Tutorial I _Counting_Q&A

Q1. A basketball team have four competitions with other teams in a month. Each match has three possible results for the NSW team — win, lose or draw. How many different sequences of results are possible in a month for the team?

Solution:

There are three possible results for each competition. Hence for two matches, there are 3^2 possible results. Continuing in this way we see that there are $3^4 = 81$ different sequences possible for the month.

Q2. Computer the result of each term below:

(1) ${}_6C_3$ **; (2)** ${}_6P_3$ **; (3)** ${}_{10}C_3$ **; (4)** ${}_6P_3$

Solution:

(1) $${}_6C_3 = \frac{6!}{3!3!} = 20$$

(2) $${}_6P_3 = \frac{6!}{3!} = 120$$

(3) $${}_{10}C_3 = \frac{10!}{7!3!} = 120$$

(4) $${}_{10}P_3 = \frac{10!}{7!} = 720$$

Q3. Find the number of words, with or without meaning, that can be formed with the letters of the following word

(1) 'CHAIR'

(2) "INDIA"

(3) "SWIMMING"

Solution:

(1) "CHAIR' contains 5 letters. Therefore, the number of words that can be formed with these 5 letters is ${}_5P_5 = 5! = 120$;

(2) The word 'INDIA' contains 5 letters and 'I' comes twice. Then, this is to find the distinguishable permutations. The number of words formed by 'INDIA' = 5!/2! = 60

(3) The word 'SWIMMING' contains 8 letters and both of 'I' and "M" comes twice. Then, this is to find the distinguishable permutations. The number of words formed by 'SWIMMING' = 8!/(2! *2!) = 10080

Q4. Considering 5 boys and 3 girls. Find the number of permutations of these 8 children that girls cannot adjacent. (e.g. "boy, girl, boy, girl")

Solution:

In this question, we may need find the permutation of boys first and then insert the girls into the sequenced boys.

For 5 boys, the number of permutations is ${}_5P_5 = 5! = 120$.
Among the five boys, there are 6 intervals shown below:

Boy Boy Boy Boy Boy
$\Delta \quad \Delta \quad \Delta \quad \Delta \quad \Delta \quad \Delta$

Since the girls are not allowed to adjacent. One way to achieve this is insert the three girls in to the six intervals with order.
Then, the number of way to insert 3 girls in to 6 intervals is: ${}_6P_3 = 6!/3! = 120$.
Therefore, total number of permutations possible = 120*120=14400

Q5. Among a set of 5 black balls and 3 red balls, how many selections of 5 balls can be made such that at least 3 of them are black balls.

Solution:
Selecting at least 3 black balls from a set of 5 black balls in a total selection of 5 balls can be
3 B and 2 R
4 B and 1 R and
5 B and 0 R balls.
Therefore, the solution expression is: ${}_5C_3 * {}_3C_2 + {}_5C_4 * {}_3C_1 + {}_5C_5 * {}_3C_0 = 46$

Q6. Use the binomial theorem to expand $(2x+3y)^4$

Solution:
Step 1: determine the number of terms, there are: $(2x)^4$, $(2x)^3 3y$, $(2x)^2(3y)^2$, $(2x)^1(3y)^3$, $(3y)^4$
Step 2: determine the binomial coefficient for each term: ${}_4C_4=1$, ${}_4C_3=4$, ${}_4C_2=6$, ${}_4C_1=4$, ${}_4C_0=1$
Step 3: write the expansion:

$$(2x + 3y)^4 = (2x)^4 + 4(2x)^3 3y + 6(2x)^2(3y)^2 + (2x)^1(3y)^3 + (3y)^4$$
$$= 16x^4 + 96x^3y + 216x^2y^2 + 54xy^3 + 81y^4$$