CITY UNIVERSITY OF HONG KONG

Course code and title : MA1201 Calculus and Basic Linear Algebra II

Session : Semester A, 2016/2017

Time allowed : Three hours

This paper has **THREE** pages (including this cover page and the attached table).

Instructions to candidates:

1. Attempt all **SIX** questions in this paper.
2. Start each question on a new page.
3. Show all steps clearly in order to get full credits.

---

*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable calculators

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorised materials or aids are found on them.

NOT TO BE TAKEN AWAY

Question 1

Compute the following elementary integrals.

(a) $\int_{-1}^{2} \sqrt{3x+5}\, dx$ [4]

(b) $\int \frac{4x-1}{(2x+3)^2}\, dx$ [5]

(c) $\int \frac{3x-2}{x^2+4x+13}\, dx$ [6]

Question 2

Evaluate the following indefinite integrals.

(a) $\int \frac{1}{(x^2+4)^2}\, dx$ [7]

(b) $\int \sin^{-1} x\, dx$ [5]

(c) $\int \frac{-13x-14}{(x+2)^3(x-1)}\, dx$ [8]

Question 3

(a) Find volume of the solid generated by revolving the region bounded by the two parabolas $y=2x^2$ and $y=x^2+4$ about the y-axis. [7]

(b) Calculate the arc length of the segment of the parabola $y=x^2$, $0 \le x \le 1$. [8]

Question 4

(a) Given $\vec{a}=\vec{i}-2\vec{j}+3\vec{k}$, $\vec{b}=-3\vec{i}+\vec{j}+2\vec{k}$ and $\vec{c}=2\vec{i}+3\vec{j}-\vec{k}$, find the volume of the parallelepiped with $\vec{a}$, $\vec{b}$ and $\vec{c}$ as three adjacent sides. [6]

(b) Convert the complex number $\frac{3-2i}{-2+i}$ into the Cartesian form. [4]

(c) Given $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ and $B=\begin{pmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{pmatrix}$, compute the matrix product AB^T if it is compatible; otherwise state why it doesn't exist. [5]

Question 5

(a) Use the vector approach to find the shortest distance from a point $P(1,2,3)$ to the plane containing $A(1,-1,2)$, $B(-2,0,3)$ and $C(0,-1,-2)$. [10]

(b) Solve the complex equation $z^4+1=i$ and list all possible solutions in Euler's form with principal arguments. [10]

Question 6

Given a system of linear equations as follows.

$$\begin{aligned} -x - 2y + 3z + 4w &= 5 \\ 2x + 3y - 4z - 5w &= -6 \\ -3x - 8y + 13z + 18w &= 23 \end{aligned}$$

(a) Solve the above linear system by the Gaussian elimination and express the solution in vector form. [11]

(b) Write down the associated homogeneous system and provide the largest possible set of linearly independent solutions to the homogeneous system from (a) without resolving it. [4]

Brief Table of Integrals

$\int x^p\,dx = \frac{x^{p+1}}{p+1} + C, \quad p \neq -1$	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$
$\int e^x\,dx = e^x + C$	$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln\lvert \sec x + \tan x\rvert + C$
$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$
$\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert + C$	$\int \csc x\,dx = -\ln\lvert \csc x + \cot x\rvert + C$
$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$	$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

- END -