Assignment 2

Q1. Determine if the follow functions are injective, surjective, or bijective.

a. $f: N \to N, f(x) = x^2$ (10%)
b. $f: R \to R, f(x) = x^2$ (10%)
c. $f: N \to N, f(x) = x + 2$ (10%)
d. $f: R \to R, f(x) = 2x - 3$ (20%)

Solution:

a. Injective
b. Injective
c. Surjective
d. Bijective

Proof d.:

If $f(x_1) = f(x_2)$ then $2x_1 - 3 = 2x_2 - 3 \Rightarrow \mathrm{x}_1 = \mathrm{x}_2$. Hence injective.

$2x - 3 = y$, so $x = \frac{y+5}{3}$, which belongs to R and $f(x) = y$. Hence surjective.

$$\therefore \text{Injective \& Surjective}$$

$$\therefore Bijective$$

Q2.

a. $f(x) = 2x + 3, g(x) = -x^2 + 5$. Find $(g \circ f)(x)$. (10%)
b. $f(x) = \frac{3}{5}x + 4, g(x) = 2x^2 - 5x + 9$. Find $(f \circ g)\left(\frac{1}{2}\right)$. (10%)

Solution:

a. $$\begin{aligned}(g \circ f)(x) &= -(2x + 3)^2 + 5 \\ &= -4x^2 - 12x - 9 + 5 \\ &= -4x^2 - 12x - 4\end{aligned}$$

b. $(f \circ g)(x) = \frac{3}{5}(2x^2 - 5x + 9) + 4$

$(f \circ g)(x) = \frac{6x^2}{5} - 3x + \frac{27}{5} + 4$

$(f \circ g)\left(\frac{1}{2}\right) = \frac{6}{5\times 4} - \frac{3}{2} + \frac{27}{5} + 4 = \frac{6}{20} - \frac{3}{2} + \frac{27}{5} + 4 = \frac{6-30+108+80}{20}$

$(f \circ g)\left(\frac{1}{2}\right) = \frac{164}{20} = \frac{41}{5}$

Q3. Define $f, g: R \to R, f(x) = 3^x, g(x) = x^3$. Prove g is surjective and f is not surjective. (20%)

("onto") $\forall y \in Y, \exists x \in X$, such that $y = f(x)$.

Proof:

Since $x \in R$, then 3^x is always positive.

But there are some $b \leq 0$, when b is the co-domain of f.

$\therefore f$ is not surjective.

On the other hand, for any $b \in R$, the $b = g(x)$ has a solution (namely x $= \sqrt[3]{b}$), so b has a preimage under g.

$\therefore g$ is surjective.

Q4. Use contrapositive proof to prove: If x and $y \in Z$, $x + y$ is even, then x and y have the same parity (either both are even, or both are odd). (10%)

Proof:

Contrapositive.

Prov If both x and y do not have the same parity, then $x + y$ is odd.

Assume: x is odd and y is even.

Then $\exists m \in Z$, such that $x = 2m + 1$

$\exists n \in Z$, such that $y = 2n$

$\therefore x + y = (2m + 1) + 2n = 2(m + n) + 1$

$\therefore x + y$ must be odd.

Proved. ■