### LP Oracle Document

#### Pendle Labs

#### Introduction

We provided an Oracle implied rate for the asset in our pool by calculating the TWAP of the implied rate from the pool's state. In order to use the LP token for other purpose (such as collateral), we also want to have an Oracle price for the token (in term of the asset) that immune to data manipulation attacks. We also want this price to match the Oracle implied rate, i.e. this price comes from a pool's state that has the implied rate equals to the Oracle implied rate.

This document explains how we estimate the Oracle LP token price. Having the Oracle implied rate, which is often different from the current implied rate of the pool, we want to find the corresponding LP token price.

With each implied rate, we can calculate the corresponding exchange rate e and proportion p by the formulas:

$$exchangeRate = impliedRate^{yearsToExpiry}$$

$$exchangeRate = \frac{\ln(\frac{p}{1-p})}{rateScalar} + rateAnchor$$

Therefore, we can work with these numbers instead of the implied rate.

# Approach

We have the last exchange rate of the pool e and the corresponding pool's state (current state) with proportion p that allows us to compute the LP token price. In order to find the LP token price, we want to find the hypothetical pool's state (expected state) with the exchange rate of e' and proportion p' (corresponding to the Oracle implied rate) that having "similar" state to the current state.

Our solution is to find a hypothetical trade that transform the current state to the expected state, where the impact of the trading fee is minimized. Without loss of generalization, we assume e < e', so we want to trade  $d_{PT}$  PT for  $d_{asset} < d_{PT}$  asset in the hypothetical trade to raise the exchange rate (raise p to p').

# Implicit fee revisit

There are 2 types of fee in our protocol: explicit fee and implicit fee. The former is easy to eliminate, whereas the latter is harder to minimized. The implicit fee is due to how we calculate the exchange rate of each swap. It is calculated based on  $p_{trade}$  which is

$$p_{trade} = \frac{n_{PT} + d_{PT}}{n_{PT} + n_{asset}}$$

Note that  $p = \frac{n_{PT}}{n_{PT} + n_{asset}}$ ,  $p' = \frac{n_{PT} + d_{PT}}{n_{PT} + n_{asset} + (d_{PT} - d_{asset})} < p_{trade}$ , meaning we have to trade with a exchange rate larger than p'. This discrepancy makes an implicit fee for large size swap.

If instead of doing this one trade, we trade multiple times to move the proportion very slowly until the expected proportion reached, the average exchange rate is much smaller, roughly  $\frac{e+e'}{2}$ . We argue that, using this exchange rate  $\frac{e+e'}{2}$  for the hypothetical trade mentioned above would minimize the impact of implicit fee.

With that spirit, let find out which size of the trade  $d = d_P T$  we need to raise p to p' and get the expected implied rate.

## Swap size calculation

Let say we swap d PT for  $\frac{d}{e^*}$  asset, where  $e^* = \frac{e+e'}{2}$  is the exchange rate used for the hypothetical trade. We can find the closed form of the solution for d as follows:

$$e' = \frac{\ln(\frac{n_{PT} + d}{n_{asset} - \frac{d}{e^*}})}{S} + A$$

where S is the rate scalar, A is the rate anchor.

Derive this we get  $d = \frac{C \times n_{asset} - n_{PT}}{1 + \frac{C}{e^*}}$  with  $C = \exp(S(e' - A)).$