Differentiated Bertrand

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Grad IO

Differentiated Products Bertrand

Consider the multi-product Bertrand problem where firms solve:
$\arg\max_{p\in\mathcal{J}_f} \pi_f(\mathbf{p}) = \sum_{j\in\mathcal{J}_f}(p_j - c_j)\cdot q_j(\mathbf{p})$:

$$0 = q_j(\mathbf{p}) + \sum_{k\in\mathcal{J}_f}(p_k - c_k)\frac{\partial q_k}{\partial p_j}(\mathbf{p})$$

$$\rightarrow p_j = q_j(\mathbf{p})\left[-\frac{\partial q_j}{\partial p_j}(\mathbf{p})\right]^{-1} + c_j + \sum_{k\in\mathcal{J}_f\setminus j}(p_k - c_k)\underbrace{\frac{\partial q_k}{\partial p_j}(\mathbf{p})\left[-\frac{\partial q_j}{\partial p_j}(\mathbf{p})\right]^{-1}}_{D_{jk}(\mathbf{p})}$$

$$p_j(p_{-j}) = \underbrace{\frac{1}{1+1/\epsilon_{jj}(\mathbf{p})}}_{\text{Markup}}\left[c_j + \sum_{k\in\mathcal{J}_f\setminus j}(p_k - c_k)\cdot D_{jk}(\mathbf{p})\right].$$

We call $D_{jk}(\mathbf{p}) = \frac{\frac{\partial q_k}{\partial p_j}(\mathbf{p})}{\left|\frac{\partial q_j}{\partial p_j}(\mathbf{p})\right|}$ the diversion ratio.

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We can also re-write the best-response in the Lerner Index form:

$$p_j(p_{-j}) = \underbrace{\frac{1}{1 + 1/\epsilon_{jj}(\mathbf{p})}}_{\text{Markup}} \left[c_j + \underbrace{\sum_{k \in \mathcal{J}_f \setminus j} (p_k - c_k) \cdot D_{jk}(\mathbf{p})}_{\text{opportunity cost}} \right]$$

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It is helpful to define the matrix Ω with entries:

$$\Omega_{(j,k)}(\mathbf{p}) = \left\{ \begin{array}{ll} -\frac{\partial q_j}{\partial p_k}(\mathbf{p}) & \text{for } (j,k) \in \mathcal{J}_f \\ 0 & \text{for } (j,k) \notin \mathcal{J}_f \end{array} \right\}$$

We can re-write the FOC in matrix form:

$$q(\mathbf{p}) = \Omega(\mathbf{p}) \cdot (\mathbf{p} - \mathbf{mc})$$

What do we want to learn from demand systems?

We can recover markups and marginal costs:

$$(\mathbf{p} - \mathbf{mc}) = \Omega(\mathbf{p})^{-1} q(\mathbf{p}) \Rightarrow \mathbf{mc} = \mathbf{p} + \Omega(\mathbf{p})^{-1} q(\mathbf{p})$$

What do we want to learn from demand systems?

- We need two objects from the demand system:
  - The (vector of) predicted sales at the (vector of) prices $\mathbf{q}(\mathbf{p})$.
  - The derivatives (or elasticities) of the demand function $\frac{\partial q_k}{\partial p_j}(\mathbf{p})$
  - Or the diversion ratios: $D_{jk}(\mathbf{p})$.
  - Welfare is often related to substitution to no purchase or "outside goods".
- Ideally, these objects depend on the full vector of prices $\mathbf{p}$.
  - We can impose restrictions on the demand curve, such as constant slope or constant elasticity.
- With these in hand, we can:
  - Recover estimates of marginal cost (remember these are proprietary firm information and accounting estimates are often unreliable).

Overview

Multiproduct Demand system estimation is probably the most important contribution from the New Empirical IO literature

- Demand is an important primitive (think about Econ 101)
- Welfare Analysis, Pass through of taxation, Value of Advertising, Price Effects of Mergers all rely on demand estimates.
- Last 5-10 years has seen successful export to other fields: Trade, Healthcare, Education, Urban Economics, Marketing, Operations Research.
- Anywhere consumers face a number of options and "prices": doctors, hospitals, schools, mutual funds, potential dates, etc.
- There are many many more applications.

Two Major Issues

- Endogeneity of Prices
  - Prices are not randomly determined, but set strategically by firms who observe the demand curve
  - The simultaneity of supply and demand creates a problem. We see the market clearing (P^*, Q^*) over several periods, but in general we do not know which curve shifted.
- Multiple Products/Flexibility
  - We want to allow for flexible (data driven) substitution across products but if we have J products, then $\partial Q_j / \partial P_k$ might have J^2 elements.
  - We may also think that $\partial Q_j / \partial P_k$ varies with P and with other covariates x.
  - We may also care about $\partial^2 Q_j / \partial P_k^2$, $\partial^3 Q_j / \partial P_k^3$ and so on.
- We will address the issues separately and then see how to put them back together.

Taxonomy of Demand Systems

- Representative Consumer vs. Heterogeneous Agents?
- Discrete Choices vs. Continuous Choices?
- Single Product vs. Many Products?
- Product Space vs. Characteristic Space?
    - Do consumers choose products in product space or in characteristic space?

Data Sources

- We can either have aggregate data (market level data on (P_j, Q_j))
  - Many supermarket "scanner' datasets: Nielsen, IRI, Dominick's
  - NPD: video games/computers/consumer electronics.
  - Many proprietary single firm sources
- or micro data panel of data with same individuals over time.
  - Best example is Nielsen Homescan Consumer Panel data.
  - Visa/Mastercard datasets
  - Medicare
- Sometimes we have a combination of both
  - Often we have aggregate purchase data plus some micro data from a survey on a subpopulation.
  - ie: asking people who purchased GM cars which other cars they were considering.
  - Using scanner data for alcohol purchases and comparing to consumption surveys by income and education.