Adding Supply Restrictions

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Grad IO

Supply

- Economic theory gives us some additional powerful restrictions.
- We may want to impose $MR = MC$.
    - Not always a good idea
    - Do we know what conduct is? Collusion? Bertrand? Cournot?
- Alternatively, we can ask – what is a good instrument for demand? something from another equation (ie: supply).

Some setup

We can break up the parameter space into three parts:

- θ_1: linear exogenous demand parameters with dimension K_1
- θ_2: parameters including price and random coefficients (endogenous / nonlinear) with dimension K_2
    - $\theta_2 = [\alpha, \widetilde{\theta}_2]$
- θ_3: linear exogenous supply parameters with dimension K_3
- $N = \sum_t \dim(\mathcal{J}_t)$ observations.

Supply Side

Consider the multi-product Bertrand FOCs:

$$\max_{p_{jt}:j\in\mathcal{J}_{ft}} \sum_{j\in\mathcal{J}_{ft}} s_{jt}\left(\boldsymbol{p}_t\right)\cdot\left(p_{jt}-c_{jt}\right)$$
$$s_{jt}\left(\boldsymbol{p}_t\right)+\sum_{k\in\mathcal{J}_{ft}} \frac{\partial s_{kt}}{\partial p_{jt}}\left(\boldsymbol{p}_t\right)\cdot\left(p_{kt}-c_{kt}\right)=0$$

It is helpful to define the multiproduct oligopoly ownership matrix $\mathcal{H}_t$ as having 1's if (j,k) have the same owner, and 0's otherwise. We can re-write the FOC in matrix form where $\odot$ denotes Hadamard product:

$$\begin{aligned}\Delta_t\left(\boldsymbol{p}_t\right) &\equiv -\mathcal{H}_t\odot\frac{\partial \boldsymbol{s}_t}{\partial \boldsymbol{p}_t}\left(\boldsymbol{p}_t\right)\\ s_t\left(\boldsymbol{p}_t\right) &= \Delta_t\left(\boldsymbol{p}_t\right)\cdot\left(\boldsymbol{p}_t-\boldsymbol{c}_t\right)\\ \underbrace{\Delta_t\left(\boldsymbol{p}_t\right)^{-1}\boldsymbol{s}_t\left(\boldsymbol{p}_t\right)}_{\eta_t\left(\boldsymbol{p}_t,\boldsymbol{s}_t,\theta_2\right)} &= \boldsymbol{p}_t-\boldsymbol{c}_t\end{aligned}$$

Recovering Marginal Costs

Recover implied markups/ marginal costs, and assume a functional form for $mc_{jt}(x_{jt}, w_{jt})$. (Again for a single market t).

$$\widehat{\mathbf{mc_t}}(\theta_2) = \mathbf{p_t} - \boldsymbol{\eta_t}(\mathbf{p_t}, \mathbf{s_t}, \theta_2)$$
$$f(mc_{jt}) = h_s(x_{jt}, w_{jt}, \theta_3) + \omega_{jt}$$

Which we can solve for ω_{jt}:

$$\omega_{jt} = f(\mathbf{p_t} - \boldsymbol{\eta_t}(\mathbf{p}, \mathbf{s}, \theta_2)) - h_s(x_{jt}, w_{jt}, \theta_3)$$

- $f(\cdot)$ is usually $\log(\cdot)$ or identity.
- $h_s(x_{jt}, w_{jt}, \theta_3) = [x_{jt},\, w_{jt}]\gamma$ is usually linear
- Use this to form additional moments: $E[\omega'_{jt} Z^s_{jt}] = 0$

Additional Details (Conlon Gortmaker RJE 2020)

If everything is linear:

$$
\begin{aligned}
y_{jt}^D &:= \widehat{\delta}_{jt}(\theta_2) + \alpha p_{jt} &= (x_{jt}\ v_{jt})'\beta + \xi_t =: x_{jt}^{D\prime}\beta + \xi_{jt} \\
y_{jt}^S &:= f(\widehat{mc}_{jt}(\theta_2)) &= (x_{jt}\ w_{jt})'\gamma + \omega_t =: x_{jt}^{S\prime}\gamma + \omega_{jt}
\end{aligned}
$$

Stacking the system across observations yields:

$$
\underbrace{\begin{bmatrix} y_D \\ y_S \end{bmatrix}}_{2N\times 1} = \underbrace{\begin{bmatrix} X_D & 0 \\ 0 & X_S \end{bmatrix}}_{2N\times(K_1+K_3)} \underbrace{\begin{bmatrix} \beta \\ \gamma \end{bmatrix}}_{(K_1+K_3)\times 1} + \underbrace{\begin{bmatrix} \xi \\ \omega \end{bmatrix}}_{2N\times 1}
$$

Note: we cannot perform independent regressions unless we are willing to assume that $Cov(\xi_{jt}, \omega_{jt}) = 0$!

Simultaneous Supply and Demand (Conlon Gortmaker RJE 2020)

(a) For each market t: solve $\mathcal{S}_{jt} = s_{jt}(\delta_{\cdot t}, \theta_2)$ for $\widehat{\delta}_{\cdot t}(\theta_2)$.
(b) For each market t: use $\widehat{\delta}_{\cdot t}(\theta_2)$ to construct $\eta_{\cdot t}(\mathbf{q_t}, \mathbf{p_t}, \widehat{\delta}_{\cdot t}(\theta_2), \theta_2)$
(c) For each market t: Recover $\widehat{mc}_{jt}(\widehat{\delta}_{\cdot t}(\theta_2), \theta_2) = p_{jt} - \eta_{jt}(\widehat{\delta}_{\cdot t}(\theta_2), \theta_2)$
(d) Stack up $\widehat{\delta}_{\cdot t}(\theta_2)$ and $\widehat{mc}_{jt}(\widehat{\delta}_{\cdot t}(\theta_2), \theta_2)$ and use linear IV-GMM to recover $[\widehat{\theta}_1(\theta_2), \widehat{\theta}_3(\theta_2)]$ following the recipe on previous slide
(e) Construct the residuals:

$$\widehat{\xi}_{jt}(\theta_2) = \widehat{\delta}_{jt}(\theta_2) - [x_{jt}\ v_{jt}]\, \widehat{\beta}(\theta_2) + \alpha p_{jt}$$
$$\widehat{\omega}_{jt}(\theta_2) = f(\widehat{mc}_{jt}(\theta_2)) - [x_{jt}\ w_{jt}]\, \widehat{\gamma}(\theta_2)$$

(f) Construct sample moments

$$g_n^D(\theta_2) = \frac{1}{N} \sum_{jt} Z_{jt}^{D\prime} \widehat{\xi}_{jt}(\theta_2)$$
$$g_n^S(\theta_2) = \frac{1}{N} \sum_{jt} Z_{jt}^{S\prime} \widehat{\omega}_{jt}(\theta_2)$$

(g) Construct GMM objective $Q_n(\theta_2) = \begin{bmatrix} g_n^d(\theta_2) \\ g_n^s(\theta_2) \end{bmatrix}' W \begin{bmatrix} g_n^d(\theta_2) \\ g_n^s(\theta_2) \end{bmatrix}$

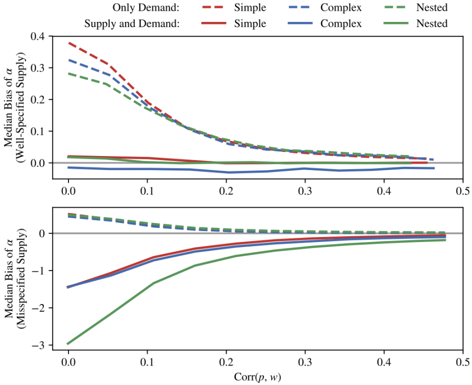
What's the Point (Conlon Gortmaker RJE 2020)

- A well-specified supply side can make it easier to estimate θ_2 parameters (price in particular).
- Imposing the supply side only helps if we have information about the marginal costs / production function that we would like to impose
- May want to enforce some economic constraints: ($mc_{jt} > 0$ is a good one).
- But assuming the wrong conduct $\mathcal{H}_t$ can lead to misspecification!

				True Value				Median Bias				Median Absolute Error			
Simulation	Supply	Instruments	Seconds	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ	α	σ_x	σ_p	ρ
Simple	No	Own	0.6	-1	3			0.126	-0.045			0.238	0.257		
Simple	No	Sums	0.6	-1	3			0.224	-0.076			0.257	0.208		
Simple	No	Local	0.6	-1	3			0.181	-0.056			0.242	0.235		
Simple	No	Quadratic	0.6	-1	3			0.206	-0.085			0.263	0.239		
Simple	No	Optimal	0.8	-1	3			0.218	-0.049			0.250	0.174		
Simple	Yes	Own	1.4	-1	3			0.021	0.006			0.226	0.250		
Simple	Yes	Sums	1.5	-1	3			0.054	-0.020			0.193	0.196		
Simple	Yes	Local	1.4	-1	3			0.035	-0.006			0.207	0.229		
Simple	Yes	Quadratic	1.4	-1	3			0.047	-0.022			0.217	0.237		
Simple	Yes	Optimal	2.2	-1	3			0.005	0.012			0.170	0.171		

What about Misspecification? (Conlon Gortmaker RJE 2020)

Figure 2: Instrument Strength and Misspecification



Each plot documents how bias of the linear parameter on price, α, decreases with the strength of the cost shifter w_{jt}, which is included as a demand-side instrument. To weaken or strengthen the instrument, we vary its supply-side parameter from $\gamma_w = 0$ to $\gamma_w = 1$, and report the correlation this induces between w_{jt} and prices p_{jt}. Reported bias values are medians across 1,000 different simulations. The top plot reports results for the simulation configurations described in Section 5. In the bottom plot, we simulate data according to perfect competition (i.e., prices are set equal to marginal costs instead of those that satisfy Bertrand-Nash first order conditions), but continue to estimate the model under the assumption of imperfect competition. For all problems, we use the "approximate" version of the feasible optimal instruments and a Gauss-Hermite product rule that exactly integrates polynomials of degree 17 or less.