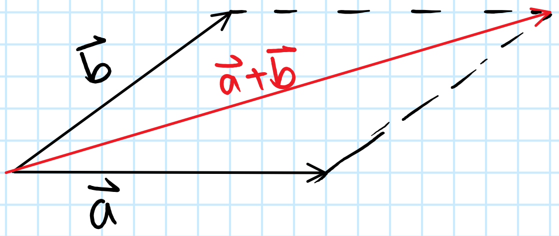
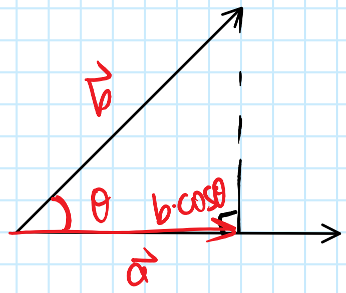
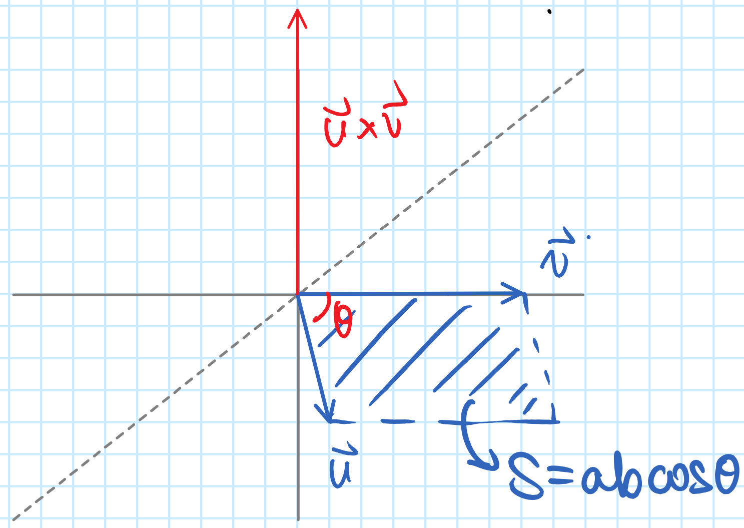
RC 1.

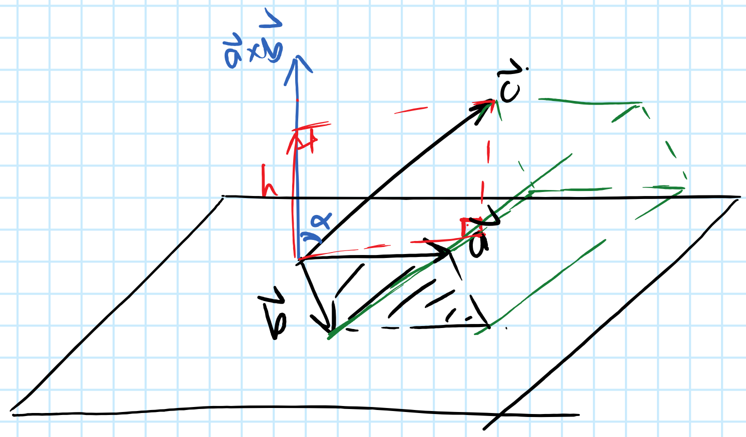
2020年5月17日 19:46

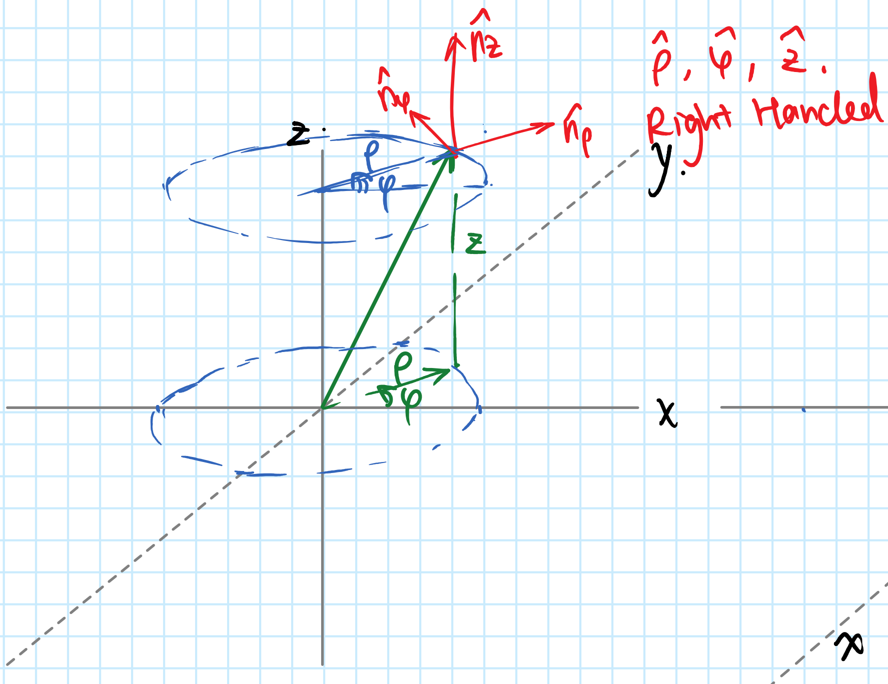
Vector Calculus.

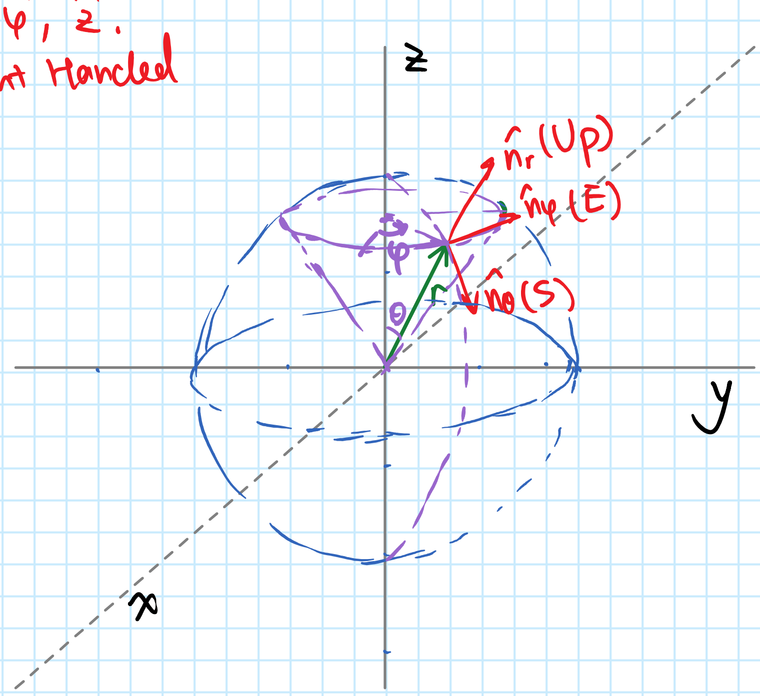












1. Problem 1.62 or 1.63

2. Check that in the Cartesian coordinates the dot product of two vectors $\mathbf{u} = (u_x, u_y, u_z)$ and $\mathbf{w} = (w_x, w_y, w_z)$ can be equivalently found either as $\mathbf{u} \cdot \mathbf{w} = u_x w_x + u_y w_y + u_z w_z$ or as $\mathbf{u} \cdot \mathbf{w} = uw \cos\alpha$, where α is the smaller angle between $\mathbf{u}$ and $\mathbf{w}$.

3. (**12th edition of the textbook**) Problem 1.90 (p. 34).

4. Consider two vectors $\mathbf{u} = 3\hat{n}_x + 4\hat{n}_y$ and $\mathbf{w} = 6\hat{n}_x + 16\hat{n}_y$. Find (a) the components of the vector $\mathbf{w}$ that are, respectively, parallel and perpendicular to the vector $\mathbf{u}$, (b) the angle between $\mathbf{w}$ and $\mathbf{u}$.

5. Is it possible to find a vector $\mathbf{u}$, such that $(2\hat{n}_x - 3\hat{n}_y + 4\hat{n}_z) \times \mathbf{u} = (4\hat{n}_x + 3\hat{n}_y - \hat{n}_z)$? What is a quick way to check it?

6. A particle moves along a straight line with non-constant acceleration $a_x(t) = -A\omega^2 \cos\omega t$, where A and ω are positive constants (what are their units?). At the instant of time $t = 0$ its velocity $v_x(0) = 3$ [m/s] and position $x(0) = 4$ [m]. Find $v_x(t)$ and $x(t)$ at any instant of time. Sketch the graphs of $x(t)$, $v_x(t)$, and $a_x(t)$. What kind of motion may these results describe?

7. A particle is moving along a straight line with velocity $v_x(t) = -\beta A\omega e^{-\beta t} \cos\omega t$, where A, ω, and β are positive constants.

   a) What are the units of these constants?

   b) Find acceleration $a_x(t)$ and position $x(t)$ of the particle, assuming that $x(0) = 5$ [m].

   c) Sketch $x(t)$, $v_x(t)$, and $a_x(t)$.

   d) What kind of motion could these results refer to (qualitatively)?

8. A car is moving in one direction along a straight line. Find the average velocity of the car if: (a) it travels *half of the journey time* with velocity v_1 and the other half with velocity v_2, (b) it covers *half of the distance* with velocity v_1 and the other one with velocity v_2. Both v_1 and v_2 are constants.

9. In a certain motion of a particle along a straight line, the acceleration turns out to be related to the position of the particle according to the formula $a_x = \sqrt{kx}$, where $k > 0$ is a constant and $x > 0$. How does the velocity depend on x, if we know that for $v_x(x_0) = v_0$?

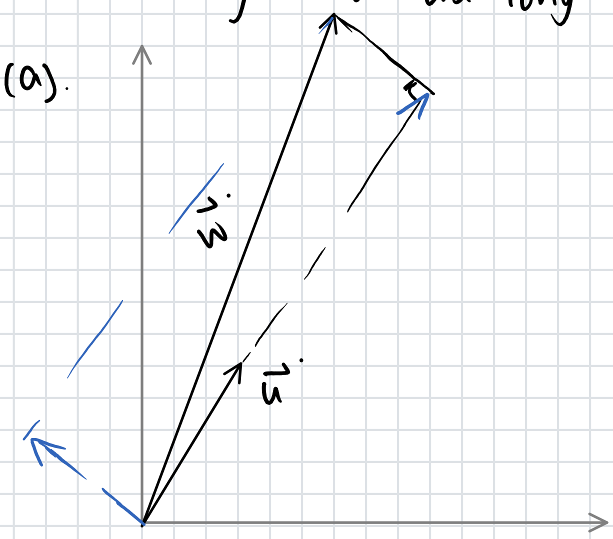
10. (**12th edition of the textbook**) Problem 2.72 (p. 68). Do you need any additional information to correctly sketch the $x(t)$ curve?

11. Problem 2.98.

Problem 4.

$\vec{u} = 3\hat{n}_x + 4\hat{n}_y$ $\quad$ $\vec{w} = 6\hat{n}_x + 16\hat{n}_y$

(a).



$$\vec{w}_{\parallel} = \langle \vec{w} \cdot (\tfrac{\vec{u}}{u}) \rangle (\tfrac{\vec{u}}{u}) = \tfrac{1}{5} \langle \begin{pmatrix} 6 \\ 16 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix} \rangle \cdot \tfrac{1}{5} \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 9.84 \\ 13.12 \end{pmatrix}$$

$$\vec{w}_{\perp} = \vec{w} - \vec{w}_{\parallel} = \begin{pmatrix} 6 \\ 16 \end{pmatrix} - \begin{pmatrix} 9.84 \\ 13.12 \end{pmatrix} = \begin{pmatrix} -3.84 \\ 2.88 \end{pmatrix}$$

(b) $$\cos\alpha = \frac{\vec{w} \cdot \vec{u}}{w \cdot u} = \frac{\langle \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 6 \\ 16 \end{pmatrix} \rangle}{5 \cdot \sqrt{6^2 + 16^2}} = 0.9597.$$

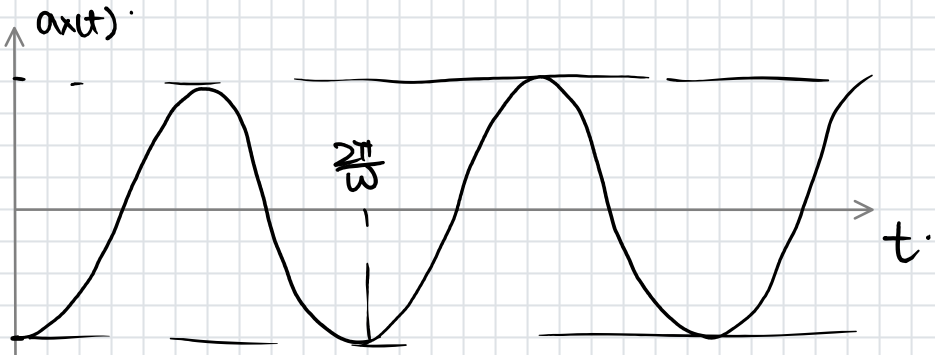
Problem 5

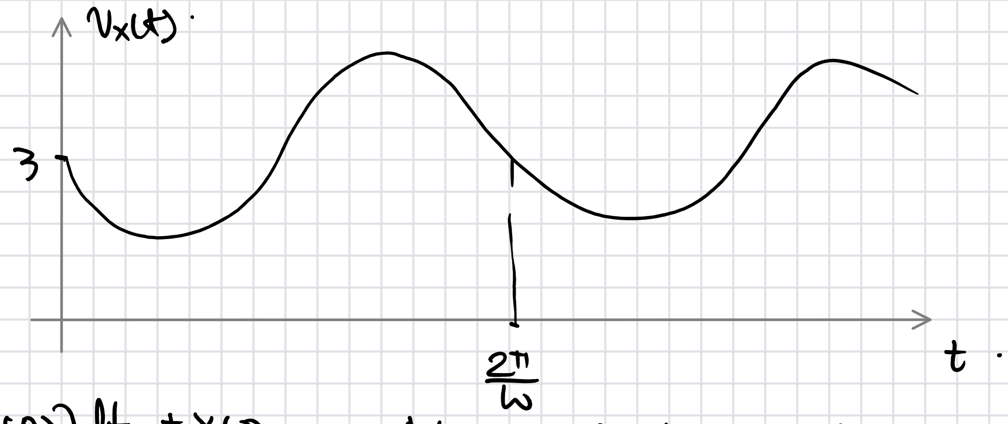
$$\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \times \vec{u} = \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}.$$

Check $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \perp \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$? $\quad \langle \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \rangle = 8 - 9 - 4 \neq 0.$

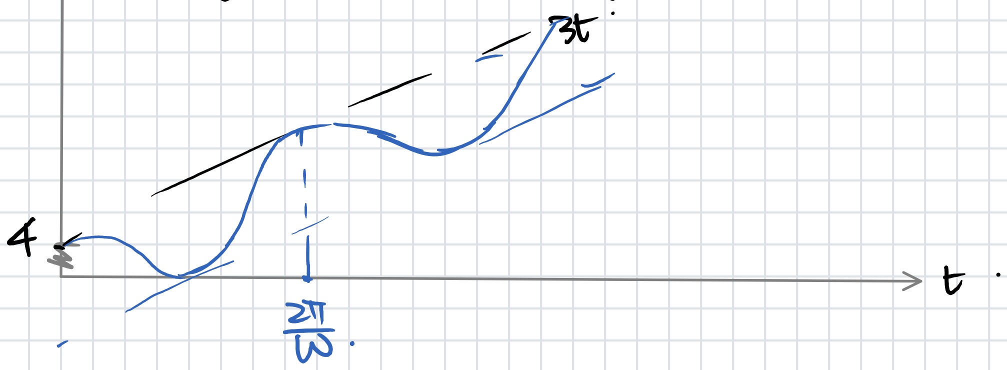
Problem 6

$a_x(t) = -A\omega^2 \cos\omega t.$ $\quad$ $V_x(t) = \int_0^t a_x(t)\,dt + v_x(0) = -A\omega \sin\omega t + v_x(0).$





$$x(t) = \int_0^t V_x(t) + x(0) = \int_0^t (-A\omega \sin\omega t + V_x(0))\,dt + x(0) = A(\cos\omega t - 1) + V_x(0)t + x(0)$$



$$\int \sin\omega t \, d\omega t = -\cos\omega t.$$

Problem 7

a) $v_x(t) = -\beta A \omega e^{-\beta t} \cos\omega t$.

s^{-1} $m \cdot s$ ← Appear on exp: must be const. $\beta: s^{-1}$.

$m \cdot s^{-1}$. s^{-1}. $\underline{\text{rad/s}}$ = 无量纲数

b)

$$a_x(t) = \frac{d}{dt} v_x(t) = -\beta A\omega \cdot \left(-e^{-\beta t} \cdot \omega \sin\omega t - \beta e^{-\beta t} \cos\omega t\right)$$

$$= -\beta A\omega e^{-\beta t}\left(-\omega \sin\omega t - \beta \cos\omega t\right)$$

$$x(t) = \int_0^t v_x(t)\, dt + x(0)$$

$$\int_0^t -\beta A\omega e^{-\beta t} \cos\omega t \, dt. \quad X$$

$$= \int_0^t \omega A \cos\omega t \, d e^{-\beta t}.$$

$$= \omega A \cos\omega t \cdot e^{-\beta t}\Big|_0^t - \int_0^t e^{-\beta t} \, d\omega A \cos\omega t.$$

$$= \omega A(\cos\omega t \, e^{-\beta t} - 1) + \int_0^t e^{-\beta t} \omega^2 A \sin\omega t \, dt.$$

$$= \omega A(\cos\omega t \, e^{-\beta t} - 1) + \left(\frac{1}{-\beta}\right) \cdot \int_0^t \omega^2 A \sin\omega t \, d e^{-\beta t}$$

$$= \omega A(\cos\omega t \, e^{-\beta t} - 1) - \frac{1}{\beta} \cdot \left(\omega^2 A \sin\omega t \, e^{-\beta t}\Big|_0^t - \int_0^t e^{-\beta t} \omega^2 A \cos\omega t \cdot \omega dt\right).$$

$$= \omega A \cos\omega t e^{-\beta t} - \omega A - \frac{\omega^2 A}{\beta} \sin\omega t \, e^{-\beta t} + \frac{\omega^3 A}{\beta} \int_0^t e^{-\beta t} \cos\omega t \, dt. \quad X$$

$$-\beta A\omega X = \omega A \cos\omega t e^{-\beta t} - \omega A - \frac{\omega^2 A}{\beta} \sin\omega t \, e^{-\beta t} + \frac{\omega^3 A}{\beta} X$$

$$X = \frac{\beta}{\omega^3 A + \beta^2 A\omega}\left(\omega A \cos\omega t \, e^{-\beta t} - \omega A - \frac{\omega^2 A}{\beta} \sin\omega t \, e^{-\beta t}\right)$$

$$x(t) = \frac{\cancel{\omega A} \cdot \beta \cdot (-\beta A\omega)}{\cancel{\omega A}(\omega^2 + \beta^2)}\left(\cos\omega t \, e^{-\beta t} - 1 - \frac{\omega}{\beta} \sin\omega t \, e^{-\beta t}\right) + \underset{=5}{x(0)}$$

Problem 8