Tutorial 4

Gauss's Law

Lecture Outline

- **Chapter 23**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Gauss's Law
  – Electric flux
  – Gauss's law and Coulomb's law
  – Applications of Gauss' law

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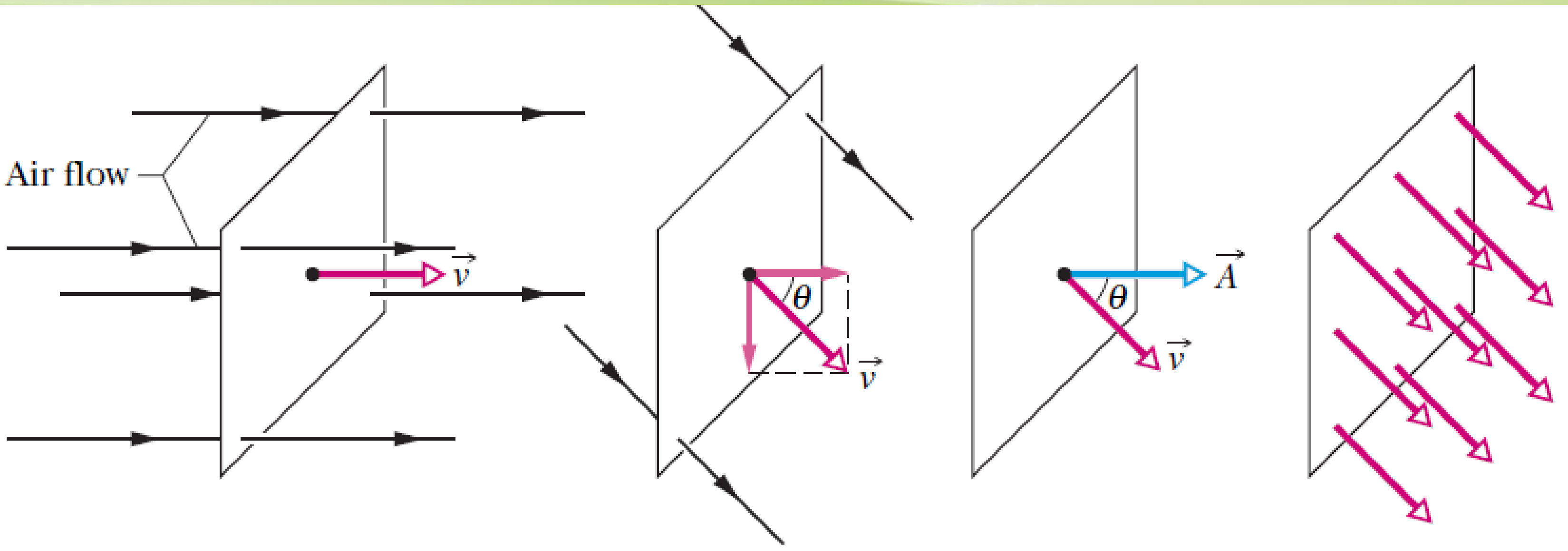
Lecture 04 – Review
Electric Flux

- In Lecture 04, we introduced the concept of electric flux Φ, which is a measure of the flow of the electric field through a given area.

- In order to determine the electric flux, we need to represent the given area by its area vector $\vec{A}$, whose direction is perpendicular (normal) to the face of the area surface and magnitude equals to the area of the surface.

- With the definition of area vector, the electric flux is simply

$$\Phi = \vec{E} \cdot \vec{A} = EA\cos\theta$$



(a) (b) (c) (d)

Fig. 23-2 (a) A uniform airstream of velocity is perpendicular to the plane of a square loop of area ***A***. (b) The component of perpendicular to the plane of the loop is *v cos θ*, where *θ* is the angle between ***v*** and a normal to the plane. (c) The area vector ***A*** is perpendicular to the plane of the loop and makes an angle *θ* with ***v***. (d) The velocity field intercepted by the area of the loop. The **rate of volume flow** through the loop is Φ= (*v cos θ*)*A*.

This rate of flow through an area is an example of a flux—a *volume flux* in this situation.

$$\Phi = vA\cos\theta = \vec{v}\cdot\vec{A},$$

#The actual physical quantity referred to by the term 'flux' can be different in different occasions.

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Lecture 04 – Review
Gauss's Law

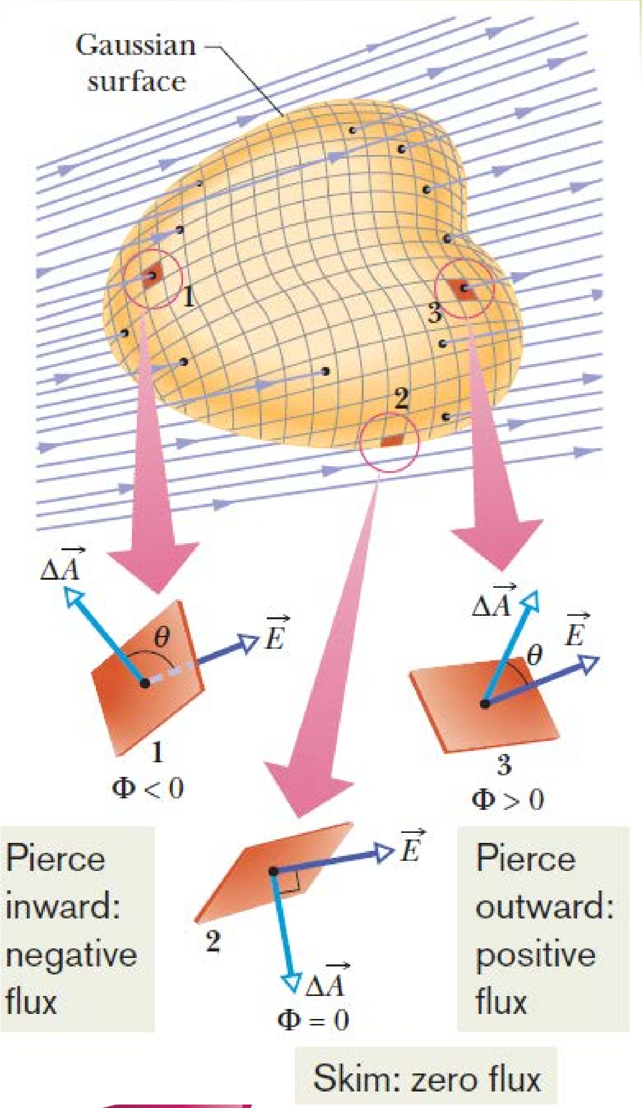
- A Gaussian surface is a closed surface in 3D space.
- Gauss' law states that the net charge enclosed in a volume is equal to the product ε_0 and Φ through the Gaussian surface.

$$\Phi_E = \vec{E} \bullet \Delta\vec{A}$$

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A}.$$

$$\varepsilon_0 \Phi = q_{enc}$$

or $$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q_{enc}$$



Lecture 04 – Review
Gauss's Law and Coulomb's Law

- We shown that one may derive Coulomb's law from Gauss' law as the two laws are equivalent.

Gauss' Law for point charge q_1

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}$$

Coulomb's Law for point charges q_1 & q_2

$$F = q_2 E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

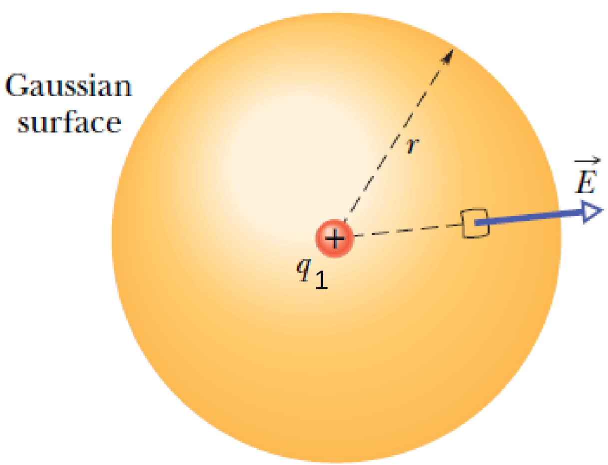


Fig. 23-8 A spherical Gaussian surface centered on a point charge q_1

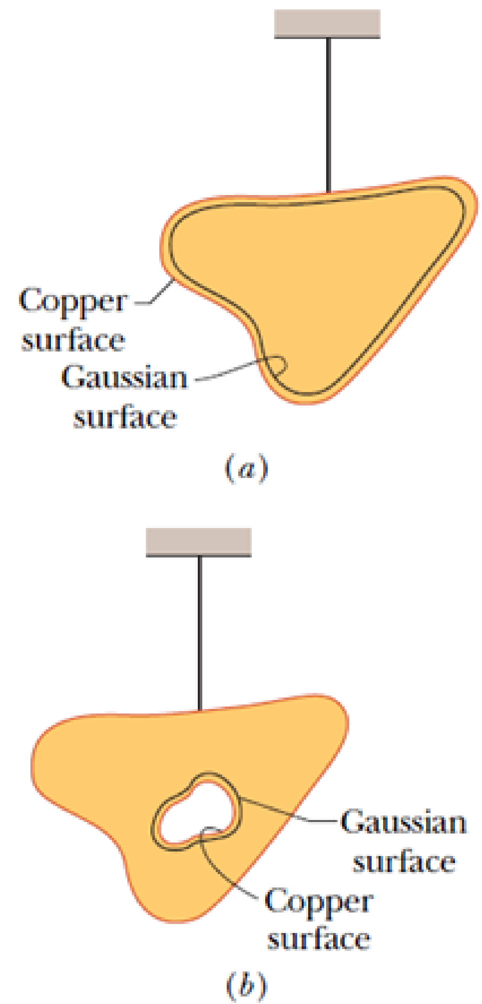
$$\varepsilon_0 \oiint \vec{E}\cdot d\vec{A} = q$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

Lecture 04 – Review
E field vanishes inside a conductor

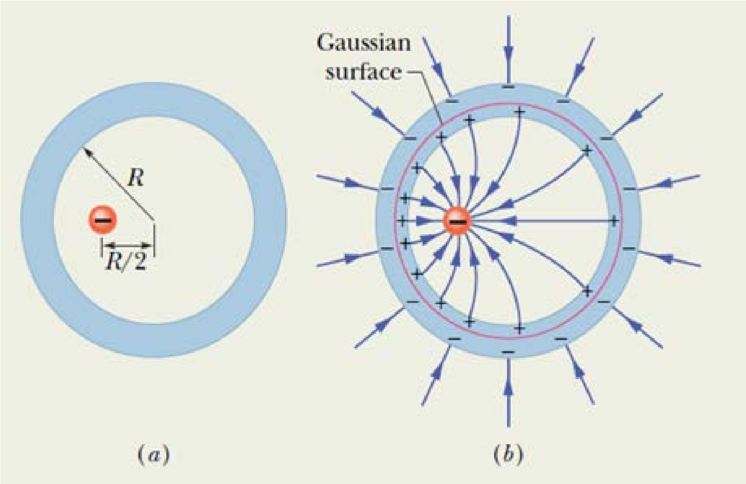
- Using Gauss' law, we argue that when an excess charge is placed on an isolated conductor, the charge must remains on the outer surface of the conductor as the static electric field inside the conductor must be zero.
- In a conductor, the charge is free to move. If the E field inside the conductor is non-zero, the charges inside the conductor will move. They will only stop moving, when they rearrange themselves in such a way that the E field inside the conductor vanishes. Therefore, these charges must reside on the conductor surface.



Lecture 04 – Review

Distribution of charge on a conducting spherical shell

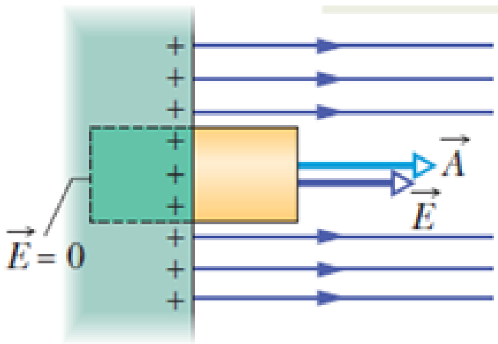
- The charge inside the conducting shell will induce the opposite charges on the inside surface of the shell. The distribution of the induced charges will be non-uniform, if the charge is positioned off the center of the shell. However, the distribution of the charges on the outer surface of the sphere will be uniform, since these charges are shielded from (the charges on the outer surface are not affected by those in the inner surface or the original charge) the non-uniform charge distribution inside the shell, because the E field in the conductor is zero.



- Combining with symmetric consideration, Gauss Law provides much simpler solution than that of Coulumb Law for symmetric charge distributions.

Lecture 04 – Review
E Field on a conducting surface

- If $E=0$ inside the conductor, the electric field on the conducting surface can be determined by constructing a small Gaussian surface that overlaps both inside and outside the surface and show the field is normal to the surface and proportional to the surface charge density σ



$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface)}$$

23.8 Applying Gauss' Law, Planar Symmetry
Non-conducting Sheet:

$$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q_{enc}$$

$$\varepsilon_0 (EA + EA) = \sigma A$$

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(sheet of charge).}$$

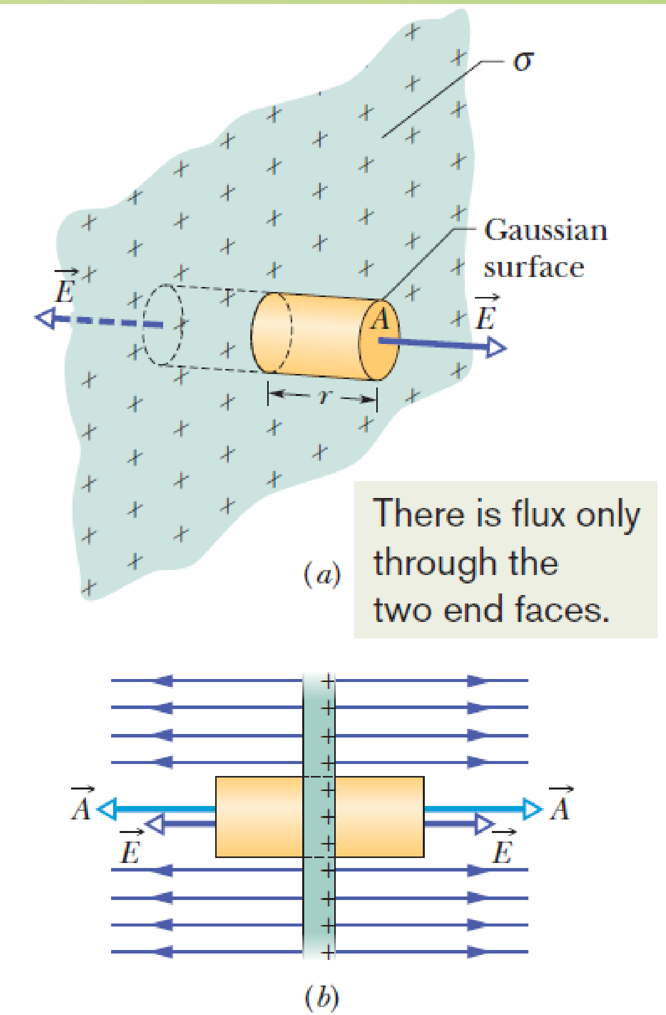


Fig. 23-15 (*a*) Perspective view and (*b*) side view of a portion of a very large, thin plastic sheet, uniformly charged on one side to surface charge density σ. A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

Two Conducting Plates:

$$E = \frac{2\sigma_1}{\varepsilon_0}$$

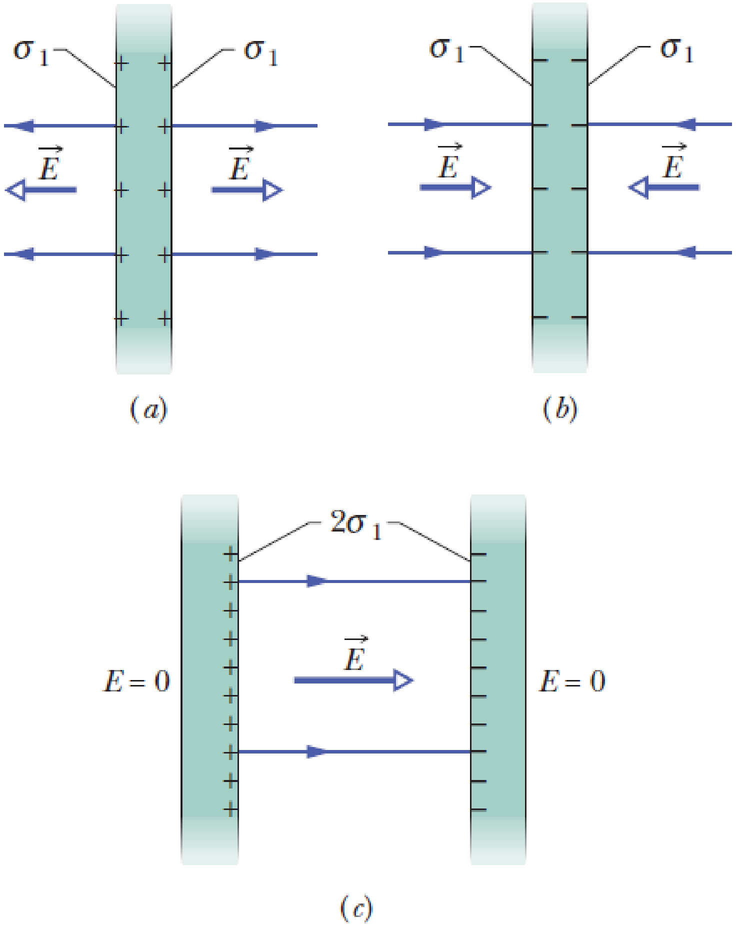
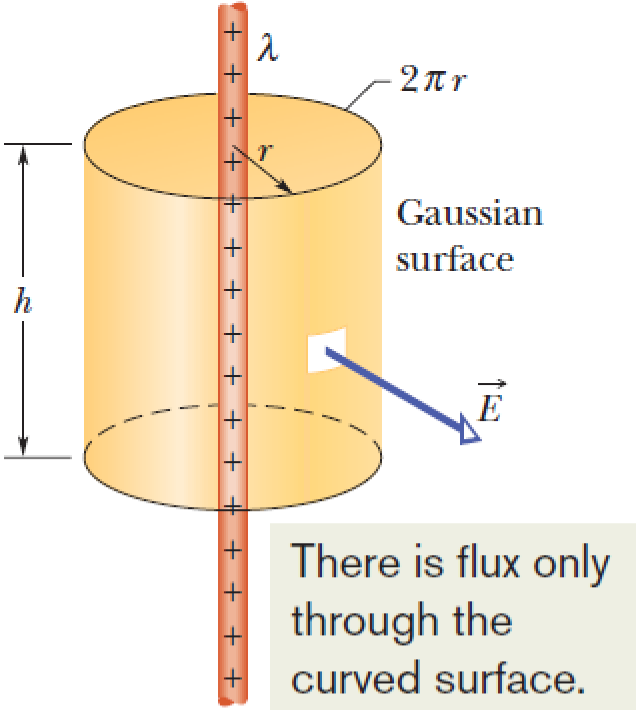


Fig. 23-16 (*a*) A thin, very large conducting plate with excess positive charge. (*b*) An identical plate with excess negative charge. (*c*) The two plates arranged so they are parallel and close.

Lecture 04 – Review
E field from a line of charge

- When we apply the Gauss's law on a line of charge with charge density λ, we found that

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \text{(line of charge)}$$



23.9 Applying Gauss' Law, Spherical Symmetry:

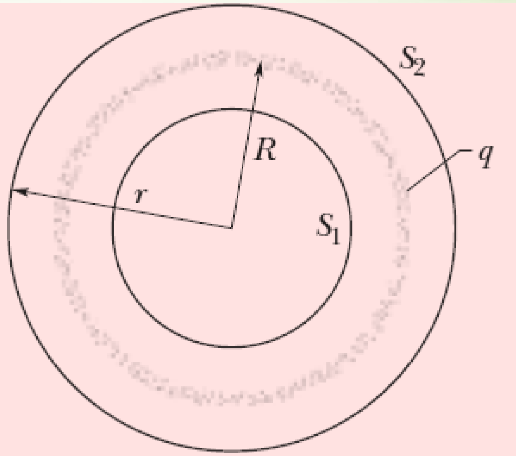
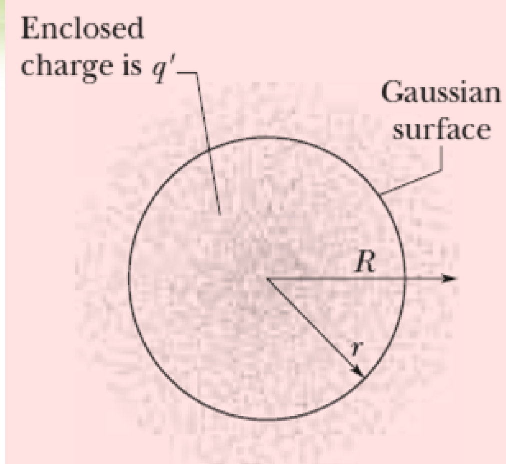


Fig. 23-18 A thin, uniformly charged, spherical shell with total charge q, in cross section. Two Gaussian surfaces S_1 and S_2 are also shown in cross section. Surface S_2 encloses the shell, and S_1 encloses only the empty interior of the shell.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{(spherical shell, field at } r \geq R\text{).}$$

$$E = 0 \qquad \text{(spherical shell, field at } r < R\text{),}$$

Recall the case for gravitation.



$$r \leq R;\ q' = \frac{4\pi r^3}{3}\rho$$

$$r > R;\ q' = \frac{4\pi R^3}{3}\rho$$

Fig. 23-19 The dots represent a spherically symmetric distribution of charge of radius R, whose volume charge density ρ is a function only of distance from the center. The charged object is not a conductor, and therefore the charge is assumed to be fixed in position. A concentric spherical Gaussian surface with $r < R$ is shown.

$$r \leq R;\ E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{4\pi r^3}{3}\rho = \frac{\rho}{3\varepsilon_0}r$$

$$r > R;\ E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{4\pi R^3}{3}\rho = \frac{R^3\rho}{3\varepsilon_0 r^2}$$

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Halliday/Resnick/Walker
Fundamentals of Physics 8th edition

Classroom Response System Questions

Chapter 23 Gauss' Law

Interactive Lecture Questions

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23.2.1. The end of a garden hose is enclosed in a mesh sphere of radius 4 cm. If the hose delivers five liters per minute, how much water flows through the sphere each minute?

a) 0.0013 liters

b) 0.67 liters

c) 3.2 liters

d) 5.0 liters

e) 20 liters

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e) 20 liters

23.2.2. The a solid brass sphere of radius 3 cm is placed 0.5 m directly below a water faucet. The flow of water from the faucet is two liters per minute. How much water flows through the sphere each minute?

a) zero liters

b) 0.018 liters

c) 0.09 liters

d) 2 liters

e) 6 liters

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23.2.3. In July, Joe set up his fixed array of solar panels to maximize the amount of electricity output from the array when the Sun was high in the sky. Unfortunately, Joe finds that the electricity output from the array during the winter months is much lower, even though there is nothing physical wrong with the array. What is the most likely cause of Joe's winter problem?

a) Less sunlight reaches the Earth during the winter months.

b) The sun is lower in the sky during the winter, so sunlight strikes the solar panels at an angle.

c) The average temperature is much colder during the winter months.

d) More sunlight is absorbed by the atmosphere during the winter months because the Sun is much lower in the sky.

e) The Sun is not as bright during winter months as it is during the summer months.

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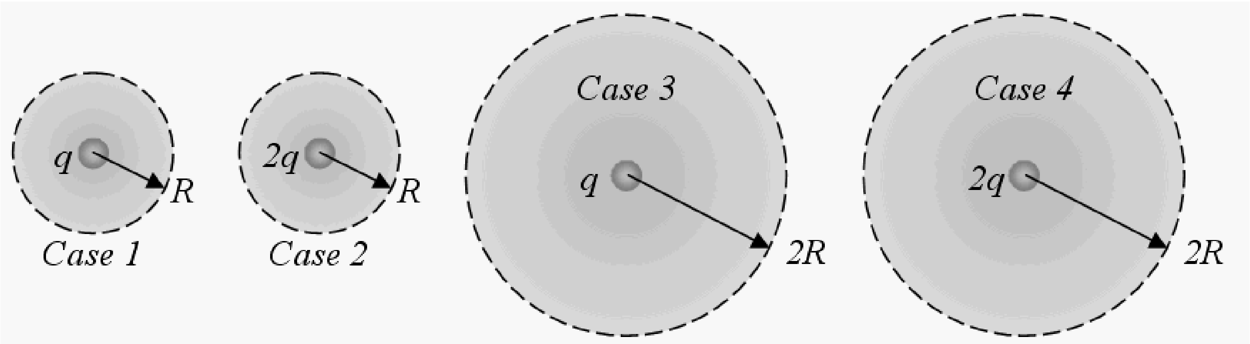
23.3.2. Consider the five situations shown. Each one contains either a charge q or a charge $2q$. A Gaussian surface surrounds the charged particle in each case. Considering the electric flux through each of the Gaussian surfaces, which of the following comparative statements is correct?



a) $\Phi_2 = \Phi_4 > \Phi_1 = \Phi_3$

b) $\Phi_1 = \Phi_3 > \Phi_2 = \Phi_4$

c) $\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3$

d) $\Phi_3 = \Phi_4 > \Phi_2 = \Phi_1$

e) $\Phi_4 > \Phi_3 > \Phi_2 > \Phi_1$

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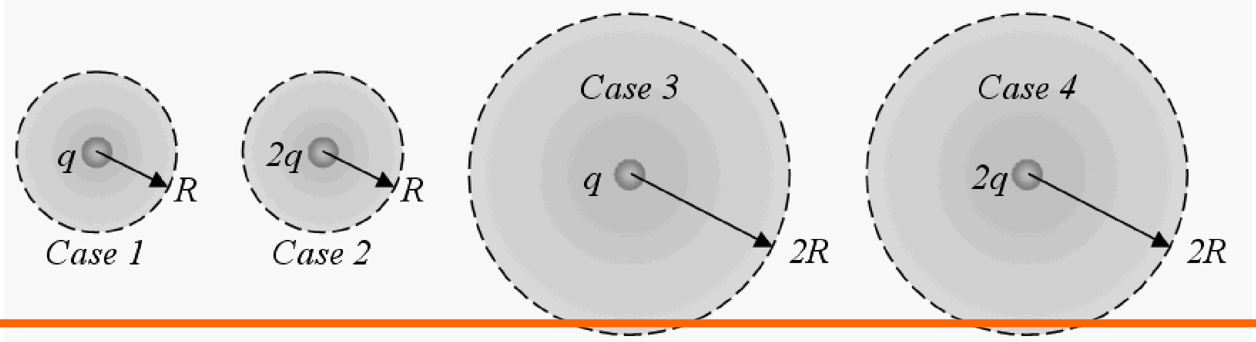

a) $\Phi_2 = \Phi_4 > \Phi_1 = \Phi_3$

b) $\Phi_1 = \Phi_3 > \Phi_2 = \Phi_4$

c) $\Phi_2 > \Phi_1 > \Phi_4 > \Phi_3$

d) $\Phi_3 = \Phi_4 > \Phi_2 = \Phi_1$

e) $\Phi_4 > \Phi_3 > \Phi_2 > \Phi_1$

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23.4.3. Gauss' law may be written: $\Phi = \oiint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_0}$. Which of the following statements concerning the charge q is true?

a) The charge q is the sum of all charges.

b) The charge q is the sum of all charges on the Gaussian surface.

c) The charge q is the sum of all charges inside the Gaussian surface.

d) The electric field due to q is zero inside the Gaussian surface.

e) The charge q is the amount of charge present whenever the electric field is constant.

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23.6.1. A conducting shell with an outer radius of 2.5 cm and an inner radius of 1.5 cm has an excess charge of 1.5×10^{-7} C. What is the surface charge density on the inner wall of the shell?

a) 1.5×10^{-9} C/m^2

b) 2.9×10^{-10} C/m^2

c) 4.8×10^{-10} C/m^2

d) 8.5×10^{-9} C/m^2

e) None of the above answers is correct.

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e) None of the above answers is correct.

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23.7.2. A straight, copper wire has a length of 0.50 m and an excess charge of -1.0×10^{-5} C distributed uniformly along its length. Find the magnitude of the electric field at a point located 7.5×10^{-3} m from the midpoint of the wire.

a) 1.9×10^{10} N/C

b) 7.3×10^{8} N/C

c) 6.1×10^{13} N/C

d) 1.5×10^{6} N/C

e) 4.8×10^{7} N/C

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b) 7.3×10^{8} N/C

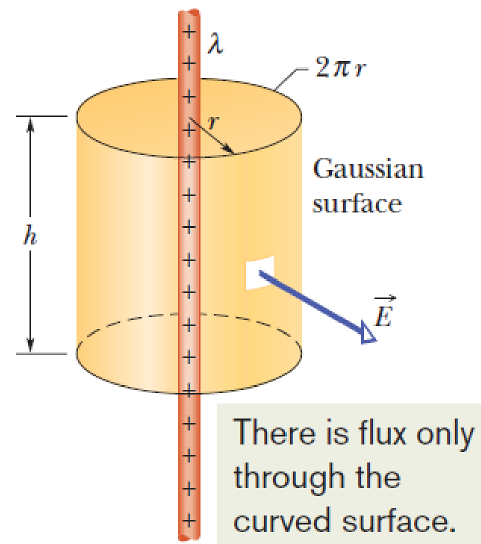
c) 6.1×10^{13} N/C

d) 1.5×10^{6} N/C

e) $\boxed{4.8 \times 10^{7} \text{ N/C}}$

$$E = \frac{-1.0\times10^{-5}/0.5}{2\pi\left(8.854\times10^{-12}\right)\left(7.5\times10^{-3}\right)}$$

$$= 4.79\times10^{7} \text{ N/C}$$



$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \text{(line of charge)}$$

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23.8.1. An infinite slab of electrically insulating material has a thickness *t*. The slab has a uniform volume charge density *ρ*. Which one of the following expressions gives the electric field at a point P at a depth *t* – *d* relative to the surface?

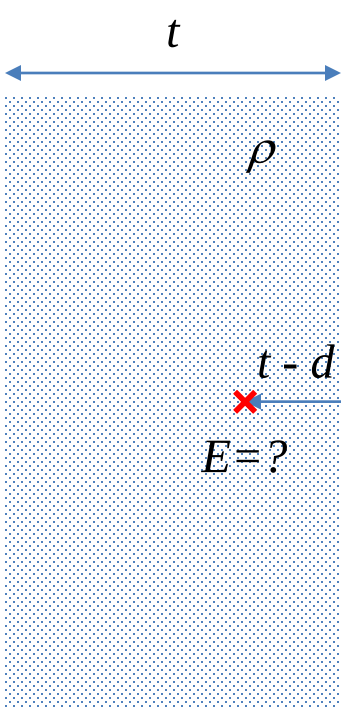
a) $E = \dfrac{\rho t}{2\varepsilon_0}$

b) $E = \dfrac{\rho d}{2\varepsilon_0}$

c) $E = \dfrac{\rho}{2(t-d)\varepsilon_0}$

d) $E = \dfrac{\rho(2d-t)}{2\varepsilon_0}$

e) $E = \dfrac{\rho\left(t/2 - d\right)}{2\varepsilon_0}$



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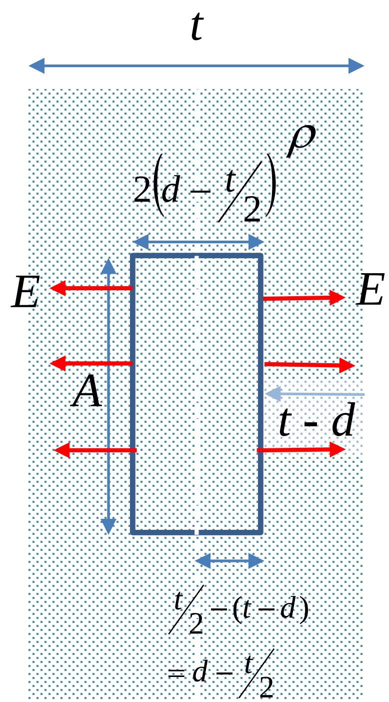

Total enclosed charge

$$2\rho A\left(d - \frac{t}{2}\right)$$

Total electric flux

$$2EA$$

Electric field

$$2EA = \frac{2\rho A\left(d - \frac{t}{2}\right)}{\varepsilon_0}$$

$$E = \frac{\rho(2d-t)}{2\varepsilon_0}$$

23.8.2. A large sheet of electrically insulating material has a uniform charge density σ. Let's compare the electric field produced by the insulating sheet with that produced by a thin metal (electrically conducting) slab with $\sigma/2$ charge density distributed on one large surface of the slab and $\sigma/2$ distributed over the surface on the opposite side. How does the electric field at a distance *d* from each surface compare?

a) The electric field near the insulating sheet is four times that near the conducting slab.

b) The electric field near the insulating sheet is twice that near the conducting slab.

c) The electric field near the insulating sheet is the same as that near the conducting slab.

d) The electric field near the insulating sheet is one half that near the conducting slab.

e) The electric field near the insulating sheet is one fourth that near the conducting slab.

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c) The electric field near the insulating sheet is the same as that near the conducting slab.

d) The electric field near the insulating sheet is one half that near the conducting slab.

e) The electric field near the insulating sheet is one fourth that near the conducting slab.

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