EE 1001 Assignment 4

Q1. Write the first five terms of the sequences with nth term $a_n = (-1)^{n-1} 5^{n+1}$

Solution Q1:

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$
$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$
$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$
$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$
$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$

Therefore, the required terms are 25, –125, 625, –3125, and 15625.

Q2.

(1) 8 members are to be selected from a group of 9 males and 7 females. In how many ways will the members with at most 3 females and at least 4 males be selected?

(2) How many three letter words (with or without meaning) can be formed using the letters of the word "PRACTICES"?

(3) A postmaster wants to get delivered 6 letters at six different addresses. In the post office there are 2 postmen then in how many ways can the postmaster send the letters at different addresses through the postmen?

(4) Five people out of whom only two can drive are to be seated in a five-seater car with two seats in front and three in the rear. The people who know driving don't sit together. Only someone who knows driving can sit on the driver's seat. Find the number of ways the five people can be seated.

Solution Q2:

(1) These 8 members can be formed by:
5 males and 3 females: ${}_9C_5 * {}_7C_3 = 4410$;
6 males and 2 females: ${}_9C_6 \times {}_7C_2 = 1764$;
7 males and 1 female: ${}_9C_7 \times {}_7C_1 = 252$;
8 males: ${}_9C_8 = 9$;
Total number of ways = 4410 + 1764 + 252 + 9 = 6435

(2) "C" exists twice.
For the word contains 1 "C", the number of words are ${}_8P_3 = 336$;
For the word contains 2 "p", the number of words are: ${}_7C_1 * 3!/(2!) = 21$
So, there are total 336+21=357 possible words.

(3) Each letter can be delivered at the six different addresses in 2 different ways
Hence, the required number of ways = 26 = 64

(4) Number of ways of selecting driver = ${}_2C_1$;
The other person seat in the front can only be from the non-driving people, which is ${}_3C_1$

who knows driving can be seated only in the rear three seats in 3 ways

Number of ways of seating remaining = ${}_3P_3$

So, there are total ${}_2C_1$ * ${}_3C_1$ * ${}_3P_3$ = 36

Q3. Given m, n are positive integers, $f(x) = (1+x)^m+(1+x)^n$. It is known that the coefficients of the terms x and x^2 are 7 and 9 respectively. Compute:

(1) The values of *m* and *n*;

(2) The coefficient of the term x^3;

(3) Use the binomial theorem to compute $(1.01)^4$

Solution Q3:

(1) With the binomial theorem, we have: m+n=7; ${}_mC_2+{}_nC_2=(m^2+n^2-m-n)/2=9$;

It is computed: m=4, n=3, or m=3, n=4;

(2) No matter m=4, n=3, or m=3, n=4; $f(x) = (1+x)^3+(1+x)^4$;

Thus, the coefficient of the term x3 is ${}_3C_3+{}_4C_3 = 5$;

(3) $(1.01)^4=(1+0.01)^4={}_4C_0(1)^4(0.01)^0 + {}_4C_1(1)^3(0.01)^1 + {}_4C_2(1)^2(0.01)^2 + {}_4C_3(1)^1(0.01)^3 + {}_4C_1(1)^0(0.01)^4$

$= 1+ 0.04+0.0006+0.000004+0.00000001=1.04060401$

Q4. Let A= {2, 3, 5, 6, 7, 9}; B = {3, 6, 9}, and C = {2, 4, 5, 6, 8}. Find each of the following:

(1) A ∪ B

(2) A ∩ B

(3) A ∪ C

(4) A ∩ C

(5) A - B

(6) B - A

(7) B ∪ C

(8) B ∩ C

Solution Q4:

(1) {2, 3, 5, 6, 7, 9}

(2) {3, 6, 9}

(3) {2, 3, 4, 5, 6, 7, 8, 9}

(4) {2, 5, 6}

(5) {2 ,5, 7}

(6) $\varnothing$

(7) {2, 3, 4, 5, 6, 8, 9}

(8) {6}

Q5. A large software development company employs 100 computer programmers. Amongst them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and

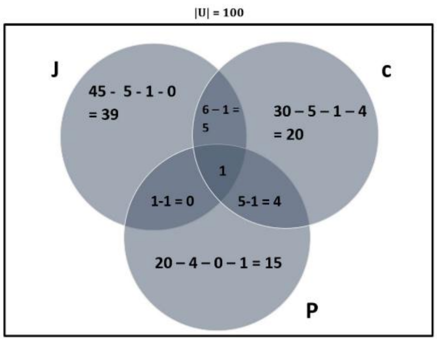
Python, five in C# and Python, and just one programmer is proficient in all three languages above.
Determine the number of computer programmers that are not proficient in any of these three languages.

Solution Q5:

Let U denotes the set of all employed computer programmers and let J, C and P denotes respectively the set of programmers proficient in Java, C# and Python, respectively. Thus:

$$|U| = 100 \quad |J| = 45 \quad |C| = 30 \quad |P| = 20$$
$$|J \cap C| = 6 \quad |J \cap P| = 1 \quad |C \cap P| = 5 \quad |J \cap C \cap P| = 1$$

With Venn diagram, it is easy to obtain:



we need to determine the complement of the set $J \cup C \cup P$.

Calculate $|J \cup C \cup P|$ first before determining the complement value:

$|J \cup C \cup P| = 39 + 5 + 20 + 4 + 15 + 1 = 84$

Now calculate the complement: $|(J \cup C \cup P)'| = |U| - |J \cup C \cup P| = 100 - 84 = 16$

16 programmers are not proficient in any of the three languages.

Q6.

6-1) A drawer contains 12 red and 12 blue socks, all unmatched. A person takes socks out at random in the dark. How many socks must be taken out to ensure that he has at least two blue socks?

6-2) Three students are running for a student government. There are 202 students voting, what is the minimum number of votes required to win the election?

6-3) Three students are running for a student government. There are 202 students voting, what is the minimum number of votes required to ensure the winning of the election?

Solution Q6:

(1) Given 12 red and 12 blue socks so, in order to take out at least 2 blue socks, first we need to take out 12 shocks (which might end up red in worst case) and then take out 2 socks (which would be definitely blue). Thus, we need to take out total 14 socks.

(2) By pigeonhole, there exists a person who has gotten at least set $\lceil 202/3 \rceil = 68$ votes. So, someone could win with a $67 - 67 - 68$ split.

(3) To ensure the wining, the one need more than 50% vote, which is 202*50% +1=102