VP160 RC2 3D Kinetics

topics: 2D polar coordinates; quick review and problems on 3D kinetics

$\dot{\mathbf{v}}$ vs. $\dot{v}$

acceleration vs. speed change rate (will discuss later in nature coordinates

3D Motion in Cartesian Coordinates

- $\vec{r} = x\hat{n}_x + y\hat{n}_y + z\hat{n}_z$
- $\vec{v} = \dot{x}\hat{n}_x + \dot{y}\hat{n}_y + \dot{z}\hat{n}_z$
- $\vec{a} = \ddot{x}\hat{n}_x + \ddot{y}\hat{n}_y + \ddot{z}\hat{n}_z$

3D Cylindrical Coordinates

- $$\begin{pmatrix} \dot{\hat{n}}_\rho \\ \dot{\hat{n}}_\varphi \\ \dot{\hat{n}}_z \end{pmatrix} = \begin{pmatrix} \dot{\varphi}\hat{n}_\varphi \\ -\dot{\varphi}\hat{n}_\rho \\ 0 \end{pmatrix}$$
- $\vec{r} = \rho\hat{n}_\rho + z\hat{n}_z$
- $\vec{v} = \dot{\rho}\hat{n}_\rho + \rho\dot{\varphi}\hat{n}_\varphi + \dot{z}\hat{n}_z$
- $\vec{a} = (\ddot{\rho} - \rho\dot{\varphi}^2)\hat{n}_\rho + (\rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi})\hat{n}_\varphi + \ddot{z}\hat{n}_z$

3D Spherical Coordinates

- $$\begin{pmatrix} \dot{\hat{n}}_r \\ \dot{\hat{n}}_\varphi \\ \dot{\hat{n}}_\theta \end{pmatrix} = \begin{pmatrix} \dot{\theta}\hat{n}_\theta + \dot{\varphi}sin\theta\hat{n}_\varphi \\ -\dot{\varphi}\sin\theta\hat{n}_r - \dot{\varphi}cos\theta\hat{n}_\theta \\ -\dot{\theta}\hat{n}_r + \dot{\varphi}cos\theta\hat{n}_\varphi \end{pmatrix}$$
- $\vec{r} = r\hat{n}_r$
- $\vec{v} = \dot{r}\hat{n}_r + r\dot{\theta}\hat{n}_\theta + r\dot{\varphi}\sin\theta\hat{n}_\varphi$
- $\vec{a} = \ddot{r}\hat{n}_r + \dot{r}\dot{\hat{n}}_r + \dot{r}\dot{\theta}\hat{n}_\theta + r\ddot{\theta}\hat{n}_\theta + r\dot{\theta}\dot{\hat{n}}_\theta + \dot{r}\dot{\varphi}sin\theta\hat{n}_\varphi + r\ddot{\varphi}sin\theta\hat{n}_\varphi + r\dot{\varphi}\dot{\theta}cos\theta\hat{n}_\varphi + r\dot{\varphi}sin\theta\dot{\hat{n}}_\varphi$

Polar Coordinates

> **transverse**: along n_φ
>
> **radial**: along n_ρ

- change r while keeping φ constant. $ds = dr$
- change φ while keeping r constant $ds = rd\varphi$
- change both at the same time $(ds)^2 = (dr)^2 + (d\varphi)^2$

$$(\frac{ds}{dt})^2 = (\frac{dr}{dt})^2 = (\frac{d\varphi}{dt})^2$$

= magnitude of $\vec{v} = \dot{r}\hat{n}_r + \rho\dot{\varphi}\hat{n}_\varphi$

Nature Coordinates

- established based on the trajectory (rely on the trajectory, cannot describe the trajectory, but the motion along the trajectory)
    - $\hat{n_\tau} \times \hat{n_n} = \hat{n_b}$ normal, tangential, normal, binormal (normal vector of the plane the trajectory is in locally)
- Acceleration under nature coordinates
    - a_t: **tangential** component. 'speed change rate' $\dot{v} = a_t$
    - a_n: **normal** component. contributes to 'turning'
- **Curvature**: 'local radius'
    - physic way of finding curvature: $R = \frac{v^2}{a_n}$