

CITY UNIVERSITY OF HONG KONG

Course code and title : MA1201 Calculus and Basic Linear Algebra II

Session : Semester A, 2017/2018

Time allowed : Three hours

This paper has **THREE** pages (including this cover page and the attached table).

Instructions to candidates:

1. Attempt all **SIX** questions in this paper.
 2. Start each question on a new page.
 3. Show all steps clearly in order to get full credits.
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*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable calculators

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorised materials or aids are found on them.

NOT TO BE TAKEN AWAY

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BUT FORWARDED TO LIB**

Question 1 [15]

Compute the following elementary integrals.

(a) $\int \frac{3x+4}{2x-1} dx$ [5]

(b) $\int x\sqrt{3x^2-7} dx$ [5]

(c) $\int_0^{\pi/2} \sin 3x \cos 3x dx$ [5]

Question 2 [22]

Evaluate the following indefinite integrals.

(a) $\int \sqrt{9-4x^2} dx$ [8]

(b) $\int \frac{1}{x^3} \ln x dx$ [5]

(c) $\int \frac{13x^2}{(x-1)(x^2+2x+10)} dx$ [9]

Question 3 [13]

(a) Find the area of the region bounded by the parabola $y=3-x^2$ and the straight line $y=4x-2$. [7]

(b) Calculate the arc length of the curve $y=\frac{1}{4}(e^{2x}+e^{-2x})$, $0 \leq x \leq 1$. [6]

Question 4 [15]

(a) Find the angle between two vectors $\vec{a}=\vec{i}+2\vec{j}-3\vec{k}$ and $\vec{b}=-2\vec{i}-3\vec{j}+\vec{k}$. [5]

(b) Simplify the complex expression $(2-3i)^{-2}$ into the Cartesian form. [5]

(c) Find the matrix inverse $\begin{pmatrix} 2 & 5 \\ -1 & -3 \end{pmatrix}^{-1}$ by the adjoint method. [5]

Question 5 [20]

(a) Determine the shortest distance from a point $P(-3, 2, 1)$ to a plane containing $A(-2, 1, 3)$, $B(3, -2, 4)$ and $C(1, 0, 2)$. [10]

(b) Solve the complex equation $z^3+\sqrt{2}=\sqrt{6}i$ and list all the solutions in Euler's form with principal arguments. [10]

Question 6 [15]

Consider the system of linear equations as follows.

$$y+5z-3w=8$$

$$x-2y+3z-4w=5$$

$$2x-3y+z-5w=2$$

(a) Solve the above linear system by the Gaussian elimination and express the general solution in vector form. [11]

(b) Write down the corresponding homogeneous system explicitly and provide a possible largest set of linear independent solutions from (a) without resolving it. [4]

Brief Table of Integrals

$\int x^p dx = \frac{x^{p+1}}{p+1} + C, \quad p \neq -1$	$\int \frac{1}{x} dx = \ln x + C$
$\int e^x dx = e^x + C$	$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln \sec x + \tan x + C$
$\int \sin x dx = -\cos x + C$	$\int \cos x dx = \sin x + C$
$\int \sec^2 x dx = \tan x + C$	$\int \csc^2 x dx = -\cot x + C$
$\int \sec x \tan x dx = \sec x + C$	$\int \csc x \cot x dx = -\csc x + C$
$\int \sec x dx = \ln \sec x + \tan x + C$	$\int \csc x dx = -\ln \csc x + \cot x + C$
$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$