VP160 RC2 3D Kinetics

topics: 2D polar coordinates; quick review and problems on 3D kinetics

$\dot{\mathbf{v}}$ vs. $\dot{v}$

acceleration vs. speed change rate (will discuss later in nature coordinates

3D Motion in Cartesian Coordinates

- $\vec{r} = x\hat{n_x} + y\hat{n_y} + z\hat{n_z}$
- $\vec{v} = \dot{x}\hat{n_x} + \dot{y}\hat{n_y} + \dot{z}\hat{n_z}$
- $\vec{a} = \ddot{x}\hat{n_x} + \ddot{y}\hat{n_y} + \ddot{z}\hat{n_z}$

3D Cylindrical Coordinates

- $$\left(\begin{matrix} \dot{\hat{n_\rho}} \\ \dot{\hat{n_\varphi}} \\ \dot{\hat{n_z}} \end{matrix}\right) = \left(\begin{matrix} \dot{\varphi}\hat{n_\varphi} \\ -\dot{\varphi}\hat{n_\rho} \\ 0 \end{matrix}\right)$$
- $\vec{r} = \rho\hat{n_\rho} + z\hat{n_z}$
- $\vec{v} = \dot{\rho}\hat{n_\rho} + \rho\dot{\varphi}\hat{n_\varphi} + \dot{z}\hat{n_z}$
- $\vec{a} = (\ddot{\rho} - \rho\dot{\varphi}^2)\hat{n_\rho} + (\rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi})\hat{n_\varphi} + \ddot{z}\hat{n_z}$

3D Spherical Coordinates

- $$\left(\begin{matrix} \dot{\hat{n_r}}\\\dot{\hat{n_\varphi}}\\\dot{\hat{n_\theta}} \end{matrix}\right) = \left(\begin{matrix} \dot{\theta} \hat{n}_{\theta}+\dot{\varphi} \sin \theta \hat{n}_{\varphi}\\-\dot{\varphi} \sin \theta \hat{n}_{r}-\dot{\varphi} \cos \theta \hat{n}_{\theta}\\-\dot{\theta} \hat{n}_{r}+\dot{\varphi} \cos \theta \hat{n}_{\varphi}\end{matrix}\right)$$
- $\vec{r} = r\hat{n_r}$
- $\vec{v} = \dot{r}\hat{n}_{r}+r\dot{\theta}\hat{n}_{\theta}+r\dot{\varphi}\sin\theta\hat{n}_{\varphi}$
- $\vec{a} = \ddot{r}\hat{n}_{r}+\dot{r}\dot{\hat{n}}_{r}+\dot{r}\dot{\theta}\hat{n}_{\theta}+r\ddot{\theta}\hat{n}_{\theta}+r\dot{\theta}\dot{\hat{n}}_{\theta}+\dot{r}\dot{\varphi}\sin\theta\hat{n}_{\varphi}+r\ddot{\varphi}\sin\theta\hat{n}_{\varphi}+r\dot{\varphi}\dot{\theta}\cos\theta\hat{n}_{\varphi}+r\dot{\varphi}\sin\theta\dot{\hat{n}}_{\varphi}$

Polar Coordinates

> **transverse**: along n_φ
>
> **radial**: along n_ρ

- change r while keeping φ constant. $ds = dr$
- change φ while keeping r constant $ds = rd\varphi$
- change both at the same time $(ds)^2 = (dr)^2 + (d\varphi)^2$

$$(\frac{ds}{dt})^2 = (\frac{dr}{dt})^2 = (\frac{d\varphi}{dt})^2$$

= magnitude of $\vec{v} = \dot{r}\hat{n_r} + \rho\dot{\varphi}\hat{n_\varphi}$

Nature Coordinates

- established based on the trajectory (rely on the trajectory, cannot describe the trajectory, but the motion along the trajectory)
  - $\hat{n_\tau} \times \hat{n_n} = \hat{n_b}$ normal, tangential, normal, binormal (normal vector of the plane the trajectory is in locally)
- Acceleration under nature coordinates
  - a_t: **tangential** component. 'speed change rate' $\dot{v} = a_t$
  - a_n: **normal** component. contributes to 'turning'
- **Curvature**: 'local radius'
  - physic way of finding curvature: $R = \frac{v^2}{a_n}$