pi f_c \tau_i)$$

$$y = -\sum_{i=0}^{L-1} b_i \sin(2\pi f_c \tau_i)$$

Statistics of this variables in complex h is char. by Gaussian R.V.

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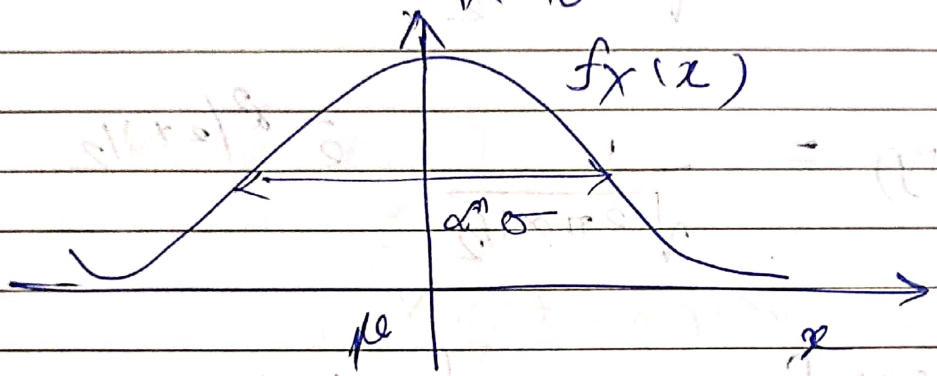
Gaussian R.V.

$X \sim \mathcal{N}(\mu_x, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu_x)^2/2\sigma^2}$$



* Standard Normal R.V. ; $\mu = 0$.

$X \sim \mathcal{N}(0, 1)$

$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ } PDF of the Standard Gaussian Random variable

Now, complex fading coefficient: →

$h = x + jy$

[sum of a large number which are random]

follows the C.L.T., can be assumed to be a $\boxed{\text{Gaussian R.V.}}$

X and Y assumed to be Normal R.V. as $X \sim N(0, 1/2)$; $Y \sim N(0, 1/2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi \times 1/2}} e^{-x^2/2\times 1/2}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi\times 1/2}} e^{-y^2/2\times 1/2}$$

$\because$ x and y are independent r.v.

$$f_X(x) f_Y(y) = f_{XY}(x,y)$$

$$\boxed{f_{XY}(x,y) = \frac{1}{\pi} e^{-(x^2+y^2)}}$$

⇒ Now convert this fading into polar form

$$h = X + JY = a e^{J\phi}$$

$x = a\cos\phi$

$y = a\sin\phi$

joint distribution of a and ϕ will give you the ~~power~~ amp and ~~delay~~ phase of the signal power

$$f_{A,\phi}(a,\phi) = f_A(a)\, f_\phi(\phi)$$

$$f_{XY}(x,y) = \frac{1}{\pi} e^{-(x^2+y^2)}$$

$x^2 + y^2 = a^2\cos^2\phi + a^2\sin^2\phi$

a^2 = gain of fading coefficient

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$$f_{A,\phi} = \frac{1}{\pi} e^{-a^2} \det(J_{XY})$$

Jacobian matrix → scaling term

$$J_{XY} = \begin{bmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial a} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} \end{bmatrix}$$

$$\begin{bmatrix} x = a\cos\phi \\ y = a\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos\phi & \sin\phi \\ -a\sin\phi & a\cos\phi \end{bmatrix}$$

$$\det(J_{XY}) = a\cos^2\phi + a\sin^2\phi$$

$$\boxed{\det(J_{XY}) = a}$$

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$$\boxed{f_{A,\phi} = \frac{a}{\pi} e^{-a^2}}$$

Now, we derive the marginal PDF of a as

$$F_A(a) = \int_{-\pi}^{\pi} f_{A,\phi}(a,\phi)\, d\phi$$

$$= \int_{-\pi}^{\pi} \frac{a}{\pi} e^{-a^2} d\phi$$

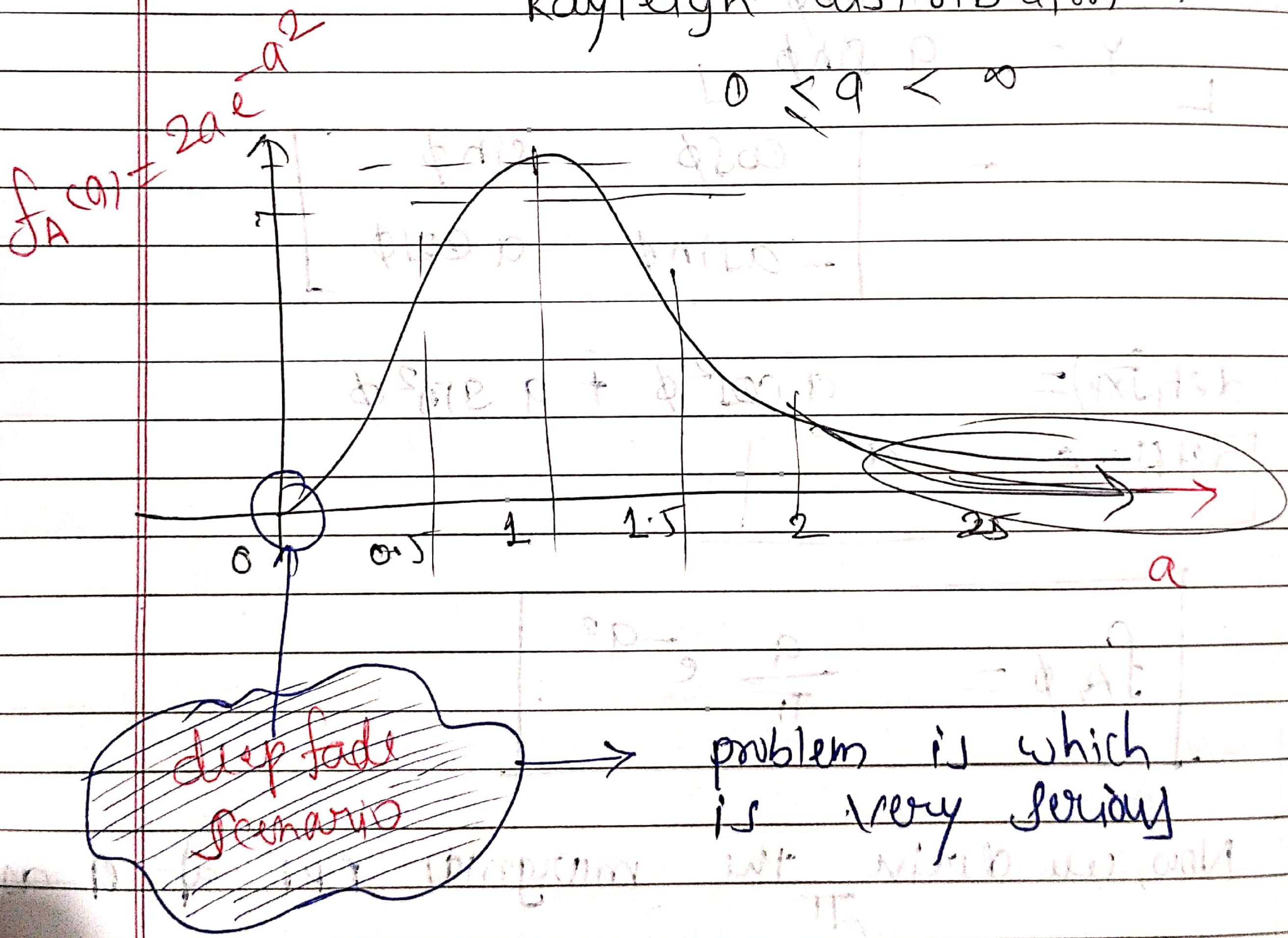
$$= \frac{a}{\cancel{\pi}} e^{-a^2} \times 2\cancel{\pi} = 2ae^{-a^2}$$

Now marginal distribution of a; i.e., envelope of the fading channel.

$$\boxed{f_A(a) = 2a\,e^{-a^2}}$$

Rayleigh distribution :

$0 \leq a < \infty$



$\rightarrow$ problem is which is very serious

Marginal PDF $f_\phi(\phi)$? $\Rightarrow$

$$f_\phi(\phi) = \int_0^\infty \frac{a}{\pi} e^{-a^2} da$$

$$= \frac{1}{2\pi} \int_0^\infty 2a\, e^{-a^2} da$$

$$-a^2 = x$$

$$-2a\, da = dx$$

$$da = dx/-2a$$

$$= \frac{1}{2\pi} \int_0^\infty e^{+x} dx$$

$$\boxed{f_\phi(\phi) = \frac{1}{2\pi}} \longrightarrow \text{uniform distribution over } -\pi \text{ to } \pi$$

Thus,

$$f_{A\phi}(a,\phi) = \frac{a}{\pi} . e^{-a^2}$$

$$= \underbrace{\frac{1}{2\pi}}_{f_\phi(\phi)} . \underbrace{2a\, e^{-a^2}}_{f_A(a)}$$

$$\boxed{f_{A\phi}(a,\phi) = f_\phi(\phi) \times f_A(a)}$$

A, ϕ are independent r.v.

* Recalling $y_b(t) = h S_b(t)$

$h = a e^{j\phi}$

complex fading coefficient having

density of a : $2a e^{-a^2}$ $0 \le a < \infty$

density of ϕ = $\frac{1}{2\pi}$ $-\pi \le \phi \le \pi$

Example

(1) Problem:— What is the prob. that a transmitted signal is attenuated more than -20 dB. ?

If g is the Gain of the channel

$10 \log_{10} g \le -20$ dB

$\log_{10} g \le -2$

$\Rightarrow g \le 10^{-2}$

$\Rightarrow g \le 0.01$

$\Rightarrow a^2 \le 0.01$

$\Rightarrow a \le 0.1$

$$p(a \leq 0.1) = \int_0^{0.1} 2a\, e^{-a^2}\, da$$

$$= e^{-a^2} \Big|_0^{0.1}$$

$$= 1 - e^{-0.01}$$

$$p(a \leq 0.1) = 0.1$$

Then the prob. that the attenuation is worse than -20 dB is 0.1.

(2) <u>problem:-</u> What is the prob. that the phase ϕ lies b/w $-\pi/3$ to $\pi/3$

$$\phi \in [-\pi/3, \pi/3]$$

$$P_r(-\pi/3 \leq \phi \leq \pi/3) = \int_{-\pi/3}^{\pi/3} 1/2\pi \, d\phi$$

$$= \frac{1}{2\pi}\left(\pi/3 + \pi/3\right)$$

$$= 1/3$$

$$\boxed{P_r(-\pi/3 \leq \phi \leq \pi/3) = 1/3}$$

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* performanu of Wireless and Wireline communication system

BER :— Bit error rate

1 0 0 1 1 1 0 1 0 0

1 0 0 $\boxed{0}$ $\boxed{0}$ 1 0 1 0 $\boxed{1}$

↑ ↑ ↑

Bits are having error

[The prob. of bit error in the information stream]

if 10,000 bits
100 Received in error

$$BER = \frac{100}{10,000} = 0.01$$

* BER of Wireline communication system:–

$$y = S_b = 1.x + n$$

$$y = x + n$$

↑ noise

System model for wireline communication

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n is Additive White Gaussian Noise (AWGN)

$$n \sim \mathcal{N}(0, \sigma_n^2)$$

bit 1 : $+1 \times \sqrt{P}$

bit 0 : $-1 \times \sqrt{P}$

For 1 : $\sqrt{P}$ volt

0 : $-\sqrt{P}$ volt

BPSK

Since the cases are symmetric, we take a case:- where bit '0' was transmitted from the TX.

$$y = -\sqrt{P} + n$$

Bit error occur if $y > 0$

$$n - \sqrt{P} > 0$$

$$n > \sqrt{P}$$

$$n \sim \mathcal{N}(0, \sigma_n^2)$$

$$P(n > \sqrt{P}) = \int_{\sqrt{P}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-\frac{n^2}{2\sigma_n^2}} \, dn$$

Let $\frac{x}{\sigma_n} = t \Rightarrow dx = \sigma_n \, dt$

$$= \int_{\sqrt{P/\sigma_n^2}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-t^2/2} \, \sigma_n \, dt$$

$$P(n > \sqrt{P}) = \int_{\sqrt{P/\sigma_n^2}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \, dt$$

$$\boxed{Q(V) = \int_{V}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \, dt}$$

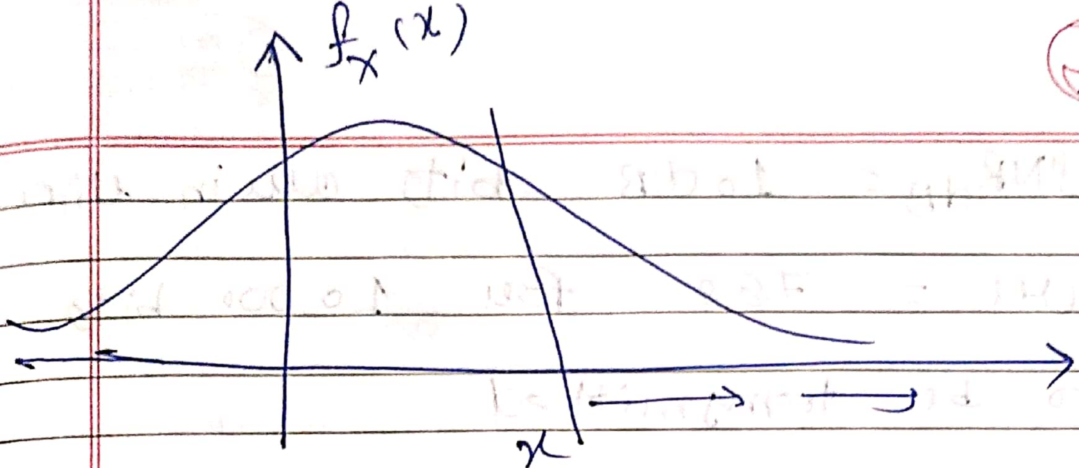
$$\boxed{P(n > \sqrt{P}) = Q\left(\sqrt{P/\sigma_n^2}\right)}$$

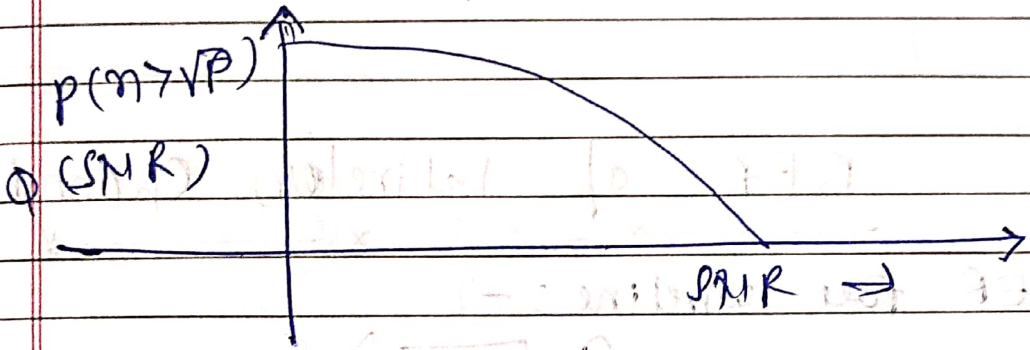
Bit error rate for wire line channel is

$$\boxed{P(n > \sqrt{P}) = Q\left(\sqrt{\frac{P}{\sigma_n^2}}\right)}$$

$$\boxed{P(n > \sqrt{P}) = Q\left(\sqrt{SNR}\right)}$$

For wireline communication system





* **Example**

Prob :- At SNR = 10 dB, what is the BER of wireline comm.

$$SNR = 10 \log_{10} SNR = 10$$

$$\log_{10} SNR = 1$$

$$SNR = 10$$

$$BER = Q(\sqrt{10})$$

$$= 7.82 \times 10^{-4}$$

bits in error in 10,000 bits

$$= 7.82 \times 10^{-4} \times 10\,000$$

$$= 7.82 \text{ bits are in error}$$

Imp

(a) $SNR_{dB} = 10\,dB$ bits are in error are $= 7.82$ for $10{,}000$ bits to be transmitted

lect4

BER of Wireless Channel

BER for wireline :-

$$BER = Q\left(\sqrt{\frac{P}{\sigma_n^2}}\right)$$

Example 2 : for Wireline channel

compute the $SNR|_{dB}$ required for the probability for $BER = 10^{-6}$

$$10^{-6} = Q\left\{\sqrt{\frac{P}{\sigma_n^2}}\right\}$$

$$\Rightarrow \sqrt{SNR} = Q^{-1}\left\{10^{-6}\right\}$$

$$\Rightarrow SNR = \left[Q^{-1}\left(10^{-6}\right)\right]^2$$

$$\Rightarrow SNR = (4.7534)^2$$

$$\Rightarrow SNR = 22.595$$

$$(SNR)_{dB} = 10\log_{10}(22.59)$$

$$\boxed{(SNR)_{dB} = 13.6\ dB}$$

→ BER Analysis of Wireless channel

$$y = hx + n$$

h: fadding coefficient; n: noise

$$h = a_i e^{j\alpha}$$

a_i: attenuation; α: delay

power of the signal $= P$
power of the noise $= \sigma_n^2$

Recieved power $= P \times |h|^2$

$$= P \times a^2$$

a: mag of h

Received SNR

$$= \frac{Pa^2}{\sigma_n^2} =$$

Wired BER	Wireless BER
$SNR = \frac{P}{\sigma_n^2}$	$SNR = \frac{a^2 P}{\sigma_n^2}$
$Q\left(\sqrt{\frac{P}{\sigma_n^2}}\right)$	$Q\left(\sqrt{\frac{a^2 P}{\sigma_n^2}}\right)$

$$BER = \int_{\sqrt{\frac{a^2 P}{\sigma_n^2}}}^{\infty} \frac{1}{2\pi} e^{-x^2/2}\, dx$$

a is a random quantity

* To get the average performance, since a is random

$$\text{Average of } g(a) = \int_0^{\infty} g(a)\, f_A(a)\, da$$

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* The average bit-error-rate :—

BER of a wireless system $= \int_0^\infty Q\left(\sqrt{\frac{a^2 P}{\sigma_n^2}}\right) \cdot f_A(a)$

$$= \int_0^\infty Q\left(\sqrt{\frac{a^2 P}{\sigma_n^2}}\right) \underbrace{2a e^{-a^2}}_{P} da$$

Rayligh fading

$$= \int_0^\infty Q\left(\sqrt{\frac{a^2 P}{\sigma_n^2}}\right) 2a e^{-a^2} da$$

$$= \int_0^\infty \int_{\sqrt{a^2\mu}}^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \; 2a e^{-a^2} da$$

$\mu = \frac{P}{\sigma_n^2} = SNR$ $\quad \cancel{\mu = \sqrt{\frac{a^2 P}{\sigma_n^2}}}$

Let $\frac{x}{a\sqrt{\mu}} = u$

$dx = a\sqrt{\mu}\, du$

$$= \int_0^\infty \int_1^\infty 2a e^{-a^2} a\sqrt{\mu}\, e^{-a^2 \mu u^2/2} du\, da$$

$$= \frac{\sqrt{\mu}}{\sqrt{2\pi}} \int_1^\infty \int_0^\infty \frac{2a^2}{\sqrt{2\pi}} e^{-\frac{a^2}{2}(2+\mu u^2)} \, da \, du$$

Let

$$\because \int_0^\infty \frac{2y^2}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} = \sigma^2$$

$$\Rightarrow \int_0^\infty \frac{2y^2}{\sqrt{2\pi}} e^{-y^2/2\sigma^2} = \sigma^3$$

$$\int_0^\infty \frac{2a^2}{\sqrt{2\pi}} e^{-\frac{a^2}{2}(2+\mu u^2)} \, da$$

$$a = y$$

$$\frac{1}{\sigma^2} = 2+\mu u^2$$

$$\sigma = \frac{1}{\sqrt{2+\mu u^2}}$$

$$\sigma^3 = \left(\frac{1}{2+\mu u^2}\right)^{3/2}$$

$$= \sqrt{\mu} \int_{1}^{\infty} \left(\frac{1}{2+\mu u^2} \right)^{3/2} du$$

substitution :-

$$u = \sqrt{\frac{2}{\mu}} \tan\theta$$

$$du = \sqrt{\frac{2}{\mu}} \sec^2\theta \, d\theta$$

$$2+\mu u^2 = 2+\mu \cdot \frac{2}{\mu} \tan^2\theta$$

$$= 2\sec^2\theta \, d\theta$$

$$u = 1, \quad \sqrt{\frac{2}{\mu}} \tan\theta = 1$$

$$\theta = \tan^{-1}\left(\sqrt{\mu/2}\right)$$

$$u = \infty, \quad \sqrt{\frac{2}{\mu}} \tan\theta = \infty$$

$$\theta = \tan^{-1}(\infty)$$

$$\theta = \pi/2$$

$$= \cancel{\sqrt{\mu}} \int_{\tan^{-1}(\sqrt{\mu/2})}^{\pi/2} \frac{1}{(2\sec^2\theta)^{3/2}} \cancel{\sqrt{\frac{2}{\mu}}} \sec^2\theta \, d\theta$$

$$= \frac{1}{2} \int_{\tan^{-1}\sqrt{\mu/2}}^{\pi/2} \frac{\sec^2\theta}{\sec^3\theta} \, d\theta$$

$$= \frac{1}{2}\left[\sin\theta\right]_{\tan^{-1}(\sqrt{\mu/2})}^{\pi/2}$$

$$= \frac{1}{2}\left[1 - \sin\tan^{-1}\sqrt{\mu/2}\right]$$

$$\because \sin\theta = \sqrt{\frac{\tan^2\theta}{1+\tan^2\theta}}$$

$$\Rightarrow \sin\tan^{-1}\left(\sqrt{\mu/2}\right) = \sqrt{\frac{\tan^2\tan^{-1}\sqrt{\mu/2}}{1+\tan^2\tan^{-1}\sqrt{\mu/2}}}$$

$$= \sqrt{\frac{\mu/2}{1+\mu/2}}$$

$$BER = \frac{1}{2}\left[1 - \sqrt{\frac{\mu}{2+\mu}}\right]$$

$$\boxed{BER = \frac{1}{2}\left[1 - \sqrt{\frac{SNR}{2+SNR}}\right]}$$

BER =

Wired channel	Wireless channel
$y = x + n$	$y = hx + n$
$BER = Q(\sqrt{SNR})$	$BER = \frac{1}{2}\left[1 - \sqrt{\frac{SNR}{2+SNR}}\right]$

$$= \frac{1}{2}\left[1 - \frac{1}{\sqrt{1 + \frac{2}{SNR}}}\right]$$

$$\simeq \frac{1}{2}\left[1 - \left(1 - \frac{1}{2}\frac{2}{SNR}\right)\right]$$

$$\boxed{BER \simeq \frac{1}{2}\frac{1}{SNR}}$$

BER for wireless channel at high SNR

$$\boxed{BER \simeq \frac{1}{2\,SNR}}$$

* Example 1 for wireless

compute the BER of a WC system at SNR = 20 dB

$$10\log_{10} SNR = 20 \text{ dB}$$

$$\log_{10} SNR = 2$$

$$SNR = 10^2$$

$$BER = \frac{1}{2 \times SNR}$$

$$= \frac{1}{200}$$

$$BER = 5 \times 10^{-3} = 50 \times 10^{-4}$$

(Has very high BER)

* compare with wireline communication

$$SNR = 10 \text{ dB}$$

$$BER = 7.8 \times 10^{-4}$$

Example ② - previous

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Example :— To compute the required $(SNR)_{dB}$ for BER of 4 10^{-6} order? for wireless communication

$$10^{-6} = \frac{1}{2SNR}$$

$$SNR = 0.5 \times 10^{+6}$$
$$= 5 \times 10^{+7}$$
$$= 10\log_{10}(5 \times 10^{7})$$
$$= +70 \log_{10} 5$$

$$\boxed{SNR = 57\ dB}$$

However, in wirline comm. SNR required

SNR = 13.6 dB

so the difference = 57 − 13.6
= 43 dB !

This is because of distructive interference

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$$10\log_{10} \frac{SNR_{wireless}}{SNR_{wireline}} = 43\ dB$$

$$\frac{SNR_{wireless}}{SNR_{wired}} = 10^{4.3} \approx 10^{4}$$

$$= 10 \times 10^{3}$$

$\Rightarrow SNR_{wireless} = SNR \times 10{,}000$

$\Rightarrow \boxed{P_{wireless} = 10{,}000 \times P_{wired}}$

* If we want to achive ~~more~~ same bit-error-rate as in wired communication system, in Wlc we need a huge amount of power.

* This is due to the fact of fading behavior of Wlc channel

* The BER expression for wired and wireless system

Wireless system

$$BER \approx \frac{1}{2\,SNR}$$

$$\tan^2 \tan^{-1} x = \theta$$

$$\tan(\tan^{-1} x) = \sqrt{\theta}$$

$$\tan^{-1} x = \tan^{-1}\sqrt{\theta} \Rightarrow \sqrt{x} = \sqrt{\theta}$$

$$x = \theta$$

Exact expression

$$= \frac{1}{2}\left[1 - \sqrt{\frac{SNR}{2+SNR}}\right]$$

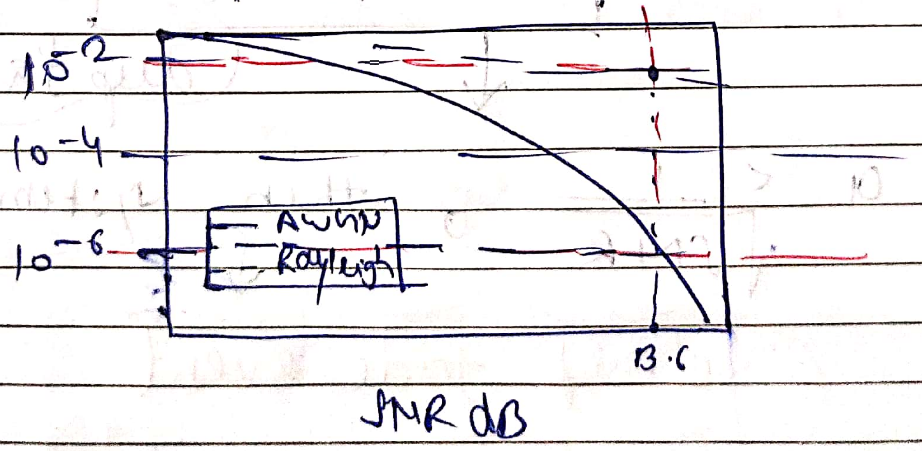
* In wired communication

$$BER \Rightarrow Q\{\sqrt{SNR}\}$$

$$\boxed{Q(\sqrt{x}) \leq \frac{1}{2} e^{-x/2}}$$

** Decreasing exponentially in wired. **

* plot on ~~matl~~ MATLAB



* Wireless communication system

$$y = hx + n$$

fading coefficient

noise → σ_n^2

$|h|^2 P$

performance of wireless system is worsed BAD when the Rx power < the noise power

Thus, performance is bad, when:

$$|h|^2 P = a^2 P < \sigma_n^2$$

perf

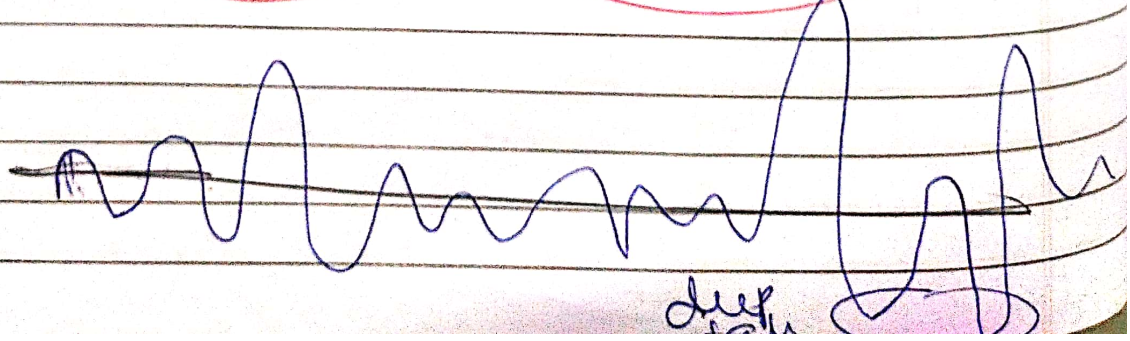
$$\Rightarrow a^2 < \frac{\sigma_n^2}{P} = \frac{1}{SNR}$$

$$\Rightarrow \boxed{a < \frac{1}{\sqrt{SNR}}}$$

deep fade

when $a < \frac{1}{\sqrt{SNR}}$, then system is in

deep fade event



Vehant

Bionic

$$f_A(a) = 2ae^{-a^2}$$

* Prob. of deep-fade event is

$$Pr\left(a < \frac{1}{\sqrt{SNR}}\right)$$

$$= \int_{-\infty}^{1/\sqrt{SNR}} f_A(a)\,da = \int_{-\infty}^{1/\sqrt{SNR}} 2a\,e^{-a^2}\,da$$

for high SNR

$$\simeq \int_{0}^{1/\sqrt{SNR}} 2a\,da = \frac{1}{SNR}$$

prob of deep fade even is

$$= \frac{1}{SNR}$$

$$BER = \frac{1}{2SNR}$$

$$\simeq \frac{\text{prob of deep fade}}{2}$$

or $\boxed{BER \simeq \text{prob. of deep fade}}$

~~PoE~~ =

poor performance of wireless system is arising from the deep fade

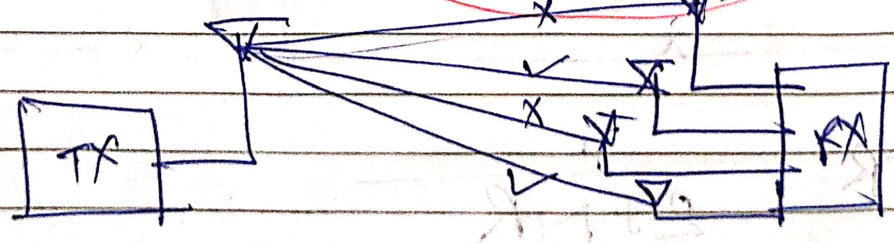
Destructive interference is the cause for this

* How to improve the performance of the system is ?

Ans is :- one way is

Diversity

multiple antenna system



$Y = HX + n$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} x + \underbrace{\begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}}_{\bar{n}} \leftarrow \text{noise vector}$$

$$\boxed{\bar{y} = \bar{h} x + \bar{n}}$$

Vector notation for multiple ant.

Lect 09

delay spread and coherence BW

$$h(t) = \sum_{i=0}^{L-1} a_i \, \delta(t - \tau_i)$$

i^{th} path = $\begin{cases} a_i & \text{attenuation factor} \\ \tau_i & \text{delay factor} \end{cases}$

* power profile of the wireless channel:—

$$\phi(\tau) = |h(\tau)|^2$$

$$= \sum_{i=0}^{L-1} \underbrace{|a_i|^2}_{\text{arriving power}} |\delta(t - \tau_i)|^2$$

$$= \sum_{i=0}^{L-1} g_i \, \delta(t - \tau_i)$$

g_i is the gain of the i^{th} path.

Time domain characteristics

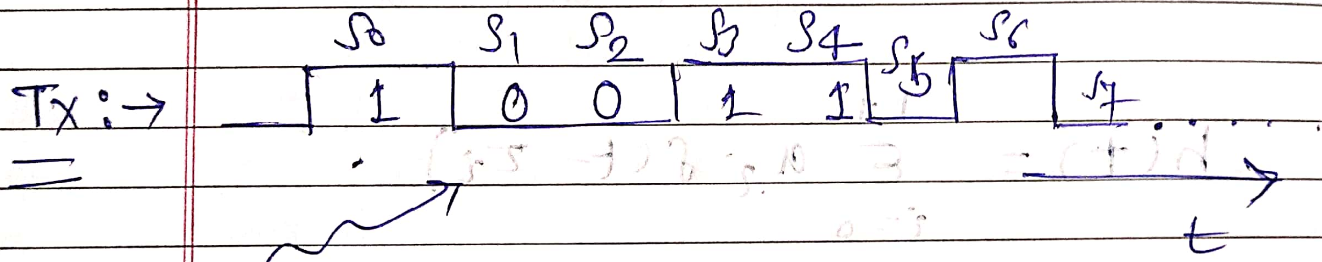
Freq. domain characteristics

* Delay spread and coherence BW.

* ① Max delay spread
* ② RMS delay spread

* What is the impact or significance of delay spread on wireless communications

* Let us consider transmitted signal



Transmitted Signal :- comprising various symbols $S_0, S_1, S_2, \ldots S_6$

* Symbol time = T time / symbol

* Now, let us look at a simple wireless multipath channel

$L = 2$ multipath components

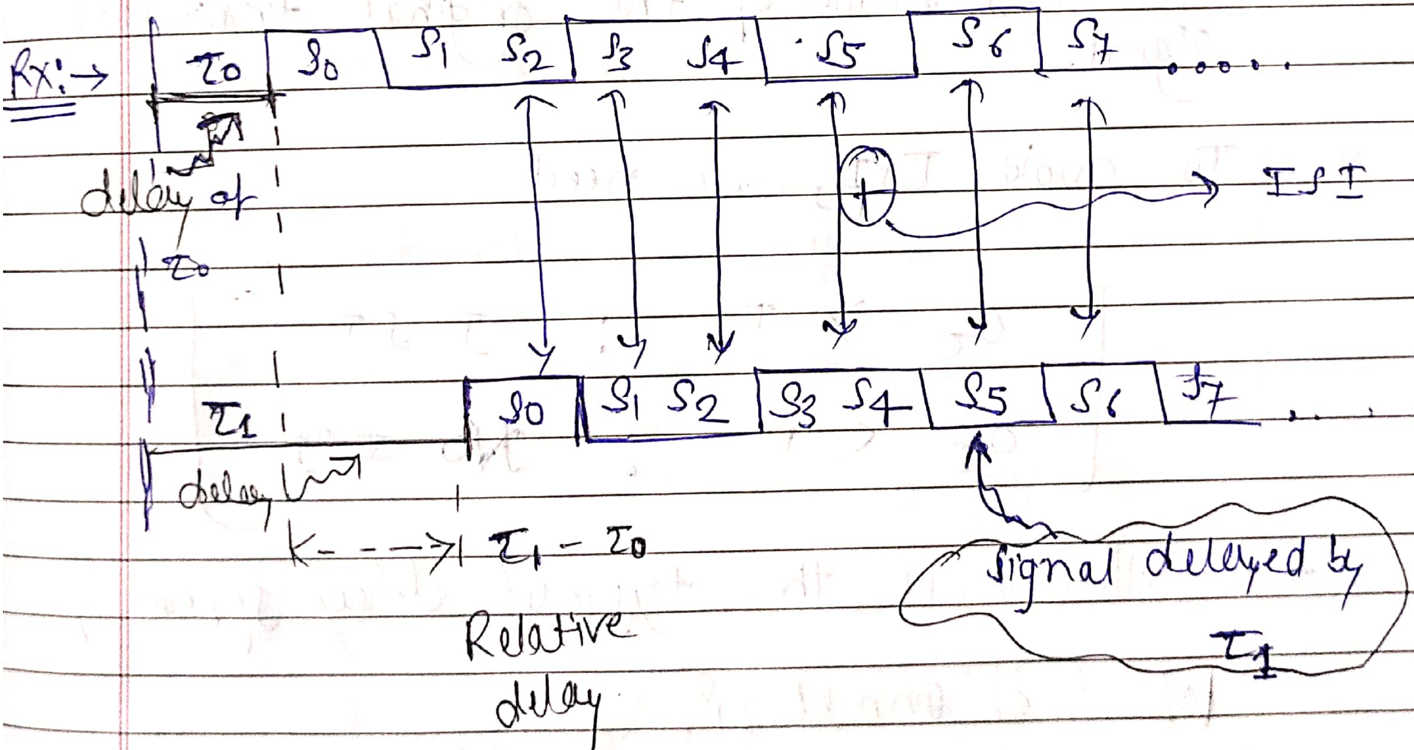
$$h(t) = a_0\,\delta(t - \tau_0) + a_1\,\delta(t - \tau_1)$$

$$a_0 = a_1 = 1$$

Therefore channel model will become

$$h(t) = \underbrace{\delta(t-\tau_0)}_{\text{Let us direct path component}} + \underbrace{\delta(t-\tau_1)}_{\text{scattered or reflected component}}$$

Now the Rx^d signal correspond to the multipath channel



* Different symbols are interfering each other called as ISI

classmate

Date ____

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* When $\underbrace{\tau_i - \tau_0}_{10\sigma_\tau} > T$

$10\sigma_\tau > T$

Then different symbols interfere with each other

(ISI)

* delay spread is larger than the symbol time leading to ISI

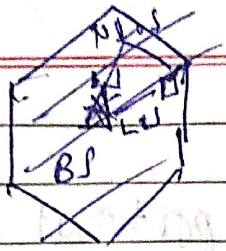
* ISI is undesirable since it leads to distortion of the original transmitted signal.

* To avoid ISI, we need

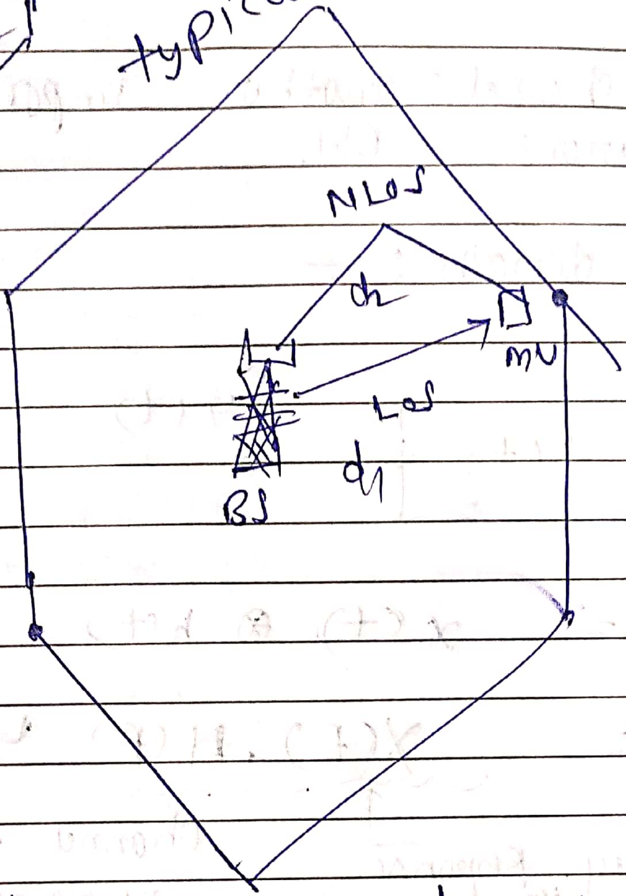
$$\left[\begin{array}{lcl} \sigma_\tau > T & : & ISI \\ \sigma_\tau < T & : & \text{No ISI} \end{array}\right]$$

* What is the typical delay spread of WC channel ?

PT 3M



Let us consider a typical hexagonal structure cell



LOS

NLOS in Km

Δd = difference in the distances $= d_1 - d_2$

$V = d/t$

Δd is of the order of Km

* Now therefore the delay spread is given as

= differences in the time of arrival

Delay spread $= \dfrac{\Delta d}{c} = \dfrac{1\text{ Km}}{3 \times 10^8} \approx \cancel{3}\ \mu s \approx 3.33\ \mu s$

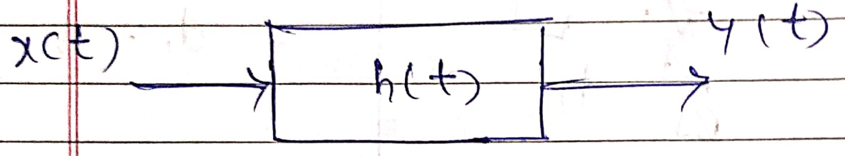
$\approx$ ~~1-10~~ μs

$$\boxed{\begin{array}{ll} T < \sigma_\tau \approx 2\ \mu s \rightarrow & ISI \\ T > \sigma_\tau \approx 2\ \mu s & NO\ ISI \end{array}}$$

* * Coherence B.W.

In Wireless channel another important term
i) Coherence B.W.

↳ Time domain :-



$$y(t) = x(t) \circledast h(t)$$

$$Y(f) = X(F) \cdot H(F)$$

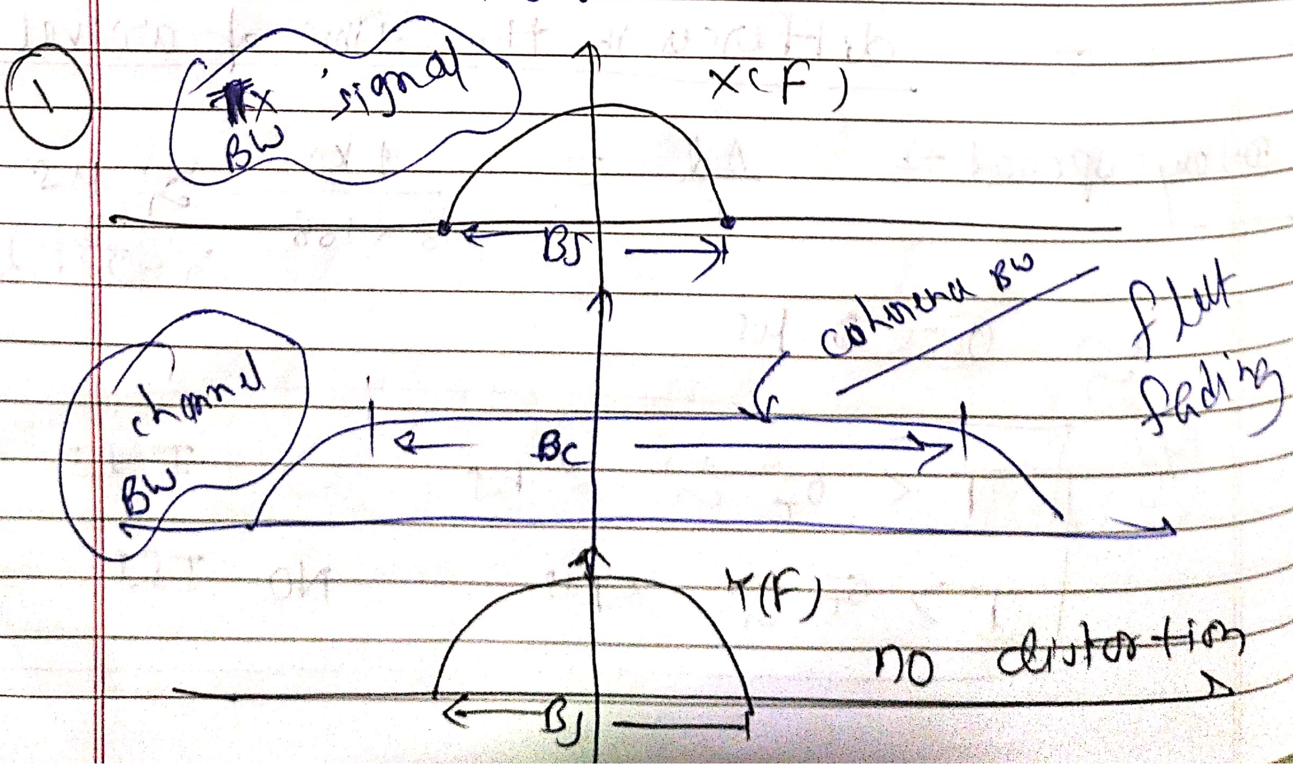
Freq. response of RX

freq. Response of i/p signal Tx

Channel freq. response

Two scenarios :-

(1)

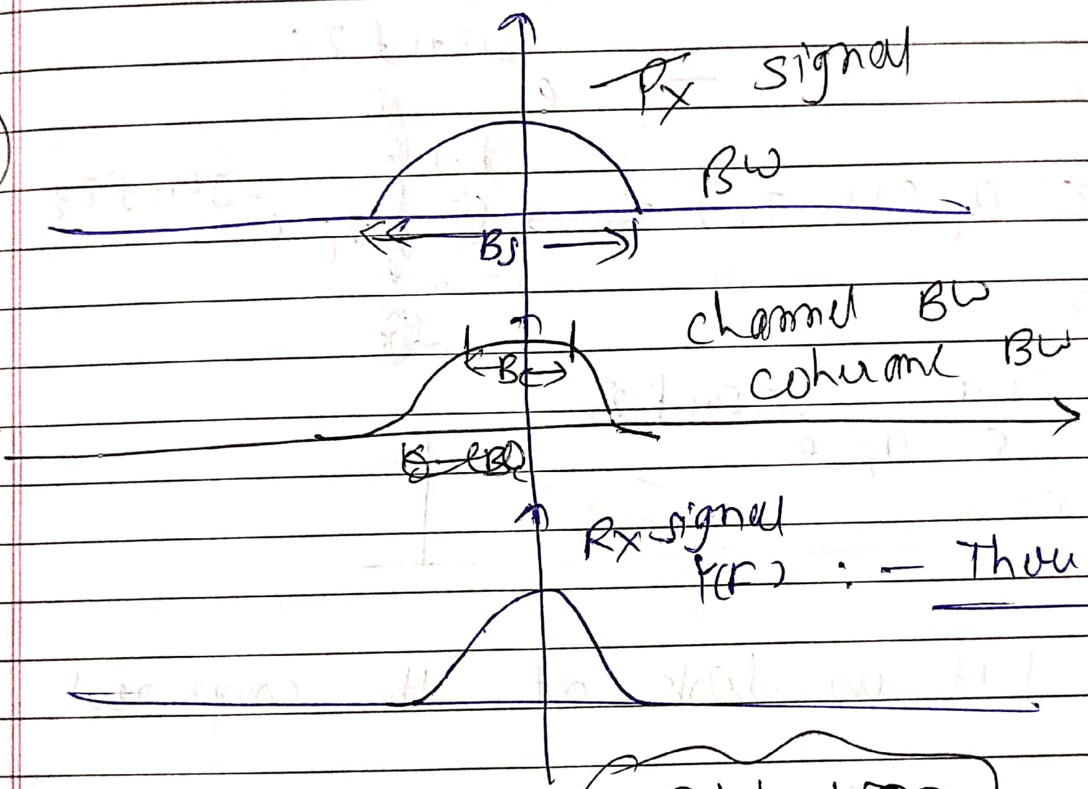


i.e., $B_S < B_C$

signal BW $\quad$ Coherence BW.

$$\boxed{Y(F) = X(F)\,H(F)}$$

②



Freq. selective fading $\rightarrow$ Distortion $B_S > B_C$

other than this

Flat fading with no distortion $\rightarrow$ if $B_S < B_C$

→ * Impulse response of the channel

$$h(t) = \sum_{i=0}^{L-1} a_i \, \delta(t-\tau_i)$$

$$\delta(t-\tau_i) \Longleftrightarrow \int_{-\infty}^{\infty} \delta(t-\tau_i)\, e^{-j2\pi f t}\, dt$$

$$= e^{-j2\pi f \tau_i}$$

$$\sum_{i=0}^{L-1} a_i \delta(t-\tau_i) \longleftrightarrow \sum_{i=0}^{L-1} a_i \, e^{-j2\pi f \tau_i}$$

$$\boxed{H(f) = \sum_{i=0}^{L-1} a_i \, e^{-j2\pi f \tau_i}}$$

* Let us look at the component

$e^{-j2\pi F \tau_i}$

At $F = 0$ phase $2\pi F \tau_i = 0$

at $F = 1/2\tau_i$ phase $2\pi \frac{1}{2\tau_i} \tau_i = \pi$

phase has changed significantly

~~at F=~~

Freq. response has changed significantly

↳ $F = 0$ → $\phi \Rightarrow 0$

↳ $F = 1/2\tau_i$ → $\phi \rightarrow \pi$

* In General, approximately the BW of channel

$$BW = \frac{1}{\tau_i} \equiv \frac{1}{\sigma_\tau}$$

τ_i ← delay spread

$$B.W. = \frac{1}{\sigma_\tau} \leftarrow \text{delay spread}$$

↑ Coherance BW

$\sigma_\tau = 2\mu s$

$$\boxed{BW = \frac{1}{2\mu s} = 500KHz}$$

* Freq. selective distortion occurs if

✓ $\boxed{B_s > B_c}$ ISI distortion in freq.

$$\Rightarrow \frac{1}{T} > \frac{1}{\sigma_\tau}$$

$$\Rightarrow \boxed{\sigma_\tau > T}$$ ✓ ISI in time

$$\sigma_\tau = \frac{1}{B_c} > \frac{1}{B_s}$$

$$\boxed{B_s > B_c}$$

Summary

If $B_s > B_c \Rightarrow \sigma_\tau > T$

$\Rightarrow$ Frequency selective Fading

$\Rightarrow$ ISI

If $B_s < B_c \Rightarrow \sigma_\tau < T$

$\Rightarrow$ Freq. Flat fading

No ISI