Introduction to signals, time and frequency domains, Fourier series

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Contents

What is signal?

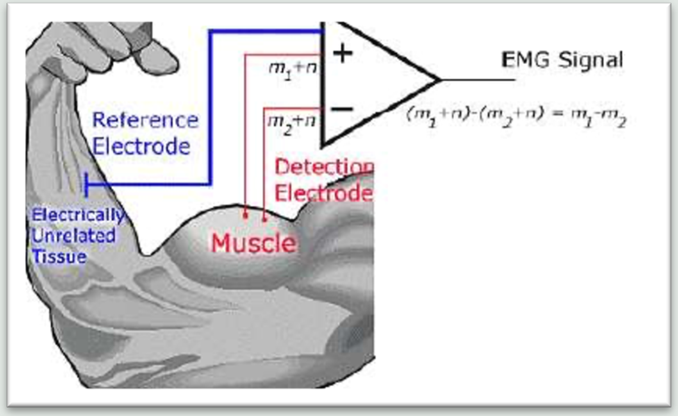
- A **signal** describes how one parameter varies with another parameter, such as current variation over time in an electric circuit.
- A **system** produces an output **signal** in response to an input **signal**.
- Mathematically, a signal is a function, usually a function of time. It is used to represent some sort of phenomenon, for example, audio signals.

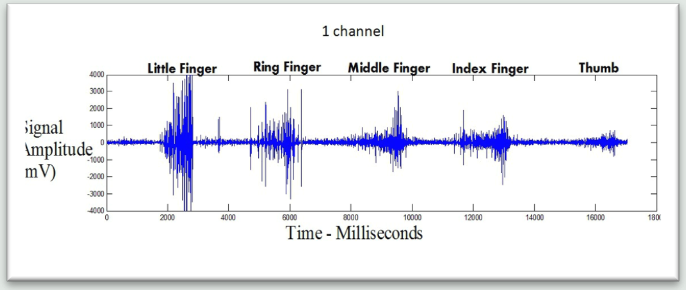
Example – Signals from body

- EMG (Electromyography) – electrical activity by skeletal muscles
- ECG (Electrocardiography) – electrical activity of the heart
- EEG (Electroencephalography) – electrical activity of the brain
- Pace signal – Signal from pacemaker
- Respiration – study if impedance created from inhale / exhale

Example - EMG

Muscles create many action potentials during a movement, like moving your arm or closing your hands. This creates a signal similar to the image below, which shows an increase in signal due to different fingers moving.





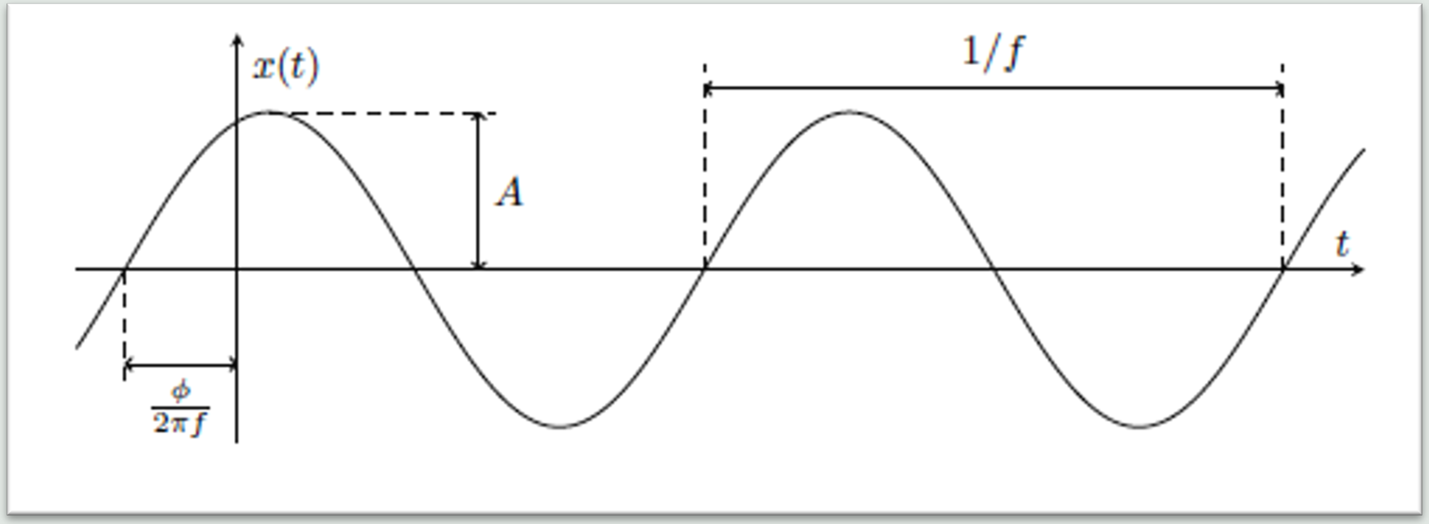
Periodic signals

- A signal $x(t)$ is said to be periodic if there exists some number T such that, for all t,

$$x(t) = x(t + T)$$

- The number T is known as the period of the signal.
- The smallest T satisfying $x(t) = x(t + T)$ is known as the fundamental period. Then, $1 / T$ is the fundamental frequency.

Sinsuoidal signals



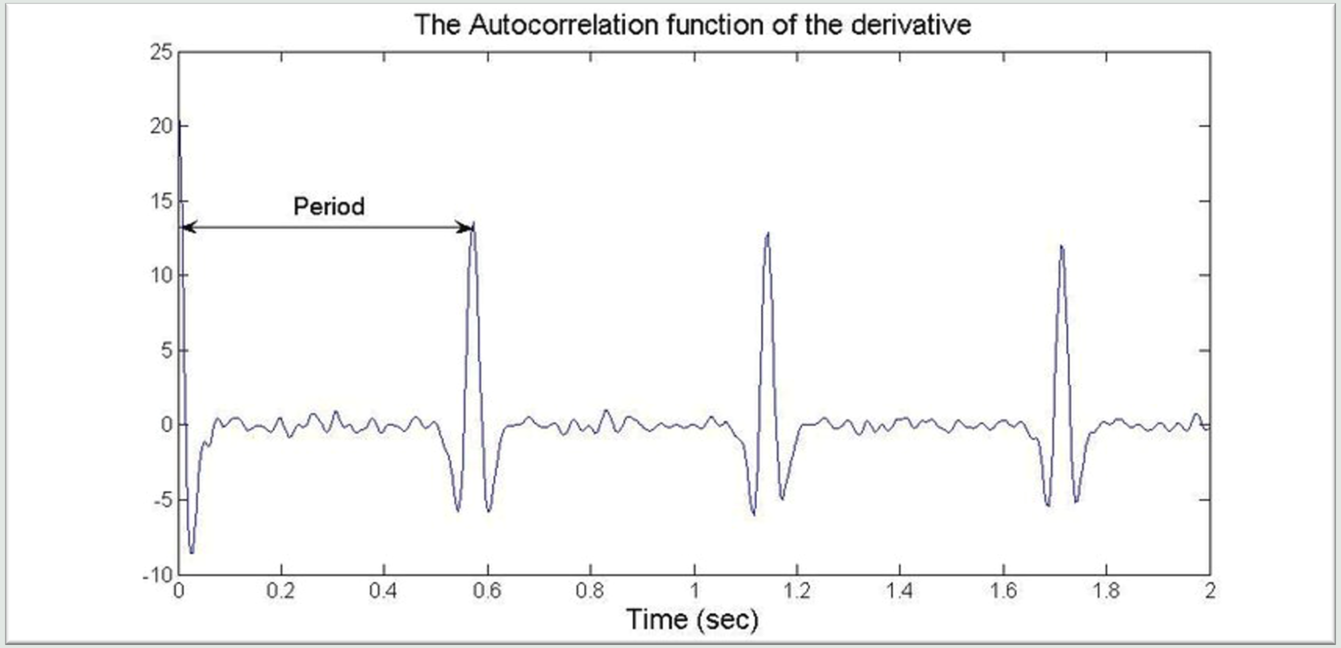
A single sinusoid means a function of the form

$$x(t) = A \sin(2\pi f t + \phi)$$

A : amplitude, f : frequency (Hz), and ϕ : phase.

Periodic, non-sinusoidal signals

Non-sinusoidal signals appear in real world.



Question

We understand sinusoidal function very well. We use

Amplitude, frequency, and phase

to describe a sinusoid.

How about periodic non-sinusoidal functions?

Fourier series

A French mathematician called *Joseph Fourier*, who showed that a periodic signal can be represented by a sum of sine and cosine functions.

This idea gave rise to what is now known as the frequency domain. Signals are considered as function of frequency, as opposed to a function of time.

General form of Fourier series

The Fourier series states that any practical periodic function (period T or frequency $\omega_o = 2\pi / T$) can be represented as an infinite sum of sinusoidal waveforms (or sinusoids) that have frequencies which are an integral multiple of ω_o .

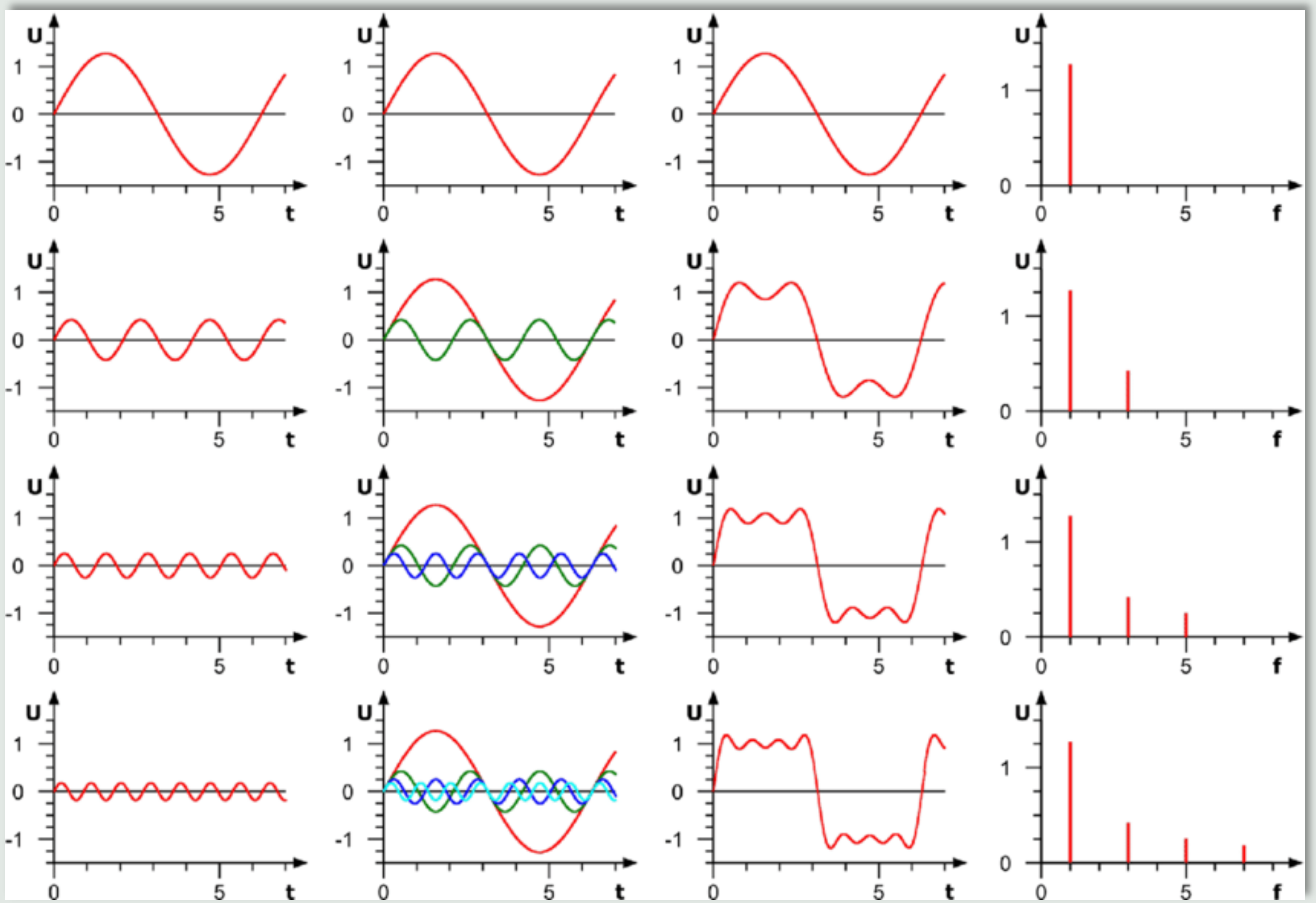
$$f(t) = A_o / 2 + A_1 \cos \omega_o t + A_2 \cos 2 \omega_o t + A_3 \cos 3 \omega_o t + \dots$$
$$+ B_1 \sin \omega_o t + B_2 \sin 2 \omega_o t + B_3 \sin 3 \omega_o t + \dots$$

$$f(t) = A_o / 2 + C_1 \sin (\omega_o t + \phi_1) + C_2 \sin (2\omega_o t + \phi_2) + C_3 \sin (2\omega o\, t + \phi_3) + \dots$$

Questions:

1. Express C_k and ϕ_k in terms of A_k and B_k ?
2. How to interpret the second equation?

Example:
Square wave



There are fundamental component, 3rd harmonic, 5th harmonic, and 7th harmonic components.

Interesting Fourier Series Animations

[Square wave]

https://www.youtube.com/watch?v=k8FXF1KjzY0

[Saw wave]

https://www.youtube.com/watch?v=YUBe-ro89I4

Instruments for observing signals

[Time waveform]

If we want to observe how *voltage changes over time* by displaying a waveform of electronic signals, we use an oscilloscope.

[Frequency spectrum]

A spectrum analyzer measures the *magnitude of an input signal* versus *frequency* within the full frequency range of the instrument.

Online resource: https://www.youtube.com/watch?v=l5eXMQB_gxA&t=51s

Signals in time and frequency domains

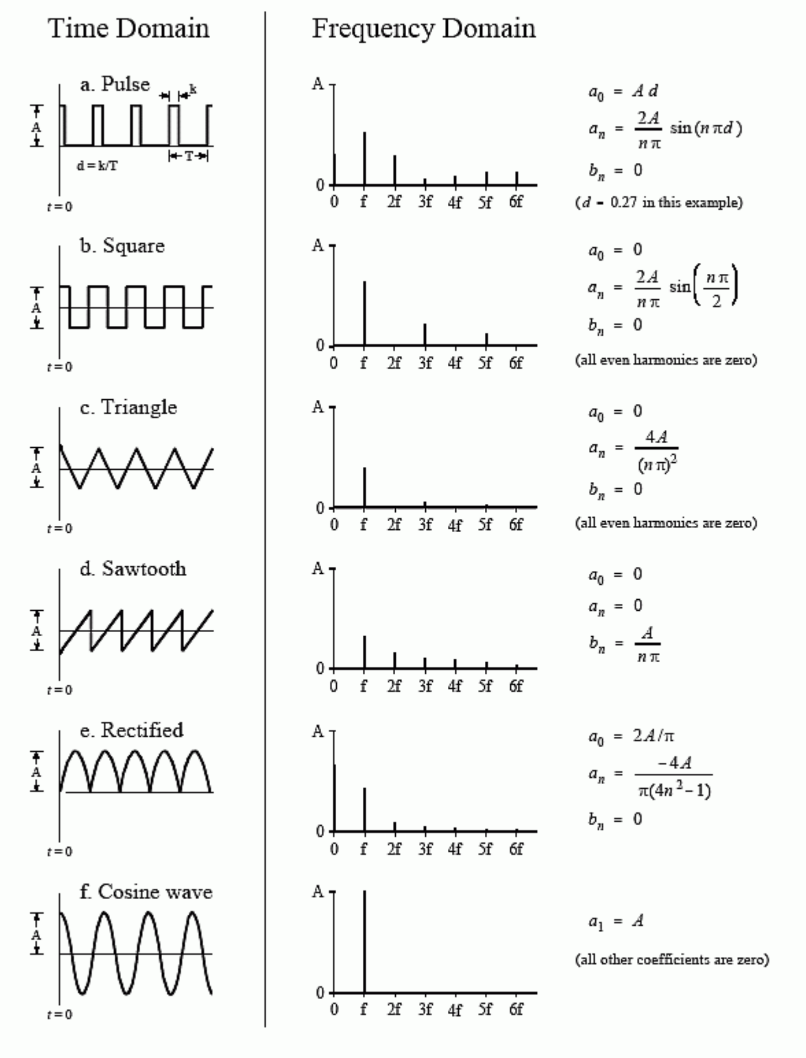
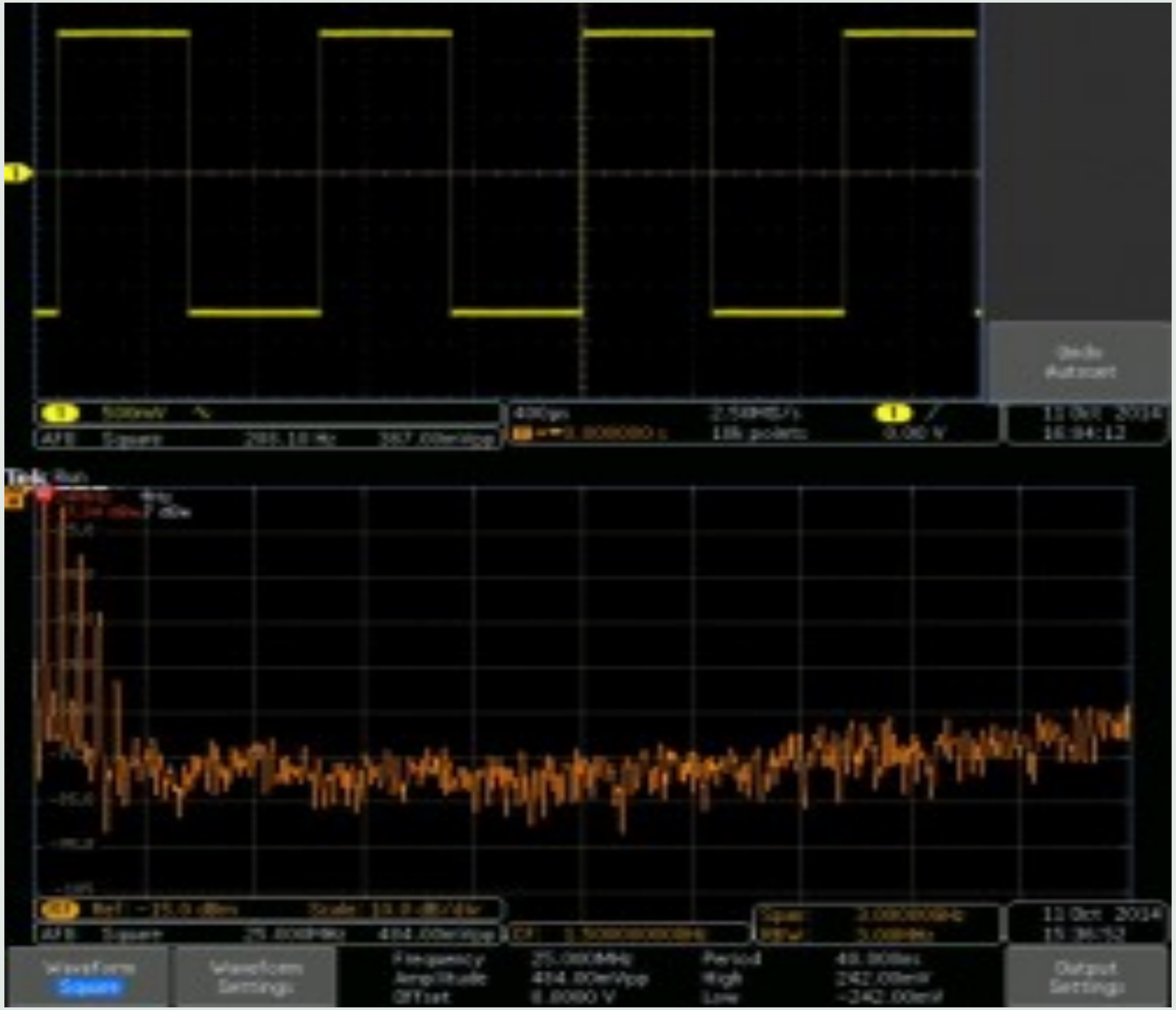
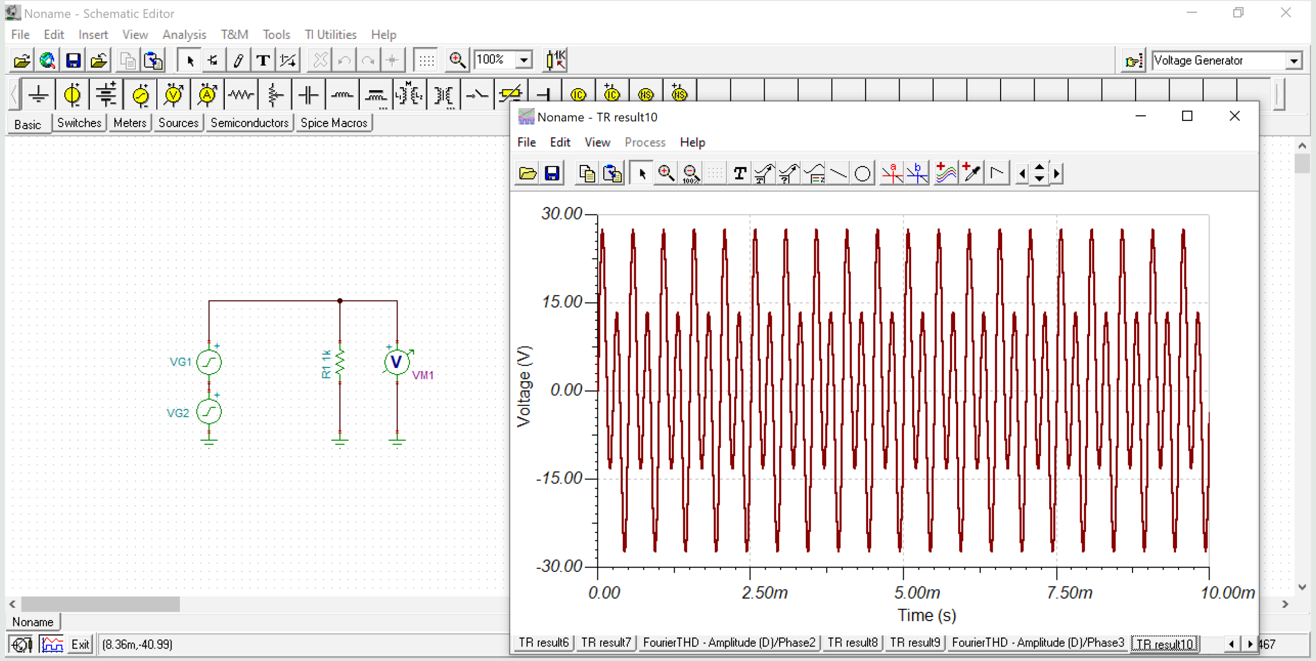


FIGURE 13-10
Examples of the Fourier series. Six common time domain waveforms are shown, along with the equations to calculate their "a" and "b" coefficients.

What do you see on spectrum analyzer?



Computer simulation on Tina



[Download] SPICE-based analog simulation program

https://www.ti.com/tool/TINA-TI

Test with different amplitudes & phases

Perform some simulations and observe the time waveforms and spectra of the following combinations

(a) $V_1 = 10V$, $f_1 = 2kHz$, $\phi_1 = 0^o$ and $V_2 = 20V$, $f_2 = 4kHz$, $\phi_2 = 0^o$

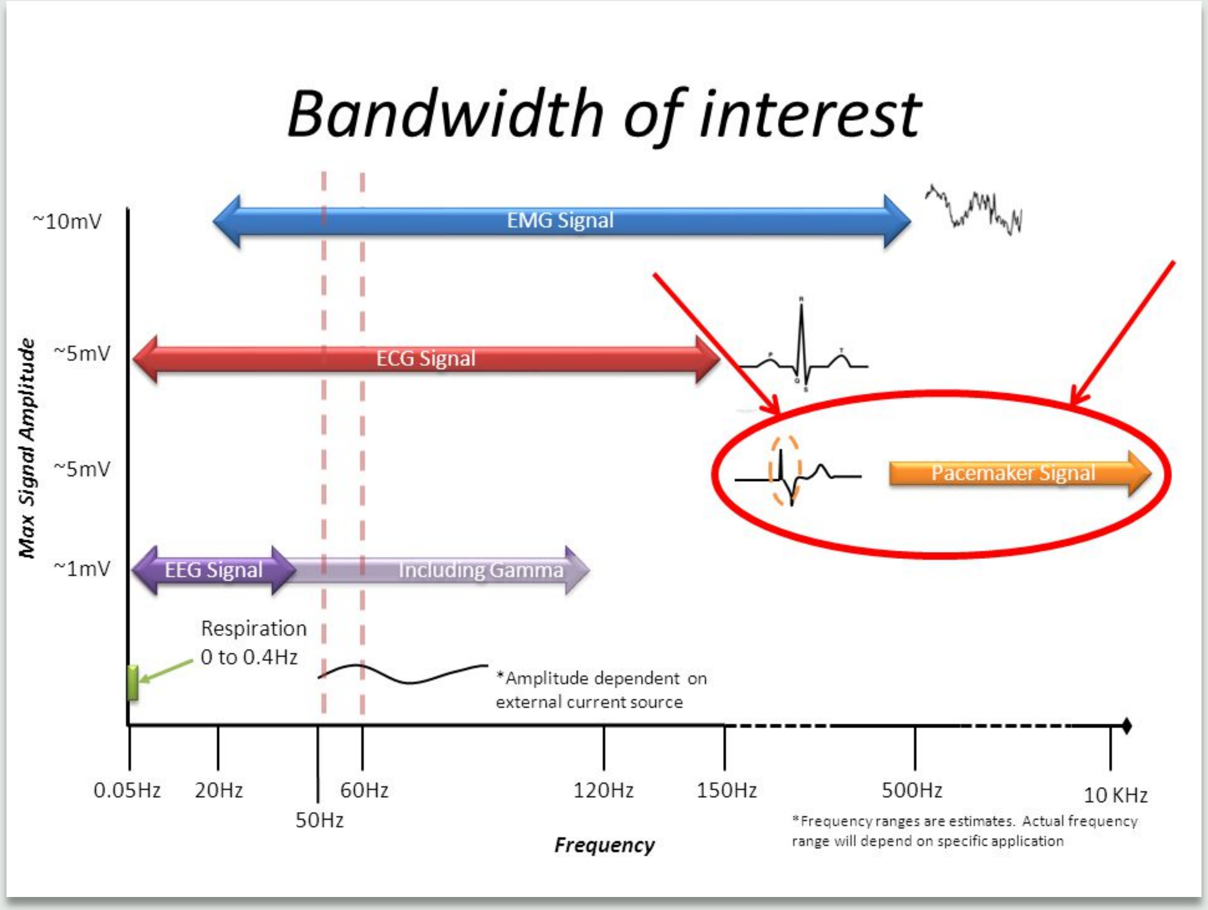
(b) (a) $V_1 = 10V$, $f_1 = 2kHz$, $\phi_1 = 0^o$ and $V_2 = 5V$, $f_2 = 4kHz$, $\phi_2 = 0^o$

(c) (a) $V_1 = 10V$, $f_1 = 2kHz$, $\phi_1 = 0^o$ and $V_2 = 20V$, $f_2 = 4kHz$, $\phi_2 = 90^o$

(d) (a) $V_1 = 10V$, $f_1 = 2kHz$, $\phi_1 = 0^o$ and $V_2 = 5V$, $f_2 = 4kHz$, $\phi_2 = 90^o$

What are their similarities and differences? Draw the conclusions.

Frequency range of different kinds of signals



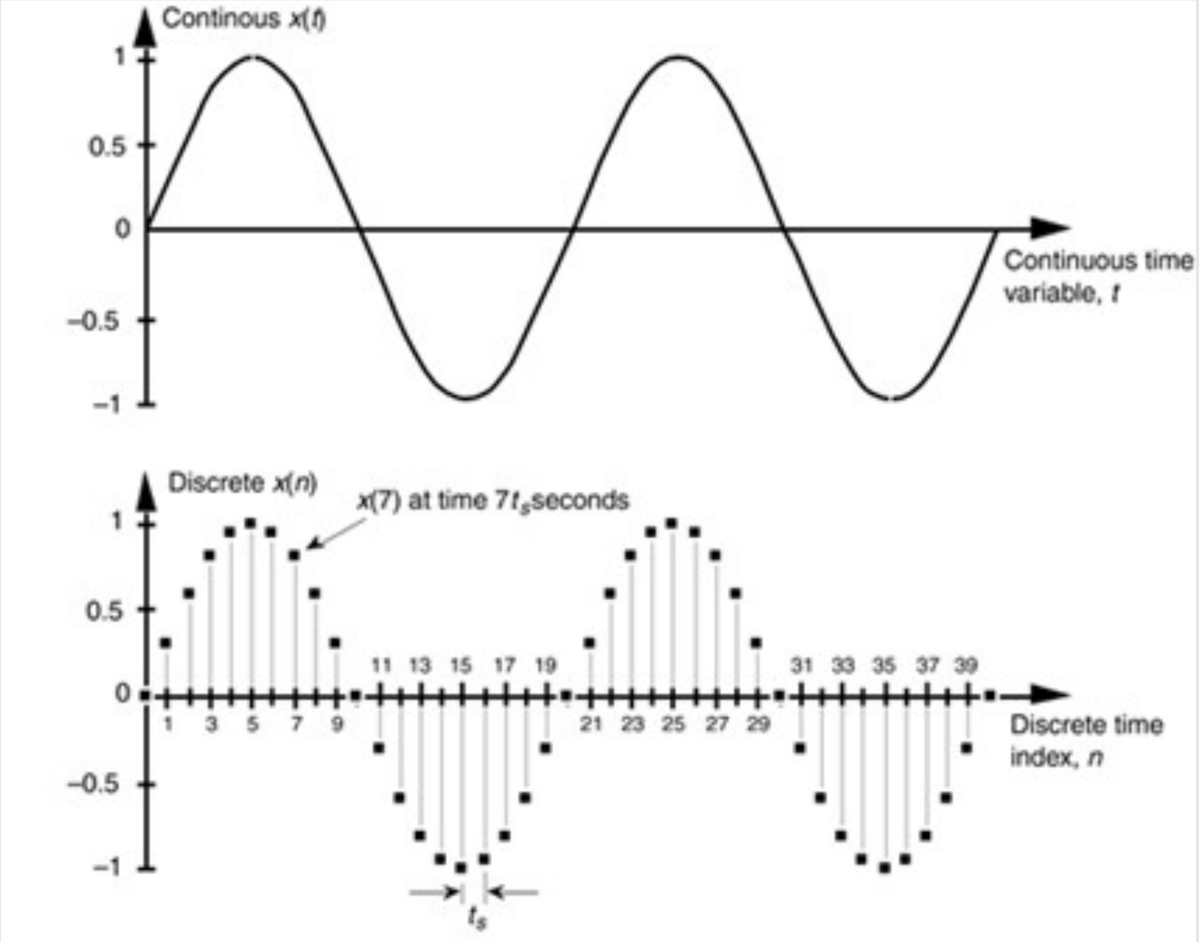
References

Schaum's Outline of Electric Circuits, Sixth Edition

Chapter 6 : Waveforms and signals

Chapter 17 : Fourier Method of Waveform Analysis

End of this unit



Continous x(t)
1
0.5
0
−0.5
−1
Continuous time variable, t
Discrete x(n)
x(7) at time 7t_s seconds
1
0.5
0
−0.5
−1
1 3 5 7 9
11 13 15 17 19
21 23 25 27 29
31 33 35 37 39
Discrete time index, n
t_s