Distributed Graph Coloring

Based on the Japanese tree frog calling behaviour

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Introduction

- Graph coloring problem.
- Practical applications.
- Implementation of the FrogSim algorithm.
- Results.

Description of the chromatic number problem

- Working on a graph G, with a set of node V and a set of edge E.

$$G = (V, E)$$

- A valid k-coloring of a graph is an assignation of a color (within k colors) to each node such that there is not two adjacent nodes with the same color.
- Here we want to find a valid coloring using the minimum k needed[1].
- NP-Hard problem.

[1] Called the chromatic number of G, $\chi(G)$

Concrete applications

- Frequency assignation for wireless networks.
  - graph: a network topology.
  - nodes: the emitters.
  - edges: between the emitters near enough to interfere.
  - colors: the frequencies used by the emitters.
- Jobs scheduling.
  - graph: a set of jobs to schedule.
  - nodes: the jobs.
  - edges: the shared resources of the system between two jobs.
  - colors: the time slots.

Other algorithm

This is a NP-hard problem. Then, we use heuristic to solve it.
Centralized algorithm:

- Genetic algorithm.
- DSATUR algorithm.

Initialization of the system

Each node contains:

- phase value θ_i.
- color value c_i.
- relevance value r_i.
- a value controlling the amplitude of change α_i.
- a stack M_i containing the messages sent by the neighbours.

The system contains:

- a spanning tree.
- the root of that tree.
- a memorised coloring.
- a convergence rate value ρ.

Event during phase 1 execution

When an event occurs for a node i:

1. $\theta_i = \theta_i + \alpha_i \times \sum r_m * inc(\theta_m - \theta_i)$, where:

$$inc(x) = \begin{cases} x - 0.5 & \text{if } x \geq 0 \\ x + 0.5 & \text{if } x < 0 \end{cases}$$

2. $c_i = \min \{c \in \mathbb{N} | c \neq c_m \forall m \in M_i\}$
3. $m = \langle \theta_m, c_m, relevance_m \rangle$
4. $\alpha_i = \frac{\alpha_i}{\rho}$

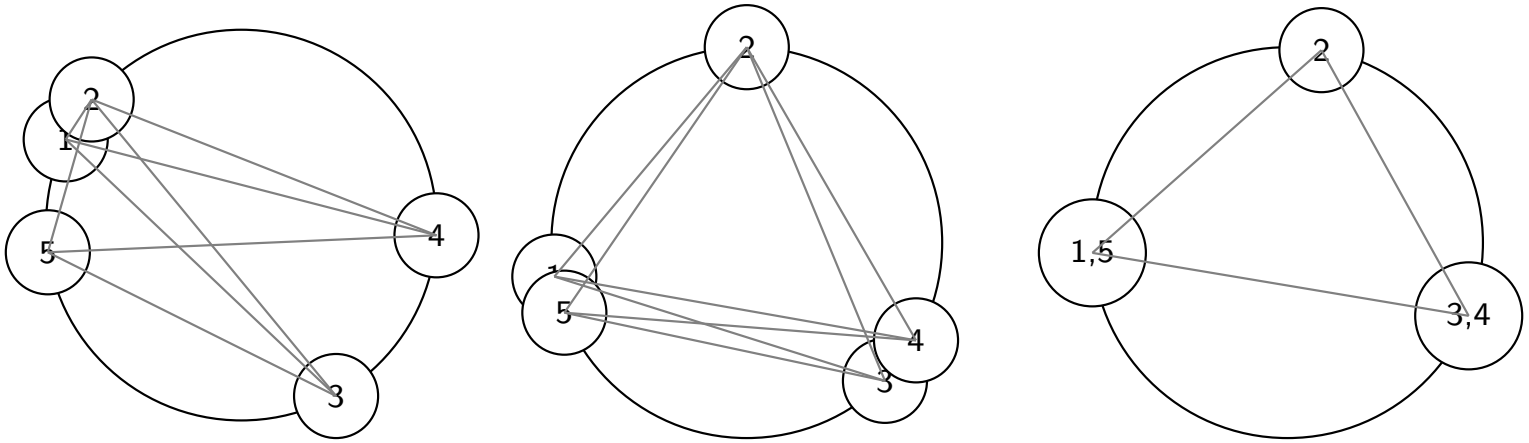


Figure : Example for two oscillators.

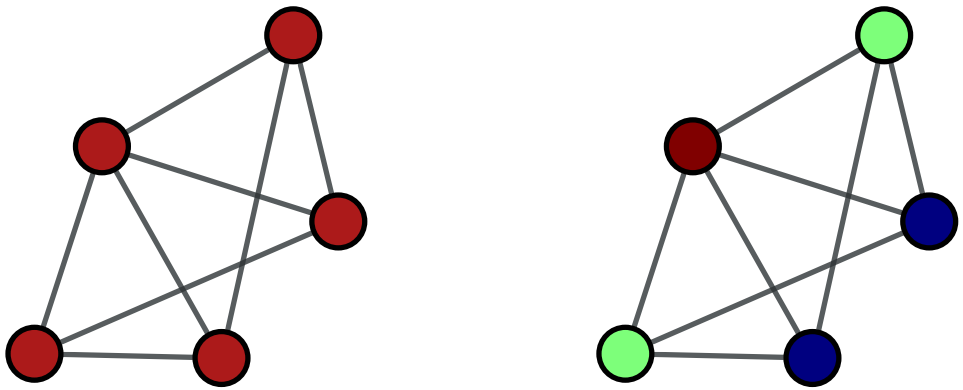


Figure : Example for two oscillators.

Event during phase 2 execution

An event for a node i during a communication round:

- For the first round:

$$p_i = \begin{cases} r, \text{ with } r \text{ random} \mid r \in \mathbb{N} & \text{if } p_i = 1 \\ 0 & \text{if } p_i > 1 \end{cases}$$

- The other communication rounds:
  1. $c_i = \min\{c \in \mathbb{N} \mid (c \neq c_m \wedge p_m \geq p_i)\, \forall m \in M_i\}$
  2. $p_i = \max\{p_m \mid m \in M_i\}$
  3. $m = \langle c_m, p_m \rangle$

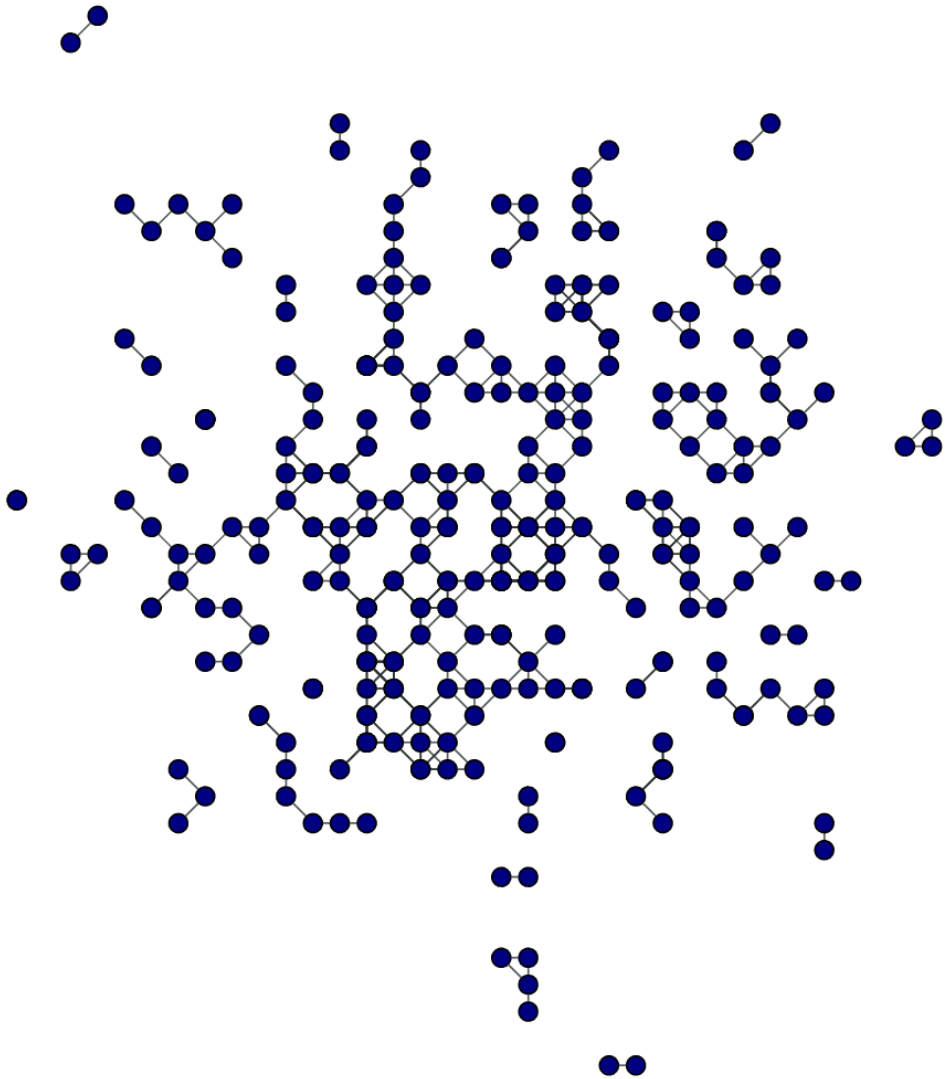


Figure : Coloration at $round = 1$

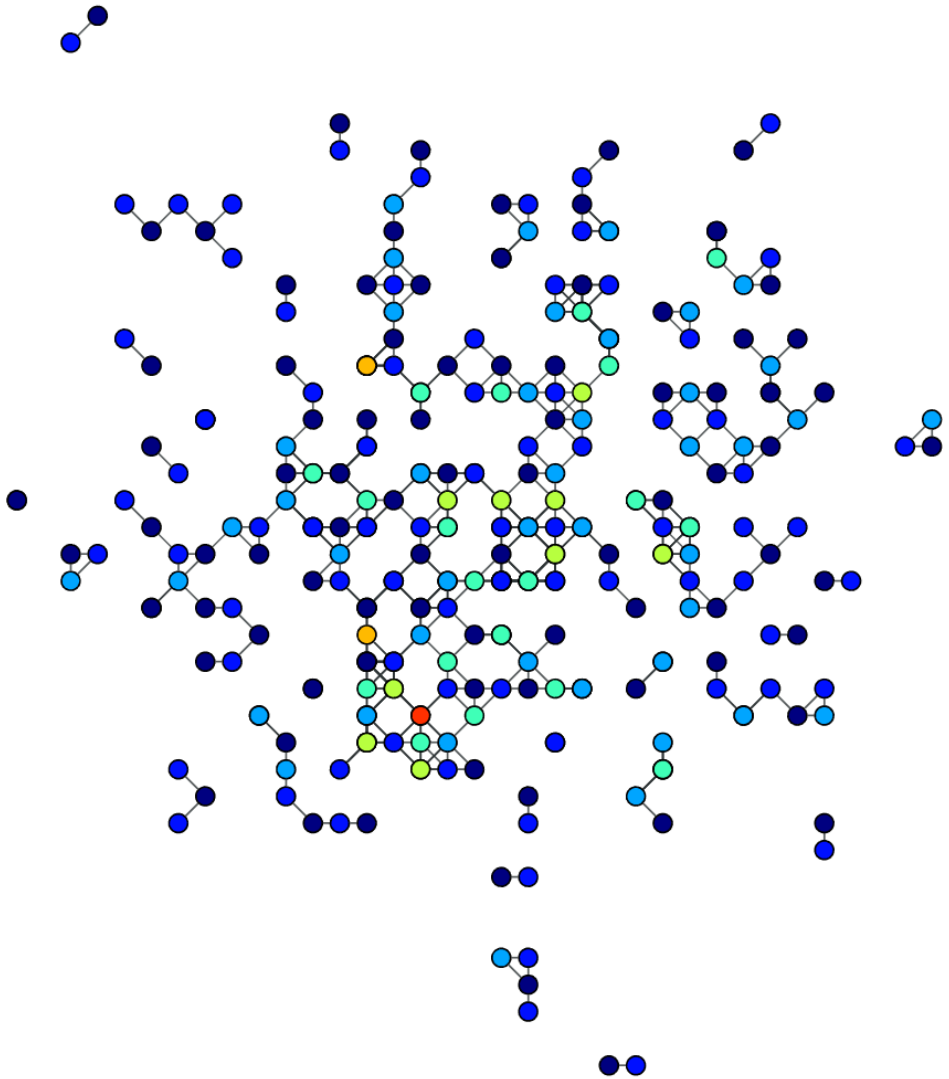


Figure : Coloration at *round* = 2

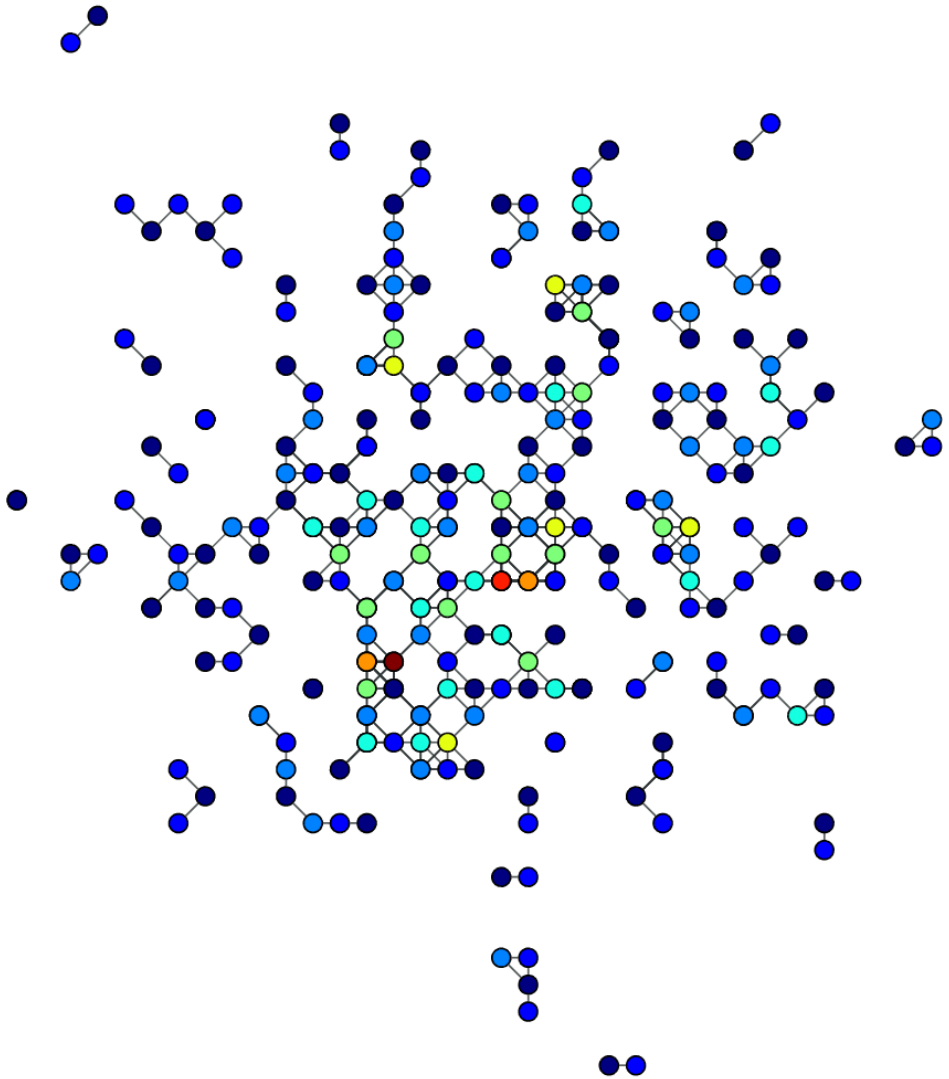


Figure : Coloration at *round* = 3

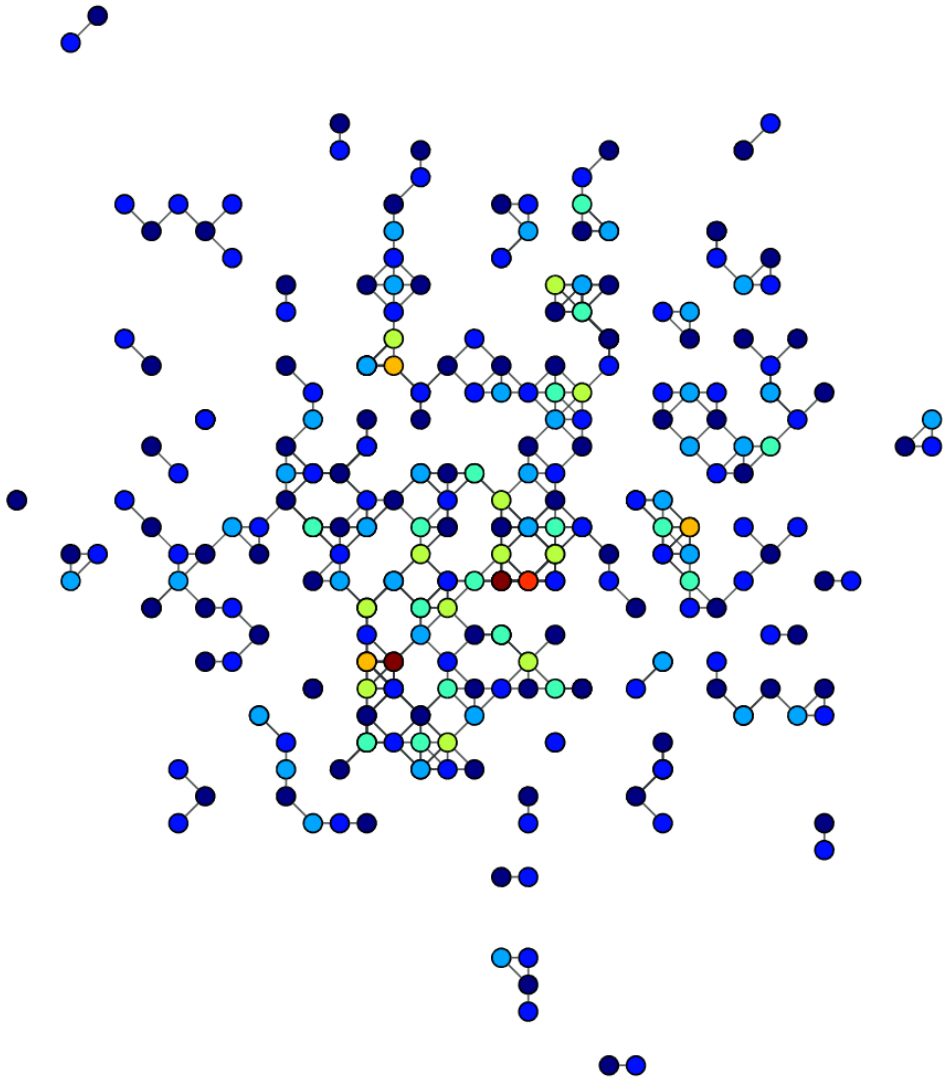


Figure : Coloration at $round = 4$

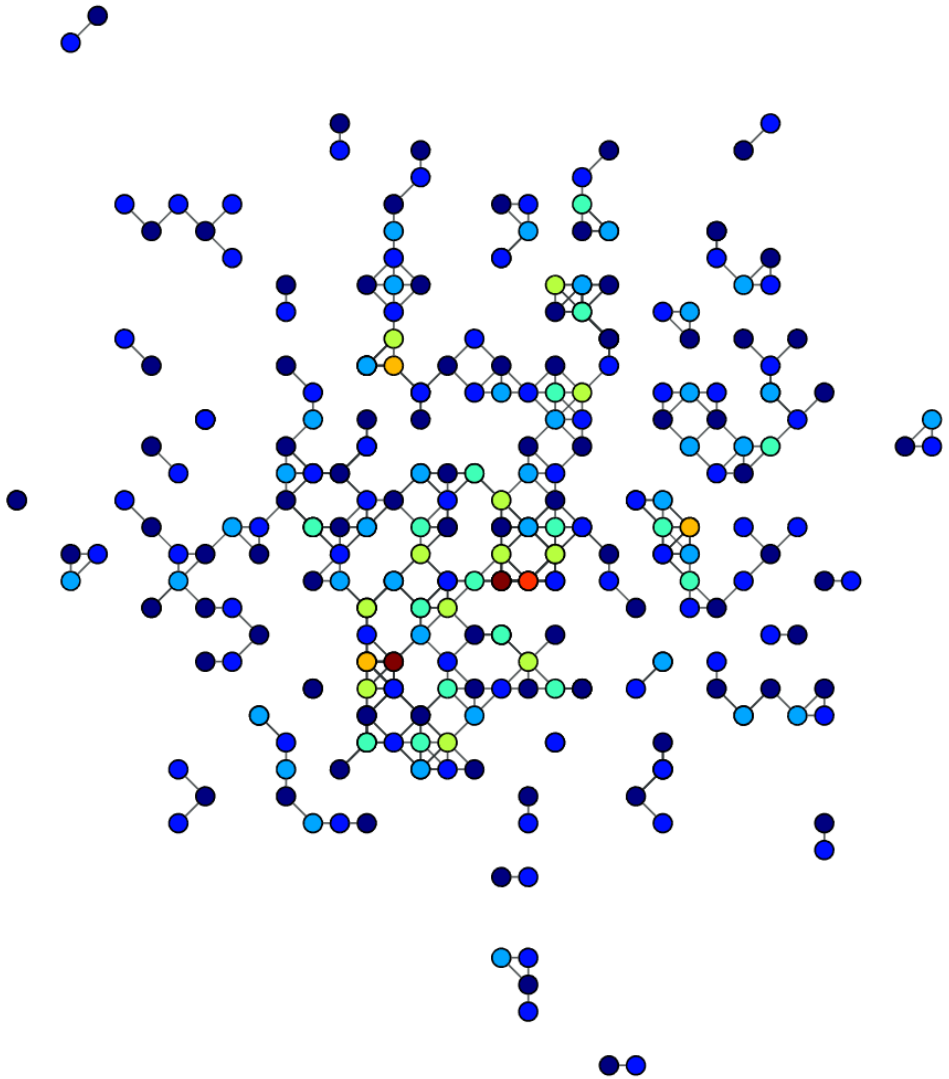


Figure : Coloration at $round = 5$

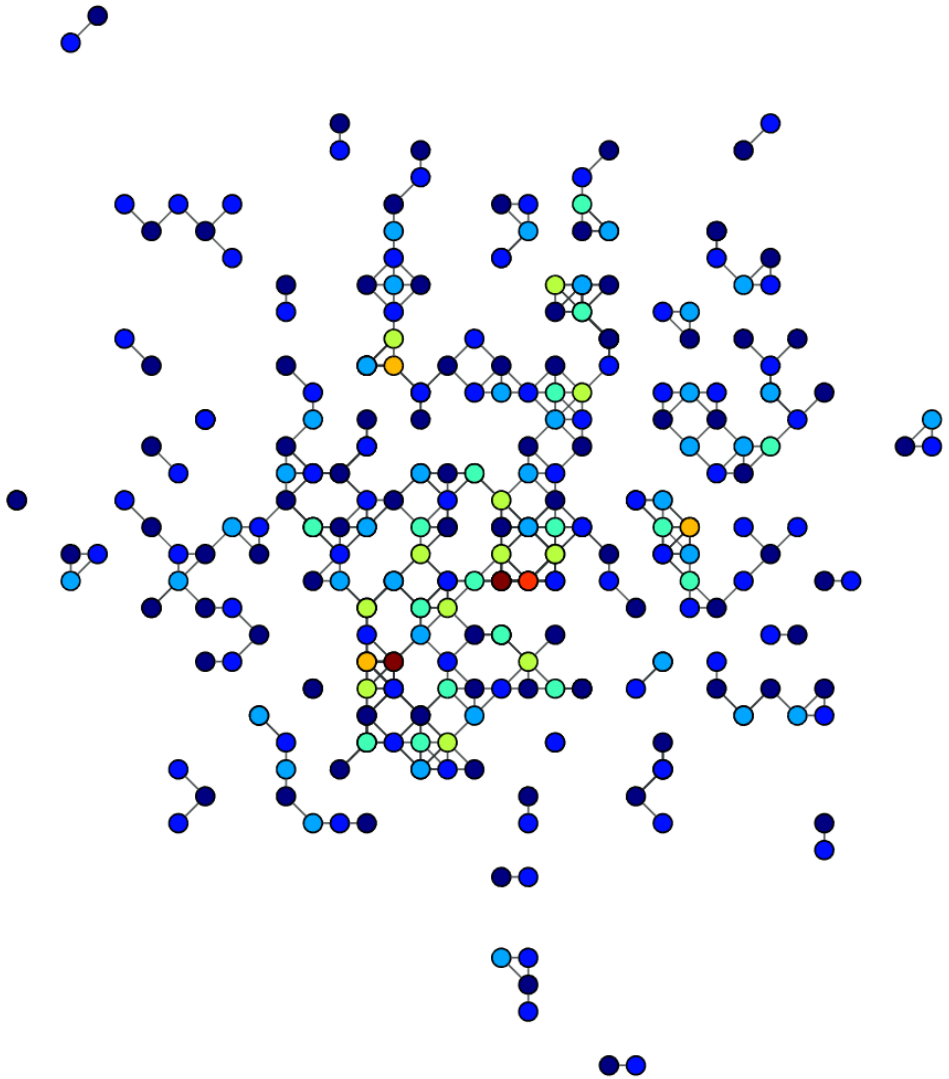


Figure : Coloration at *round* = 6

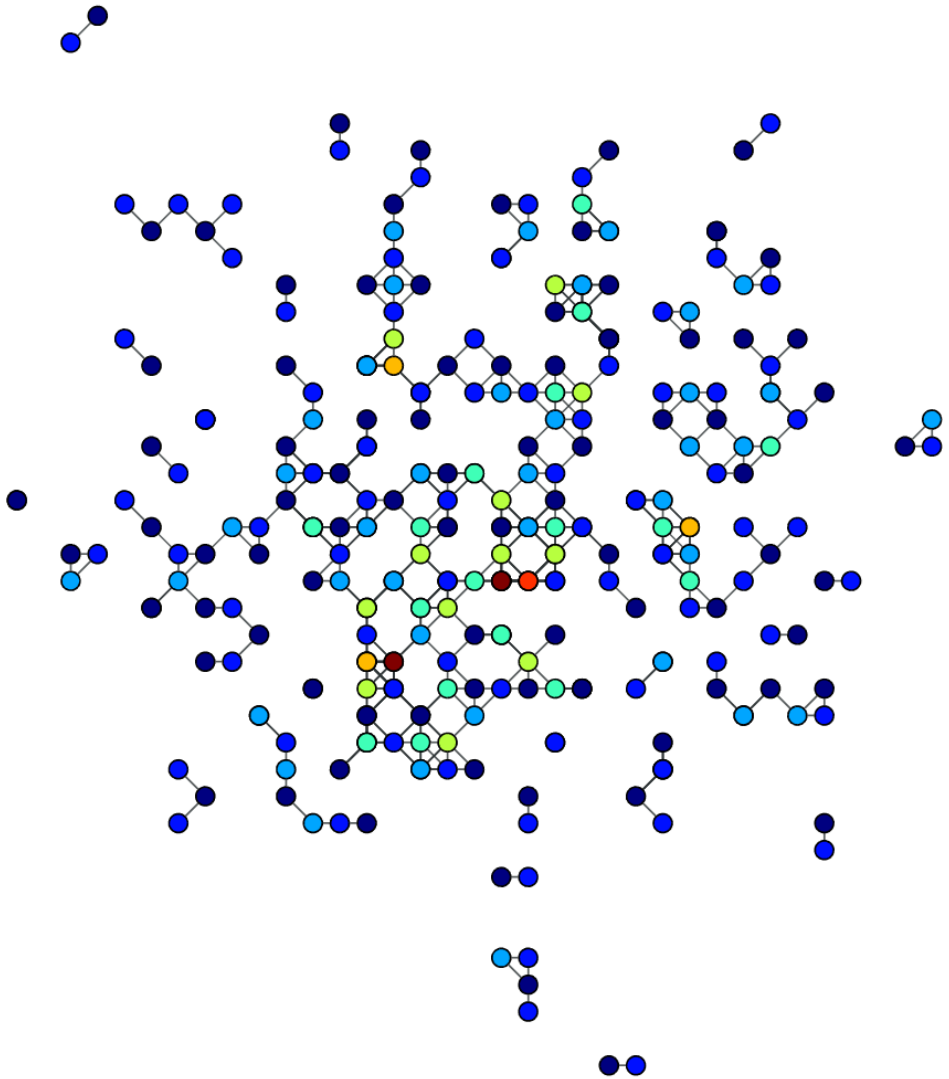


Figure : Coloration at $round = 7$

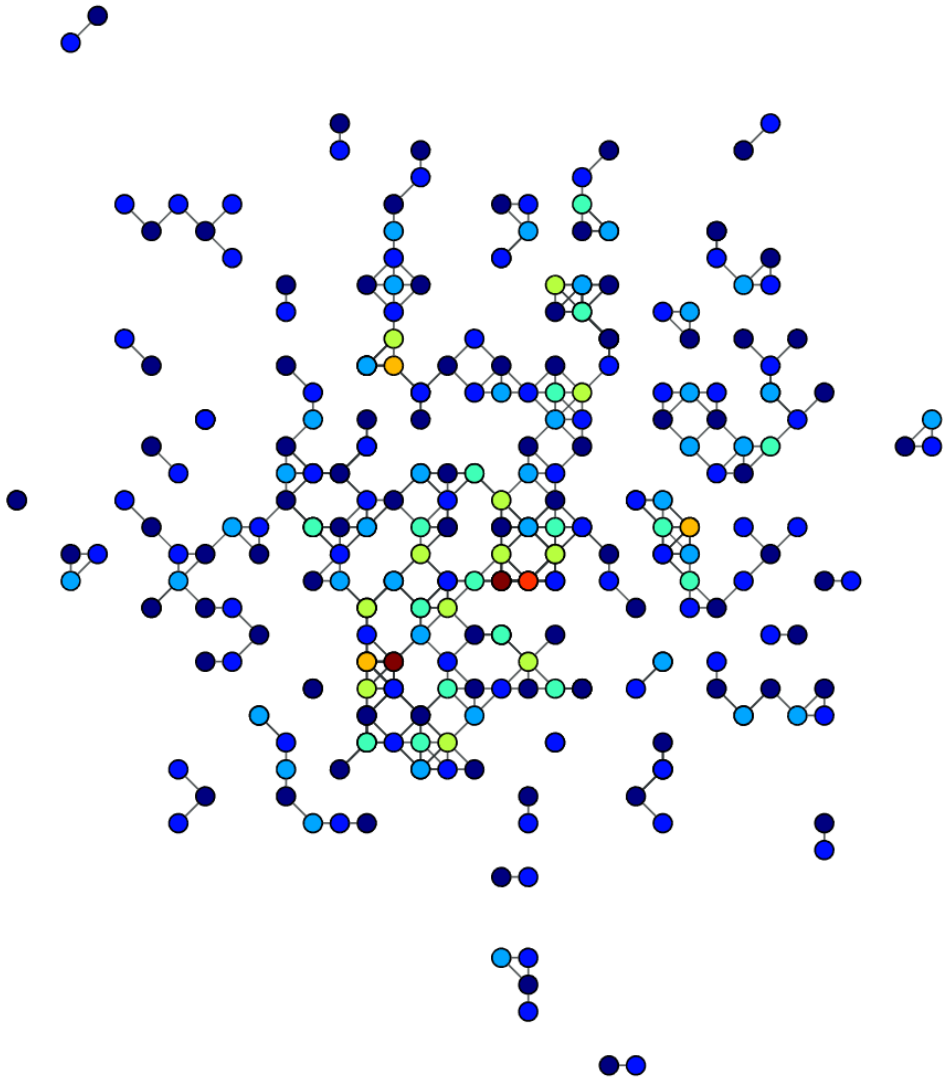


Figure : Coloration at *round* = 8

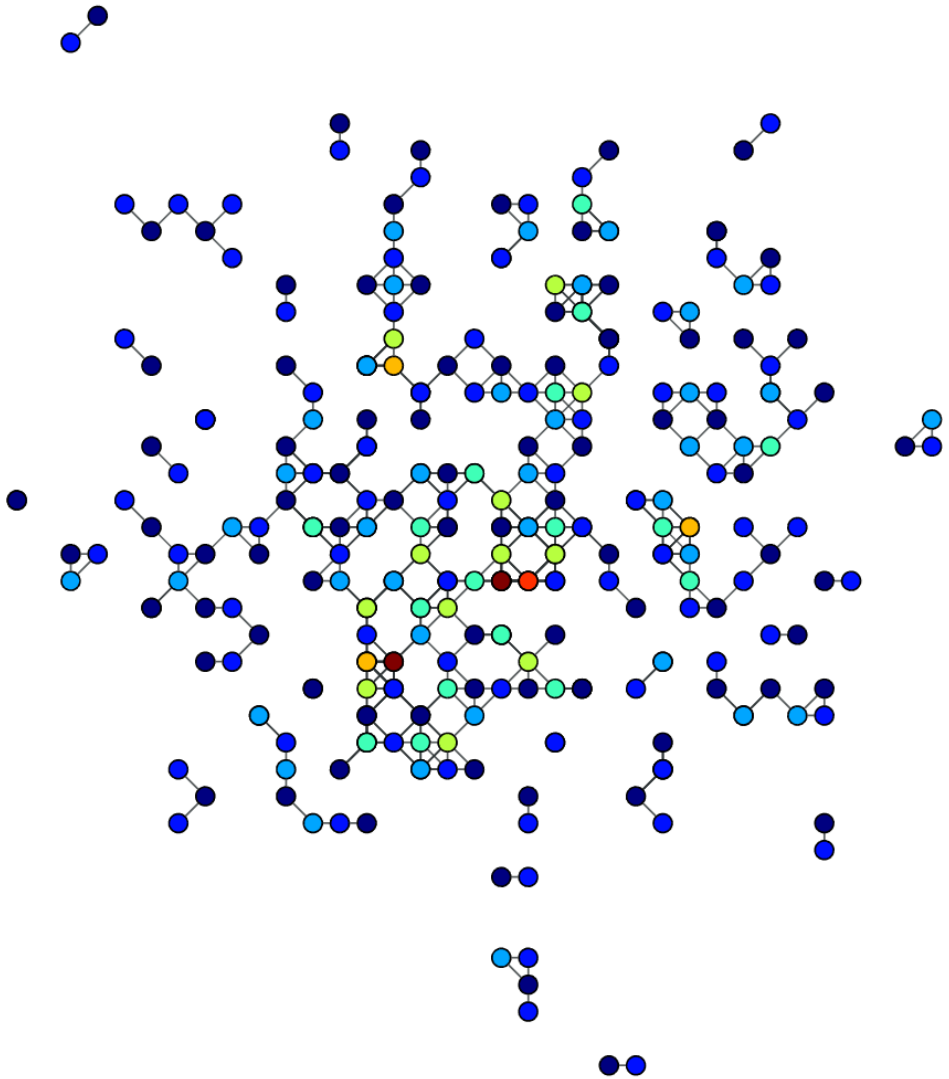


Figure : Coloration at *round* = 9

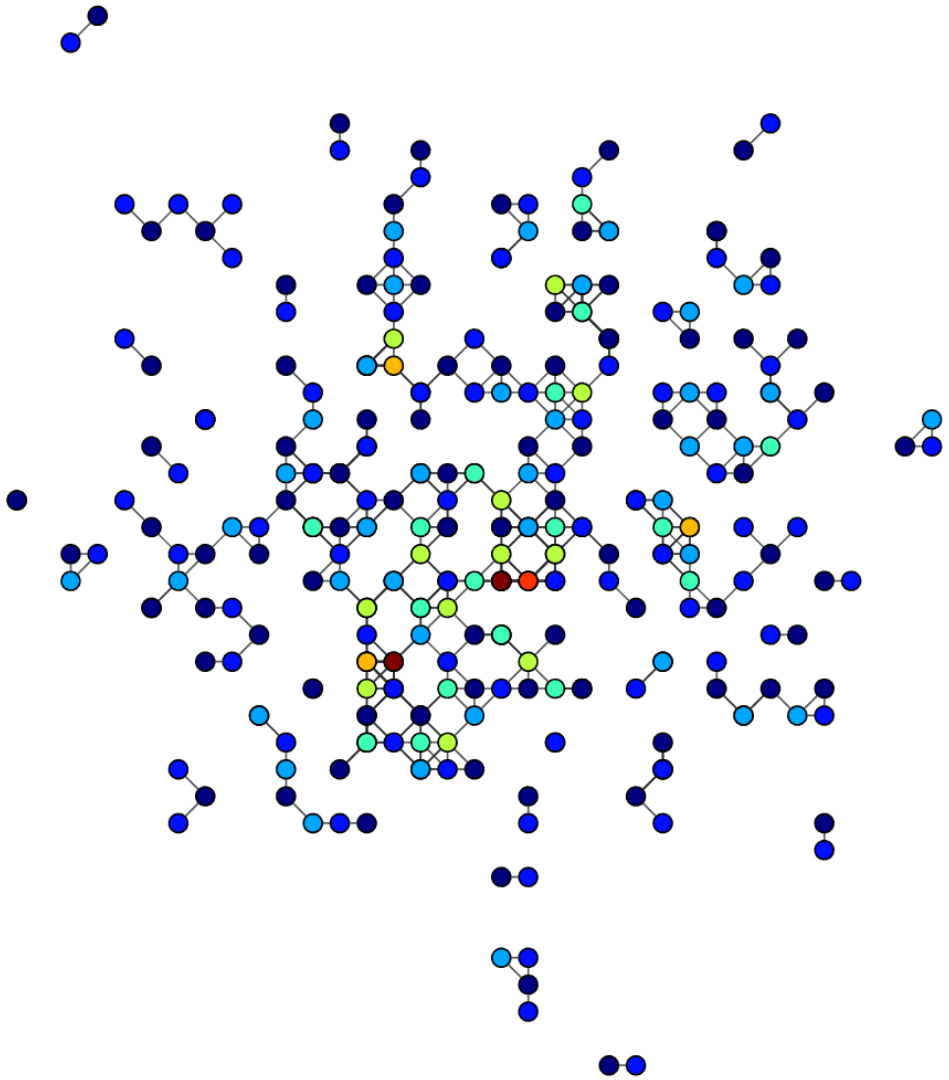


Figure : Coloration at *round* = 10

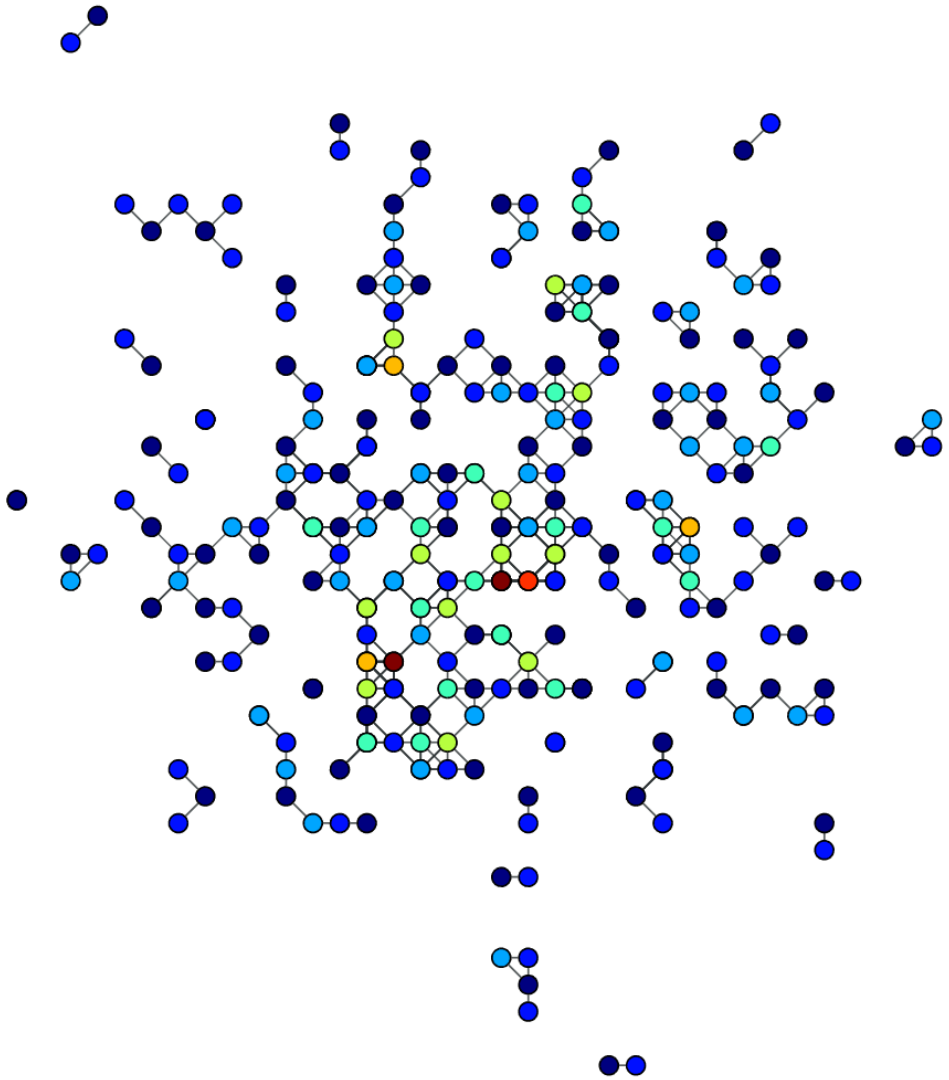


Figure : Coloration at *round* = 11

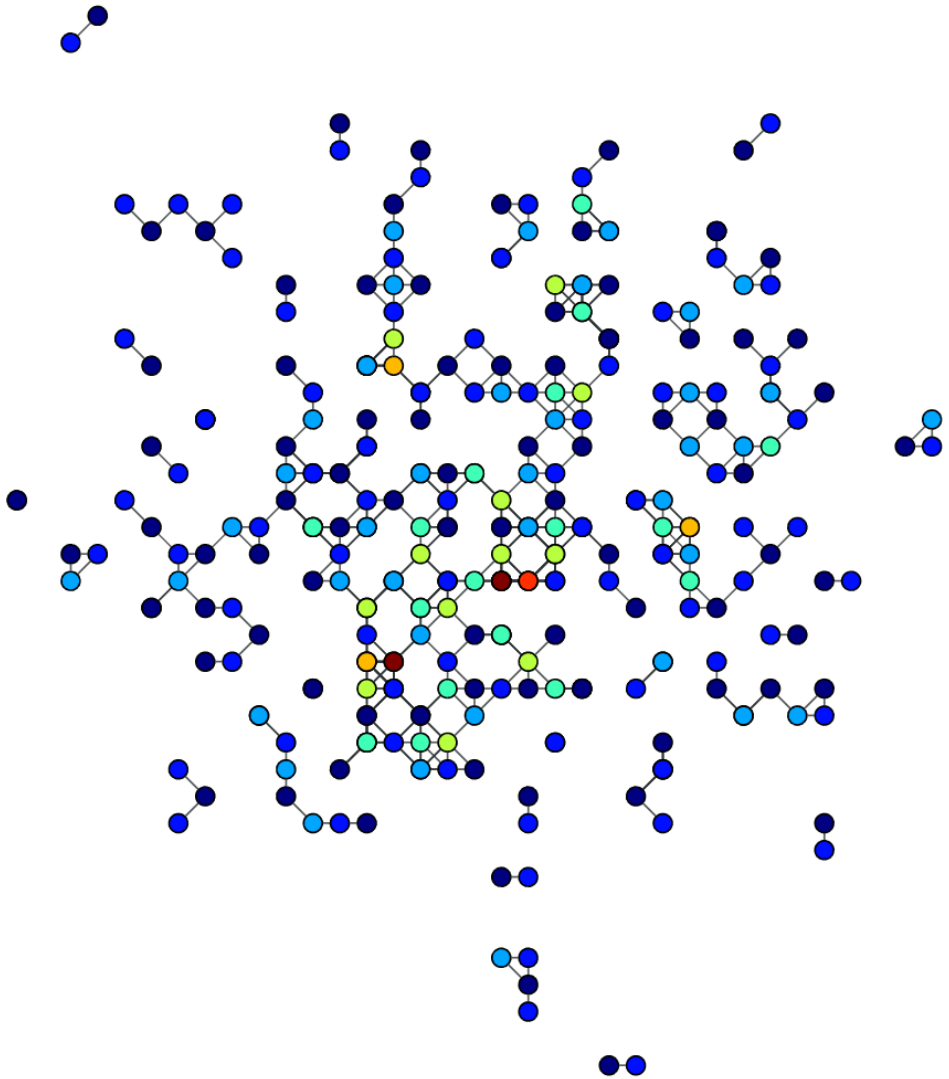


Figure : Coloration at $round = 12$

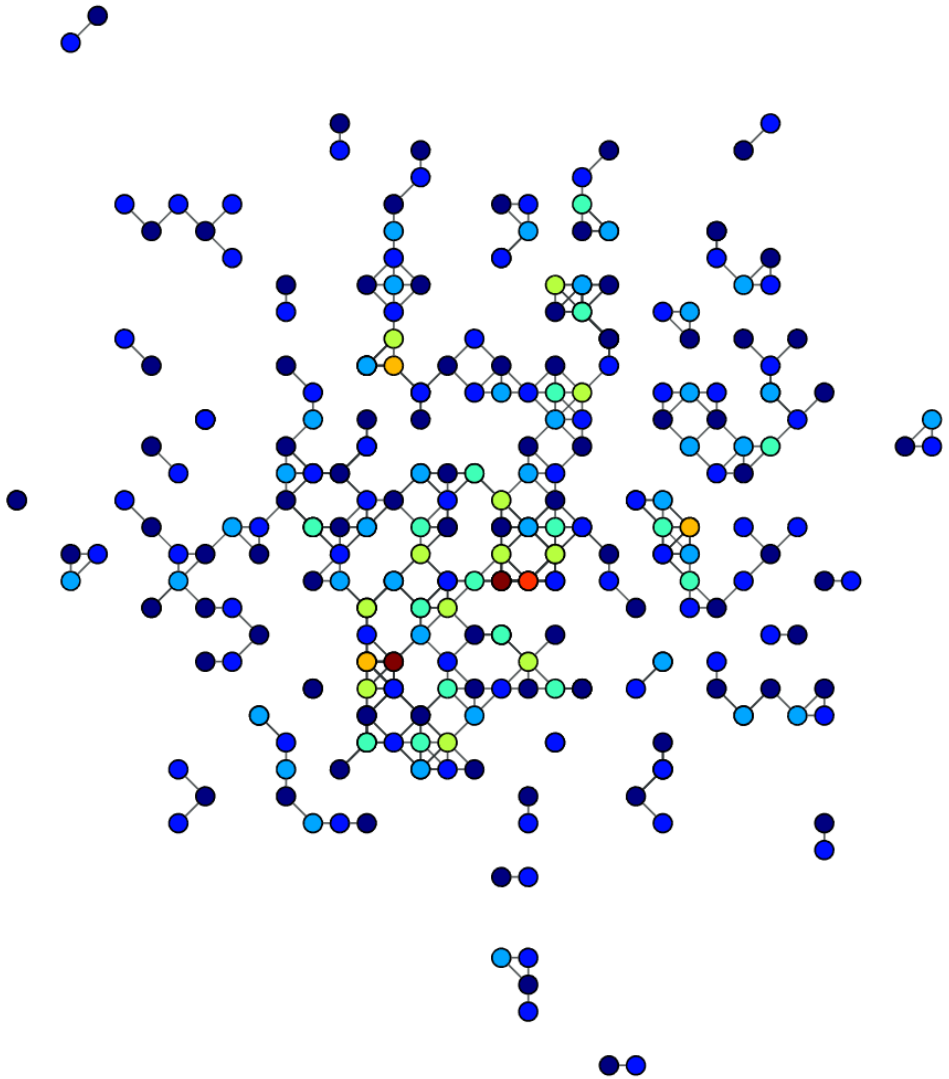


Figure : Coloration at *round* = 13

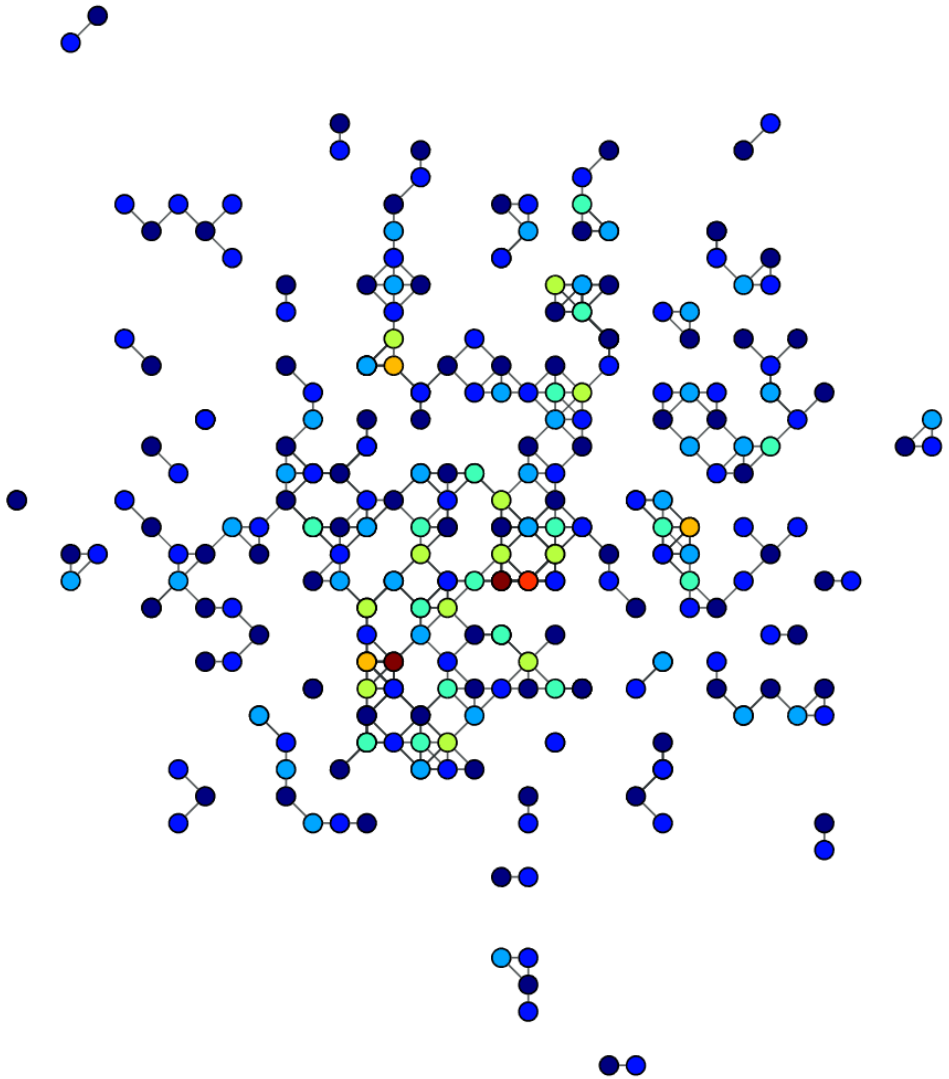


Figure : Coloration at *round* = 14

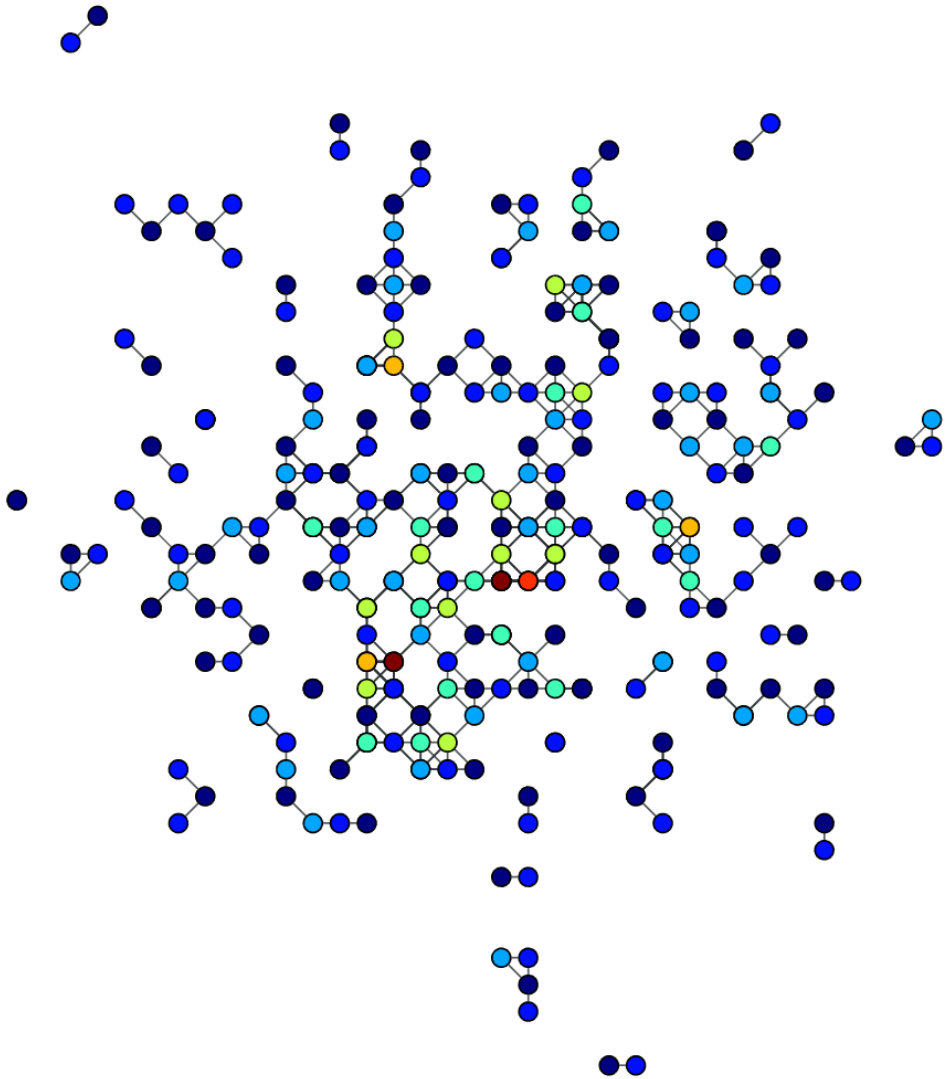


Figure : Coloration at *round* = 15

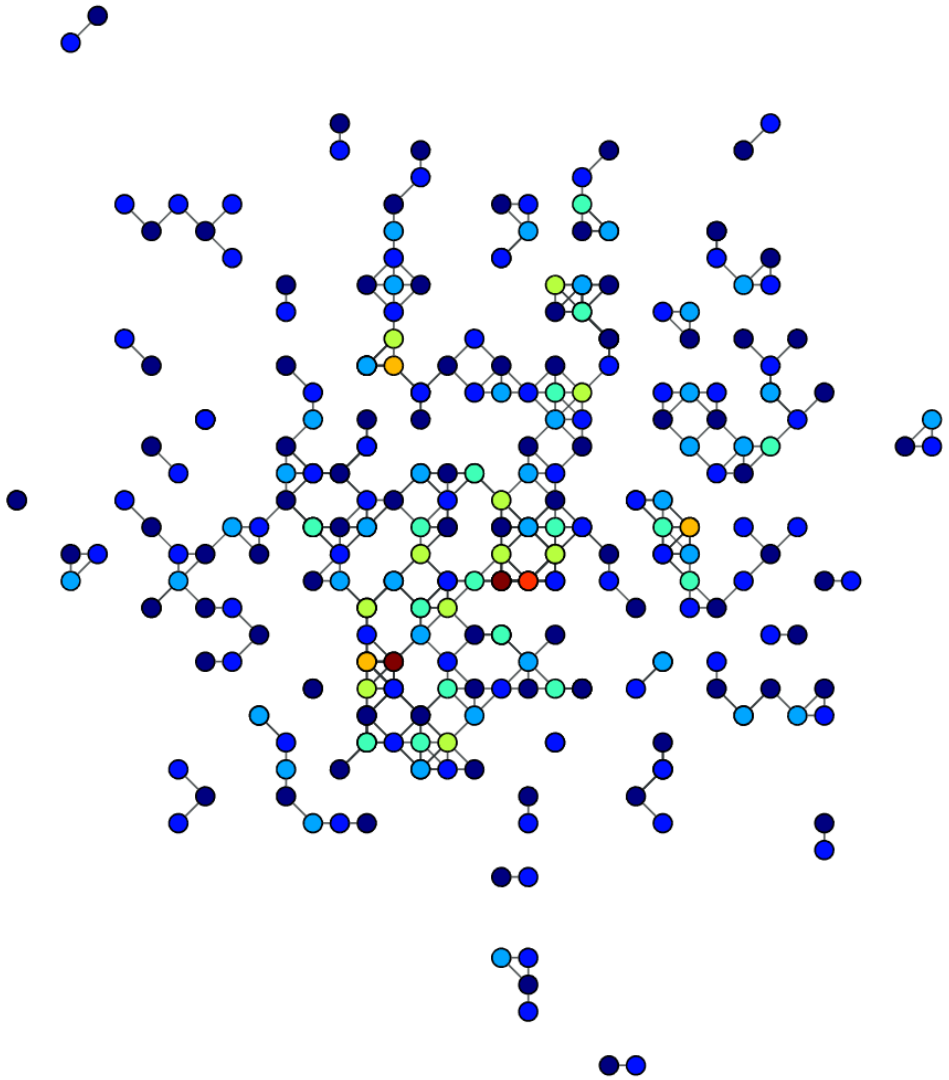


Figure : Coloration at *round* = 16

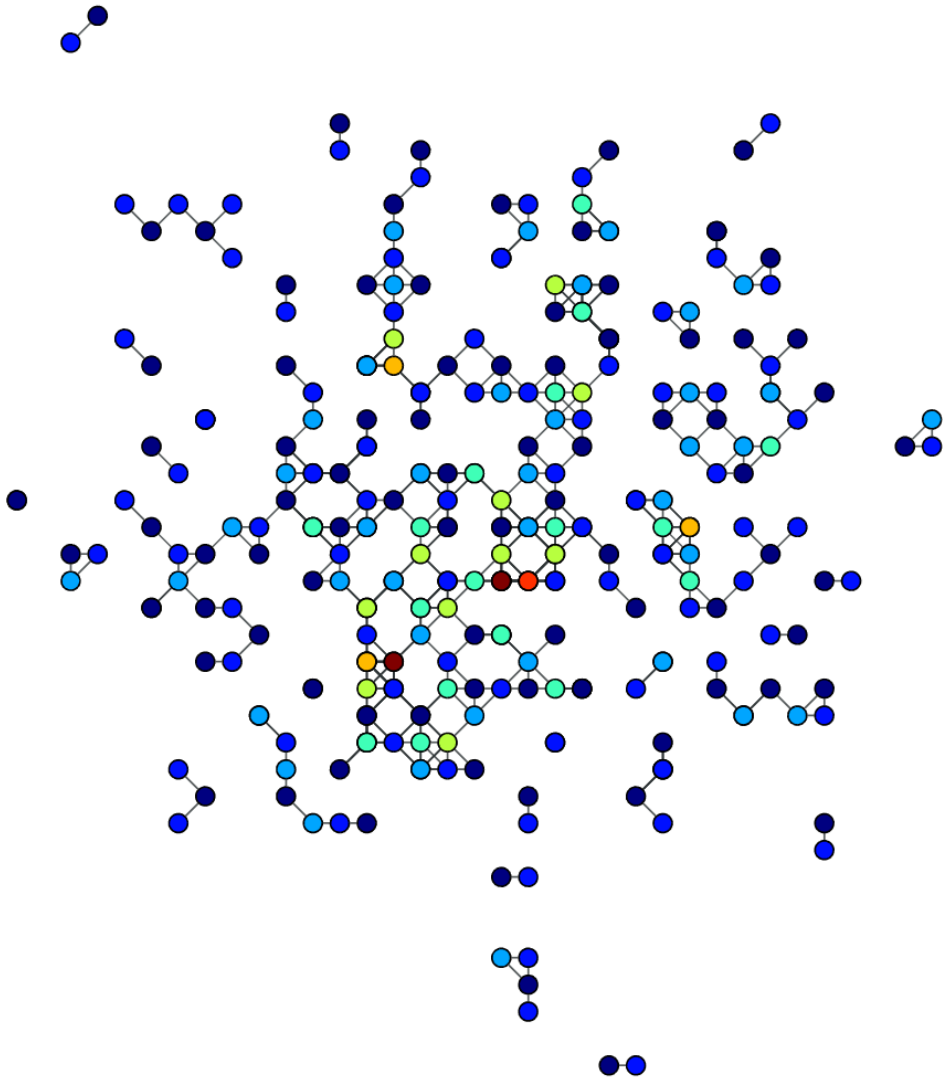


Figure : Coloration at *round* = 17

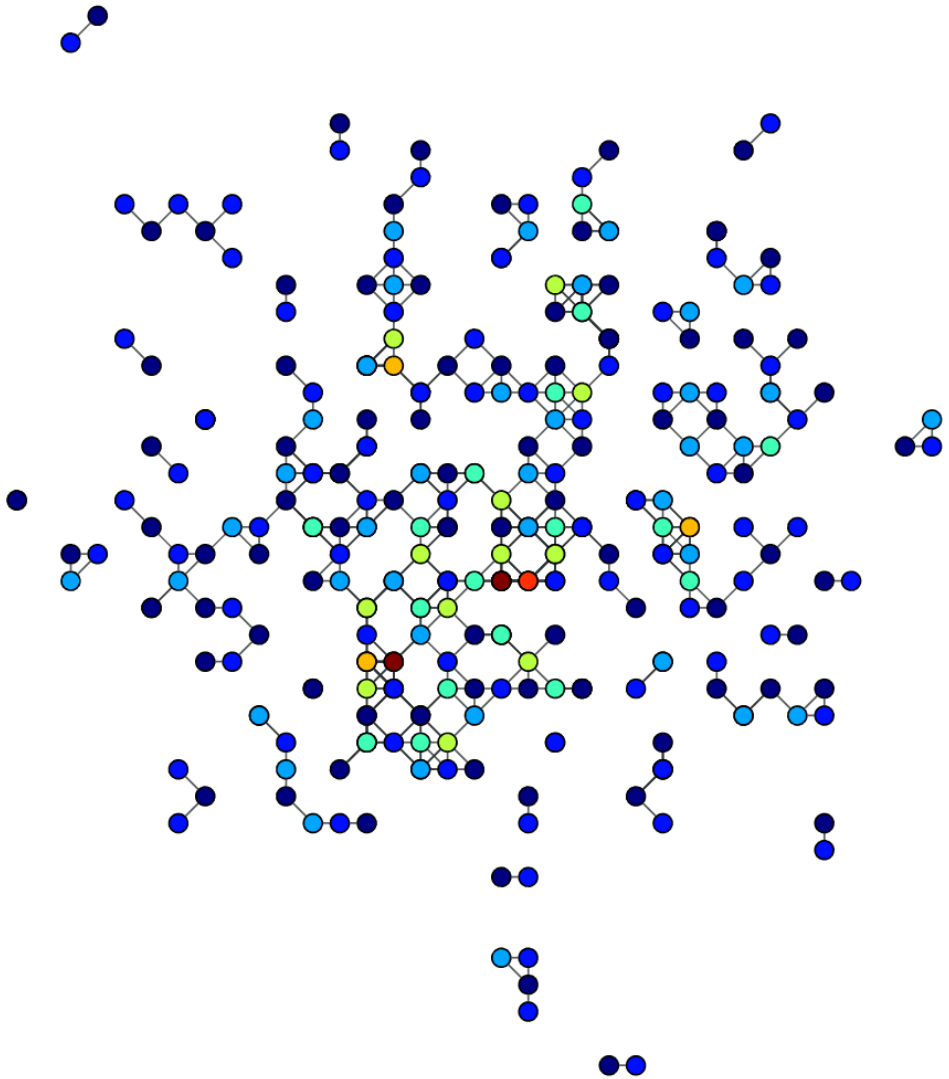


Figure : Coloration at *round* = 18

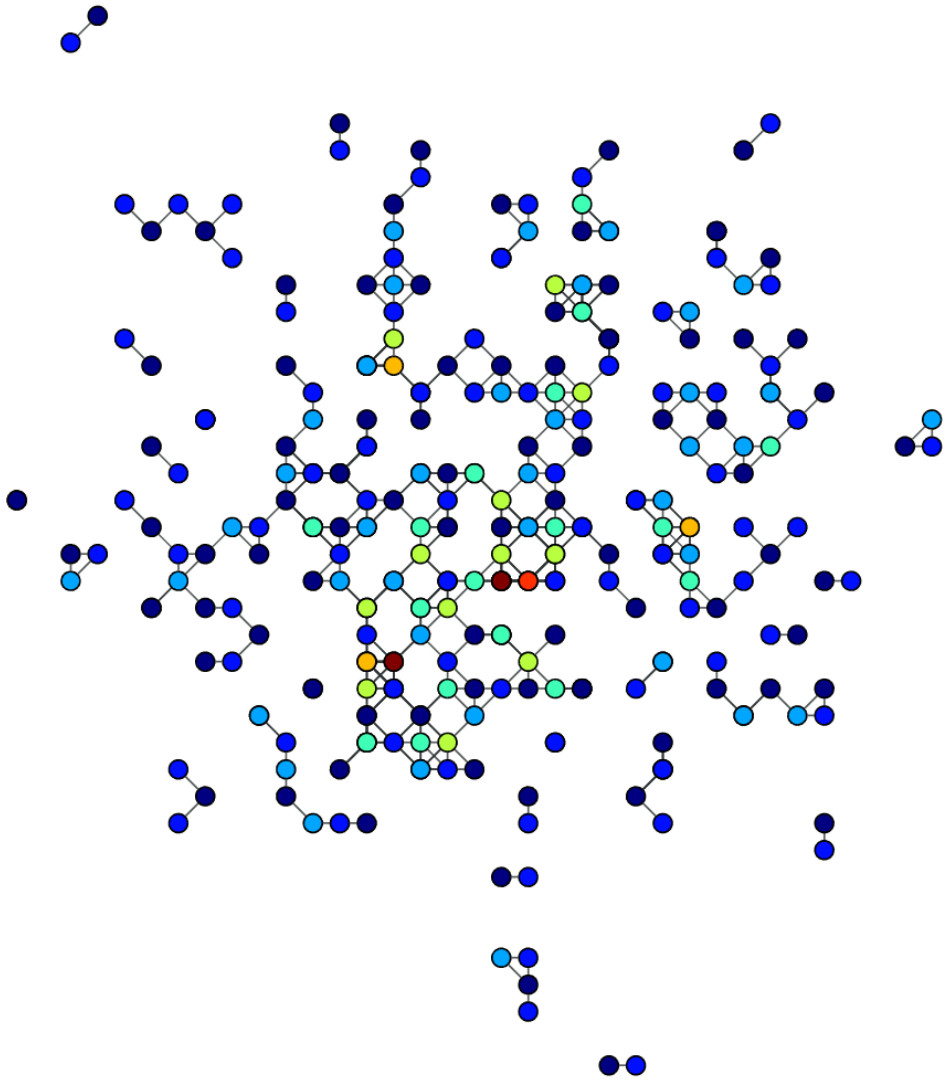


Figure : Coloration at *round* = 19

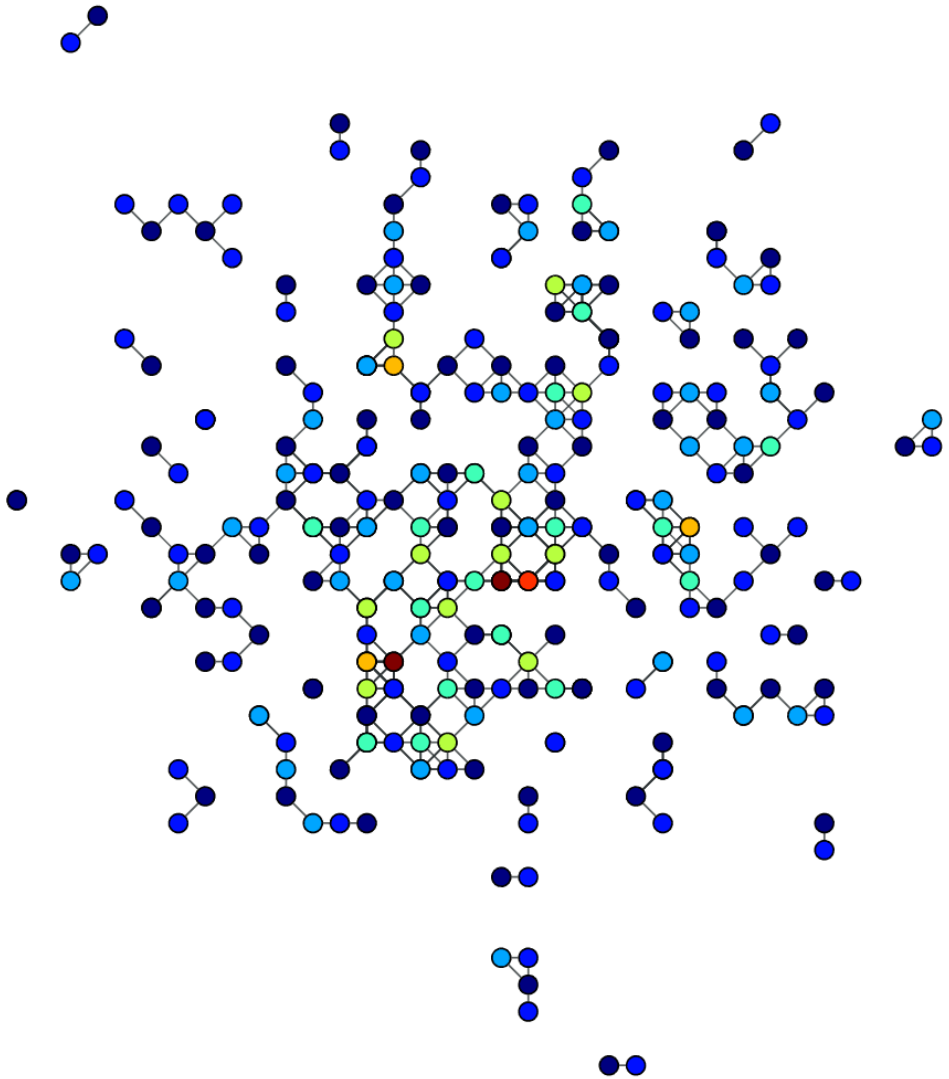
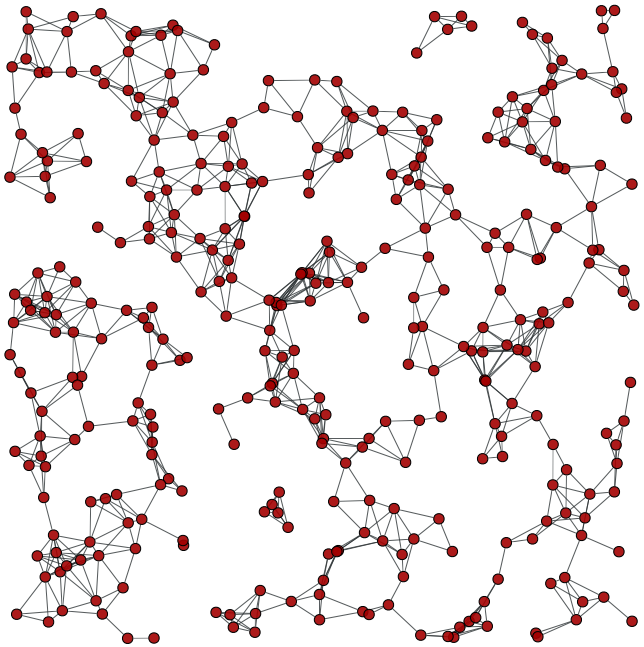
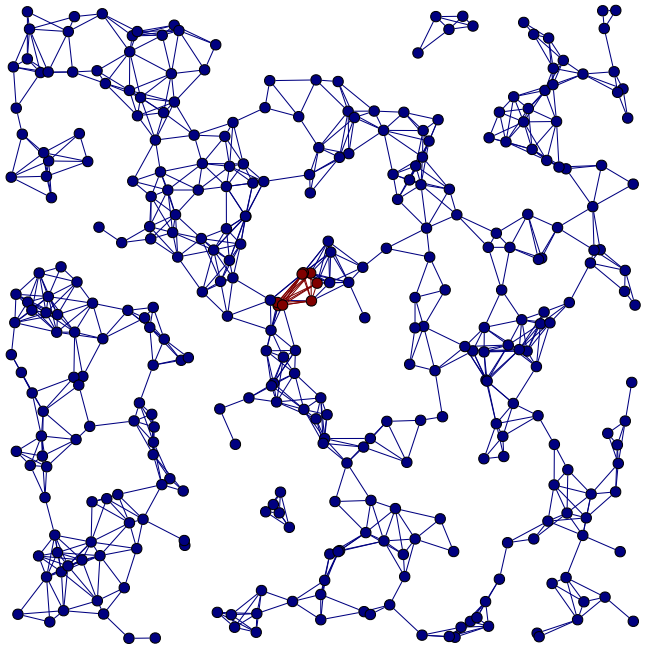


Figure : Coloration at *round* = 20

Geometric topology



(a) Instance of a geometric toplogy of 350 nodes.



(b) k-clique of the graph with k maximal.

Geometric topology

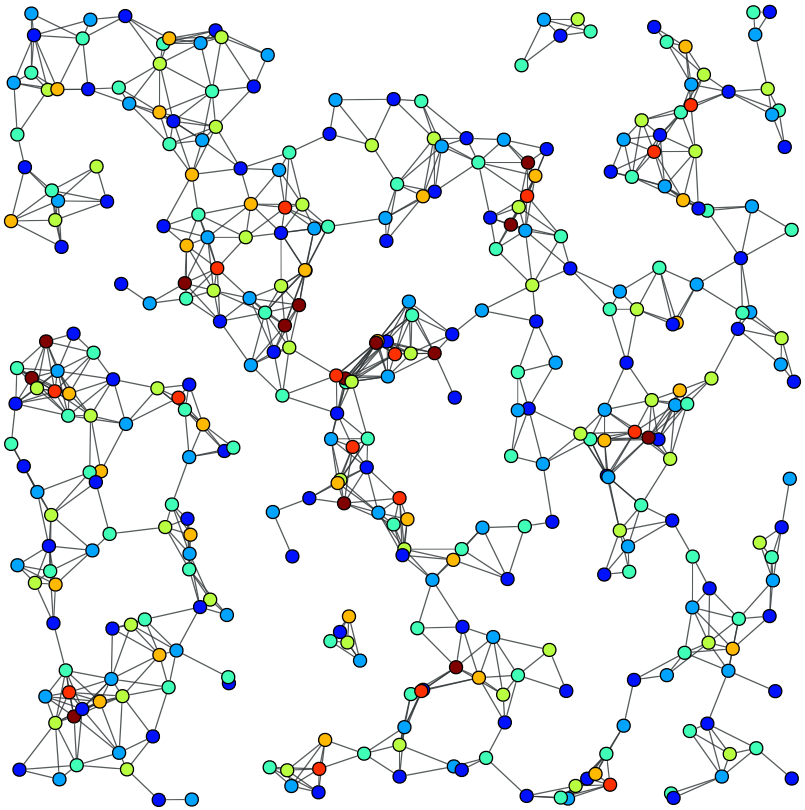
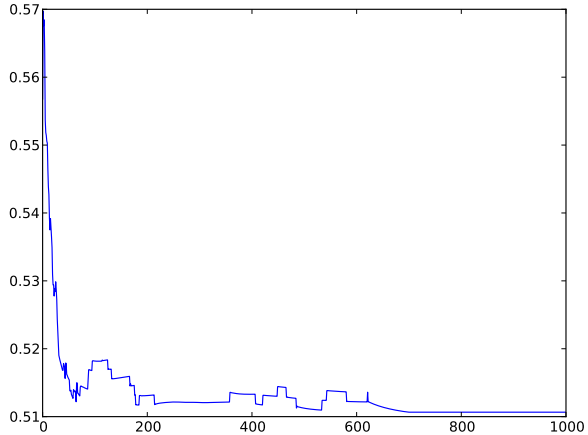
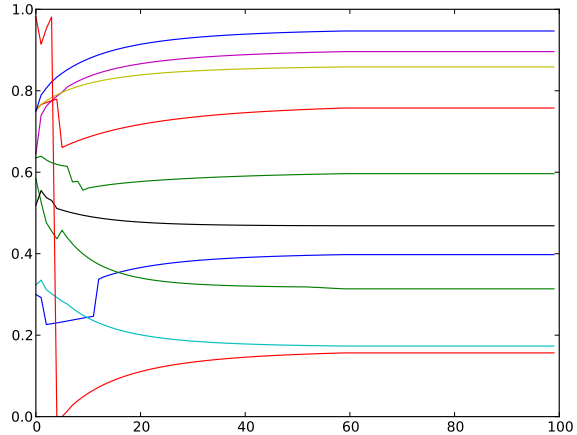


Figure : Result coloring returned by the FrogSim algorithm.



(a) Average of all the phase of a system.



(b) Phases of a fully connected graphe of 10 nodes.

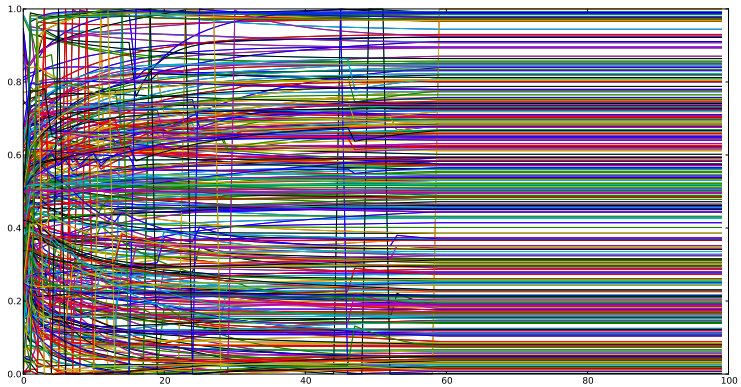
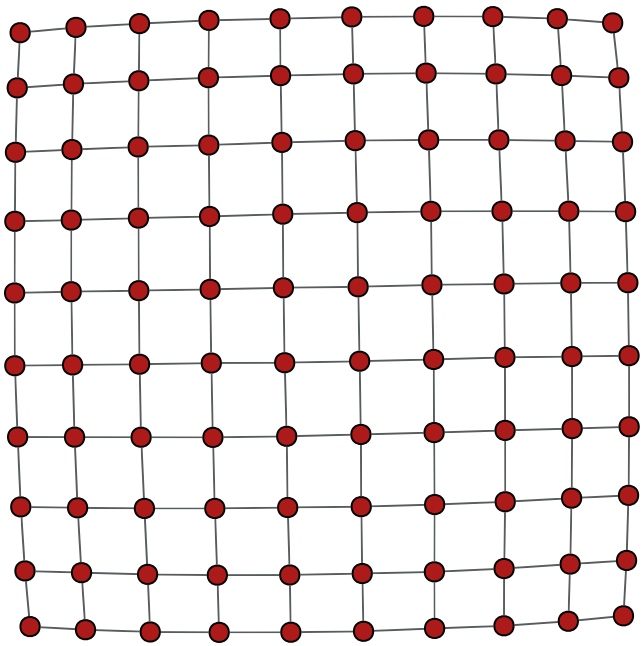
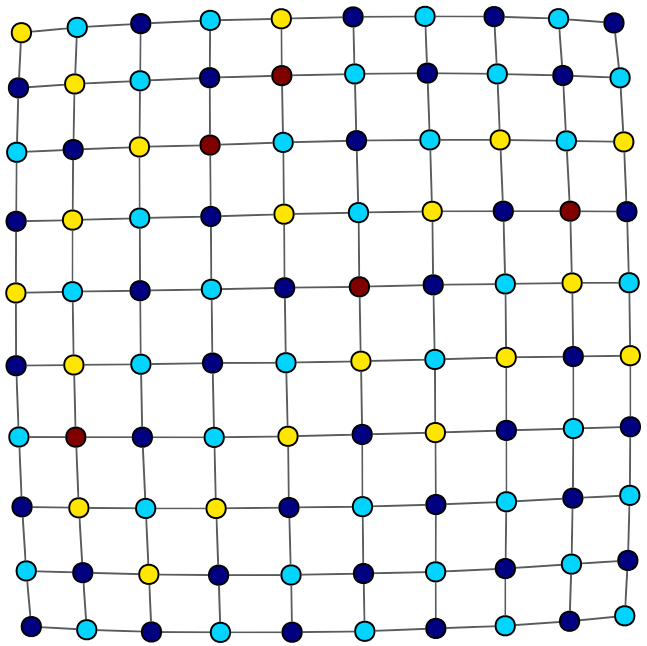


Figure : Phase value for all the nodes of a system.

Grid topology



(a) Instance of a grid topology $10x10$.



(b) Result coloring returned by the fromSim.

The END

Thanks you for listening.

Any questions ?