EE1002 Principles of Electronic Engineering Test 1

Part I: Multiple choice (15 questions, 3 points per question, total 45 minutes)

1. If $z = \frac{a+4j}{2-3j}$ is a real number, where $a \in R$, find a.

Answer: a = -8/3

2. If $z = \left(\frac{1+j}{1-j}\right)^4$, find z.

Answer: 1

3. If $y = \sqrt{3 - \cos^2 x}$, then find the derivative of y.

Answer: $\frac{\cos x \sin x}{\sqrt{3-\cos^2 x}}$

4. If $\sin(x) + \cos(x) = \frac{1}{5}$ $\left(-\frac{\pi}{4} \leq x < 0\right)$, find $z = \cos^2(x)$.

Answer: 16/25

5. If $\sin(\alpha) + \sin(\beta) = 1, \cos(\alpha) - \cos(\beta) = \frac{1}{2}$, find $\cos(\alpha + \beta)$

Answer: 3/8

6. Evaluate $\int (1 - \sin^2 \frac{x}{2}) dx$

Answer: $\frac{1}{2}(x + \sin x) + c$

7. Evaluate $\int \frac{x}{x^4-1} dx$

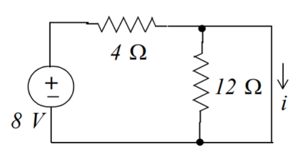
[Hint: $\frac{1}{y^2-1} = \frac{1}{2}\left(\frac{1}{y-1} - \frac{1}{y+1}\right)$]

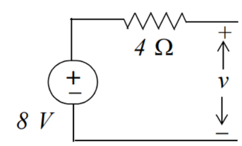
Answer: $\frac{1}{4} \ln \left|\frac{x^2-1}{x^2+1}\right| + c$

8. Find the general solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 0$

Answer: $y = Ae^{-4x} + Be^{3x}$

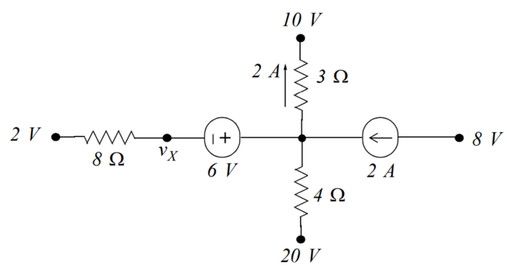
9. For the following circuits, find the value of the current i and the value of the voltage v.





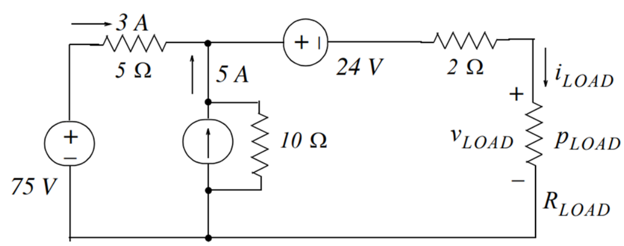
Answer: $i = 2\ A, v = 8\ V$.

10. Find the node voltage v_x of the following circuit.



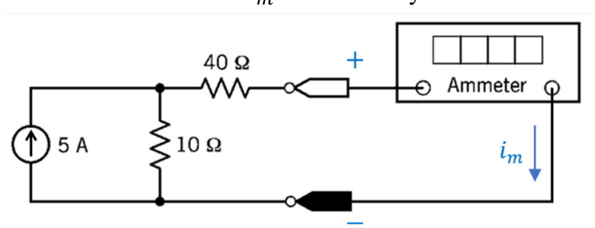
Answer: $10\ V$

11. In the following circuit, R_{LOAD} is the load. Find the current i_{LOAD}, voltage v_{LOAD}, and power p_{LOAD} of R_{LOAD}.



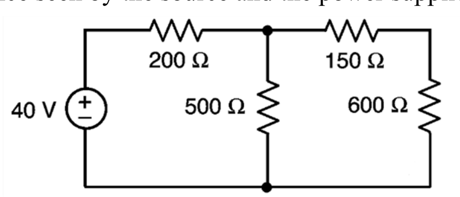
Answer: $i_{LOAD} = 8\ A, v_{LOAD} = 20\ V,\ p_{LOAD} = 160\ W.$

12. Determine the current i_m measured by the actual ammeter (**Not ideal**) in the following circuit.



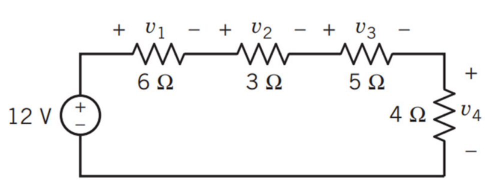
Answer: $i_m < 1\ A$

13. Find the equivalent resistance seen by the source and the power supplied by the source.



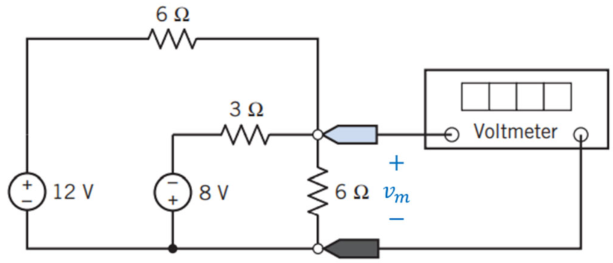
Answer: $R_{eq} = 500\ \Omega, P = 3.2\ W.$

14. Determine the voltages v_1, v_2, v_3, and v_4 in the following circuit.



Answer: $v_1 = 4\ \text{V}, v_2 = 2\ V, v_3 = \frac{10}{3}\ \text{V}, v_4 = \frac{8}{3}\ V.$

15. Determine the value of the voltage measured by the ideal voltmeter v_m.



Answer: $v_m = -1\,V$

Part II: Written Questions (2 questions, 25 points per question, total 40 minutes and 6 minutes per question)

Question 1

(a) Find the derivative of y.

$$y = \frac{1}{2}\ln(\mathrm{e}^{cos4x} + 1)$$

(b) Evaluate the following integrals.

1) $\int \frac{\ln(\tan x)}{\cos x \sin x} dx$

2) $\int \frac{1+\ln x}{(x\ln x)^2} dx$

Answer:

a) $\frac{-2\sin(4x)\mathrm{e}^{cos4x}}{\mathrm{e}^{cos4x}+1}$

b)

1) $\frac{1}{2}(\ln\tan x)^2 + C$

$$\int \frac{\ln\tan x}{\cos x \sin x} dx = \int \frac{\ln\tan x}{\cos^2 x \tan x} dx = \int \frac{\ln\tan x}{\tan x}\sec^2 x\, dx = \int \frac{\ln\tan x}{\tan x} d(\tan x)$$
$$= \int \ln\tan x\, d(\ln\tan x) = \frac{1}{2}(\ln\tan x)^2 + C$$

2) $-\frac{1}{x\ln x} + C$

$$\int \frac{1+\ln x}{(x\ln x)^2} dx = \int \frac{1}{(x\ln x)^2} d(x\ln x) = -\frac{1}{x\ln x} + C$$

(c) Find the general solution of the equation by using the integrating factor method:

$$x^2\frac{dy}{dx} - y = \frac{x^2}{(1-e^x)e^{\frac{1}{x}}}$$

Answer:

$$y = e^{-1/x}\left(-\ln|e^{-x} - 1| + C\right)$$

$$\because \frac{dy}{dx} - \frac{y}{x^2} = \frac{1}{(1-e^x)e^{\frac{1}{x}}}$$

$$\therefore p(x) = -\frac{1}{x^2}, Q(x) = \frac{1}{(1-e^x)e^{\frac{1}{x}}}$$

$IF = e^{P(x)}$, where $P(x) = \int p(x)\,dx = \int -\frac{1}{x^2}dx = \frac{1}{x}$

$$\therefore IF = e^{\frac{1}{x}}$$

Multiplying both sides of the original equation by the integration factor gives:

$$e^{\frac{1}{x}}\frac{dy}{dx} - e^{\frac{1}{x}}\frac{y}{x^2} = e^{\frac{1}{x}} \cdot \frac{1}{(1-e^x)e^{\frac{1}{x}}} = \frac{1}{1-e^x}$$

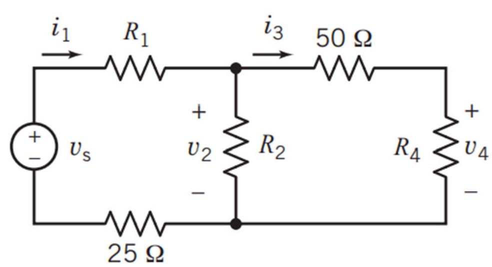
$$\therefore ye^{\frac{1}{x}} = \int \frac{dx}{1-e^x} = \int \frac{e^{-x}dx}{e^{-x}-1} = -\int \frac{1}{e^{-x}-1}d(e^{-x}) = -\int \frac{1}{e^{-x}-1}d(e^{-x}-1)$$

$$\therefore ye^{\frac{1}{x}} = -\ln|e^{-x}-1| + C$$

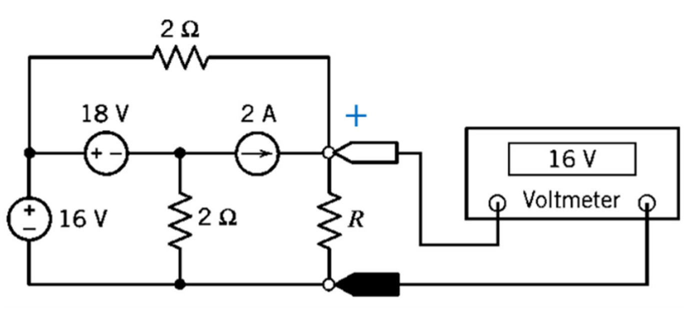
$$\therefore y = e^{-1/x}(-\ln|e^{-x}-1| + C)$$

Question 2

(a) Consider the following circuit. Given $v_2 = \frac{1}{3}v_s$, $i_3 = \frac{1}{5}i_1$, and $v_4 = \frac{3}{8}v_2$, determine the values of R_1, R_2, and R_4.



(b) Consider the circuit shown in the following figure (Ideal Voltmeter). Find the value of the resistance R.



Answer:

a) $R_1 = 7\ \Omega, R_2 = 20\ \Omega, R_4 = 30\ \Omega$

Based on Voltage dividing method:

$$v_2 = \frac{1}{3}v_s \rightarrow (50\ \Omega + R_4) \parallel R_2 = \frac{R_2 + 25\ \Omega}{2}$$

$$v_4 = \frac{3}{8}v_2 \rightarrow \frac{R_4}{R_4 + 50\ \Omega} = \frac{3}{8} \rightarrow R_4 = 30\ \Omega$$

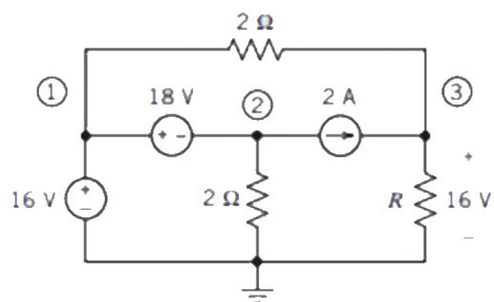
Based on the current dividing method:

$$i_3 = \frac{1}{5}i_1 \rightarrow \frac{R_2}{R_4 + 50\ \Omega} = \frac{1}{4} \rightarrow R_2 = 20\ \Omega$$

Then

$$R_1 = 7\ \Omega$$

b) $8\ \Omega$



Apply KVL,

$$16 = v_1 - 0 \rightarrow v_1 = 16\ V$$

$$18 = v_1 - v_2 \rightarrow 18 = 16 - v_2 \rightarrow v_2 = -2\ V$$

$$v_3 = 16\ V$$

Apply KCL at node 3

$$\frac{v_1 - v_3}{2} + 2 = \frac{v_3}{R}$$

Then

$$R = 8\ \Omega$$