Learning Models and Experience Goods

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Fall 2016

Uncertainty and Learning

- We have already looked at models with forward looking consumers
- Consumers faced uncertainty about the price, but understood the characteristics and the utility received from the good up to the IID ϵ.
- In many cases, consumers do not fully understand their preferences over goods until they sample the goods themselves.
- Changes to brands, introduction of new brands, price cuts, coupons, or advertising may induce consumers to resample.
- We would like to incorporate persistence in brand choice but also experiential learning

Uncertainty and Learning

We examine three papers dealing with uncertainty and learning:

- Ackerberg (2001) looks at whether advertising lets consumers learn about new brands and distinguishes between informative and prestige effects
- Erdem and Keane (1996) extends models of brand choice to allow for Bayesian learning about experience goods
- Crawford and Shum (2005) look at how doctor's learn about patient's types as well as drug efficacy in a model of experiential learning.

Ackerberg 2001: Advertising and Yoplait 150

- **Informative** about product existence and search characteristics. Stigler (1961), Butters (1977), Grossman Shapiro (1984) should not affect behavior of experienced users.
- **Signalling** Nelson (1974), Kihlstrom and Riordan (1984), Milgrom and Roberts (1986).
  1. If consumer perfectly learns about brand's experience characteristics after consumption $\rightarrow$ does not affect behavior of experienced users
  2. If consumer continues to learn about experience characteristics after consumption $\rightarrow$ should be decreasing in number of consumption experiences.
- **Prestige** Becker or Becker and Murphy (1993) does not depend on whether or not consumers have experienced the good but enters utility.

Ackerberg 2001: Advertising and Yoplait 150

- Ackerberg exploits panel data following advertising and grocery purchases over time.
- Hypothesis is that informative advertising has a larger effect on consumers with no brand experience.
- Prestige affects all consumers equally independent of experience.
- Looks at a new product introduction to get around initial conditions problem

Ackerberg 2001: Data

- AC Neilsen *Scanner Data* matched upw ith TV meters
- 1986-1989 covers 2000 households and 80% of area drugstores and supermarkets.
- Two cities: Sioux Falls, SD (SF) and Springfield, MO (SP)
- He chooses yogurt because it is not easily storable (Hendel Nevo 2007).
- Introduction of Yoplait 150 by the #2 manufacturer
- Heavily advertised, first low-fat, low-calorie yogurt by Yoplait!

Table 1: Descriptive Statistics

Variable	SF	SP
Households	950	825
Average shopping trips per household	70.58 (33.39)	65.82 (31.82)
Average price of Yoplait 150 (cents)	.645 (.060)	.663 (.079)
Shopping trips with Yoplait 150 purchase	302	656
Manufacturers' coupons redeemed for Yoplait 150	16	238
Shopping trips with other Yogurt purchase	5,432	3,863
Households trying Yoplait 150	123	184
Households trying other yogurts	648	512
Commercial exposures per household	13.60 (10.81)	15.22 (9.96)
Advertising share of Yoplait 150	.35	.37
Market share of Yoplait 150	.05	.14

Table 2: Descriptive Correlations

TABLE 2 **Weekly Correlations**

Variable	SF	SP
p_t, q_t	$-.326^{**}$	$-.499^{**}$
p_t, a_t	.106	$.285^{*}$
q_t, a_t	.122	.030
q_t, a_{t-1}	.028	.194
p_t, p_{t-1}	$.274^{*}$	$.744^{**}$
p_t, a_{t-1}	.141	.249
a_t, p_{t-1}	.216	.216
a_t, a_{t-1}	$.486^{**}$	$.387^{**}$

Note: **.01 significance, *.05 significance.

Table 3: Reduced Form Results

	Dependent Variable: Initial Purchases				Dependent Variable: Repeat Purchases			
	1	2	3	4	1	2	3	4
N	918	918	678	918	918	918	678	918
R^2	.066	.085	.107	.066	.162	.149	.120	.162
Market	.222	.002	.224	.223	.700	.006	.832	.700
Dummy	(.062)	(.000)	(.069)	(.062)	(.089)	(.000)	(.111)	(.089)
Price	−5.298	−.038	−7.388	−5.354	−3.954	−.029	−5.512	−3.942
	(1.568)	(.013)	(1.726)	(1.585)	(1.829)	(.014)	(2.207)	(1.838)
Ads	.044	.030	.042	.044	.020	.014	.014	.016
	(.022)	(.015)	(.021)	(.022)	(.023)	(.017)	(.024)	(.024)
t-value	1.981	1.925	2.046	1.988	.873	.818	.596	.679

Notes: Unit of observation is a market day. Constant term and third-order polynomial in time not reported. SEs corrected for serial correlation using Newey-West.

Model

Reduced form for discrete choice that consumer i purchases Yoplait 150 on trip t

$$c_{it} = \begin{cases} 1 & \text{IFF } \alpha_i + X_{it}\beta_1 - \gamma p_{it} + \epsilon_{1it} > Z_{it}\beta_2 + \epsilon_{2it} \\ 0 & \text{o.w.} \end{cases}$$

- First term may proxy for static utility or choice specific value function of YP150 purchase
- Second term represents utility of outside option
- α_i is a random effect (persistent heterogeneity) for YP150.
- X_{it} contains advertising, household and consumer characteristics, and functions of previous purchases of YP150, coupon, time trend.
- Z_{it} contains an index of other competitors' prices

Likelihood

$$
\begin{aligned}
L_i(\theta) &= Pr[c_{i1},\ldots,c_{iT_i}|W_i^t,Z_i^t,p_i^t;\theta] \\
&= \int Pr[c_{i1},\ldots,c_{iT_i}|W_i^t,Z_i^t,p_i^t;a_i;\theta]f(d\alpha_i|\theta) \\
&= \int \prod_{t=1}^{T_i} Pr[c_{it}|X_{it}(c_i^{t-1}),Z_{it},p_{it};a_i;\theta]f(d\alpha_i|\theta)
\end{aligned}
$$

- c_i^{t-1} is your entire purchase history
- W_i^t is the subset of explanatory variables X_{it} that are completely exogenous
- Choice probabilities determined by ϵ IID logit.

Table 4: Structural Parameters

Parameter	Simple Logit	Normal Random Effect	Simple Logit	Normal Random Effect	Flexible Ad Coefs	.5 Logit	With Mean Advertising	Extra Promotional Variables
Advertising *	2.04073	2.30566	—	—	2.32360	—	—	—
Inexperienced	(.72313)	(.77561)			(.78683)			
Advertising *	.90371	.43304	—	—	1.33200	—	—	—
Experienced	(.63504)	(1.21180)			(1.39850)			
t-statistic on diffference	1.47662	1.58703	—	—	—	—	—	—
Advertising	—	—	1.71550	2.01370	—	2.10570	1.73080	2.40619
			(.76392)	(.79037)		(.85627)	(.82047)	(.89738)
Advertising *	—	—	−.14812	−.35627	−.29487	−.27106	−.35253	−.39207
Num prev pur			(.06282)	(.10803)	(.12079)	(.14411)	(.10904)	(.11248)
Mean	—	—	—	—	—	—	2.48400	—
ads							(2.40050)	
Own price	−4.89980	−5.58440	−4.89500	−5.61630	−5.61890	−7.21680	−5.60710	−5.02189
	(.33114)	(.34993)	(.33501)	(.35604)	(.35541)	(.43486)	(.35583)	(.38633)
Store	2.72990	2.88690	2.73590	2.87050	2.88770	3.23160	2.88460	2.91887
coupon	(.74368)	(.85073)	(.74214)	(.85707)	(.85558)	(.95421)	(.86097)	(.86565)
Competitor	.76070	.76116	.76215	.76848	.76809	1.00150	.76963	.63461
price	(.19214)	(.21745)	(.19180)	(.21904)	(.21889)	(.24940)	(.21953)	(.23211)
Number prev	.10810	−.26717	.10314	−.27046	−.27303	−.55373	−.27129	−.27843
purchases	(.06370)	(.09312)	(.06227)	(.09152)	(.09235)	(.15038)	(.09161)	(.09715)
Number prev	−.00360	.00085	−.00340	.00110	.00117	.00019	.00119	.00130
purchases^2	(.00053)	(.00096)	(.00057)	(.00099)	(.00099)	(.00124)	(.00099)	(.00106)
Never	−2.78400	−.81135	−2.72150	−.58661	−.70453	−.22113	−.655 61	−.59998
purchased	(.11685)	(.22343)	(.11042)	(.21866)	(.22804)	(.29160)	(.21907)	(.22796)
Once	−.59088	−.08104	−.59857	.00169	−.06915	.11842	−.07050	−.03513
purchased	(.11515)	(.15986)	(.11430)	(.16046)	(.16103)	(.18864)	(.16181)	(.16683)
Prev purch/	.84429	.46907	.84135	.46784	.46557	0.85689	0.46457	0.46080
time	(.08562)	(.10757)	(.08571)	(.10882)	(.10903)	(.16457)	(.10940)	(.11785)
Purchased	.17144	.47774	.19047	.51778	.51009	1.12970	.51200	.51312
last s. trip	(.10042)	(.15667)	(.09691)	(.15421)	(.15550)	(.28121)	(.15559)	(.16910)
Days since	−.00577	−.00487	−.00582	−.00511	−.00499	−.00470	−.00504	−.00552
last purch	(.00072)	(.00091)	(.00073)	(.00092)	(.00092)	(.00103)	(.00092)	(.00096)
Time trend	−1.65580	−.36393	−1.64200	−.26339	−.30594	−.19387	−.28784	−.01729
	(.17406)	(.26303)	(.17325)	(.27417)	(.27314)	(.30920)	(.27332)	(.29203)
Constant	.27671	−3.83780	.22409	−4.18620	−4.03510	−3.05580	−4.26380	−4.32983
	(.29693)	(.60556)	(.29907)	(.62472)	(.62341)	(.72518)	(.64286)	(.68434)

Discussion

- Adv*Exp insignificant image and prestige
- Adv*Inexp - Adv*Exp: significant informative
- 30-sec commercial each week is like 10 cent price decrease
- Adv*NPurch: decreasing returns to advertising

Erdem Keane

- Many markets are characterized by lots of new brands, price changes, and brand repositioning (especially CPG).
- Nevo (2001) has hundreds of cereal brands enter and exit, similar in laundry detergent
- Consumers may spend time experimenting with different brands to learn about them.
- After learning takes place there may be state dependence until new brands are introduced or price cuts.

Guardini Little (Pre-Dynamics)

$$E[U_{ij}|I_i(t)] = a_j - w_P P_j + w_E \sum_{s=0}^{t} D_{1ijs} + w_{Ad} \sum_{s=t_0}^{t} D_{2ijs}$$

- a_j mean brand taste for j
- D_{1ijt}: dummy of whether consumer purchases brand j or not
- D_{2ijt}: dummy of whether consumers receives an advertising signal of brand j or not
- w are utility weights (Lancaster 1966)

Erdem Keane: Decision-making Under Uncertainty

- Consumer i chooses among J products in T periods of time.
- $d_{ij}(t) = 1$ if consumer chooses j (0 o.w.)
- Includes an *other brand* option
- $E[U_{ij}(t)|I_i(t)]$ is current period expected utility conditional on information set $I_i(t)$.

Consumers maximize a discounted stream of expected utilities producing the Bellman:

$$\begin{aligned} V_{ij}(I_i(t), t) &= E[U_{ij}(t)|I_i(t)] + \beta E[V(I(t+1), t+1)|I(t)] \\ V_i(I(t), t) &= \max_j V_j(I_j(t), t) \end{aligned}$$

Attribute Uncertainty

- $A_{ijt} = A_j + \xi_{ijt}$ with i.i.d. mean zero shock ξ_{ijt}
- Consumers don't immediately learn about attribute levels, instead:
- $A_{E_{ijt}} = A_{ijt} + \eta_{ijt}$ with mean zero i.i.d disturbance η_{ijt}.
- $A_{E_{ijt}} = A_j + \delta_{ijt}$ where $\delta_{ijt} = \xi_{ijt} + \eta_{ijt}$.
- Empirically can't differentiate between private value ξ_{ijt} and experience shock η_{ijt}.

Consumer Expected Utility

Additive Compensatory Multiattribute utility model. (Fishbein 1967) (Lancaster 1966)

$$
\begin{aligned}
U_{ijt} &= -w_p P_{ijt} + w_A A_{E_{ijt}} - w_A r A^2_{E_{ijt}} + e_{ijt} \\
E[U_{ijt}|I_i(t)] &= -w_j P_{ijt} + w_A E[A_{E_{ijt}}|I(t)] - w_A r E[A_{E_{ijt}}|I_i(t)]^2 \\
&\quad - w_A r E[A_{E_{ijt}} - E[A^2_{E_{ijt}}|I_i(t)]]^2 + e_{ijt}
\end{aligned}
$$

Where r is your risk parameter: $r > 0$ risk averse

$$
\begin{aligned}
EU_{i0t} &= \Phi_O + \Phi_{Ot} + \epsilon_{i0t} \\
EU_{iNPt} &= \Phi_{NP} + \Phi_{NPt} + \epsilon_{iNPt}
\end{aligned}
$$

For outside good or other good.

Bayesian Learning

With no experience initial variability δ_{ijt}, and advertising signal S_{ijt}

$$\delta_{ijt} \sim N(0, \sigma_\delta^2), \qquad A_j \sim N(A, \sigma_A^2(0))$$
$$S_{ijt} = A_j + \zeta_{ijt}, \qquad \zeta_{ijt} \sim N(0, \sigma_\zeta^2)$$

Consumers update:

$$\begin{aligned} E[A_{E_{ij,t+1}}|I_i(t)] &= E[A_{E_{ijt}}|I_i(t-1)] \\ &- D_{1ijt}\beta_{1ij}(t)[A_{E_{ijt}} - E[A_{E_{ijt}}|I_i(t-1)]] \\ &+ D_{2ijt}\beta_{2ij}(t)[S_{E_{ijt}} - E[S_{E_{ijt}}|I_i(t-1)]] \end{aligned}$$

Bayesian Learning

- D_{1ijt}: dummy of whether consumer purchases brand j or not
- D_{2ijt}: dummy of whether consumers receives an advertising signal of brand j or not
- Kalman Filter Update

$$\beta_{1ijt} = \frac{\sigma^2_{vij}(t)}{\sigma^2_{vij}(t) + \sigma^2_{\delta}}, \qquad \beta_{2ijt} = \frac{\sigma^2_{vij}(t)}{\sigma^2_{vij}(t) + \sigma^2_{\zeta}}$$

$$v_{ij} = E[A_{ij}|I_{ij}(t)] - A_j$$

- And

$$\begin{aligned} A_j &= E[A_j|I_{ij}(t)] + v_{ij}(t) \\ A_{E_{ijt}} &= A_j + \delta_{ijt}, \quad S_{ijt} = A_j + \zeta_{ijt} \end{aligned}$$

Bayesian Learning

$$
\begin{aligned}
v_{ijt}(t) &= v_{ij}(t-1) + D_{1ijt}\beta_{1ij}(t)[-v_{ij}(t-1) + \delta_{ijt}] \\
&+ D_{2ijt}\beta_{2ij}(t)[-v_{ij}(t-1) + \zeta_{jt}] \\
\sigma^2_{vij}(t) &= \frac{1}{\frac{1}{\sigma^2_v(0)} + \frac{\sum_{s=0}^{t} D_{1ijs}}{\sigma^2_\delta} + \frac{\sum_{s=0}^{t} D_{2ijs}}{\sigma^2_\zeta}}
\end{aligned}
$$

And expected utilities:

$$
\begin{aligned}
E[U_{ij}|I_i(t)] &= w_A A_j - w_A r A_j^2 - w_A r \sigma^2_\delta - w_P P_{ij} \\
&- -w_A r \sigma^2_{vij}(t) - w_A r v_{ij}(t)^2 - w_A v_{ij}(t) - 2 w_A r A_j v_{ij}(t) \\
&+ e_{ijt} \\
E[V_{ij}|I_i(t)] &= E[U_{ij}|I_i(t)] + \beta E[V_{ij}|I_i(t+1)|d_{ijt} = 1, I_i(t)]
\end{aligned}
$$

Choice Probabilities

For the Static and Dynamic case:

$$P_i^s(I(t),t) = \int \frac{\exp[E[U_{ij}|I_i(t)]]}{\sum_k \exp[E[U_{ik}|I_i(t)]]} f(v)dv$$
$$P_i^d(I(t),t) = \int \frac{\exp[E[V_{ij}|I_i(t)]]}{\sum_k \exp[E[V_{ik}|I_i(t)]]} f(v)dv$$

- Static model allows choices to depend on current knowledge of attribute
- Static model does not incorporate value of learning for future consumption
- Logit choice probabilities but with time varying random coefficients
- Everything about learning in is in the distribution of v

Data

- Laundry detergent scanner data from 1986-1988.
- 3000 HH's w/ 20 purchases (7 liquid)
- Lots of advertising
- Only liquids (80% of market)
- Many new brands
- TVs measures ad exposure
  - Percentage of weeks household saw brand j's ad.
  - Saw at least one ad during that week

Table 2: Static Model No Learning

Table 2 GL Model Estimates

Parameter	Estimate	t-statistic
price coefficient ($-w_p$)	−1.077	−18.10
"brand loyalty" parameter (w_E)	3.363	53.18
advertising coefficient (w_{Ad})	0.144	0.31
brand intercepts (a_j):		
a_{Dash}	0.000	–
a_{Cheer}	1.115	8.87
a_{Solo}	0.917	7.22
a_{Surf}	1.382	14.43
a_{Era}	1.601	11.03
a_{Wisk}	1.102	6.78
a_{Tide}	1.700	12.29
"Other Brands" intercept (Φ_O)	−0.633	−2.98
"Other Brands" time trend (Ψ_O)	0.011	4.87
"No Purchase" intercept (Φ_{NP})	1.636	8.02
"No Purchase" time trend (Ψ_{NP})	0.005	1.35
"Brand Loyalty" smoothing coefficient (α_E)	0.770	50.62
advertising smoothing coefficient (α_{AD})	0.788	2.95

Table 3: Dynamic Model

Table 3 Structural Model Estimates

Parameter	Immediate Utility Maximization[1] ($\gamma = 0$) Estimate	t-statistic	Forward-looking Dynamic Structural Model[2] ($\gamma = 0.995$) Estimate	t-statistic
price coefficient ($-w_p$)	−0.790	−12.26	−0.795	−12.31
utility weight (w_A)	28.356	1.73	34.785	1.84
risk coefficient (r)	3.625	2.08	4.171	2.25
initial variance ($\sigma_v^2(t)$)	0.053	4.64	0.040	4.21
mean attribute levels (A_j):				
A_{Dash}	0.049	0.74	0.040	0.74
A_{Cheer}	0.019	0.27	0.012	0.21
A_{Solo}	0.056	0.84	0.047	0.87
A_{Surf}	0.105	1.65	0.089	1.77
A_{Era}	0.137	2.41	0.120	2.64
A_{Wisk}	0.040	0.59	0.029	0.53
A_{Tide}	0.138	–	0.120	–
"Other Brands" intercept (Φ_O)	−17.657	−7.98	−17.267	−7.59
"Other Brands" time trend (Ψ_O)	0.018	8.53	0.018	8.91
"No Purchase" intercept (Φ_{NP})	−15.408	−6.99	−19.537	−8.55
"No Purchase" time trend (Ψ_{NP})	0.011	3.17	0.012	3.42
experience variability (σ_δ)	0.374	9.17	0.33	8.37
advertising variability (δ_ς)	3.418	6.29	3.08	5.57

[1] $-LL = 7312.09$ $AIC = 7324.09$ $BIC = 7384.49$

[2] $-LL = 7306.05$ $AIC = 7322.05$ $BIC = 7378.45$

Results

- Static model has no effect of advertising (!)
- Consumers are risk averse
- Price coefficient negative and significant
- Utility weight is huge (latent attribute – cleaning power?)
- Attribute levels are not significant (maybe differences are?)
- Advertising more variable than experience
- relatively small initial variance
- Dynamic model shows more willingness to try new brands

Uncertainty and Learning in Pharmaceutical Demand

Crawford and Shum (2005)

- Italian anti-ulcer data: 34,972 patients (and a total of 98,634 prescription episodes)
- Patients receive, on average, 2.8 prescriptions for 1.2 drugs over a period of just under 6 months.
- Break up data into *spells* or a sequence of one or more prescriptions of a single drug.
  - A patient has 1.2 spells on average
  - An average spell is around 2.37 prescriptions
- Probability of switching drugs is not constant over time
  1. Early Switching: Experimentation - about 10% after first prescription
  2. Late Switching: Learning rise in switching at the end, especially for long-treatment length patients

SWITCHING PROBABILITIES OVER THE COURSE OF TREATMENT[a]

Prescription Number	Total Treatment Length 5	6	7	8	9	10
2	14.3	13.6	10.9	10.0	7.8	9.2
3	11.6	11.6	6.3	8.8	7.8	6.6
4	8.9	5.6	5.4	3.1	7.8	3.9
5	13.4	7.9	10.0	8.8	4.9	5.3
6		11.3	6.3	5.7	2.9	5.3
7			9.5	10.0	7.8	11.8
8				8.1	4.9	11.8
9					7.8	5.3
10						11.8

[a] The (i, j)th entry is the percentage of treatment sequences of length j in which a switch was observed during the ith $(i \leq j)$ prescription.

Model Setup

- Patients, j. Drugs, $n = 5$, types $k = 4$ (known to doctor-patient but not econometrician).
- Treatment is characterized by two match values (μ_{jn}, ν_{jn}) and two corresponding signals (x_{jnt}, y_{jnt}) that correspond to the side-effects or curative probabilities respectively.
- Patient's utility $u(\cdot)$ depends on side effects x_{int}
- Cure probability $w(\cdot)$ depends on y_{jnt}
- Don't know your match value (μ_{jn}, ν_{jn}) only the signal (x_{jnt}, y_{jnt}), or treatment length $\tau = 1, \ldots, T$
- Consumers have both signals (x, y) and priors (μ, ν) about side effects and cure probability

$$\begin{pmatrix} x_{jnt} \\ y_{jnt} \end{pmatrix} \sim N\left(\begin{matrix} \mu_{jn} \\ \nu_{jnt} \end{matrix}, \begin{matrix} \sigma^2_{jn} \\ \tau^2_{jnt} \end{matrix} \right)$$

$$\begin{pmatrix} \mu_{jn} \\ \nu_{jn} \end{pmatrix} \sim N\left(\begin{matrix} \overline{\mu}_{nk} \\ \overline{\nu}_{nk} \end{matrix}, \begin{matrix} \overline{\sigma}^2_{n} \\ \overline{\tau}^2_{n} \end{matrix} \right)$$

- Where $k = 1, \ldots, 4$ indexes the type specific priors.

Model Setup

- Doctors (without incentive problems) solve:

$$\max_{D=\{(d_{jnt})_{n=1}^{N}\}_{t=1}^{\infty}} E_D \sum_{t=1}^{\infty} \beta^t d_{jnt} u_{jnt} (1 - w_{j,t-1})$$

- Patients have CARA utility

$$u(x_{jnt}, p_n, \epsilon_{jnt}) = -\exp(r * x_{jnt}) - \alpha p_n + \epsilon_{jnt}$$

- Derive the expected utility as:

$$\begin{aligned} \tilde{EU}(\mu_{jn}(t), \nu_{jn}(t), p_n, \epsilon_{jnt}) &= -\exp(r * \mu_{jn}(t) + \frac{1}{2} r^2 (\sigma)(\sigma_n^2 + V_{jn}(t)) \\ &\quad -\alpha p_n + \epsilon_{jnt} \\ &= EU(\mu_{jn}(t), V_{jn}(t), p_n) + \epsilon_{jnt} \end{aligned}$$

State Space

- State Variables S_t:
  - $(\mu_{jnt}, \nu_{jnt}), I_{jnt}$ for $n = 1, \ldots, 5$ drugs.
  - $h_{j,t-1}$ (cure probability)
  - ϵ_{jnt}
- Recovery probability follows a Markov Process:

$$h_{jt}(h_{j,t-1}, y_{jnt}) = \frac{\left(\frac{h_{j,t-1}}{1-h_{j,t-1}}\right) + d_{jnt}y_{jnt}}{1 + \left(\frac{h_{j,t-1}}{1-h_{j,t-1}}\right) + d_{jnt}y_{jnt}}$$

- Beliefs follow Bayesian updating depending on I_{jnt} the number of times patient j takes drug n at time t.

Dynamic Decision Problem (DDP)

Doctors face choice specific value function (infinite horizon, recovery state absorbing):

$$
\begin{aligned}
W(S_t) &= \max_n [\exp(-r\mu_{jnt} + 0.5r^2(\sigma_n^2 + V_{jnt})) - \alpha p_n + \epsilon_{jnt} \\
&\quad + \beta E[(1 - h_{jt}(h_{j,t-1}, y_{jnt}) E[W(S_{t+1}) | x_{jnt}, y_{jnt}, d_n = 1] | S_t] \\
&= \log[\sum_n \exp[\tilde{EU}(s) + \beta E[(1 - h(s'))W(s') | d_n = 1] | S_t] \\
&= \max_n \{W_n(S_t)\}
\end{aligned}
$$

Value Function

$$
\begin{aligned}
W(S_t) &= \max_n [\exp(-r\mu_{jnt} + 0.5r^2(\sigma_n^2 + V_{jnt})) - \alpha p_n + \epsilon_{jnt} \\
&\quad +\beta E[(1 - h_{jt}(h_{j,t-1}, y_{jnt})E[W(S_{t+1})|x_{jnt}, y_{jnt}, d_n = 1]|S_t] \\
&= \log[\sum_n \exp[\tilde{EU}(s) + \beta E[(1 - h(s'))W(s')|d_n = 1]|S_t] \\
&= \max_n \{W_n(S_t)\}
\end{aligned}
$$

VFI + Simulate + Interpolate: (Keane Wolpin 1994):

1. Define discrete grid $S^* \in S$
2. For each state $s \in S^*$ make an initial guess at the value function $W^0(s)$.
3. Run regression $W^0(s) = G(s)'\theta^0 + \varepsilon$
4. Draw the M random signals $\{x_{jn}^m, y_{jn}^m\}$
5. Compute the expected value of choosing drug n for each $s \in S^* S$, where s^m is state corresponding to random draw m and drug n being chosen.

$$E[W(s|d_n = 1, s)] = \frac{1}{M} \sum_m (1 - h(s^m)) W^0(s^m)$$

6. Update the value function for each $s \in S^*$
7. Iterate until convergence

Likelihood

For $I_{=}0$ and $I_j = 1$ censored and uncensored observations for patient j.

$$\sum_{k=1}^{K} p_k E_{\overline{x}_{jnT_j},k|h_{0,j,k}} \left[\prod_{t=1}^{T_j - 1} \left((1 - h_{jt,k}) \prod_n \lambda_{jnt,k}^{d_{jnt}} \right) \right] \cdot h_{jT_j,k} \prod_n \lambda_{jnt,k}^{d_{jnt}}$$

$$\sum_{k=1}^{K} p_k E_{\overline{x}_{jnT_j},k|h_{0,j,k}} \left[\prod_{t=1}^{T_j - 1} \left((1 - h_{jt,k}) \prod_n \lambda_{jnt,k}^{d_{jnt}} \right) \right] \cdot \prod_n \lambda_{jnt,k}^{d_{jnt}}$$

(λ is logit choice probability)
We need to calculate expectations of joint distribution of $(\overline{x}, h)$ by drawing $S = 30$ sequences per patient.

Dynamic Model Parameters: Sick vs. Not so Sick

Parameter	Est.	Std. Err.	Est.	Std. Err.
Illness heterogeneity distribution	Recovery Probability		Type Probability	
θ_1 (Type 1)	0.433	0.003	0.593	0.006
θ_2 (Type 2)	0.127	0.003	0.335	0.006
θ_3 (Type 3)	0.199	0.007	0.043	0.001
θ_4 (Type 4)	0.432	0.011	0.029	0.002
Means, symptom match values[b]	Type 1		Type 2	
$\underline{\mu}_1$	0.927	0.282	1.195	0.369
$\underline{\mu}_2$[c]	0.928	0.287	0.428	0.166
$\underline{\mu}_3$	0.481	0.197	−0.028	0.178
$\underline{\mu}_4$	0.335	0.161	−0.145	0.079
$\underline{\mu}_5$	0.451	0.174	−0.483	0.137
Means, curative match values[b]	Type 1		Type 2	
$\underline{\nu}_1$	0.014	0.003	0.006	0.000
$\underline{\nu}_2$[c]	0.015	0.005	0.006	0.001
$\underline{\nu}_3$	0.013	0.030	0.006	0.095
$\underline{\nu}_4$	0.013	0.084	0.014	0.009
$\underline{\nu}_5$	−0.034	0.000	−0.038	0.000
Std. dev., symptom match values				
$\underline{\sigma}$	1.574	0.448		
Std. devs., symptom signals				
σ_1	0.998	0.287		
σ_2	1.134	0.326		
σ_3	1.375	0.395		
σ_4	1.159	0.333		
σ_5	0.931	0.268		
Std. dev., curative match values				
$\underline{\tau}$	0.007	0.000		
Std. dev., curative signals				
τ	0.007	0.001		
Price coefficient, α[a]	1.080	0.091		
Risk-aversion parameter, r	0.990	0.274		
Discount rate, β	0.950	Fixed		
Number of observations	34,972			
Number of similar draws	30			
Log likelihood function	−124,484.34			

Dynamic Model Parameters: Omeprazole (All types)

	Type 1		Type 2		Type 3		Type 4	
Parameter	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.	Est.	Std. Err.
Match values, all types								
Symptom match values								
$\underline{\mu}_1$	0.927	0.282	1.195	0.369	0.489	0.163	0.151	0.091
$\underline{\mu}_2$[a]	0.928	0.287	0.428	0.166	0.577	0.198	0.573	0.199
$\underline{\mu}_3$	0.481	0.197	−0.028	0.178	1.762	0.531	0.013	0.167
$\underline{\mu}_4$	0.335	0.161	−0.145	0.079	−0.111	0.305	0.504	0.184
$\underline{\mu}_5$	0.451	0.174	−0.483	0.137	−0.113	0.125	−0.561	0.220
Curative match values								
$\underline{\nu}_1$	0.014	0.003	0.006	0.000	0.011	0.002	0.014	0.010
$\underline{\nu}_2$[a]	0.015	0.005	0.006	0.001	0.011	0.006	0.015	0.003
$\underline{\nu}_3$	0.013	0.030	0.006	0.095	0.004	0.001	0.013	0.329
$\underline{\nu}_4$	0.013	0.084	0.014	0.009	−0.035	0.214	0.012	0.003
$\underline{\nu}_5$	−0.034	0.000	−0.038	0.000	−0.037	0.054	−0.034	0.409
Time-varying priors for omeprazole								
Symptom match value, $\underline{\mu}_2$								
Period 1	0.805	0.258	0.306	0.140	0.454	0.171	0.451	0.172
Period 2	0.910	0.285	0.411	0.166	0.560	0.197	0.556	0.198
Period 3	0.722	0.237	0.223	0.122	0.371	0.151	0.368	0.152
Period 4	0.979	0.301	0.480	0.181	0.628	0.212	0.625	0.214
Period 5[a]	0.928	0.287	0.428	0.166	0.577	0.198	0.573	0.199
Curative match value, $\underline{\nu}_2$								
Period 1	−0.007	0.011	−0.016	0.010	−0.011	0.011	−0.007	0.010
Period 2	−0.001	0.012	−0.011	0.011	−0.006	0.012	−0.001	0.011
Period 3	0.015	0.016	0.005	0.015	0.011	0.016	0.015	0.016
Period 4	0.013	0.017	0.004	0.016	0.009	0.017	0.013	0.017
Period 5[a]	0.015	0.005	0.015	0.001	0.011	0.006	0.015	0.003

Results

- Coefficient of risk aversion is high (switching costs?)
- Learning happens very fast (variance falls from 2.48 to 0.7 after only one prescription).
- Learning slows after first prescription
- Counterfactual (Complete Information): You know your match values which you draw from the same distribution but your perceived variance $V_{jn}^{t} = R_{jn}^{=} 0$.
  - Leads to more drugs 1.9 instead of 1.4.
  - Substitution away from market leader (no reason to stay with first drug). Lower HHI
  - Welfare up 9%. Treatment up 80%, cost up 60%.
- Counterfactual (Ban Experimenting): You are stuck with your first drug forever.
  - Utility down 6% but treatment length and costs about the same.
  - Wasn't much experimentation to begin with
- Counterfactual (No Diagnostic Matching): Doctors can't learn types.
  - Utility down 11% and costs and length up 30-40%.