EE1004 Tutorial 4 (Part 2)

1. Consider a trial in which a jury must decide between the hypothesis that the defendant is guilty and the hypothesis that he or she is innocent.

(a) In the framework of hypothesis testing and the Hong Kong legal system, which of the hypotheses should be the null hypothesis?

(b) What do you think would be an appropriate significance level in this situation?

Answer 1. (a) The null hypothesis should be the defendant is innocent.

(b) The significance level should be relatively small, say α = 0.01.

2. A colony of laboratory mice consists of several thousand mice. The average weight of all the mice is 32 grams with a standard deviation of 4 grams. A laboratory assistant was asked by a scientist to select 25 mice for an experiment. However, before performing the experiment the scientist decided to weigh the mice as an indicator of whether the assistant's selection constituted a random sample or whether it was made with some unconscious bias (perhaps the mice selected were the ones that were slowest in avoiding the assistant, which might indicate some inferiority about this group). If the sample mean of the 25 mice was 30.4, would this be significant evidence, at the 5 percent level of significance, against the hypothesis that the selection constituted a random sample?

2. If the selection was random, then the data would constitute a sample of size 25 from a normal population with mean 32 and standard deviation 4.

Define Hypothesis

- Null hypothesis H_0: e.g. the mean value = μ = 32
- Alternative H_1: e.g. the mean value $\neq \mu$ = 32

Then

- Calculate the z-score from the data set: $z = |\frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}| = |\frac{30.4-32}{\frac{4}{\sqrt{25}}}| = 2$
- Use **Normal Distribution Calculator** to obtain the cumulative probability P($Z < z$) = 0.977.
- This is a **two-tailed** test so the P-value is obtained from

  $P = 2[1 - P(Z < z)] = 0.046$

  Since $P < \alpha = 0.05$, we reject the null hypothesis H_0.

3. A population distribution is known to have standard deviation 20. Determine the *p*-value of a test of the hypothesis that the population mean is equal to 50, if the average of a sample of 64 observations is (a) 52.5; (b) 55.0; (c) 57.5.

3 (a) $z = |\frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}| = |\frac{52.5 - 50}{\frac{20}{\sqrt{64}}}| = 1$

P(Z < z) = 0.841

P = $2[1 - \mathrm{P}(Z < z)] = 0.318$

(b) $z = |\frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}| = |\frac{55 - 50}{\frac{20}{\sqrt{64}}}| = 2$

P(Z < z) = 0.977

P = $2[1 - \mathrm{P}(Z < z)] = 0.046$

(c) $z = |\frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}| = |\frac{57.5 - 50}{\frac{20}{\sqrt{64}}}| = 3$

P(Z < z) = 0.999

P = $2[1 - \mathrm{P}(Z < z)] = 0.002$

[**Quiz**] 4. In a certain chemical process, it is very important that a particular solution that is to be used as a reactant have a pH of exactly 8.20. A method for determining pH that is available for solutions of this type is known to give measurements that are normally distributed with a mean equal to the actual pH and with a standard deviation of 0.02. Suppose 10 independent measurements yielded the following pH values:

8.18, 8.17, 8.16, 8.15, 8.17, 8.21, 8.22, 8.16, 8.19, 8.18

(a) What conclusion can be drawn at the $\alpha = 0.10$ level of significance?

(b) What about at the $\alpha = 0.05$ level of significance?

4. $\bar{x}$ = 8.179.

- Calculate the z-score from the data set: $z = |\frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}| = |\frac{8.179 - 8.2}{\frac{0.02}{\sqrt{10}}}| = 3.320$
- Use **Normal Distribution Calculator** to obtain the cumulative probability P(Z < z) = 1.000.
- This is a **two-tailed** test so the *P*-value is obtained from

  P = $2[1 - \mathrm{P}(Z < z)] = 0.000$

  Since $P < \alpha$ at both levels, we reject the null hypothesis for both levels.

5. The mean breaking strength of a certain type of fiber is required to be at least 200 psi. Past experience indicates that the standard deviation of breaking strength is 5 psi. If a sample of 8 pieces of fiber yielded breakage at the following pressures,

210, 198, 195, 202, 197.4, 196, 199, 195.5

would you conclude, at the 5 percent level of significance, that the fiber is unacceptable? What about at the 10 percent level of significance?

5. $\bar{x}$ = 199.1125.

- Calculate the z-score from the data set: $z = |\frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}| = |\frac{199.1125 - 200}{\frac{5}{\sqrt{8}}}| = 0.5020$
- Use **Normal Distribution Calculator** to obtain the cumulative probability P(Z < z) = 0.692.
- This is a **one-tailed** test so the *P*-value is obtained from

  $P = 1 - \mathrm{P}(Z < z) = 0.308$

  Since $P \geq \alpha$ at both levels, we accept the null hypothesis for both levels.

[**Assignment 3**] 6. A producer specifies that the mean lifetime of a certain type of battery is at least 240 hours. A sample of 18 such batteries yielded the following data.

237, 242, 232, 242, 248, 230, 244, 243, 254, 262, 234, 220, 225, 236, 232, 218, 228, 240

Assuming that the life of the batteries is approximately normally distributed, do the data indicate that the specifications are not being met at the $\alpha = 0.05$ level of significance?

7. An advertisement for a new toothpaste claims that it reduces cavities of children in their cavity-prone years. Cavities per year for this age group are normal with mean 3 and standard deviation 1. A study of 2,500 children who used this toothpaste found an average of 2.95 cavities per child. Assume that the standard deviation of the number of cavities of a child using this new toothpaste remains equal to 1.

(a) Are these data strong enough, at the 5 percent level of significance, to establish the claim of the toothpaste advertisement? Note that we assume that the null hypothesis: $\mu \geq 3.00$.

(b) Do the data convince you to switch to this new toothpaste?

7. (a) $\bar{x}$ = 2.95.

- Calculate the z-score from the data set: $z = |\frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}| = |\frac{2.95-3}{\frac{1}{\sqrt{2500}}}| = 2.5$
- Use **Normal Distribution Calculator** to obtain the cumulative probability P(Z < z) = 0.994.
- This is a **one-tailed** test so the *P*-value is obtained from

  $P = 1 - P(Z < z) = 0.006$

  Since $P < \alpha = 0.05$, we reject the null hypothesis. Thus, we can conclude that the new toothpaste will reduce cavities of children.

(b) However, since it also suggests that the mean drop is of the order of 0.05 cavities (less than 2%), it is probably not large enough to convince most users to switch to this new toothpaste.