VP160 RC2 3D Kinetics

topics: 2D polar coordinates; quick review and problems on 3D kinetics

$\dot{\mathbf{v}}$ vs. \dot{v}

acceleration vs. speed change rate (will discuss later in nature coordinates

3D Motion in Cartesian Coordinates

- $ullet \vec{r} = x\hat{n_x} + y\hat{n_y} + z\hat{n_z}$
- $ullet ec{v} = \dot{x}\hat{n_x} + \dot{y}\hat{n_y} + \dot{z}\hat{n_z}$
- ullet $ec{a}=\ddot{x}\hat{n_x}+\ddot{\ddot{y}}\hat{n_y}+\ddot{z}\hat{n_z}$

3D Cylindrical Coordinates

•

$$egin{pmatrix} \dot{\hat{n_
ho}} \ \dot{\hat{n_
ho}} \ \dot{\hat{n_z}} \end{pmatrix} = egin{pmatrix} \dot{arphi}\hat{n_
ho} \ -\dot{arphi}\hat{n_
ho} \ 0 \end{pmatrix}$$

- $ullet \ ec r =
 ho \hat{n_
 ho} + z \hat{n_z}$
- $ullet ec{v} = \dot{
 ho}\hat{n_
 ho} +
 ho\dot{arphi}\hat{n_arphi} + \dot{z}\hat{n_z}$
- $ullet \ ec a = (\ddot
 ho
 ho \dot arphi^2) \hat{n_
 ho} + (
 ho \ddot arphi + 2 \dot
 ho \, \dot arphi) \hat{n_arphi} + \ddot z \, \hat{n_z}$

3D Spherical Coordinates

$$egin{pmatrix} \dot{\hat{n_r}} \ \dot{\hat{n_ heta}} \ \dot{\hat{n_ heta}} \end{pmatrix} = egin{pmatrix} \dot{ heta}\hat{n_ heta} + \dot{arphi}sin heta\hat{n_arphi} \ -\dot{arphi}sin heta\hat{n_r} - \dot{arphi}cos heta\hat{n_ heta} \ -\dot{artheta}\hat{n_r} + \dot{arphi}cos heta\hat{n_arphi} \end{pmatrix}$$

- \bullet $\vec{r} = r\hat{n}$
- $ullet \ ec{v} = \dot{r} \hat{n_r} + r \dot{ heta} \hat{n_ heta} + r \dot{arphi} \sin heta \hat{n_arphi}$
- $\bullet \quad \vec{a} = \ddot{r}\hat{n_r} + \dot{r}\dot{\hat{n_r}} + \dot{r}\dot{\theta}\hat{n_\theta} + r\ddot{\theta}\hat{n_\theta} + r\dot{\theta}\dot{\hat{n_\theta}} + \dot{r}\dot{\varphi}sin\theta\hat{n_\varphi} + r\ddot{\varphi}sin\theta\hat{n_\varphi} + r\dot{\varphi}\dot{\theta}cos\theta\hat{n_\varphi} + r\dot{\varphi}sin\theta\hat{\hat{n_\varphi}}$

Polar Coordinates

transverse: along n_{arphi}

radial: along n_o

- ullet change r while keeping arphi constant. ds=dr
- ullet change arphi while keeping r constant ds=rdarphi
- ullet change both at the same time $(ds)^2=(dr)^2+(darphi)^2$

$$(rac{ds}{dt})^2 = (rac{dr}{dt})^2 = (rac{darphi}{dt})^2$$

= magnitude of $ec{v}=\dot{r}\hat{n_r}+
ho\dot{arphi}\hat{n_{arphi}}$

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Nature Coordinates

• established based on the trajectory (rely on the trajectory, cannot describe the trajectory, but the motion along the trajectory)

- ° $\hat{n_{ au}} imes \hat{n_n} = \hat{n_b}$ normal, tangential, normal, binormal (normal vector of the plane the trajectory is in locally)
- Acceleration under nature coordinates
 - $\circ \ \ a_t$: **tangential** component. 'speed change rate' $\dot{v}=a_t$
 - $\circ \ a_n$: **normal** component. contributes to 'turning'
- Curvature: 'local radius'
 - \circ physic way of finding curvature: $R=rac{v^2}{a_n}$