Lecture 7

Current and Resistance

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Lecture 06 Review

- In Lecture 6, we learnt that the common passive electrical component Capacitor is used to store static electric potential energy.
- One can construct a capacitor by simply placing two conductors (plates) in isolation.
- A charged capacitor is a capacitor with equal magnitude but opposite charges, $+q$ and $-q$, on its plates.
- The capacitor stores the electric potential via the charges that it stores.
- From Lecture 5, we realize that the two plates of the capacitor are at different electric potentials, so there is a potential difference across the capacitor.
- The potential difference between the two plates V is a function of the amount of charges q on the plates and the geometry of the plates.

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Lecture 06 Review

- In fact, there is linear relationship between q and V, where $q = CV$.
- The proportionality constant C is called the capacitance of the capacitor. Note that C depends only on the plates geometry.
- From the relationship of charge and potential, one can see that the higher the capacitance, the more charges that the capacitor can store for the same amount of potential difference.
- The amount of energy stored by the capacitor is the same as the amount of energy needs to separate these charges. Thus, more charges means more energy store.
- We also studied the parallel plates capacitor in depth, which has a uniform electric field across the two plates.

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- We derived the formulations for capacitors in series and in parallel.
- When one inserts a dielectric between the plates of the capacitor, it increases the capacitance of the capacitor due to the induced dielectric dipoles.
- The induced dipoles create a microscopic electric field that is in the opposite direction of the initial field. The weaker E field means more charges are needed to create the same potential difference across the plate.
- The addition of the dielectric allows the capacitor to store more charges or more energy.

$$U = \frac{q^2}{2C} \qquad U = \tfrac{1}{2}CV^2 \qquad \text{(potential energy).}$$

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Lecture Outline

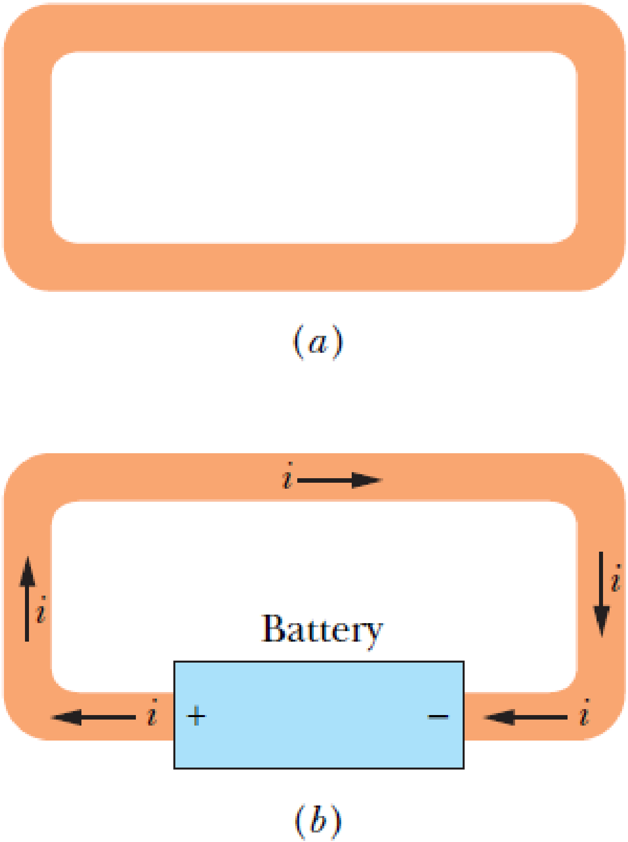
- **Chapter 26/27**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Current and Resistance
  - Electric current
  - Current density and drift velocity
  - Resistance and resistivity
  - Ohm's law
  - Power in electric circuits

a. A loop of conducting wire is in electrostatic equilibrium. Because the entire loop is at the same potential there is no movement of charge within the loop.

b. If a battery is added to the loop imposes an electric potential difference between the terminals of the battery, positive charges will move from the positive terminal to the negative terminal and resulting in a movement of charges.

c. This movement of charges is referred to as current *i*.



In fact, the free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of 10^6 m/s. But they move in all directions and there is no *net* transport of charge and thus no current through the wire.

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Electric current *i* is defined as the time rate of flow of electric charges

$$i = \frac{dq}{dt}$$

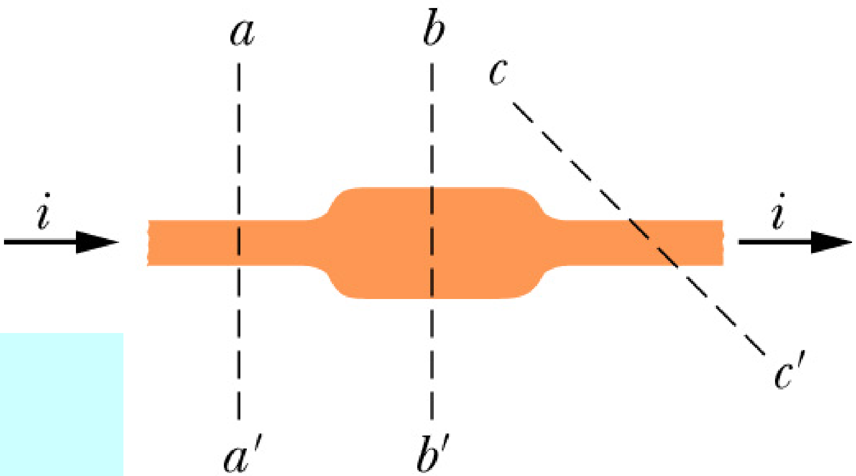
If *i(t)* is known, we can calculate the total amount of charges that passes through a cross-section in a time interval from 0 to *t* :

$$q = \int dq = \int_0^t i\,dt$$

Unit for electric current:

1 ampere = 1 A = 1 coulomb per second = 1 C/s

The current *i* through the conductor has the same value at planes *aa*′ , *bb*′ , and *cc*’ because the same amount of charges flow through these planes over the same time interval.



26.2: Electric Current, Conservation of Charge, and Direction of Current:

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

The relation $i_0 = i_1 + i_2$ is true at junction *a* no matter what the orientation in space of the three wires. *Currents are scalars, not vectors.*

If currents are vectors, then the magnitude of i_1 and i_2 will depends on their orientations with i_0 !!!

The current arrow which is shown next to the wire is only indicating the direction that of flow for the positive charges along the wire (conductor), it is not showing a direction in space.

The current into the junction must equal the current out (charge is conserved).

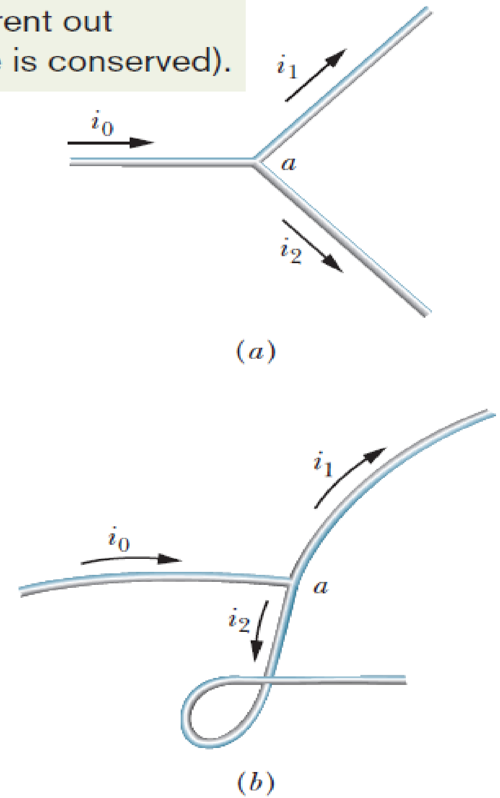


Fig. 26-3 The relation $i_0 = i_1 + i_2$ is true at junction *a* no matter what the orientation in space of the three wires. Currents are scalars, not vectors.

Example:

Water flows through a garden hose at a volume flow rate dV/dt of 450 cm^3/s. What is the current of negative charge?

Calculations: We can write the current in terms of the number of molecules that pass through such a plane per second as

$$i = \begin{pmatrix}\text{charge}\\\text{per}\\\text{electron}\end{pmatrix}\begin{pmatrix}\text{electrons}\\\text{per}\\\text{molecule}\end{pmatrix}\begin{pmatrix}\text{molecules}\\\text{per}\\\text{second}\end{pmatrix}$$

or

$$i = (e)(10)\frac{dN}{dt}.$$

We substitute 10 electrons per molecule because a water (H_2O) molecule contains 8 electrons in the single oxygen atom and 1 electron in each of the two hydrogen atoms.

We can express the rate dN/dt in terms of the given volume flow rate dV/dt by first writing

$$\begin{pmatrix}\text{molecules}\\\text{per}\\\text{second}\end{pmatrix} = \begin{pmatrix}\text{molecules}\\\text{per}\\\text{mole}\end{pmatrix}\begin{pmatrix}\text{moles}\\\text{per unit}\\\text{mass}\end{pmatrix}$$

$$\times \begin{pmatrix}\text{mass}\\\text{per unit}\\\text{volume}\end{pmatrix}\begin{pmatrix}\text{volume}\\\text{per}\\\text{second}\end{pmatrix}.$$

$$\frac{dN}{dt} = N_A\left(\frac{1}{M}\right)\rho_{\text{mass}}\left(\frac{dV}{dt}\right) = \frac{N_A\rho_{\text{mass}}}{M}\frac{dV}{dt}.$$

Substituting this into the equation for i, we find

$$i = 10eN_AM^{-1}\rho_{\text{mass}}\frac{dV}{dt}.$$

We know that Avogadro's number N_A is 6.02×10^{23} molecules/mol, or 6.02×10^{23} mol^{-1}, and from Table 15-1 we know that the density of water ρ_{mass} under normal conditions is 1000 kg/m^3. We can get the molar mass of water from the molar masses listed in Appendix F (in grams per mole): We add the molar mass of oxygen (16 g/mol) to twice the molar mass of hydrogen (1 g/mol), obtaining 18 g/mol = 0.018 kg/mol. So, the current of negative charge due to the electrons in the water is

$$\begin{aligned} i &= (10)(1.6\times10^{-19}\ \text{C})(6.02\times10^{23}\ \text{mol}^{-1}) \\ &\quad \times (0.018\ \text{kg/mol})^{-1}(1000\ \text{kg/m}^3)(450\times10^{-6}\ \text{m}^3/\text{s}) \\ &= 2.41\times10^{7}\ \text{C/s} = 2.41\times10^{7}\ \text{A} \\ &= 24.1\ \text{MA}. \qquad\qquad \text{(Answer)}\end{aligned}$$

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26.3: Current Density:

The magnitude of **current density, *J*,** is equal to the current per unit area through any element of cross section. It provides the localized view of the current flow, the total current flow is.

$$i = \int \vec{J} \cdot d\vec{A}.$$

Current density, ***J*,** is a vector quantity, if the current is uniform across the surface and parallel to ***dA***, then ***J*** is also uniform and parallel to ***dA***, and then

$$i = \int J\, dA = J \int dA = JA$$

$$J = \frac{i}{A},$$

Here, *A* is the total area of the surface.
The SI unit for current density is the ampere per square meter (A/m^2).

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26.3: Current Density:

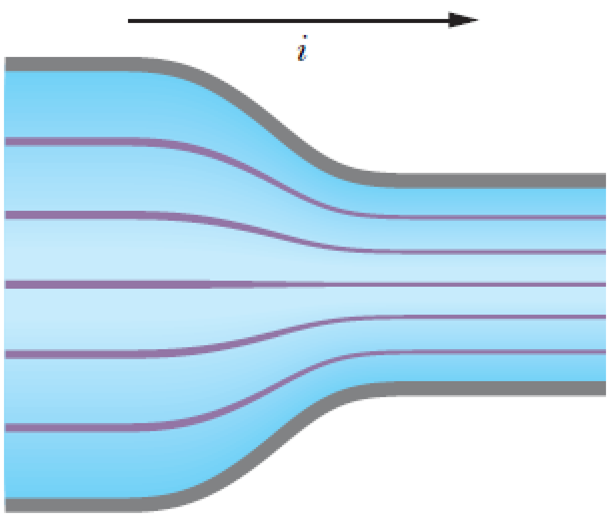

Fig. 26-4 Streamlines representing current density in the flow of charge through a constricted conductor.

Figure 26-4 shows how current density can be represented with a similar set of lines, which we can call *streamlines*.

The current, which is toward the right, makes a transition from the wider conductor at the left to the narrower conductor at the right. Since charge is conserved during the transition, the amount of charge and thus the amount of current cannot change.

However, the current density changes—it is greater in the narrower conductor.

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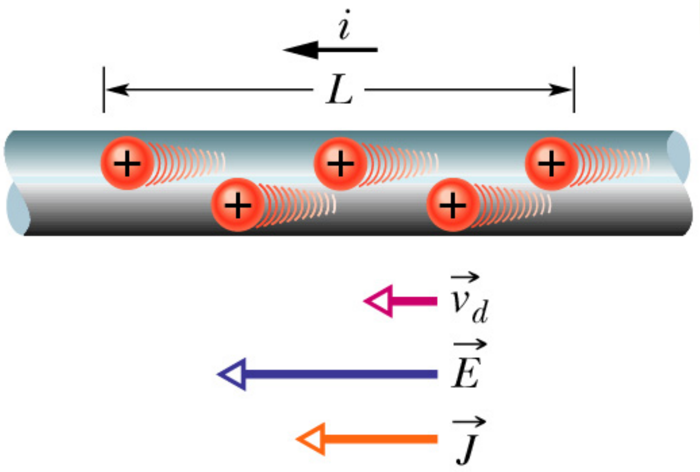
What really happens with an electric current flowing in a conducting wire?

a. Thermal agitation from heat will cause movement of charges in a conductor even in the absence of current flow, however, this type of movements is random and without a prefer direction.

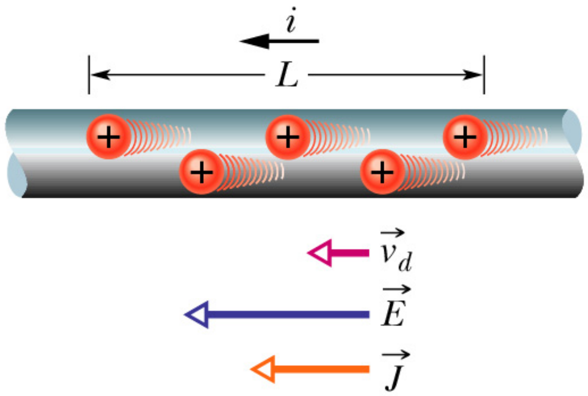
b. When a potential difference is setup in the conductor so there is a current through it, these charges will migrate or drift in the direction of the electric field. Although there is still random motion within their movement, but there is a net *drift* with a **drift speed** v_d in the direction of the electric field.



Positive charge carriers drift at speed v_d in the direction of the applied electric field **E**. By convention, the direction of the current density **J** and the sense of the current arrow are drawn in that same direction.

Let us assume that these charge carriers all move with the same drift speed v_d and that the current density J is uniform across the wire's cross-sectional area A. The number of charge carriers in a length L of the wire is nAL, where n is the number of carriers per unit volume. The total charge of the carriers in the length L, each with charge e, is then

$$\Delta q = (nAL)e$$

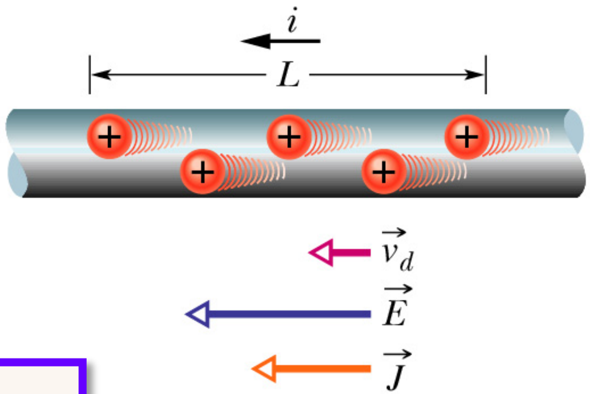


Because the charge carriers all move along the wire at the same speed v_d, the time required to move all these charges over the length of wire L is

$$\Delta t = \frac{L}{v_d}$$

We can relate the drift velocity to the electric current density from the definition of current flow

$$i = \frac{dq}{dt} = \frac{\Delta q}{\Delta t} = \frac{(nAL)e}{L/v_d} = nAev_d$$



$$\Rightarrow \quad v_d = \frac{i}{nAe} = \frac{J}{ne} \quad \Rightarrow \quad \boxed{\vec{J} = (ne)\vec{v}_d.}$$

$$\vec{J}\left(\frac{\mathrm{A}}{\mathrm{m}^2}\right) = n\left(\frac{\#\,\text{of carriers}}{\text{volume}\,\mathrm{m}^3}\right)e(\mathrm{C})\vec{v}_d\left(\frac{\mathrm{m}}{\mathrm{s}}\right) = \left(\frac{\mathrm{C}}{\mathrm{s}} \cdot \frac{1}{\mathrm{m}^2}\right) = \frac{\mathrm{A}}{\mathrm{m}^2}$$

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Example, Current Density, Uniform and Nonuniform:

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m^2. What is the current through the outer portion of the wire between radial distances $R/2$ and R (Fig. 26-6a)?

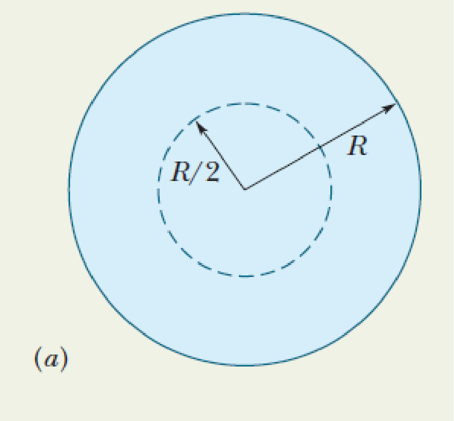


Fig. 26-6

Calculations: We want only the current through a reduced cross-sectional area A' of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$
$$= \frac{3\pi}{4}(0.0020\text{ m})^2 = 9.424 \times 10^{-6}\text{ m}^2.$$

So, we rewrite Eq. 26-5 as

$$i = JA'$$

and then substitute the data to find

$$i = (2.0 \times 10^5\text{ A/m}^2)(9.424 \times 10^{-6}\text{ m}^2)$$
$$= 1.9\text{ A}. \qquad \text{(Answer)}$$

Example, Current Density, Uniform and Nonuniform, cont.:

(b) Suppose, instead, that the current density through a cross section varies with radial distance r as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m^4 and r is in meters. What now is the current through the same outer portion of the wire?

Calculations: The current density vector $\vec{J}$ (along the wire's length) and the differential area vector $d\vec{A}$ (perpendicular to a cross section of the wire) have the same direction. Thus,

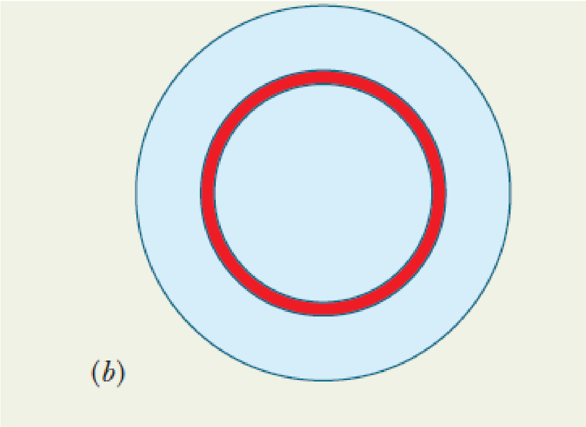
$$\vec{J} \cdot d\vec{A} = J\,dA \cos 0 = J\,dA.$$

We need to replace the differential area dA with something we can actually integrate between the limits $r = R/2$ and $r = R$. The simplest replacement (because J is given as a function of r) is the area $2\pi r\,dr$ of a thin ring of circumference $2\pi r$ and width dr (Fig. 26-6b). We can then integrate

$$\begin{aligned} i &= \int \vec{J} \cdot d\vec{A} = \int J\,dA \\ &= \int_{R/2}^{R} ar^2\, 2\pi r\,dr = 2\pi a \int_{R/2}^{R} r^3\,dr \\ &= 2\pi a \left[\frac{r^4}{4}\right]_{R/2}^{R} = \frac{\pi a}{2}\left[R^4 - \frac{R^4}{16}\right] = \frac{15}{32}\pi a R^4 \\ &= \frac{15}{32}\pi(3.0 \times 10^{11}\ \text{A/m}^4)(0.0020\ \text{m})^4 = 7.1\ \text{A}. \end{aligned}$$

(Answer)

Fig. 26-6



26.4: Resistance and Resistivity:

How fast can the charge carriers move down a block of material depends on the geometry and the material properties.

Resistance is a measure used to determine the ease of flow of charges. Similar to capacitance, it's value is associated with the object.

We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results. The resistance R is then

$$\begin{aligned} 1 \text{ ohm} &= 1\ \Omega = 1 \text{ volt per ampere} \\ &= 1 \text{ V/A}. \end{aligned}$$

The SI unit for resistance that follows from Eq. 26-8 is the volt per ampere. This has a special name, the **ohm** (symbol Ω):

$$R = \frac{V}{i} \qquad \text{(definition of } R\text{)}.$$

In a circuit diagram, we represent a resistor and a resistance with the symbol .

26.4: Resistance and Resistivity:

The resistivity, ρ, of a resistor is defined as:

$$\rho = \frac{E}{J} \Rightarrow \vec{E} = \rho\vec{J}.$$

This is a local property.
The SI unit for ρ is $\Omega\, m$.

The conductivity σ of a material is the reciprocal of its resistivity:

$$\sigma = \frac{1}{\rho} \Rightarrow \vec{J} = \sigma\vec{E}.$$

Similar to resistance R, which relates voltage V and current i, *resistivity* ρ is used to relate electric field **E** and current density **J**.

Table 26-1

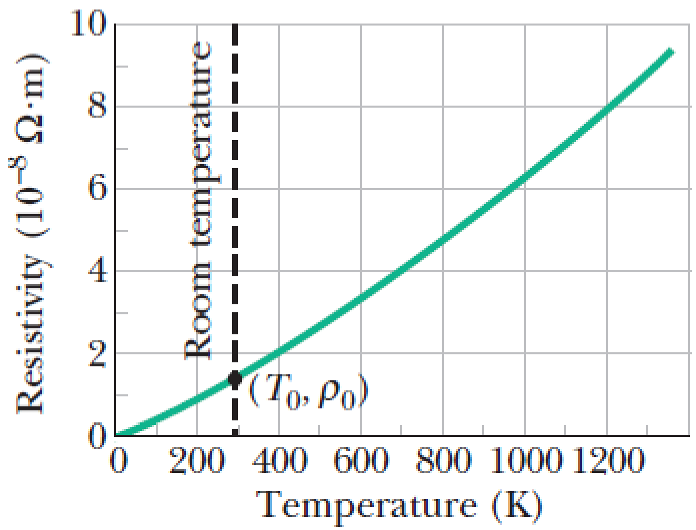
Resistivities of Some Materials at Room Temperature (20°C)

Material	Resistivity, ρ ($\Omega\cdot$m)	Temperature Coefficient of Resistivity, α (K^{-1})
	Typical Metals	
Silver	1.62×10^{-8}	4.1×10^{-3}
Copper	1.69×10^{-8}	4.3×10^{-3}
Gold	2.35×10^{-8}	4.0×10^{-3}
Aluminum	2.75×10^{-8}	4.4×10^{-3}
Manganin[a]	4.82×10^{-8}	0.002×10^{-3}
Tungsten	5.25×10^{-8}	4.5×10^{-3}
Iron	9.68×10^{-8}	6.5×10^{-3}
Platinum	10.6×10^{-8}	3.9×10^{-3}
	Typical Semiconductors	
Silicon, pure	2.5×10^{3}	-70×10^{-3}
Silicon, *n*-type[b]	8.7×10^{-4}	
Silicon, *p*-type[c]	2.8×10^{-3}	
	Typical Insulators	
Glass	$10^{10} - 10^{14}$	
Fused quartz	$\sim 10^{16}$	

$$\rho = \rho_0(1 + \alpha(T - T_0))$$

26.4: Resistance and Resistivity, Variation with Temperature:

Fig. 26-10 The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature $T_0 = 293$ K and resistivity $\rho_0 = 1.69 \times 10^{-8}\ \Omega \cdot \text{m}$.



Resistivity can depend on temperature.

The relation between temperature and resistivity for copper—and for metals in general—is fairly linear over a rather broad temperature range. For such linear relations we can write an empirical approximation that is good enough for most engineering purposes:

$$\boxed{\rho - \rho_0 = \rho_0 \alpha (T - T_0).}$$

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26.4: Resistance and Resistivity, Calculating Resistance from Resistivity:

Resistance is a property of an object. Resistivity is a property of a material.

$$E = V/L \quad \text{and} \quad J = i/A.$$

$$\rho = \frac{E}{J} = \frac{V/L}{i/A}.$$

$$\boxed{R = \rho \frac{L}{A}.}$$

Current is driven by a potential difference.

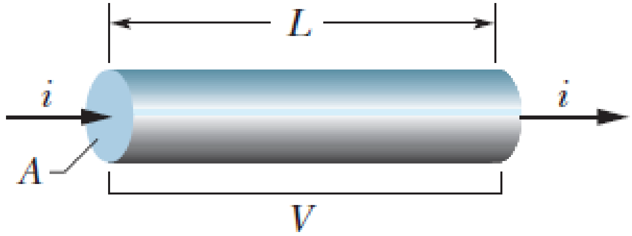

Fig. 26-9 A potential difference V is applied between the ends of a wire of length L and cross section A, establishing a current i.

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Example, A material has resistivity, a block of the material has a resistance.:

A rectangular block of iron has dimensions 1.2 cm × 1.2 cm × 15 cm. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8*b*). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions 1.2 cm × 1.2 cm) and (2) two rectangular sides (with dimensions 1.2 cm × 15 cm)?

KEY IDEA

The resistance R of an object depends on how the electric potential is applied to the object. In particular, it depends on the ratio L/A, according to Eq. 26-16 ($R = \rho L/A$), where A is the area of the surfaces to which the potential difference is applied and L is the distance between those surfaces.

Calculations: For arrangement 1, we have $L = 15$ cm $= 0.15$ m and

$$A = (1.2\ \text{cm})^2 = 1.44 \times 10^{-4}\ \text{m}^2.$$

Substituting into Eq. 26-16 with the resistivity ρ from Table 26-1, we then find that for arrangement 1,

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(0.15\ \text{m})}{1.44 \times 10^{-4}\ \text{m}^2} 3$$

$$= 1.0 \times 10^{-4}\ \Omega = 100\ \mu\Omega. \qquad \text{(Answer)}$$

Similarly, for arrangement 2, with distance $L = 1.2$ cm and area $A = (1.2\ \text{cm})(15\ \text{cm})$, we obtain

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega\cdot\text{m})(1.2 \times 10^{-2}\ \text{m})}{1.80 \times 10^{-3}\ \text{m}^2}$$

$$= 6.5 \times 10^{-7}\ \Omega = 0.65\ \mu\Omega. \qquad \text{(Answer)}$$

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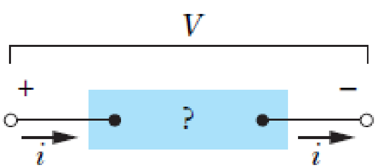
26.5: Ohm's Law:

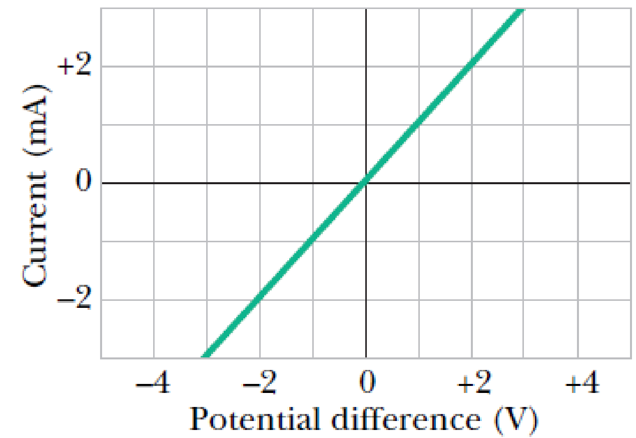
Ohm's law is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

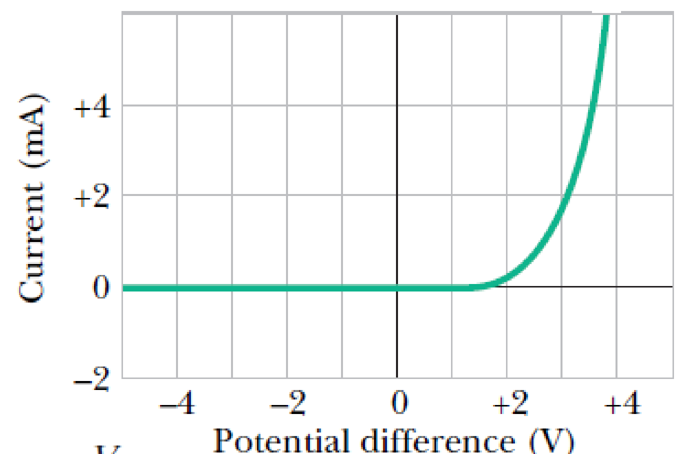
A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.



(a)



(b)



(c)

Fig. 26-11 (*a*) A potential difference V is applied to the terminals of a device, establishing a current i. (*b*) A plot of current i versus applied potential difference V when the device is a 1000 Ω resistor. (*c*) A plot when the device is a semiconducting *pn* junction diode.

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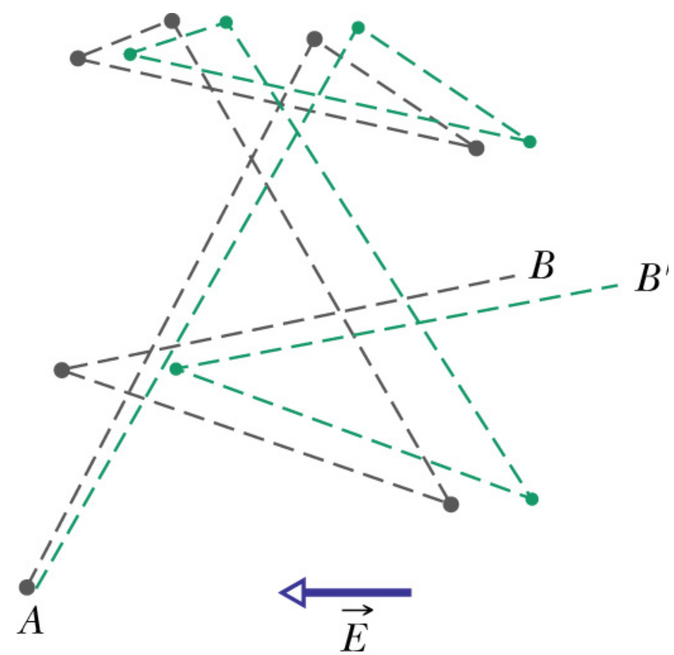
When we apply an electric field to a metal sample, the electrons modify their random motions slightly and drift very slowly—in a direction opposite that of the field—with an average drift speed v_d.

The motion of the conduction electrons in an electric field **E** is thus a combination of the motion due to random collisions and that due to **E**. When we consider all the free electrons, their random motions average to zero and make no contribution to the drift speed. Thus, the drift speed is due only to the effect of the electric field on the electrons.

The grey lines show an electron moving from *A* to *B*, making six collisions en route. The green lines show what its path might be in the presence of an applied electric field **E**. Note the steady drift in the direction of **-E**. (Actually, the green lines should be slightly curved, to represent the parabolic paths followed by the electrons between collisions, under the influence of an electric field.)



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26.7: Power in Electric Circuits:

Charge dq moves through a decrease in potential of magnitude V, and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq\, V = i\, dt\, V.$$

The power P associated with that transfer is the rate of transfer dU/dt, given by

$$P = iV \qquad \text{(rate of electrical energy transfer).}$$

$$P = i^2 R \qquad \text{(resistive dissipation)}$$

$$P = \frac{V^2}{R} \qquad \text{(resistive dissipation).}$$

The unit of power is the volt-ampere (V A).

$$1\ \mathrm{V \cdot A} = \left(1\,\frac{\mathrm{J}}{\mathrm{C}}\right)\left(1\,\frac{\mathrm{C}}{\mathrm{s}}\right) = 1\,\frac{\mathrm{J}}{\mathrm{s}} = 1\ \mathrm{W}.$$

The battery at the left supplies energy to the conduction electrons that form the current.

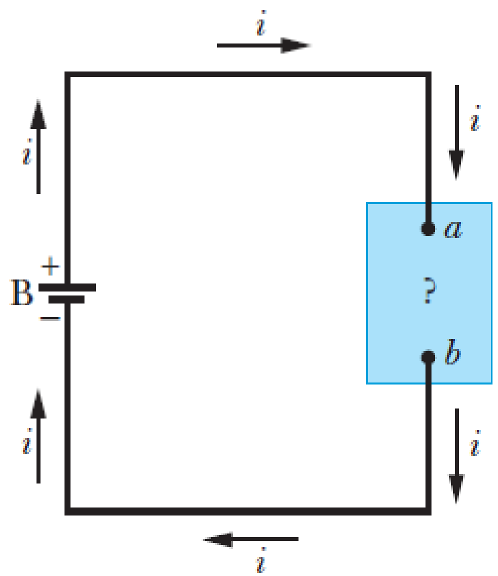

Fig. 26-13 A battery B sets up a current i in a circuit containing an unspecified conducting device.

Example, Rate of Energy Dissipation in a Wire Carrying Current:

You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of 72 Ω. At what rate is energy dissipated in each of the following situations? (1) A potential difference of 120 V is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of 120 V is applied across the length of each half.

KEY IDEA

Current in a resistive material produces a transfer of mechanical energy to thermal energy; the rate of transfer (dissipation) is given by Eqs. 26-26 to 26-28.

Calculations: Because we know the potential V and resistance R, we use Eq. 26-28, which yields, for situation 1,

$$P = \frac{V^2}{R} = \frac{(120\text{ V})^2}{72\ \Omega} = 200\text{ W}. \qquad \text{(Answer)}$$

In situation 2, the resistance of each half of the wire is (72 Ω)/2, or 36 Ω. Thus, the dissipation rate for each half is

$$P' = \frac{(120\text{ V})^2}{36\ \Omega} = 400\text{ W},$$

and that for the two halves is

$$P = 2P' = 800\text{ W}. \qquad \text{(Answer)}$$

This is four times the dissipation rate of the full length of wire. Thus, you might conclude that you could buy a heating coil, cut it in half, and reconnect it to obtain four times the heat output. Why is this unwise? (What would happen to the amount of current in the coil?)

Table 26-2

Some Electrical Properties of Copper and Silicon

Property	Copper	Silicon
Type of material	Metal	Semiconductor
Charge carrier density, m^{-3}	8.49×10^{28}	1×10^{16}
Resistivity, $\Omega \cdot m$	1.69×10^{-8}	2.5×10^{3}
Temperature coefficient of resistivity, K^{-1}	$+4.3 \times 10^{-3}$	-70×10^{-3}

Pure silicon has a high resistivity and it is effectively an insulator. However, its resistivity can be greatly reduced in a controlled way by adding minute amounts of specific "impurity" atoms in a process called *doping*.

A semiconductor is like an insulator except that the energy required to free some electrons is not quite so great. The process of doping can supply electrons or positive charge carriers that are very loosely held within the material and thus are easy to get moving. Also, by controlling the doping of a semiconductor, one can control the density of charge carriers that are responsible for a current.

The resistivity in a conductor is given by: $\rho = \frac{m}{e^2 n \tau}$,

In a semiconductor, *n* is small but increases very rapidly with temperature as the increased thermal agitation makes more charge carriers available. This causes a decrease of resistivity with increasing temperature. The same increase in collision rate that is noted for metals also occurs for semiconductors, but its effect is swamped by the rapid increase in the number of charge carriers.

26.9: Superconductors:

A disk-shaped magnet is levitated above a superconducting material that has been cooled by liquid nitrogen. The goldfish is along for the ride. *(Courtesy Shoji Tonaka/International Superconductivity Technology Center, Tokyo, Japan)*

For Advanced Standing Students

In 1911, Dutch physicist Kamerlingh Onnes discovered that the resistivity of mercury absolutely disappears at temperatures below about 4 K .This phenomenon is called **superconductivity**, and it means that charge can flow through a superconducting conductor without losing its energy to thermal energy.

One explanation for superconductivity is that the electrons that make up the current move in coordinated pairs. One of the electrons in a pair may electrically distort the molecular structure of the superconducting material as it moves through, creating nearby a short-lived concentration of positive charge. The other electron in the pair may then be attracted toward this positive charge. Such coordination between electrons would prevent them from colliding with the molecules of the material and thus would eliminate electrical resistance. New theories appear to be needed for the newer, higher temperature superconductors.

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who made the historic discovery of superconductivity above 77 K in YBCO in 1987