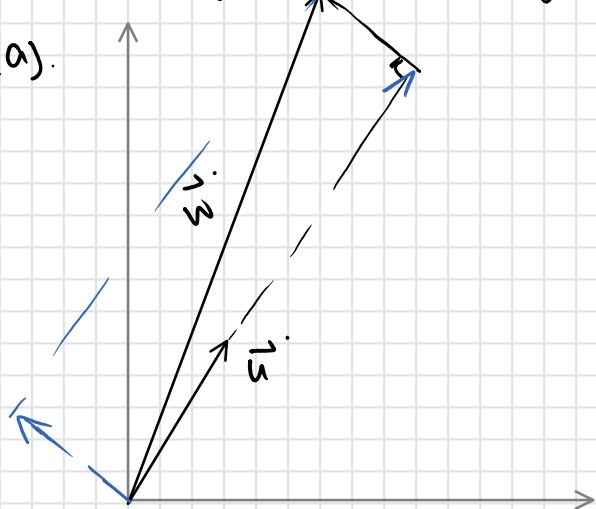


1. Problem 1.62 or 1.63
2. Check that in the Cartesian coordinates the dot product of two vectors  $\mathbf{u} = (u_x, u_y, u_z)$  and  $\mathbf{w} = (w_x, w_y, w_z)$  can be equivalently found either as  $\mathbf{u} \cdot \mathbf{w} = u_x w_x + u_y w_y + u_z w_z$  or as  $\mathbf{u} \cdot \mathbf{w} = uw \cos \alpha$ , where  $\alpha$  is the smaller angle between  $\mathbf{u}$  and  $\mathbf{w}$ .
3. **(12th edition of the textbook)** Problem 1.90 (p. 34).
4. Consider two vectors  $\mathbf{u} = 3\hat{n}_x + 4\hat{n}_y$  and  $\mathbf{w} = 6\hat{n}_x + 16\hat{n}_y$ . Find (a) the components of the vector  $\mathbf{w}$  that are, respectively, parallel and perpendicular to the vector  $\mathbf{u}$ , (b) the angle between  $\mathbf{w}$  and  $\mathbf{u}$ .
5. Is it possible to find a vector  $\mathbf{u}$ , such that  $(2\hat{n}_x - 3\hat{n}_y + 4\hat{n}_z) \times \mathbf{u} = (4\hat{n}_x + 3\hat{n}_y - \hat{n}_z)$ ? What is a quick way to check it?
6. A particle moves along a straight line with non-constant acceleration  $a_x(t) = -A\omega^2 \cos \omega t$ , where  $A$  and  $\omega$  are positive constants (what are their units?). At the instant of time  $t = 0$  its velocity  $v_x(0) = 3$  [m/s] and position  $x(0) = 4$  [m]. Find  $v_x(t)$  and  $x(t)$  at any instant of time. Sketch the graphs of  $x(t)$ ,  $v_x(t)$ , and  $a_x(t)$ . What kind of motion may these results describe?
7. A particle is moving along a straight line with velocity  $v_x(t) = -\beta A\omega e^{-\beta t} \cos \omega t$ , where  $A$ ,  $\omega$ , and  $\beta$  are positive constants.
  - a) What are the units of these constants?
  - b) Find acceleration  $a_x(t)$  and position  $x(t)$  of the particle, assuming that  $x(0) = 5$  [m].
  - c) Sketch  $x(t)$ ,  $v_x(t)$ , and  $a_x(t)$ .
  - d) What kind of motion could these results refer to (qualitatively)?
8. A car is moving in one direction along a straight line. Find the average velocity of the car if: (a) it travels *half of the journey time* with velocity  $v_1$  and the other half with velocity  $v_2$ , (b) it covers *half of the distance* with velocity  $v_1$  and the other one with velocity  $v_2$ . Both  $v_1$  and  $v_2$  are constants.
9. In a certain motion of a particle along a straight line, the acceleration turns out to be related to the position of the particle according to the formula  $a_x = \sqrt{kx}$ , where  $k > 0$  is a constant and  $x > 0$ . How does the velocity depend on  $x$ , if we know that for  $v_x(x_0) = v_0$ ?
10. **(12th edition of the textbook)** Problem 2.72 (p. 68). Do you need any additional information to correctly sketch the  $x(t)$  curve?
11. Problem 2.98.

### Problem 4.

$$\vec{u} = 3\hat{n}_x + 4\hat{n}_y \quad \vec{w} = 6\hat{n}_x + 16\hat{n}_y$$

(a).



$$\vec{w}_{||} = \langle \vec{w} \cdot \left(\frac{\vec{u}}{u}\right) \rangle \left(\frac{\vec{u}}{u}\right) = \frac{1}{5} \langle \begin{pmatrix} 6 \\ 16 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix} \rangle \cdot \frac{1}{5} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 9.84 \\ 13.12 \end{pmatrix}$$

$$\vec{w}_{\perp} = \vec{w} - \vec{w}_{||} = \begin{pmatrix} 6 \\ 16 \end{pmatrix} - \begin{pmatrix} 9.84 \\ 13.12 \end{pmatrix} = \begin{pmatrix} -3.84 \\ 2.88 \end{pmatrix}$$

(b)  $\cos \alpha = \frac{\vec{w} \cdot \vec{u}}{w \cdot u} = \frac{\langle \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 \\ 16 \end{pmatrix} \rangle}{5 \cdot \sqrt{6^2 + 16^2}} = 0.9597.$

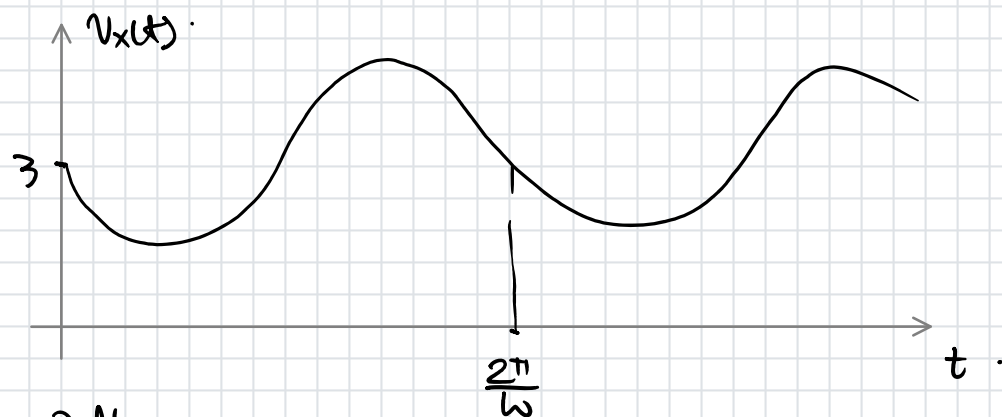
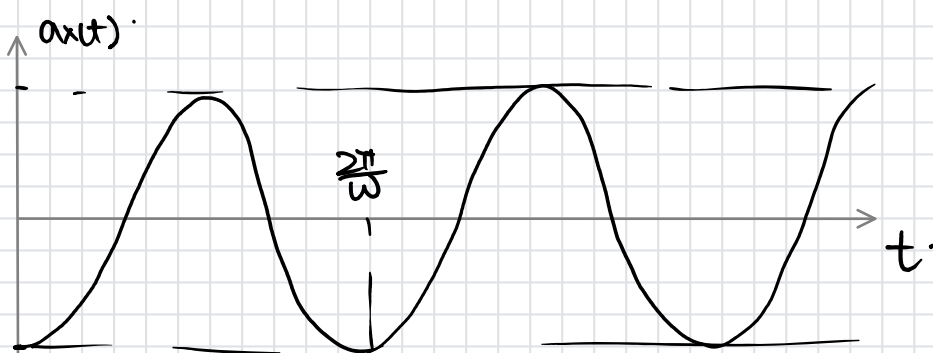
### Problem 5

$$\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \times \vec{u} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \quad \text{check } \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \perp \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} ? \quad \langle \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \rangle = 8 - 9 - 4 \neq 0.$$

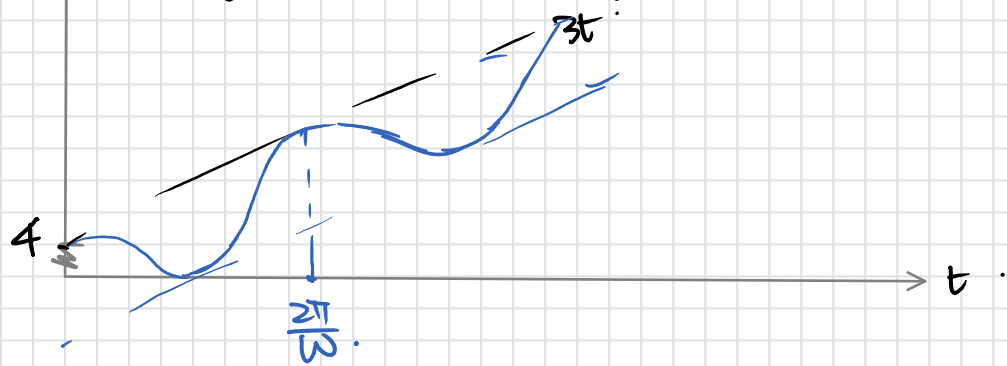
### Problem 6

$$a_x(t) = -A\omega^2 \cos \omega t.$$

$$v_x(t) = \int_0^t a_x(t) dt + v_x(0) = -A\omega \sin \omega t + v_x(0).$$



$$x(t) = \int_0^t v_x(t) dt + x(0) = \int_0^t (-A\omega \sin \omega t + v_x(0)) dt + x(0) = A(\cos \omega t - 1) + v_x(0)t + x(0)$$



$$\sin \omega t d\omega t = -\cos \omega t.$$

# Problem 7

a)  $v_x(t) = -\beta A \omega e^{-\beta t} \cos \omega t$

Annotations:  $\beta: s^{-1}$ ,  $\omega: rad/s$ ,  $\beta \omega: s^{-1}$ ,  $\beta: s^{-1}$ ,  $\omega: rad/s$ ,  $\beta \omega: s^{-1}$

b)  $a_x(t) = \frac{d}{dt} v_x(t) = -\beta A \omega \cdot (-e^{-\beta t} \cdot \omega \sin \omega t - \beta e^{-\beta t} \cos \omega t)$

$= -\beta A \omega e^{-\beta t} (-\omega \sin \omega t - \beta \cos \omega t)$

$x(t) = \int_0^t v_x(t) dt + x(0)$

$\int_0^t -\beta A \omega e^{-\beta t} \cos \omega t dt$

$= \int_0^t \omega A \cos \omega t d e^{-\beta t}$

$= \omega A \cos \omega t \cdot e^{-\beta t} \Big|_0^t - \int_0^t e^{-\beta t} d \omega A \cos \omega t$

$= \omega A (\cos \omega t e^{-\beta t} - 1) + \int_0^t e^{-\beta t} \omega^2 A \sin \omega t dt$

$= \omega A (\cos \omega t e^{-\beta t} - 1) + (-\frac{1}{\beta}) \cdot \int_0^t \omega^2 A \sin \omega t d e^{-\beta t}$

$= \omega A (\cos \omega t e^{-\beta t} - 1) - \frac{1}{\beta} \cdot (\omega^2 A \sin \omega t e^{-\beta t} \Big|_0^t - \int_0^t e^{-\beta t} \omega^3 A \cos \omega t \cdot \omega dt)$

$= \omega A \cos \omega t e^{-\beta t} - \omega A - \frac{\omega^2 A}{\beta} \sin \omega t e^{-\beta t} + \frac{\omega^3 A}{\beta} \int_0^t e^{-\beta t} \cos \omega t dt$

$- \beta A \omega X = \omega A \cos \omega t e^{-\beta t} - \omega A - \frac{\omega^2 A}{\beta} \sin \omega t e^{-\beta t} + \frac{\omega^3 A}{\beta} X$

$X = \frac{\beta}{\omega^3 A + \beta^2 A \omega} (\omega A \cos \omega t e^{-\beta t} - \omega A - \frac{\omega^2 A}{\beta} \sin \omega t e^{-\beta t})$

$x(t) = \frac{\omega A \cdot \beta \cdot (-\beta A \omega)}{\omega A (\omega^2 + \beta^2)} (\cos \omega t e^{-\beta t} - 1 - \frac{\omega}{\beta} \sin \omega t e^{-\beta t}) + x(0)$

$\approx 5$

# Problem 8

