Lecture 11

AC Circuits

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Lecture 10 Review

- The Biot-Savart Law in electromagnetism describes the magnetic field generated by an electric current.
- The law is experimentally deduced, it describes the magnitude of the field dB produced at point P at distance r by a current length element $i\,ds$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$$

- Using the Biot-Savart law, we can determine the net magnetic field due to a variety of current configurations:
  - ***B*** due to a long straight wire

$$B = \frac{\mu_0 i}{2\pi R}$$

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– ***B*** due to the circular current arc of angle ϕ

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

– ***B*** at center of a full circle of current.

$$B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}$$

- When we place two current carrying wires next to each other, each wire will encounter a force with the same magnitude that is caused by the magnetic field generated by the other wire

$$\boxed{F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}.}$$

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- Ampere's Law states that The line integral of the ***B*** field around any closed loop is proportional to the electric current passing through the area enclosed by the loop.

$$\boxed{\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}}$$

- Using the Ampere's Law and with the knowledge of the current, one can determine the magnetic field around current carrying structures such as cylinder, solenoids and toroids.
- A solenoid is just a coil or wire, however the ***B*** field in the current carrying coil is magnified by the number of turns, for the ideal solenoid,

$$B = \mu_0 i n \qquad \text{(ideal solenoid).}$$

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- A current carrying coil can be treated as a magnetic dipole that produces a dipole magnetic field similar to the electric field from an electric dipole.
- The magnetic field as a function of z, along the axial distance is

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}}$$

- One can also use the moment μ to define the strength of the magnetic dipole, where for a N current loop

$$\mu = NiA = Ni\pi R^2$$

- Thus the B field due to the magnetic dipole is $\vec{B}(z) = \dfrac{\mu_0}{2\pi}\dfrac{\vec{\mu}}{z^3}$

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- The motion of moving a magnet through a wire loop produces a current in the wire.
- The current appears only if there is relative motion between the loop and the magnet.
- The motion direction of the magnet dictates the direction of the current in the loop.
- The current produced in the loop is called induced current.
- The work done per unit charge produce that current is called and *emf*.
- The process of producing the current and emf is called ***induction.***

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Lecture 10 Review

- One can induce current and emf in a loop with the magnetic field generated by a current carrying loop placed close to the first loop.
- This process is described by the ***Faraday's Law of Induction***, which states the magnitude of the emf $\mathcal{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)},$$

- For a coil of N turns, the induced emf is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

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- An inductor (symbol  a component in an electrical circuit that can be used to produce a desired magnetic field.
- The inductance is defined as the ratio between the total magnetic flux and the current

$$L = \frac{N\Phi_B}{i} \qquad \text{(inductance defined)}$$

- Similar to resistance and capacitance, the inductance is a property of the geometry, in this case it measure the ability of the geometry to generate magnetic flux with a given current *i*.

$$N\Phi_B = Li.$$

$$\mathcal{E}_L = -L\frac{di}{dt} \qquad \text{(self-induced emf).}$$

$$\mathcal{E}_L = -\frac{d(N\Phi_B)}{dt}.$$

- The SI unit of inductance is the tesla–square meter per ampere (T m^2/A). We call this the henry (H), after American physicist Joseph Henry.
- The inductance per unit length near the center of a solenoid is

$$\frac{L}{l} = \mu_0 n^2 A \qquad \text{(solenoid).}$$

Lecture Outline

- **Chapter 31**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- AC Circuits
  - LC oscillations
  - RLC circuits
  - Simple AC circuits

31.2: LC Oscillations:

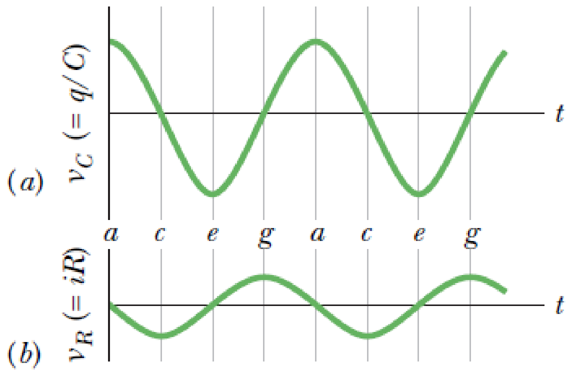


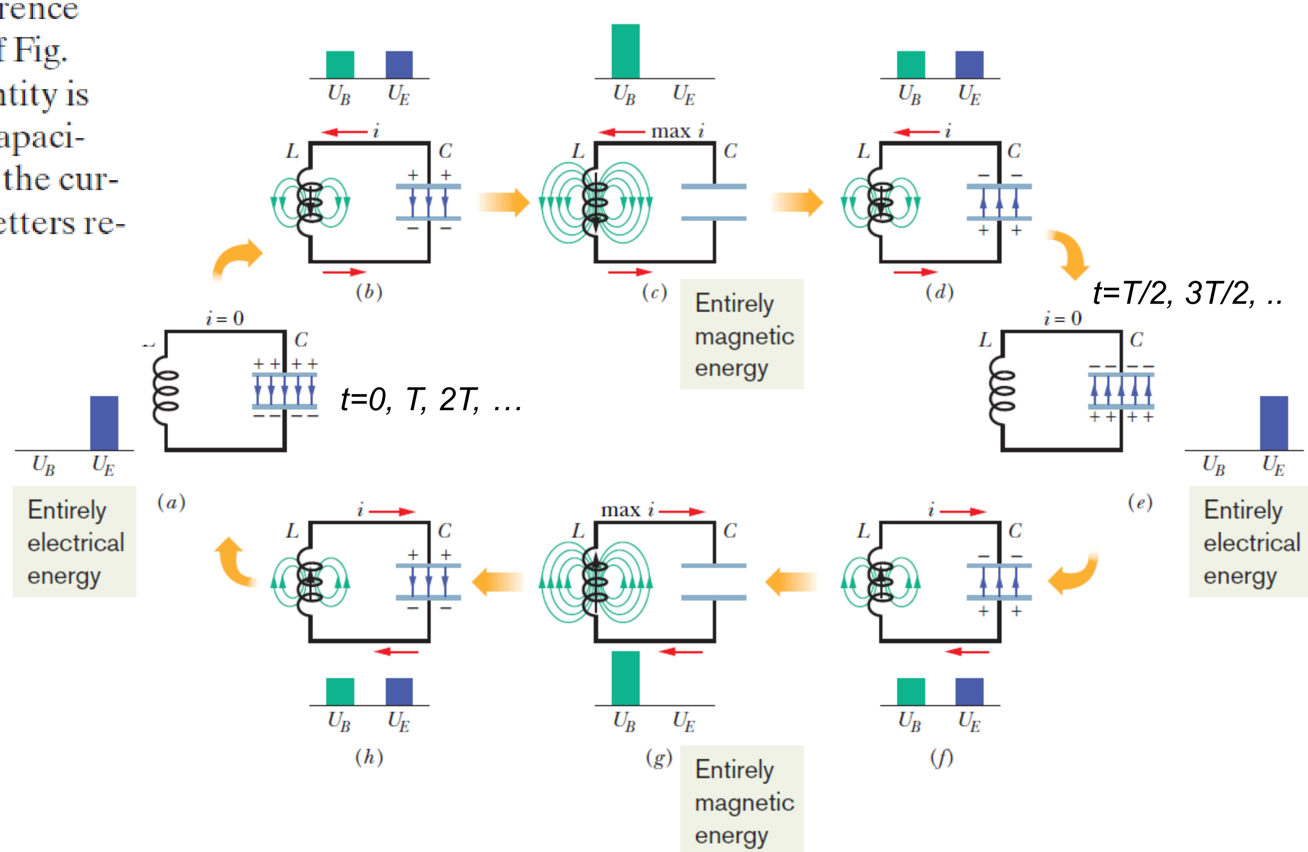
Fig. 31-2 (*a*) The potential difference across the capacitor of the circuit of Fig. 31-1 as a function of time. This quantity is proportional to the charge on the capacitor. (*b*) A potential proportional to the current in the circuit of Fig. 31-1. The letters refer to the correspondingly labeled oscillation stages in Fig. 31-1.

$$U_E = \frac{q^2}{2C},$$

Electric Energy in the Capacitor

$$U_B = \frac{Li^2}{2},$$

Magnetic Energy in the Inductor



31.3: The Electrical Mechanical Analogy:

q corresponds to x, $\quad 1/C$ corresponds to k,
i corresponds to v, and $\quad L$ corresponds to m.

Table 31-1

Comparison of the Energy in Two Oscillating Systems

Block–Spring System		LC Oscillator	
Element	Energy	Element	Energy
Spring	Potential, $\frac{1}{2}kx^2$	Capacitor	Electrical, $\frac{1}{2}(1/C)q^2$
Block	Kinetic, $\frac{1}{2}mv^2$	Inductor	Magnetic, $\frac{1}{2}Li^2$
$v = dx/dt$		$i = dq/dt$	

The angular frequency of oscillation for an ideal (resistanceless) LC is:

$$\omega = \frac{1}{\sqrt{LC}} \qquad (LC \text{ circuit}).$$

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31.4: LC Oscillations, Quantitatively:

The Block-Spring Oscillator:

$$U = U_b + U_s = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2,$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0.$$

$$m\frac{d^2x}{dt^2} + kx = 0 \longrightarrow x = X\cos(\omega t + \phi) \qquad \text{(displacement)}$$

$$\omega = \sqrt{\frac{k}{m}}$$

The LC Oscillator:

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C},$$

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0.$$

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \qquad (LC\ \text{oscillations}).$$

$$q = Q\cos(\omega t + \phi) \qquad \text{(charge)} \longrightarrow i = \frac{dq}{dt} = -\omega Q\sin(\omega t + \phi) \qquad \text{(current)}.$$

$$\boxed{i = -I\sin(\omega t + \phi).} \qquad I = \omega Q, \qquad \boxed{\omega = \frac{1}{\sqrt{LC}}.}$$

31.4: LC Oscillations, Quantitatively:

The electrical energy stored in the LC circuit at time t is,

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi).$$

The magnetic energy is:

$$U_B = \tfrac{1}{2}Li^2 = \tfrac{1}{2}L\omega^2Q^2\sin^2(\omega t + \phi).$$

But
$$\omega = \frac{1}{\sqrt{LC}} \qquad (LC \text{ circuit}).$$

Therefore
$$U_B = \frac{Q^2}{2C}\sin^2(\omega t + \phi).$$

The electrical and magnetic energies vary but the total is constant.

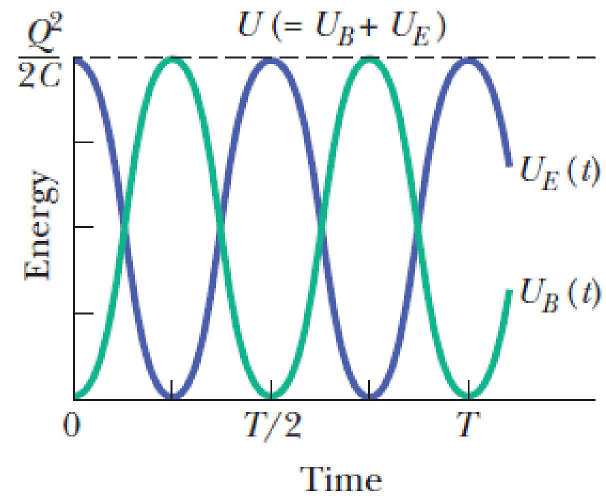

Fig. 31-4 The stored magnetic energy and electrical energy in the circuit of Fig. 31-1 as a function of time. Note that their sum remains constant. T is the period of oscillation.

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Example, LC oscillator, potential charge, rate of current change

A 1.5 μF capacitor is charged to 57 V by a battery, which is then removed. At time $t = 0$, a 12 mH coil is connected in series with the capacitor to form an LC oscillator (Fig. 31-1).

(a) What is the potential difference $v_L(t)$ across the inductor as a function of time?

Calculations: At any time t during the oscillations, the loop rule and Fig. 31-1 give us

$$v_L(t) = v_C(t); \qquad (31\text{-}18)$$

that is, the potential difference v_L across the inductor must always be equal to the potential difference v_C across the capacitor, so that the net potential difference around the circuit is zero. Thus, we will find $v_L(t)$ if we can find $v_C(t)$, and we can find $v_C(t)$ from $q(t)$ with Eq. 25-1 ($q = CV$).

Because the potential difference $v_C(t)$ is maximum when the oscillations begin at time $t = 0$, the charge q on the capacitor must also be maximum then. Thus, phase constant ϕ must be zero; so Eq. 31-12 gives us

$$q = Q \cos \omega t. \qquad (31\text{-}19)$$

$$\frac{q}{C} = \frac{Q}{C} \cos \omega t,$$

$$v_C = V_C \cos \omega t. \qquad (31\text{-}20)$$

$$v_L = V_C \cos \omega t. \qquad (31\text{-}21)$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{[(0.012\ \text{H})(1.5 \times 10^{-6}\ \text{F})]^{0.5}}$$

$$= 7454\ \text{rad/s} \approx 7500\ \text{rad/s}.$$

Thus, Eq. 31-21 becomes

$$v_L = (57\ \text{V}) \cos(7500\ \text{rad/s})t. \qquad \text{(Answer)}$$

(b) What is the maximum rate $(di/dt)_{max}$ at which the current i changes in the circuit?

Calculations: Taking the derivative, we have

$$\frac{di}{dt} = \frac{d}{dt}(-\omega Q \sin \omega t) = -\omega^2 Q \cos \omega t.$$

We can simplify this equation by substituting CV_C for Q (because we know C and V_C but not Q) and $1/\sqrt{LC}$ for ω according to Eq. 31-4. We get

$$\frac{di}{dt} = -\frac{1}{LC} CV_C \cos \omega t = -\frac{V_C}{L} \cos \omega t.$$

This tells us that the current changes at a varying (sinusoidal) rate, with its maximum rate of change being

$$\frac{V_C}{L} = \frac{57\ \text{V}}{0.012\ \text{H}} = 4750\ \text{A/s} \approx 4800\ \text{A/s}. \qquad \text{(Answer)}$$

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31.5: Damped Oscillations in an RLC Circuit:

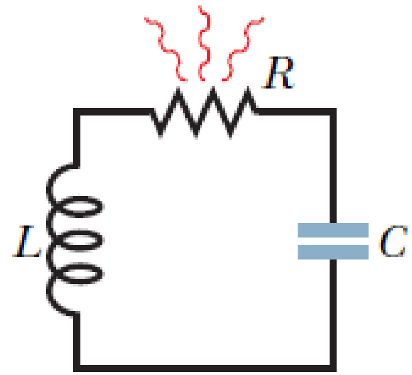

Fig. 31-5 A series *RLC* circuit. As the charge contained in the circuit oscillates back and forth through the resistance, electromagnetic energy is dissipated as thermal energy, damping (decreasing the amplitude of) the oscillations.

Analysis:

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}$$

$$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2R.$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \qquad (RLC\text{ circuit}),$$

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi),$$

Where $\omega' = \sqrt{\omega^2 - (R/2L)^2},$

And $\omega = 1/\sqrt{LC}$

$$U_E = \frac{q^2}{2C} = \frac{[Qe^{-Rt/2L}\cos(\omega' t + \phi)]^2}{2C} = \frac{Q^2}{2C}e^{-Rt/L}\cos^2(\omega' t + \phi)$$

Example, Damped RLC Circuit:

Damped *RLC* circuit: charge amplitude

A series *RLC* circuit has inductance $L = 12$ mH, capacitance $C = 1.6\ \mu$F, and resistance $R = 1.5\ \Omega$ and begins to oscillate at time $t = 0$.

(a) At what time t will the amplitude of the charge oscillations in the circuit be 50% of its initial value? (Note that we do not know that initial value.)

KEY IDEA

The amplitude of the charge oscillations decreases exponentially with time t: According to Eq. 31-25, the charge amplitude at any time t is $Qe^{-Rt/2L}$, in which Q is the amplitude at time $t = 0$.

Calculations: We want the time when the charge amplitude has decreased to $0.50Q$, that is, when

$$Qe^{-Rt/2L} = 0.50Q.$$

We can now cancel Q (which also means that we can answer the question without knowing the initial charge). Taking the natural logarithms of both sides (to eliminate the exponential function), we have

$$-\frac{Rt}{2L} = \ln 0.50.$$

Solving for t and then substituting given data yield

$$t = -\frac{2L}{R}\ln 0.50 = -\frac{(2)(12\times 10^{-3}\ \mathrm{H})(\ln 0.50)}{1.5\ \Omega}$$

$$= 0.0111\ \mathrm{s} \approx 11\ \mathrm{ms}. \qquad \text{(Answer)}$$

(b) How many oscillations are completed within this time?

KEY IDEA

The time for one complete oscillation is the period $T = 2\pi/\omega$, where the angular frequency for LC oscillations is given by Eq. 31-4 ($\omega = 1/\sqrt{LC}$).

Calculation: In the time interval $\Delta t = 0.0111$ s, the number of complete oscillations is

$$\frac{\Delta t}{T} = \frac{\Delta t}{2\pi\sqrt{LC}}$$

$$= \frac{0.0111\ \mathrm{s}}{2\pi[(12\times 10^{-3}\ \mathrm{H})(1.6\times 10^{-6}\ \mathrm{F})]^{1/2}} \approx 13. \qquad \text{(Answer)}$$

Thus, the amplitude decays by 50% in about 13 complete oscillations. This damping is less severe than that shown in Fig. 31-3, where the amplitude decays by a little more than 50% in one oscillation.

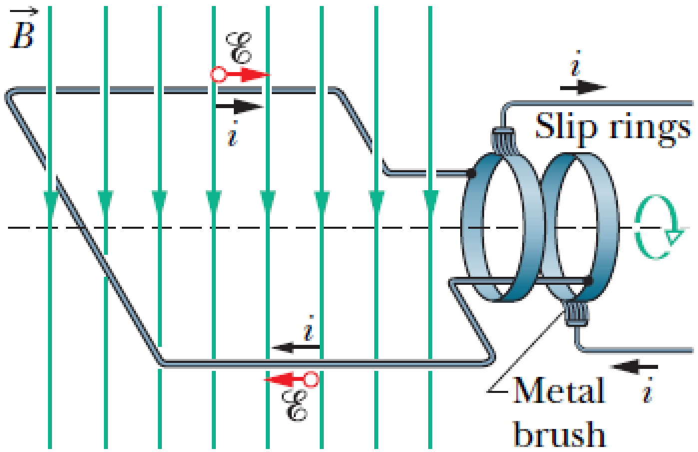

Fig. 31-6 The basic mechanism of an alternating-current generator is a conducting loop rotated in an external magnetic field. In practice, the alternating emf induced in a coil of many turns of wire is made accessible by means of slip rings attached to the rotating loop. Each ring is connected to one end of the loop wire and is electrically connected to the rest of the generator circuit by a conducting brush against which the ring slips as the loop (and it) rotates.

$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t.$$

$$i = I \sin(\omega_d t - \phi),$$

ω_d is called the driving angular frequency, and I is the amplitude of the driven current.

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31.6: Forced Oscillations:

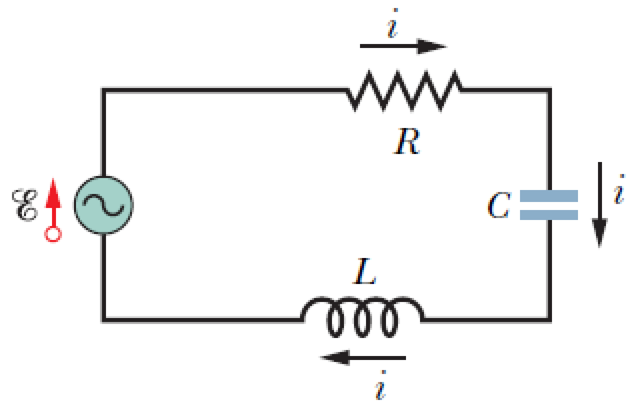

Fig. 31-7 A single-loop circuit containing a resistor, a capacitor, and an inductor. A generator, represented by a sine wave in a circle, produces an alternating emf that establishes an alternating current; the directions of the emf and current are indicated here at only one instant.

Whatever the natural angular frequency ω of a circuit may be, forced oscillations of charge, current, and potential difference in the circuit always occur at the driving angular frequency ω_d.

31.7: Three Simple Circuits:
i. A Resistive Load:

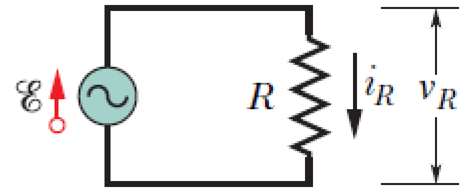

Fig. 31-8 A resistor is connected across an alternating-current generator.

$$\mathscr{E} - v_R = 0.$$

$$v_R = \mathscr{E}_m \sin \omega_d t. = V_R \sin \omega_d t.$$

$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin \omega_d t.$$

$$= I_R \sin(\omega_d t - \phi),$$

For a purely resistive load the phase constant $\phi = 0°$.

We are concerned with the potential drop (voltage) along the current flow, and the phase lag of the current w.r.t. the voltage, which is in phase with the driving AC emf.

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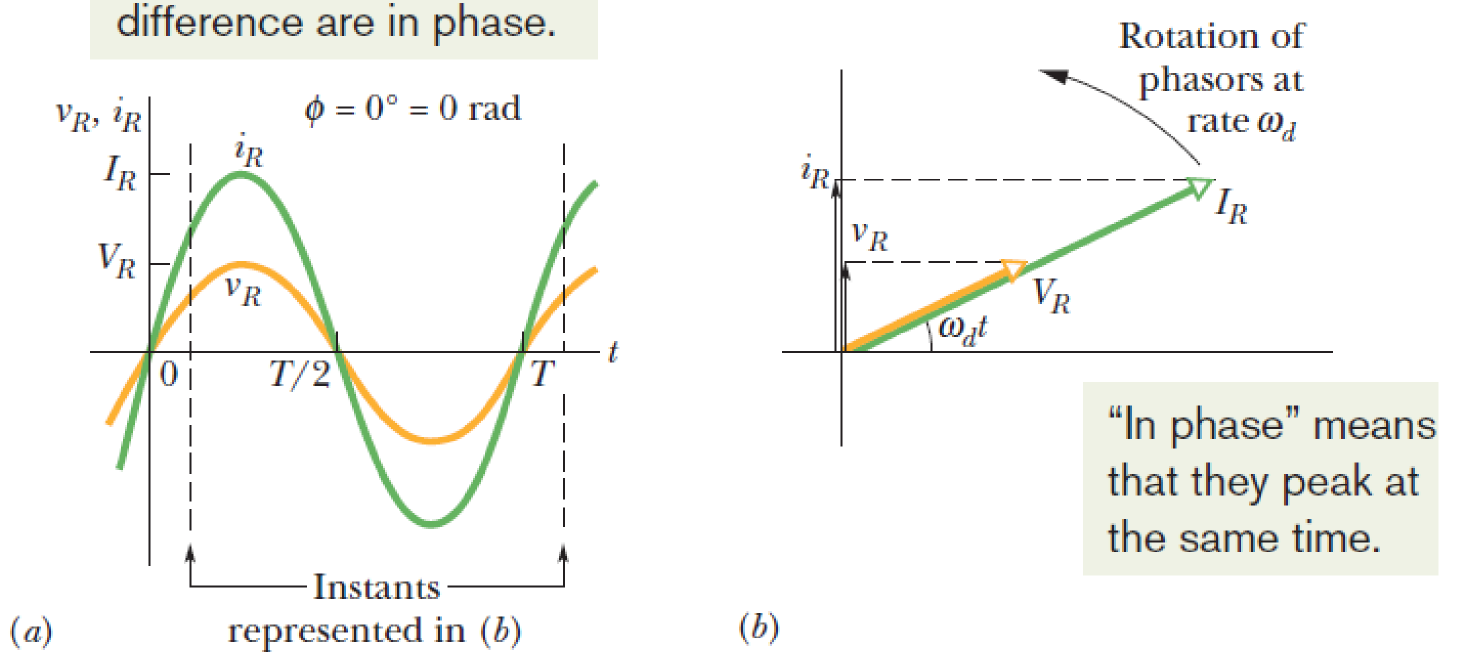
For a resistive load, the current and potential difference are in phase.



"In phase" means that they peak at the same time.

Fig. 31-9 (a) The current i_R and the potential difference v_R across the resistor are plotted on the same graph, both versus time t. They are in phase and complete one cycle in one period T. (b) A phasor diagram shows the same thing as (a).

Example, Purely resistive load: potential difference and current

In Fig. 31-8, resistance R is 200 Ω and the sinusoidal alternating emf device operates at amplitude $\mathscr{E}_m = 36.0$ V and frequency $f_d = 60.0$ Hz.

(a) What is the potential difference $v_R(t)$ across the resistance as a function of time t, and what is the amplitude V_R of $v_R(t)$?

KEY IDEA

In a circuit with a purely resistive load, the potential difference $v_R(t)$ across the resistance is always equal to the potential difference $\mathscr{E}(t)$ across the emf device.

Calculations: Here we have $v_R(t) = \mathscr{E}(t)$ and $V_R = \mathscr{E}_m$. Since $\mathscr{E}_m$ is given, we can write

$$V_R = \mathscr{E}_m = 36.0 \text{ V}. \qquad \text{(Answer)}$$

To find $v_R(t)$, we use Eq. 31-28 to write

$$v_R(t) = \mathscr{E}(t) = \mathscr{E}_m \sin \omega_d t \qquad (31\text{-}34)$$

and then substitute $\mathscr{E}_m = 36.0$ V and

$$\omega_d = 2\pi f_d = 2\pi(60 \text{ Hz}) = 120\pi$$

to obtain

$$v_R = (36.0 \text{ V}) \sin(120\pi t). \qquad \text{(Answer)}$$

We can leave the argument of the sine in this form for convenience, or we can write it as (377 rad/s)t or as (377 s^{-1})t.

(b) What are the current $i_R(t)$ in the resistance and the amplitude I_R of $i_R(t)$?

KEY IDEA

In an ac circuit with a purely resistive load, the alternating current $i_R(t)$ in the resistance is *in phase* with the alternating potential difference $v_R(t)$ across the resistance; that is, the phase constant ϕ for the current is zero.

Calculations: Here we can write Eq. 31-29 as

$$i_R = I_R \sin(\omega_d t - \phi) = I_R \sin \omega_d t. \qquad (31\text{-}35)$$

From Eq. 31-33, the amplitude I_R is

$$I_R = \frac{V_R}{R} = \frac{36.0 \text{ V}}{200\ \Omega} = 0.180 \text{ A}. \qquad \text{(Answer)}$$

Substituting this and $\omega_d = 2\pi f_d = 120\pi$ into Eq. 31-35, we have

$$i_R = (0.180 \text{ A}) \sin(120\pi t). \qquad \text{(Answer)}$$

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31.7: Three Simple Circuits:
ii. A Capacitive Load:

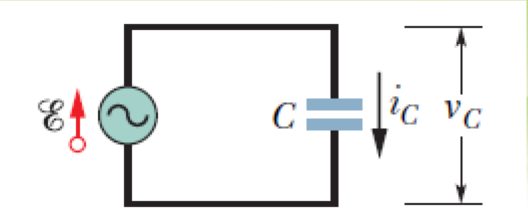

Fig. 31-10 A capacitor is connected across an alternating-current generator.

$$v_C = V_C \sin \omega_d t,$$

$$q_C = Cv_C = CV_C \sin \omega_d t.$$

$$i_C = \frac{dq_C}{dt} = \omega_d C V_C \cos \omega_d t.$$

$$i_C = \left(\frac{V_C}{X_C}\right) \sin(\omega_d t + 90°).$$

$$X_C = \frac{1}{\omega_d C} \qquad \text{(capacitive reactance).}$$

X_C is called the **capacitive reactance of a capacitor**. The SI unit of X_C is the *ohm*, just as for resistance R.

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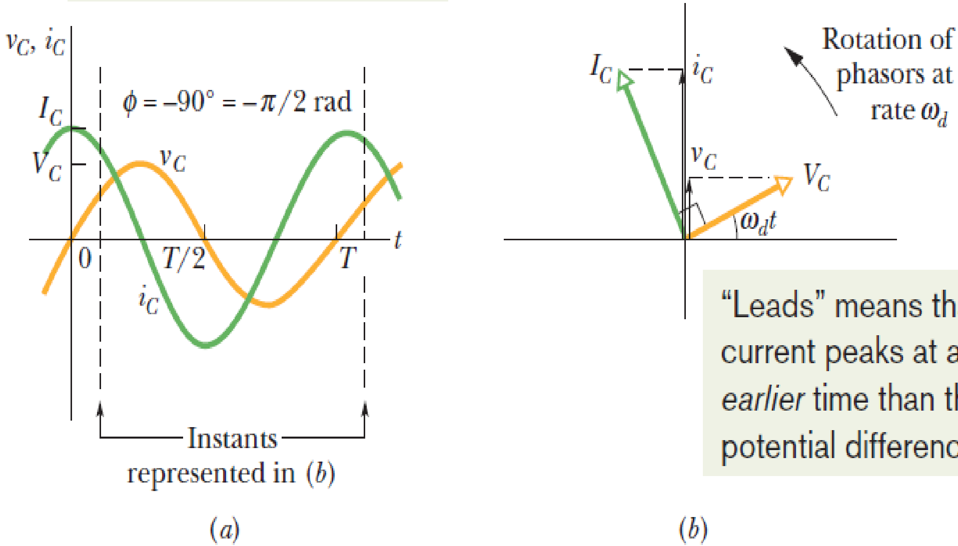
31.7: Three Simple Circuits:
ii. A Capacitive Load:

For a capacitive load, the current leads the potential difference by 90°.



"Leads" means that the current peaks at an *earlier* time than the potential difference.

$$\cos \omega_d t = \sin(\omega_d t + 90°).$$

$$i_C = \left(\frac{V_C}{X_C}\right) \sin(\omega_d t + 90°).$$

$$i_C = I_C \sin(\omega_d t - \phi),$$

$$V_C = I_C X_C \qquad \text{(capacitor).}$$

Fig. 31-11 (*a*) The current in the capacitor leads the voltage by 90° (= $\pi/2$ rad). (*b*) A phasor diagram shows the same thing.

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Example, Purely capacitive load: potential difference and current

In Fig. 31-10, capacitance C is 15.0 μF and the sinusoidal alternating emf device operates at amplitude $\mathscr{E}_m = 36.0$ V and frequency $f_d = 60.0$ Hz.

(a) What are the potential difference $v_C(t)$ across the capacitance and the amplitude V_C of $v_C(t)$?

KEY IDEA

In a circuit with a purely capacitive load, the potential difference $v_C(t)$ across the capacitance is always equal to the potential difference $\mathscr{E}(t)$ across the emf device.

Calculations: Here we have $v_C(t) = \mathscr{E}(t)$ and $V_C = \mathscr{E}_m$. Since $\mathscr{E}_m$ is given, we have

$$V_C = \mathscr{E}_m = 36.0 \text{ V}. \qquad \text{(Answer)}$$

To find $v_C(t)$, we use Eq. 31-28 to write

$$v_C(t) = \mathscr{E}(t) = \mathscr{E}_m \sin \omega_d t. \qquad (31\text{-}43)$$

Then, substituting $\mathscr{E}_m = 36.0$ V and $\omega_d = 2\pi f_d = 120\pi$ into Eq. 31-43, we have

$$v_C = (36.0 \text{ V}) \sin(120\pi t). \qquad \text{(Answer)}$$

(b) What are the current $i_C(t)$ in the circuit as a function of time and the amplitude I_C of $i_C(t)$?

KEY IDEA

In an ac circuit with a purely capacitive load, the alternating current $i_C(t)$ in the capacitance leads the alternating potential difference $v_C(t)$ by 90°; that is, the phase constant ϕ for the current is $-90°$, or $-\pi/2$ rad.

Calculations: Thus, we can write Eq. 31-29 as

$$i_C = I_C \sin(\omega_d t - \phi) = I_C \sin(\omega_d t + \pi/2). \qquad (31\text{-}44)$$

We can find the amplitude I_C from Eq. 31-42 ($V_C = I_C X_C$) if we first find the capacitive reactance X_C. From Eq. 31-39 ($X_C = 1/\omega_d C$), with $\omega_d = 2\pi f_d$, we can write

$$X_C = \frac{1}{2\pi f_d C} = \frac{1}{(2\pi)(60.0 \text{ Hz})(15.0 \times 10^{-6} \text{ F})}$$
$$= 177\ \Omega.$$

Then Eq. 31-42 tells us that the current amplitude is

$$I_C = \frac{V_C}{X_C} = \frac{36.0 \text{ V}}{177\ \Omega} = 0.203 \text{ A}. \qquad \text{(Answer)}$$

Substituting this and $\omega_d = 2\pi f_d = 120\pi$ into Eq. 31-44, we have

$$i_C = (0.203 \text{ A}) \sin(120\pi t + \pi/2). \qquad \text{(Answer)}$$

31.7: Three Simple Circuits:
iii. An Inductive Load:

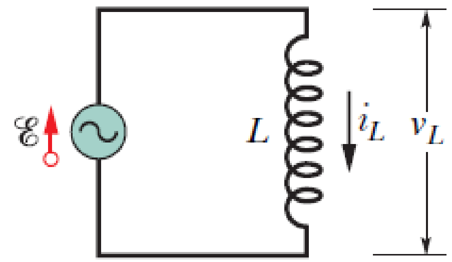


Fig. 31-12 An inductor is connected across an alternating-current generator.

$$v_L = V_L \sin \omega_d t, \qquad v_L = L \frac{di_L}{dt}.$$

$$\frac{di_L}{dt} = \frac{V_L}{L} \sin \omega_d t.$$

$$i_L = \int di_L = \frac{V_L}{L} \int \sin \omega_d t \, dt = -\left(\frac{V_L}{\omega_d L}\right) \cos \omega_d t.$$

$$X_L = \omega_d L \qquad \text{(inductive reactance).}$$

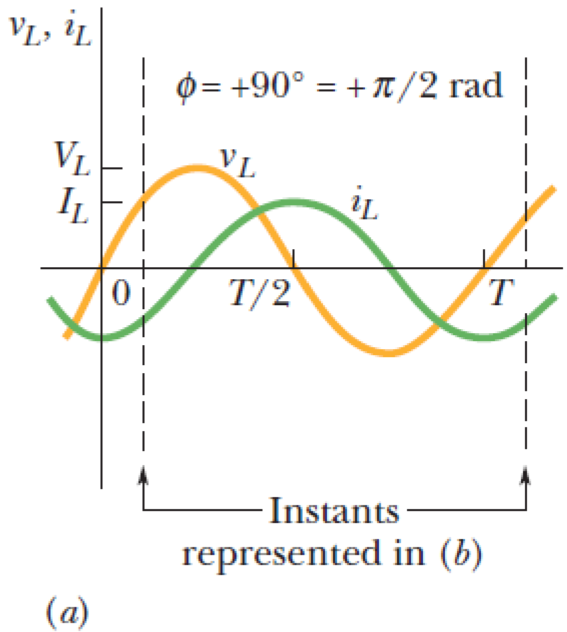
$$i_L = \left(\frac{V_L}{X_L}\right) \sin(\omega_d t - 90^\circ). \qquad i_L = I_L \sin(\omega_d t - \phi),$$

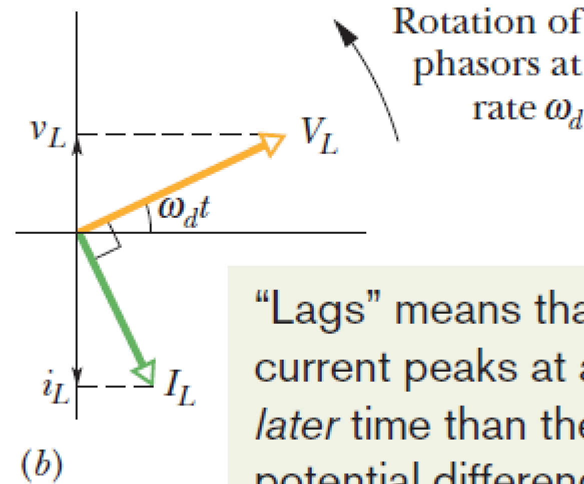
$$V_L = I_L X_L \qquad \text{(inductor).}$$

The X_L is called **the inductive reactance** of an inductor.
The SI unit of X_L is the *ohm*.

31.7: Three Simple Circuits:
iii. An Inductive Load:

For an inductive load, the current lags the potential difference by 90°.





"Lags" means that the current peaks at a *later* time than the potential difference.

Fig. 31-13 (a) The current in the inductor lags the voltage by 90° ($= \pi/2$ rad). (b) A phasor diagram shows the same thing.

Example, Purely inductive load: potential difference and current

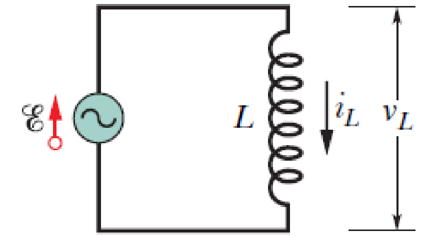


Fig. 31-12 An inductor is connected across an alternating-current generator.

In Fig. 31-12, inductance L is 230 mH and the sinusoidal alternating emf device operates at amplitude $\mathscr{E}_m = 36.0$ V and frequency $f_d = 60.0$ Hz.

(a) What are the potential difference $v_L(t)$ across the inductance and the amplitude V_L of $v_L(t)$?

KEY IDEA

In a circuit with a purely inductive load, the potential difference $v_L(t)$ across the inductance is always equal to the potential difference $\mathscr{E}(t)$ across the emf device.

Calculations: Here we have $v_L(t) = \mathscr{E}(t)$ and $V_L = \mathscr{E}_m$. Since $\mathscr{E}_m$ is given, we know that

$$V_L = \mathscr{E}_m = 36.0 \text{ V}. \qquad \text{(Answer)}$$

To find $v_L(t)$, we use Eq. 31-28 to write

$$v_L(t) = \mathscr{E}(t) = \mathscr{E}_m \sin \omega_d t. \qquad (31\text{-}53)$$

Then, substituting $\mathscr{E}_m = 36.0$ V and $\omega_d = 2\pi f_d = 120\pi$ into Eq. 31-53, we have

$$v_L = (36.0 \text{ V}) \sin(120\pi t). \qquad \text{(Answer)}$$

(b) What are the current $i_L(t)$ in the circuit as a function of time and the amplitude I_L of $i_L(t)$?

KEY IDEA

In an ac circuit with a purely inductive load, the alternating current $i_L(t)$ in the inductance lags the alternating potential difference $v_L(t)$ by 90°. (In the mnemonic of the problem-solving tactic, this circuit is "positively an *ELI* circuit," which tells us that the emf *E* leads the current *I* and that ϕ is *positive.*)

Calculations: Because the phase constant ϕ for the current is $+90°$, or $+\pi/2$ rad, we can write Eq. 31-29 as

$$i_L = I_L \sin(\omega_d t - \phi) = I_L \sin(\omega_d t - \pi/2). \qquad (31\text{-}54)$$

We can find the amplitude I_L from Eq. 31-52 ($V_L = I_L X_L$) if we first find the inductive reactance X_L. From Eq. 31-49 ($X_L = \omega_d L$), with $\omega_d = 2\pi f_d$, we can write

$$X_L = 2\pi f_d L = (2\pi)(60.0 \text{ Hz})(230 \times 10^{-3} \text{ H}) = 86.7\ \Omega.$$

Then Eq. 31-52 tells us that the current amplitude is

$$I_L = \frac{V_L}{X_L} = \frac{36.0 \text{ V}}{86.7\ \Omega} = 0.415 \text{ A}. \qquad \text{(Answer)}$$

Substituting this and $\omega_d = 2\pi f_d = 120\pi$ into Eq. 31-54, we have

$$i_L = (0.415 \text{ A}) \sin(120\pi t - \pi/2). \qquad \text{(Answer)}$$

31.7: Three Simple Circuits:

Table 31-2

Phase and Amplitude Relations for Alternating Currents and Voltages

Circuit Element	Symbol	Resistance or Reactance	Phase of the Current	Phase Constant (or Angle) ϕ	Amplitude Relation
Resistor	R	R	In phase with v_R	$0°$ (= 0 rad)	$V_R = I_R R$
Capacitor	C	$X_C = 1/\omega_d C$	Leads v_C by 90° (= $\pi/2$ rad)	$-90°$ (= $-\pi/2$ rad)	$V_C = I_C X_C$
Inductor	L	$X_L = \omega_d L$	Lags v_L by 90° (= $\pi/2$ rad)	$+90°$ (= $+\pi/2$ rad)	$V_L = I_L X_L$

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