

**Основні правила диференціювання**

$$(u + v)' = u' + v'$$

$$(u - v)' = u' - v'$$

$$(uv)' = u'v + uv'$$

$$(Cu)' = Cu'$$

$$(uvw)' = u'vw + uv'w + uvw'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}, \quad v \neq 0$$

$$\left(\frac{C}{v}\right)' = -C \frac{v'}{v^2}, \quad v \neq 0$$

$$\left(\frac{u}{C}\right)' = \frac{1}{C}u'$$

$$f(g(x))' = f'(g(x)) \cdot g'(x)$$

$$y'_x = \frac{y'_t}{x'_t}$$

$$y' = u^v \ln u \cdot v' + vu^{v-1}u'$$

$$dy = f'(x)dx$$

**Основні похідні**

$$C' = 0$$

$$(u^n)' = nu^{n-1}u'$$

$$(\sqrt{u})' = \frac{1}{2\sqrt{u}}u'$$

$$\left(\frac{1}{u}\right)' = -\frac{1}{u^2}u'$$

$$(a^u)' = a^u \ln a \cdot u'$$

$$(\arccos u)' = -\frac{1}{\sqrt{1-u^2}}$$

$$(\log_a u)' = \frac{1}{u \ln a}u'$$

$$(\ln u)' = \frac{1}{u}u'$$

$$(\sin u)' = \cos u \cdot u'$$

$$(\cos u)' = -\sin u \cdot u'$$

$$(\operatorname{tg} u)' = \frac{1}{\cos^2 u}u'$$

$$(\operatorname{ctg} u)' = \frac{1}{\sin^2 u}u'$$

$$(\arcsin u)' = \frac{1}{\sqrt{1-u^2}}u'$$

$$(e^u)' = e^u u'$$

$$(\operatorname{arctg} u)' = \frac{1}{1+u^2}u'$$

$$(\operatorname{arccotg} u)' = -\frac{1}{1+u^2}u'$$

$$(\operatorname{sh} u)' = \operatorname{ch} u \cdot u'$$

$$(\operatorname{ch} u)' = \operatorname{sh} u \cdot u'$$

$$(\operatorname{th} u)' = \frac{1}{\operatorname{ch}^2 u}u'$$

$$(\operatorname{cth} u)' = -\frac{1}{\operatorname{sh}^2 u}u'$$

**Основні інтеграли**

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$$

$$\int du = u + C$$

$$\int \frac{du}{\cos^2 u} = \operatorname{tg} u + C$$

$$\int \frac{du}{u} = \ln |u| + C$$

$$\int \operatorname{ctg} u du = \ln |\sin u| + C$$

$$\int e^u du = e^u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \frac{du}{\sin u} = \ln \left| \operatorname{tg} \frac{u}{2} \right| + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{du}{\sin^2 u} = -\operatorname{ctg} u + C$$

$$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{u}{a} + C$$

$$\int 0 du = C$$

$$\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln \left| u + \sqrt{u^2 \pm a^2} \right| + C$$

$$\int \operatorname{sh} u du = \operatorname{ch} u + C$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$$

$$\int \operatorname{ch} u du = \operatorname{sh} u + C$$

$$\int \frac{du}{\cos u} = \ln \left| \operatorname{tg} \left( \frac{u}{2} + \frac{\pi}{4} \right) \right| + C$$

$$\int \frac{du}{\operatorname{sh}^2 u} = -\operatorname{cth} u + C$$

$$\int \operatorname{tg} u du = -\ln |\cos u| + C$$

$$\int a^n du = \frac{a^u}{\ln a} + C$$

$$\int \cos u du = \sin u + C$$

$$\int \frac{du}{\operatorname{ch}^2 u} = \operatorname{th} u + C$$

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a) - \Phi. \text{ Н.-П.}$$

## Еквівалентні нескінченно малі функції

$$x \sim \sin x \sim \operatorname{tg} x \sim \arcsin x \sim \arctg x \sim e^x - 1 \sim \ln(1+x), \quad 1 - \cos x \sim \frac{x^2}{2}, \quad a^x - 1 \sim x \ln a, \quad (a+x)^k - 1 \sim kx$$

$$\sqrt{1+x} - 1 \sim x/2, \quad \ln \cos x \sim -x^2/2, \quad \log_a(1+x) \sim x \log_a e, \quad \text{якщо } x \rightarrow 0$$

## Важливі границі

$$\lim_{x \rightarrow 0} \frac{\sin kx}{x} = k \quad \lim_{x \rightarrow \infty} \frac{\operatorname{tg} kx}{x} = k \quad \lim_{x \rightarrow 0} \frac{\arcsin kx}{x} = k \quad \lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x = e^k \quad \lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \log_a e$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2} \quad \lim_{x \rightarrow 0} \frac{(1+x)^k - 1}{x} = k \quad \lim_{x \rightarrow 0} \frac{\arctg kx}{x} = k \quad \lim_{t \rightarrow 0} (1+t)^{\frac{1}{t}} = e \quad \lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$$

## Прогресії

$$a_n = a_1 + d(n-1), \quad S_n = \frac{a_1 + a_n}{2} \cdot n - \text{арифм.}$$

$$b_n = b_1 \cdot q^{n-1}, \quad S_n = \frac{b_1(q^n - 1)}{q - 1}, \quad q \neq 1 - \text{геомтр.}$$

## Комбінаторика

$$P_n = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n = n! \quad C_n^k = \frac{n!}{k! \cdot (n-k)!} \quad A_n^k = \frac{n!}{(n-k)!} \quad \overline{C_n^k} = C_{n+k-1}^k = \frac{(n+k-1)!}{k! \cdot (n-k)!} \quad \overline{A_n^k} = n^k$$

## Логарифми

### Інші формули

$$a, b, c > 0, \quad a \neq 1, \quad k \neq 0$$

$$a^{\log_a b} = b, \quad \log_a a = 1$$

$$\log_a(a \cdot b) = \log_a b + \log_a c$$

$$\log \frac{b}{c} = \log_a b - \log_a c$$

$$\log_a b^n = n \cdot \log_a b$$

$$\log_{a^k} b = \frac{1}{k} \cdot \log_a b$$

$$\ln \frac{1}{a} = -\ln a$$

$$\log_a 1 = 0$$

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1} - \Phi. \text{ М.}$$

$$\operatorname{grad} u(M) = \frac{\partial u(M)}{\partial x} \vec{i} + \frac{\partial u(M)}{\partial y} \vec{j} + \frac{\partial u(M)}{\partial z} \vec{k} - \text{означення градієнту}$$

$$z^n = |z|^n (\cos n\varphi + i \sin n\varphi), \quad \frac{z_1}{z_2} = \frac{\rho_1}{\rho_2} (\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2))$$

$$\int u dv = uv - \int v du, \quad \sqrt[n]{z} = \sqrt[n]{|z|} \left( \cos \frac{\varphi + 2\pi k}{n} + i \sin \frac{\varphi + 2\pi k}{n} \right)$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2), \quad (a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

$$S_{\text{кола}} = \pi r^2, \quad l_{\text{кола}} = 2\pi r, \quad V_{\text{сфери}} = \frac{4}{3}\pi r^3, \quad S_{\text{сфери}} = 4\pi r^2$$

## Криві другого порядку

$$x^2 + y^2 = r^2 - \text{коло} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \text{еліпс} \quad \pm \frac{y^2}{b^2} \mp \frac{x^2}{a^2} = 1 - \text{гіпербола} \quad y^2 = 2px - \text{парабола}$$