VP160 RC1

Physical Quantities, Coordinate Sys, 1D Kinetics

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Github: https://github.com/joydddd/VP160-2020-SU-NOTES

you may need chrome + MathJax Plugin for Github to view properly

Github version will be the most up to date one.

Concepts

Physical Quantities: ALWAYS number + unit

Scale / Vector ?

Numbers

- Scientific notation: 6.02×10^{23}
- significant figures: (Simple Approach)
  - addition/subtraction: round to least most decimal point
  - multiplication/division: round to least number of SF
  - complicated calculation: Always round to
  - REAL WORLD: round to uncertainty dimension
- uncertainty: (We don't ask for uncertainty analysis in VP160)
  - e.g. $1.259 \pm 0.001 \mu A$ (always only one significant figure)
  - $u_{\alpha X} = \alpha \cdot u_X$
  - $u_{X \pm Y} = \sqrt{u_X^2 + u_Y^2}$
  - $u_{r,XY} = u_{r,X/Y} = \sqrt{u_{r,X}^2 + u_{r,Y}^2}$, r denotes relative uncertainty
  - $u_{r,X^k} = k \cdot u_{r,X}$
  - etc. (ref. Uncertainty Analysis Handbook, VP141/VP241)

Units

- unit prefixes:

n	μ	**m**	*****	**k**	**M**	**G**	**T**	**P**
nano	micor	mili	/	kilo	mega	giga	tera	peta
10^{-9}	10^{-6}	10^{-3}	10^{0}	10^{3}	10^{6}	10^{9}	10^{12}	10^{15}

- unit conversions

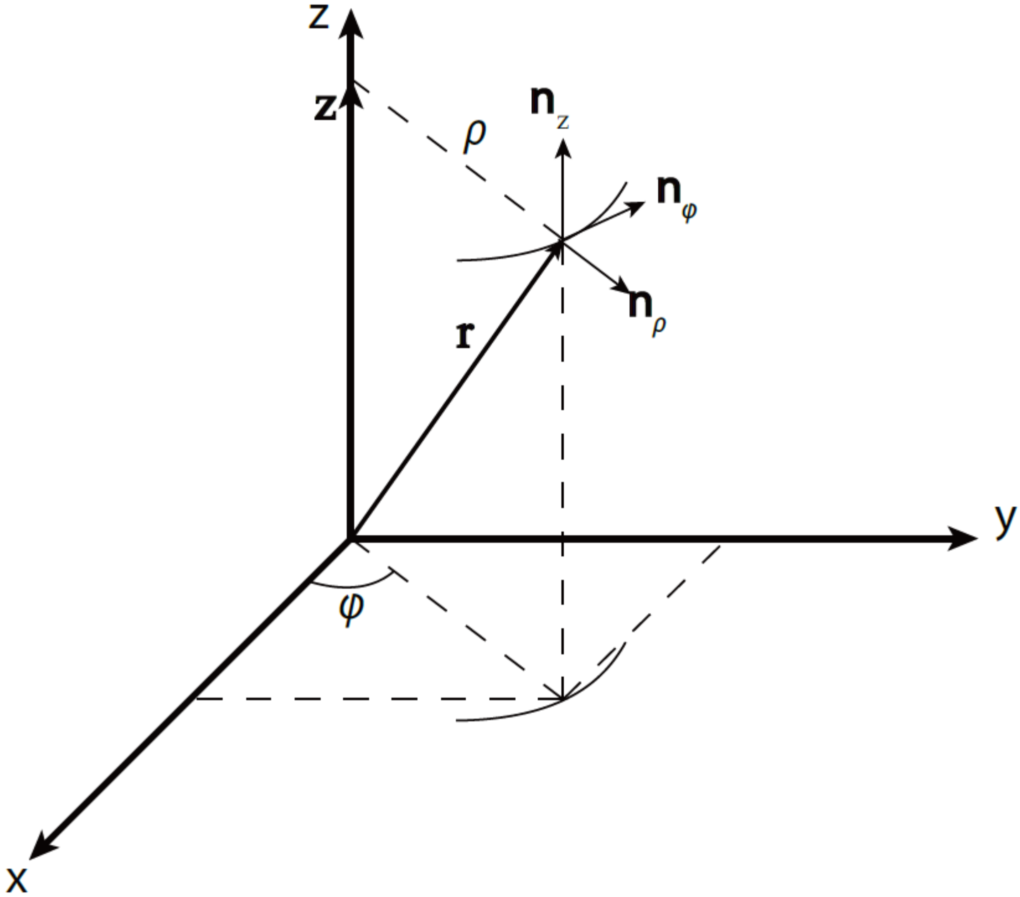
Vectors

- addition/ constant multiplication/ subtraction --> vector calculus
- dot product: vector . vector --> scale
  - $$\overrightarrow{u} \cdot \overrightarrow{v} = \left\langle \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}, \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \right\rangle = u_x v_x + u_y v_y + u_z v_z$$
  - e.g. $P = \overrightarrow{F} \cdot \overrightarrow{v}$ = $|\overrightarrow{F}||\overrightarrow{v}|cos\theta$
- cross product: vector x vector --> vector
  - $$\overrightarrow{u} \times \overrightarrow{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ u_x & u_y & u_z \\ v_z & v_y & v_z \end{vmatrix} = \begin{vmatrix} u_y & v_z \\ v_y & v_z \end{vmatrix} \hat{x} - \begin{vmatrix} u_x & v_z \\ v_x & v_z \end{vmatrix} \hat{y} + \begin{vmatrix} u_x & v_y \\ v_x & v_y \end{vmatrix} \hat{z}$$
  - e.g. $\overrightarrow{F} = I\overrightarrow{L} \times \overrightarrow{B}$
  - length: the cross section area of two vector $|\overrightarrow{F}| = I|\overrightarrow{L}||\overrightarrow{B}|sin\theta$
  - direction: right handed rule
    - $\overrightarrow{u} \times \overrightarrow{v} = -\overrightarrow{v} \times \overrightarrow{u}$
    - $\vec{a} \times \vec{b} \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
    - $(\vec{a} \times \vec{b}) \cdot \vec{c} = (\vec{c} \times \vec{a}) \cdot \vec{b} = (\vec{b} \times \vec{c}) \cdot \vec{a}$
- Differentiation/Integration w.r.t. time

Coordinate Systems

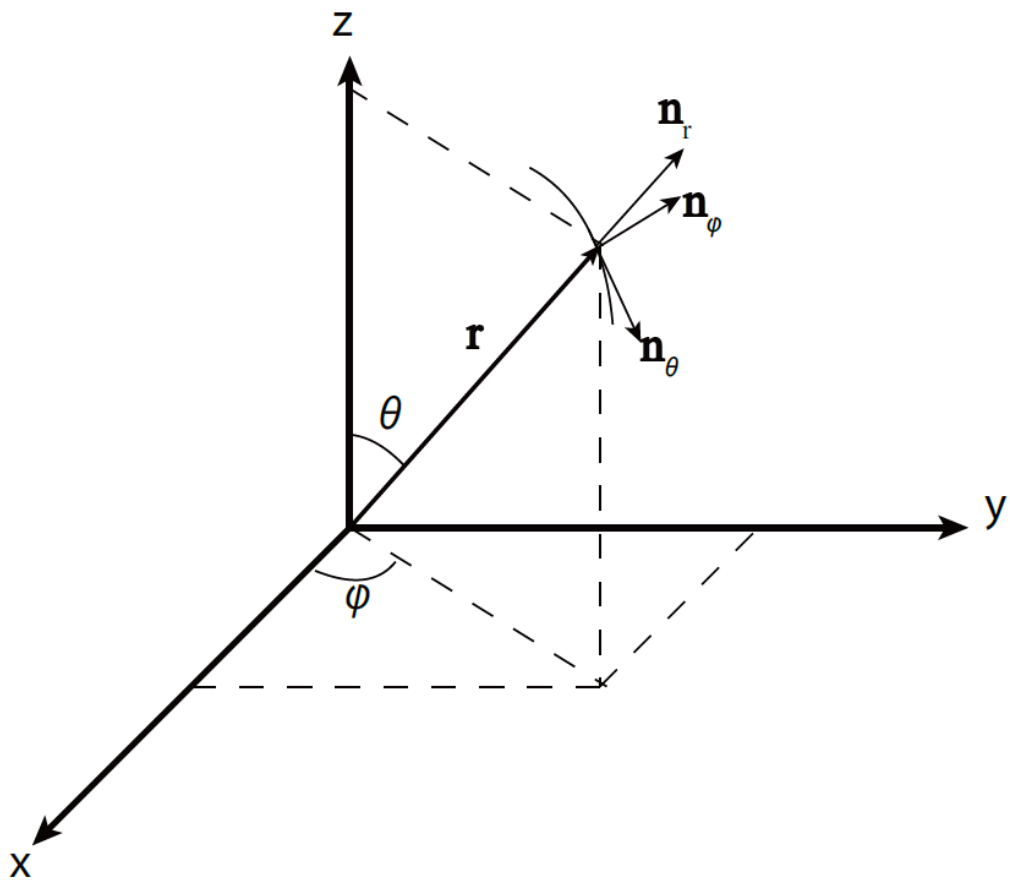
- Cartesian
  - $|\overrightarrow{w}| = \sqrt{w_x^2 + w_y^2 + w_z^2}$
  - {$\hat{n_x}$, $\hat{n_y}$, $\hat{n_z}$} / {$\hat{i}$, $\hat{j}$, $\hat{k}$} Range: {$[0, \infty)$,$[0, \infty)$, $[0, \infty)$}
    - mutually perpendicular $\hat{n_x} \cdot \hat{n_y} = 0$
    - unit length $|\hat{n_x}| = 1$
    - Right-hand Rule $\hat{n_x} \times \hat{n_y} = \hat{n_z}$
  - $\overrightarrow{r} = x\hat{n_x} + y\hat{n_y} + z\hat{n_z}$
    - differentiate: $\frac{\mathrm{d}\overrightarrow{u}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(u_x(t)\hat{n_x} + u_y(t)\hat{n_y} + u_z(t)\hat{n_z}) = \dot{u_x}(t)\hat{n_x} + \dot{u_y}(t)\hat{n_y} + \dot{u_z}(t)\hat{n_z}$ \
    - integrate
    - dot product $\overrightarrow{u} \cdot \overrightarrow{w} = u_x w_x + u_y w_y + u_z w_z$
    - cross product $\overrightarrow{u} \times \overrightarrow{w} = (u_y w_z - u_z w_y)\hat{n_x} + (u_z w_x - u_x w_z)\hat{n_y} + (u_x w_y - u_y w_x)\hat{n_z}$

- Cylindrical



  - $\{\hat{n_\rho}$, $\hat{n_\varphi}$, $\hat{n_z}\}$ Range: $\{[0,\infty)$, $[0,2\pi)$, $[0,\infty)\}$
    - $\rho=\sqrt{x^2+y^2}$
    - $\varphi=\arctan\frac{y}{x}(+\pi)$
    - $z=z$
    - $x=\rho\cos\varphi$
    - $y=\rho\sin\varphi$
  - $\overrightarrow{r}=\rho\hat{n_\rho}+z\hat{n_z}$
    - NOT directly differentiable!!! Will discuss later

- Spherical



  - longitude and latitude system
  - $\{\hat{n_r}, \hat{n_\varphi}, \hat{n_\theta}\}$ Range: $\{[0,\infty), [0,2\pi), [0,\pi)\}$
    - $\rho = \sqrt{x^2+y^2+z^2}$
    - $\varphi = \arctan\frac{y}{x}(+\pi)$
    - $\theta = \arctan\frac{\sqrt{x^2+y^2}}{z}(+\pi)$
    - $x = r\sin\theta\cos\varphi$
    - $y = r\sin\theta\sin\varphi$
    - $z = r\cos\theta$
  - $\overrightarrow{r} = r\hat{n_r}$
    - NOT directly differentiable!!! Will discuss later
- 2D polar coordinates
  - Cylindrical coordinates with $z = 0$
  - Spherical coordinates with $\theta = 0$

1D kinematics

Average vs. Instantaneous

Velocity

- average velocity:
  - $v_{\mathrm{av,x}} = \frac{x(t+\Delta t)-x(t)}{\Delta t}$
- velocity
  - When the time interval Δt -> 0
  - $\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \dot{x}(t) \stackrel{\mathrm{def}}{=} v_x(t)$

- velocity is location change rate w.r.t time

Acceleration

- average acceleration
  - $a_{\mathrm{av,x}} = \frac{v_x(t+\Delta t)-v_x(t)}{\Delta t}$
- acceleration
  - When time interval Δt -> 0
  - $a_x(t) = \frac{\mathrm{d}v_x(t)}{\mathrm{d}t} = \dot{v_x}(t) = \frac{\mathrm{d}^2 x(t)}{\mathrm{d}t^2} = \ddot{x}(t)$
  - acceleration is velocity change rate w.r.t and twice differentiation of position w.r.t time.

see lecture notes for pics

Relativity of Velocity/acceleration