Name:

Total: 100 points. Time: 60 min.

1. (25 points) Given that the equation of a conic section is

$$x^2 + 4y^2 - 24y + 20 = 0.$$

a) Using completing square, identify the type of the conic section;
b) Hence, sketch the graph of this conic section, including foci, center and vertices.

2. (20 points) Find the largest possible domains and the ranges of the following functions:

a)

$$f_1(x) = \ln(1 - x^2);$$

b)

$$f_2(x) = \frac{1}{10^x - 100}.$$

3. (20 points) Does the inverse of $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = 2 + 10^x$ exist? Why? Find the inverse if it exists.

4. (20 points) Resolve into partial fractions

$$\frac{x^3 - 2x^2 - 3x - 5}{x^3 - 1}.$$

5. (15 points) Prove the identity

$$4 \sin A \sin\left(\frac{2\pi}{3} + A\right) \sin\left(\frac{2\pi}{3} - A\right) = \sin 3A.$$

End of Test

Question 1

Show that if $f(x)$ and $g(x)$ are odd functions of x, then $(f+g)(x)$ is an odd function of x. (10 marks)

Question 2

Find the centre and radius of the circle $x^2+y^2-4x-5=0$. (10 marks)

Question 3

A straight line passes through the point (-5, 2) and has equal x-intercept and y-intercept. Find the equation of the straight line.

(10 marks)

Question 4

Let $F(x)$ and $G(x)$ be two functions defined by

$F(x)=\dfrac{x^2-1}{x^2-9}$,

$G(x)=x+1$.

(a) Find their largest possible domains.

(b) Find $(\frac{F}{G})(x)$ and state its largest possible domain (15 marks)

Question 5

Let $h(x)=\dfrac{x^2-2x-2}{x+1}$, $x\in R\backslash\{-1\}$.

(a) Show that the function $h(x)$ has no value between -6 and -2.
(Hint: Let $y=h(x)$.)

(b) Determine the largest possible range of $h(x)$. (15 marks)

Question 6

If the equation of an ellipse is $\dfrac{(x-1)^2}{9}+\dfrac{(y-2)^2}{5}=1$, find the coordinates of its centre, vertices and foci. Sketch its graph.

(20 marks)

Question 7

Express $\dfrac{2x^2+17}{(x+3)(x^2+2x+2)}$ in partial fractions. (20 marks)

- END -

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Test 2.

Question 1

(a) It is given that $\sin A = \frac{4}{5}$ and $\cos B = \frac{5}{13}$. Without using a calculator, show that one possible value of $\sin(A+B)$ is $\frac{56}{65}$, and find all the other possible values.

(Hint: $\sin(A+B) = \sin A \cos B + \cos A \sin B$)

(b) (i) Prove that $\tan x + \cot x = 2\operatorname{cosec} 2x$.

(ii) Find the general solution, in radians, of the equation $1 + 2\operatorname{cosec} 2x = \cot x$. (20 marks)

Question 2

(a) Without the use of De L'Hôpital rule, prove that, for all positive rational number n,

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}.$$

(Hint: Consider two cases.

1. For n be a positive integer and $n>1$, we have
$x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \ldots + a^r x^{n-r-1} + \ldots + a^{n-1})$.
2. For $n = \frac{p}{q}$, where p and q are positive integers, and let $y = x^{\frac{1}{q}}$ and $b = a^{\frac{1}{q}}$.)

(b) Evaluate the limit $\lim_{x \to 0} \frac{\sin(2x)}{\sin\left(\frac{x}{2}\right)}$.

(20 marks)

Question 3

(a) Let

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1}, & \text{if } x \neq 1 \\ c, & \text{if } x = 1 \end{cases}.$$

Find the value of c for which $f(x)$ is continuous at $x = 1$. Give your reason.

(b) Let

$$g(x) = |\tan x|, \text{ for } x \in \left[-\frac{\pi}{3}, \frac{\pi}{3}\right].$$

Determine whether $g(x)$ is differentiable at $x = 0$. Give your reason.

(20 marks)

Question 4

(a) Given that $\cosh x = \frac{1}{2}(e^x + e^{-x})$ and $\sinh x = \frac{1}{2}(e^x - e^{-x})$, prove that

(i) $\cosh^2 x - \sinh^2 x = 1$,

(ii) the inverse of $\sinh x = \sinh^{-1} x = \log_e\left(x + \sqrt{1+x^2}\right)$.

(b) Solve the equation $\cosh^2 x - 2\sinh x = 0$, giving your answer as natural logarithms.

(20 marks)

Question 5

(a) Solve the equation $2\log_{10} x = 1 + \log_{10}\left(\frac{2(2x+5)}{5}\right)$.

(b) The functions F and G are defined by
$F(x) = \log_e(1+x)$, for $x \in R^+$,
$G(x) = e^{-x}$, for $x \in R^+$.

(i) Give the ranges of $F(x)$ and $G(x)$.

(ii) Give definitions of the inverse functions $F^{-1}(x)$ and $G^{-1}(x)$ in a form similar to the above definitions.

(20 marks)

- END -