1. Problem 1.62 or 1.63

2. Check that in the Cartesian coordinates the dot product of two vectors $\mathbf{u} = (u_x, u_y, u_z)$ and $\mathbf{w} = (w_x, w_y, w_z)$ can be equivalently found either as $\mathbf{u} \cdot \mathbf{w} = u_x w_x + u_y w_y + u_z w_z$ or as $\mathbf{u} \cdot \mathbf{w} = uw \cos \alpha$, where $\alpha$ is the smaller angle between $\mathbf{u}$ and $\mathbf{w}$.

3. (**12th edition of the textbook**) Problem 1.90 (p. 34).

3

4. Consider two vectors $\mathbf{u} = 3\hat{n}_x + 4\hat{n}_y$ and $\mathbf{w} = 6\hat{n}_x + 16\hat{n}_y$. Find (a) the components of the vector $\mathbf{w}$ that are, respectively, parallel and perpendicular to the vector $\mathbf{u}$, (b) the angle between $\mathbf{w}$ and $\mathbf{u}$.

5. Is it possible to find a vector $\mathbf{u}$, such that $(2\hat{n}_x - 3\hat{n}_y + 4\hat{n}_z) \times \mathbf{u} = (4\hat{n}_x + 3\hat{n}_y - \hat{n}_z)$? What is a quick way to check it?

6. A particle moves along a straight line with non-constant acceleration $a_x(t) = -A\omega^2 \cos \omega t$, where $A$ and $\omega$ are positive constants (what are their units?). At the instant of time $t = 0$ its velocity $v_x(0) = 3$ [m/s] and position $x(0) = 4$ [m]. Find $v_x(t)$ and $x(t)$ at any instant of time. Sketch the graphs of $x(t)$, $v_x(t)$, and $a_x(t)$. What kind of motion may these results describe?

7. A particle is moving along a straight line with velocity $v_x(t) = -\beta A \omega e^{-\beta t} \cos \omega t$, where $A, \omega$, and $\beta$ are positive constants.

   a) What are the units of these constants?

   b) Find acceleration $a_x(t)$ and position $x(t)$ of the particle, assuming that $x(0) = 5$ [m].

   c) Sketch $x(t)$, $v_x(t)$, and $a_x(t)$.

   d) What kind of motion could these results refer to (qualitatively)?

8. A car is moving in one direction along a straight line. Find the average velocity of the car if: (a) it travels *half of the journey time* with velocity $v_1$ and the other half with velocity $v_2$, (b) it covers *half of the distance* with velocity $v_1$ and the other one with velocity $v_2$. Both $v_1$ and $v_2$ are constants.

9. In a certain motion of a particle along a straight line, the acceleration turns out to be related to the position of the particle according to the formula $a_x = \sqrt{kx}$, where $k > 0$ is a constant and $x > 0$. How does the velocity depend on $x$, if we know that for $v_x(x_0) = v_0$?

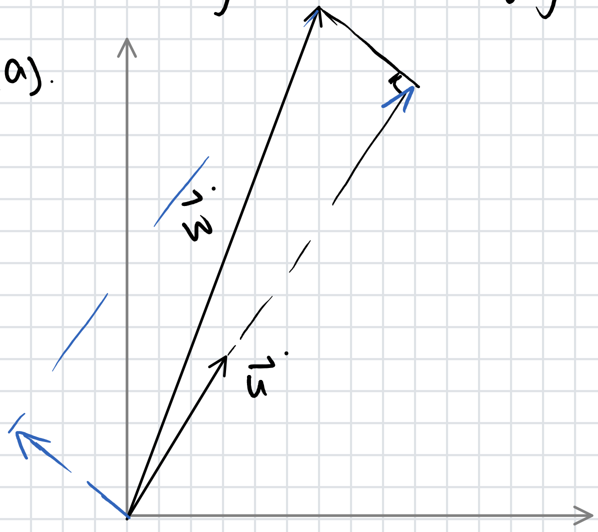
10. (**12th edition of the textbook**) Problem 2.72 (p. 68). Do you need any additional information to correctly sketch the $x(t)$ curve?

11. Problem 2.98.

## Problem 4.

$\vec{u} = 3\hat{n}_x + 4\hat{n}_y$ $\quad$ $\vec{w} = 6\hat{n}_x + 16\hat{n}_y$

(a).




$$\vec{w}_{\parallel} = \langle \vec{w} \cdot \left(\frac{\vec{u}}{u}\right) \rangle \left(\frac{\vec{u}}{u}\right) = \frac{1}{5} \langle \begin{pmatrix}6\\16\end{pmatrix}, \begin{pmatrix}3\\4\end{pmatrix} \rangle \cdot \frac{1}{5} \cdot \begin{pmatrix}3\\4\end{pmatrix}$$

$$= \begin{pmatrix}9.84\\13.12\end{pmatrix}$$

$$\vec{w}_{\perp} = \vec{w} - \vec{w}_{\parallel} = \begin{pmatrix}6\\16\end{pmatrix} - \begin{pmatrix}9.84\\13.12\end{pmatrix} = \begin{pmatrix}-3.84\\2.88\end{pmatrix}$$

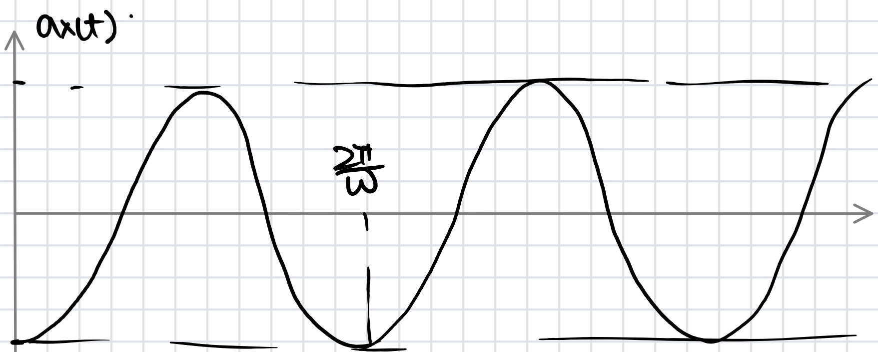
(b) $\cos\alpha = \dfrac{\vec{w} \cdot \vec{u}}{w \cdot u} = \dfrac{\langle \begin{pmatrix}3\\4\end{pmatrix}, \begin{pmatrix}6\\16\end{pmatrix} \rangle}{5 \cdot \sqrt{6^2+16^2}} = 0.9597.$

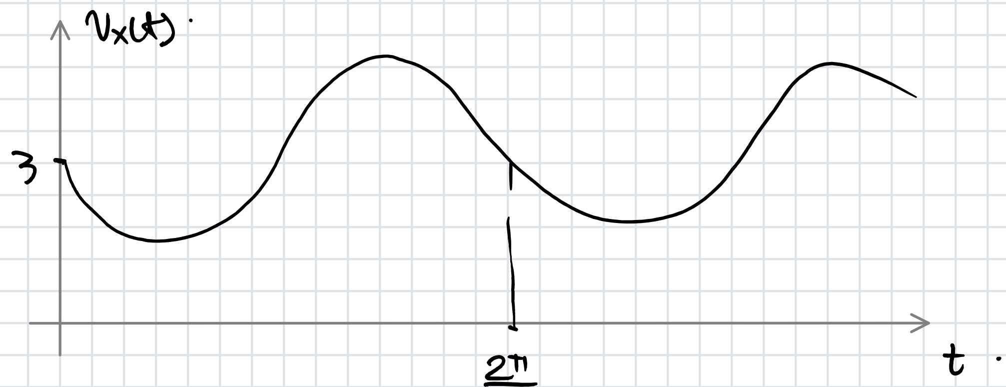
## Problem 5

$$\begin{pmatrix}2\\-3\\4\end{pmatrix} \times \vec{u} = \begin{pmatrix}4\\3\\-1\end{pmatrix}. \quad \text{check } \begin{pmatrix}2\\3\\4\end{pmatrix} \perp \begin{pmatrix}4\\3\\-1\end{pmatrix} ? \quad \langle \begin{pmatrix}2\\3\\4\end{pmatrix}, \begin{pmatrix}4\\3\\-1\end{pmatrix} \rangle = 8-9-4 \neq 0.$$

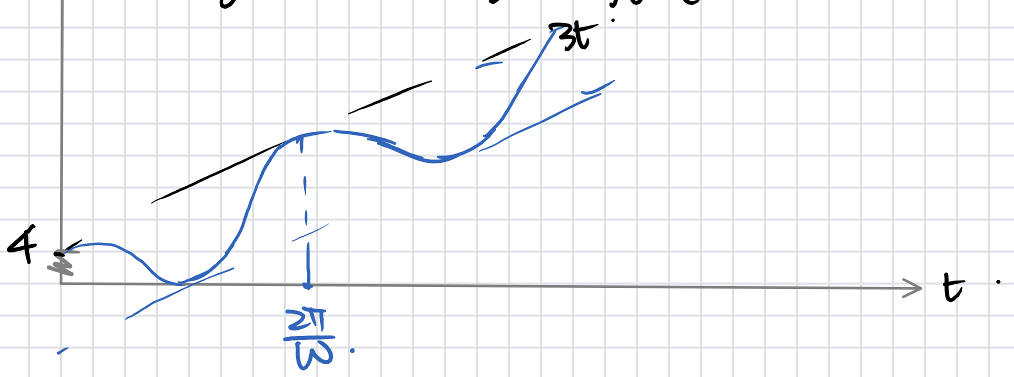
## Problem 6

$a_x(t) = -A\omega^2 \cos\omega t.$ $\quad$ $V_x(t) = \int_0^t a_x(t)\,dt + v_x(0) = -A\omega \sin\omega t + v_x(0).$




$x(t) = \int_0^t V_x(t) + x(0) = \int_0^t (-A\,\omega \sin\omega t + v_x(0))\,dt + x(0) = A(\cos\omega t - 1) + V_x(0)t + x(0)$




$\int \sin\omega t\, d\omega t = -\cos\omega t.$

# Problem 7

a) $v_x(t) = -\beta A \omega e^{-\beta t} \cos \omega t$.

$m \cdot s^{-1}$ ; $s^{-1}$ ; $m \cdot s$ ; $s^{-1}$ ; rad/s. ; 无量纲数

↙ Appear on exp: must be const.

$\beta : s^{-1}$.

b)

$$a_x(t) = \frac{d}{dt} v_x(t) = -\beta A \omega \cdot \left(-e^{-\beta t} \cdot \omega \sin \omega t - \beta e^{-\beta t} \cos \omega t\right)$$

$$= -\beta A \omega e^{-\beta t} \left(-\omega \sin \omega t - \beta \cos \omega t\right)$$

$$x(t) = \int_0^t v_x(t)\, dt + x(0)$$

$$\int_0^t -\beta A \omega e^{-\beta t} \cos \omega t \, dt \quad X$$

$$= \int_0^t \omega A \cos \omega t \, d e^{-\beta t}$$

$$= \omega A \cos \omega t \cdot e^{-\beta t} \Big|_0^t - \int_0^t e^{-\beta t} \, d \omega A \cos \omega t$$

$$= \omega A (\cos \omega t \, e^{-\beta t} - 1) + \int_0^t e^{-\beta t} \omega^2 A \sin \omega t \, dt$$

$$= \omega A (\cos \omega t \, e^{-\beta t} - 1) + \left(\frac{1}{-\beta}\right) \cdot \int_0^t \omega^2 A \sin \omega t \, d e^{-\beta t}$$

$$= \omega A (\cos \omega t \, e^{-\beta t} - 1) - \frac{1}{\beta} \left( \omega^2 A \sin \omega t \, e^{-\beta t} \Big|_0^t - \int_0^t e^{-\beta t} \omega^2 A \cos \omega t \cdot \omega dt \right).$$

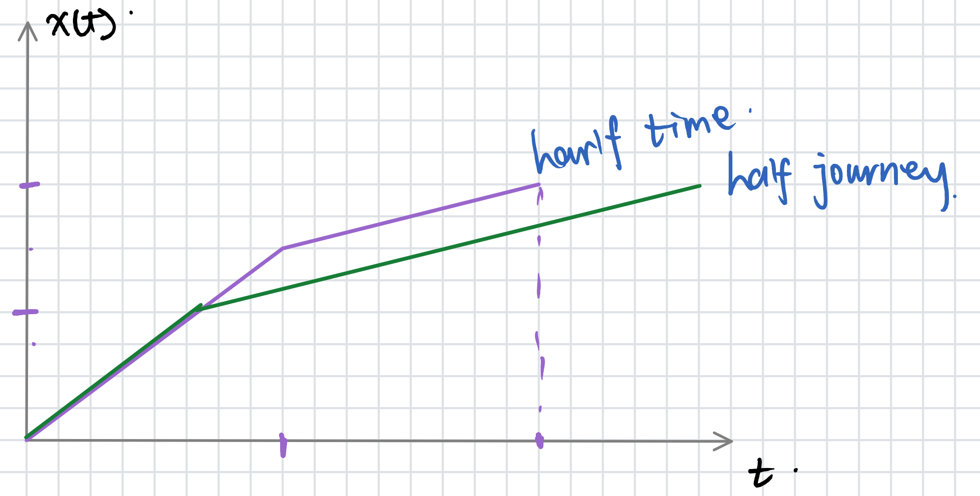
$$= \omega A \cos \omega t \, e^{-\beta t} - \omega A - \frac{\omega^2 A}{\beta} \sin \omega t \, e^{-\beta t} + \frac{\omega^3 A}{\beta} \int_0^t e^{-\beta t} \cos \omega t \, dt \quad X$$

$$-\beta A \omega X = \omega A \cos \omega t \, e^{-\beta t} - \omega A - \frac{\omega^2 A}{\beta} \sin \omega t \, e^{-\beta t} + \frac{\omega^3 A}{\beta} X$$

$$X = \frac{\beta}{\omega^3 A + \beta^2 A \omega} \left( \omega A \cos \omega t \, e^{-\beta t} - \omega A - \frac{\omega^2 A}{\beta} \sin \omega t \, e^{-\beta t} \right)$$

$$x(t) = \frac{\cancel{\omega A} \cdot \beta \cdot (-\beta A \omega)}{\cancel{\omega A} (\omega^2 + \beta^2)} \left( \cos \omega t \, e^{-\beta t} - 1 - \frac{\omega}{\beta} \sin \omega t \, e^{-\beta t} \right) + \underset{=5}{x(0)}$$

# Problem 8



Problem 9

$a_x = \sqrt{kx}$

$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$. Chain Rule: $\frac{dx}{dt} = \frac{dx}{dv} \cdot \frac{dv}{dt}$.

$v_x = \frac{dx}{dt}$.

Goal: variable seperation

$f(x)dx = f(v)dv$. $\Rightarrow$ then, integration.

$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx} \cdot \frac{dx}{dt} = \frac{dv_x}{dx} \cdot v_x$.

we want to eliminate $dt$.

$\frac{dv_x}{dx} v_x = \sqrt{kx} \Rightarrow v_x \, dv_x = k^{\frac{1}{2}} x^{\frac{1}{2}} dx$.

$$\int_{v_0}^{v_x} v_x dv_x = \int_{x_0}^{x} k^{\frac{1}{2}} x^{\frac{1}{2}} dx$$

$$\frac{1}{2}v_x^2 - \frac{1}{2}v_0^2 = k^{\frac{1}{2}} \cdot \frac{2}{3}\left(x^{\frac{3}{2}} - x_0^{\frac{3}{2}}\right)$$