EE1001
Foundations of Digital Techniques

Logic

Assignment #3

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Please submit assignment #3 on or before 28 March 2021 Sunday, 23:59

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Foundations of Digital Techniques

Logic

Assignment 3

Validity and Soundness of Argument

Propositional Logic

Conditionals

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Q1

- Q1)
- A = {4, 14, 66, 70}, x ∈ A such that x is an odd number. Determine whether the statement is T or F.

- Solution Q1)
- Consider A = {4, 14, 66, 70}. Let p: ∃ x ∈ A such that x is an odd number. Here, the statement p uses the quantifier 'there exists' (∃). This statement is true if at least one element of set A satisfies the condition 'x is an odd number' and is false otherwise. Here, the given statement is false as none of the elements of set A satisfy the condition, 'x ∈ A such that x is an odd number'.

Q2

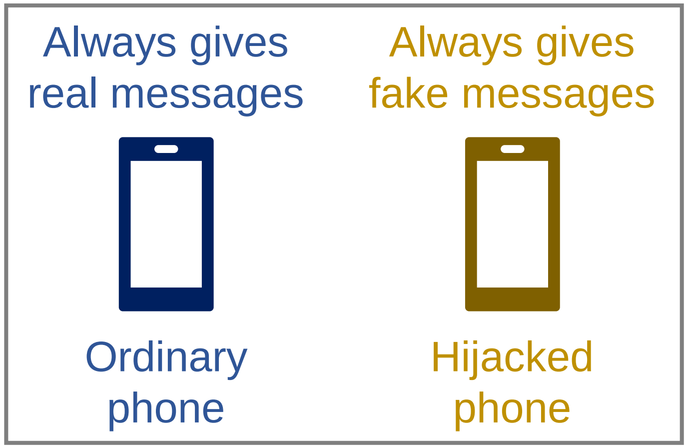
- Q2)
- A = {1, 2, 3}, p: $\forall$ x $\in$ A, x < 4. Determine whether the statement is T or F.
-
- Solution Q2)
- A = {1, 2, 3} Let p: $\forall$ x $\in$ A, x < 4 Here, the statement p uses the quantifier 'for all'($\forall$). This statement is true if and only if each and every element of set A satisfies the condition 'x < 4' and is false otherwise. Here, the given statement is true for all the elements of set A, as 1, 2, 3 satisfy the condition, 'x $\in$ A, x < 4'.

Q3

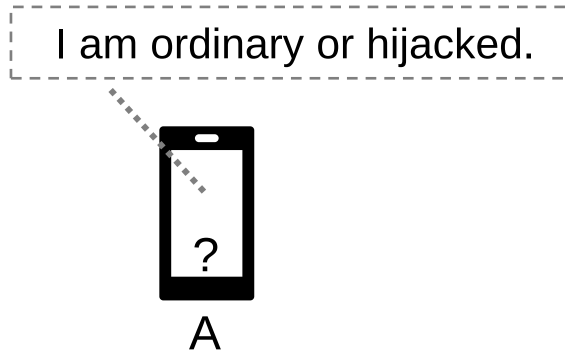
- Q3)
- Write the negations of following statements.
- i. $\forall$ n $\in$ N, n + 1 > 2.
- ii. $\forall$ x $\in$ N, x 2 + x is even number.
- solution
- i. $\exists$ x $\in$ N, such that x 2 + x is not an even number.
- ii. $\forall$ n $\in$ N, n2 $\neq$ n.

Q4

Q4. There are two types of phones:



The Message given by A:



Question:
Is A ordinary or hijacked? Use truth table to justify.

Ans:

Let p = "A is an ordinary phone"
The statement "A is ordinary or hijacked" can be formulated as **"**$p \vee \sim p$**"**
The condition is satisfied only when the truth value of the phone and the truth value of the phone's message are **the same** (= **if and only if (iff)**), i.e.,

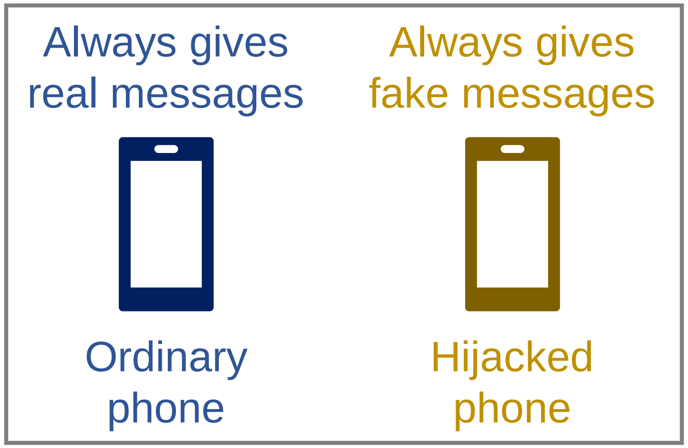
$$\boldsymbol{p \leftrightarrow (p \vee \sim p) = \text{True}}$$

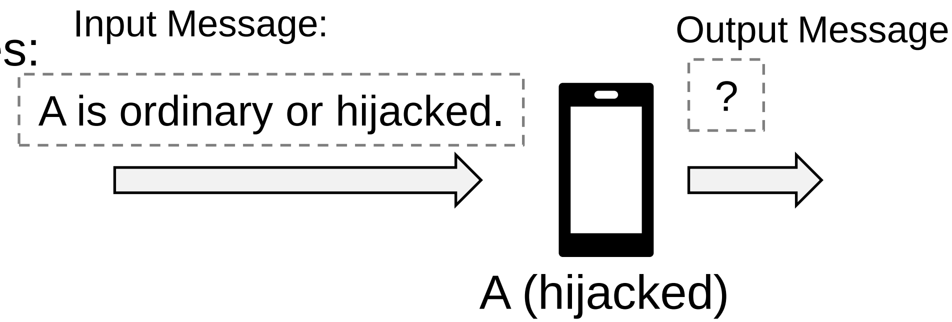
p	$\sim p$	$p \vee \sim p$	$p \leftrightarrow \sim p$
T	F	T	**T**
F	T	T	F

∴ A is an ordinary phone.

Q5

Q5. There are two types of phones:





Question:
A is a hijacked phone, and my input message is "A is ordinary or hijacked".
<u>What message will A output</u>?

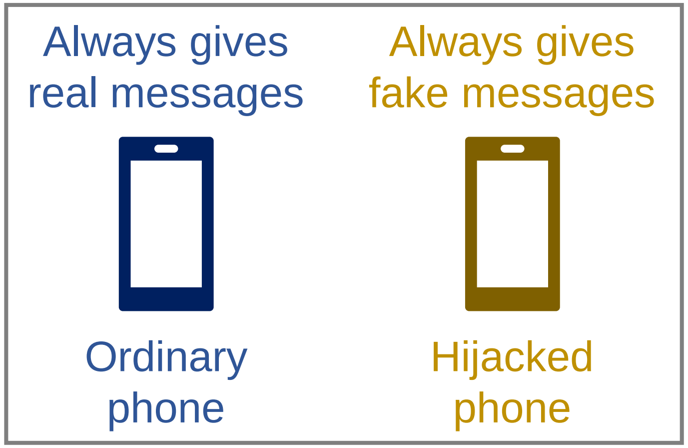
Ans:

Let p = "A is an ordinary phone"
The statement "A is ordinary or hijacked" can be formulated as **"**$p \vee \sim p$**"**
Given A is a hijacked phone, its output message should be the <u>**negation**</u> of the input message, i.e.,

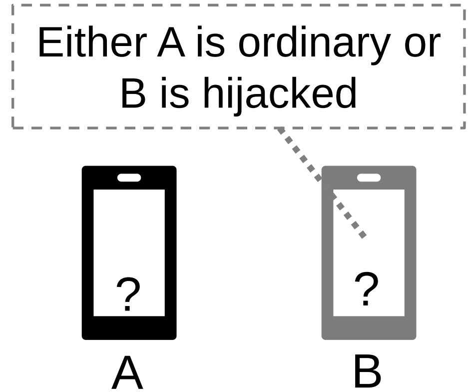
Output message $= \sim(p \vee \sim p)$
$= \sim p \wedge \sim(\sim p)$ (De Morgan's laws)
$= \sim p \wedge p$
= A is hijacked and ordinary

Q6

Q6. There are two types of phones:



The Message given by B:



Question:
Are A and B ordinary or hijacked? Use truth table to justify.

Ans:
The more systematic formulation of "either A or B" is "**Exclusive Or (XOR)**"
<u>XOR means either A or B is true, but not both.</u>

<u>Truth Table of XOR</u>

A	B	A ⊕ B
T	T	F
T	F	T
F	T	T
F	F	F

Let p = "A is an ordinary phone", and q = "B is an ordinary phone".
Therefore, the statement "either A is ordinary or B is hijacked" can be formulated as **"$p \oplus \sim q$"**
The condition is satisfied only when the <u>truth value of the phone</u> and <u>the truth value of the phone's message</u> are <u>**the same**</u> (= **if and only if (iff)**), i.e.,

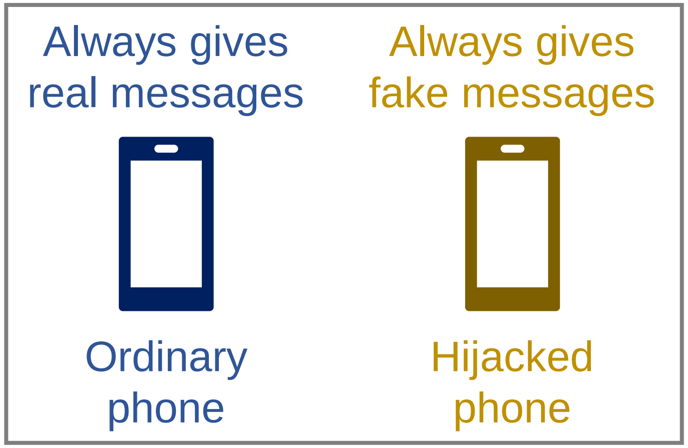
$$q \leftrightarrow (p \oplus \sim q) = \text{True}$$

p	q	$\sim q$	$p \oplus \sim q$	$q \leftrightarrow (p \oplus \sim q)$
T	T	F	T	**T**
T	F	T	F	**T**
F	T	F	F	F
F	F	T	T	F

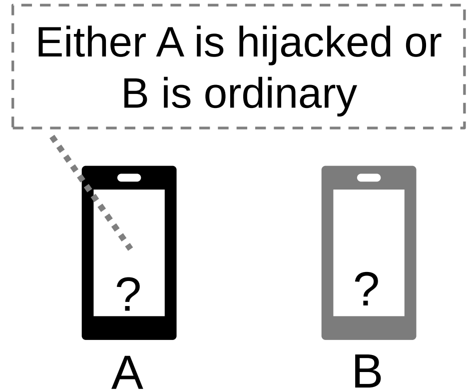
<u>∴ Two possible solutions: (1) "A and B are ordinary phones"; (2) "A is ordinary and B is hijacked."</u>

Q7

Q7. There are two types of phones:



The Message given by A:



Question:
Are A and B ordinary or hijacked? Use truth table to justify.

Ans:

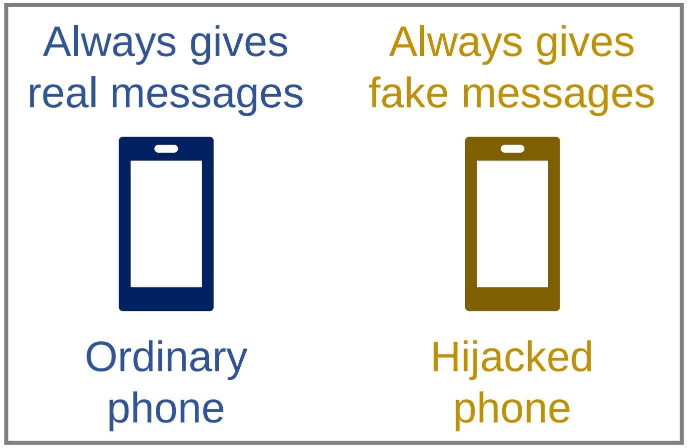
Let p = "A is an ordinary phone", and q = "B is an ordinary phone".
Therefore, the statement "either A is hijacked or B is ordinary" can be formulated as **"$\sim p \oplus q$"**
The condition is satisfied only when **$p \leftrightarrow (\sim p \oplus q)$ = True**

p	q	$\sim p$	$\sim p \oplus q$	$p \leftrightarrow (\sim p \oplus q)$
T	T	F	T	**T**
T	F	F	F	F
F	T	T	F	**T**
F	F	T	T	F

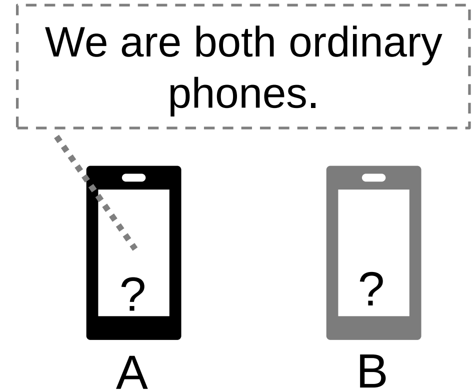
∴ Two possible solutions: (1) "A and B are ordinary phones"; (2) "A is hijacked and B is ordinary."

Q8

Q8. There are two types of phones:



The Message given by A:



Question:
Are A and B ordinary or hijacked? Use truth table to justify.

Ans:
Let p = "A is an ordinary phone", and q = "B is an ordinary phone".
Therefore, the statement "A and B are ordinary" can be formulated as **"**$p \land q$**"**
The condition is satisfied only when $\boldsymbol{p \leftrightarrow (p \land q)}$ **= True**

p	q	$p \land q$	$p \leftrightarrow (p \land q)$
T	T	T	**T**
T	F	F	F
F	T	F	**T**
F	F	F	**T**

∴ Three possible solutions

- \- END --