Assignment (part 3)

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Question 1

EXE 3 Question 4

(5 marks)

(a) $B^T = \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 2 & -1 \end{bmatrix}$

$AB^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3 & -3 \\ 0 & 3 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^TB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 2 \\ -2 & 3 & -1 \\ 1 & 0 & 2 \end{bmatrix}$

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(b) Determinant of AB^T = (3 x 3) – (-3 x 0) = 9
Determinant of A^TB = 1(3x2—1x0) – 0(-2x2-(-1x1)) + 2(-2x0 – 3x1) = 0
Therefore AB^T is invertible but A^TB is not invertible.

Question 2

EXE 3 Question 8

(5 marks)

(a) mn+n , O(mn)

(b) Method 1 : $O(n^2) + O(n^2)$
Method 2 : $O(n^3) + O(n^2)$
Therefore method 1 is better.

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Question 3

EXE 3 Question 10

(5 marks)

(a) $e = Ce_j$

(b) $S = (Ce_j)(Me_k)^T$

2

Question 4

EXE 3 Question 26

(5 marks)

(a) True.

$|\boldsymbol{AB}|=|\boldsymbol{A}||\boldsymbol{B}|$

$|\boldsymbol{AB}| = 0$

Therefore inverse does not exist.

(b) False.

Let A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and B = $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

A – B = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ - $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

$= \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$

$|\boldsymbol{A}\text{-}\boldsymbol{B}|$= (((a-e)(d-h))-((b-f)(c-g))) = (ad-ed-ah+eh)-(bc-fc-bg+fg)

$|\boldsymbol{A}|$ = (ad) – (bc)

$|\mathrm{B}|$ = (eh) – (fg)

$|\boldsymbol{A}|$ - $|\mathbf{B}|$ = (ad) – (bc) - (eh) + (fg)

Because the answer of $|\boldsymbol{A}\text{-}\boldsymbol{B}|$ and $|\boldsymbol{A}|$ - $|\mathbf{B}|$ are not equal

Therefore False.

(c) True.

AB = $\begin{bmatrix} ae & bf \\ cg & dh \end{bmatrix}$

BA = $\begin{bmatrix} ea & fb \\ gc & hd \end{bmatrix}$

Determinant of AB = (aedh)-(bfcg)

Determinant of BA = (eahd)-(fbgc)

Because the answer of the determinant of AB and BA are equal,

Therefore True.

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Question 5

EXE 3 Question 30

(5 marks)

(a) a = 0
b = 0
c = 1
d = 0

(b) a = 1
b = 1
c = 0
d = 0

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Question 6

EXE 3 Question 35

(5 marks)

(a) $a_i = \sum_{j=i}^{n} xj\ aij$
therefore $x^T(Ax) = \sum_{i=1}^{n} xi(\sum_{j=i}^{n} xj\ aij)$
$x^TAx = \sum_{i=1}^{n} \sum_{j=1}^{n} xi\ xj\ aij$

(b) $x^TA^Tx = (Ax)^Tx$
$= x^T(Ax)$

(c) $\frac{1}{2}x^T(A^T + A)x = \frac{1}{2}x^T(A^Tx + Ax)$
$= \frac{1}{2}(x^TAx + x^TAx)$
$= x^TAx$

(d) $x\begin{pmatrix} x1 \\ x2 \end{pmatrix}$, $A\begin{bmatrix} 2 & \frac{3}{2} \\ \frac{3}{2} & 3 \end{bmatrix}$

$x^TAx = (x1 \quad x2)\begin{bmatrix} 2 & \frac{3}{2} \\ \frac{3}{2} & 3 \end{bmatrix}\begin{pmatrix} x1 \\ x2 \end{pmatrix}$
$= 2x_1^2 + 3x_1x_2 + 3x_2^2$

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Question 7

EXE 3 Question 38

(15 marks)

(a) x_i=1, others equals to 0
multiply the vector to matrix one by one, therefore using x^TAAAAAAA rather than A^l

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(b) By $x^T A^m$, m equals to the number of nodes.

Question 8

EXE 3 Question 39

(15 marks)

(a) For column 1 = (1+2)/2 = 1.5
For column 2 = (2.2+1.8)/2 = 2
For column 3 = (2.8+2.2)/2 = 2.5
For column 4 = (4+4)/2 = 4

Therefore the mean vector for this data is $\begin{pmatrix} 1.5 \\ 2 \\ 2.5 \\ 4 \end{pmatrix}$

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Mean vector should be 2x1

(b) Covariance vector = $\begin{bmatrix} \frac{1}{4} & -0.1 & -0.15 & 0 \\ -0.1 & 0.04 & 0.06 & 0 \\ -0.15 & 0.06 & 0.09 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(c) Eigenvalues = 0 and $\frac{19}{50}$

Eigenvectors = $\begin{pmatrix} \frac{2}{5}x2 + \frac{3}{5}x3 \\ x2 \\ x3 \\ x4 \end{pmatrix}$ and $\begin{pmatrix} \frac{-5}{3}x3 \\ \frac{2}{3}x3 \\ x3 \\ 0 \end{pmatrix}$

(d) =

(e)

(f)