CHAPTER 1

P.P.1.1 A proton has 1.602×10^{-19} C. Hence, 6 million protons have

$+1.602 \times 10^{-19} \times 6 \times 10^{6} =$ $\underline{\mathbf{9.612 \times 10^{-13} C}}$

P.P.1.2 $i = dq/dt = -10(-2)e^{-2t}$ mA

At $t = 1.0$ sec, $i = 20e^{-2} =$ $\underline{\mathbf{2.707\ mA}}$

P.P.1.3 $q = \int i dt = \int_0^1 4dt + \int_1^2 4t^2 dt = 4t \Big|_0^1 + (4/3)t^3 \Big|_1^2$
$= 4 + 28/3 =$ $\underline{\mathbf{13.333\ C}}$

P.P.1.4 (a) $V_{ab} = w/q = -30/6 =$ $\underline{\mathbf{-5\ V}}$

The negative sign indicates that point a is at a higher potential than point b.

(b) $V_{ab} = w/q = -30/-3 =$ $\underline{\mathbf{10\ V}}$

P.P.1.5 (a) $v = 2\,i = 10 \cos(60\,\pi\,t)$

$p = v\,i = 50 \cos^2(60\,\pi\,t)$

At $t = 5$ ms, $p = 50 \cos^2(60\,\pi\, 5\times10^{-3}) = 50 \cos^2(0.3\,\pi)$

$=$ $\underline{\mathbf{17.27\ watts}}$

(b) $v = 10 + 5\int_0^t i dt = 10 + \int_0^t 25 \cos 60\,\pi\, t\, dt = 10 + \frac{25}{60\pi} \sin 60\,\pi\, t$

$p = vi = 5 \cos(60\,\pi t)[10 + (25/(60\,\pi)) \sin(60\,\pi\, t)]$

At $t = 5$ ms, $p = 5 \cos(0.3\pi)\{10 + (25/(60\,\pi)) \sin(0.3\,\pi)\}$

$=$ $\underline{\mathbf{29.7\ watts}}$

P.P.1.6 $p = v\,i = 15 \times 240 = 3600$ watts; $w = p \times t$

therefore, $t = w/p = (180\text{x}10^3)/3600 =$ **<u>50 seconds</u>**

P.P.1.7 $p_1 = 5(-9) =$ **<u>–45w</u>**

$p_2 = 2(9) =$ **<u>18w</u>**

$p_3 = 0.6\text{xI}(4) = 0.6(5)(4) =$ **<u>12w</u>**

$p_4 = 3(5) =$ **<u>15w</u>**

Note that all the absorbed power adds up to zero as expected.

P.P.1.8 $i = dq/dt = e\dfrac{dn}{dt} = -1.6 \times 10^{-19} \times 10^{13} = -1.6 \times 10^{-6}$ A

$p = v_0\, i = 30 \times 10^3 \times (1.6 \times 10^{-6}) =$ **<u>48mW</u>**

P.P.1.9

Minimum monthly charge	= \$12.00
First 100 kWh @ \$0.16/kWh	= \$16.00
Next 200 kWh @ \$0.10/kWh	= \$20.00
Remaining 50 kWh @ \$0.06/kWh	= <u>\$3.00</u>
Total Charge	= \$51.00

Average cost = \$51/[100+200+50] = **<u>14.571 cents/kWh</u>**

P.P.1.10 This assigned practice problem is to apply the detailed problem solving technique to some of the more difficult problems of Chapter 1.

CHAPTER 2

P.P.2.1 $i = V/R = 110/15 = \underline{\mathbf{7.333\ A}}$

P.P.2.2 (a) $v = iR = 3\ mA[10\ kohms] = \underline{\mathbf{30\ V}}$

(b) $G = 1/R = 1/10\ kohms = \underline{\mathbf{100\ \mu S}}$

(c) $p = vi = 30\ volts[3\ mA] = \underline{\mathbf{90\ mW}}$

P.P.2.3 $p = vi$ which leads to $i = p/v = [30\cos^2(t)\ mW]/[15\cos(t)\ mA]$

or $i = \underline{\mathbf{2\cos(t)\ mA}}$

$R = v/i = 15\cos(t)V/2\cos(t)mA = \underline{\mathbf{7.5\ k\Omega}}$

P.P.2.4 5 branches and 3 nodes. The 1 ohm and 2 ohm resistors are in parallel. The 4 ohm resistor and the 10 volt source are also in parallel.

P.P.2.5 Applying KVL to the loop we get:

$-32 + 4i - (-8) + 2i = 0$ which leads to $i = 24/6 = 4A$

$v_1 = 4i = \underline{\mathbf{16\ V}}$ and $v_2 = -2i = \underline{\mathbf{-8\ V}}$

P.P.2.6 Applying KVL to the loop we get:

$-70 + 10i + 2v_x + 5i = 0$

But, $v_x = 10i$ and $v_0 = -5i$. Hence,

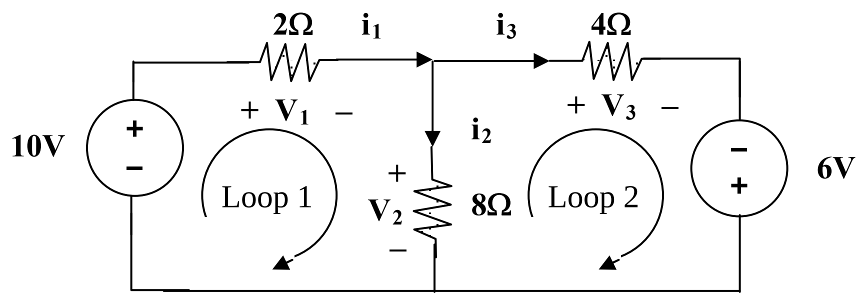
$-70 + 10i + 20i + 5i = 0$ which leads to $i = 2$ A.

Thus, $v_x = \underline{\mathbf{20V}}$ and $v_0 = \underline{\mathbf{-10\ V}}$

P.P.2.7 Applying KCL, $0 = -9 + i_0 + [i_0/4] + [v_0/8]$, but $i_0 = v_0/2$

Which leads to: $9 = (v_0/2) + (v_0/8) + (v_0/8)$ thus, $v_0 = \underline{\mathbf{12\ V}}$ and $i_0 = \underline{\mathbf{6\ A}}$

P.P.2.8



At the top node, $0 = -i_1 + i_2 + i_3$ or $i_1 = i_2 + i_3$ (1)

For loop 1 $-10 + V_1 + V_2 = 0$
or $V_1 = 10 - V_2$ (2)

For loop 2 $-V_2 + V_3 - 6 = 0$
or $V_3 = V_2 + 6$ (3)

Using (1) and Ohm's law, we get

$$(V_1/2) = (V_2/8) + (V_3/4)$$

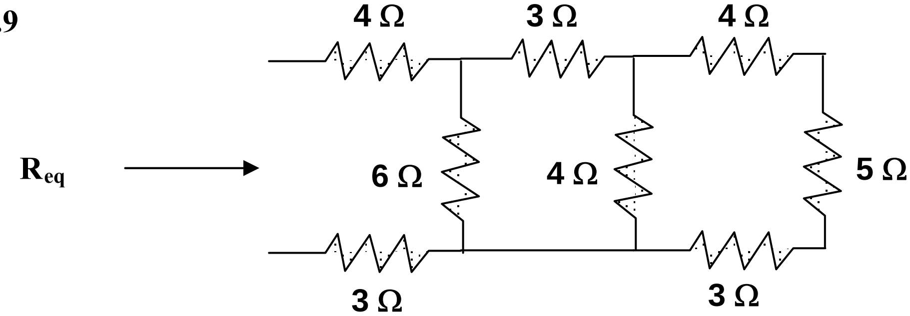
and now using (2) and (3) in the above yields

$$[(10 - V_2)/2] = (V_2/8) + (V_2 + 6)/4$$

or $[7/8]V_2 = 14/4$ or $V_2 = \underline{\mathbf{4\ V}}$

$V_1 = 10 - V_2 = \underline{\mathbf{6\ V}}$, $V_3 = 4+6 = \underline{\mathbf{10\ V}}$, $i_1 = (10-4)/2 = \underline{\mathbf{3\ A}}$,
$i_2 = 4/8 = \underline{\mathbf{500\ mA}}$, $i_3 = \underline{\mathbf{2.5\ A}}$

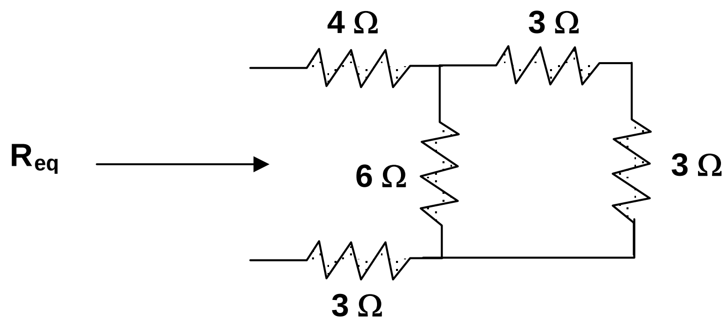
P.P.2.9



Combining the 4 ohm, 5 ohm, and 3ohm resistors in series gives 4+3+5 = 12.

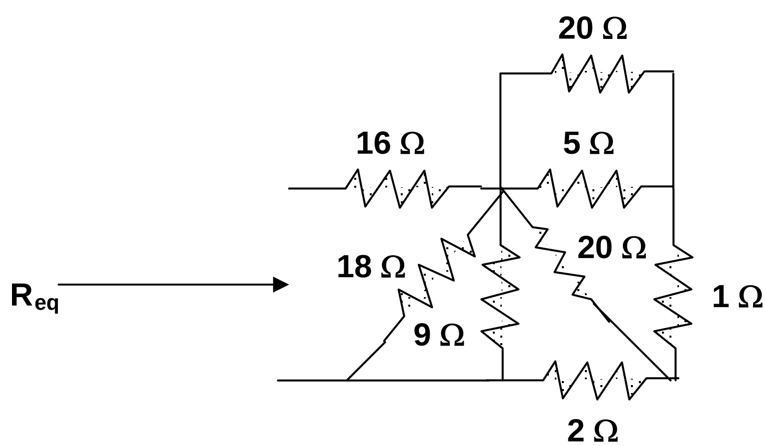
But, 4 in parallel with 12 produces [4x12]/[4+12] = 48/16 = 3ohm.

So that the equivalent circuit is shown below.



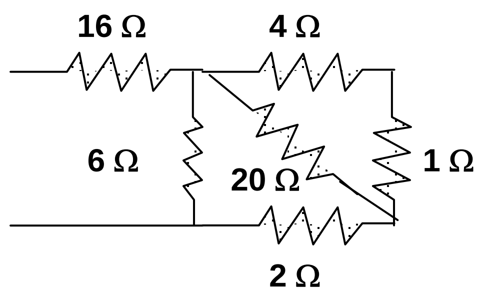
Thus, $\mathbf{R_{eq}}$ = 4 + 3 + [6x6]/[6+6] = **<u>10 Ω</u>**

P.P.2.10

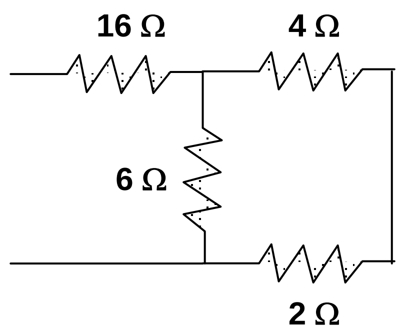


Combining the 9 ohm resistor and the 18 ohm resistor yields [9x18]/[9+18] = 6 ohms.

Combining the 5 ohm and the 20 ohm resistors in parallel produces [5x20/(5+20)] = 4 ohms We now have the following circuit:



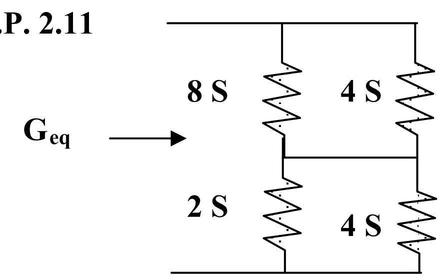
The 4 ohm and 1 ohm resistors can be combined into a 5 ohm resistor in parallel with a 20 ohm resistor. This will result in [5x20/(5+20)] = 4 ohms and the circuit shown below:



The 4 ohm and 2 ohm resistors are in series and can be replaced by a 6 ohm resistor. This gives a 6 ohm resistor in parallel with a 6 ohm resistor, [6x6/(6+6)] = 3 ohms. We now have a 3 ohm resistor in series with a 16 ohm resistor or 3 + 16 = 19 ohms. Therefore:

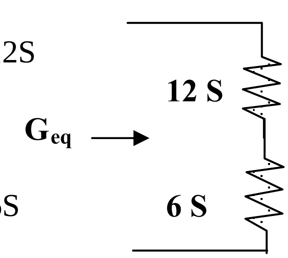
$$\mathbf{R_{eq} = \underline{19\ ohms}}$$

P.P. 2.11



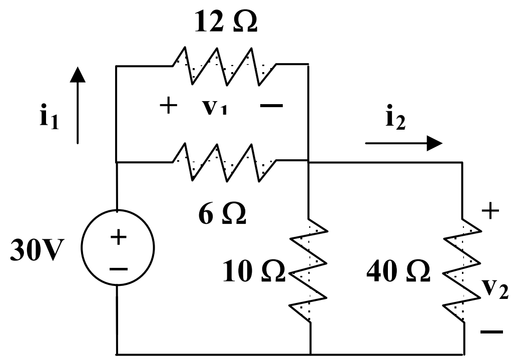
8||4 = 8+4 = 12S

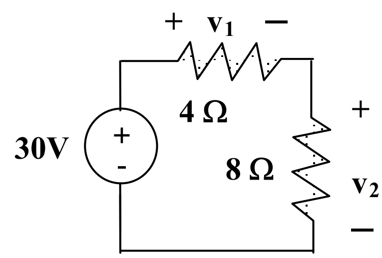
2||4 = 2+4 = 6S



12 S in series with 6 S = {12x6/(12+6)] = 4 or: $\mathbf{G_{eq} = \underline{4\ S}}$

P.P.2.12





6||12 = [6x12/(6+12)] = 4 ohm and 10||40 = [10x40/(10+40)] = 8 ohm.

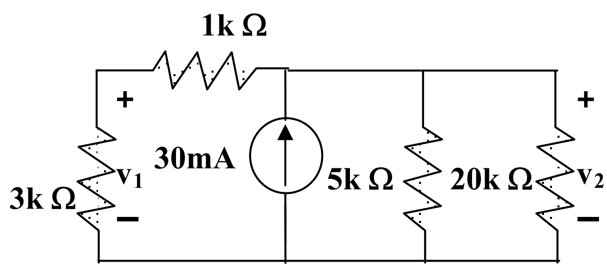
Using voltage division we get:

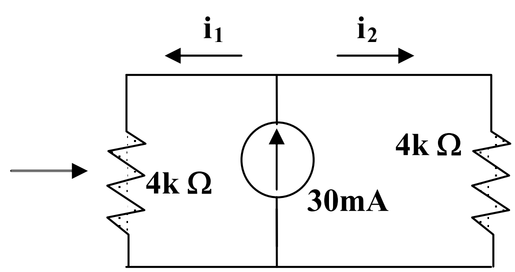
$\mathbf{v_1}$ = [4/(4+8)] (30) = **<u>10 volts</u>**, $\mathbf{v_2}$ = [8/12] (30) = **<u>20 volts</u>**

$\mathbf{i_1}$ = v_1/12 = 10/12 = **<u>833.3 mA</u>**, $\mathbf{i_2}$ = v_2/40 = 20/40 = **<u>500 mA</u>**

$\mathbf{P_1}$ = v_1 i_1 = 10x10/12 = **<u>8.333 watts</u>**, $\mathbf{P_2}$ = v_2 i_2 = 20x0.5 = **<u>10 watts</u>**

P.P.2.13





Using current division, i_1 = i_2 = (30 mA)(4 kohm/(4 kohm + 4 kohm)) = 15mA

(a) $\mathbf{v_1}$ = (3 kohm)(15 mA) = **<u>45 volts</u>**
$\mathbf{v_2}$ = (4 kohm)(15 mA) = **<u>60 volts</u>**

(b) For the 3k ohm resistor, $\mathbf{P_1}$ = v_1 x i_1 = $45 \times 15 \times 10^{-3}$ = **<u>675 mw</u>**
For the 20k ohm resistor, $\mathbf{P_2}$ = $(v_2)^2$ /20k = **<u>180 mw</u>**

(c) The total power supplied by the current source is equal to:

P $= v_2 \times 10$ mA $= 60 \times 30 \times 10^{-3}$ = **<u>1.8 W</u>**

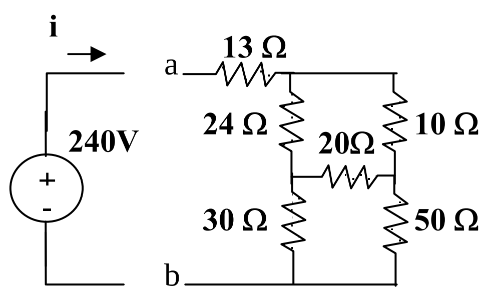
P.P.2.14

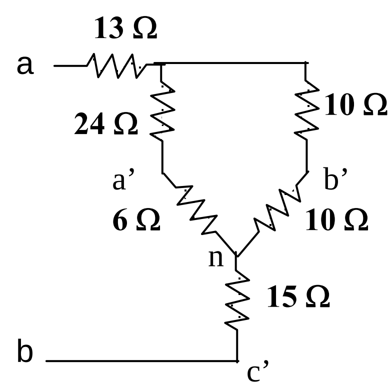
$\mathbf{R_a} = [R_1 R_2 + R_2 R_3 + R_3 R_1]/ R_1 = [10 \times 20 + 20 \times 40 + 40 \times 10]/10 =$ **<u>140 ohms</u>**

$\mathbf{R_b} = [R_1 R_2 + R_2 R_3 + R_3 R_1]/ R_2 = 1400/20 =$ **<u>70 ohms</u>**

$\mathbf{R_c} = [R_1 R_2 + R_2 R_3 + R_3 R_1]/ R_3 = 1400/40 =$ **<u>35 ohms</u>**

P.P.2.15 We first find the equivalent resistance, R. We convert the delta sub-network to a wye connected form as shown below:





$R_{a'n} = 20 \times 30/[20 + 30 + 50] = 6$ ohms, $R_{b'n} = 20 \times 50/100 = 10$ ohms
$R_{c'n} = 30 \times 50/100 = 15$ ohms.

Thus, $R_{ab} = 13 + [(24 + 6)||(10 + 10)] + 15 = 28 + 30 \times 20/(30 + 20) =$ **<u>40 ohms.</u>**

i $= 240/ R_{ab} = 240/40 =$ **<u>6 amps</u>**

P.P.2.16 **For the parallel case, v = v_0 = 110volts.**
p = vi ⟹ i = p/v = 40/110 = <u>364 mA</u>

For the series case, $v = v_0/N = 110/10 = 11$ volts
i $= p/v = 40/11 =$ **<u>3.64 amps</u>**

P.P.2.17 We use equation (2.61)

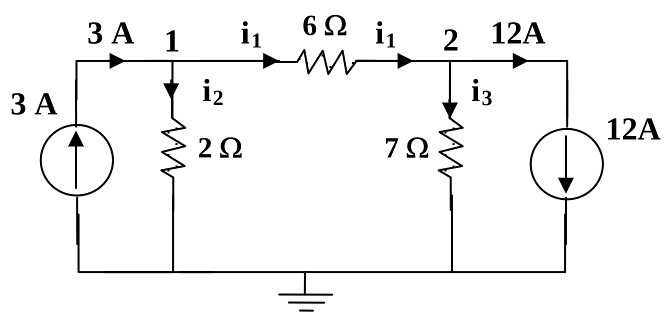
(a) $\mathbf{R_1} = 50 \times 10^{-3}/ (1-10^{-3}) = 0.05/999 =$ **<u>50 mΩ (shunt)</u>**

(b) $\mathbf{R_2} = 50 \times 10^{-3}/(100 \times 10^{-3} - 10^{-3}) = 50/99 =$ **<u>505 mΩ (shunt)</u>**

(c) $\mathbf{R_3} = 50x10^{-3}/(10x10^{-3}-10^{-3}) = 50/9 =$ **<u>5.556 Ω (shunt)</u>**

CHAPTER 3

P.P.3.1



At node 1,

$$-3 + i_1 + i_2 = 0 \text{ or } \frac{v_1 - v_2}{6} + \frac{v_1 - 0}{2} = 3$$

$$\text{or } 4v_1 - v_2 = 18 \qquad (1)$$

At node 2,

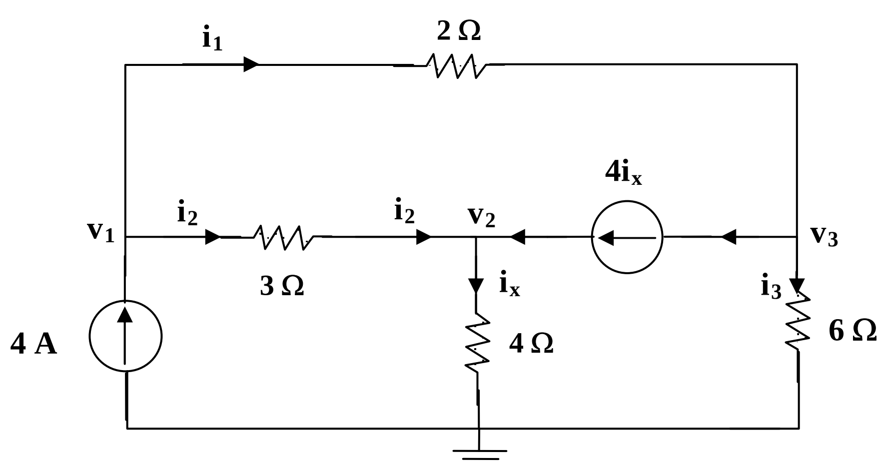
$$-i_1 + i_3 + 12 = 0 \text{ or } i_1 = 12 + i_3 \text{ or } \frac{v_1 - v_2}{6} = 12 + \frac{v_2 - 0}{7}$$

$$\text{or } 7v_1 - 13v_2 = 504 \qquad (2)$$

Solving (1) and (2) gives

v_1 = **–6 V**, v_2 = **–42 V**

P.P.3.2



At node 1,

$$-4 + i_1 + i_2 = 0 = -4 + \frac{v_1 - v_3}{2} + \frac{v_1 - v_2}{3}$$

or $5v_1 - 2v_2 - 3v_3 = 24$ (1)

At node 2,

$-i_2 + i_x - 4i_x = 0 = -i_2 - 3i_x = 0$ where $i_x = [(v_2 - 0)/4]$ or

$$\frac{v_1 - v_2}{3} + 3\frac{v_2}{4} = 0 \text{ which leads to } 4v_1 + 5v_2 = 0 \quad (2)$$

At node 3,

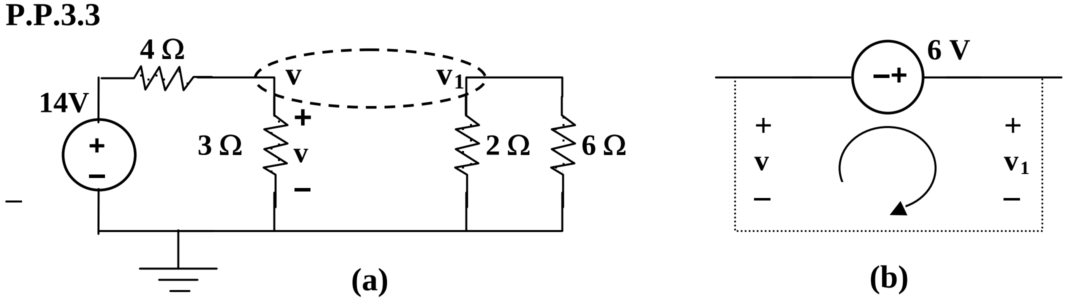
$$-i_1 + i_3 + 4i_x = 0 = \frac{v_3 - v_1}{2} + \frac{v_3 - 0}{6} + 4\frac{v_2}{4}$$

or $-3v_1 + 6v_2 + 4v_3 = 0$ (3)

Solving (1) to (3) gives

v_1 = **32 V**, v_2 = **–25.6 V**, v_3 = **62.4 V**

P.P.3.3



(a) **(b)**

At the supernode in Fig. (a),

$$\frac{14 - v}{4} = \frac{v}{3} + \frac{v_1}{2} + \frac{v_1}{6}$$

or $42 = 7v + 8v_1$ (1)

Applying KVL to the loop in Fig. (b),

$-v - 6 + v_1 = 0 \longrightarrow v_1 = v + 6$ (2)

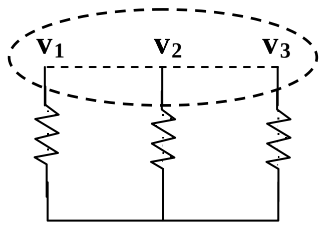
Solving (1) and (2),

$$v = \mathbf{-400\ mV}$$

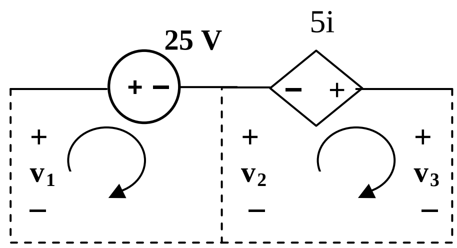
$$v_1 = v + 6 = 5.6,\ i_1 = \frac{v_1}{2} = 2.8$$

$$i_1 = \mathbf{2.8\ A}$$

P.P.3.4



(a)



(b)

From Fig. (a),

$$\frac{v_1}{2} + \frac{v_2}{4} + \frac{v_3}{3} = 0 \longrightarrow 6v_1 + 3v_2 + 4v_3 = 0 \qquad (1)$$

From Fig. (b),

$$-v_1 + 25 + v_2 = 0 \longrightarrow v_1 = v_2 + 25 \qquad (2)$$

$$-v_2 - 5i + v_3 = 0 \longrightarrow v_3 = v_2 + 2.5v_1 \qquad (3)$$

Solving (1) to (3), we obtain

$$v_1 = \mathbf{7.608\ V},\ v_2 = \mathbf{-17.39\ V},\ v_3 = \mathbf{1.6305\ V}$$

P.P.3.5 We apply KVL to the two loops and obtain

$$-45 + 2i_1 + 12(i_i - i_2) + 4i_1 = 0 \text{ or}$$

$$-45 + 18i_i - 12i_2 = 0 \text{ which leads to } 3i_i - 2i_2 = 7.5 \qquad (1)$$

$$12(i_2 - i_1) + 9i_2 + 30 + 3i_2 = 0 \text{ or}$$

$$30 + 24i_2 - 12i_1 = 0 \text{ which leads to } -3i_1 + 6i_2 = -7.5 \qquad (2)$$

From (1) and (2) we get

$$i_1 = \mathbf{2.5\ A},\ i_2 = \mathbf{0A}$$

P.P.3.6 For mesh 1,

$$-16 + 6i_1 - 2i_2 - 4i_3 = 0 \longrightarrow 3i_1 - i_2 - 2i_3 = 8 \qquad (1)$$

For mesh 2,

$$10i_2 - 2i_1 - 8i_3 - 10i_0 = 0 = -i_1 + 5i_2 - 9i_3 \qquad (2)$$

But $i_0 = i_3$,

$$18i_3 - 4i_1 - 8i_2 = 0 \longrightarrow -2i_1 - 4i_2 + 9i_3 = 0 \qquad (3)$$

From (1) to (3),

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 5 & -9 \\ -2 & -4 & 9 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 5 & -9 \\ -2 & -4 & 9 \\ 3 & -1 & -2 \\ -1 & 5 & -9 \end{vmatrix} = 135 - 8 - 18 - 20 - 108 - 9 = -28$$

$$\Delta_1 = \begin{vmatrix} 8 & -1 & -2 \\ 0 & 5 & -9 \\ 0 & -4 & 9 \\ 8 & -1 & -2 \\ 0 & 5 & -9 \end{vmatrix} = 360 - 288 = 72$$

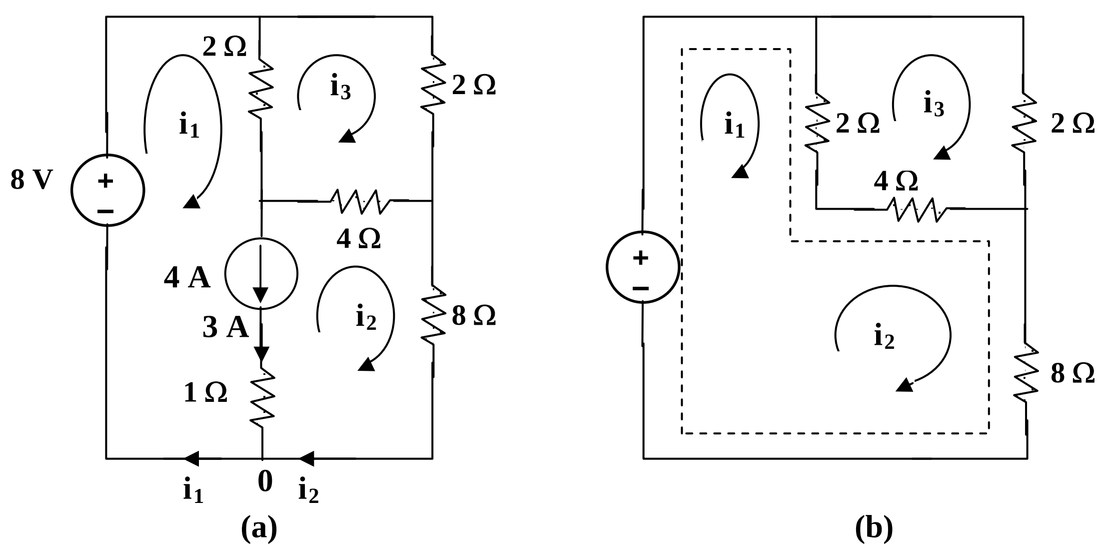
$$\Delta_2 = \begin{vmatrix} 3 & 8 & -2 \\ -1 & 0 & -9 \\ -2 & 0 & 9 \\ 3 & 8 & -2 \\ -1 & 0 & -9 \end{vmatrix} = 144 + 72 = 216$$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 8 \\ -1 & 5 & 0 \\ -2 & -4 & 0 \\ 3 & -1 & 8 \\ -1 & 5 & 0 \end{vmatrix} = 32 + 80 = 112$$

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{72}{-28} = -2.571, \; i_2 = \frac{\Delta_2}{\Delta} = \frac{216}{-28} = -7.714, \; i_3 = \frac{\Delta_3}{\Delta} = \frac{112}{-28} = -4\text{A}$$

$$I_o = i_3 = \mathbf{-4\ A}$$

P.P.3.7



For the supermesh,

$-8 + 2i_1 - 2i_3 + 12i_2 - 4i_3 = 0$ or $i_1 + 6i_2 - 3i_3 = 4$ (1)

For mesh 3,

$8i_3 - 2i_1 - 4i_2 = 0$ or $-i_1 - 2i_2 + 4i_3 = 0$ (2)

At node 0 in Fig. (a),

$i_1 = 4 + i_2 \longrightarrow i_1 - i_2 = 4$

Solving (1) to (3) yields

$i_1 =$ **4.632 A**, $i_2 =$ **631.6 mA**, $i_3 =$ **1.4736 A**

P.P.3.8 $G_{11} = 1/(1) + 1/(20) + 1/(5) = 1.25$, $G_{12} = –1/(5) = –0.2$,
$G_{33} = 1/(4) + 1 = 1.25$, $G_{44} = 1/(1) + 1/(4) = 1.25$,
$G_{12} = –1/(5) = –0.2$, $G_{13} = –1$, $G_{14} = 0$,
$G_{21} = –0.2$, $G_{23} = 0 = G_{26}$,
$G_{31} = –1$, $G_{32} = 0$, $G_{34} = –1/4 = –0.25$,
$G_{41} = 0$, $G_{42} = 0$, $G_{43} = 0.25$,
$i_1 = 0$, $i_2 = 3+2 = 5$, $i_3 = –3$, $i_4 = 2$.

Hence,

$$\begin{bmatrix} \mathbf{1.25} & \mathbf{-0.2} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-0.2} & \mathbf{0.2} & \mathbf{0} & \mathbf{0} \\ \mathbf{-1} & \mathbf{0} & \mathbf{1.25} & \mathbf{-0.25} \\ \mathbf{0} & \mathbf{0} & \mathbf{-0.25} & \mathbf{1.25} \end{bmatrix} \begin{bmatrix} \boldsymbol{v_1} \\ \boldsymbol{v_2} \\ \boldsymbol{v_3} \\ \boldsymbol{v_4} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{5} \\ \mathbf{-3} \\ \mathbf{2} \end{bmatrix}$$

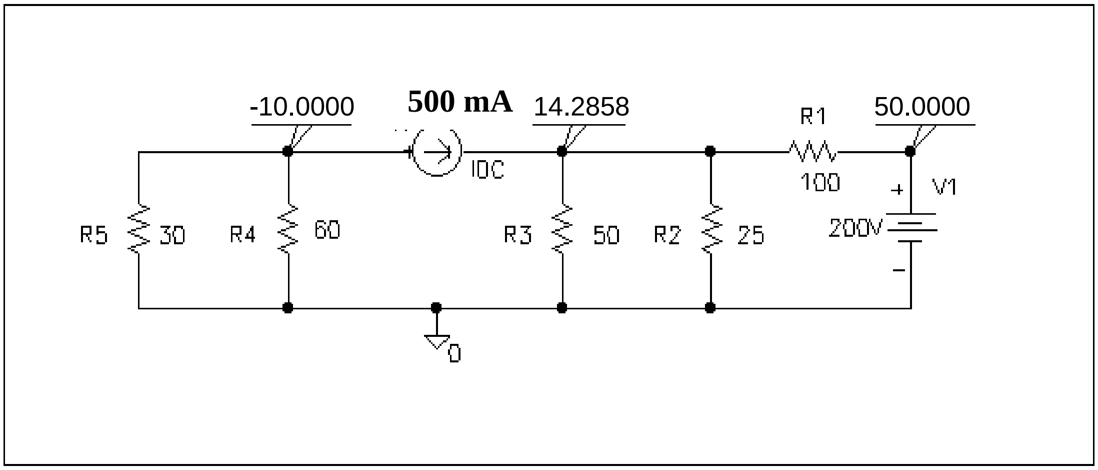
P.P.3.9 $R_{11} = 50 + 20 + 80 = 150$, $R_{22} = 20 + 30 + 15 = 65$,
$R_{33} = 30 + 20 = 50$, $R_{44} = 15 + 80 = 95$,
$R_{55} = 20 + 60 = 80$, $R_{12} = –40$, $R_{13} = 0$, $R_{14} = –80$,
$R_{15} = 0$, $R_{21} = –40$, $R_{23} = –30$, $R_{24} = –15$, $R_{25} = 0$,
$R_{31} = 0$, $R_{32} = –30$, $R_{34} = 0$, $R_{35} = –20$,
$R_{41} = –80$, $R_{42} = –15$, $R_{43} = 0$, $R_{45} = 0$,
$R_{51} = 0$, $R_{52} = 0$, $R_{53} = –20$, $R_{54} = 0$,
$v_1 = 30$, $v_2 = 0$, $v_3 = –12$, $v_4 = 20$, $v_5 = –20$

Hence the mesh-current equations are

$$\begin{bmatrix} \mathbf{150} & \mathbf{-40} & \mathbf{0} & \mathbf{-80} & \mathbf{0} \\ \mathbf{-40} & \mathbf{65} & \mathbf{-30} & \mathbf{-15} & \mathbf{0} \\ \mathbf{0} & \mathbf{-30} & \mathbf{50} & \mathbf{0} & \mathbf{-20} \\ \mathbf{-80} & \mathbf{-15} & \mathbf{0} & \mathbf{95} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-20} & \mathbf{0} & \mathbf{80} \end{bmatrix} \begin{bmatrix} \boldsymbol{i_1} \\ \boldsymbol{i_2} \\ \boldsymbol{i_3} \\ \boldsymbol{i_4} \\ \boldsymbol{i_5} \end{bmatrix} = \begin{bmatrix} \mathbf{30} \\ \mathbf{0} \\ \mathbf{-12} \\ \mathbf{20} \\ \mathbf{-20} \end{bmatrix}$$

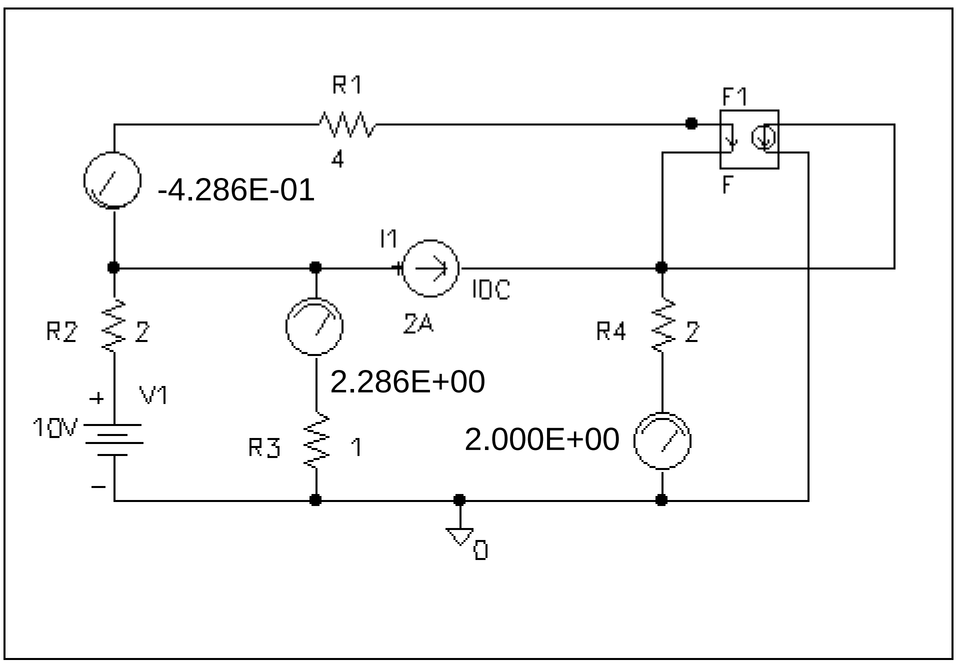
P.P.3.10 The schematic is shown below. It is saved and simulated by selecting Analysis/Simulate. The results are shown on the viewpoints:

$v_1 =$ **–10 V**, $v_2 =$ **14.286 V**, $v_3 =$ **50 V**



P.P.3.11 The schematic is shown below. After saving it, it is simulated by choosing Analysis/Simulate. The results are shown on the IPROBES.

i_1 = **–428.6 mA**, i_2 = **2.286 A**, i_3 = **2 A**



P.P.3.12 For the input loop,

$$-5 + 10 \times 10^3 I_B + V_{BE} + V_0 = 0 \qquad (1)$$

For the outer loop,

$$-V_0 - V_{CE} - 500 I_0 + 12 = 0 \qquad (2)$$

But $V_0 = 200 I_E$ (3)

Also $I_C = \beta I_B = 100 I_B,\ \alpha = \beta/(1+\beta) = 100/(101)$

$$I_C = \alpha I_E \longrightarrow I_E = I_C/(\alpha) = \beta I_B/(\alpha)$$

$$I_E = 100\,(101/(100))\, I_R = 101\, I_B \qquad (4)$$

From (1), (3) and (4),

$$10{,}000\, I_B + 200(101)\, I_R = 5 - V_{BE}$$

$$I_B = \frac{5-0.7}{10{,}000+20{,}000} = 142.38\mu A$$

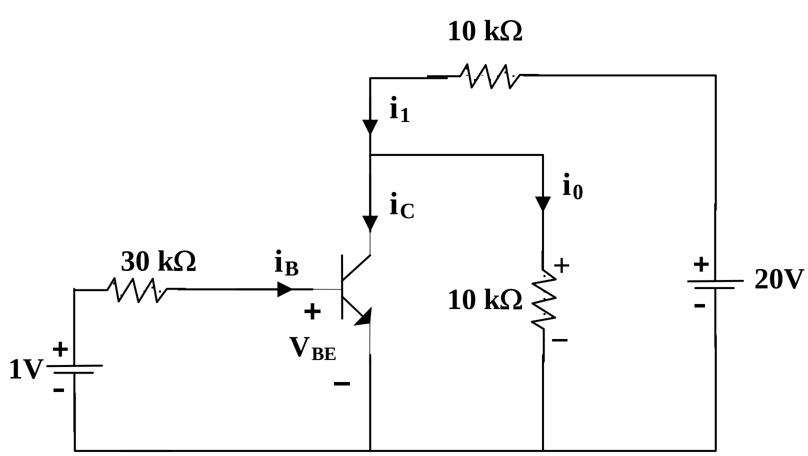
$V_0 = 200 I_E = 20{,}000 I_B =$ **2.876 V**

From (2),

$$V_{CE} = 12 - V_0 - 500 I_C = 9.124 - 500 \times 100 \times 142.38 \times 10^{-6}$$

$V_{CE} =$ **1.984 V {often, this is rounded to 2.0 volts}**

P.P.3.13



$$i_B = \frac{1-0.7}{30k} = 10\mu A,\ \ i_C = \beta i_B = 0.8 \text{ mA}$$

$$i_1 = i_C + i_0 \qquad (1)$$

Also, $-10ki_0 - 10ki_1 + 20 = 0 \longrightarrow i_1 = 2 \text{ mA} - i_0$ (2)

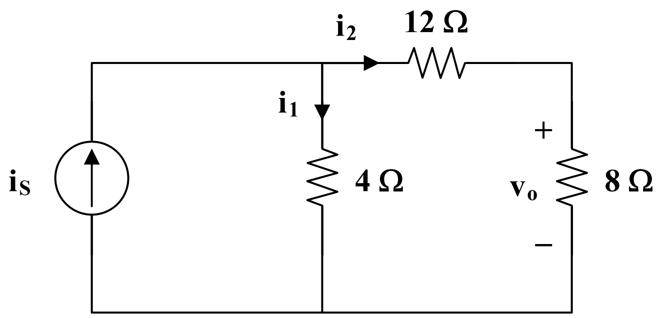
Equating (1) and (2),

$$2 \text{ mA} - i_0 = 0.8 \text{ mA} + i_0 \longrightarrow i_0 = \mathbf{600\ \mu A}$$

$$v_0 = 20\ ki_0 = 20x10^3\ x600x10^{-6} = \mathbf{12\ V}$$

CHAPTER 4

P.P.4.1



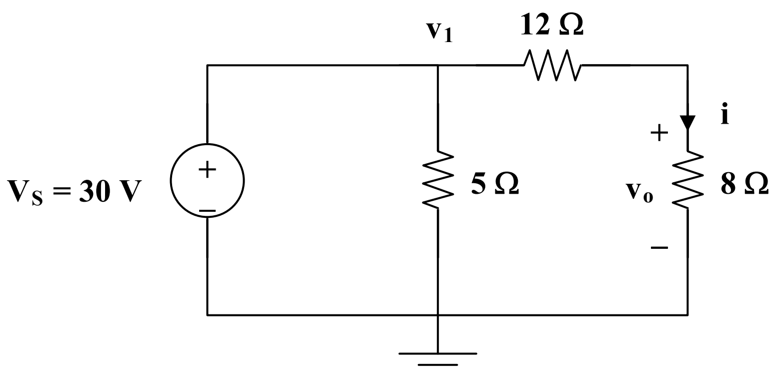
By current division, $i_2 = \dfrac{4}{4+12+8} i_s = \dfrac{1}{6} i_s$

$$v_0 = 8i_2 = \frac{4}{3} i_s$$

When i_s = 30 A, $v_0 = \dfrac{4}{3}(30) = \mathbf{40\ V}$

When i_s = 45 A, $v_0 = \dfrac{4}{3}(45) = \mathbf{60\ V}$

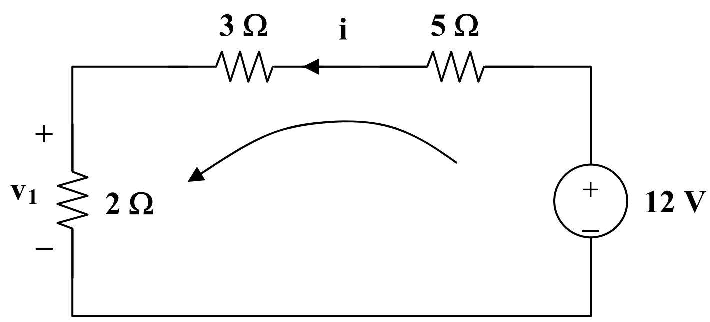
P.P.4.2



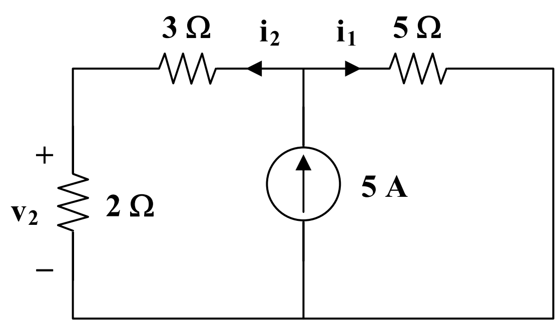
Let v_o = 1 volt. Then $i = \dfrac{1}{8}$ and $v_1 = \dfrac{1}{8}(12 + 8) = 2.5$
giving v_s = 2.5V.

If v_s = 40V, then v_0 = (40/2.5)(1) = **16 V**

P.P.4.3 Let $v_0 = v_1 + v_2$, where v_1 and v_2 are contributions to the 12–V and 5–A sources respectively.



(a)



(b)

To get v_1, consider the circuit in Fig. (a).

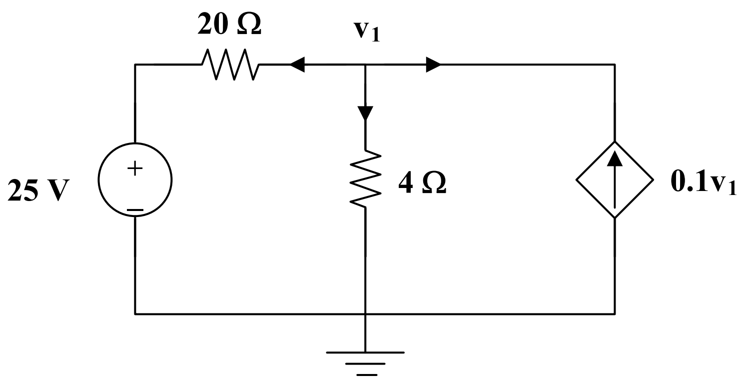
$(2 + 3 + 5)i – 12 = 0$ or $i = 12/(10) = 1.2$ A
$v_1 = 2i = 2.4$ V

To get v_2, consider the circuit in Fig. (b).

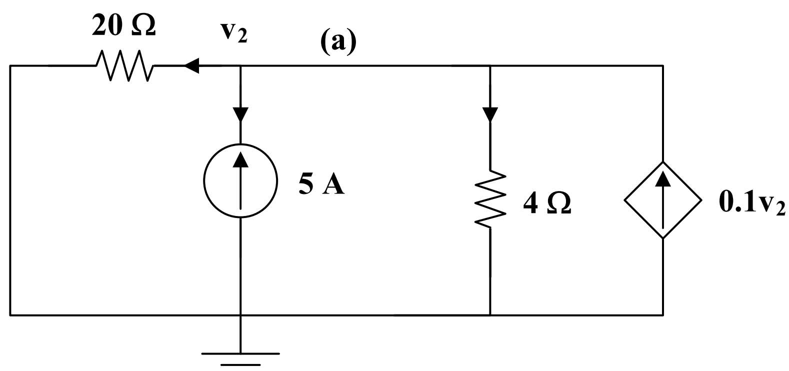
Since the resistors are equal (5 = 2 + 3) then the current divides equally and
$i_1 = i_2 = 5/2 = 2.5$ and $v_2 = 2i_2 = 5$ V

Thus,
$v = v_1 + v_2 = 2.4 + 5 = \mathbf{7.4\ V}$

P.P.4.4 Let $v_x = v_1 + v_2$, where v_1 and v_2 are due to the 25-V and 5-A sources respectively.



(a)



(b)

To obtain v_1, consider Fig. (a).

$$-0.1v_1 + \frac{v_1 - 25}{20} + \frac{v_1 - 0}{4} = 0 \quad \text{or} \quad 0.2v_1 = 25/20 = 1.25 \text{ or } v_1 = 6.25 \text{ V}$$

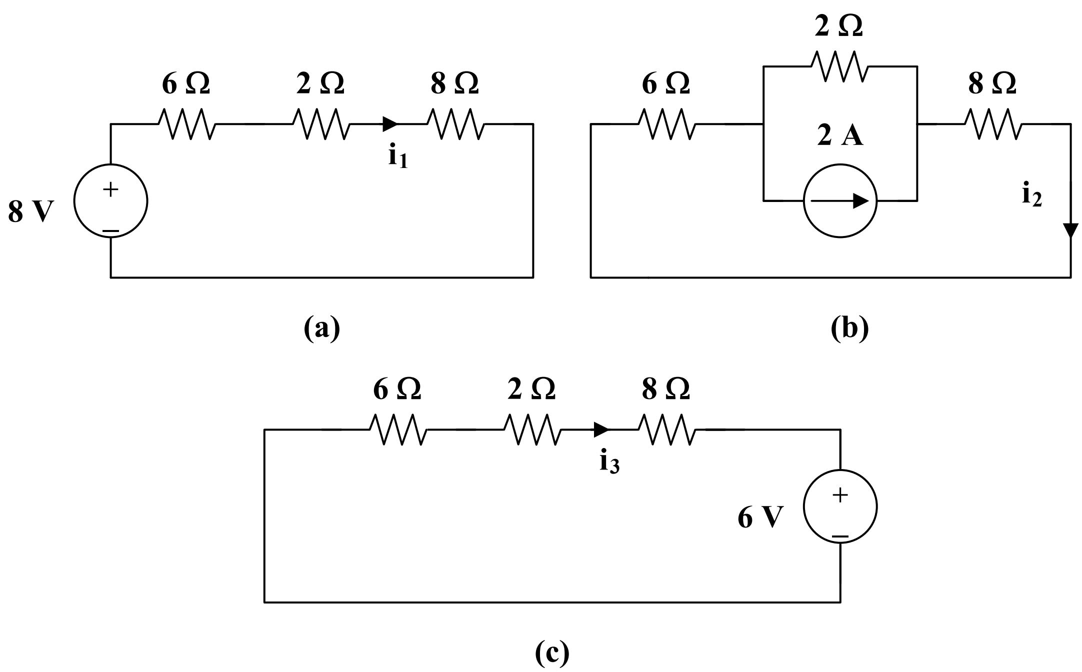
For v_2, consider Fig. (b).

$$-5 - 0.1v_2 + \frac{v_2 - 0}{20} + \frac{v_2 - 0}{4} = 0 \text{ or } 0.2v_2 = 5 \text{ or } v_2 = 25 \text{ V}$$

$v_x = v_1 + v_2 =$ **31.25 V**

P.P.4.5 Let $i = i_1 + i_2 + i_3$

where i_1, i_2, and i_3 are contributions due to the 8-V, 2-A, and 6-V sources respectively.



For i_1, consider Fig. (a), $i_1 = \dfrac{8}{6+2+8} = 0.5\,\text{A}$

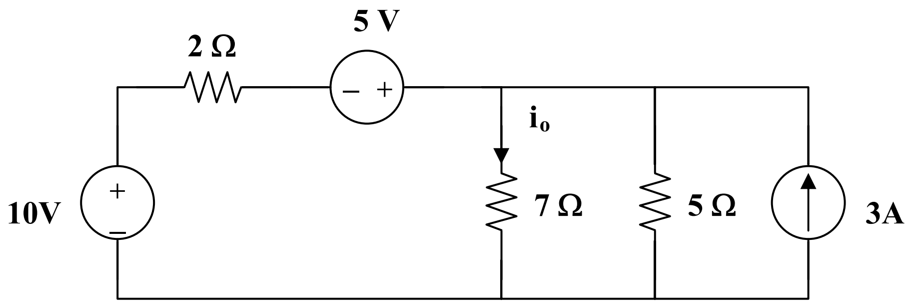
For i_2, consider Fig. (b). By current division, $i_2 = \dfrac{2}{2+14}(2) = 0.25$

For i_3, consider Fig. (c), $i_3 = \dfrac{-6}{16} = -0.375\text{A}$

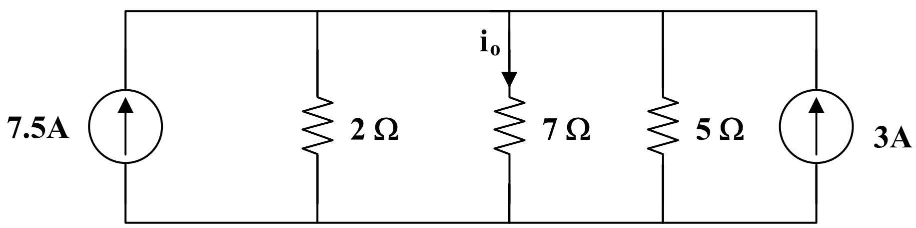
Thus, $i = i_1 + i_2 + i_3 = 0.5 + 0.25 - 0.375 =$ **375 mA**

P.P.4.6 Combining the 6-Ω and 3-Ω resistors in parallel gives $6 \| 3 = \dfrac{6\text{x}3}{9} = 2\Omega$.

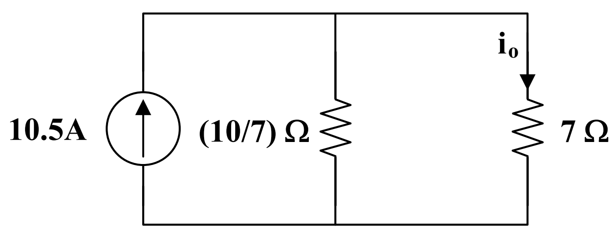
Adding the 1-Ω and 4-Ω resistors in series gives 1 + 4 = 5Ω. Transforming the left current source in parallel with the 2-Ω resistor gives the equivalent circuit as shown in Fig. (a).



(a)



(b)



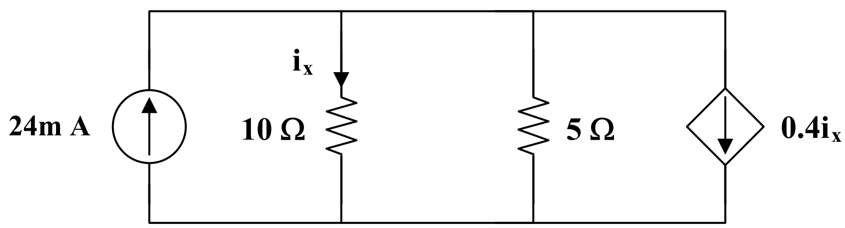
(c)

Adding the 10-V and 5-V voltage sources gives a 15-V voltage source. Transforming the 15-V voltage source in series with the 2-Ω resistor gives the equivalent circuit in Fig. (b). Combining the two current sources and the 2-Ω and 5-Ω resistors leads to the circuit in Fig. (c). Using circuit division,

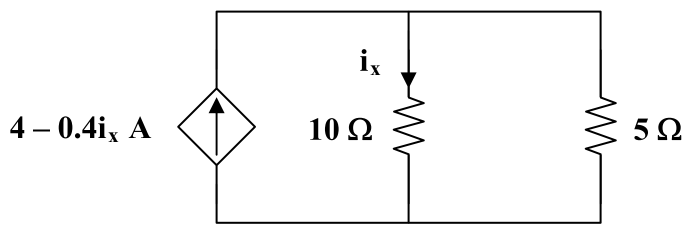
$$i_o = \frac{\frac{10}{7}}{\frac{10}{7} + 7}(10.5) \quad = \mathbf{1.78\ A}$$

P.P.4.7 We transform the dependent voltage source as shown in Fig. (a). We combine the two current sources in Fig. (a) to obtain Fig. (b). By the current division principle,

$$i_x = \frac{5}{15}(0.024 - 0.4i_x) \longrightarrow i_x = \mathbf{7.059\ mA}$$

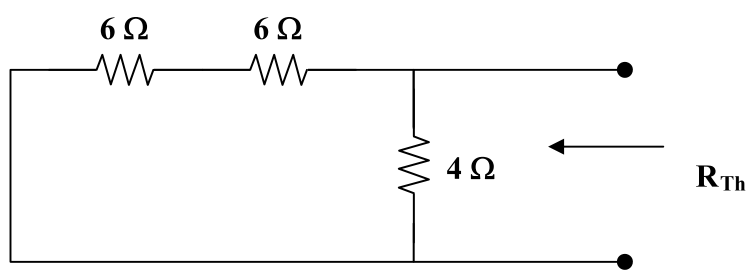


(a)

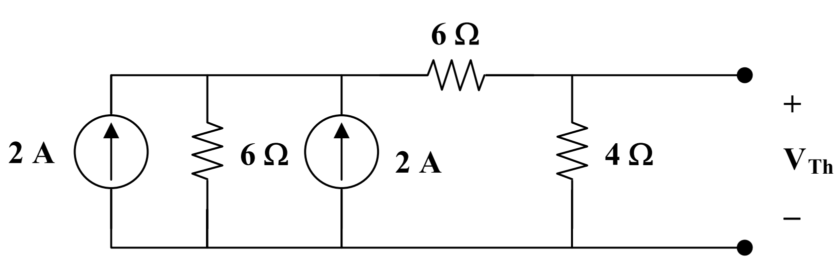


(b)

P.P.4.8 To find R_{Th}, consider the circuit in Fig. (a).



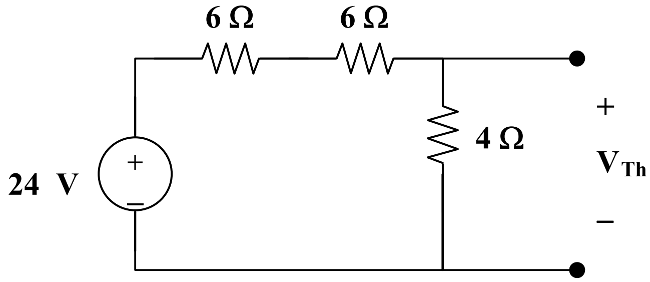
(a)



(b)

$$R_{Th} = (6+6)\|4 = \frac{12x4}{18} = \mathbf{3\Omega}$$

To find V_{Th}, we use source transformations as shown in Fig. (b) and (c).



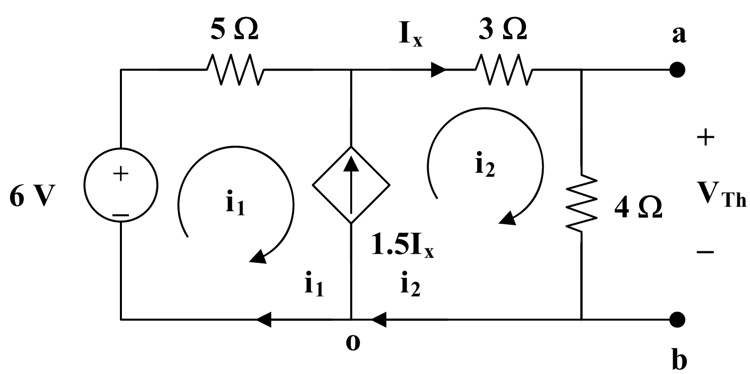
(c)

Using current division in Fig. (c),

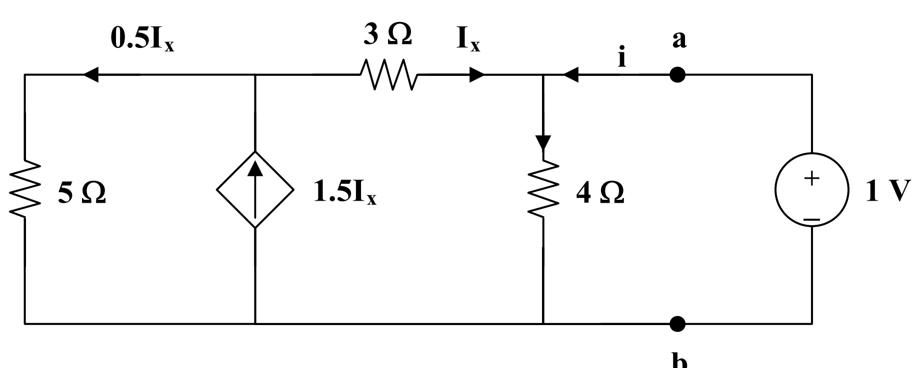
$$V_{Th} = \frac{4}{4+12}(24) = \mathbf{6\ V}$$

$$i = \frac{V_{Th}}{R_{Th}+1} = \frac{6}{3+1} = \mathbf{1.5\ A}$$

P.P.4.9 To find V_{Th}, consider the circuit in Fig. (a).



(a)



(b)

$I_x = i_2$

$i_2 - i_1 = 1.5I_x = 1.5i_2 \longrightarrow i_2 = -2i_1$ (1)

For the supermesh, $-6 + 5i_1 + 7i_2 = 0$ (2)

From (1) and (2), $i_2 = 4/(3)$A

$V_{Th} = 4i_2 = \mathbf{5.333V}$

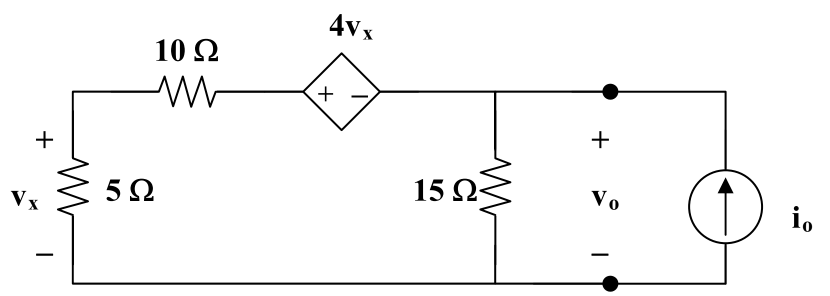
To find R_{Th}, consider the circuit in Fig. (b). Applying KVL around the outer loop,

$$5(0.5I_x) - 1 - 3I_x = 0 \longrightarrow I_x = -2$$

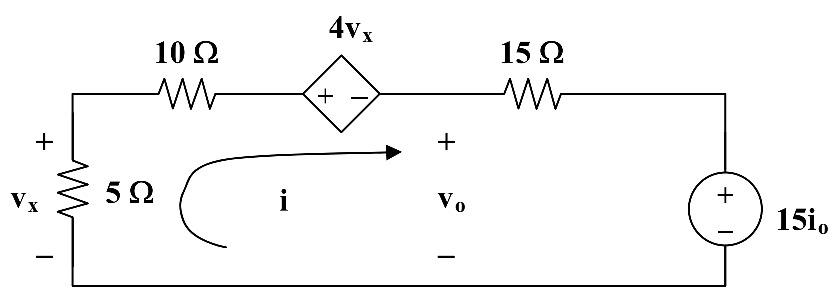
$$i = \frac{1}{4} - I_x = 2.25$$

$$R_{Th} = \frac{1}{i} = \frac{1}{2.25} = \mathbf{444.4\ m\Omega}$$

P.P.4.10 Since there are no independent sources, $V_{Th} = \mathbf{0}$



(a)

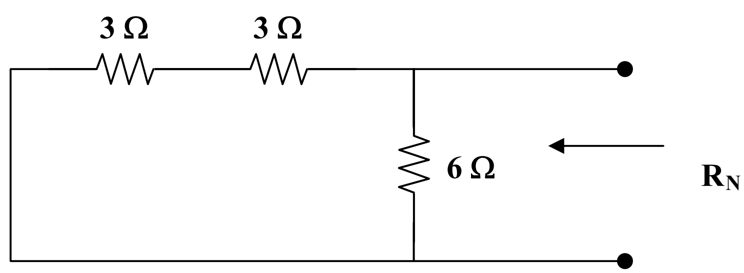


(b)

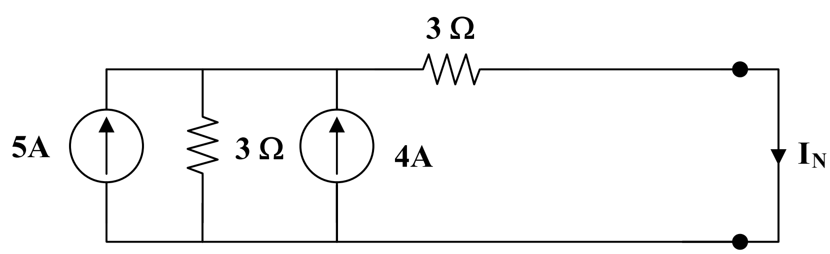
To find R_{Th}, consider Fig.(a). Using source transformation, the circuit is transformed to that in Fig. (b). Applying KVL,).

But v_x = -5i. Hence, 30i - 20i + 15i_o = 0 ⟶ 10i = -15i_o
$v_o = (15i + 15i_o) = 15(-1.5i_o + i_o) = -7.5i_o$
$R_{Th} = v_o/(i_o) =$ **–7.5Ω It needs to be noted that this negative resistance indicates we must have an active source (a dependent source).**

P.P.4.11



(a)

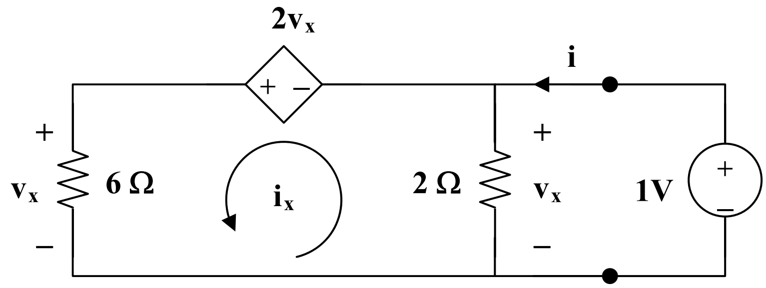


(b)

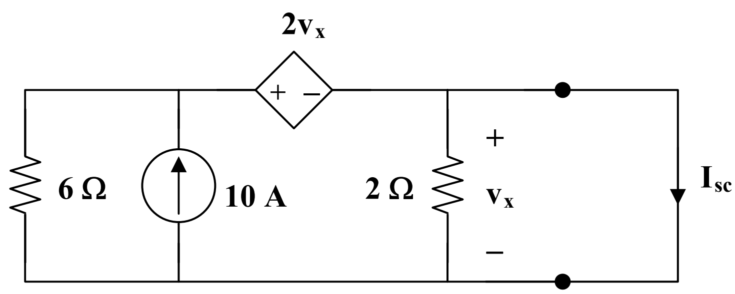
From Fig. (a), $R_N = (3+3)\|6 =$ **3 Ω**

From Fig. (b), $I_N = \frac{1}{2}(5+4) =$ **4.5A**

P.P.4.12



(a)



(b)

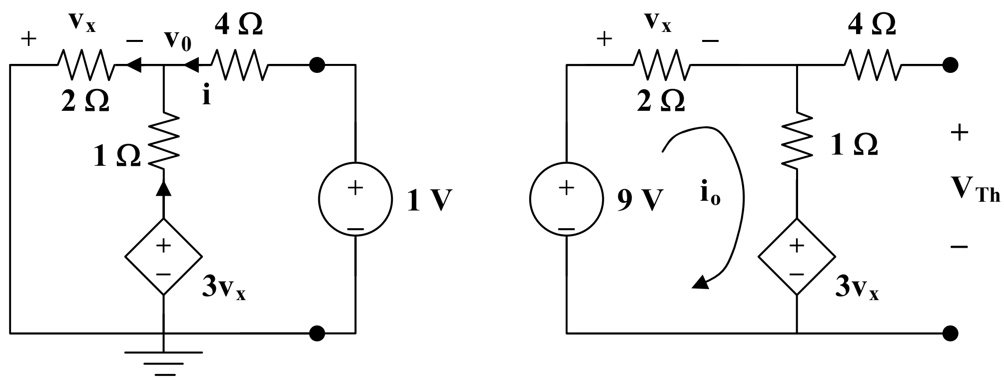
To get R_N consider the circuit in Fig. (a). Applying KVL, $6i_x - 2v_x - 1 = 0$
But $v_x = 1$, $\quad 6i_x = 3 \longrightarrow \quad i_x = 0.5$

$$i = i_x + \frac{v_x}{2} = 0.5 + 0.5 = 1$$

$$R_N = R_{Th} = \frac{1}{i} = \mathbf{1\Omega}$$

To find I_N, consider the circuit in Fig. (b). Because the 2Ω resistor is shorted, $v_x = 0$ and the dependent source is inactive. Hence, $I_N = i_{sc} =$ **10A**.

P.P.4.13 We first need to find R_{Th} and V_{Th}. To find R_{Th}, we consider the circuit in Fig. (a).



(a) **(b)**

Applying KCL at the top node gives

$$\frac{1-v_o}{4}+\frac{3v_x-v_o}{1}=\frac{v_o}{2}$$

But $v_x = -v_o$. Hence

$$\frac{1-v_o}{4}-4v_o=\frac{v_o}{2} \longrightarrow v_o = 1/(19)$$

$$i=\frac{1-v_o}{4}=\frac{1-\frac{1}{19}}{4}=\frac{9}{38}$$

$R_{Th} = 1/i = 38/(9) = 4.222\Omega$

To find V_{Th}, consider the circuit in Fig. (b),

$$-9 + 2i_o + i_o + 3v_x = 0$$

But $v_x = 2i_o$. Hence,

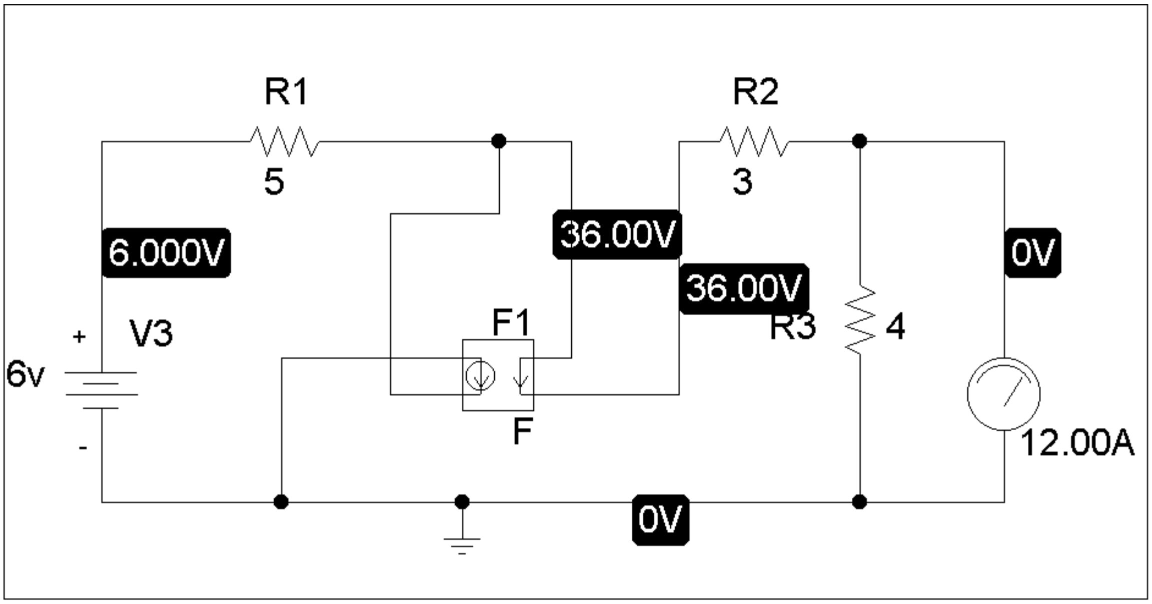
$$9 = 3i_o + 6i_o = 9i_o \longrightarrow i_o = 1A$$

$$V_{Th} = 9 - 2i_o = 7V$$

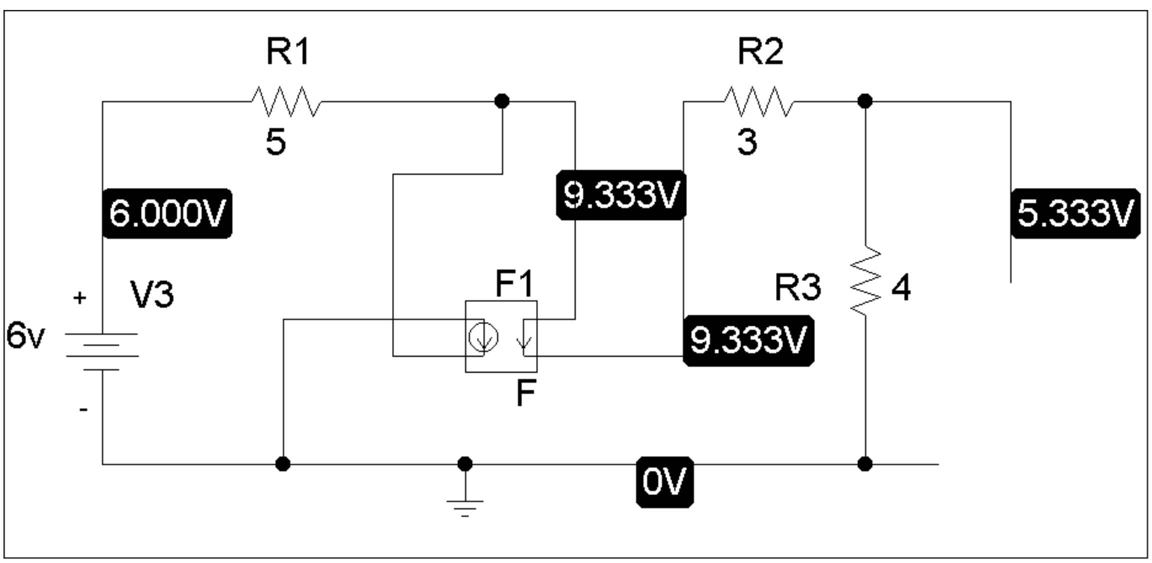
$R_L = R_{Th} =$ **4.222 Ω**

$$P_{max} = \frac{v_{Th}^2}{4R_L}=\frac{49}{4(4.222)}=\mathbf{2.901\ W}$$

P.P.4.14 We will use PSpice to find V_{oc} and I_{sc} which then can be used to find V_{Th} and R_{th}.

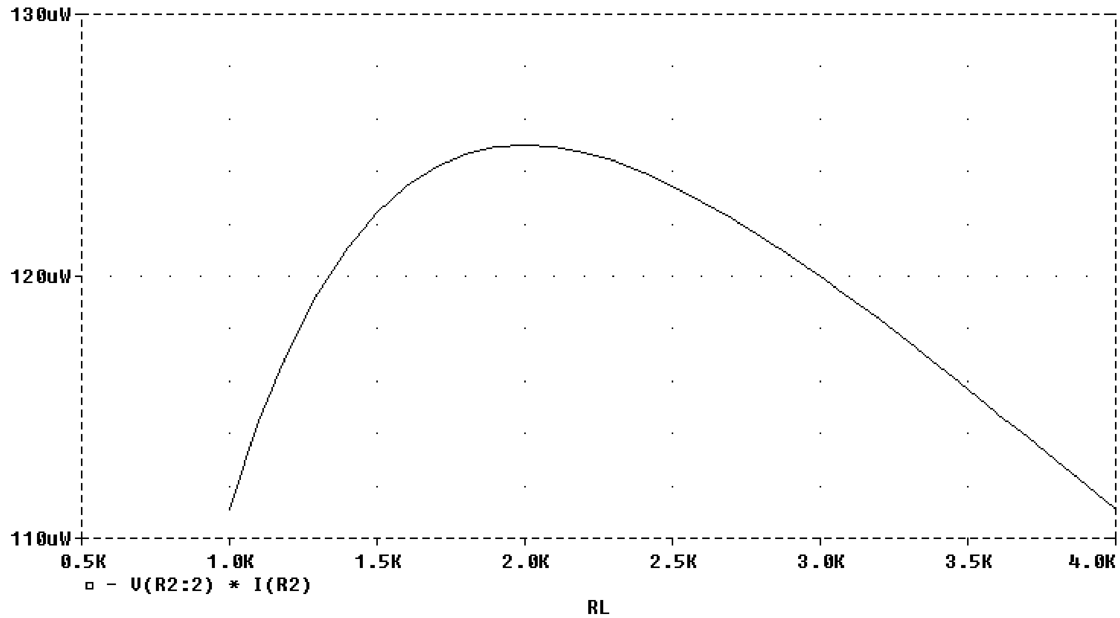


Clearly I_{sc} = 12 A

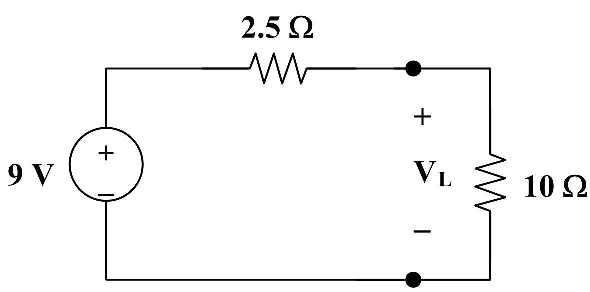


Clearly V_{Th} = I_{oc} = **5.333 volts**. R_{Th} = Voc/Isc = 5.333/12 = **444.4 mΩ**.

P.P.4.15 The schematic is the same as that in Fig. 4.56 except that the 1-kΩ resistor is replaced by 2-kΩ resistor. The plot of the power absorbed by R_L is shown in the figure below. From the plot, it is clear that the maximum power occurs when R_L = 2kΩ and it is **125 μW**.



P.P.4.16 V_{Th} = 9V, $R_{Th} = \left(v_{oc} - V_L\right)\dfrac{R_L}{V_L} = (9-1)\dfrac{20}{8} = 2.5\Omega$



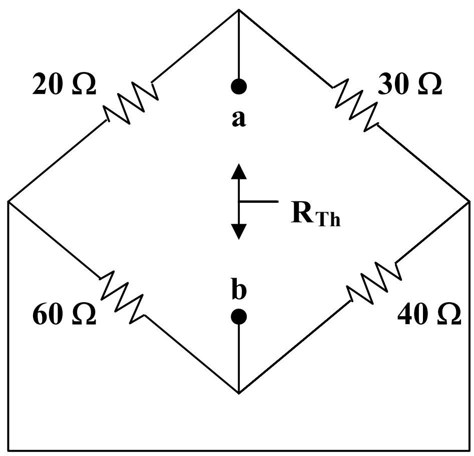
$$V_L = \frac{10}{10+2.5}(9) = \mathbf{7.2\ V}$$

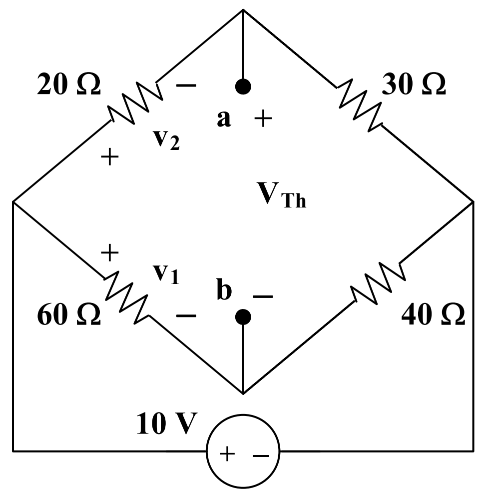
P.P.4.17 $R_1 = R_3 = 1k\Omega$, $R_2 = 3.2k\Omega$

$$R_x = \frac{R_3}{R_1}R_2 = R_2 = \mathbf{3.2\ k\Omega}$$

P.P.4.18 We first find R_{Th} and V_{Th}. To get R_{Th}, consider the circuit in Fig. (a).

$$R_{Th} = 20\|30 + 60\|40 = \frac{20x30}{50} + \frac{60x40}{100}$$
$$= 12 + 24 = 36\Omega$$





(a) **(b)**

To find V_{Th}, we use Fig. (b). Using voltage division,

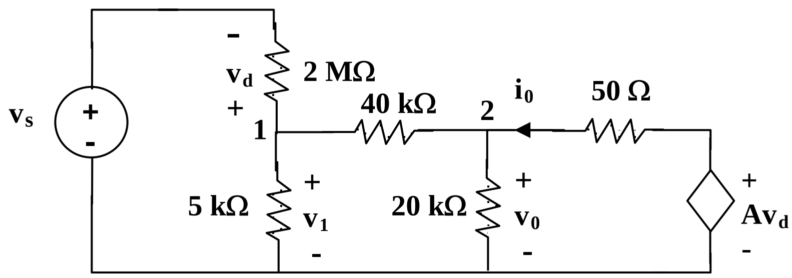
$$v_1 = \frac{60}{100}(16) = 9.6, \quad v_2 = \frac{20}{50}(16) = 6.4$$

But $-v_1 + v_2 + v_{Th} = 0$ $\longrightarrow$ $v_{Th} = v_1 - v_2 = 9.6 - 6.4 = 32V$

$$I_G = \frac{V_{Th}}{R_{Th} + R_m} = \frac{3.2}{3.6 + 1.4} = \mathbf{64mA}$$

CHAPTER 5

P.P.5.1 The equivalent circuit is shown below:



At node 1, $\frac{v_s - v_1}{2x10^6} = \frac{v_1}{5x10^3} + \frac{v_1 - v_0}{40x10^3} \longrightarrow v_1 = \frac{v_S + 50v_0}{451}$ (1)

At node 2, $\frac{Av_d - v_0}{50} + \frac{v_1 - v_0}{40x10^3} = \frac{v_0}{20x13^3}$

But $v_d = v_1 - v_S$.

$[2 \text{ x } 10^5 (v_1 - v_S) - v_0]\ 4000/(5) + v_1 - v_0 = 2v_0$

$1600 \text{ x } 10^5 (v_S - v_1) + 803v_0 \cong 0$ (2)

Substituting v_1 in (1) into (2) gives

$1.5914523 \text{ x } 10^8\ v_S - 17737556v_0 = 0$

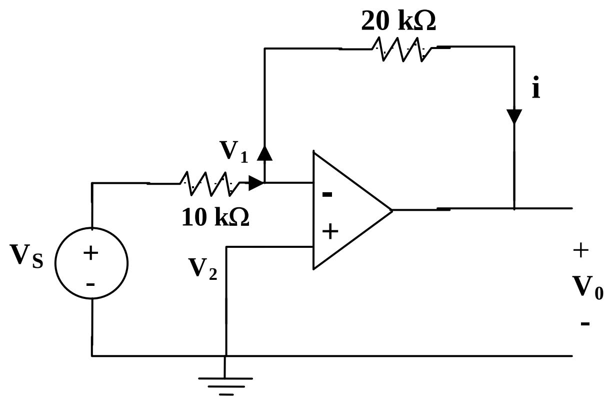
$$\frac{v_0}{v_S} = \frac{1.5964523x10^8}{17737556} = \mathbf{9.00041}$$

If $v_S = 1$ V, $v_0 = 9.00041$ V, $v_1 = 1.0000455$

$v_d = v_S - v_1 = -4.545 \text{ x } 10^{-5}$

$Av_d = -9.0909$, $i_0 = \frac{Av_d - v_0}{50} =$ **657 μA**

P.P.5.2



At node 1, $\frac{v_S - v_1}{10} = \frac{v_1 - v_0}{20}$

But $v_1 = v_2 = 0$,

$$\frac{v_S}{10} = -\frac{v_0}{20} \longrightarrow \frac{v_0}{v_S} = \mathbf{-2}$$

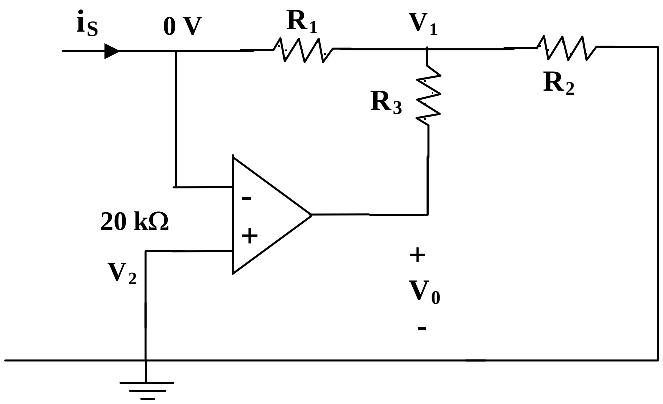
$$i_0 = \frac{0 - v_0}{20x10^3} = -\frac{v_0}{20x10^3}$$

When v_s = 2V, v_0 = -4, $i_0 = \frac{4x10^{-3}}{20}$ = **200 μA**

P.P.5.3 $\quad v_0 = -\frac{R_2}{R_1} v_i = \frac{-280}{4}(45\text{mV}) =$ **–3.15 V**

$$i = \frac{0 - v_0}{120k} = \mathbf{26.25\ \mu A}$$

P.P.5.4 $\quad$ (a) $i_S = \frac{0 - v_0}{R} \longrightarrow \boldsymbol{\frac{v_0}{i_S} = -R}$



(b) At node 2, $i_S = \dfrac{0 - v_1}{R_1} \longrightarrow \quad v_1 = -i_S R_1$ (1)

At node 1, $\dfrac{0 - v_1}{R_1} = \dfrac{v_1 - 0}{R_2} + \dfrac{v_1 - v_0}{R_3}$

$$-v_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{-v_0}{R_3}$$

$$v_0 = -i_S R_1 R_3 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\boldsymbol{\frac{v_0}{i_S} = -R_1\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right)}$$

P.P.5.5 By voltage division

$$v_1 = \frac{8}{4+8}(3) = 2\text{V}$$

where v_1 is the voltage at the top end of the 8kΩ resistor. Using the formula for noninverting amplifier,

$$v_0 = \left(1 + \frac{5}{2}\right)(2) = \mathbf{7\ V}$$

P.P.5.6 This is a summer.

$$v_0 = -\left[\frac{8}{20}(1.5) + \frac{8}{10}(2) + \frac{8}{6}(1.2)\right] = \mathbf{-3.8\ V}$$

$$i_0 = \frac{v_0}{8} + \frac{v_0}{4} = -\frac{3.8}{8} - \frac{3.8}{4} = \mathbf{-1.425\ mA}$$

P.P.5.7 If the gain is 7.5, then

$$\frac{R_2}{R_1} = 7.5 \longrightarrow R_2 = 7.5R_1$$

But $\frac{R_2}{R_1} = \frac{R_4}{R_3} \longrightarrow R_4 = 7.5R_3$

If we select $R_1 = R_3 =$ **20kΩ**, then $R_2 = R_4 =$ **150 kΩ**.

P.P.5.8 $v_0 = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$

$R_3 = 0$, $R_4 = \infty$, $R_2 = 40\text{k}\Omega$, $R_1 = 20\text{k}\Omega$

$$v_0 = \frac{40}{20}(6.98 - 7) = -0.04\,\textit{volts.}$$

$$i_0 = \frac{v_0}{50} = \frac{-0.04}{50} = \mathbf{-800\ \mu A}.$$

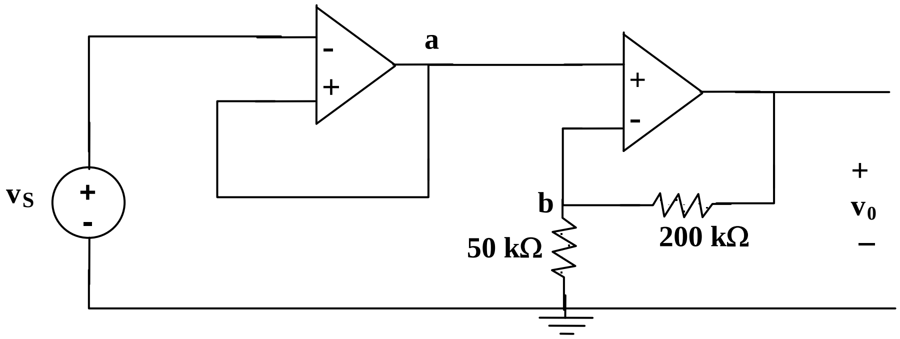
P.P.5.9 Due to the voltage follower

$v_a = 1.2$ V

For the noninverting amplifier,

$$v_o = \left(1 + \frac{200}{50}\right)v_a \quad v_o = (1 + 4)\,(1.2) = \mathbf{6\ V}.$$

$$i_0 = \frac{v_b}{50}\,\text{mA}$$



But $v_b = v_a = 4$

$$i_0 = \frac{1.2}{50x10^3} = \mathbf{24\ \mu A}.$$

P.P.5.10 As a voltage follower,

$$v_a = v_1 = 7\text{ V}$$

where v_a is the voltage at the left end of the 20 kΩ resistor.

As an inverter, $v_b = -\frac{50}{10}v_2 = -15.5\text{V}$

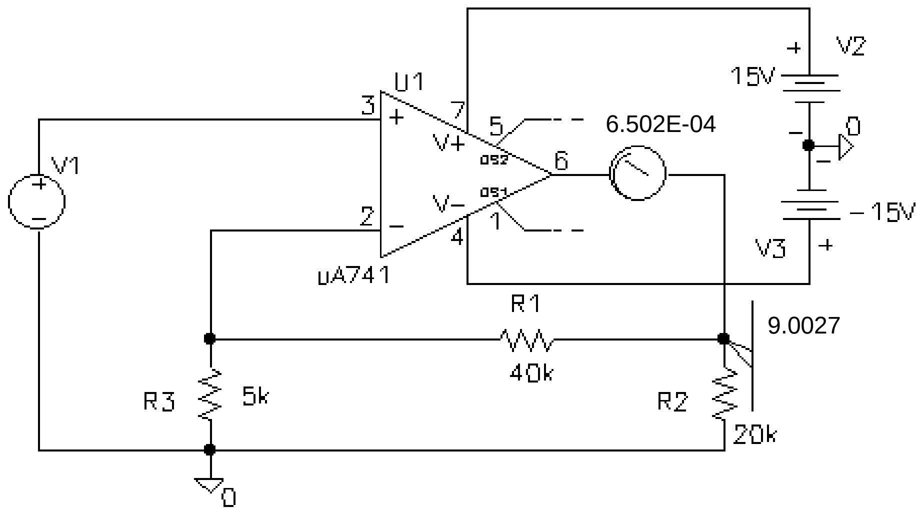
Where v_b is the voltage at the right end of the 50 kΩ resistor. As a summer

$$v_0 = -\left[\frac{60}{20}v_a + \frac{60}{30}v_b\right]$$

$$= [-21 + 31] = \mathbf{10\ V}.$$

P.P.5.11 The schematic is shown below. When it is saved and run, the results are displayed on 1PROBE and VIEWPOINT as shown. By making $v_s = 1\text{V}$, we obtain

$$v_0 = \mathbf{9.0027V} \text{ and } i_0 = \mathbf{650.2\ \mu A}$$



P.P.5.12 $-V_0 = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3$

or $\quad |V_0| = V_1 + 0.5V_2 + 0.25V_3$

(a) If $[V_1V_2V_3] = [010]$, $|V_0| =$ **0.5V**

(b) If $[V_1V_2V_3] = [110]$, $|V_0| = 1 + 0.5 =$ **1.5V**

(c) If $|V_0| = 1.25$, then $V_1 = 1$, $V_2 = 0$, $V_3 = 1$, i.e.
$[V_1V_2V_3] =$ **[101]**

(d) $|V_0| = 1.75$, then $V_1 = 1$, $V_2 = 1$, $V_3 = 1$, i.e.
$[V_1V_2V_3] =$ **[111]**

P.P.5.13 $A_v = 1 + \frac{2R}{R_G} \longrightarrow R_G = \frac{2R}{A_v - 1}$

$$R_G = \frac{2x25x10^3}{142-1} = \mathbf{354.6\ \Omega}$$

CHAPTER 6

P.P.6.1 $v = \frac{q}{C} = \frac{120x10^{-6}}{4.5x10^{-6}} =$ **26.67 V**

$w = \frac{1}{2}Cv^2 = \frac{1}{2}x4.5x10^{-6}x711.1 =$ **1.6 mJ**.

P.P.6.2 $i(t) = C\frac{dv}{dt} = 10x10^{-6}\frac{d}{dt}(75\sin(2000t))$

= **1.5cos(2000t) A**.

P.P.6.3 $v = \frac{1}{C}\int_0^t idt = \frac{10^{-3}}{0.1x10^{-3}}\int_0^t 50\sin 120\pi t\ dt\ V$

$= -\frac{500}{120\pi}\cos 120\pi t\Big|_0^t = \frac{50}{12\pi}(1-\cos 120\pi t)V$

v(t = 1ms) = $\frac{50}{12\pi}(1-\cos 0.12\pi) =$ **93.14 mV**

v(t = 5ms) = $\frac{50}{12\pi}(1-\cos 0.6\pi) =$ **1.736 V**

P.P.6.4 $i(t) = \begin{cases} 50t, & 0 \langle\ t \ \langle\ 2 \\ 100, & 2 \langle\ t \ \langle\ 6 \end{cases}$

$v = \frac{1}{C}\int idt = \frac{1}{10^{-3}}\int idt \cdot 10^{-3} = \int idt$

For 0< t <2, $v = \frac{1}{C}\int_0^t 50t\ dt = 25t^2 \text{ x } 10^3$

For 2< t <6, $v = \frac{1}{C}\int_2^t 100dt + v(2) = (100t - 0.2 + 0.1)$

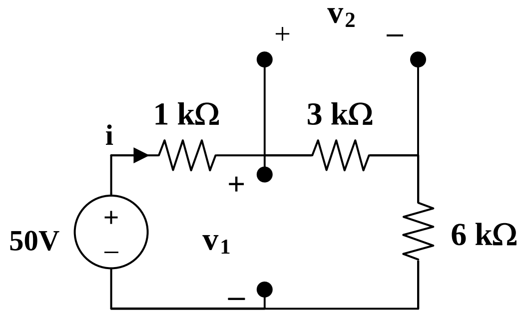
= (100t - 0.1)V

At t = 2ms, v = **100 mV**

At t = 5ms, v = (500 - 100)mV

= **400 mV**

P.P.6.5 Under dc conditions, the capacitors act like open-circuits as shown below:



$$i = \frac{50}{1+3+6} = 5\text{mA}$$

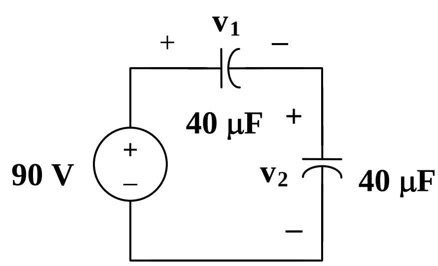
$$v_1 = (3k + 6k)i = 45\text{V}$$
$$v_2 = (3k)i = 15\text{V}$$
$$w_1 = \frac{1}{2}C_1 v_1^2 = \frac{1}{2}\left(20\text{x}10^{-6}\right)(45)^2 = \mathbf{20.25\ mJ}$$
$$w_2 = \frac{1}{2}C_2 v_2^2 = \frac{1}{2}\left(30\text{x}10^{-6}\right)(15)^2 = \mathbf{3.375\ mJ}.$$

P.P.6.6 Combining 60 and 120μF in series = $\frac{60\text{x}120}{180}$ = 40μF

40μF in parallel with 20μF = 40 + 20 = 60μF
50μF in parallel with 70μF = 50 + 70 = 120μF
60μF in series with 120μF = $\frac{60\text{x}120}{180}$ = **40μF**

P.P.6.7 Before we solve this, we need to assume that the initial charge on each capacitor is equal to zero.

60μF in series with 30μF = $\frac{60\text{x}30}{90}$ = 20μF
20μF in parallel with 20μF = 40μF



From the Figure, $v_1 = v_2 = 90/2 =$ **45 V**; $q_1 = 45\text{x}40\text{x}10^{-6} = 1.8$ mC; $q_2 = 45\text{x}20\text{x}10^{-6} =$ 0.9 mC $= q_3 = q_4$ leading to $v_3 = 0.0009/(60\text{x}10^{-6}) =$ **15 V** and
$v_4 = 0.0009/(30\text{x}10^{-6}) =$ **30 V**.

P.P.6.8 $v = L\frac{di}{dt} = 10^{-3}\frac{d}{dt}(60\cos(100t))\cdot 10^{-3}$

= **–6sin(100t) mV**

$$w = \frac{1}{2}Li^2 = \frac{1}{2}x10^{-3}\left(3600\cos^2(100t)\right)\cdot 10^{-6}$$

= **1.8cos ^{2}(100t) μJ**.

P.P.6.9 $i = \frac{1}{L}\int_{t_0}^{t} v(t)dt + i(t_0) = \frac{1}{2}\int_{0}^{t} 10(1-t)dt + 2$

$$= 5\left(t - \frac{t^2}{2}\right) + 2$$

At t = 4, i = 5(4 - 8) + 2 = **–18 A**

p = vi = 10(1 - t) $\left[5t - \frac{5}{2}t^2 + 2\right]$

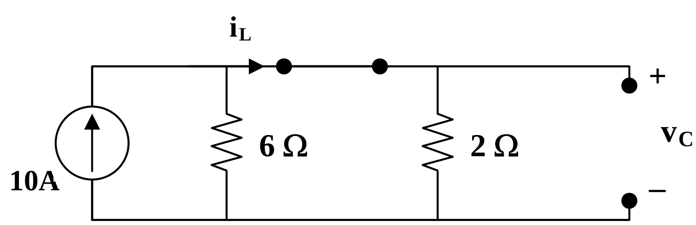
= 20 + 30t - 75t^2 + 25t^3

w = $\int_0^4 p\,dt$ = [20t + 15t^2 - 25t^3 + $25t^4/4$]$\Big|_0^4$

= 80 + 15 x 16 - 1600 + 1600

w = **320 J**

P.P.6.10 Under dc conditions, the circuit is equivalent to that shown below



$$i_L = \frac{3}{1+3}(10) = \mathbf{7.5\ A}$$

$v_C = 2i_C$ = **15 V**

$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(4)(15)^2 =$ **450 J**

$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(6)(7.5)^2 =$ **168.75 J**.

P.P.6.11 40mH in series with 20mH = 40 + 20 = 60mH
60mH in parallel with 30mH = 30 x 60/(90) = 20mH
20mH in series with 100mH = 120mH
120mH in parallel with 40mH = 40 x 120/(160) = 30mH
30mH in series with 20mH = 50mH
50mH in parallel with 50mH = 25mH

L_{eq} = **25mH**

P.P.6.12 (a) $i_2 = i - i_1 \longrightarrow i_2(0) = i(0) - i_1(0) = 1.4 - 0.6 =$ **800 mA**

(b) $v_1 = 6\dfrac{di_1}{dt} = 6(0.6)(-2)e^{-2t} = -7.2e^{-2t}$

$$i_2 = \frac{1}{3}\int_0^t v_1 dt + i_2(0) = \frac{1}{3}\frac{(-7.2)}{(-2)}e^{-2t}\Big|_0^t + 0.8$$

$= \mathbf{(-0.4 + 1.2e^{-2t})\ A}$

$i = i_1 + i_2 = \mathbf{(-0.4 + 1.8e^{-2t})\ A}$

(c) From (b),

$v_1 = \mathbf{-7.2e^{-2t}V}$

$v_2 = 8\dfrac{di}{dt} = 8(-2)(1.8)e^{-2t} = \mathbf{-28.8e^{-2t}V}$

$v = v_1 + v_2 = \mathbf{-36e^{-2t}V}$

P.P.6.13 $RC = 100 \times 10^3 \times 20 \times 10^{-6} = 2$

$$v_o = -\frac{1}{RC}\int_o^t v_i(t) + v_o(0) = -\frac{1}{2}\int_o^t 2.5dt \text{ mV} + 0$$

$=$ **–1.25t mV**.

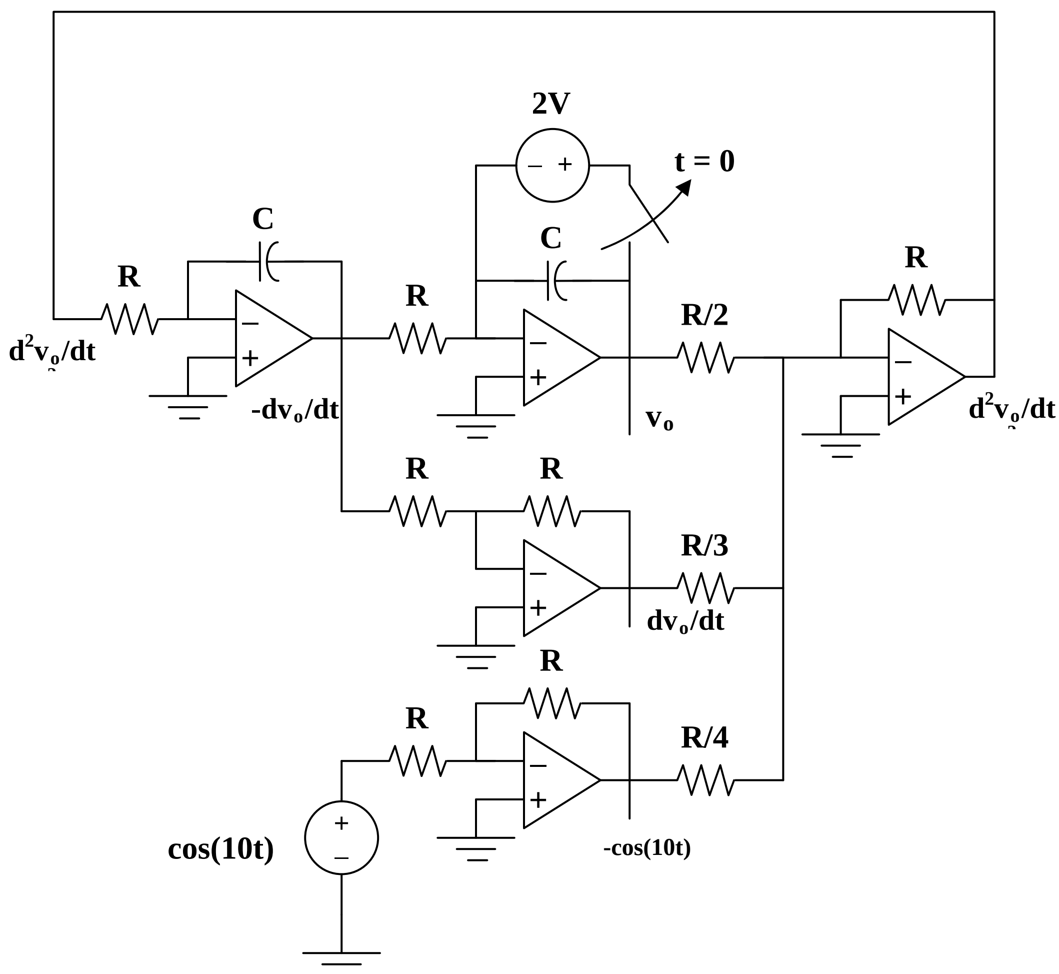
P.P.6.14 $RC = 100 \times 10^3 \times 0.1 \times 10^{-6} = 10 \times 10^{-3}$

$$v_o = -RC\frac{dv_i}{dt} = -10x10^{-3}\frac{d}{dt}(1.25t)$$

$v_o =$ **–12.5 mV**.

P.P.6.15 $$\frac{dv_o^2}{dt^2} = 4\cos 10t - 3\frac{dv_o}{dt} - 2v_o$$

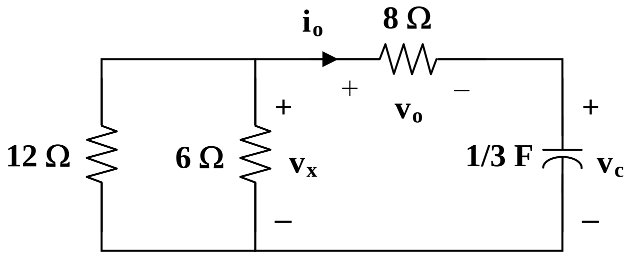
Using this we obtain the analog computer as shown below. We may let RC = 1s.



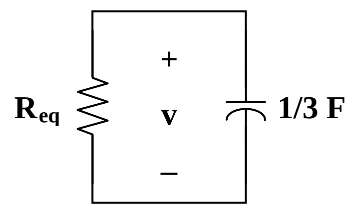
2V
t = 0
C
C
R
R
R
R/2
d2vo/dt
-dvo/dt
vo
d2vo/dt
R
R
R/3
dvo/dt
R
R
R/4
cos(10t)
-cos(10t)

CHAPTER 7

P.P.7.1 The circuit in Fig. (a) is equivalent to the one shown in Fig. (b).



(a)



(b)

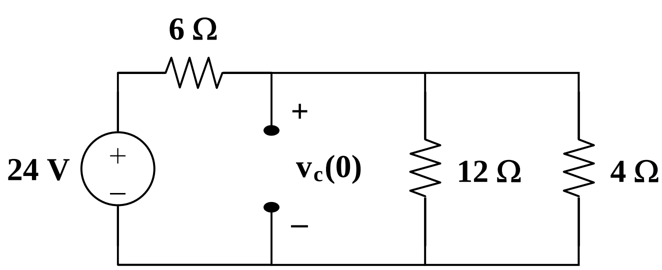
$$R_{eq} = 8 + 12 \,||\, 6 = 12\ \Omega$$
$$\tau = R_{eq}C = (12)(1/3) = 4\text{ s}$$
$$v_c = v_c(0)\,e^{-t/\tau} = 60\,e^{-t/4} = \mathbf{60e^{-0.25t}\ V}$$

$$v_x = \frac{4}{4+8}v_c = \mathbf{20e^{-0.25t}\ V}$$

$$v_x = v_o + v_c \quad \longrightarrow \quad v_o = v_x - v_c = -40\,e^{-0.25t}\ V$$

$$i_o = \frac{v_o}{8} = \mathbf{-5e^{-0.25t}\ A}.$$

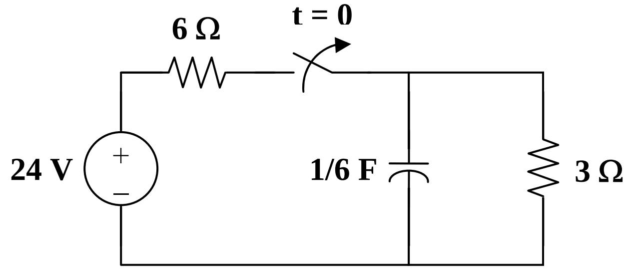
P.P.7.2 When t < 0, the switch is closed as shown in Fig. (a).



(a)

$$R_{eq} = 4 \,||\, 12 = 3\ \Omega \qquad v_c(0) = \frac{3}{3+6}(24) = 8\text{ V}$$

When t > 0, the switch is open as shown in Fig. (b).



(b)

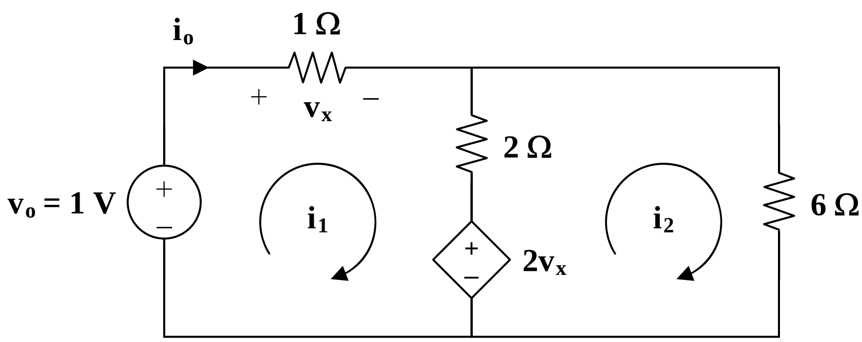
$\tau = R_{eq}C = (3)(1/6) = 1/2$ s

$v(t) = v_c(0)\,e^{-t/\tau} = \mathbf{8e^{-2t}\ V}$

$w_c(0) = \frac{1}{2}Cv_c^2(0) = \frac{1}{2}\times\frac{1}{6}\times 64 = \mathbf{5.333J}$

P.P.7.3 This can be solved in two ways.

Method 1: Find R_{th} at the inductor terminals by inserting a voltage source.



Applying mesh analysis gives

Loop 1: $-1 + 3i_1 - 2i_2 + 2v_x = 0$, where $v_x = 1i_1$

$5i_1 - 2i_2 = 1$ (1)

Loop 2: $8i_2 - 2i_1 - 2v_x = 0 = 8i_2 - 2i_1 - 2i_1$

$i_2 = \frac{1}{2}i_1$ (2)

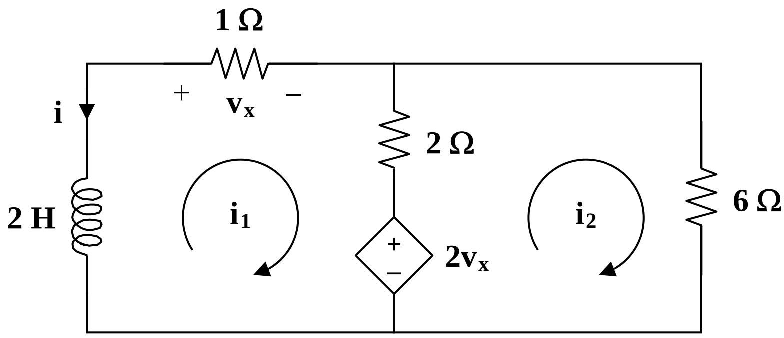
From (1) and (2), $5i_1 - 1i_1 = 1$ or

$i_o = i_1 = (1/4)$ A

$$R_{th} = \frac{v_o}{i_o} = 4\,\Omega, \qquad \tau = \frac{L}{R} = \frac{2}{4} = \frac{1}{2}\text{ s}$$

$$i(t) = 12e^{-2t}\ A$$

<u>Method 2</u>: We can obtain i using mesh analysis.



Applying KVL to the loops, we obtain

Loop 1: $$2\frac{di_1}{dt} + 3i_1 - 2i_2 + 2v_x = 0 \qquad \text{where } v_x = 1i_1$$

$$2\frac{di_1}{dt} + 5i_1 - 2i_2 = 0 \qquad (3)$$

Loop 2: $$8i_2 - 2i_1 - 2v_x = 0$$

$$i_2 = \frac{1}{2}i_1 \qquad (4)$$

Substituting (4) into (3) yields

$$2\frac{di_1}{dt} + 5i_1 - 1i_1 = 0$$

or $$\frac{di_1}{dt} + 2i_1 = 0$$

$$i_1 = Ae^{-2t}$$

$$i = -i_1 = Be^{-2t}$$
$$i(0) = 12 = B$$

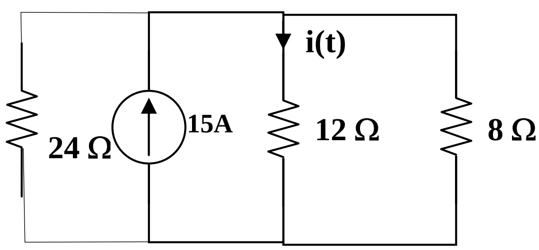
$$i(t) = 12e^{-2t}\ A$$

Therefore,

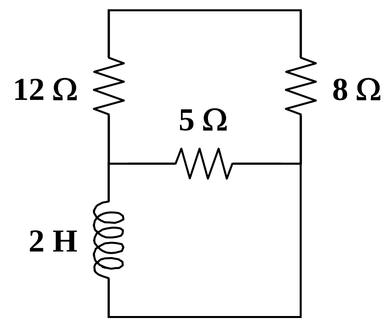
$$i(t) = \mathbf{12e^{-2t}\ A}$$

and $v_x(t) = -1i(t) = \mathbf{-12e^{-2t}\ V}$ for all $t > 0$.

P.P.7.4 For t < 0, the equivalent circuit is shown in Fig. (a).



(a)



(b)

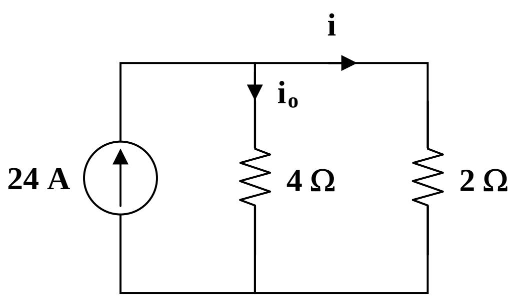
$$i(0) = 15[1/\{(1/24)+(1/12)+(1/8)\}]/12 = (15x24/6)/12 = 5\ A$$

For t > 0, the current source and 24-ohm is cut off and the RL circuit is shown in Fig. (b).

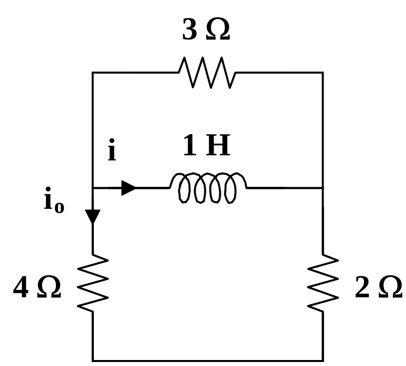
$$R_{eq} = (12+8)\ ||\ 5 = 20\ ||\ 5 = 4\,\Omega, \qquad \tau = \frac{L}{R_{eq}} = \frac{2}{4} = 0.5$$

$$\text{i(t)} = \text{i(0)e}^{-2t} = \mathbf{5e^{-2t}\ amps}, \text{ for all } t > 0.$$

P.P.7.5 For t < 0, the switch is closed. The inductor acts like a short so the equivalent circuit is shown in Fig. (a).



(a)



(b)

$$i = \frac{4}{4+2}(24) = 16\text{ A}, \qquad i_o = 24 - 16 = 8\text{ A}, \qquad v_o = 2i = 32\text{ V}$$

For t > 0, the current source is cut off so that the circuit becomes that shown in Fig. (b). The Thevenin equivalent resistance at the inductor terminals is

$$R_{th} = (4+2)\ ||\ 3 = 2\,\Omega, \qquad \tau = \frac{L}{R_{th}} = \frac{1}{2}$$

$$i_o = \frac{3(\text{-}i)}{6+3} = \frac{-1}{3}i = -5.333e^{-2t}\text{ A} \qquad \text{and} \qquad v_o = -2i_o = 10.667e^{-2t}\text{V}$$

Thus,

$$i = \begin{cases} 16\text{ A} & t<0 \\ 16e^{-2t}\text{ A} & t>0 \end{cases} \quad i_o = \begin{cases} 8\text{ A} & t<0 \\ -5.333e^{-2t}\text{ A} & t>0 \end{cases} \quad v_o = \begin{cases} 32\text{ V} & t<0 \\ 10.667e^{-2t}\text{V} & t>0 \end{cases}$$

P.P.7.6 $$i(t) = \begin{cases} 0 & t<0 \\ 10 & 0<t<2 \\ -10 & 2<t<4 \end{cases}$$

$$i(t) = 10[u(t) - u(t-2)] - 10[u(t-2) - u(t-4)]$$
$$i(t) = \mathbf{10[u(t) - 2u(t-2) + u(t-4)]A}$$

Let $I = \int_{-\infty}^{t} i\,dt$.

For $t < 0$, $I = 0$.

For $0 < t < 2$, $I = \int_0^t 10\,dt = 10t$

For $2 < t < 4$, $I = \int_0^2 10\,dt - 10\int_2^t dt = 20 - 10t\Big|_2^t = 40 - 10t$

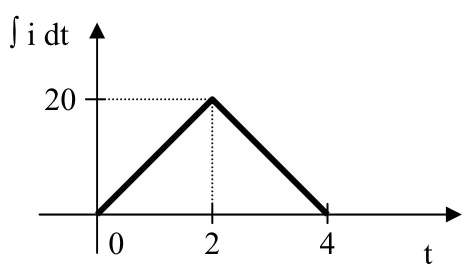
For $t > 4$, $I = 20 - 10t\Big|_2^4 = 0$

Thus,

$$I = \begin{cases} 0 & t<0 \\ 10t & 0<t<2 \\ 40-10t & 2<t<4 \\ 0 & t>4 \end{cases}$$

or $I = \mathbf{10[r(t) - 2r(t-2) + r(t-4)]\underline{A}}$

which is sketched below



P.P.7.7 $$i(t) = \begin{cases} 2-2t & 0<t<2 \\ -6+2t & 2<t<3 \\ 0 & \text{otherwise} \end{cases}$$

$$i(t) = (2-2t)[\,u(t) - u(t-2)] + (-6+2t)[\,u(t-2) - u(t-3)]$$

$$i(t) = 2\,u(t) - 2t\,u(t) + 4(t-2)\,u(t-2) - 2(t-3)\,u(t-3)$$
$$i(t) = \mathbf{[2\,\boldsymbol{u(t)} - 2\,\boldsymbol{r(t)} + 4\,\boldsymbol{r(t-2)} - 2\,\boldsymbol{r(t-3)}]\,A}$$

Remember the singularity function, r(t), is a ramp function equal to t for all values of t > 0 and equal to zero for all values of t < 0.

P.P.7.8 $h(t) = -4[u(t) - u(t-2)] + (3t-8)[u(t-2) - u(t-6)]$

$h(t) = -4u(t) + 4u(t-2) + 3tu(t-2) - 8u(t-2) - 3tu(t-6) + 8u(t-6)$

$h(t) = -4u(t) + (4-8+6)u(t-2) + 3(t-2)u(t-2) - 3(t-6)u(t-6)$
$+ (-18+8)u(t-6)$

$h(t) = -4u(t) + 2u(t-2) + 3(t-2)u(t-2) - 3(t-6)u(t-6) - 10u(t-6)$

$h(t) = \mathbf{-4u(t) + 2u(t-2) + 3r(t-2) - 10u(t-6) - 3r(t-6)}$.

P.P.7.9 (a) $$\int_{-\infty}^{\infty}\left(t^3 + 5t^2 + 10\right)\delta(t+3)\,dt = t^3 + 5t^2 + 10\Big|_{t=-3}$$
$$= -27 + 45 + 10 = \mathbf{28}$$

(b) $$\int_{0}^{10}\delta(t-\pi)\cos(3t)\,dt = \cos(3\pi) = \mathbf{-1}$$

P.P.7.10 For t < 0, the capacitor acts like an open circuit.

$$v(0^-) = v(0^+) = v(0) = 15$$

For t > 0, $[(v(\infty)-15)/2] + [(v(\infty)-(-7.5))/6] = 0$ or $(4/6)v(\infty) = 7.5-1.25 = 6.25$ or

$v(\infty) = 9.375$ V

$$R_{th} = 2\,||\,6 = \frac{3}{2}\,\Omega, \qquad \tau = R_{th}C = \frac{3}{2}\times\frac{1}{3} = \frac{1}{2}$$

$$v(t) = v(\infty) + \left[\,v(0) - v(\infty)\right]\,e^{-t/\tau} = 9.375 + (15-9.375)e^{-2t}$$

$$v(t) = \mathbf{(9.375 + 5.625e^{-2t})\ V\ for\ all\ t > 0}$$

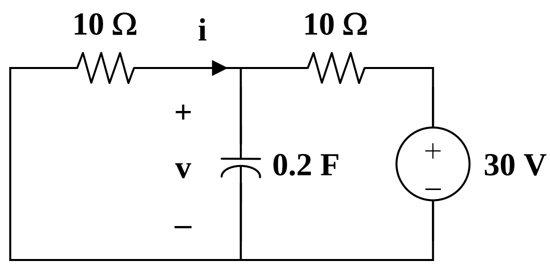
At t = 0.5, $v(0.5) = 6.25 + 3.75e^{-1} = 6.25 + 1.3795 = \mathbf{7.63\ V}$

P.P.7.11 For t < 0, only the left portion of the circuit is operational at steady state.

$$v(0^-) = v(0^+) = v(0) = 20, \qquad i(0) = 0$$

For t > 0, $20u(-t) = 0$ so that the voltage source is replaced by a short circuit.

Transforming the current source leads to the circuit below.



$$v(\infty) = \frac{5}{15}(30) = 10$$

$$R_{th} = 5 \,||\, 10 = \frac{10}{3}\,\Omega, \qquad \tau = R_{th}C = \frac{10}{3} \times 0.2 = \frac{2}{3}$$

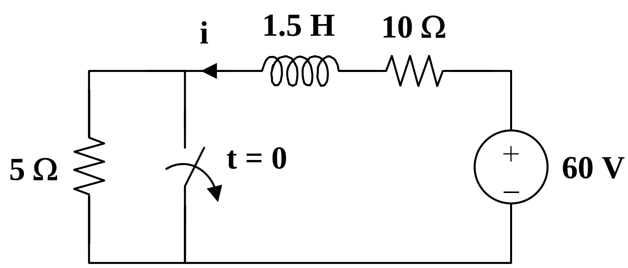
$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right]\, e^{-t/\tau}$$
$$v(t) = 10 + (20 - 10)\, e^{-3t/2}$$
$$v(t) = 10\left(1 + e^{-1.5t}\right)$$

$$i(t) = \frac{-\,v(t)}{5} = -2\left(1 + e^{-1.5t}\right)$$

$$i(t) = \begin{cases} \mathbf{0} & \mathbf{t < 0} \\ \mathbf{-\,2\left(1 + e^{-1.5t}\right) A} & \mathbf{t > 0} \end{cases}$$

$$v(t) = \begin{cases} \mathbf{20\ V} & \mathbf{t < 0} \\ \mathbf{10\left(1 + e^{-1.5t}\right) V} & \mathbf{t > 0} \end{cases}$$

P.P.7.12 Applying source transformation, the circuit is equivalent to the one below.



At t < 0, the switch is closed so that the 5 ohm resistor is short circuited.

$$i(0^-) = i(0) = \frac{60}{10} = 6\ \text{A}$$

For t > 0, the switch is open.

$$R_{th} = 10 + 5 = 15, \qquad \tau = \frac{L}{R_{th}} = \frac{1.5}{15} = 0.1$$

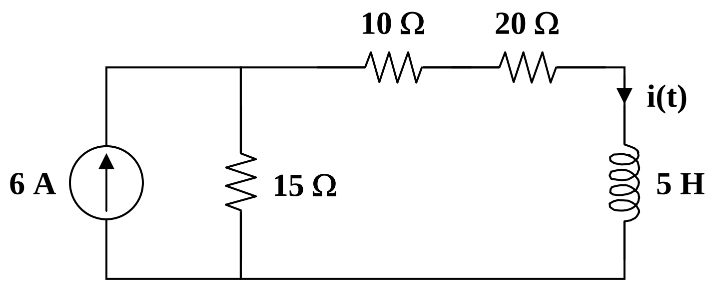
$$i(\infty) = \frac{60}{10+5} = 4\text{A}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 4 + (6-4)e^{-10t}$$

i(t) = **(4 + 2e^{-10t}) A for all t > 0**

P.P.7.13 For 0 < t < 2, the given circuit is equivalent to that shown below.



Since switch S_1 is open at $t = 0^-$, $i(0^-) = 0$. Also, since i cannot jump, $i(0) = i(0^-) = 0$.

$$i(\infty) = \frac{90}{15+10+20} = 2\text{ A}$$

$$R_{th} = 45\,\Omega, \quad \tau = \frac{L}{R_{th}} = \frac{5}{45} = \frac{1}{9}$$

$$i(t) = i(\infty) + \left[i(0) - i(\infty) \right] e^{-t/\tau}$$

$$i(t) = 2 + (0-2)e^{-9t}$$

$$i(t) = 2(1 - e^{-9t})\text{ A}$$

When switch S_2 is closed, the 20 ohm resistor is short-circuited.

$$i(2^+) = i(2^-) = 2(1 - e^{-18}) \cong 2$$

This will be the initial current

$$i(\infty) = \frac{90}{15+10} = 3.6\text{ A}$$

$$R_{th} = 25\,\Omega, \quad \tau = \frac{5}{25} = \frac{1}{5}$$

$$i(t) = i(\infty) + \left[i(2^+) - i(\infty) \right] e^{-(t-2)/\tau}$$

$$i(t) = 3.6 + (2 - 3.6)e^{-5(t-2)}$$

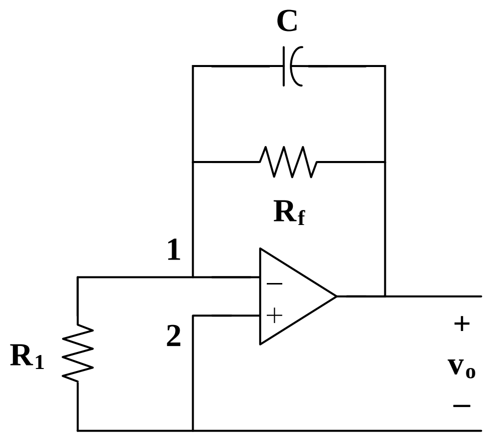
$$i(t) = 3.6 - 1.6e^{-5(t-2)}$$

Thus, $i(t) = \begin{cases} \mathbf{0} & \mathbf{t < 0} \\ \mathbf{2(1 - e^{-9t})\ A} & \mathbf{0 < t < 2} \\ \mathbf{3.6 - 1.6e^{-5(t-2)}\ A} & \mathbf{t > 2} \end{cases}$

At $t = 1$, $i(1) = 2(1 - e^{-9}) = \mathbf{1.9997\ A}$

At $t = 3$, $i(3) = 3.6 - 1.6e^{-5} = \mathbf{3.589\ A}$

P.P.7.14 The op amp circuit is shown below.



Since nodes 1 and 2 must be at the same potential, there is no potential difference across R_1. Hence, no current flows through R_1. Applying KCL at node 1,

$$\frac{v}{R_f} + C\frac{dv}{dt} = 0 \quad \longrightarrow \quad \frac{dv}{dt} + \frac{v}{CR_f} = 0$$

which is similar to Eq. (7.4).

Hence,

$$v(t) = v_o\, e^{-t/\tau},\ \tau = R_f C$$

$$v(0) = v_o = 4, \qquad \tau = (50\times10^3)(10\times10^{-6}) = 0.5$$

$$v(t) = 4e^{-2t}\ \text{V}, \quad t > 0$$

Alternatively, since no current flows through R_1, the feedback loop forms a first order RC circuit with $v(0) = 4$ and $\tau = R_f C = 0.5$. Hence,

$$v(t) = 4e^{-2t}\ \text{V}, \quad t > 0$$

To get to v_o from v, we notice that v is the potential difference between node 1 and the output terminal, i.e.

$$0 - v_o = v \quad \longrightarrow \quad v_o = -v \quad \text{or} \quad v_o(t) = \mathbf{-4e^{-2t}\ V, \quad t > 0}$$

P.P.7.15 Let v_1 be the potential at the inverting terminal.

$$v(t) = v(\infty) + \left[v(0) - v(\infty) \right] e^{-t/\tau}$$

where $\tau = RC = 100 \times 10^3 \times 10^{-6} = 0.1$, $\qquad v(0) = 0$

$$v_1 = 0 \text{ for all t}$$

$$v_1 - v_o = v \tag{1}$$

For t > 0, the switch is closed and the op amp circuit is an inverting amplifier with

$$v_o(\infty) = \frac{-100}{10}(4\text{ mV}) = -40\text{ mV}$$

From (1),

$$v(\infty) = 0 - v_o(\infty) = 40\text{ mV}$$

Thus, $\qquad v(t) =$ $\mathbf{40(1 - e^{-10t})u(t)}$ **mV**

$$v_o = v_1 - v = -v$$

$v_o =$ $\mathbf{40(e^{-10t} - 1)u(t)}$ **mV**

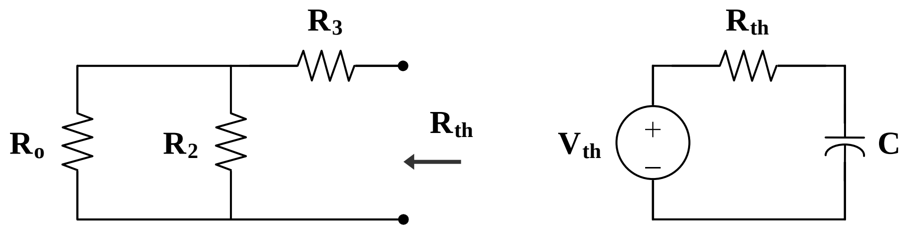
P.P.7.16 This is a noninverting amplifier so that the output of the op amp is

$$v_a = \left(1 + \frac{R_f}{R_1}\right) v_i$$

$$v_{th} = v_a = \left(1 + \frac{R_f}{R_1}\right) v_i = \left(1 + \frac{40}{20}\right) 4.5\,u(t) = 13.5\,u(t)$$

To get R_{th}, consider the circuit shown in Fig. (a), where R_o is the output resistance of the op amp. For an ideal op amp, $R_o = 0$ so that

$$R_{th} = R_3 = 10\text{ k}\Omega$$



(a) (b)

$$\tau = R_{th}C = 10 \times 10^3 \times 2 \times 10^{-6} = \frac{1}{50}$$

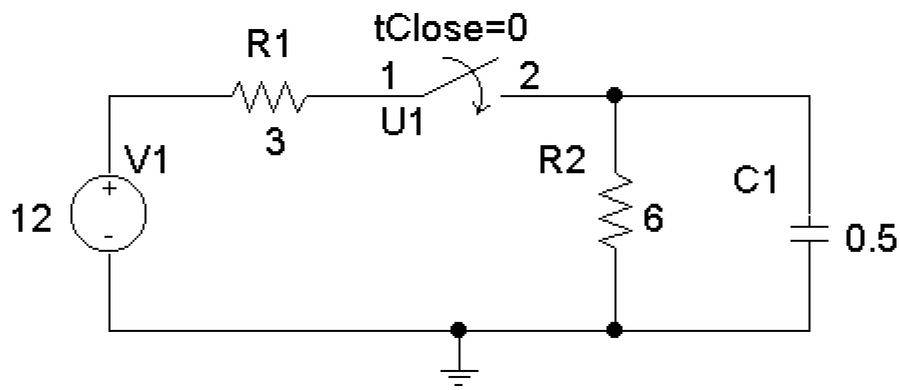
The Thevenin equivalent circuit is shown in Fig. (b), which is a first order circuit.

Hence,

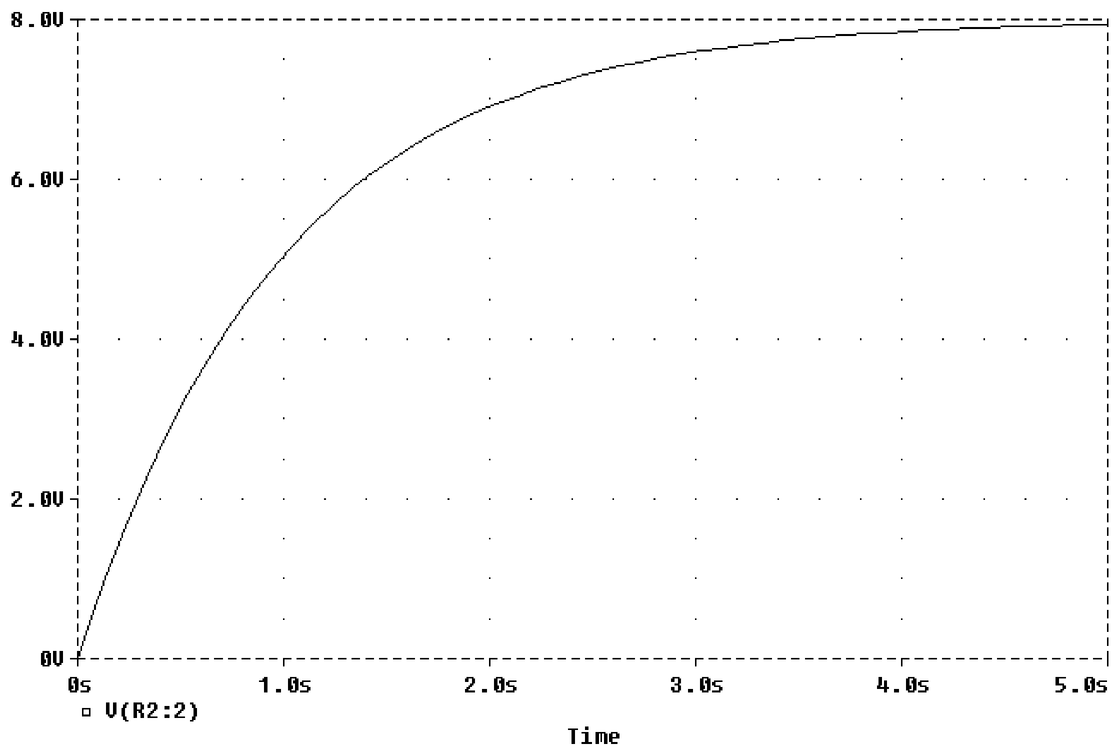
$$v_o(t) = 13.5\left(1 - e^{-t/\tau}\right)u(t)$$

$$v_o(t) = \mathbf{13.5(1 - e^{-50t})u(t)\ V}$$

P.P.7.17 The schematic is shown in Fig. (a). Construct and save the schematic. Select Analysis/Setup/Transient to change the Final Time to 5 s. Set the Print Step slightly greater than 0 (20 ns is default). The circuit is simulated by selecting Analysis/ Simulate. In the Probe menu, select Trace/Add and display V(R2:2) as shown in Fig. (b).

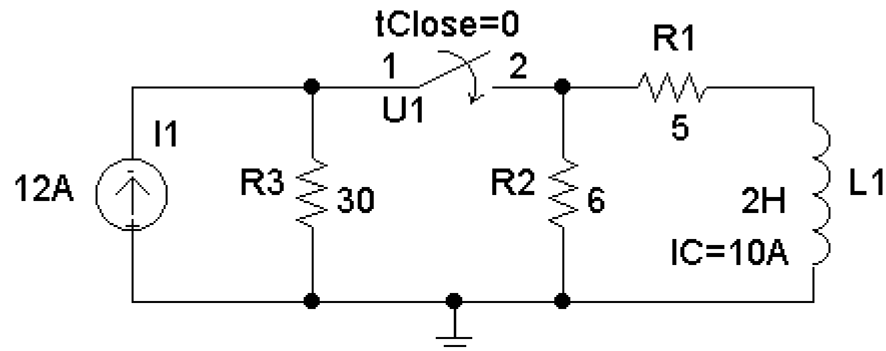


(a)

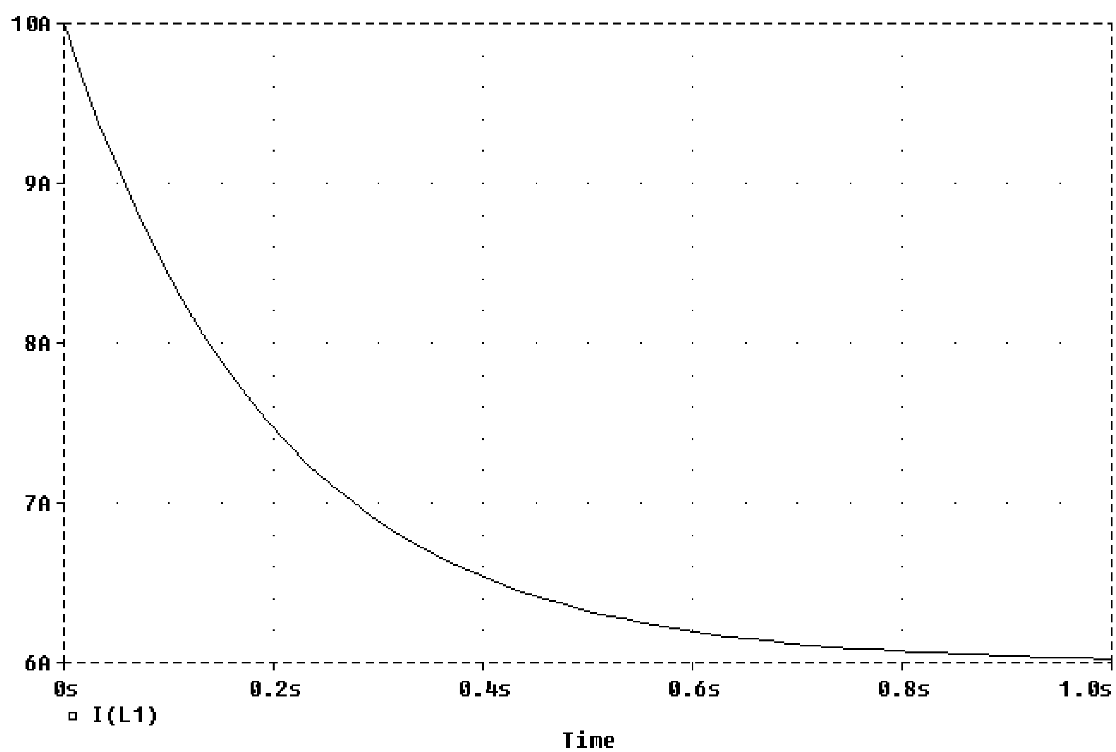


(b)

P.P.7.18 The schematic is shown in Fig. (a). While constructing the circuit, rotate L1 counterclockwise through 270° so that current i(t) enters pin 1 of L1 and set IC = 10 for L1. After saving the schematic, select Analysis/Setup/Transient to change the Final Time to 1 s. Set the Print Step slightly greater than 0 (20 ns is default). The circuit is simulated by selecting Analysis/ Simulate. After simulating the circuit, select Trace/Add in the Probe menu and display I(L1) as shown in Fig. (b).

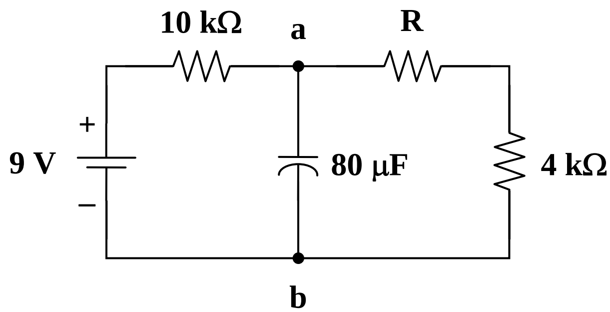


(a)



(b)

P.P.7.19 $v(0) = 0$. When the switch is closed, we have the circuit shown below.



We find the Thevenin equivalent at terminals a-b.

$$R_{th} = (R+4) \,||\, 10 = \frac{10(R+4)}{R+14}$$

$$v_{th} = v(\infty) = \frac{R+4}{R+14}(9)$$

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}, \qquad \tau = R_{th}C$$

$$v(t) = v(\infty)\left(1 - e^{-t/\tau}\right)$$

Since $v(0) = 0$,

$$i(t) = \frac{v(t)}{R+4} = \frac{9}{R+4}\left(1 - e^{-t/\tau}\right) \text{ mA}$$

Assuming R is in kΩ,

$$120 \times 10^{-6} = \frac{9}{R+14}\left(1 - e^{-t_0/\tau}\right) \times 10^{-3}$$

$$(0.12)\frac{R+14}{9} = 1 - e^{-t_0/\tau}$$

or $$e^{-t_0/\tau} = 1 - \frac{0.12R + 1.68}{9} = \frac{7.32 - 0.12R}{9}$$

$$t_0 = \tau \ln\left(\frac{9}{7.32 - 0.12R}\right)$$

$$t_0 = \frac{10(R+4)}{R+14} \times 80 \times 10^{-6} \times \ln\left(\frac{9}{7.32 - 0.12R}\right)$$

When R = 0,

$$t_0 = \frac{40 \times 80 \times 10^{-6}}{14} \times \ln\left(\frac{9}{7.32}\right) = 0.04723 \text{ s}$$

When R = 6 kΩ,

$$t_0 = \frac{100}{20} \times 80 \times 10^{-6} \times \ln\left(\frac{9}{6.6}\right) = 0.124 \text{ s}$$

The time delay is **between 47.23 ms and 124 ms**.

P.P.7.20 (a) $q = CV = (2\text{x}10^{-3})(80) =$ **160 mC**.

(b) $W = \frac{1}{2}CV^2 = \frac{1}{2}(2\times10^{-3})(6400) = \mathbf{6.4\ J}$

(c) $\Delta I = \frac{\Delta q}{\Delta t} = \frac{0.16}{0.8\times10^{-3}} = \mathbf{200\ A}$

(d) $p = \frac{\Delta w}{\Delta t} = \frac{6.4}{0.8\times10^{-3}} = \mathbf{8\ kW}$

(e) $\Delta t = \frac{\Delta q}{\Delta I} = \frac{0.16}{5\times10^{-3}} = \mathbf{32\ s}$

P.P.7.21 $\tau = \frac{L}{R} = \frac{500\times10^{-3}}{200} = 2.5 \text{ ms}$

$$i(0) = 0, \qquad i(\infty) = \frac{110}{200} = 550 \text{ mA}$$

$$i(t) = 550\left(1 - e^{-t/\tau}\right) \text{ mA}$$

$$350 \text{ mA} = i(t_0) = 550\left(1 - e^{-t_0/\tau}\right) \text{ mA}$$

$$\frac{35}{55} = 1 - e^{-t_0/\tau} \longrightarrow e^{-t_0/\tau} = \frac{20}{55}$$

$$e^{t_0/\tau} = \frac{55}{20}$$

$$t_0 = \tau \ln\left(\frac{55}{20}\right) = 2.5 \ln\left(\frac{55}{20}\right) \text{ ms}$$

$$t_0 = \mathbf{2.529\ ms}$$

P.P.7.22 (a) $t = 5\tau = \frac{5L}{R} = \frac{5\times20\times10^{-3}}{5} = \mathbf{20\ ms}$

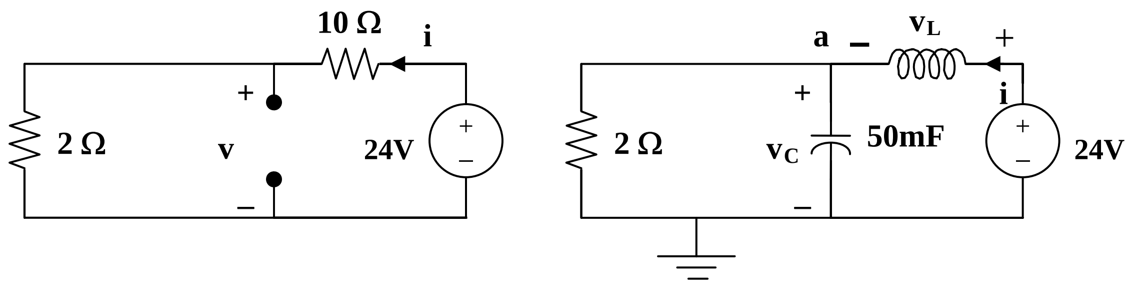
(b) $W = \frac{1}{2}LI^2 = \frac{1}{2}\left(20\times10^{-3}\right)\left(\frac{12}{5}\right)^2 = \mathbf{57.6\ mJ}$

(c) $V = L\frac{di}{dt} = 20\times10^{-3}\left(\frac{12/5}{2\times10^{-6}}\right) = \mathbf{24\ kV}$

CHAPTER 8

P.P.8.1

(a) At $t = 0^-$, we have the equivalent circuit shown in Figure (a).



(a) **(b)**

$$i(0^-) = 24/(2 + 10) = \mathbf{2\ A},\ \ v(0^-) = 2i(0^-) = 4\ V$$

$$\text{hence, } v(0^+) = v(0^-) = \mathbf{4\ V}.$$

(b) At $t = 0^+$, the switch is closed.

$$L(di/dt) = v_L, \text{ leads to } (di/dt) = v_L/L$$

But, $v_C(0^+) + v_L(0^+) = 24 = 4 + v_L(0+)$, or $v_L(0+) = 20$ V

$$(di(0^+)/dt) = 20/0.4 = \mathbf{50\ A/s}$$

$$C(dv/dt) = i_C \text{ leading to } (dv/dt) = i_C/C$$

But at node a, KCL gives $i(0^+) = i_C(0^+) + v(0^+)/2 = 2 = i_C(0^+) + 4/2$

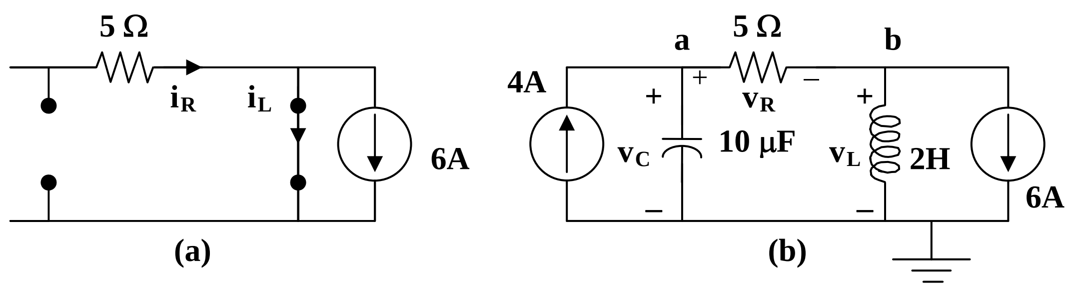
$$\text{or } i_C(0^+) = 0, \text{ hence } (dv(0^+)/dt) = \mathbf{0\ V/s}$$

(c) As t approaches infinity, the capacitor is replaced by an open circuit and the inductor is replaced by a short circuit.

$$v(\infty) = \mathbf{24\ V}, \text{ and } i(\infty) = \mathbf{12\ A}.$$

P.P.8.2

(a) At $t = 0-$, we have the equivalent circuit shown in (a).



$$i_L(0-) = -6A, \ v_L(0^-) = 0, \ v_R(0^-) = 0$$

At $t = 0^+$, we have the equivalent circuit in Figure (b). At node b,

$$i_R(0^+) = i_L(0^+) + 6, \text{ since } i_L(0^+) = i_L(0^-) = -6A, \ i_R(0^+) = 0,$$

and $v_R(0+) = 5i_R(0+) = 0$. Thus, $i_L(0) =$ **–6 A**, $v_C(0) =$ **0**, and $v_R(0^+) =$ **0**.

(b) $dv_C(0^+)/dt = i_C(0^+)/C = 4/0.2 = 20$ V/s.

To get (dv_R/dt), we apply KCL to node b, $i_R = i_L + 6$, thus $di_R/dt = di_L/dt$.

Since $v_R = 5i_R$, $dv_R/dt = 5di_R/dt = 5di_L/dt$. But $Ldi_L/dt = v_L$, $di_L/dt = v_L/L$.

$$\text{Hence, } dv_R(0^+)/dt = 5v_L(0^+)/L.$$

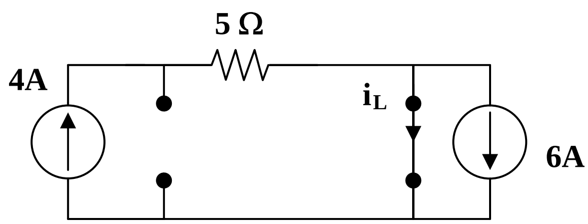
Applying KVL to the middle mesh in Figure (b),

$$-v_C(0^+) + v_R(0^+) + v_L(0^+) = 0 = 0 + 0 + v_R(0^+), \text{ or } v_R(0^+) = 0$$

$$\text{Hence, } dv_R(0^+)/dt = 0 = di_L(0^+)/dt;$$

$di_L(0^+)/dt =$ **0**, $dv_C(0^+)/dt =$ **20 V/s**, $dv_R(0^+)/dt =$ **0**.

(c) As t approaches infinity, we have the equivalent circuit shown below.



$$4 = 6 + i_L(\infty) \text{ leads to } i_L(\infty) = -2\text{A}$$

$$v_C(\infty) = v_R(\infty) = 4x5 = 20\text{V}$$

Thus, $i_L(\infty)$ = **–2 A**, $v_C(\infty) = v_R(\infty)$ = **20 V**

P.P.8.3

(a) $\alpha = R/(2L) = 10/(2x5) = \mathbf{1}$, $\omega_o = 1/\sqrt{LC} = 1/\sqrt{5x2x10^{-2}} = \mathbf{10}$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1 \pm \sqrt{1-100} = \mathbf{-1 \pm j9.95}.$$

(b) Since $\alpha < \omega_o$, we clearly have an **underdamped** response.

P.P.8.4 For t < 0, the inductor is connected to the voltage source and when the circuit reaches steady state, the inductor acts like a short circuit.

$$i(0-) = 50/10 = 5 = i(0+) = i(0)$$

The voltage across the capacitor is $0 = v(0-) = v(0+) = v(0)$.

For t > 0, we have a source-free RLC circuit.

$$\omega_o = 1/\sqrt{LC} = 1/\sqrt{1x\frac{1}{9}} = 3$$

$$\alpha = R/(2L) = 5/(2x1) = 2.5$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2.5 \pm \sqrt{6.25 - 9} = -2.5 \pm j1.6583$$

$$i(t) = e^{-2.5t}[A_1\cos 1.6583t + A_2\sin 1.6583t]$$

We now determine A_1 and A_2.

$$i(0) = 10 = A_1$$

$$di/dt = -2.5\{e^{-2.5t}[A_1\cos(1.6583t) + A_2\sin(1.6583t)]\}$$
$$+ 1.6583e^{-2.5t}[-A_1\sin(1.6583t) + A_2\cos(1.6583t)]$$

$$di(0)/dt = -(1/L)[Ri(0) + v(0)] = -2.5A_1 + 1.6583A_2$$

$$= -1[5x10+0] = -1[50] = -2.5(10) + 1.6583A_2$$

$$A_2 = -15.076$$

Thus, $i(t) = \mathbf{e^{-2.5t}[10\cos(1.6583t) - 15.076\sin(1.6583t)]\ A}$

P.P.8.5 $\alpha = 1/(2RC) = 1/(2x2x25x10^{-3}) = 10$

$$\omega_0 = 1\big/\sqrt{LC} = 1\big/\sqrt{0.4x25x10^{-3}} = 10$$

since $\alpha = \omega_o$, we have a critically damped response. Therefore,

$$v(t) = [(A_1 + A_2t)e^{-10t}]$$

$v(0) = 0 = A_1 + A_2x0 = A_1$, which leads to $v(t) = [A_2te^{-10t}]$.

$$dv(0)/dt = -(v(0) + Ri(0))/(RC) = -2x0.05/(2x25x10^{-3}) = -2$$

$$dv/dt = [(A_2 - 10A_2t)e^{-10t}]$$

At $t = 0$, $-2 = A_2$ therefore, $v(t) = \mathbf{(-2t)e^{-10t}u(t)\ V}$

P.P.8.6 For t < 0, the switch is closed. The inductor acts like a short circuit while the capacitor acts like an open circuit. Hence,

$$i(0) = 3\text{A} \text{ and } v(0) = 0.$$

$$\alpha = 1/(2RC) = 1/(2x20x4x10^{-3}) = 6.25$$

$$\omega_o = 1\big/\sqrt{LC} = 1\big/\sqrt{10x4x10^{-3}} = 5$$

Since $\alpha > \omega_o$, this is an overdamped response.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o} = -6.25 \pm \sqrt{(6.25)^2 - 25} = -2.5 \text{ and } -10$$

$$\text{Thus,} \quad v(t) = A_1e^{-2.5t} + A_2e^{-10t}$$

$$v(0) = 0 = A_1 + A_2, \text{ which leads to } A_2 = -A_1$$

$$dv(0)/dt = -(v(0) + Ri(0))/(RC) = -(20x4.5)12.5 = -1125$$

$$\text{But, } dv/dt = -2.5A_1e^{-2.5t} - 10A_2e^{-10t}$$

At $t = 0$, $-1125 = -2.5A_1 - 10A_2 = 7.5A_1$ since $A_1 = -A_2$

$$A_1 = -150, \quad A_2 = 150$$

$$\text{Thus,} \quad v(t) = \mathbf{150(e^{-10t} - e^{-2.5t})\ V}$$

P.P.8.7 The initial capacitor voltage is obtained when the switch is in position a.

$$v(0) = [2/(2 + 1)]18 = 12\text{ V}$$

The initial inductor current is $i(0) = 0$.

When the switch is in position b, we have the RLC circuit with the voltage source.

$$\alpha = R/(2L) = 10/(2x2.5) = 2$$

$$\omega_o = 1\big/\sqrt{LC} = 1\big/\sqrt{(5/2)x(1/40)} = 4$$

Since $\alpha < \omega_o$, we have an underdamped case.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o} = -2 \pm \sqrt{(2)^2 - 16} = -2 \pm j\,3.464$$

Thus, $v(t) = v_f + [(A_1\cos 3.464t + A_2\sin 3.464t)e^{-2t}]$

where $v_f = v(\infty) = 15$, the final capacitor voltage. We now impose the initial conditions to get A_1 and A_2.

$$v(0) = 12 = 15 + A_1 \text{ leads to } A_1 = -3$$

The initial capacitor current is the same as the initial inductor current.

$$i(0) = C(dv(0)/dt) = 0 \text{ therefore, } dv(0)/dt = 0$$

But, $dv/dt = 3.464[\{-A_1\sin(3.464t) + A_2\cos(3.464t)\}e^{-2t}]$
$-2[\{A_1\cos(3.464t) + A_2\sin(3.464t)\}e^{-2t}]$

$$dv(0)/dt = 0 - 2A_1 + 3.464A_2, \text{ which leads to } A_2 = -6/3.464 = -1.7321$$

Thus, $v(t) = \mathbf{\{15 + [(-3\cos(3.464t) - 1.7321\sin(3.464t)]e^{-2t}\}\ V}$

$$i = C(dv/dt),\ v_R = Ri = RC(dv/dt) = (1/4)dv/dt$$

$$= (1/4)[(6-6)\cos(3.464t) + (2x1.7321 + 3x3.464)\sin(3.464t)]e^{-2t}$$

$v_R(t) = \mathbf{(3.464\sin(3.464t)e^{-2t})\ V}$

P.P.8.8 When $t < 0$, $v(0) = 0$, $i(0) = 0$; for $t > 0$,

$$\alpha = 0, \omega_o = 1/\sqrt{LC} = 1/\sqrt{0.2x20} = 0.25$$

$$i(t) = i_s + A_1\cos t + A_2\sin t = 10 + A_1\cos(0.25t) + A_2\sin(0.25t)$$

$$i(0) = 0 = 10 + A_1, \text{ therefore } A_1 = -10$$

$$Ldi(0)/dt = v(0) = 0$$

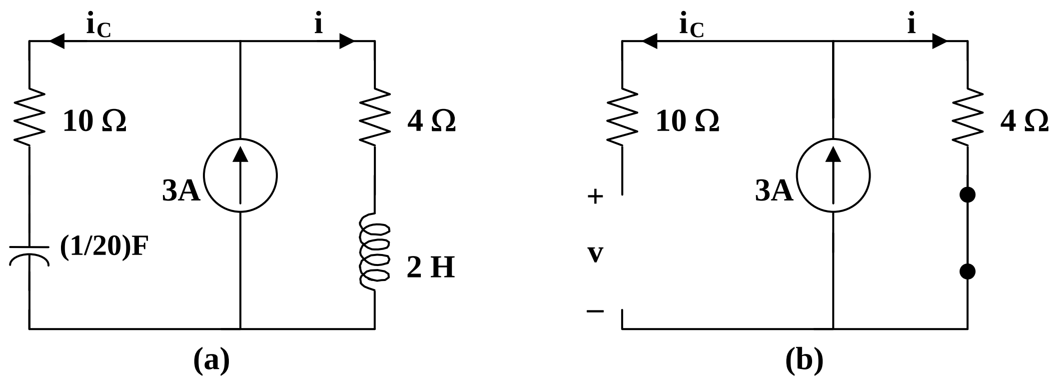
$$\text{But} \quad di/dt = -A_1\, 0.25\sin(0.25t) + A_2\, 0.25\cos(0.25t)$$

At $t = 0$, $di(0)/dt = 0 = 0 + 0.25A_2$ leading to $i(t) = \mathbf{10(1 - cos(0.25t))\ A}$

$$v(t) = Ldi/dt = 20x10x0.25\text{sint} = \mathbf{50sin(0.25t)\ V}$$

P.P.8.9 At $t = 0$, the switch is open so that $v(0) = 0$, $i(0^-) = 0$ (1)

For $t > 0$, the switch is closed. We have the equivalent circuit as in Figure (a).



$$v(0^+) = 0,\ i(0^+) = 0 \qquad (2)$$

$$-3 + i_C + i = 0 \qquad (3)$$

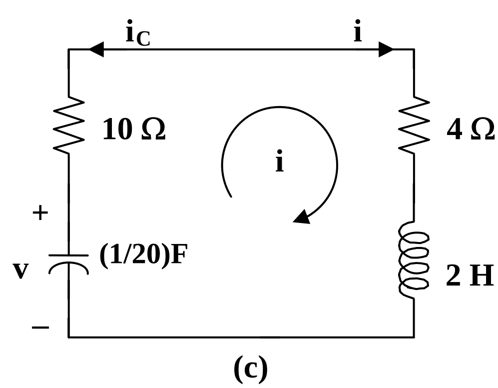
From (3), $i(0^+) = 0$ means that $i_C(0^+) = 3$, but $i_C(0^+) = Cdv(0^+)/dt$

which leads to $dv(0^+)/dt = i_C(0^+)/C = 3/(1/20) = 60$ V/s

As t approaches infinity, we have the equivalent circuit in (b).

$$i(\infty) = 3\text{ A},\ v(\infty) = 4i(\infty) = 12\text{V} \qquad (5)$$

Next we find the network response by turning off the current source as shown in Figure (c).



Applying KVL gives $\quad -v - 10i_C + 4i + 2di/dt = 0 \quad (6)$

Applying KCL to the top node, $\quad i - i_C = 0$

Namely, $\quad i = i_C = -Cdv/dt = -(1/20)dv/dt \quad (7)$

Combining (6) and (7), $\quad -v - (10/20)dv/dt - (4/20)dv/dt - (2/20)d^2v/dt^2 = 0.$

or $\quad (d^2v/dt^2) + 7(dv/dt) + 10 = 0$

The characteristic equation is $\quad s^2 + 7s + 10 = 0 = (s + 2)(s + 5)$

This means that $v_n = (Ae^{-2t} + Be^{-5t})$ and $v_f = v(\infty) = 12$ V.

Thus, $v = v_f + v_n \quad v = 12 + (Ae^{-2t} + Be^{-5t}) \quad (8)$

$v(0) = 0 = 12 + A + B$, or $A + B = -12 \quad (9)$

$dv/dt = (-2Ae^{-2t} - 5Be^{-5t})$

$dv(0)/dt = 60 = -2A - 5B$

$2A + 5B = -60 \quad (10)$

From (9) and (10), $A = 0$ and $B = -12$.

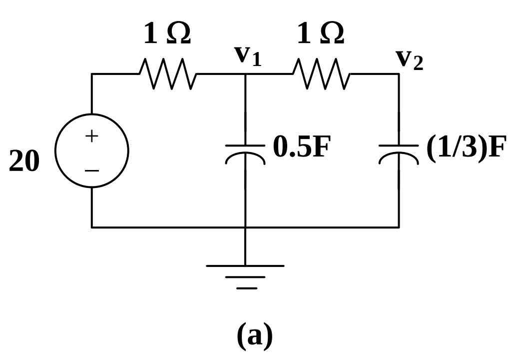
Thus, $v(t) = \mathbf{12(1 - e^{-5t})}$ **V for all t > 0**.

But, from (3), $i = 3 - i_C = 3 - (1/20)dv/dt = 3 - (1/20)(60)e^{-5t}$

$i(t) = \mathbf{3(1 - e^{-5t})}$ **A for all t > 0**.

P.P.8.10 $\quad$ For $t < 0$, $5u(t) = 0$ so that $v_1(0-) = v_2(0-) = 0 \quad (1)$

For $t > 0$, the circuit is as shown in Figure (a).



$$i_1 = C_1 dv_1/dt, \text{ or } dv_1/dt = i_1/C_1; \text{ likewise } dv_2/dt = i_2/C_2$$

$$i_2(0^+) = (v_1(0^+) - v_2(0^+))/1 = (0 - 0)/1 = 0$$

$$(20 - v_1(0^+))/1 = i_1(0^+) + i_2(0^+), \text{ or } 20 = i_1(0^+)$$

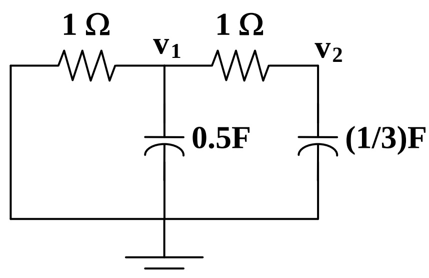
Hence, $$dv_1(0^+)/dt = 20/(1/2) = 40 \text{ V/s} \quad (2a)$$

$$dv_2(0^+)/dt = 0 \quad (2b)$$

As t approaches infinity, the capacitors can be replaced by open circuits as shown in Figure (b). Thus,

$$v_1(\infty) = v_2(\infty) = 20\text{V} \quad (3)$$

Next we obtain the network response by considering the circuit in Figure (c).



(c)

Applying KCL at node 1 gives $(v_1/1) + (1/2)(dv_1/dt) + (v_1 - v_2)/1 = 0$

or $$v_2 = 2v_1 + (1/2)dv_1/dt \quad (4)$$

Applying KCL at node 2 gives $(v_1 - v_2)/1 = (1/3)dv_2/dt$

$$\text{or } v_1 = v_2 + (1/3)dv_2/dt \quad (5)$$

Substituting (5) into (4) yields,

$$v_2 = 2v_2 + (2/3)(dv_2/dt) + (1/2)(dv_2/dt) + (1/6)d^2v_2/dt^2$$

$$\text{or,} \quad (d^2v_2/dt^2) + (7dv_2/dt) + 6v_2 = 0$$

Now we have, $s^2 + 7s + 6 = 0 = (s + 1)(s + 6)$

$$\text{Thus,}\quad v_{2n} = (Ae^{-t} + Be^{-6t}) \text{ and } v_{2f} = v_2(\infty) = 20V.$$

$$v_2 = v_{2n} + v_{2f} = 20 + (Ae^{-t} + Be^{-6t})$$

$$v_2(0) = 0 \text{ which implies that } A + B = -20 \qquad (6)$$

$$dv_2/dt = (-Ae^{-t} - 6Be^{-6t})$$

$$dv_2(0) = 0 = -A - 6B \qquad (7)$$

From (6) and (7), $A = -24$ and $B = 4$.

$$\text{Thus,}\quad v_2(t) = (20 - 24e^{-t} + 4e^{-6t}) \text{ V}$$

From (5), $v_1 = v_2 + (1/3)dv_2/dt$

$$\text{Thus,}\quad v_1(t) = (20 - 16e^{-t} - 4e^{-6t}) \text{ V}$$

Now we can find, $v_o = v_1 - v_2 =$ $\mathbf{8(e^{-t} - e^{-6t})\ V,\ t > 0}$

P.P.8.11 Let v_1 equal the voltage at non-inverting terminal of the op amp. Then v_o is equal to the output of the op amp.

At the non-inverting terminal, $(v_s - v_o)/R_1 = C_1dv_1/dt$ (1)

At the output terminal of the op amp, $(v_1 - v_o)/R_2 = C_2dv_o/dt$ (2)

We now eliminate v_1 from (2), $v_1 = v_o + R_2C_2dv_o/dt$ (3)

From (1) $v_s = v_1 + R_1C_1dv_1/dt$ (4)

Substituting (3) into (4) gives

$$v_s = v_o + R_2C_2dv_o/dt + R_1C_1dv_o/dt + R_1C_1R_2C_2d^2v_o/dt^2$$

$$\text{or}\quad d^2v_o/dt^2 + [(1/(R_1C_1)) + (1/(R_2C_2))]dv_o/dt + v_o/(R_1R_2C_1C_2) = v_s/(R_1R_2C_1C_2)$$

With the given parameters,

$$(R_1R_2C_1C_2) = 10^4x10^4x20x10^{-6}x100x10^{-6} = 2x10^{-2}$$

$$1/(R_1R_2C_1C_2) = 5$$

$$[(1/(R_1C_1)) + (1/(R_2C_2))] = 10^{-4}[(1/20x10^{-6}) + (1/200x10^{-6})] = 6$$

$$\text{Hence, we now have } s^2 + 6s + 5 = 0 = (s+1)(s+5)$$

$$\text{Therefore } v_{on} = Ae^{-t} + Be^{-5t}, \text{ and } v_{of} = 10\text{V}$$

Thus,
$$v_o = 10 + Ae^{-t} + Be^{-5t} \qquad (5)$$

$$\text{For } t < 0,\ v_s = 0,\ v_1(0^-) = 0 = v_o(0^-)$$

$$\text{For } t > 0,\ v_s = 10, \text{ but}$$

$$v_1(0^+) - v_o(0^+) = 0 \qquad (6)$$

From (2),
$$dv_o(0^+)/dt = [v_1(0^+) - v_o(0^+)]/R_2C_2 = 0 \qquad (7)$$

Imposing these conditions on $v_o(t)$,

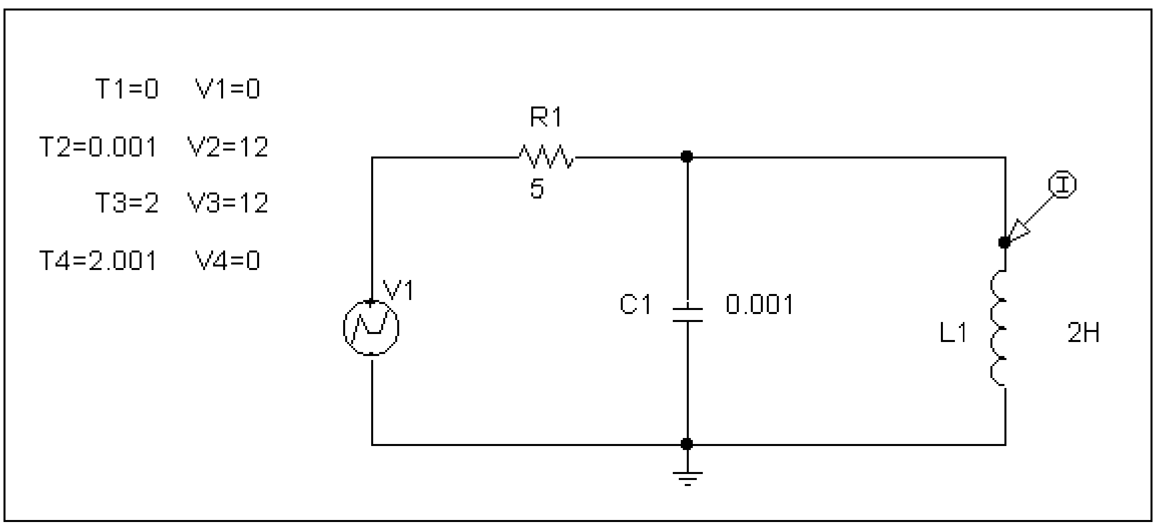
$$0 = 10 + A + B \qquad (8)$$

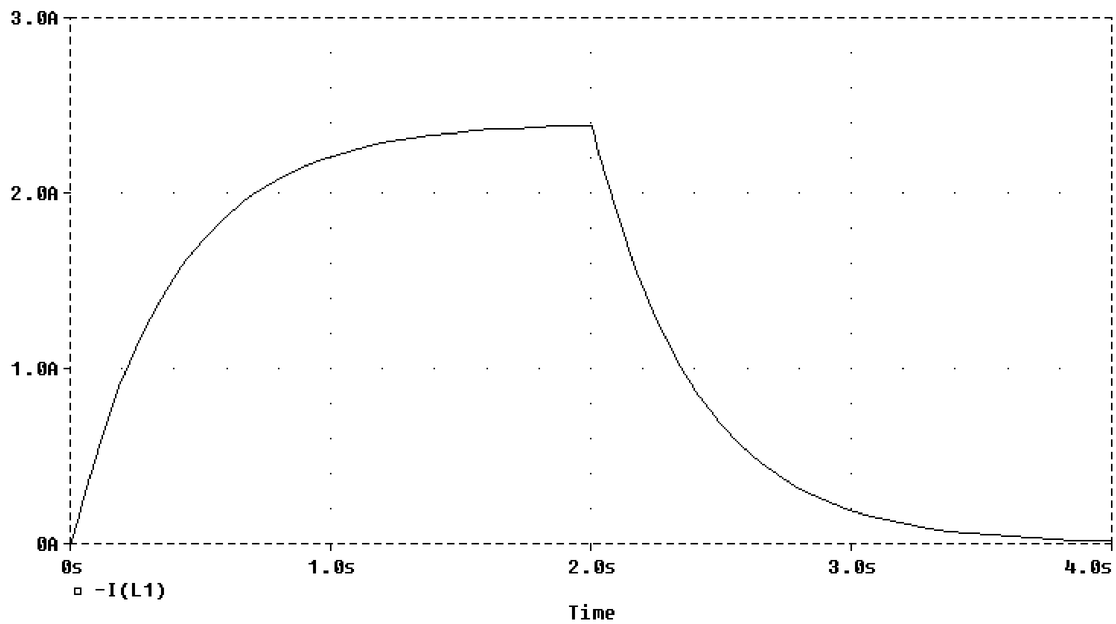
$$0 = -A - 5B \text{ or } A = -5B \qquad (9)$$

From (8) and (9), $A = -12.5$ and $B = 2.5$

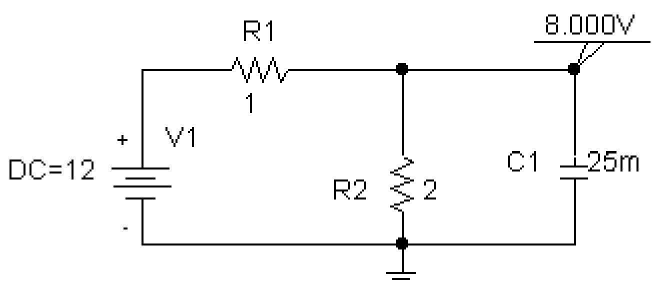
$$v_o(t) = \mathbf{(10 - 12.5e^{-t} + 2.5e^{-5t})\ V,\ \ t > 0}$$

P.P.8.12 We follow the same procedure as in Example 8.12. The schematic is shown in Figure (a). The current marker is inserted to display the inductor current. After simulating the circuit, the required inductor current is plotted in Figure (b).



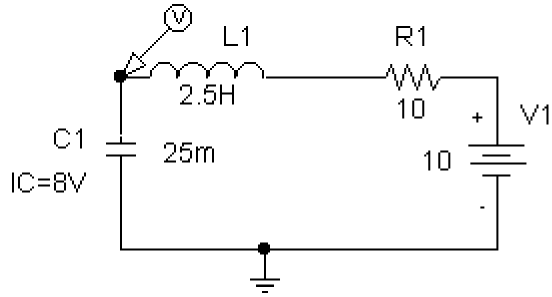


P.P.8.13 When the switch is at position a, the schematic is as shown in Figure (a). We carry out dc analysis on this to obtain initial conditions. It is evident that $v_C(0) = 8$ volts.

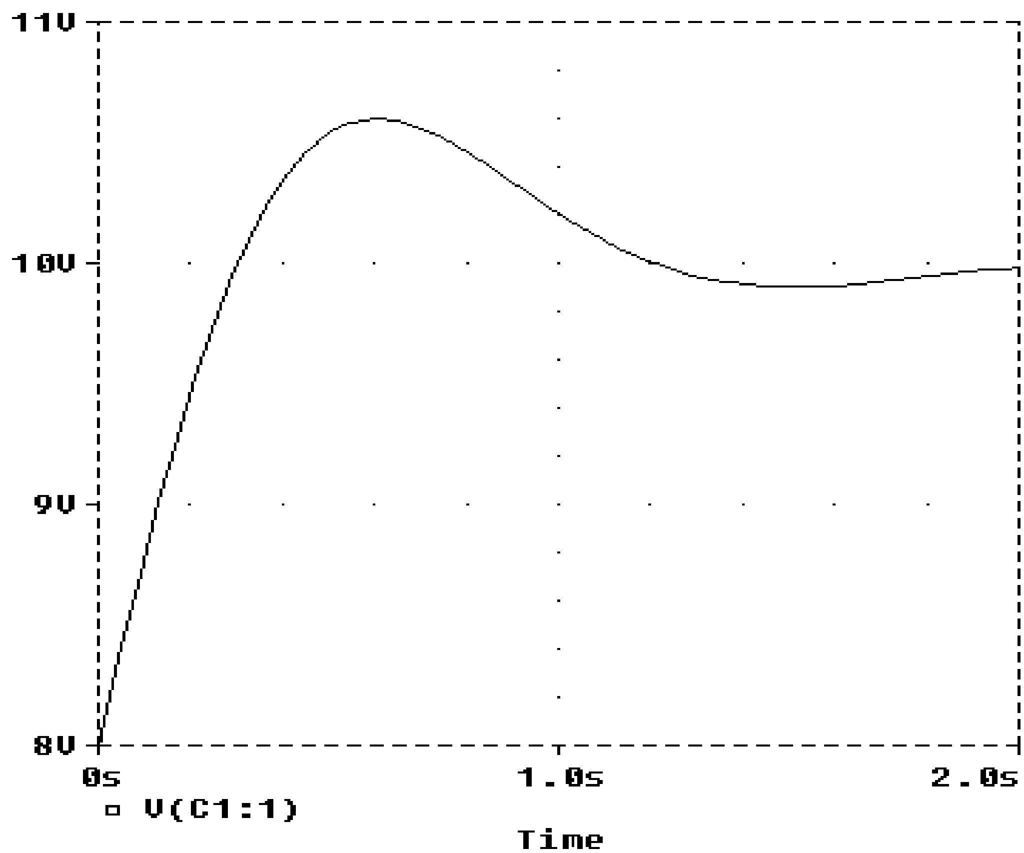


(a)

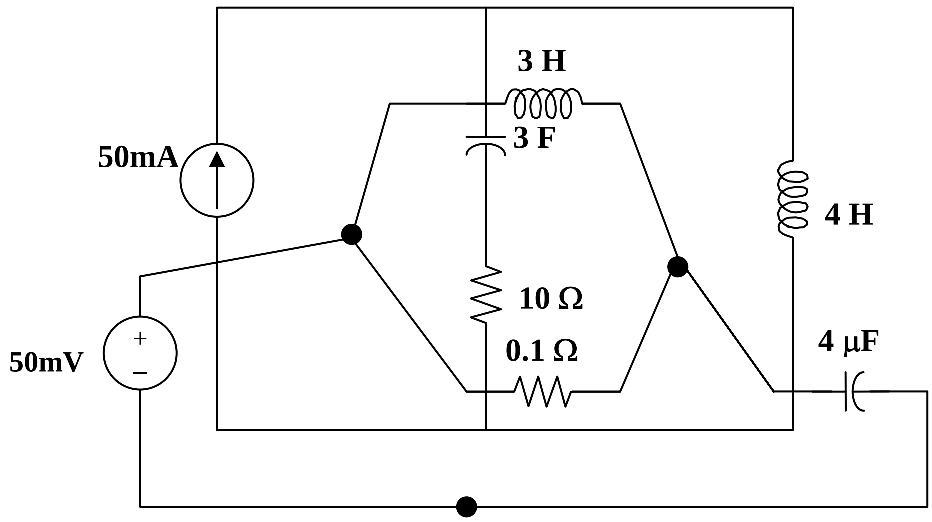
With the switch in position b, the schematic is as shown in Figure (b). A voltage marker is inserted to display the capacitor voltage. When the schematic is saved and run, the output is as shown in Figure (c).



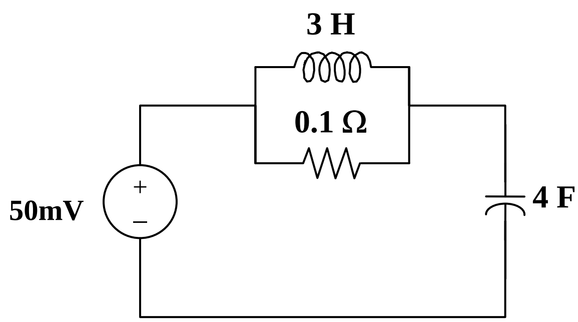
(b)



P.P.8.14 The dual circuit is obtained from the original circuit as shown in Figure (a). It is redrawn as shown in Figure (b).

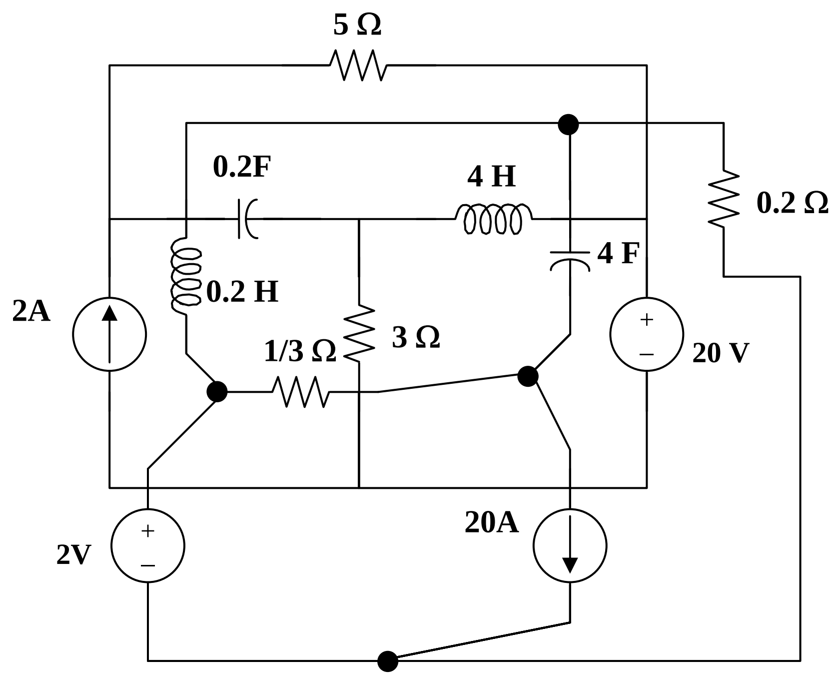


(a)

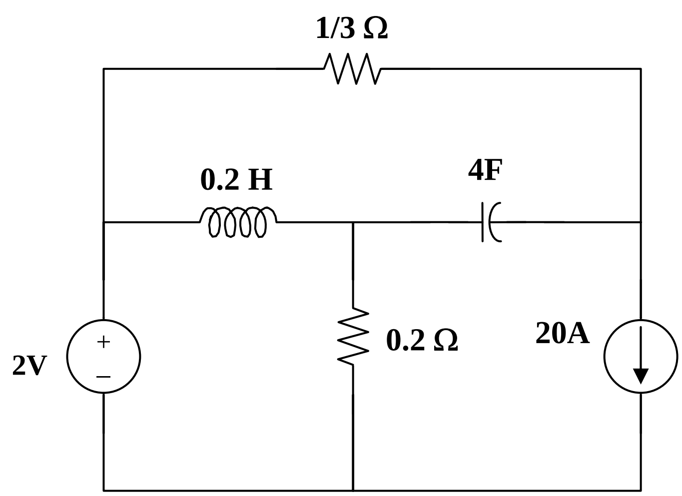


(b)

P.P.8.15 The dual circuit is obtained in Figure (a) and redrawn in Figure (b).



(a)



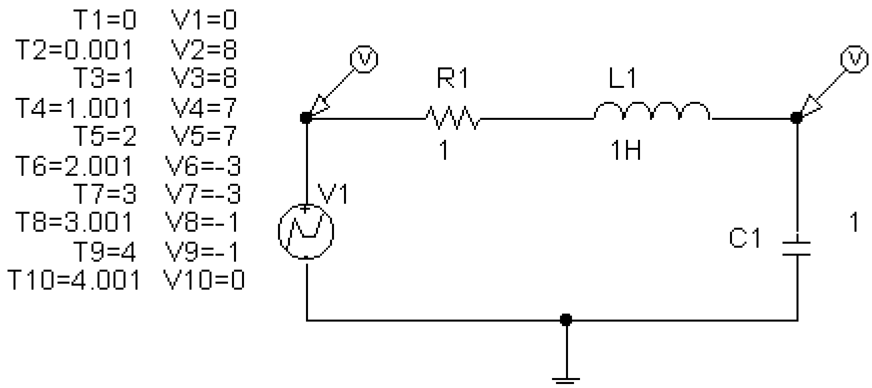
(b)

P.P.8.16 Since $12 = 4i + v_L + v_C$ or $v_C = 12 - 4i - v_L$

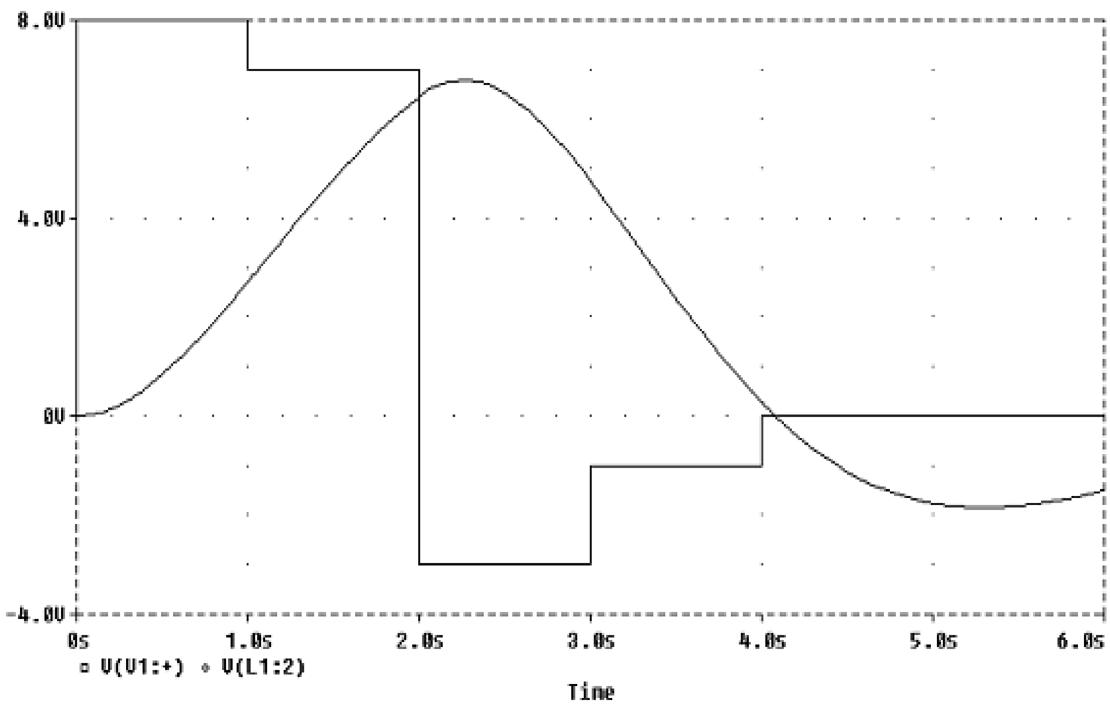
$$-(v_C - 12) = 4i + v_L = e^{-250t}(12\cos\omega_d t + 0.2684\sin\omega_d t - 268\sin\omega_d t)$$

$$v_C(t) = \mathbf{[12 - 12e^{-250t}\cos(11{,}180t) + 267.7e^{-250t}\sin(11{,}180t)]\ V}$$

P.P.8.17 We follow the same procedure as in Example 8.17. The schematic is as shown in Figure (a) with two voltage markers to display both input and output voltages. When the schematic is saved and run, the result is as displayed in Figure (b).



(a)



(b)

CHAPTER 9

P.P.9.1 amplitude = **30**

phase = **–75°**

angular frequency (ω) = 4π = **12.57 rad/s**

period (T) $= \dfrac{2\pi}{\omega}$ = **0.5 s**

frequency (f) $= \dfrac{1}{T}$ = **2 Hz**

P.P.9.2

$$i_1 = -4\sin(\omega t + 55°) = 4\cos(\omega t + 55° + 90°)$$

$$i_1 = 4\cos(\omega t + 145°), \qquad \omega = 377 \text{ rad/s}$$

Compare this with

$$i_2 = 5\cos(\omega t - 65°)$$

indicates that the phase angle between i_1 and i_2 is

145° + 65° = 210°

Thus, **i_1 leads i_2 by 210°**

P.P.9.3 (a)

(5 + j2)(-1 + j4) = -5 + j20 – j2 – 8 = -13 + j18

5∠60° = 2.5 + j4.33

(5 + j2)(-1 + j4) – 5∠60° = -15.5 + j13.67

[(5 + j2)(-1 + j4) – 5∠60]* = **-15.5 – j13.67** = **20.67∠221.41°**

(b)

3∠40° = 2.298 + j1.928

10 + j5 + 3∠40° = 12.298 + j6.928 = 14.115∠29.39°

–3 + j4 = 5∠126.87°

$$\frac{10 + j5 + 3\angle 40°}{-3 + j4} = \frac{14.115\angle 29.39°}{5\angle 126.87°} = 2.823\angle -97.48°$$

2.823∠-97.48° = –0.3675 – j2.8

10∠30° = 8.66 + j5

$$\frac{10 + j5 + 3\angle 40°}{-3 + j4} + 10\angle 30° + j5 = \mathbf{8.293 + j7.2}$$

P.P.9.4 (a) v = 7 cos(2t + 40°)

The phasor form is

V = 7∠40° V

(b) Since –sin(A) = cos(A + 90°),

$i = -4\sin(10t + 10°) = 4\cos(10t+10° + 90°)$

$i = 4\cos(10t + 100°)$

The phasor form is

$\mathbf{I} = \mathbf{4\angle 100°\ A}$

P.P.9.5 (a) Since $-1 = 1\angle\pm 180°$ (we can use either sign)

$\mathbf{V} = -25\angle 40° = 25\angle(40°-180°) = 25\angle -140°$

The sinusoid is

$v(t) = \mathbf{25\cos(\omega t - 140°)\ V}$ or $\mathbf{25\cos(\omega t+220°)\ V}$

(b) $\mathbf{I} = j(12 - j5) = 5 + j12 = 13\angle 67.38°$

The sinusoid is

$\mathbf{i(t) = 13\cos(\omega t + 67.38°)\ A}$

P.P.9.6 Let $v(t) = -10\sin(\omega t-30°) + 20\cos(\omega t+45°)$

$= 10\cos(\omega t-30°+90°) + 20\cos(\omega t+45°)$

Taking the phasor of each term

$\mathbf{V} = 10\angle 60° + 20\angle 45°$

$\mathbf{V} = 5 + j8.66 + 14.142 + j14.142$

$\mathbf{V} = 19.142 + j22.8 = 29.77\angle 49.98°$

Converting **V** to the time domain

$v(t) = \mathbf{29.77\cos(\omega t + 49.98°)\ V}$

P.P.9.7 Given that

$$2\frac{dv}{dt} + 5v + 10\int v\,dt = 50\cos(5t - 30°)$$

we take the phasor of each term to get

$$2j\omega\,\mathbf{V} + 5\,\mathbf{V} + \frac{10}{j\omega}\mathbf{V} = 50\angle{-30°}, \qquad \omega = 5$$

$\mathbf{V}\,[j10 + 5 - j(10/5)] = \mathbf{V}\,(5 + j8) = 50\angle{-30°}$

$$\mathbf{V} = \frac{50\angle{-30°}}{5 + j8} = \frac{50\angle{-30°}}{9.434\angle 58°}$$

$\mathbf{V} = 5.3\angle{-88°}$

Converting **V** to the time domain

$v(t) = \mathbf{5.3\cos(5t - 88°)V}$

P.P.9.8 For the capacitor,

$\mathbf{V} = \mathbf{I}/(j\omega C)$, where $\mathbf{V} = 10\angle 30°$, $\omega = 100$

$\mathbf{I} = j\omega C\,\mathbf{V} = (j100)(50\text{x}10^{-6})(10\angle 30°)$

$\mathbf{I} = 50\angle 120°$ mA

$i(t) = \mathbf{50\cos(100t + 120°)\ mA}$

P.P.9.9 $\mathbf{V}_s = 20\angle 30°$, $\omega = 10$

$\mathbf{Z} = 4 + j\omega L = 4 + j2$

$\mathbf{I} = \mathbf{V}_s / \mathbf{Z} = \dfrac{20\angle 30°}{4 + j2} = \dfrac{20\angle 30°(4 - j2)}{16 + 4} = 4.472\angle 3.43°$

$\mathbf{V} = j\omega L\,\mathbf{I} = j2\,\mathbf{I} = (2\angle 90°)(4.472\angle 3.43°) = 8.944\angle 93.43°$

Therefore, $v(t) = \mathbf{8.944\ sin(10t + 93.43°)\ V}$
$i(t) = \mathbf{4.472\ sin(10t + 3.43°)\ A}$

P.P.9.10

Let $\mathbf{Z}_1$ = impedance of the 1-mF capacitor in series with the 100-Ω resistor
$\mathbf{Z}_2$ = impedance of the 1-mF capacitor
$\mathbf{Z}_3$ = impedance of the 8-H inductor in series with the 200-Ω resistor

$$\mathbf{Z}_1 = 100 + \frac{1}{j\omega C} = 100 + \frac{1}{j(10)(1\times 10^{-3})} = 80 - j100$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C} = \frac{1}{j(10)(1\times 10^{-3})} = -j100$$

$$\mathbf{Z}_3 = 200 + j\omega L = 200 + j(10)(8) = 200 + j80$$

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \parallel \mathbf{Z}_3 = \mathbf{Z}_1 + \mathbf{Z}_2\,\mathbf{Z}_3 / (\mathbf{Z}_2 + \mathbf{Z}_3)$$

$$\mathbf{Z}_{in} = 100 - j100 + \frac{-j100x(200 + j80)}{-j100 + 200 + j80}$$

$$\mathbf{Z}_{in} = 100 - j100 + 49.52 - j95.04$$

$$\mathbf{Z}_{in} = \mathbf{[149.52 - j195]\ \Omega}$$

P.P.9.11 In the frequency domain,

the voltage source is $\mathbf{V}_s = 20\angle 100°$

the 0.5-H inductor is $j\omega L = j(10)(0.5) = j5$

the $\dfrac{1}{20}$-F capacitor is $\dfrac{1}{j\omega C} = \dfrac{1}{j(10)(1/20)} = -j2$

Let $\mathbf{Z}_1$ = impedance of the 0.5-H inductor in parallel with the 10-Ω resistor
and $\mathbf{Z}_2$ = impedance of the (1/20)-F capacitor

$$\mathbf{Z}_1 = 10 \parallel j5 = \frac{(10)(j5)}{10 + j5} = 2 + j4 \qquad \text{and} \qquad \mathbf{Z}_2 = -j2$$

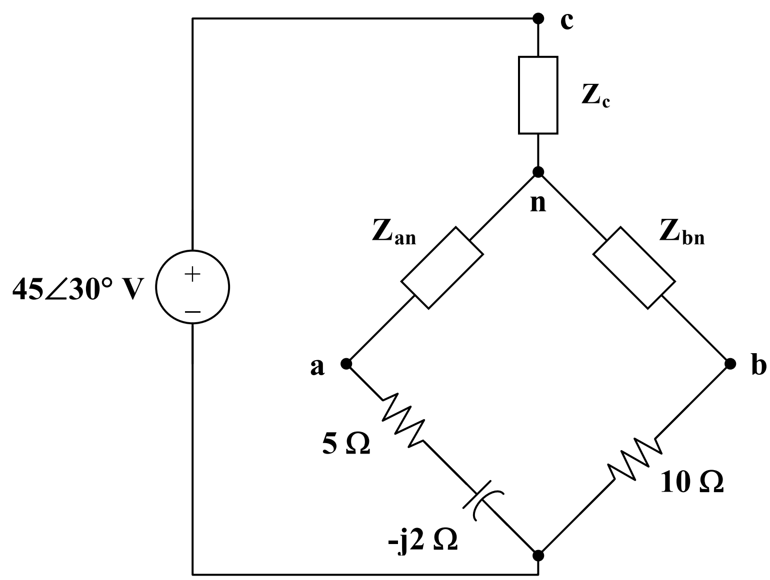
$$\mathbf{V}_o = \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2)\,\mathbf{V}_s$$

$$\mathbf{V}_o = \frac{-j2}{2 + j4 - j2}(50\angle 30°) = \frac{-j(50\angle 30°)}{1 + j} = \frac{50\angle(30° - 90°)}{\sqrt{2}\angle 45°}$$

$$\mathbf{V}_o = 35.36\angle -105°$$

$v_o(t) = \mathbf{35.36\ cos(10t - 105°)\ V}$

P.P.9.12 We need to find the equivalent impedance via a delta-to-wye transformation as shown below.



$$\mathbf{Z}_{an} = \frac{j4(8+j5)}{j4+8+j5-j3} = \frac{4(-5+j8)}{8+j6} = 0.32 + j3.76$$

$$\mathbf{Z}_{bn} = \frac{-j3(8+j5)}{8+j6} = \frac{3(5-j8)(8-j6)}{100} = -0.24 - j2.82$$

$$\mathbf{Z}_{cn} = \frac{j4(-j3)}{8+j6} = \frac{12(8-j6)}{100} = 0.96 - j0.72$$

The total impedance from the source terminals is

$$\mathbf{Z} = \mathbf{Z}_{cn} + (\mathbf{Z}_{an} + 5 - j2) \parallel (\mathbf{Z}_{bn} + 10)$$

$$\mathbf{Z} = \mathbf{Z}_{cn} + (5.32 + j1.76) \parallel (9.76 - j2.82)$$

$$\mathbf{Z} = \mathbf{Z}_{cn} + \frac{(5.32+j1.76)(9.76-j2.82)}{(5.32+j1.76)+(9.76-j2.82)}$$

$$\mathbf{Z} = 0.96 - j0.72 + 3.744 + j0.4074$$

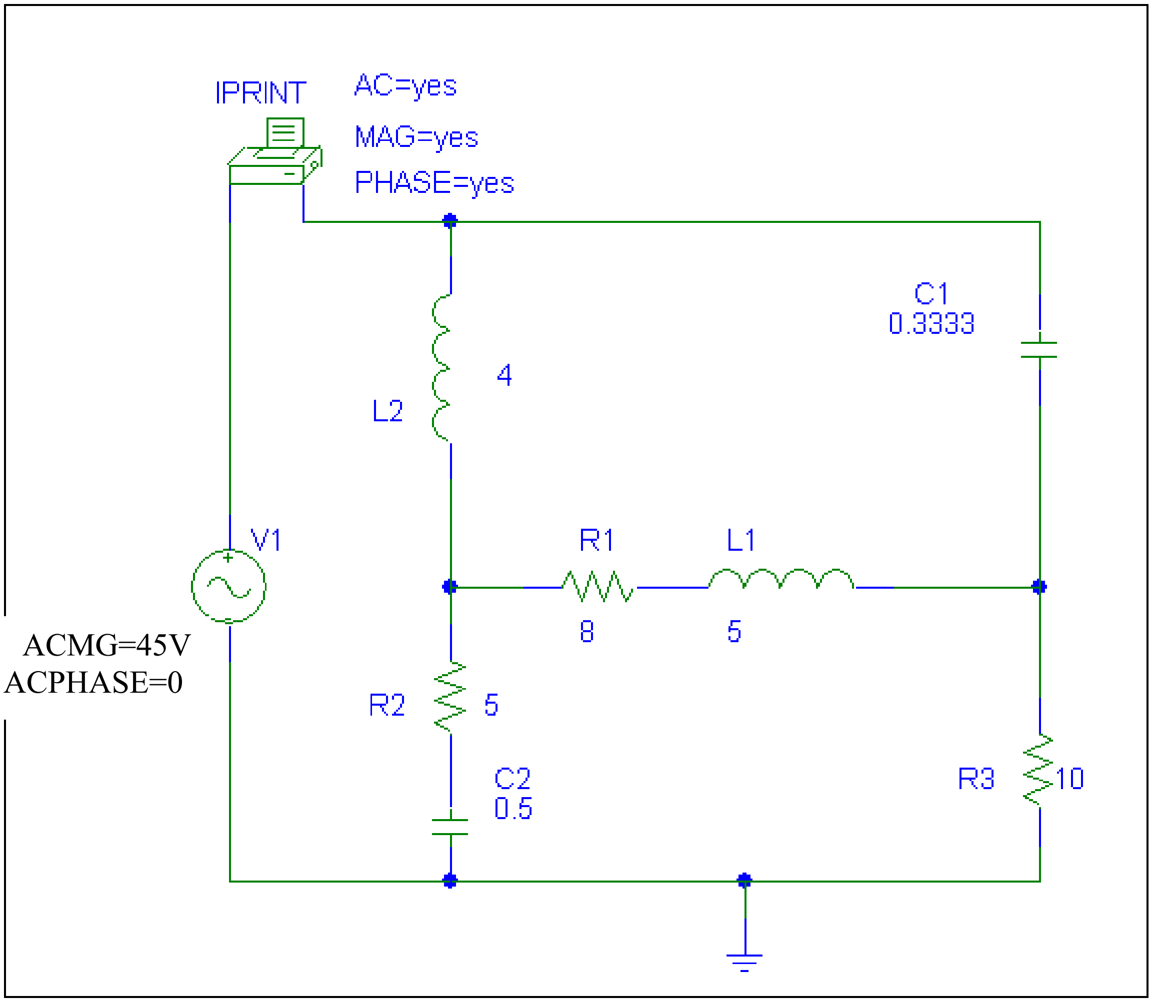
$$\mathbf{Z} = 4.704 - j0.3126 = 4.714\angle\text{-}3.802°$$

Therefore,

$$\mathbf{I} = \mathbf{V} / \mathbf{Z} = \frac{45\angle 30°}{4.714\angle -3.802°}$$

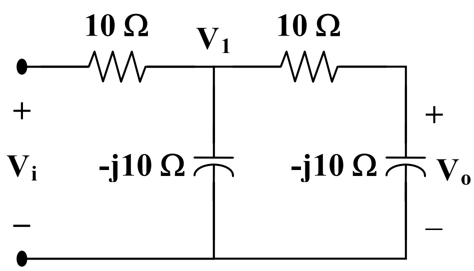
$$\mathbf{I} = \mathbf{9.546\ \angle 33.8°\ A}$$

Let us now check this using PSpice. The solution produces the magnitude of I = 9.946E+00, and the phase angle = 33.803E+00, which agrees with the above answer.

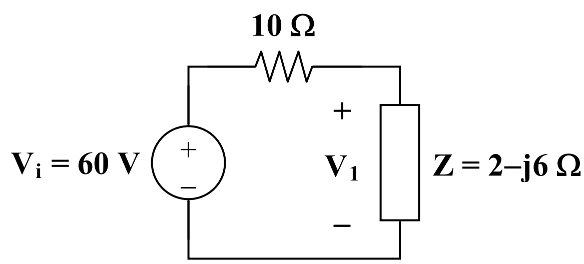


P.P.9.13 To show that the circuit in Fig. (a) meets the requirement, consider the equivalent circuit in Fig. (b).

$$\mathbf{Z} = \text{-j10} \parallel (10 - \text{j10}) = \frac{\text{-j10}(10 - \text{j10})}{10 - \text{j20}} = \frac{\text{-j}(10 - \text{j10})}{1 - \text{j2}} = 2 - \text{j6}\ \Omega$$



(a)



(b)

$$\mathbf{V}_1 = \frac{2-j6}{10+2-j6}(60) = \frac{60}{3}(1-j)$$

$$\mathbf{V}_o = \frac{-j10}{10-j10}\mathbf{V}_1 = \left(\frac{-j}{1-j}\right)\left(\frac{60}{3}\right)(1-j) = -j20$$

$$\mathbf{V}_o = 20\angle -90^\circ$$

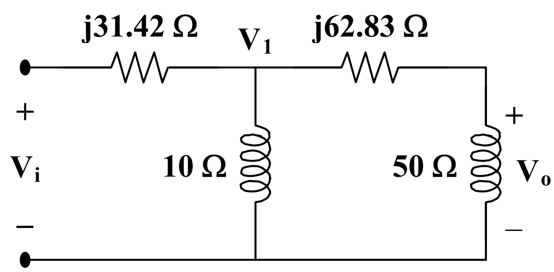
This implies that the RC circuit provides a 90° lagging phase shift.
The output voltage is = **20 V**

P.P.9.14

the 1-mH inductor is $j\omega L = j(2\pi)(5\times10^3)(1\times10^{-3}) = j31.42$

the 2-mH inductor is $j\omega L = j(2\pi)(5\times10^3)(2\times10^{-3}) = j62.83$

Consider the circuit shown below.



$$\mathbf{Z} = 10 \,||\, (50 + j62.83) = \frac{(10)(50+j62.83)}{60+j62.83}$$

$$\mathbf{Z} = 9.205 + j0.833 = 9.243\angle 5.17^\circ$$

$$\mathbf{V}_1 = \mathbf{Z}/(\mathbf{Z}+j31.42)\,\mathbf{V}_i = \frac{9.243\angle 5.17^\circ}{9.205+j32.253}(10)$$

$$= [(9.243\angle 5.17^\circ)/(33.54\angle 74.07^\circ)]10 = 2.756\angle -68.9^\circ$$

$$\mathbf{V}_o = \frac{50}{50+j62.83}\mathbf{V}_1 = \frac{50(2.756\angle -68.9^\circ)}{80.297\angle 51.49^\circ} = 1.7161\angle -120.39^\circ$$

Therefore,

magnitude = **1.7161 V**
phase = **120.39°**
phase shift is **lagging**

P.P.9.15 $\mathbf{Z}_x = (\mathbf{Z}_3 / \mathbf{Z}_1)\, \mathbf{Z}_2$

$\mathbf{Z}_3 = 12\ \text{k}\Omega$

$\mathbf{Z}_1 = 4.8\ \text{k}\Omega$

$\mathbf{Z}_2 = 10 + j\omega L = 10 + j(2\pi)(6\times10^{6})(0.25\times10^{-6}) = 10 + j9.425$

$$\mathbf{Z}_x = \frac{12k}{4.8k}(10 + j9.425) = 25 + j23.5625\ \Omega$$

$R_x = 25, \qquad X_x = 23.5625 = \omega L_x$

$$L_x = \frac{X_x}{2\pi f} = \frac{23.5625}{2\pi(6\times10^{6})} = 0.625\ \mu\text{H}$$

i.e. **a 25-Ω resistor in series with a 0.625-μH inductor**.

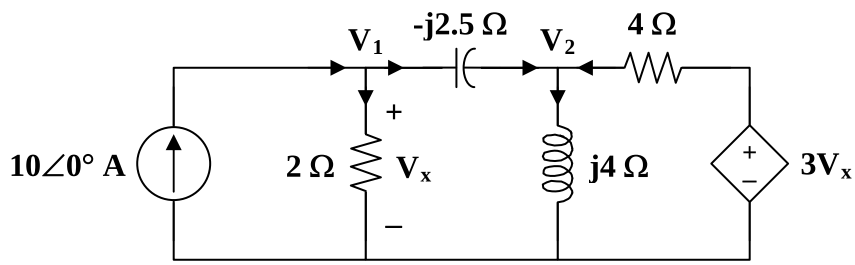
CHAPTER 10

P.P.10.1 $10\cos(2t) \longrightarrow 10\angle 0°, \quad \omega = 2$

$$2\text{ H} \longrightarrow j\omega L = j4$$

$$0.2\text{ F} \longrightarrow \frac{1}{j\omega C} = -j2.5$$

Hence, the circuit in the frequency domain is as shown below.



At node 1, $\quad 10 = \dfrac{\mathbf{V}_1}{2} + \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{-j2.5}$

$$100 = (5 + j4)\,\mathbf{V}_1 - j4\mathbf{V}_2 \qquad (1)$$

At node 2, $\quad \dfrac{\mathbf{V}_2}{j4} = \dfrac{\mathbf{V}_1 - \mathbf{V}_2}{-j2.5} + \dfrac{3\mathbf{V}_x - \mathbf{V}_2}{4} \qquad$ where $\mathbf{V}_x = \mathbf{V}_1$

$$-j2.5\mathbf{V}_2 = j4(\mathbf{V}_1 - \mathbf{V}_2) + 2.5(3\mathbf{V}_1 - \mathbf{V}_2)$$

$$0 = -(7.5 + j4)\,\mathbf{V}_1 + (2.5 + j1.5)\,\mathbf{V}_2 \qquad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 5 + j4 & -j4 \\ -(7.5 + j4) & 2.5 + j1.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

where $\Delta = (5 + j4)(2.5 + j.15) - (-j4)(-(7.5 + j4)) = 22.5 - j12.5 = 25.74\angle -29.05°$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \frac{\begin{bmatrix} 2.5 + j1.5 & j4 \\ 7.5 + j4 & 5 + j4 \end{bmatrix}}{22.5 - j12.5} \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$\mathbf{V}_1 = \frac{2.5 + j1.5}{22.5 - j12.5}(100) = \frac{2.915\angle 30.96°}{25.74\angle -29.05°}(100) = 11.325\angle 60.01° V$$

$$\mathbf{V}_2 = \frac{7.5 + j4}{22.5 - j12.5}(100) = \frac{8.5\angle 28.07°}{25.74\angle -29.05°}(100) = 33.02\angle 57.12° V$$

In the time domain,

$v_1(t) =$ **11.325cos(2t + 60.01°) V**

$v_2(t) =$ **33.02cos(2t + 57.12°) V**

P.P.10.2 The only non-reference node is a supernode.

$$\frac{75 - \mathbf{V}_1}{4} = \frac{\mathbf{V}_1}{j4} + \frac{\mathbf{V}_2}{-j} + \frac{\mathbf{V}_2}{2}$$

$$75 - \mathbf{V}_1 = -j\,\mathbf{V}_1 + j4\mathbf{V}_2 + 2\mathbf{V}_2$$

$$75 = (1 - j)\mathbf{V}_1 + (2 + j4)\mathbf{V}_2 \qquad (1)$$

The supernode gives the constraint of

$$\mathbf{V}_1 = \mathbf{V}_2 + 100\angle 60° \qquad (2)$$

Substituting (2) into (1) gives

$$75 = (1 - j)(40\angle 60°) + (3 + j3)\mathbf{V}_2$$

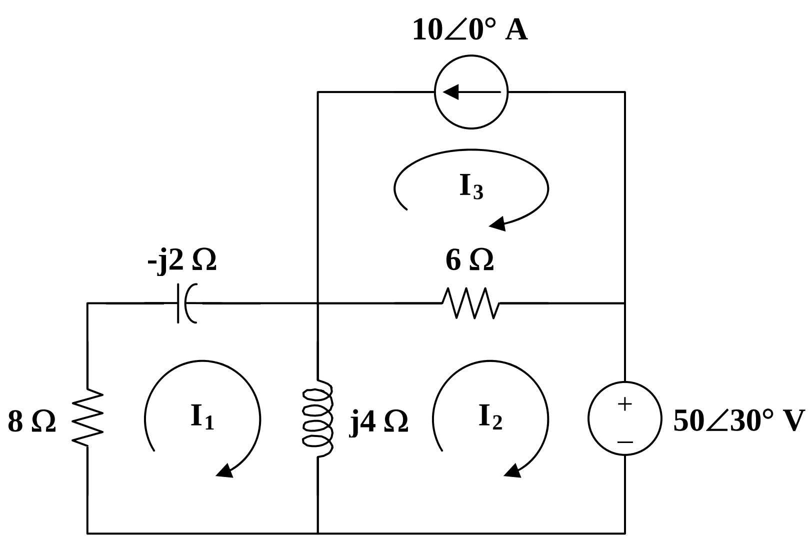
$$\mathbf{V}_2 = \frac{75 - (1 - j)(100\angle 60°)}{3 + j3} = \frac{71.62\angle 210.72°}{4.243\angle 45°} = 16.881\angle 165.72°$$

$$\mathbf{V}_1 = \mathbf{V}_2 + 100\angle 60° = (-16.358 + j4.17) + (50 + j86.6)$$

$$\mathbf{V}_1 = 33.64 + j90.77$$

Therefore, $\mathbf{V}_1 = \mathbf{96.8\angle 69.66°\ V}$, $\mathbf{V}_2 = \mathbf{16.88\angle 165.72°\ V}$

P.P.10.3 Consider the circuit below.



For mesh 1, $(8 - \mathrm{j}2 + \mathrm{j}4)\mathbf{I}_1 - \mathrm{j}4\mathbf{I}_2 = 0$

$$(8 + \mathrm{j}2)\mathbf{I}_1 = \mathrm{j}4\mathbf{I}_2 \qquad (1)$$

For mesh 2, $(6 + j4)\mathbf{I}_2 - j4\mathbf{I}_1 - 6\mathbf{I}_3 + 50\angle 30° = 0$

For mesh 3, $\mathbf{I}_3 = -10$

Thus, the equation for mesh 2 becomes

$$(6 + j4)\mathbf{I}_2 - j4\mathbf{I}_1 = -60 - 50\angle 30° \qquad (2)$$

From (1), $\mathbf{I}_2 = \dfrac{8 + \text{j}2}{\text{j}4}\mathbf{I}_1 = (0.5 - \text{j}2)\mathbf{I}_1 \qquad (3)$

Substituting (3) into (2),

$$(6 + j4)(0.5 - j2)\mathbf{I}_1 - j4\mathbf{I}_1 = -60 - 50\angle 30°$$

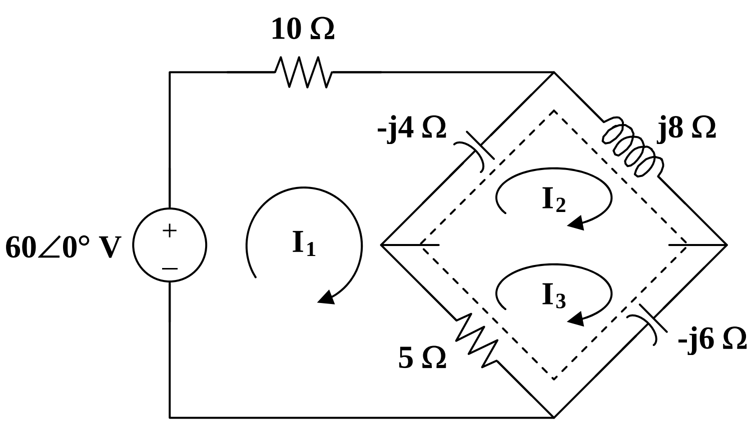
$$(11 - j14)\mathbf{I}_1 = -(103.3 + j25)$$

$$\mathbf{I}_1 = \frac{-(103.3 + j25)}{11 - j14}$$

Hence, $\mathbf{I}_o = \mathbf{-I}_1 = \dfrac{103.3 + j25}{11 - j14} = \dfrac{106.28\angle 13.605°}{17.804\angle -51.843°}$

$$\mathbf{I}_o = \mathbf{5.969\angle 65.45°\ A}$$

P.P.10.4 Meshes 2 and 3 form a supermesh as shown in the circuit below.



For mesh 1, $-60 + (15 - j4)\mathbf{I}_1 - (-j4)\mathbf{I}_2 - 5\mathbf{I}_3 = 0$

$$(15 - j4)\mathbf{I}_1 + j4\mathbf{I}_2 - 5\mathbf{I}_3 = 60 \qquad (1)$$

For the supermesh, $(\text{j}8 - \text{j}4)\mathbf{I}_2 + (5 - \text{j}6)\mathbf{I}_3 - (5 - \text{j}4)\mathbf{I}_1 = 0 \qquad (2)$

Also, $\mathbf{I}_3 = \mathbf{I}_2 + 2.4 \qquad (3)$

Eliminating $\mathbf{I}_3$ from (1) and (2)

$$(15 - j4)\mathbf{I}_1 + (-5 + j4)\mathbf{I}_2 = 72 \qquad (4)$$

$$(-5 + j4)\mathbf{I}_1 + (5 - j2)\mathbf{I}_2 = -12 + j14.4 \qquad (5)$$

From (4) and (5),

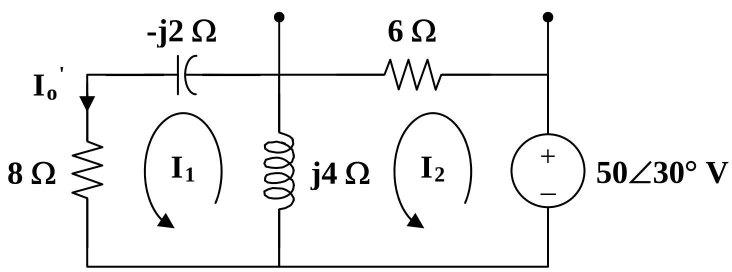
$$\begin{bmatrix} 15 - j4 & -5 + j4 \\ -5 + j4 & 5 - j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 72 \\ -12 + j14.4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 15 - j4 & -5 + j4 \\ -5 + j4 & 5 - j2 \end{vmatrix} = 58 - j10 = 58.86\angle -9.78°$$

$$\Delta_1 = \begin{vmatrix} 72 & -5 + j4 \\ -12 + j14.4 & 5 - j2 \end{vmatrix} = 357.6 - j24 = 358.4\angle -3.84°$$

Thus, $\mathbf{I}_o = \mathbf{I}_1 = \dfrac{\Delta_1}{\Delta} =$ $\mathbf{6.089\angle 5.94°\ A}$

P.P.10.5 Let $\mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o''$, where $\mathbf{I}_o'$ and $\mathbf{I}_o''$ are due to the voltage source and current source respectively. For $\mathbf{I}_o'$ consider the circuit in Fig. (a).



(a)

For mesh 1, $(8 + j2)\mathbf{I}_1 - j4\mathbf{I}_2 = 0$

$$\mathbf{I}_2 = (0.5 - j2)\mathbf{I}_1 \qquad (1)$$

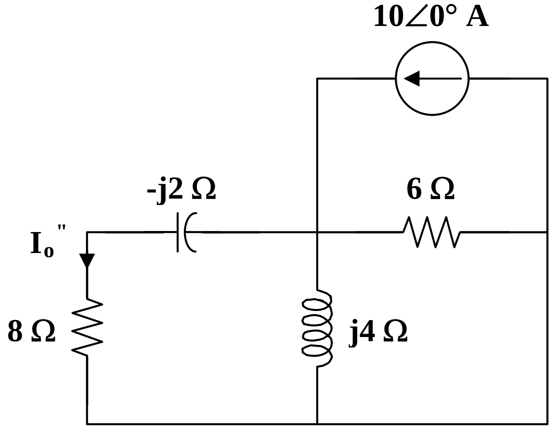
For mesh 2, $(6 + j4)\mathbf{I}_2 - j4\mathbf{I}_1 - 50\angle 30° = 0 \qquad (2)$

Substituting (1) into (2),

$$(6 + j4)(0.5 - j2)\mathbf{I}_1 - j4\mathbf{I}_1 = 50\angle 30°$$

$$\mathbf{I}_o' = \mathbf{I}_1 = \frac{50\angle 30°}{11 - j14} = 0.4 + j2.78$$

For $\mathbf{I}_o''$ consider the circuit in Fig. (b).



(b)

Let $\quad \mathbf{Z}_1 = 8 - j2\,\Omega, \qquad \mathbf{Z}_2 = 6 \,||\, j4 = \dfrac{j24}{6 + j4} = 1.846 + j2.769\,\Omega$

$$\mathbf{I}_o'' = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(10) = \frac{(10)(1.846 + j2.769)}{9.846 + j0.77} = 2.082 + j2.65$$

Therefore, $\quad \mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = 2.48 + j5.43$

$$\mathbf{I}_o = \mathbf{5.97\angle 65.45°\ A}$$

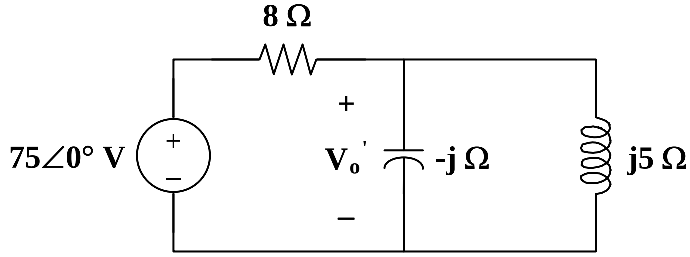
P.P.10.6 Let $v_o = v_o' + v_o''$, where v_o' is due to the voltage source and v_o'' is due to the current source. For v_o', we remove the current source.

$$75\sin(5t) \longrightarrow 75\angle 0°, \quad \omega = 5$$

$$0.2\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(5)(0.2)} = -j$$

$$1\text{ H} \longrightarrow j\omega L = j(5)(1) = j5$$

The circuit in the frequency domain is shown in Fig. (a).



(a)

Note that $\quad -j \| j5 = \text{-}j1.25$

By voltage division,

$$\mathbf{V}_o^{'} = \frac{-j1.25}{8 - j1.25}(75) = 11.577\angle -81.12°$$

Thus, $\quad v_o^{'} = 11.577\sin(5t - 81.12°)V$

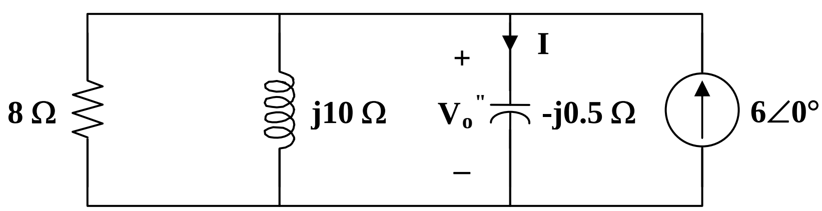
For $v_o^{''}$, we remove the voltage source.

$$6\cos(10t) \longrightarrow 6\angle 0°, \quad \omega = 10$$

$$0.2\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.2)} = \text{-}j0.5$$

$$1\text{ H} \longrightarrow j\omega L = j(10)(1) = j10$$

The corresponding circuit in the frequency domain is shown in Fig (b).



(b)

Let $\quad \mathbf{Z}_1 = \text{-}j0.5, \quad \mathbf{Z}_2 = 8 \| j10 = \frac{j80}{8 + j10} = 4.878 + j3.9$

By current division,

$$\mathbf{I} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(6)$$

$$\mathbf{V}_o^{''} = \mathbf{I}(\text{-}j0.5) = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(6)(\text{-}j0.5) = \frac{-j(14.631 + j11.7)}{4.878 + j3.4}$$

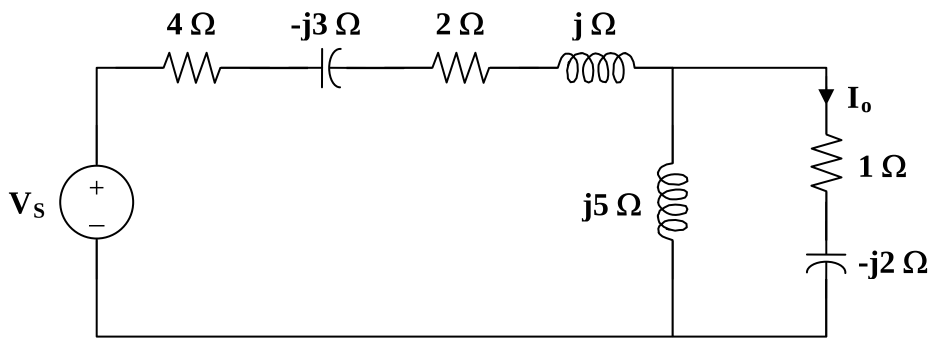
$$\mathbf{V}_o^{''} = \frac{18.735\angle\text{-}51.36°}{5.94\angle 34.88°} = 3.154\angle\text{-}86.24°$$

Thus, $\quad v_o^{''} = 3.154\cos(10t - 86.24°)$

Therefore, $\quad v_o = v_o^{'} + v_o^{''}$

$v_o =$ **[11.577sin(5t – 81.12°) + 3.154cos(10t – 86.24°)] V**

P.P.10.7 If we transform the current source to a voltage source, we obtain the circuit shown in Fig. (a).

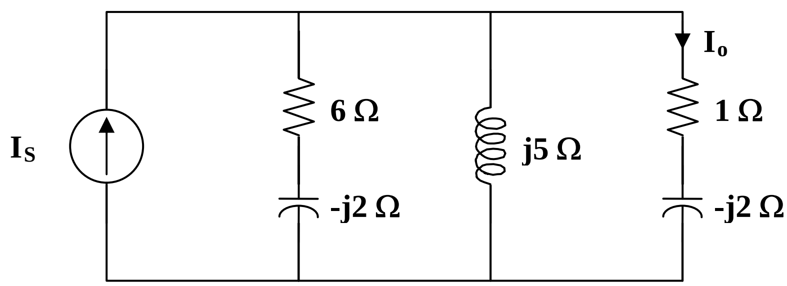


(a)

$$\mathbf{V}_s = \mathbf{I}_s\,\mathbf{Z}_s = (j12)(4 - j3) = 36 + j48$$

We transform the voltage source to a current source as shown in Fig. (b).

Let $\mathbf{Z} = 4 - \mathrm{j}3 + 2 + \mathrm{j} = 6 - \mathrm{j}2$. Then, $\mathbf{I}_s = \dfrac{\mathbf{V}_s}{\mathbf{Z}} = \dfrac{36 + j48}{6 - j2} = 4.5 + j9$.



(b)

Note that $\mathbf{Z} \parallel \mathrm{j}5 = \dfrac{(6 - \mathrm{j}2)(\mathrm{j}5)}{6 + \mathrm{j}3} = \dfrac{10}{3}(1 + \mathrm{j})$.

By current division,

$$\mathbf{I}_o = \frac{\frac{10}{3}(1 + j)}{\frac{10}{3}(1 + j) + (1 - j2)}(4.5 + j9)$$

$$\mathbf{I}_o = \frac{-60 + j120}{13 + j4} = \frac{134.16\angle 116.56°}{13.602\angle 17.1°}$$

$$\mathbf{I}_\mathrm{o} = \mathbf{9.863\angle 99.46°\ A}$$

P.P.10.8 When the voltage source is set equal to zero,

$$\mathbf{Z}_{th} = 10 + (\text{-j4}) \,||\, (6 + \text{j2})$$

$$\mathbf{Z}_{th} = 10 + \frac{(\text{-j4})(6 + \text{j2})}{6 - \text{j2}}$$

$$\mathbf{Z}_{th} = 10 + 2.4 - \text{j}3.2$$

$$\mathbf{Z}_{th} = \mathbf{(12.4 - j3.2)\ \Omega}$$

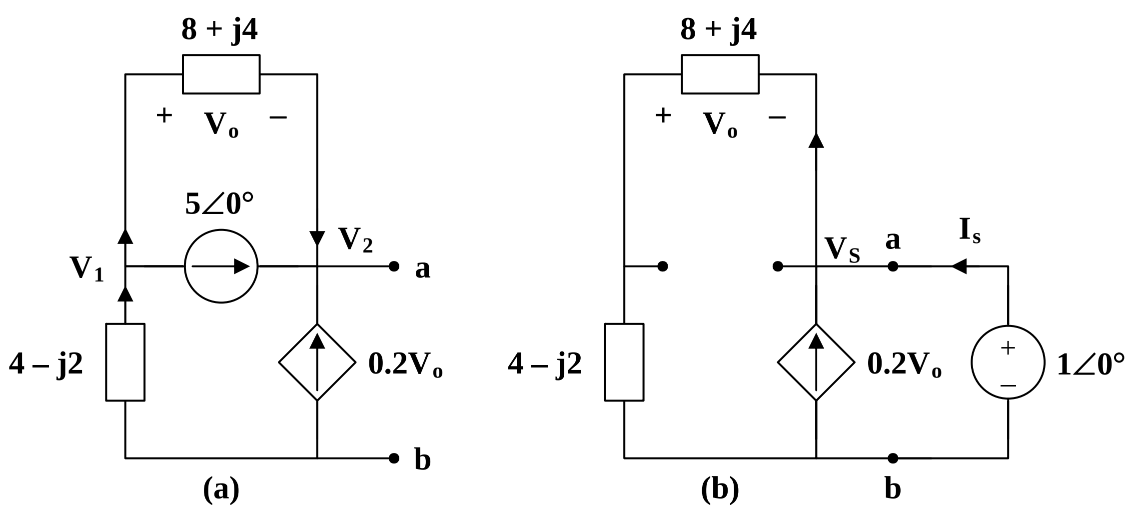
By voltage division,

$$\mathbf{V}_{th} = \frac{-j4}{6 + j2 - j4}(100\angle 20°) = \frac{(-j4)(100\angle 20°)}{6 - j2}$$

$$\mathbf{V}_{th} = \frac{(4\angle -90°)(100\angle 20°)}{6.325\angle -18.43°}$$

$$\mathbf{V}_{th} = \mathbf{63.24\angle{-51.57°}\ V}$$

P.P.10.9 To find $\mathbf{V}_{th}$, consider the circuit in Fig. (a).



At node 1, $$\frac{0 - \mathbf{V}_1}{4 - j2} = 5 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4}$$

$$-(2 + j)\mathbf{V}_1 = 50 + (1 - j0.5)(\mathbf{V}_1 - \mathbf{V}_2)$$

$$50 = (1 - j0.5)\mathbf{V}_2 - (3 + j0.5)\mathbf{V}_1 \qquad (1)$$

At node 2, $$5 + 0.2\mathbf{V}_o + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4} = 0,$$ where $\mathbf{V}_o = \mathbf{V}_1 - \mathbf{V}_2$.

Hence, the equation for node 2 becomes

$$5 + 0.2(\mathbf{V}_1 - \mathbf{V}_2) + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4} = 0$$

$$\mathbf{V}_1 = \mathbf{V}_2 - \frac{50}{3 + j0.5} \qquad (2)$$

Substituting (2) into (1),

$$50 = (1 - j0.5)\mathbf{V}_2 - (3 + j0.5)\mathbf{V}_2 + (50)\frac{3 + j0.5}{3 - j0.5}$$

$$0 = -50 - (2 + j)\mathbf{V}_2 + \frac{50}{37}(35 + j12)$$

$$\mathbf{V}_2 = \frac{-2.702 + j16.22}{2 + j} = 7.35\angle 72.9°$$

$$\mathbf{V}_{th} = \mathbf{V}_2 = \mathbf{7.35\angle 72.9° \ V}$$

To find $\mathbf{Z}_{th}$, we remove the independent source and insert a 1-V voltage source between terminals a-b, as shown in Fig. (b).

At node a, $\quad \mathbf{I}_s = -0.2\mathbf{V}_o + \dfrac{\mathbf{V}_s}{8 + j4 + 4 - j2}$

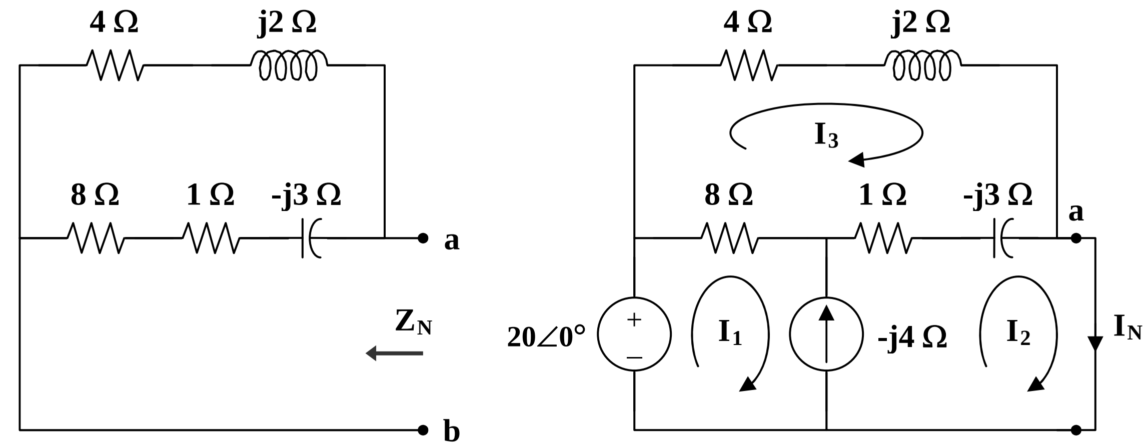
But, $\quad \mathbf{V}_s = 1 \quad$ and $\quad -\mathbf{V}_o = \dfrac{8 + j4}{8 + j4 + 4 - j2}\mathbf{V}_s$

So, $\quad \mathbf{I}_s = (0.2)\dfrac{8 + j4}{12 + j2} + \dfrac{\mathbf{1}}{12 + j2} = \dfrac{2.6 + j0.8}{12 + j2}$

and $\quad \mathbf{Z}_{th} = \dfrac{\mathbf{V}_s}{\mathbf{I}_s} = \dfrac{1}{\mathbf{I}_s} = \dfrac{12 + j2}{2.6 + j0.8} = \dfrac{12.166\angle 9.46°}{2.72\angle 17.10°}$

$$\mathbf{Z}_{th} = \mathbf{4.473\angle{-7.64}° \ \Omega}$$

P.P.10.10 To find $\mathbf{Z}_N$, consider the circuit in Fig. (a).



(a) (b)

$$\mathbf{Z}_N = (4 + j2)\,\|\,(9 - j3) = \frac{(4 + j2)(9 - j3)}{13 - j}$$

$$\mathbf{Z}_N = \mathbf{(3.176 + j0.706)\ \Omega}$$

To find $\mathbf{I}_N$, short-circuit terminals a-b as shown in Fig. (b). Notice that meshes 1 and 2 form a supermesh.

For the supermesh, $-20 + 8\mathbf{I}_1 + (1 - j3)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0$ (1)

Also, $\mathbf{I}_1 = \mathbf{I}_2 + j4$ (2)

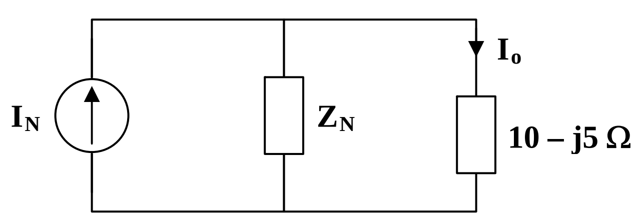
For mesh 3, $(13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0$ (3)

Solving for $\mathbf{I}_2$, we obtain

$$\mathbf{I}_N = \mathbf{I}_2 = \frac{50 - j62}{9 - j3} = \frac{79.65\angle\text{-}51.11^\circ}{9.487\angle\text{-}18.43^\circ}$$

$$\mathbf{I}_N = \mathbf{8.396\angle\text{-}32.68^\circ\ A}$$

Using the Norton equivalent, we can find $\mathbf{I}_o$ as in Fig. (c).



(c)

By current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + 10 - j5}\mathbf{I}_N = \frac{3.176 + j0.706}{13.176 - j4.294}(8.396\angle -32.68°)$$

$$\mathbf{I}_o = \frac{(3.254\angle 12.53°)(8.396\angle -32.68°)}{13.858\angle -18.05°}$$

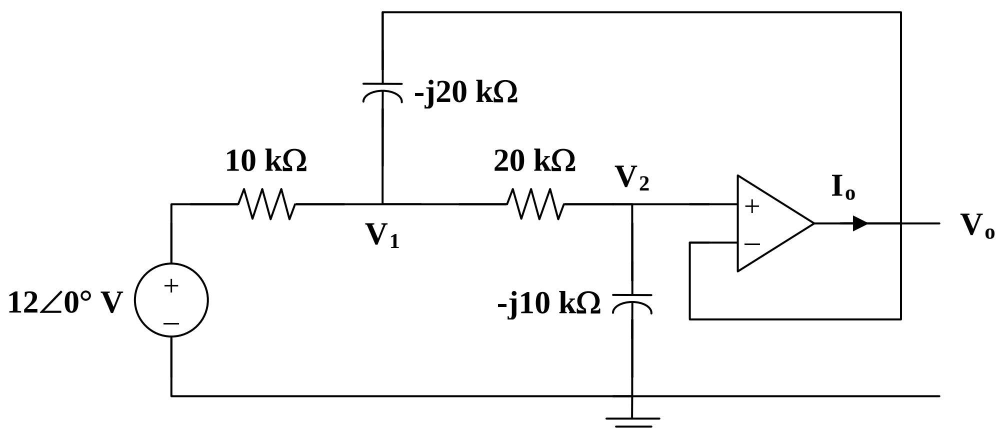
$$\mathbf{I}_o = \mathbf{1.9714\angle{-2.10}°\ A}$$

P.P.10.11

$$10\text{ nF} \longrightarrow \frac{1}{j\omega C_1} = \frac{1}{j(5\times 10^3)(10\times 10^{-9})} = \text{-j20 k}\Omega$$

$$20\text{ nF} \longrightarrow \frac{1}{j\omega C_2} = \frac{1}{j(5\times 10^3)(20\times 10^{-9})} = \text{-j10 k}\Omega$$

Consider the circuit in the frequency domain as shown below.



As a voltage follower, $\mathbf{V}_2 = \mathbf{V}_o$

At node 1, $$\frac{12 - \mathbf{V}_1}{10} = \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j20} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{20}$$

$$24 = (3 + j)\mathbf{V}_1 - (1 + j)\mathbf{V}_o \qquad (1)$$

At node 2, $$\frac{\mathbf{V}_1 - \mathbf{V}_o}{20} = \frac{\mathbf{V}_o - 0}{-j10}$$

$$\mathbf{V}_1 = (1 + j2)\mathbf{V}_o \qquad (2)$$

Substituting (2) into (1) gives

$$24 = j6\mathbf{V}_o \quad \text{or} \quad \mathbf{V}_o = 4\angle -90°$$

Hence, $v_o(t) = 4\cos(5000t - 90°)\,V$

$\mathrm{v_o(t) =}$ **4sin(5,000t) V**

Now, $\mathbf{I}_o = \dfrac{\mathbf{V}_o - \mathbf{V}_1}{\text{- j20k}}$

But from (2) $\mathbf{V}_o - \mathbf{V}_1 = \text{-}j2\mathbf{V}_o = -8$

$$\mathbf{I}_o = \frac{\text{-}8}{\text{-}\,j20k} = \text{-}\,j400\ \mu A$$

Hence, $i_o(t) = 400\cos(5000t - 90°)\ \mu A$

$\mathrm{i_o(t) =}$ **400sin(5,000t) μA**

P.P.10.12 Let $\mathrm{Z = R \parallel \dfrac{1}{j\omega C} = \dfrac{R}{1 + j\omega RC}}$

$$\frac{\mathrm{V_s}}{\mathrm{V_o}} = \frac{\mathrm{R}}{\mathrm{R + Z}}$$

The loop gain is

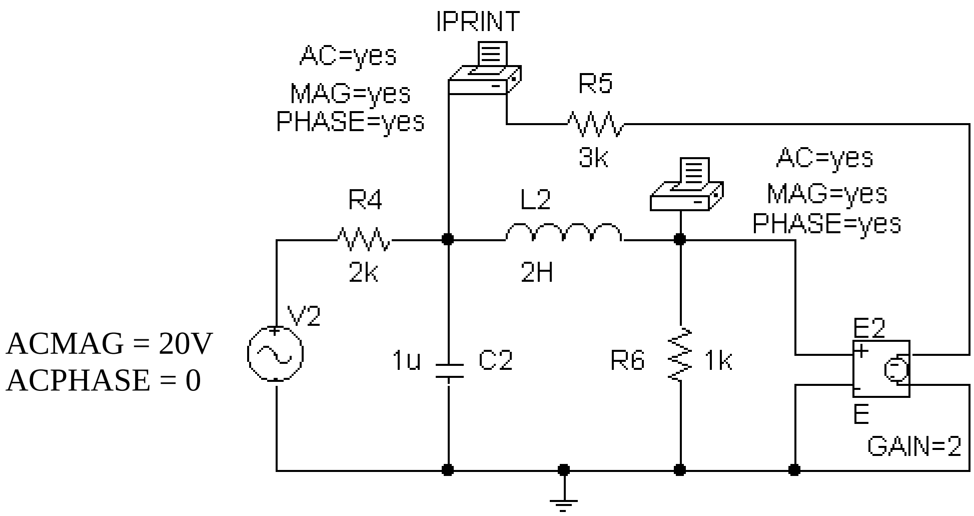
$$\mathrm{1/G = \frac{V_S}{V_O} = \frac{R}{R + Z} = \frac{R}{R + \dfrac{R}{1 + j\omega RC}} = \frac{1 + j\omega RC}{2 + j\omega RC}}$$

where $\mathrm{\omega RC = (1000)(10 \times 10^3)(1 \times 10^{-6}) = 10}$

$$\mathrm{1/G = \frac{1 + j10}{2 + j10} = \frac{10.05\angle 84.29°}{10.2\angle 78.69°}}$$

G = **1.0147∠–5.6°**

P.P.10.13 The schematic is shown below.



Since $\omega = 2\pi f = 3000$ rad / s $\longrightarrow$ f = 477.465 Hz . Setup/Analysis/AC Sweep as Linear for 1 point starting and ending at a frequency of 447.465 Hz. When the schematic is saved and run, the output file includes

Frequency	IM(V_PRINT1)	IP(V_PRINT1)
4.775E+02	1.088E-03	-5.512E+01

Frequency	VM($N_0005)	VP($N_0005)
4.775E+02	5.364E-01	-1.546E+02

From the output file, we obtain

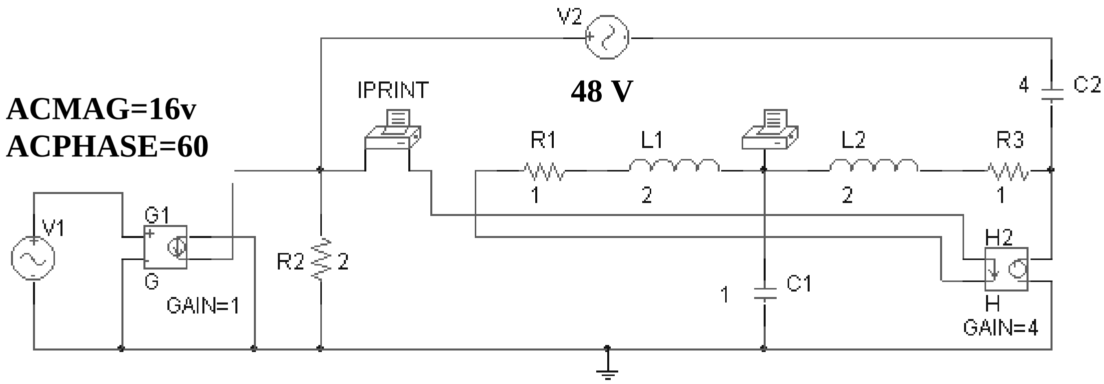
$$\mathbf{V}_o = 0.2682\angle\text{-}154.6° \text{ V} \qquad \text{and} \qquad \mathbf{I}_o = 0.544\angle\text{-}55.12° \text{ mA}$$

Therefore,

$$v_o(t) = \mathbf{536.4\ cos(3{,}000t - 154.6°)\ mV}$$

$$i_o(t) = \mathbf{1.088\ cos(3{,}000t - 55.12°)\ mA}$$

P.P.10.14 The schematic is shown below.



Since PSpice does not allow the use of complex impedances, we need to convert the complex impedances into values of capacitance and inductance. We select ω = 1 rad/s which generates f = 0.15915 Hz. We use this to obtain the values of capacitances, where $C = 1/\omega X_c$, and inductances, where $L = X_L/\omega$. Since AC current sources in PSpice does not allow the use of phase angles but AC voltages do, we can replace the current source with a voltage controlled current source. Thus we not have created an AC current source with a magnitude and a phase.

To obtain the desired output use Setup/Analysis/AC Sweep as Linear for 1 point starting and ending at a frequency of 0.15915 Hz. When the schematic is saved and run, the output file includes

Frequency	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	10.336E+00	1.580E+02
Frequency	VM($N_0004)	VP($N_0004)
1.592E-01	39.368E+00	4.478E+01

From the output file, we obtain

$$\mathbf{V_x = 39.37\angle 44.78° \ V} \qquad \text{and} \qquad \mathbf{I_x = 10.336\angle 158° \ A}$$

P.P.10.15 $$C_{eq} = \left(1 + \frac{R_2}{R_1}\right)C = \left(1 + \frac{10\times10^6}{10\times10^3}\right)(10\times10^{-9}) = \mathbf{10\ \mu F}$$

P.P.10.16 If $R = R_1 = R_2 = 2.5\ k\Omega$ and $C = C_1 = C_2 = 1\ nF$

$$f_o = \frac{1}{2\pi RC} = \frac{1}{(2\pi)(2.5\times10^3)(1\times10^{-9})} = \mathbf{63.66\ kHz}$$

CHAPTER 11

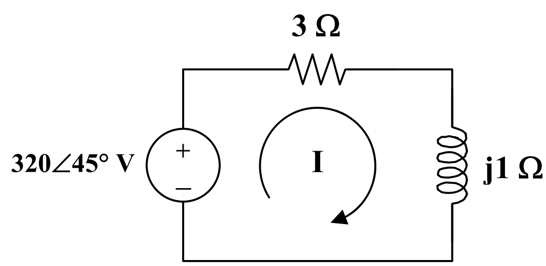
P.P.11.1 $i(t) = 33\sin(10t + 60°) = 33\cos(10t - 30°)$
$v(t) = 330\cos(10t + 20°)$

$$p(t) = v(t)\,i(t) = (330)(33)\cos(10t + 20°)\cos(10t - 30°)$$
$$p(t) = \frac{1}{2}\cdot 10890[\cos(20t + 20° - 30°) + \cos(20 - (-30°))]$$
$$p(t) = \mathbf{(3.5 + 5.445\cos(20t{-}10°))\ kW}$$
$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \mathbf{3.5\ kW}$$

P.P.11.2 $\mathbf{V} = \mathbf{I}\,\mathbf{Z} = 1320\angle 8°$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$
$$P = \frac{1}{2}(1320)(33)\cos(8° - 30°) = \mathbf{20.19\ kW}$$

P.P.11.3



$$\mathbf{I} = \frac{320\angle 45°}{3 + j} = 101.19\angle 26.57°$$

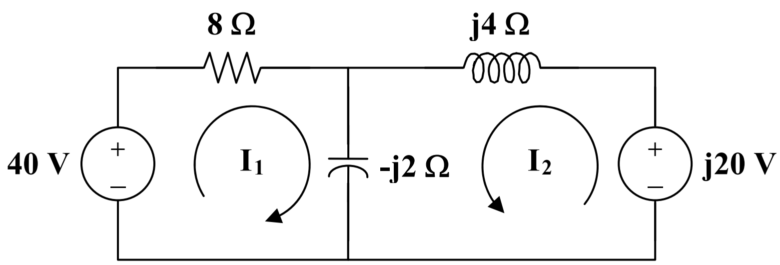
For the resistor,
$$\mathbf{I}_R = \mathbf{I} = 101.19\angle 26.57°$$
$$\mathbf{V}_R = 3\mathbf{I} = 303.6\angle 26.57°$$
$$P_R = \frac{1}{2}V_m I_m = \frac{1}{2}(303.6)(101.19) = \mathbf{15.361\ kW}$$

For the inductor,

$$\mathbf{I}_L = 101.19\angle 26.57°$$
$$\mathbf{V}_L = j\,\mathbf{I}_L = 101.19\angle(26.57° + 90°) = 101.19\angle 116.57°$$
$$P_L = \frac{1}{2}(101.19)^2 \cos(90°) = \mathbf{0\ W}$$

The average power supplied is
$$P = \frac{1}{2}(320)(101.19)\cos(45° - 26.57°) = \mathbf{15.361\ kW}$$

P.P.11.4 Consider the circuit below.



For mesh 1,
$$-40 + (8 - j2)\,\mathbf{I}_1 + (-j2)\,\mathbf{I}_2 = 0$$
$$(4 - j)\,\mathbf{I}_1 - j\,\mathbf{I}_2 = 20 \qquad (1)$$

For mesh 2,
$$-j20 + (j4 - j2)\,\mathbf{I}_2 + (-j2)\,\mathbf{I}_1 = 0$$
$$-j\,\mathbf{I}_1 + j\,\mathbf{I}_2 = j10 \qquad (2)$$

In matrix form,
$$\begin{bmatrix} 4 - j & -j \\ -j & j \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 20 \\ j10 \end{bmatrix}$$

$$\Delta = 2 + j4, \qquad \Delta_1 = -10 + j20, \qquad \Delta_2 = 10 + j60$$

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = 5\angle 53.14° \quad \text{and} \quad \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = 13.6\angle 17.11°$$

For the 40-V voltage source,
$$\mathbf{V}_s = 40\angle 0°$$
$$\mathbf{I}_1 = 5\angle 53.14°$$
$$P_s = \frac{-1}{2}(40)(5)\cos(-53.14°) = \mathbf{-60\ W}$$

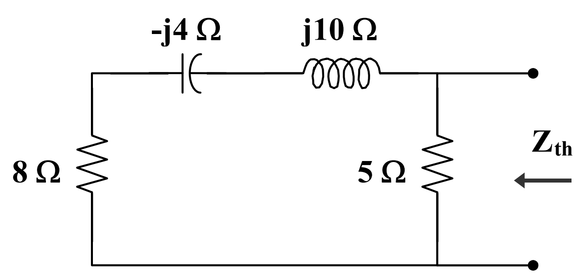
For the j20-V voltage source,

$$\mathbf{V}_s = 20\angle 90°$$
$$\mathbf{I}_2 = 13.6\angle 17.11°$$
$$P_s = \frac{-1}{2}(20)(13.6)\cos(90° - 17.11°) = \mathbf{-40\ W}$$

For the resistor,
$$I = |\mathbf{I}_1| = 5$$
$$V = 8|\mathbf{I}_1| = 40$$
$$P = \frac{1}{2}(40)(5) = \mathbf{100\ W}$$

The average power absorbed by the inductor and capacitor is **zero watts**.

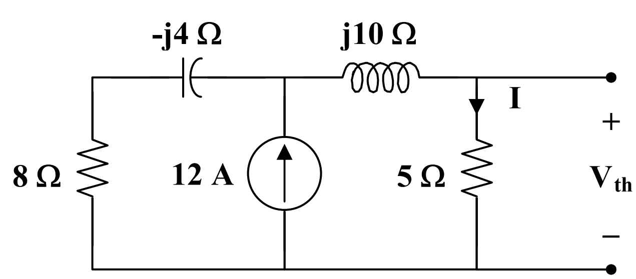
P.P.11.5 We first obtain the Thevenin equivalent circuit across $\mathbf{Z}_L$. $\mathbf{Z}_{Th}$ is obtained from the circuit in Fig. (a).



(a)

$$\mathbf{Z}_{Th} = 5 \parallel (8 - j4 + j10) = \frac{(5)(8 + j6)}{13 + j6} = 3.415 + j0.7317$$

$\mathbf{V}_{Th}$ is obtained from the circuit in Fig. (b).



(b)

By current division,
$$\mathbf{I} = \frac{8 - j4}{8 - j4 + j10 + 5}(12)$$

$$\mathbf{V}_{Th} = 5\mathbf{I} = \frac{(60)(8-j4)}{13+j6} = 37.5\angle -51.34°$$

$\mathbf{Z_L} = (\mathbf{Z_{Th}})^* = $ **[3.415–j0.7317] Ω**

$$P_{\max} = \frac{|\mathbf{V}_{Th}|^2}{8R_L} = \frac{(37.5)^2}{(8)(3.415)} = \mathbf{51.47\ W}$$

P.P.11.6 We first find $\mathbf{Z}_{\mathrm{Th}}$ and $\mathbf{V}_{\mathrm{Th}}$ across R_{L}.

Let $\mathbf{Z}_1 = 80 + \mathrm{j}60$

$$\mathbf{Z}_2 = 90 \,\|\, (\text{-j}30) = \frac{(90)(\text{-j}30)}{90-\mathrm{j}30} = 9(1-\mathrm{j}3)$$

$$\mathbf{Z}_{\mathrm{Th}} = \mathbf{Z}_1 \,\|\, \mathbf{Z}_2 = \frac{(80+\mathrm{j}60)(9-\mathrm{j}27)}{80+\mathrm{j}60+9-\mathrm{j}27} = 17.181 - \mathrm{j}24.57\ \Omega$$

$$\mathbf{V}_{\mathrm{Th}} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}(120\angle 60°) = \frac{(9)(1-\mathrm{j}3)}{89+\mathrm{j}33}(120\angle 60°)$$

$$\mathbf{V}_{\mathrm{Th}} = 35.98\angle -31.91°$$

$$\mathrm{R}_{\mathrm{L}} = |\mathbf{Z}_{\mathrm{Th}}| = \mathbf{30\ \Omega}$$

The current through the load is

$$\mathbf{I} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{Z}_{\mathrm{Th}} + \mathrm{R}_{\mathrm{L}}} = \frac{35.98\angle -31.91°}{47.181 - \mathrm{j}24.57} = 0.6764\angle -4.4°$$

The maximum average power absorbed by R_{L} is

$$\mathrm{P}_{\max} = \frac{1}{2}|\mathbf{I}|^2\,\mathrm{R}_{\mathrm{L}} = \frac{1}{2}(0.6764)^2(30) = \mathbf{6.863\ W}$$

P.P.11.7 $i(t) = \begin{cases} 16t & 0 < t < 1 \\ 32 - 16t & 1 < t < 2 \end{cases}$ T = 2

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2\,dt = \frac{1}{2}\left[\int_0^1 (16t)^2\,dt + \int_1^2 (32-16t)^2\,dt\right]$$

$$I_{rms}^2 = \frac{256}{2}\left[\int_0^1 t^2\,dt + \int_1^2 (4-4t+t^2)\,dt\right]$$

$$I_{rms}^2 = 128\left[\frac{1}{3} + \left(4t - 2t^2 + \frac{t^3}{3}\right)\Big|_1^2\right] = \frac{256}{3}$$

$$I_{rms} = \sqrt{\frac{256}{3}} = \mathbf{9.238\ A}$$

$$P = I_{rms}^2 R = \left(9.238^2\right)(9) = \mathbf{768\ w}$$

P.P.11.8 T = π , $v(t) = 100\sin(t), \quad 0 < t < \pi$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\ dt = \frac{1}{\pi}\int_0^\pi (100\sin(t))^2\ dt$$

$$V_{rms}^2 = \frac{10^4}{\pi}\int_0^\pi \frac{1}{2}\left[1 - \cos(2t)\right]dt = 5000$$

V_{rms} = **70.71 V**

$$P = \frac{V_{rms}^2}{R} = \frac{5000}{6} = \mathbf{833.3\ W}$$

P.P.11.9 The load impedance is

$$\mathbf{Z} = 60 + j40 = 72.11\angle 33.7°\ \Omega$$

The power factor is

pf = cos(33.7°) = **0.8321 lagging**

Since the load is inductive

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{320\angle 10°}{72.11\angle 33.7°} = 4.438\angle -23.69°\ A$$

The apparent power is

$$\mathbf{S} = \mathbf{V_{rms}}(\mathbf{I_{rms}})^* = 0.5(320)(4.438)\angle(10° - (-23.69°)) = \mathbf{710\angle 33.69°\ VA}$$

P.P.11.10 The total impedance as seen by the source is

$$\mathbf{Z} = 10 + j4 \parallel (8 - j6) = 10 + \frac{(j4)(8 - j6)}{8 - j2}$$

$$\mathbf{Z} = 12.69\angle 20.62°$$

The power factor is

pf = cos(20.62°) = **0.936 (lagging)**

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{165\angle 0°}{12.69\angle 20.62°} = 13.002\angle -20.62°$$

The average power supplied by the source is equal to the power absorbed by the load.

$$P = I_{rms}^2 R = (13.002)^2(11.88) = 1{,}062\ W = \mathbf{2.008\ kW}$$

or $P = V_{rms} I_{rms} pf = (165)(13.002)(0.936) = \mathbf{2.008\ kW}$

P.P.11.11

(a) $\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = (110\angle 85°)(0.4\angle -15°)$

$\mathbf{S} = \mathbf{44\angle 70°\ VA}$

$S = |\mathbf{S}| = \mathbf{44\ VA}$

(b) $\mathbf{S} = 44\angle 70° = 15.05 + j41.35$

$P = \mathbf{15.05\ W}$, $\quad Q = \mathbf{41.35\ VAR}$

(c) $\text{pf} = \cos(70°) = \mathbf{0.342}$ **(lagging)**

$$\mathbf{Z} = \frac{\mathbf{V}_{rms}}{\mathbf{I}_{rms}} = \frac{110\angle 85°}{0.4\angle -15°} = 275\angle 70°$$

$\mathbf{Z} = \mathbf{94.06 + j258.4\ \Omega}$

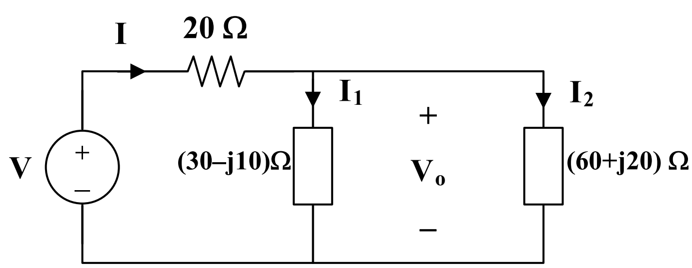
P.P.11.12

(a) If $\mathbf{Z} = 250\angle -75°$, $\quad \text{pf} = \cos(-75°) = \mathbf{0.2588}$ **(leading)**

(b) $$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{-100\ kVAR}{\sin(-75°)} = \mathbf{103.53\ kVA}$$

(c) $$S = \frac{V_{rms}^2}{|\mathbf{Z}|} \longrightarrow V_{rms} = \sqrt{S\cdot|\mathbf{Z}|} = \sqrt{(103530)(250)} = \mathbf{5.087\ kV}$$

P.P.11.13 Consider the circuit below.



Let $\mathbf{I}_2$ be the current through the 60-Ω resistor.

$$P = I_2^2 R \longrightarrow I_2^2 = \frac{P}{R} = \frac{240}{60} = 4$$

$\mathbf{I}_2 = 2$ (rms)

$$\mathbf{V}_o = \mathbf{I}_2\,(60 + j20) = 120 + j40$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_o}{30 - j10} = 3.2 + j2.4$$

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 5.2 + j2.4$$

$$\mathbf{V} = 20\mathbf{I} + \mathbf{V}_o = (104 + j48) + (120 + j40)$$
$$\mathbf{V} = 224 + j88 = \mathbf{240.7\angle 21.45^\circ\ V_{rms}}$$

For the 20-Ω resistor,

$$\mathbf{V} = 20\mathbf{I} = 204 + j48 = 114.54\angle 24.8^\circ$$
$$\mathbf{I} = 5.2 + j2.4 = 5.727\angle 24.8^\circ$$

$$\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = (114.54\angle 24.8^\circ)(5.727\angle -24.8^\circ)$$
$$\mathbf{S} = \mathbf{656\ VA}$$

For the (30 – j10)-Ω impedance,

$$\mathbf{V}_o = 120 + j40 = 126.5\angle 18.43^\circ$$
$$\mathbf{I}_1 = 3.2 + j2.4 = 4\angle 36.87^\circ$$

$$\mathbf{S}_1 = \mathbf{V}_o\,\mathbf{I}_1^* = (126.5\angle 18.43^\circ)(4\angle -36.87^\circ)$$

$$\mathbf{S_1} = 506\angle -18.44^\circ = \mathbf{[480 - j160]\ VA}$$

For the (60 + j20)-Ω impedance,

$$\mathbf{I}_2 = 2\angle 0^\circ$$

$$\mathbf{S}_2 = \mathbf{V}_o\,\mathbf{I}_2^* = (126.5\angle 18.43^\circ)(2\angle -0^\circ)$$

$$\mathbf{S_2} = 253\angle 18.43^\circ = \mathbf{[240 + j80]\ VA}$$

The overall complex power supplied by the source is

$$\mathbf{S}_T = \mathbf{V}\,\mathbf{I}^* = (240.67\angle 21.45^\circ)(5.727\angle -24.8^\circ)$$

$\mathbf{S_T}$ = 1378.3∠–3.35° = **[1376–j80] VA**

P.P.11.14

For load 1,

$$P_1 = 2000\,, \qquad \text{pf} = 0.75 = \cos\theta_1 \longrightarrow \theta_1 = -41.41°$$

$$P_1 = S_1\cos\theta_1 \longrightarrow S_1 = \frac{P_1}{\cos\theta_1} = 2666.67$$

$$Q_1 = S_1\sin\theta_1 = -176.85$$

$$\mathbf{S}_1 = P_1 + jQ_1 = 2000 - j1763.85 \quad \text{(leading)}$$

For load 2,

$$P_2 = 4000\,, \qquad \text{pf} = 0.95 = \cos\theta_2 \longrightarrow \theta_2 = 18.19°$$

$$S_2 = \frac{P_2}{\cos\theta_2} = 4210.53$$

$$Q_2 = S_2\sin\theta_2 = 1314.4$$

$$\mathbf{S}_2 = P_2 + jQ_2 = 4000 + j1314.4 \quad \text{(lagging)}$$

The total complex power is

$$\mathbf{S = S_1 + S_2 = [6–j0.4495]\ kVA}$$

$$\text{pf} = \frac{P}{|\mathbf{S}|} = \frac{6000}{6016.18} = \mathbf{0.9972 \quad (leading)}$$

P.P.11.15 $\quad \text{pf} = 0.85 = \cos\theta \longrightarrow \theta = 31.79°$

$$Q = S\sin\theta \longrightarrow S = \frac{Q}{\sin\theta} = \frac{140}{\sin(31.79°)} = 265.8\ \text{kVA}$$

$$P = S\cos\theta = 225.93\ \text{kW}$$

For $\text{pf} = 1 = \cos\theta_1 \longrightarrow \theta_1 = 0°$

Since P remains the same,

$$P = P_1 = S_1\cos\theta_1 \longrightarrow S_1 = \frac{P_1}{\cos\theta_1} = 225.93$$

$$Q_1 = S_1\sin\theta_1 = 0$$

The difference between the new Q_1 and the old Q is Q_c.

$$Q_c = 140\ \text{kVAR} = \omega C V_{rms}^2$$

$$C = \frac{140\times10^3}{(2\pi)(60)(110)^2} = \mathbf{30.69\ mF}$$

P.P.11.16 The wattmeter measures the average power from the source.

Let $\mathbf{Z}_1 = 4 - j2$

$$\mathbf{Z}_2 = 12 \,||\, j9 = \frac{(12)(j9)}{12 + j9} = 4.32 + j5.76$$

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = 8.32 + j3.76 = 9.13\angle 24.32°$$

$$\mathbf{S} = \mathbf{V}\,\mathbf{I}^* = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(120)^2}{9.13\angle -24.32°} = 1577.2\angle 24.32°\ VA$$

$$P = |\mathbf{S}|\cos\theta = \mathbf{1.437\ kW}$$

P.P.11.17 Demand charge $= \$5 \times 32{,}000 = \$160{,}000$
Energy charge for the first $50{,}000$ kWh $= \$0.08 \times 50{,}000 = \$4{,}000$
The remaining energy $= 500{,}000 - 50{,}000 = 450{,}000$ kWh
Charge for this bill $= \$0.05 \times 450{,}000 = \$22{,}500$
Total bill $= \$160{,}000 + \$4{,}000 + \$22{,}500 =$ **\$186,500**

P.P.11.18 Energy consumed $= 800\text{ kW} \times 20 \times 26 = 416{,}000$ kWh

The power factor of 0.88 exceeds 0.85 by 3×0.01. Hence, there is a power factor credit which amounts to an energy credit of

$$416{,}000 \times \frac{0.1}{100} \times 3 = 1248\text{ kWh}$$

Total energy billed $= 416{,}000 - 1{,}248 = 414{,}752$ kWh
Energy cost $= \$0.06 \times 414{,}752 =$ **\$24,885.12**

CHAPTER 12

P.P.12.1 For the abc sequence, $\mathbf{V}_{an}$ leads $\mathbf{V}_{bn}$ by 120° and $\mathbf{V}_{bn}$ leads $\mathbf{V}_{cn}$ by 120°.

Hence,

$$\mathbf{V}_{an} = 110\angle(30° + 120°) = \mathbf{110\angle 150°\ V}$$
$$\mathbf{V}_{cn} = 110\angle(30° - 120°) = \mathbf{110\angle{-90}°\ V}$$

P.P.12.2

(a)

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 120\angle 30° - 120\angle{-90°}$$
$$\mathbf{V}_{ab} = (103.92 + j60) + j120$$
$$\mathbf{V}_{ab} = \mathbf{207.8\angle 60°\ V}$$

Alternatively, using the fact that $\mathbf{V}_{ab}$ leads $\mathbf{V}_{an}$ by 30° and has a magnitude of $\sqrt{3}$ times that of $\mathbf{V}_{an}$,

$$\mathbf{V}_{ab} = \sqrt{3}(120)\angle(30° + 30°) = \mathbf{207.8\angle 60°\ V}$$

Following the abc sequence,

$$\mathbf{V}_{bc} = \mathbf{207.8\angle{-60}°\ V}$$
$$\mathbf{V}_{ca} = \mathbf{207.8\angle \pm 180°\ V}$$

(b)

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}}$$

$$\mathbf{Z} = (0.4 + j0.3) + (24 + j19) + (0.6 + j0.7)$$
$$\mathbf{Z} = 25 + j20 = 32\angle 38.66°$$

$$\mathbf{I}_a = \frac{120\angle 30°}{32\angle 38.66°} = \mathbf{3.75\angle{-8.66}°\ A}$$

Following the abc sequence,

$$\mathbf{I}_b = \mathbf{I}_a\angle{-120°} = \mathbf{3.75\angle{-128.66}°\ A}$$
$$\mathbf{I}_c = \mathbf{I}_a\angle{-240°} = \mathbf{3.75\angle 111.34°\ A}$$

P.P.12.3
The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{\Delta}} = \frac{120\angle -20°}{20\angle 40°} = \mathbf{6\angle{-60}° \ A}$$

$$\mathbf{I}_{\mathrm{BC}} = \mathbf{I}_{\mathrm{AB}}\angle -120° = \mathbf{6\angle 180° \ A}$$

$$\mathbf{I}_{\mathrm{CA}} = \mathbf{I}_{\mathrm{AB}}\angle 120° = \mathbf{6\angle 60° \ A}$$

The line currents are

$$\mathbf{I}_{a} = \mathbf{I}_{AB}\sqrt{3}\angle -30° = 6\sqrt{3}\angle -90° = \mathbf{10.392\angle{-90}° \ A}$$

$$\mathbf{I}_{\mathrm{b}} = \mathbf{I}_{\mathrm{a}}\angle -120° = \mathbf{10.392\angle 150° \ A}$$

$$\mathbf{I}_{\mathrm{c}} = \mathbf{I}_{\mathrm{a}}\angle 120° = \mathbf{10.392\angle 30° \ A}$$

P.P.12.4 In a delta load, the phase current leads the line current by 30° and has a magnitude $\frac{1}{\sqrt{3}}$ times that of the line current. Hence,

$$\mathbf{I}_{AB} = \frac{\mathbf{I}_{a}}{\sqrt{3}}\angle 30° = \frac{9.609}{\sqrt{3}}\angle 65° = \mathbf{5.548\angle 65° \ A}$$

$$\mathbf{Z}_{\Delta} = 18 + j12 = 21.63\angle 33.69° \ \Omega$$

$$\mathbf{V}_{AB} = \mathbf{I}_{AB}\,\mathbf{Z}_{\Delta} = (5.548\angle 65°)(21.63\angle 33.69°)$$
$$\mathbf{V}_{\mathrm{AB}} = \mathbf{120\angle 98.69° \ V}$$

P.P.12.5 $\mathbf{Z}_{\mathrm{Y}} = 12 + j15 = 19.21\angle 51.34°$

After converting the Δ-connected source to a Y-connected source,

$$\mathbf{V}_{\mathrm{an}} = \frac{240}{\sqrt{3}}\angle(150° - 30°) = 138.56\angle -15°$$

$$\mathbf{I}_{\mathrm{a}} = \frac{\mathbf{V}_{\mathrm{an}}}{\mathbf{Z}_{\mathrm{Y}}} = \frac{138.56\angle -15°}{19.21\angle 51.34°} = \mathbf{7.21\angle -66.34° \ A}$$

$$\mathbf{I}_{\mathrm{b}} = \mathbf{I}_{\mathrm{a}}\angle -120° = \mathbf{7.21\angle 173.66° \ A}$$
$$\mathbf{I}_{\mathrm{c}} = \mathbf{I}_{\mathrm{a}}\angle 120° = \mathbf{7.21\angle 53.66° \ A}$$

P.P.12.6
For the source,

$$\mathbf{S} = 3\,\mathbf{V}_p\ \mathbf{I}_p^* = (3)(120\angle 30°)(3.75\angle 8.66°)$$
$$\mathbf{S} = -1350\angle 38.66° = \mathbf{[-1.054.2 - j0.8433]\ kVA}$$

For the load,

$$\mathbf{S} = 3\left|\,\mathbf{I}_p\,\right|^2 \mathbf{Z}$$

where $\mathbf{Z} = 24 + j19 = 30.61\angle 38.37°$
$\mathbf{I}_p = 3.75\angle -8.66°$

$$\mathbf{S} = (3)(3.75)^2\,(30.61\angle 38.37°)$$
$$\mathbf{S} = 1291.36\angle 38.37° = \mathbf{[1.012{+}j0.8016]\ kVA}$$

P.P.12.7 $\quad \mathrm{P} = \mathrm{S}\cos\theta \quad \longrightarrow \quad \mathrm{S} = \dfrac{\mathrm{P}}{\cos\theta} = \dfrac{30\times 10^3}{0.85} = 35.29\ \text{kVA}$

$$\mathrm{S} = \sqrt{3}\,\mathrm{V_L I_L} \quad \longrightarrow \quad \mathrm{I_L} = \frac{\mathrm{S}}{\sqrt{3}\,\mathrm{V_L}} = \frac{35.29\times 10^3}{\sqrt{3}\,(440)} = \mathbf{46.31\ A}$$

Alternatively,

$$\mathrm{P_p} = \frac{30\times 10^3}{3} = 10\ \text{kW}, \qquad \mathrm{V_p} = \frac{440}{\sqrt{3}}\ \text{V}$$

$$\mathrm{P_p} = \mathrm{V_p\,I_p}\cos\theta$$

$$\mathrm{I_p} = \frac{\mathrm{P_p}}{\mathrm{V_p}\cos\theta} = \frac{(10\times 10^3)\sqrt{3}}{(440)(0.85)} = \mathbf{46.31\ A}$$

P.P.12.8
(a) For load 1,

$$\mathrm{V_p} = \frac{\mathrm{V_L}}{\sqrt{3}} = \frac{840}{\sqrt{3}}$$

$$\mathbf{I}_{a1} = \frac{\mathbf{V}_a}{\mathbf{Z}_p} = \frac{840\angle 0°}{\sqrt{3}} \cdot \frac{1}{30 + j40} = 9.7\angle -53.13°$$

$$\mathbf{S}_1 = \frac{\mathrm{V}_{rms}^2}{\mathbf{Z}^*} = \frac{(840)^2}{50\angle -53.15°} = 14.112\angle 53.13°\ \text{kVA}$$

For load 2,

$$S_2 = \frac{P_2}{\cos\theta_2} = \frac{48}{0.8} = 60 \text{ kVA}$$

$$Q_2 = S_2 \sin\theta_2 = (60)(0.6) = 36 \text{ kVAR}$$

$$\mathbf{S}_2 = 48 + j36 \text{ kVA}$$

$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$ = **[56.47 + j47.29] kVA**

$\mathbf{S} = 73.65\angle 39.94° \text{ kVA}$

with $\text{pf} = \cos(39.94°) = 0.7667$

(b) $Q_c = P(\tan\theta_{old} - \tan\theta_{new})$

$$Q_c = (56.47)(\tan 39.94° - \tan 0°) = 47.29 \text{ kVAR}$$

For each capacitor, the rating is **15.76 kVAR**

(c) At unity pf, $\mathbf{S} = P = 56.47 \text{ kVA}$

$$I_L = \frac{\mathbf{S}}{\sqrt{3}\,V_L} = \frac{56470}{\sqrt{3}\,(840)} = \mathbf{38.81\ A}$$

P.P.12.9

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{AB}} = \frac{440\angle 0°}{10 - j5} = 39.35\angle 26.56° = 35.2 + j17.595$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{BC}} = \frac{440\angle -120°}{16} = 27.5\angle -120° = -13.75 - j23.82$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{CA}} = \frac{440\angle 120°}{8 + j6} = 44\angle 83.13° = 5.263 + j43.68$$

The line currents are

$\mathbf{I_a} = \mathbf{I_{AB}} - \mathbf{I_{CA}}$ = (35.2+j17.595) – (5.263+j43.68)
= 29.94–j26.08 = **39.71∠–41.06˚ A**.
$\mathbf{I_b} = \mathbf{I_{BC}} - \mathbf{I_{AB}}$ = –48.95–j41.42 = **64.12∠–139.8˚ A**.
$\mathbf{I_c} = \mathbf{I_{CA}} - \mathbf{I_{BC}}$ = 19.013+j67.5 = **70.13∠74.27˚ A**.

P.P.12.10
The phase currents are

$$\mathbf{I}_{AB} = \frac{220\angle 0°}{-\text{j}5} = \text{j}44$$

$$\mathbf{I}_{BC} = \frac{220\angle 0°}{\text{j}10} = 22\angle 30°$$

$$\mathbf{I}_{CA} = \frac{220\angle 120°}{10} = 22\angle -120°$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = (\text{j}44) - (-11 - \text{j}19.05)$$
$$\mathbf{I}_a = 11 + \text{j}63.05 = \mathbf{64\angle 80.1°\ A}$$

$$\mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB} = (19.05 + \text{j}11) - (\text{j}44)$$
$$\mathbf{I}_b = 19.05 - \text{j}33 = \mathbf{38.1\angle -60°\ A}$$

$$\mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} = (-11 - \text{j}19.05) - (19.05 + \text{j}11)$$
$$\mathbf{I}_c = -30.05 - \text{j}30.05 = \mathbf{42.5\angle 225°\ A}$$

The real power is absorbed by the resistive load

$$\text{P} = \left|\mathbf{I}_{CA}\right|^2 (10) = (22)^2 (10) = \mathbf{4.84\ kW}$$

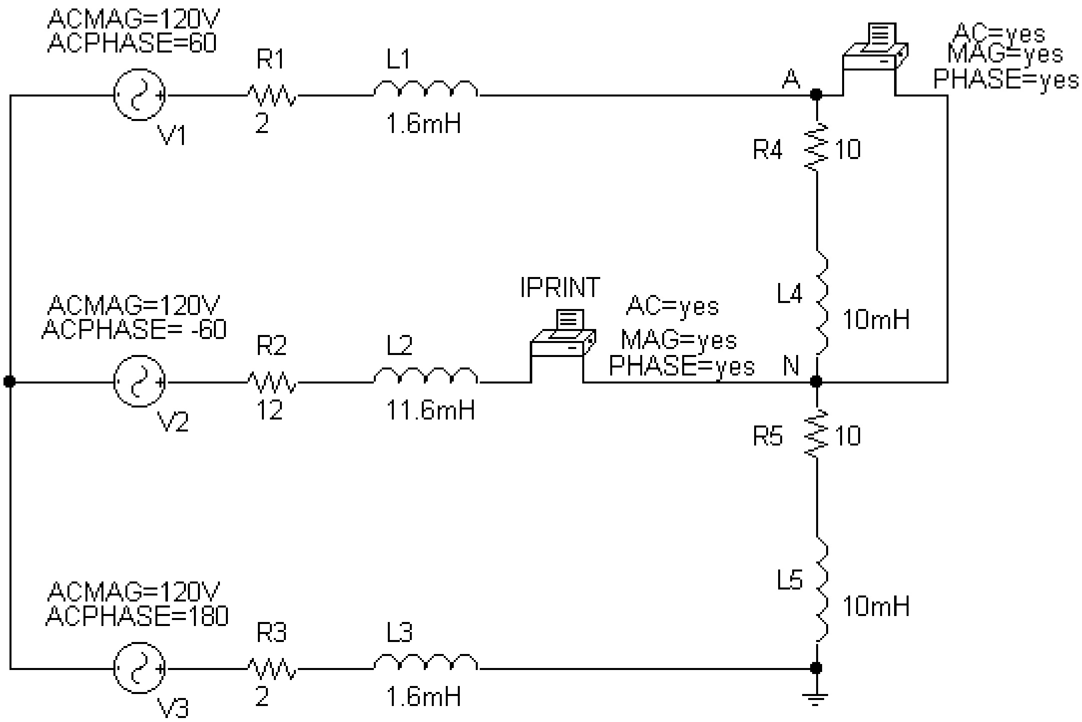
P.P.12.11 The schematic is shown below. First, use the **AC Sweep** option of the **Analysis Setup**. Choose a *Linear* sweep type with the following *Sweep Parameters* : *Total Pts* = 1, *Start Freq* = 100, and *End Freq* = 100. Once the circuit is saved and simulated, we obtain an output file whose contents include the following results.

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.000E+02	8.547E+00	-9.127E+01

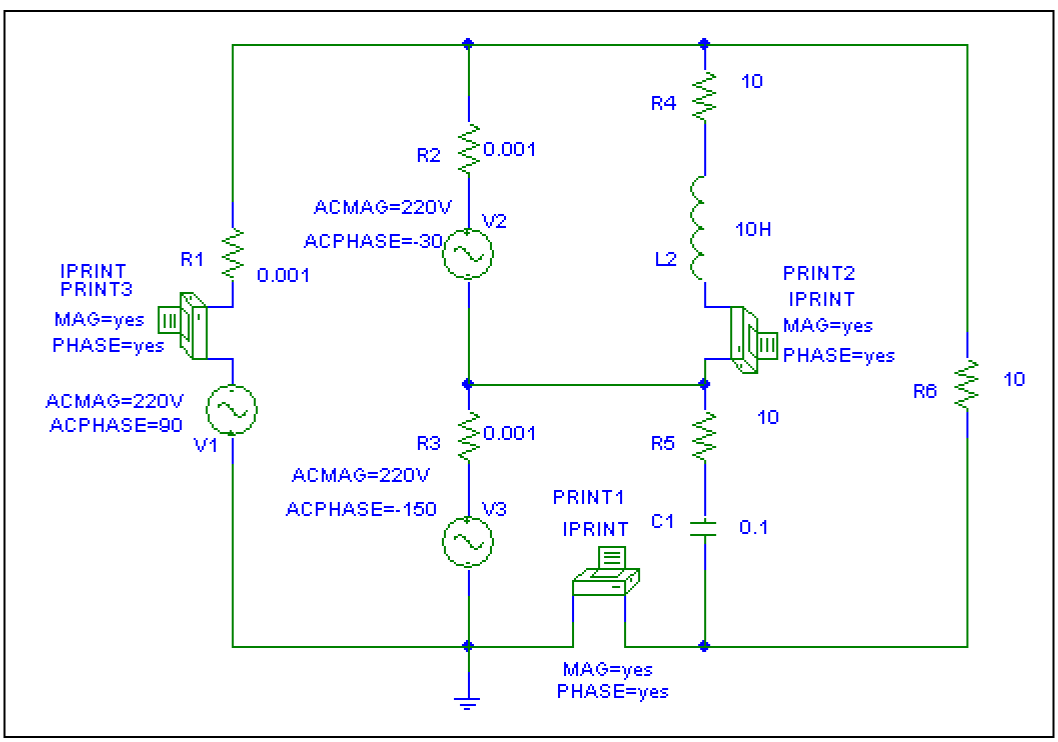
FREQ	VM(A,N)	VP(A,N)
1.000E+02	1.009E+02	6.087E+01

From this we obtain,

$$\mathbf{V}_{an} = \mathbf{100.9\angle 60.87°\ V}, \quad \mathbf{I}_{bB} = \mathbf{8.547\angle -91.27°\ A}$$



P.P.12.12 The schematic is shown below.



In this case, we may assume that $\omega = 1\ \text{rad/s}$, so that $f = 1/2\pi = 0.1592\ \text{Hz}$. Hence, $L = X_L/\omega = 10$ and $C = 1/\omega X_c = 0.1$.

Use the **AC Sweep** option of the **Analysis Setup**. Choose a *Linear* sweep type with the following *Sweep Parameters* : *Total Pts* = 1, *Start Freq* = 0.1592, and *End Freq* = 0.1592. Once the circuit is saved and simulated, we obtain an output file whose contents include the following results.

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E-01	3.724E+01	8.379E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E-01	1.555E+01	-7.501E+01

FREQ	IM(V_PRINT3)	IP(V_PRINT3)
1.592E-01	2.468E+01	-9.000E+01

From this we obtain,

$$\mathbf{I}_{ca} = \mathbf{24.68\angle -90° \ A} \qquad \mathbf{I}_{cC} = \mathbf{37.25\angle 83.79° \ A} \qquad \mathbf{I}_{AB} = \mathbf{15.55\angle{-75.01}° A}$$

P.P.12.13

(a) If point o is connected to point B, $P_2 = \mathbf{0\ W}$

$P_1 = \text{Re}(\mathbf{V}_{AB}\,\mathbf{I}_a^*)$

$P_1 = (440)(39.71)\cos(0° + 41.06°) = \mathbf{13.175kW}$

$P_3 = \text{Re}(\mathbf{V}_{CB}\,\mathbf{I}_c^*)$

where $\mathbf{V}_{CB} = \mathbf{-V}_{BC} = 240\angle(-120° + 180°) = 240\angle 60°$

$P_3 = (440)(70.13)\cos(60° - 74.27°) = \mathbf{29.91kW}$

(b) Total power is = (13.175+29.91) kW = **43.08 kW**.

P.P.12.14 $V_L = 208\ \text{V}$, $P_1 = -560\ \text{W}$, $P_2 = 800\ \text{W}$

(a) $P_T = P_1 + P_2 = -560 + 800 = \mathbf{240\ W}$

(b) $Q_T = \sqrt{3}\,(P_2 - P_1) = \sqrt{3}\,(800 + 560) = \mathbf{2.356\ kVAR}$

(c) $\tan\theta = \dfrac{Q_T}{P_T} = \dfrac{2355.6}{240} = 9.815 \longrightarrow \theta = 84.18°$

$\text{pf} = \cos\theta = \mathbf{0.1014}$ (**lagging / inductive**)

It is inductive because $P_2 > P_1$

(d) For a Y-connected load,

$$I_p = I_L, \qquad V_p = \frac{V_L}{\sqrt{3}} = \frac{208}{\sqrt{3}} = 120 \text{ V}$$

$$P_p = V_p\, I_p \cos\theta \longrightarrow I_p = \frac{80}{(120)(0.1014)} = 6.575 \text{ A}$$

$$Z_p = \frac{V_p}{I_p} = \frac{120}{6.575} = 18.25$$

$$\mathbf{Z}_p = Z_p\angle\theta = \mathbf{18.25\angle 84.18^\circ\ \Omega}$$

The impedance is **inductive**.

P.P.12.15 $\mathbf{Z}_\Delta = 30 - j40 = 50\angle -53.13^\circ$

The equivalent Y-connected load is

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} = 16.67\angle -53.13^\circ$$

$$V_p = \frac{440}{\sqrt{3}} = 254 \text{ V}$$

$$I_L = \frac{V_p}{|\mathbf{Z}_Y|} = \frac{254}{16.67} = 15.24$$

$$P_1 = V_L\, I_L \cos(\theta + 30^\circ)$$

$$P_1 = (440)(15.24)\cos(-53.13^\circ + 30^\circ) = \mathbf{6.167\ kW}$$

$$P_2 = V_L\, I_L \cos(\theta - 30^\circ)$$

$$P_2 = (440)(15.24)\cos(-53.13^\circ - 30^\circ) = \mathbf{802.1 W}$$

$$P_T = P_1 + P_2 = \mathbf{6.969\ kW}$$

$$Q_T = \sqrt{3}\,(P_2 - P_1) = \sqrt{3}\,(802.1 - 6167)$$

$$Q_T = \mathbf{-\ 9.292\ kVAR}$$

CHAPTER 13

P.P. 13.1 For mesh 1,

$$141.42+j141.42 = 4(1 + j2)\mathbf{I_1} + j\mathbf{I_2} \qquad (1)$$

For mesh 2, $0 = j\mathbf{I_1} + (10 + j5)\mathbf{I_2}$ (2)

For the matrix form $\begin{bmatrix} 141.42 + j141.42 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 + j8 & j \\ j & 10 + j5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$\Delta = j100, \quad \Delta_2 = 141.42\text{–}j141.42$$

$$\mathbf{I_2} = \Delta_2/\Delta = (141.42\text{–}j141.42)/j100$$

$$\mathbf{V_o} = 10I_2 = 10(–1.4142–j1.4142) = \mathbf{20\angle–135° \ V}$$

P.P. 13.2 Since $\mathbf{I_1}$ enters the coil with reactance 2Ω and $\mathbf{I_2}$ enters the coil with reactance 6Ω, the mutual voltage is positive. Hence, for mesh 1,

$$100\angle 60^o = (5 + j2 + j6 – j\ 3x2)\mathbf{I_1} – j6\mathbf{I_2} + \mathbf{j3I_2}$$

or $100\angle 60^o = (5 + j2)\mathbf{I_1} – j3\mathbf{I_2}$ (1)

For mesh 2, $0 = (j6 – j4)\mathbf{I_2} – j6\mathbf{I_1} + j3\mathbf{I_1}$

or $\mathbf{I_2} = 1.5\mathbf{I_1}$ (2)

Substituting this into (1), $100\angle 60^o = (5 – j2.5)\mathbf{I_1}$

$$\mathbf{I_1} = (100\angle 60^o)/(5.59\angle –26.57^o) = \mathbf{17.889\angle 86.57^o\ A}$$

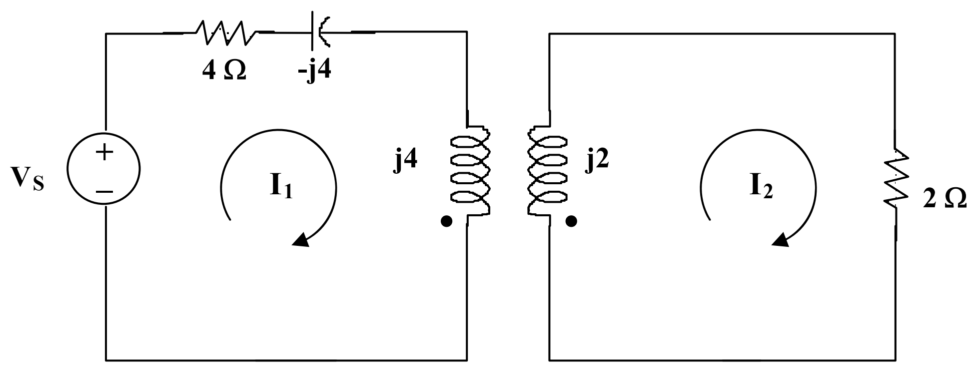
$$\mathbf{I_2} = 1.5\mathbf{I_1} = \mathbf{26.83\angle 86.57^o\ A}$$

P.P. 13.3 The coupling coefficient is, $k = m/\sqrt{L_1L_2} = 1/\sqrt{2x1} = \mathbf{0.7071}$

To obtain the energy stored, we first obtain the frequency-domain circuit shown below.

$$100\cos(\omega t) \text{ becomes } 100\angle 0^o,\ \omega = 2$$

$$1\text{H becomes } j\omega 1 = j2$$
$$2\text{H becomes } j\omega 2 = j4$$
$$(1/8)\text{ F becomes } 1/j\omega C = -j4$$



For mesh 1, $100 = (4 - j4 + j4)\mathbf{I_1} - j2\mathbf{I_2}$

or $50 = 2\mathbf{I_1} - j\mathbf{I_2}$ (1)

For mesh 2, $-j2\mathbf{I_1} + (2 + j2)\mathbf{I_2} = 0$

or $\mathbf{I_1} = (1 - j)\mathbf{I_2}$ (2)

Substituting (2) into (1), $(2 - j3)\mathbf{I_2} = 50$

$$\mathbf{I_2} = 50/(2 - j3) = 13.87\angle 56.31^\circ$$

$$\mathbf{I_1} = 19.658\angle 11.31^\circ$$

In the time domain, $i_1 = 19.658\cos(2t + 11.31^\circ)$
$i_2 = 13.87\cos(2t + 56.31^\circ)$

At $t = 1.5$, $2t = 3 \text{ rad} = 171.9^\circ$

$$i_1 = 19.658\cos(171.9^\circ + 11.31^\circ) = -19.62 \text{ A}$$
$$i_2 = 13.87\cos(171.9^\circ + 56.31^\circ) = -9.25 \text{ A}$$

The total energy stored in the coupled inductors is given by,

$$\begin{aligned} W &= 0.5L_1(i_1)^2 + 0.5L_2(i_2)^2 - 0.5M(i_1 i_2) \\ &= 0.5(2)(-19.62)^2 + 0.5(1)(-9.25)^2 - (1)(-19.62)(-9.25) \\ &= \mathbf{246.2\ J} \end{aligned}$$

P.P. 13.4 $\mathbf{Z_{in}} = 4 + j8 + [3^2/(j10 – j6 + 6 + j4)]$

$$= 4 + j8 + 9/(6 + j8)$$

$$= \mathbf{8.58\angle 58.05^o\ \Omega}$$

The current from the voltage is,

$$\mathbf{I} = \mathbf{V/Z} = 40\angle 0^o/8.58\angle 58.05^o = \mathbf{4.662\angle{-58.05^o}\ A}$$

P.P. 13.5 $L_1 = 10,\ L_2 = 4,\ M = 2$

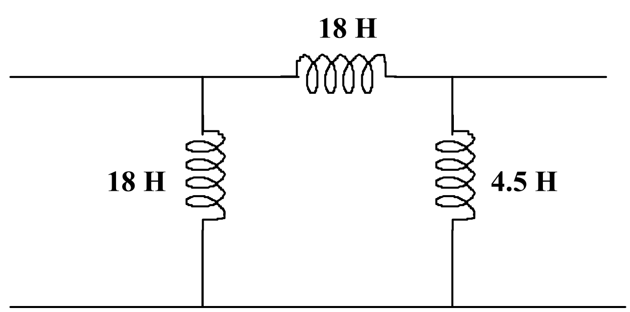
$$L_1L_2 – M^2 = 40 – 4 = 36$$

$$L_A = (L_1L_2 – M^2)/(L_2 – M) = 36/(4 – 2) = \mathbf{18\ H}$$

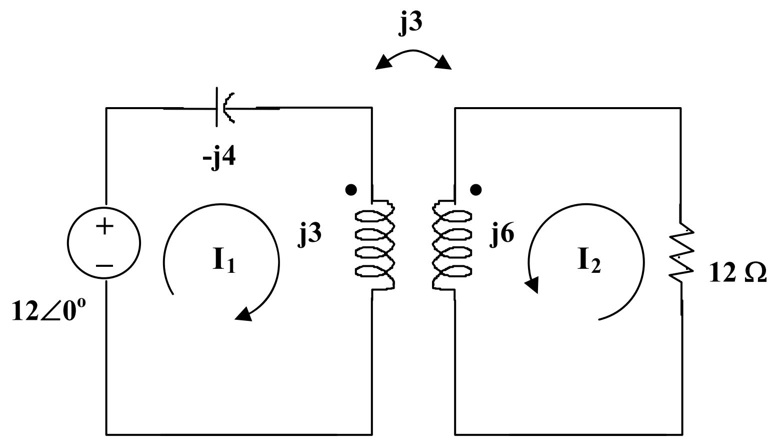
$$L_B = (L_1L_2 – M^2)/(L_1 – M) = 36/(10 – 2) = \mathbf{4.5\ H}$$

$$L_C = (L_1L_2 – M^2)/M = 36/2 = \mathbf{18\ H}$$

Hence, we get the π equivalent circuit as shown below.



P.P. 13.6 If we reverse the direction of I_2 so that we replace I_2 by $-I_2$, we have the circuit shown in Figure (a).



(a)

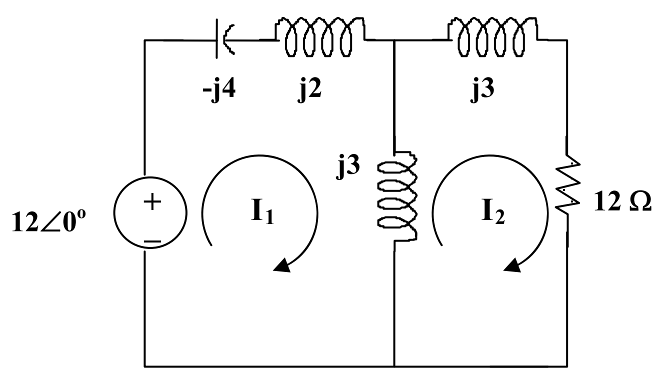
We now replace the coupled coil by the T-equivalent circuit and assume $\omega = 1$.

$$L_a = 5 - 3 = 2\text{ H}$$

$$L_b = 6 - 3 = 3\text{ H}$$

$$L_c = 3\text{ H}$$

Hence the equivalent circuit is shown in Figure (b). We apply mesh analysis.



(b)

$$12 = i_1(-j4 + j2 + j3) + j3i_2$$

$$\text{or } 12 = ji_1 + j3i_2 \qquad (1)$$

Loop 2 produces, $\quad 0 = j3i_1 + (j3 + j3 + 12)i_2$

$$\text{or } i_1 = (-2 + j4)i_2 \qquad (2)$$

Substituting (2) into (1), $\quad 12 = (-4 + j)i_2$, which leads to $i_2 = 12/(-4 + j)$

$$I_2 = -i_2 = 12/(4 - j) = \mathbf{2.91\angle 14.04^o\ A}$$

$$I_1 = i_1 = (-2 + j4)i_2 = 12(2 - j4)/(4 - j) = \mathbf{13\angle -49.4^o\ A}$$

P.P. 13.7

(a) $\quad n = V_2/V_1 = 110/2200 = \mathbf{1/20}$ (a step-down transformer)

(b) $\quad S = V_1I_1 = 2200\text{x}5 = \mathbf{11\ kVA}$

(c) $\quad I_2 = I_1/n = 5/(1/20) = \mathbf{100\ A}$

P.P. 13.8 The 16 – j24-ohm impedance can be reflected to the primary resulting in

$$Z_{in} = 2 + (16 - j24)/16 = 3 - j1.5$$

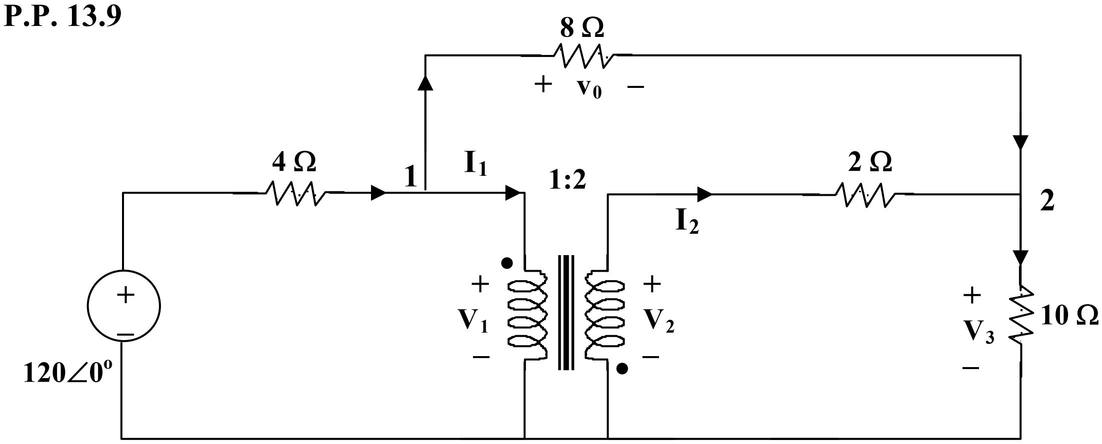
$$I_1 = 240/(3 - j1.5) = 240/(3.354\angle -26.57°) = 71.56\angle 26.57^o$$

$$I_2 = -I_1/n = -17.89\angle 26.57^o$$

$$V_o = -j24i_2 = (24\angle -90^o)(-17.89\angle 26.57^o) = \mathbf{429.4\angle 116.57^oV}$$

$$S_1 = V_1I_1 = (240)(71.56\angle 26.57^o) = \mathbf{17.174\angle -26.57^o\ kVA}.$$

P.P. 13.9



Consider the circuit shown above.

At node 1, $(120 - \mathbf{V_1})/4 = \mathbf{I_1} + (\mathbf{V_1} - \mathbf{V_3})/8$ (1)

At node 2, $[(\mathbf{V_1} - \mathbf{V_3})/8] + [(\mathbf{V_2} - \mathbf{V_3})/2] = (\mathbf{V_3})/8$ (2)

At the transformer terminals, $\mathbf{V_2} = -2\mathbf{V_1}$ and $\mathbf{I_2} = -\mathbf{I_1}/2$ (3)

But $I_2 = (\mathbf{V_2} - \mathbf{V_3})/2 = -\mathbf{I_1}/2$ which leads to $\mathbf{I_1} = (\mathbf{V_3} - \mathbf{V_2})/1 = \mathbf{V_3} + 2\mathbf{V_1}$.

Substituting all of this into (1) and (2) leads to,

$(120 - \mathbf{V_1})/4 = \mathbf{V_3} + 2\mathbf{V_1} + (\mathbf{V_1} - \mathbf{V_3})/8$ which leads $240 = 19\mathbf{V_1} + 7\mathbf{V_3}$ (4)

$[(\mathbf{V_1} - \mathbf{V_3})/8] + [(-2\mathbf{V_1} - \mathbf{V_3})/2] = \mathbf{V_3}/8$ which leads to
$\mathbf{V_3} = -7\mathbf{V_1}/6$ (5)

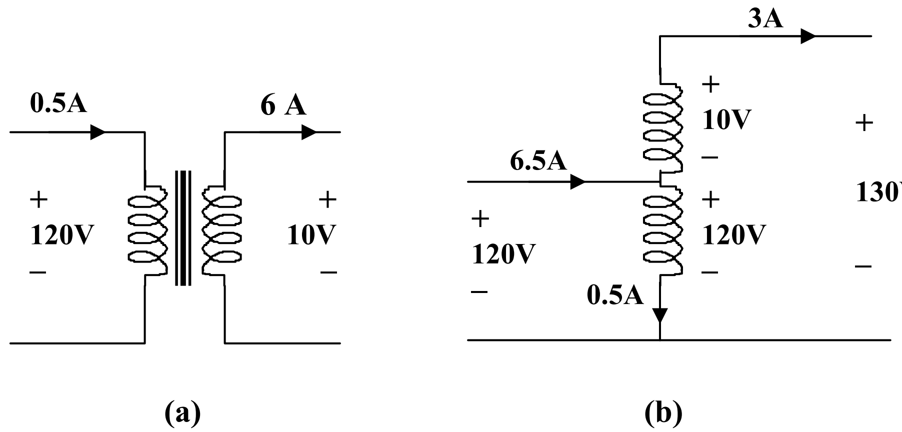
From (4) and (5),

$$240 = 10.833\mathbf{V_1} \text{ or } \mathbf{V_1} = 22.155 \text{ volts}$$

$$\mathbf{V_3} = -7\mathbf{V_1}/6 = -25.85 \text{ volts}$$

$$\mathbf{V_o} = \mathbf{V_1} - \mathbf{V_3} = \mathbf{48\ volts}$$

P.P. 13.10 We should note that the current and voltage of each winding of the autotransformer in Figure (b) are the same for the two-winding transformer in Figure (a).



For the two-winding transformer,

$$\mathbf{S_1} = 120\text{x}0.5 = 60\text{ VA}$$

$$\mathbf{S_2} = 6(10) = 60\text{ VA}$$

For the autotransformer,

$$\mathbf{S_1} = 120(6.5) = 780\text{ VA}$$

$$\mathbf{S_2} = 130(6) = \mathbf{780\text{ VA}}$$

P.P. 13.11 $(\mathbf{I_2})^* = \mathbf{S_2/V_2} = 16{,}000/1000 = \mathbf{16\text{ A}}$

Since $\mathbf{S_1} = \mathbf{V_1(I_1)}^* = \mathbf{V_2(I_2)}^* = \mathbf{S_2}$, $\mathbf{V_2/V_1} = \mathbf{I_1/I_2}$, $1000/2500 = \mathbf{I_1}/32$,

or $\mathbf{I_1} = 1000\text{x}16/2500 = \mathbf{6.4\text{ A}}$.

At the top, KCL produces $\mathbf{I_1 + I_o} = \mathbf{I_2}$, or $\mathbf{I_o} = \mathbf{I_2 - I_1} = 16 - 6.4 = \mathbf{9.6\text{ A}}$.

P.P. 13.12

(a) $S_T = (\sqrt{3})V_L I_L$, but $S_T = P_T/\cos\theta = 40x10^6/0.85 = 47.0588$ MVA

$$I_{LS} = S_T/(\sqrt{3})V_{LS} = 47.0588x10^6/[(\sqrt{3})12.5x10^3] = \mathbf{2.174\ kA}$$

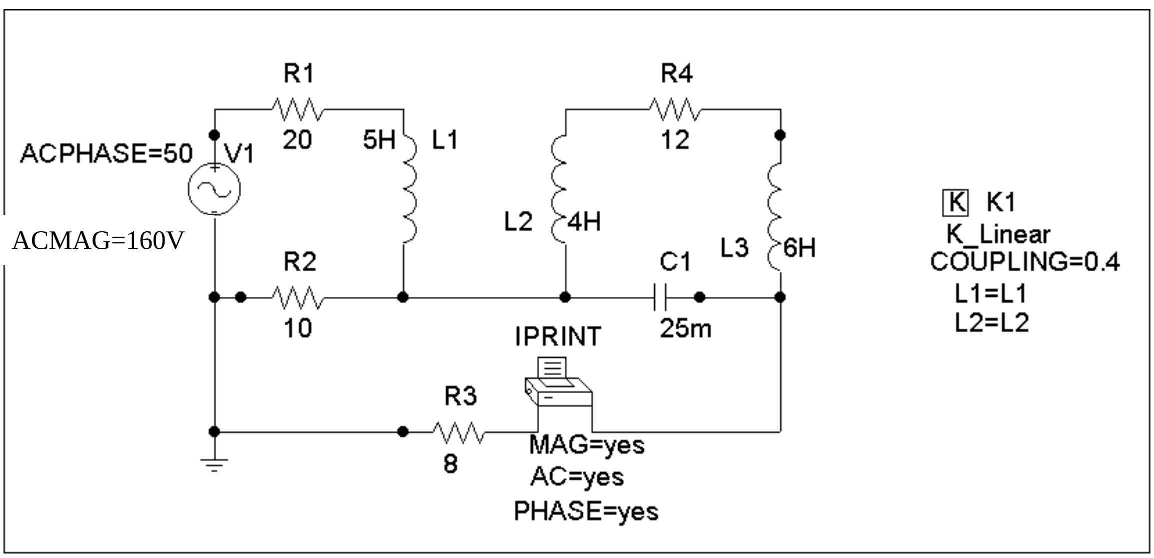
(b) $V_{LS} = 12.5$ kV, $V_{LP} = 625$ kV, $n = V_{LS}/V_{LP} = 12.5/625 = \mathbf{0.02}$

(c) $I_{LP} = nI_{LS} = 0.02x2173.6 = \mathbf{43.47\ A}$

or $I_{LP} = S_T/[(\sqrt{3})v_{LP}] = 47.0588x10^6/[(\sqrt{3})625x10^3] = \mathbf{43.47\ A}$

(d) The load carried by each transformer is $(1/3)S_T = \mathbf{15.69\ MVA}$

P.P. 13.13 The process is essentially the same as in Example 13.13. We are given the coupling coefficient, $k = 0.4$, and can determine the operating frequency from the value of $\omega = 4$ which implies that $f = 4/(2\pi) = 0.6366$ Hz.



Saving and then simulating produces,

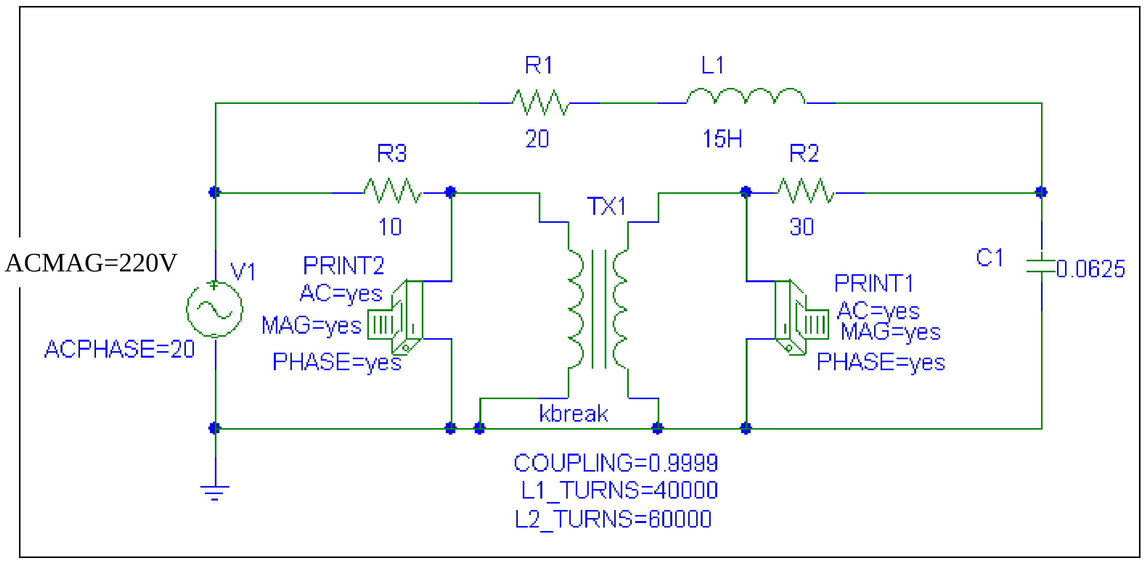
$$i_o = \mathbf{2.012cos(4t + 68.52^o)\ A}$$

P.P. 13.14 Following the same basic steps in Example 13.14, we first assume $\omega = 1$. This then leads to following determination of values for the inductor and the capacitor.

$$j15 = j\omega L \text{ leads to } L = 15\text{ H}$$

$$-j16 = 1/(\omega C) \text{ leads to } C = 62.5\text{ mF}$$

The schematic is shown below.



FREQ	VM($N_0005,0)	VP($N_0005,0)
1.592E-01	1.530E+02	2.185E+00
FREQ	VM($N_0001,0)	VP($N_0001,0)
1.592E-01	2.302E+02	2.091E+00

Thus,

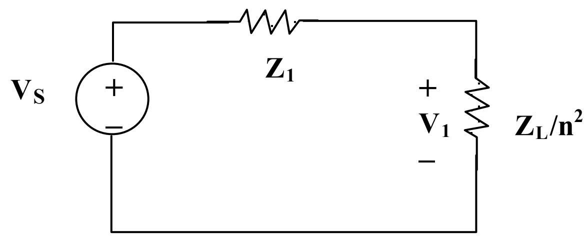
$$\mathbf{V_1 = 153\angle 2.18°\ V}$$

$$\mathbf{V_2 = 230.2\angle 2.09°\ V}$$

Note, if we divide V_2 by V_1 we get $1.5046\angle{-.09^\circ}$ which is in good agreement that the transformer is ideal with a voltage ratio of 1:1.5 (or 2:3)!

P.P. 13.15 $V_2/V_1 = 120/13{,}200 = \mathbf{1/110} = 1/n$

P.P. 13.16



As in Example 13.16, $n^2 = \mathbf{Z_L}/\mathbf{Z_1} = 400/(2.5\text{x}10^3) = 4/25$, $n = \mathbf{0.4}$

By voltage division, $\mathbf{V_1} = \mathbf{V_s}/2$ (since $\mathbf{Z_1} = \mathbf{Z_L}/n^2$), therefore $V_1 = 60/2 = 30$ volts, and

$$V_2 = nV_1 = (0.4)(30) = \mathbf{12\ volts}$$

P.P. 13.17

(a) $S = 12\text{x}60 + 350 + 4{,}500 = \mathbf{5.57\ kW}$

(b) $I_P = S/V_P$ $5570/2400 = \mathbf{2.321\ A}$

CHAPTER 14

P.P.14.1 $\mathbf{H}(\omega) = \dfrac{\mathbf{V}_o}{\mathbf{V}_s} = \dfrac{\mathbf{j\omega L}}{\mathbf{R + j\omega L}}$

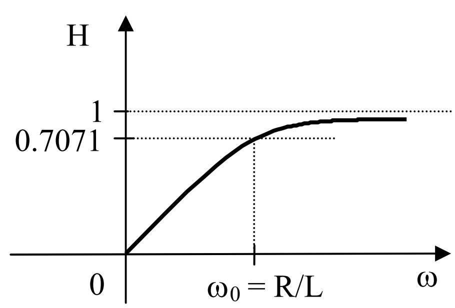
$$\mathbf{H}(\omega) = \frac{j\omega L/R}{1 + j\omega L/R} = \frac{j\omega/\omega_0}{1 + j\omega/\omega_0}$$

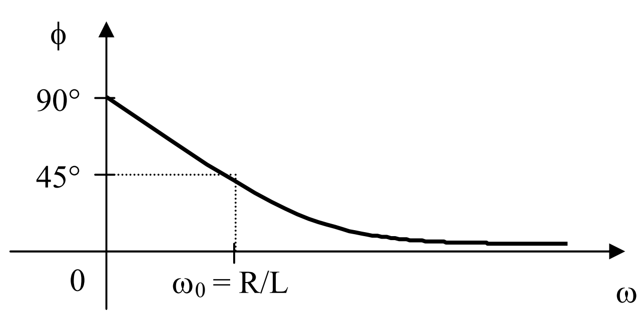
where $\omega_0 = \dfrac{R}{L}$.

$$H = \left|\mathbf{H}(\omega)\right| = \frac{\omega/\omega_0}{\sqrt{1 + (\omega/\omega_0)^2}} \qquad \phi = \angle\mathbf{H}(\omega) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

At $\omega = 0$, $H = 0$, $\phi = 90°$
As $\omega \to \infty$, $H = 1$, $\phi = 0°$
At $\omega = \omega_0$, $H = \dfrac{1}{\sqrt{2}}$, $\phi = 90° - 45° = 45°$

Thus, the sketches of H and ϕ are shown below.





P.P.14.2 The desired transfer function is the input impedance.

$$\mathbf{Z}_i(s) = \frac{\mathbf{V}_o(s)}{\mathbf{I}_o(s)} = \left(10 + \frac{1}{s/20}\right) \| (6+2s)$$

$$\mathbf{Z}_i(s) = \frac{(10+20/s)(6+2s)}{10+20/s+6+2s} = \frac{10(s+2)(s+3)}{s^2+8s+10}$$

The poles are at

$$p_{1,2} = \frac{-8 \pm \sqrt{64-40}}{2} = \mathbf{-1.5505, -6.449}$$

The zeros are at

$$z_1 = \mathbf{-2}, \qquad z_2 = \mathbf{-3}.$$

P.P.14.3 $$\mathbf{H}(\omega) = \frac{1 + j\omega/2}{(j\omega)(1+j\omega/10)}$$

$$H_{db} = 20\log_{10}\left|1+j\omega/2\right| - 20\log_{10}\left|j\omega\right| - 20\log_{10}\left|1+j\omega/10\right|$$

$$\phi = -90° + \tan^{-1}(\omega/2) - \tan^{-1}(\omega/10)$$

The magnitude and the phase plots are as shown in Fig. 14.14.

P.P.14.4 $$\mathbf{H}(\omega) = \frac{(50/400)j\omega}{(1+j\omega/4)(1+j\omega/10)^2}$$

$$H_{db} = -20\log_{10}\left|8\right| + 20\log_{10}\left|j\omega\right| - 20\log_{10}\left|1+j\omega/4\right| - 40\log_{10}\left|1+j\omega/10\right|$$

$$\phi = 90° - \tan^{-1}(\omega/4) - 2\tan^{-1}(\omega/10)$$

The magnitude and the phase plots are as shown in Fig. 14.16.

P.P.14.5 $$\mathbf{H}(\omega) = \frac{10/400}{(j\omega)\left(1 + \dfrac{j\omega 8}{40} + \left(\dfrac{j\omega}{20}\right)^2\right)}$$

$$H_{db} = -20\log_{10}\left|40\right| - 20\log_{10}\left|j\omega\right| - 20\log_{10}\left|1+j\omega/5-\omega^2/400\right|$$

$$\phi = -90° - \tan^{-1}\left(\frac{0.2\omega}{1-\omega^2/400}\right)$$

The magnitude and the phase plots are as shown in Fig. 14.18.

P.P.14.6
The gain is = 40 db = $20\log_{10}$(gain) or the gain = 100.

A zero at $\omega = 5$, $1 + j\omega/5$

A pole at $\omega = 10$, $\dfrac{1}{1 + j\omega/10}$

Two poles at $\omega = 100$, $\dfrac{1}{(1 + j\omega/100)^2}$

Hence,

$$\mathbf{H}(\omega) = \frac{100(1 + j\omega/5)}{(1 + j\omega/10)(1 + j\omega/100)^2} = \frac{100(1/5)(5 + j\omega)}{(1/100{,}000)(10 + j\omega)(100 + j\omega)^2}$$

$$\mathbf{H}(\omega) = \frac{2{,}000{,}000\,(s + 5)}{(s + 10)(s + 100)^2}$$

P.P.14.7

(a) $Q = \dfrac{\omega_0 L}{R} \longrightarrow \omega_0 = \dfrac{QR}{L} = \dfrac{(50)(4)}{25 \times 10^{-3}} = 8 \times 10^3 \text{ rad/s}$

$$\omega_0 = \frac{1}{\sqrt{LC}} \longrightarrow C = \frac{1}{\omega_0^2 L} = \frac{1}{(64 \times 10^6)(25 \times 10^{-3})}$$

C = **0.625 μF**

(b) $B = \dfrac{\omega_0}{Q} = \dfrac{8 \times 10^3}{50} = \mathbf{160\ rad/s}$

Since $Q > 10$,

$$\omega_1 = \omega_0 - \frac{B}{2} = 8000 - 80 = \mathbf{7920\ rad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 8000 + 80 = \mathbf{8080\ rad/s}$$

(c) At $\omega = \omega_0$, $P = \dfrac{V_{in}^2}{2R} = \dfrac{100^2}{8} = \mathbf{1.25\ kW}$

At $\omega = \omega_1$, $P = 0.5 \cdot \dfrac{V_{in}^2}{2R} = \mathbf{0.625\ kW}$

At $\omega = \omega_2$, $P = 0.5 \cdot \dfrac{V_{in}^2}{2R} = \mathbf{0.625\ kW}$

P.P.14.8 $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(20\times10^{-3})(5\times10^{-9})}} = 10^5 = \mathbf{100\ krad/s}$

$$Q = \frac{R}{\omega_0 L} = \frac{100\times10^3}{(10^5)(20\times10^{-3})} = \mathbf{50}$$

$$B = \frac{\omega_0}{Q} = \frac{10^5}{50} = \mathbf{2\ krad/s}$$

Since $Q > 10$,

$$\omega_1 = \omega_0 - \frac{B}{2} = 100{,}000 - 1{,}000 = \mathbf{99\ krad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 100{,}000 + 1{,}000 = \mathbf{101\ krad/s}$$

P.P.14.9 $\mathbf{Z} = j\omega 0.01 + 20 \,||\, \frac{2000}{j\omega} = j\omega 0.01 + \frac{20}{1 + j\omega/100}$

$$\mathbf{Z} = j\omega 0.01 + \frac{20(1 - j\omega/100)}{1 + \omega^2/10^4}$$

$$\mathrm{Im}(\mathbf{Z}) = 0 \longrightarrow \omega 0.01 - \frac{0.2\omega}{1 + \omega^2/10^4} = 0$$

$$\omega = \frac{20\omega}{1 + \omega^2/10^4} \longrightarrow 1 + \omega^2/10^4 = 20$$

Clearly, ω = **435.9 rad/s**

P.P.14.10 $\mathbf{H}(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2 \,||\, sL}{R_1 + R_2 \,||\, sL}$, $\quad s = j\omega$

$$\mathbf{H}(s) = \frac{sR_2L}{R_1R_2 + sR_1L + sR_2L}$$

$$\mathbf{H}(\omega) = \frac{j\omega R_2 L}{R_1R_2 + j\omega L(R_1 + R_2)}$$

$$H(0) = 0$$

$$H(\omega) = \lim_{\omega\to\infty} \frac{jR_2L}{R_1R_2/\omega + jL(R_1 + R_2)} = \frac{R_2}{R_1 + R_2}$$

i.e. a **highpass filter**.

The corner frequency occurs when $H(\omega_c) = \frac{1}{\sqrt{2}} \cdot H(\infty)$.

$$\mathbf{H}(\omega) = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{j\omega L}{j\omega L + R_1 R_2/(R_1 + R_2)}\right)$$

$$\mathbf{H}(\omega) = \left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{j\omega}{j\omega + k}\right), \qquad \text{where } k = \frac{R_1 R_2}{(R_1 + R_2)L}$$

At the corner frequency,

$$\frac{1}{\sqrt{2}} \cdot \frac{R_2}{R_1 + R_2} = \frac{R_2}{R_1 + R_2} \cdot \left|\frac{j\omega_c}{j\omega_c + k}\right|$$

$$\frac{1}{\sqrt{2}} = \frac{\omega_c}{\sqrt{\omega_c^2 + k^2}} \longrightarrow \omega_c = k = \mathbf{\frac{R_1 R_2}{(R_1 + R_2)L}}$$

Hence, $$\mathbf{H}(\omega) = \left(\mathbf{\frac{R_2}{R_1 + R_2}}\right)\left(\mathbf{\frac{j\omega}{j\omega + \omega_c}}\right)$$

and the corner frequency is

$$\omega_c = \frac{(100)(100)}{(100 + 100)(2 \times 10^{-3})} = \mathbf{25\ krad/s}$$

P.P.14.11 $B = 2\pi(20.3 - 20.1) \times 10^3 = 400\pi$

Assuming high Q,

$$\omega_0 = \frac{\omega_1 + \omega_2}{2} = \frac{(2\pi)(40.4 \times 10^3)}{2} = 40.4\pi \times 10^3 \text{ rad/s}$$

$$Q = \frac{\omega_0}{B} = \frac{40.4\pi \times 10^3}{400\pi} = \mathbf{101}$$

$$B = \frac{R}{L} \longrightarrow L = \frac{R}{B} = \frac{20 \times 10^3}{400\pi} = \mathbf{15.915\ H}$$

$$Q = \frac{1}{\omega_0 CR} \longrightarrow C = \frac{1}{\omega_0 QR}$$

$$C = \frac{1}{(40.4\pi \times 10^3)(101)(20 \times 10^3)} = \mathbf{3.9\ pF}$$

P.P.14.12 Given $H(\infty) = 5$ and $f_c = 2$ kHz

$$\omega_c = 2\pi f_c = \frac{1}{R_i C_i}$$

$$R_i = \frac{1}{2\pi f_c C_i} = \frac{1}{(2\pi)(2\times10^3)(0.1\times10^{-3})}$$

$$R_i = 795.8 \cong \mathbf{800\,\Omega}$$

$$H(\infty) = \frac{-R_f}{R_i} = -5 \quad \longrightarrow \quad R_f = 5R_i = 3{,}978 \cong \mathbf{4\,k\Omega}$$

P.P.14.13 $\quad Q = 10, \qquad \omega_0 = 20 \text{ krad/s}$

$$B = \frac{\omega_0}{Q} = 2 \text{ krad/s}$$

$$\omega_1 = \omega_0 - \frac{B}{2} = 19 \text{ krad/s}$$

$$\omega_2 = \omega_0 + \frac{B}{2} = 21 \text{ krad/s}$$

Since $\omega_1 = \dfrac{1}{C_2 R}$,

$$C_2 = \frac{1}{\omega_1 R} = \frac{1}{(19\times10^3)(10\times10^3)} = \mathbf{5.263\ nF}$$

$$C_1 = \frac{1}{\omega_2 R} = \frac{1}{(21\times10^3)(10\times10^3)} = \mathbf{4.762\ nF}$$

$$K = \frac{R_f}{R_i} = 5 \quad \longrightarrow \quad R_f = 5R_i = \mathbf{50\,k\Omega}$$

P.P.14.14 $\quad K_f = \dfrac{\omega'_c}{\omega_c} = \dfrac{2\pi\times10^4}{1} = 2\pi\times10^4$

$$C' = \frac{C}{K_m K_f} \quad \longrightarrow \quad K_m = \frac{C}{C' K_f} = \frac{1}{(15\times10^{-9})(2\pi\times10^4)} = \frac{10^4}{3\pi}$$

$$R' = K_m R = \frac{10^4}{3\pi}(1) = 1.061\text{ k}\Omega$$

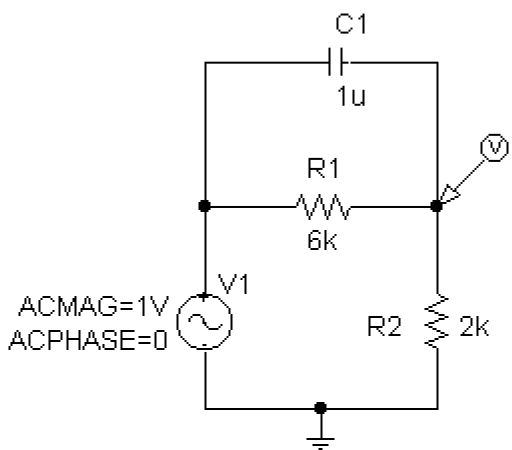
$$L' = \frac{K_m}{K_f} L = \frac{10^4}{3\pi}\cdot\frac{2}{2\pi\times10^4} = 33.77\text{ mH}$$

Therefore,

$$R'_1 = R'_2 = \mathbf{1.061\,k\Omega}$$

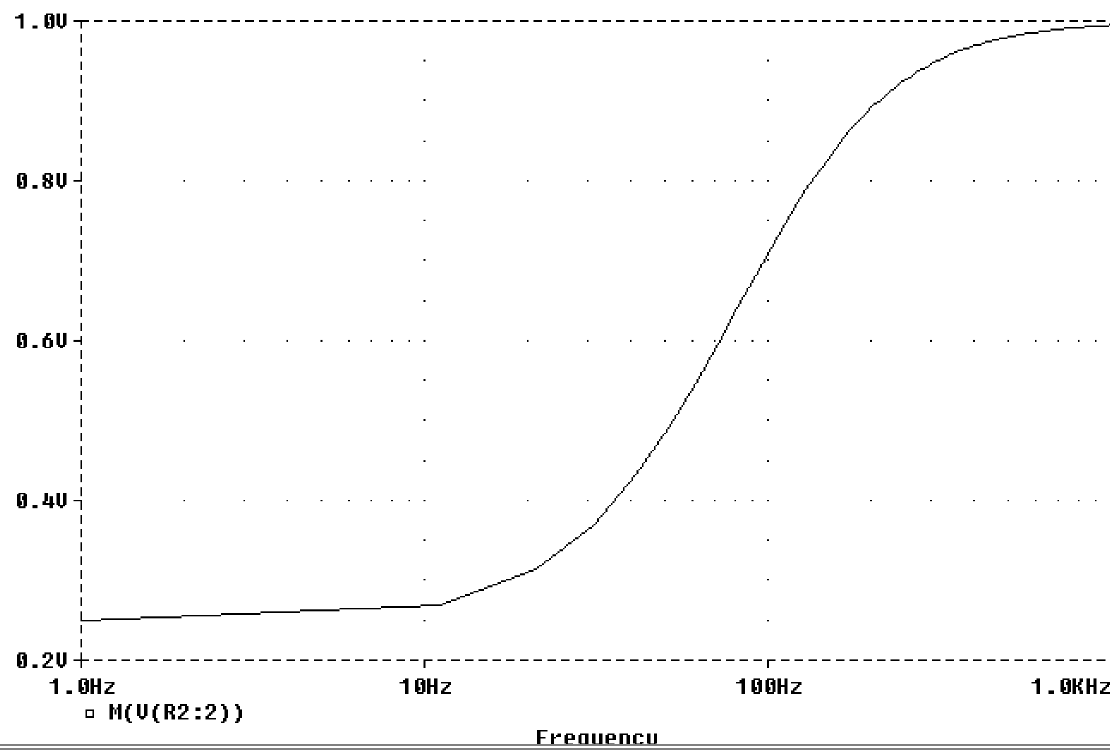
$C_1' = C_2' =$ **15 nF**
$L' =$ **33.77 mH**

P.P.14.15 The schematic is shown in Fig. (a).

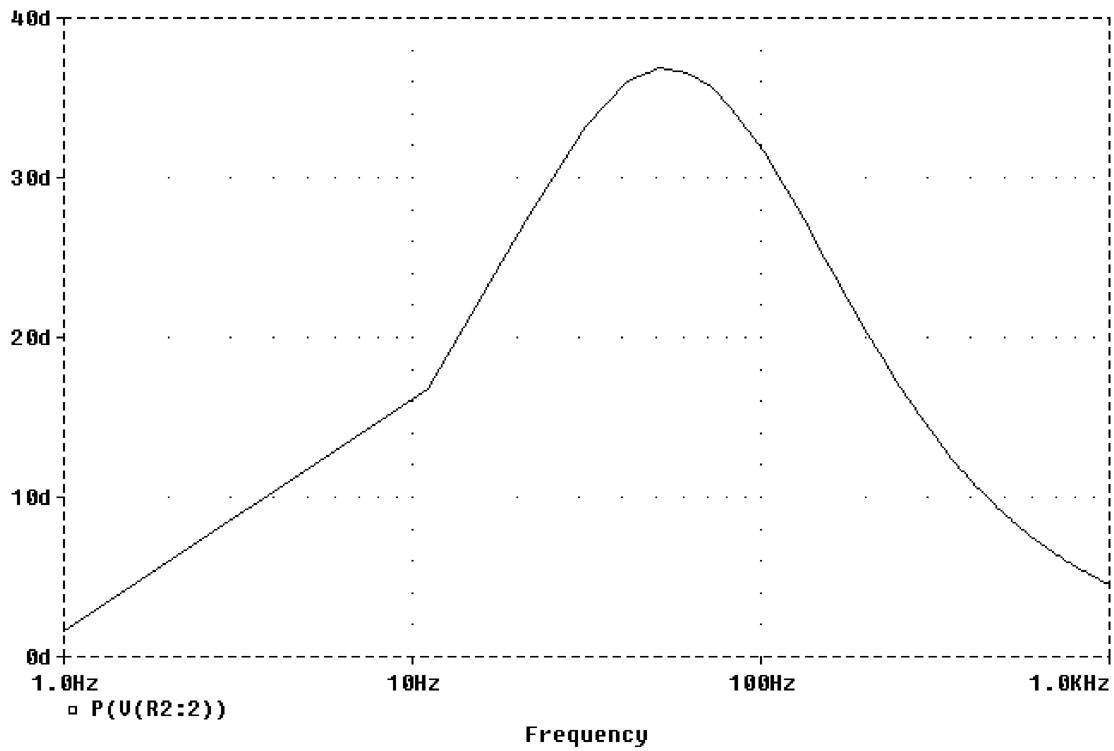


(a)

Use the **AC Sweep** option of the **Analysis Setup**. Choose a *Linear* sweep type with the following *Sweep Parameters* : *Total Pts* = 100, *Start Freq* = 1, and *End Freq* = 1K. After saving and simulating the circuit, we obtain **the magnitude and phase plots are shown in Figs. (b) and (c)**.

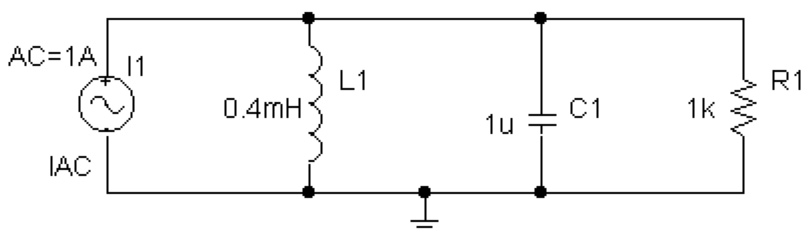


(b)



(c)

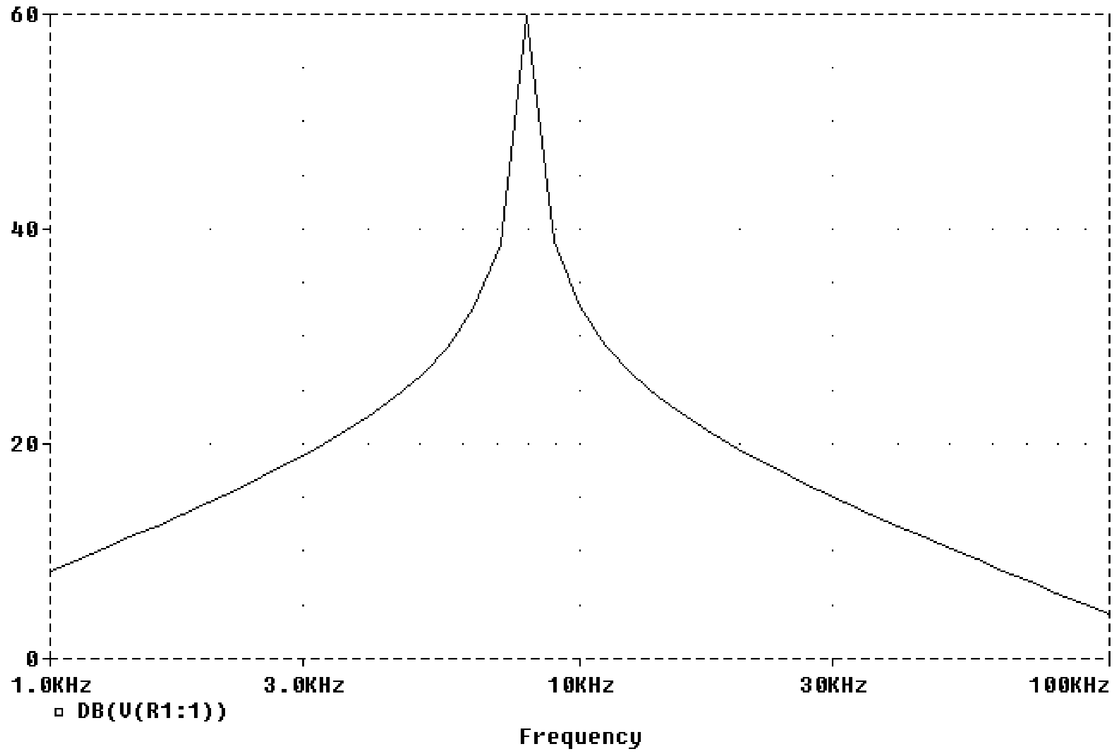
P.P.14.16 The schematic is shown in Fig. (a).



(a)

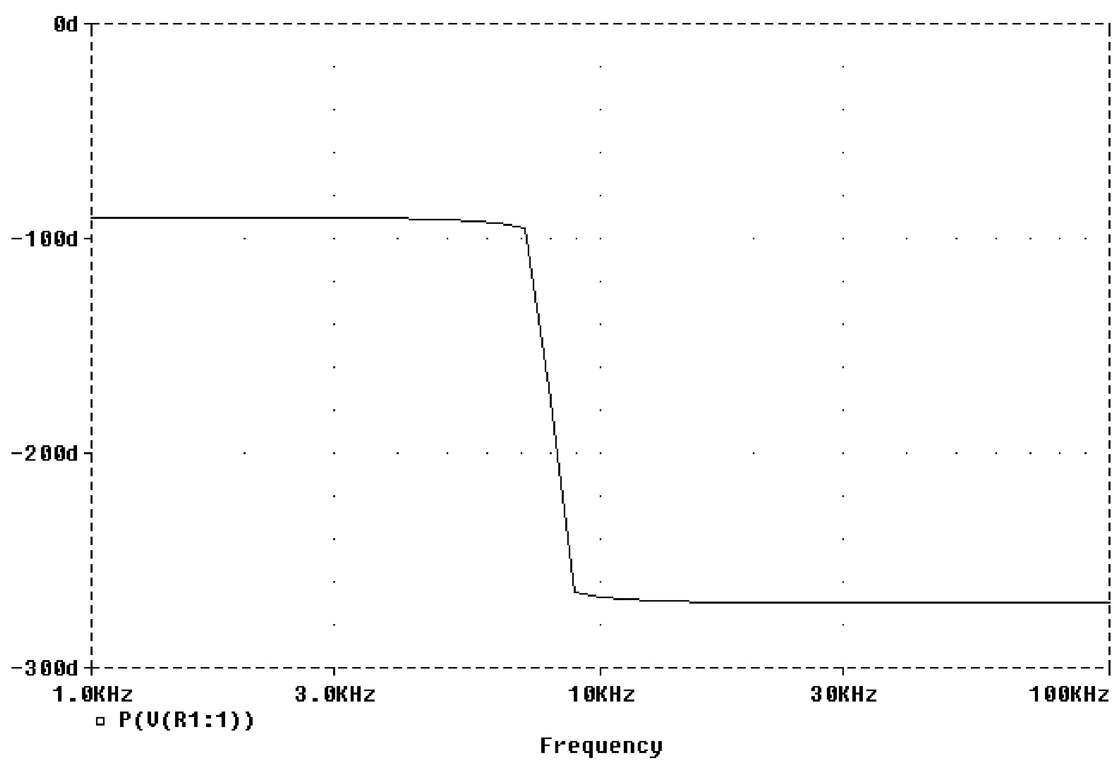
Use the **AC Sweep** option of the **Analysis Setup**. Choose a *Decade* sweep type with these *Sweep Parameters* : *Pts/Decade* = 20, *Start Freq* = 1K, and *End Freq* = 100K. Save and simulate the circuit.

For the magnitude plot, choose DB( ) from the **Analog Operators and Functions** list. Then, select the voltage V(R1:1) and OK. Another option would be to type DB(V(R1:1)) as the **Trace Expression**. For the phase plot, choose P( ) from the **Analog Operators and Functions** list. Then, select the voltage V(R1:1) and OK. Another option would be to type VP(R1:1) as the **Trace Expression**. **The resulting magnitude and phase plots are shown in Figs. (b) and (c)**.



60
40
20
0
1.0KHz
3.0KHz
10KHz
30KHz
100KHz
DB(V(R1:1))
Frequency

(b)



0d
-100d
-200d
-300d
1.0KHz
3.0KHz
10KHz
30KHz
100KHz
P(V(R1:1))
Frequency

(c)

P.P.14.17 $\omega_0 = 2\pi f_0 = \dfrac{1}{\sqrt{LC}}$

or $C = \dfrac{1}{4\pi^2 f_0^2 L}$

For the high end of the band, $f_0 = 108$ MHz

$$C_1 = \frac{1}{4\pi^2(108^2 \times 10^{12})(4 \times 10^{-6})} = 0.543 \text{ pF}$$

For the low end of the band, $f_0 = 88$ MHz

$$C_2 = \frac{1}{4\pi^2(88^2 \times 10^{12})(4 \times 10^{-6})} = 0.818 \text{ pF}$$

Therefore, C must be adjustable and be in the range **0.543 pF to 0.818 pF** .

P.P.14.18
For BP_6, $f_0 = 1336$ Hz and it passes frequencies in the range $1209 \text{ Hz} < f < 1477 \text{ Hz}$.

$$B = 2\pi(1477 - 1209) = 1683.9$$

$$L = \frac{R}{B} = \frac{600}{1683.9} = \mathbf{356\ mH}$$

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2(1336)^2(0.356)} = \mathbf{39.83\ nF}$$

P.P.14.19 $C = 10\ \mu F$ and $R_1 = R_2 = 8\ \Omega$

$$2\pi f_c = \frac{1}{R_1 C} \longrightarrow f_c = \frac{1}{2\pi R_1 C} = \frac{1}{(2\pi)(8)(10 \times 10^{-6})} = \mathbf{1.989\ kHz}$$

$$L = \frac{R_2}{2\pi f_c} = \frac{8}{(2\pi)(1.989 \times 10^3)} = \mathbf{0.64\ mH}$$

CHAPTER 15

P.P.15.1 $\mathcal{L}\left[\, t\,u(t)\right] = \int_0^\infty t\,e^{-st}\;dt$

Using integration by parts,

$$\int u\;dv = uv - \int v\;du$$

Let $u = t \longrightarrow du = dt$.

$$e^{-st}\;dt = dv \longrightarrow v = \frac{-1}{s} e^{-st}$$

$$\mathcal{L}\left[\, t\,u(t)\right] = \frac{-t}{s} e^{-st}\Big|_0^\infty + \int_0^\infty \frac{1}{s} e^{-st}\;dt = 0 + \frac{e^{-st}}{s^2}\Big|_0^\infty = \mathbf{\frac{1}{s^2}}$$

Next,

$$\mathcal{L}\left[\, Ae^{at}\;u(t)\right] = A\int_0^\infty e^{-at}e^{-st}\;dt = \frac{-A}{s+a} e^{-(s+a)t}\Big|_0^\infty$$

$$= \mathbf{A/(s+a)}$$

From this, all we need to do is to solve for the Laplace transform of $e^{-j\omega t}$ is to let a = jω in the above and B = A, we get,

$$\mathcal{L}[Be^{-j\omega t}u(t)] = \mathbf{B/(s+j\omega)}$$

P.P.15.2 $\mathcal{L}\left[50\cos(\omega t)\right] = \int_0^\infty \frac{50}{2}\left(e^{j\omega t} + e^{-j\omega t}\right)e^{-st}\;dt$

$$\mathcal{L}\left[50\cos(\omega t)\right] = 25\int_0^\infty e^{-(s-j\omega)t}\;dt + 25\int_0^\infty e^{-(s+j\omega)t}\;dt$$

$$\mathcal{L}\left[10\cos(\omega t)\right] = 25\left(\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right) = \mathbf{\frac{50\boldsymbol{s}}{\boldsymbol{s}^2 + \boldsymbol{\omega}^2}}$$

P.P.15.3 If $f(t) = \cos(2t) + e^{-4t}$,

$$F(s) = \frac{s}{s^2+4} + \frac{1}{s+4} = \frac{s^2+4s+s^2+4}{(s^2+4)(s+4)}$$

$$F(s) = \mathbf{\frac{2\boldsymbol{s}^2 + 4\boldsymbol{s} + 4}{(\boldsymbol{s}+4)(\boldsymbol{s}^2+4)}}$$

P.P.15.4 Given $f(t) = t^2 \cos(3t)$

From P.P.15.2, $\mathcal{L}\left[\cos(3t)\right] = \dfrac{s}{s^2+9}$

Using Eq. 15.34, $F(s) = \mathcal{L}\left[t^2 \cos(3t)\right] = (-1)^2 \dfrac{d^2}{ds^2}\left(\dfrac{s}{s^2+9}\right)$

$$F(s) = \frac{d^2}{ds^2}\left[s\left(s^2+9\right)^{-1}\right] = \frac{d^2}{ds^2}\left[(1)\left(s^2+9\right)^{-1} - (s)(2s)\left(s^2+9\right)^{-2}\right]$$

$$F(s) = (-2s)\left(s^2+9\right)^{-2} - (4s)\left(s^2+9\right)^{-2} + \left(4s^2\right)(2s)\left(s^2+9\right)^{-3}$$

$$F(s) = (-6s)\left(s^2+9\right)^{-2} + \left(8s^3\right)\left(s^2+9\right)^{-3} = \frac{2s^3 - 54s}{\left(s^2+9\right)^3}$$

$$F(s) = \mathbf{\frac{2s\left(s^2-27\right)}{\left(s^2+9\right)^3}}$$

P.P.15.5 $h(t) = 20\left[u(t) - u(t-4)\right] + 10\left[u(t-4) - u(t-8)\right]$

$$H(s) = 20\left(\frac{1}{s} - \frac{e^{-4s}}{s}\right) + 10\left(\frac{e^{-4s}}{s} - \frac{e^{-8s}}{s}\right)$$

$$H(s) = \boldsymbol{\frac{10}{s}\left(2 - e^{-4s} - e^{-8s}\right)}$$

P.P.15.6 $T = 5$

$$f_1(t) = u(t) - u(t-2)$$

$$F_1(s) = \frac{1}{s}\left(1 - e^{-2s}\right)$$

$$F(s) = \frac{F_1(s)}{1 - e^{-Ts}} = \mathbf{\frac{1 - e^{-2s}}{s\left(1 - e^{-5s}\right)}}$$

P.P.15.7 $g(0) = \lim\limits_{s\to\infty} sF(s) = \lim\limits_{s\to\infty} \dfrac{6s^3 + 2s + 5}{(s^2 + 4s + 4)(s+3)}$

$$g(0) = \lim_{s\to\infty} \frac{6 + \dfrac{2}{s^2} + \dfrac{5}{s^3}}{\left(1 + \dfrac{4}{s} + \dfrac{4}{s^2}\right)\left(1 + \dfrac{3}{s}\right)} = \mathbf{6}$$

Since all poles s = 0, –2, –2, –3 lie in the left-hand s-plane, we can apply the final-value theorem.

$$g(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{6s^3 + 2s + 5}{(s+2)^2(s+3)}$$

$$g(\infty) = \lim_{s\to 0} \frac{5}{(2)^2(3)} = \mathbf{0.4167}$$

P.P.15.8 $$F(s) = 5 + \frac{6}{s+4} - \frac{7s}{s^2+25}$$

$$f(t) = \mathrm{L}^{-1}\left[5\right] + \mathrm{L}^{-1}\left[\frac{6}{s+4}\right] - \mathrm{L}^{-1}\left[\frac{7s}{s^2+25}\right]$$

$$\mathrm{f}(t) = \mathbf{5\delta(t) + (6e^{-4t} - 7\cos(5t))u(t)}$$

P.P.15.9 $$\mathrm{F(s)} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$A = F(s)\,(s+1)\Big|_{s=-1} = \frac{6\,(s+2)}{(s+3)(s+4)}\Big|_{s=-1} = \frac{6}{(2)(3)} = 1$$

$$B = F(s)\,(s+3)\Big|_{s=-3} = \frac{6\,(s+2)}{(s+1)(s+4)}\Big|_{s=-3} = \frac{(6)(-1)}{(-2)(1)} = 3$$

$$C = F(s)\,(s+4)\Big|_{s=-4} = \frac{6\,(s+2)}{(s+1)(s+3)}\Big|_{s=-4} = \frac{(6)(-2)}{(-3)(-1)} = -4$$

$$F(s) = \frac{1}{s+1} + \frac{3}{s+3} - \frac{4}{s+4}$$

$$\mathrm{f}(t) = \mathbf{(e^{-t} + 3e^{-3t} - 4e^{-4t})u(t)}$$

P.P.15.10 $$G(s) = \frac{s^3 + 2s + 6}{s\,(s+1)^2(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2} + \frac{D}{s+3}$$

Multiplying both sides by $s\,(s+1)^2(s+3)$ gives

$$\begin{aligned} s^3 + 2s + 6 &= A\,(s+3)(s^2+2s+1) + Bs\,(s+1)(s+3) + Cs\,(s+3) + Ds\,(s+1)^2 \\ &= A\,(s^3 + 5s^2 + 7s + 3) + B\,(s^3 + 4s^2 + 3s) + C\,(s^2 + 3s) + D\,(s^3 + 2s^2 + s) \end{aligned}$$

Equating coefficients :

s^0: $6 = 3A \longrightarrow A = 2$ (1)

s^1: $2 = 7A + 3B + 3C + D \longrightarrow 3B + 3C + D = -12$ (2)

s^2: $0 = 5A + 4B + C + 2D \longrightarrow 4B + C + 2D = -10$ (3)

s^3: $1 = A + B + D \longrightarrow B + D = -1$ (4)

Solving (2), (3), and (4) gives

$A = 2, \qquad B = \frac{-13}{4}, \qquad C = \frac{-3}{2}, \qquad D = \frac{9}{4}$

$$G(s) = \frac{2}{s} - \frac{13/4}{s+1} - \frac{3/2}{(s+1)^2} + \frac{9/4}{s+3}$$

$$g(t) = \mathbf{(2 - 3.25\,e^{-t} - 1.5\,t\,e^{-t} + 2.25\,e^{-3t})u(t)}$$

P.P.15.11 $\quad G(s) = \dfrac{60}{(s+1)(s^2+4s+13)} = \dfrac{A}{s+1} + \dfrac{Bs+C}{s^2+4s+13}$

Multiplying both sides by $(s+1)(s^2+4s+13)$ gives

$$60 = A(s^2+4s+13) + B(s^2+s) + C(s+1)$$

Equating coefficients :

$s^2: \quad 0 = A + B \longrightarrow A = -B \qquad (1)$

$s^1: \quad 0 = 4A + B + C \longrightarrow C = -3A \qquad (2)$

$s^0: \quad 60 = 13A + C \longrightarrow 60 = 10A \qquad (3)$

Solving (1), (2), and (3) gives

$A = 6, \qquad B = -6, C = -18$

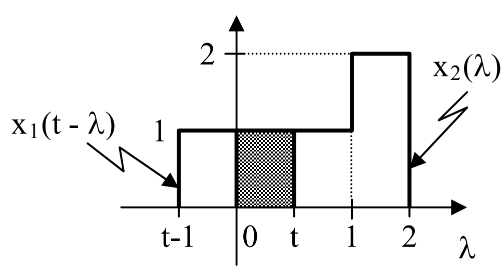
$$G(s) = \frac{6}{s+1} + \frac{-6s-18}{(s+2)^2+9} = \frac{6}{s+1} - 6\frac{s+2}{(s+2)^2+9} - \frac{(6/3)3}{(s+2)^2+9}$$

$$g(t) = \mathbf{(6e^{-t} - 6e^{-2t}\cos(3t) - 2\,e^{-2t}\sin(3t))u(t)}$$

P.P.15.12

For $0 < t < 1$, consider Fig. (a).

$$y(t) = \int_0^t (1)(1)\,d\lambda = t$$

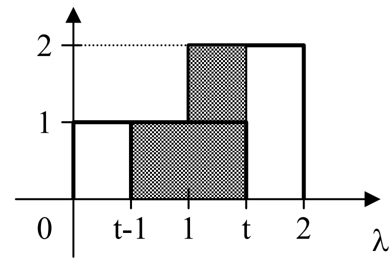


(a)

For $1 < t < 2$, consider Fig. (b).

$$y(t) = \int_{t-1}^{1} (1)(1)\, d\lambda + \int_{1}^{t} (1)(2)\, d\lambda = \lambda \Big|_{t-1}^{t} + 2\lambda \Big|_{1}^{t}$$

$$y(t) = 1 - t + 1 + 2(t-1) = t$$

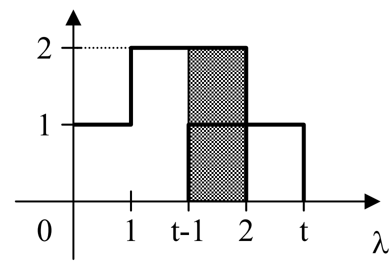


(b)

For $2 < t < 3$, consider Fig. (c).

$$y(t) = \int_{t-1}^{2} (1)(2)\, d\lambda = 2\lambda \Big|_{t-1}^{2}$$

$$y(t) = 2(2 - t + 1) = 6 - 2t$$

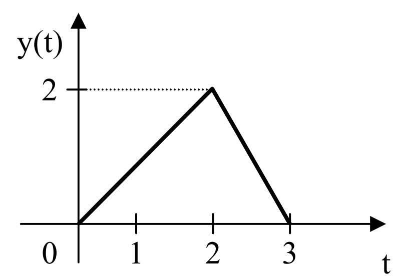


(c)

For $t > 3$, there is no overlap so $y(t) = 0$.

Thus,

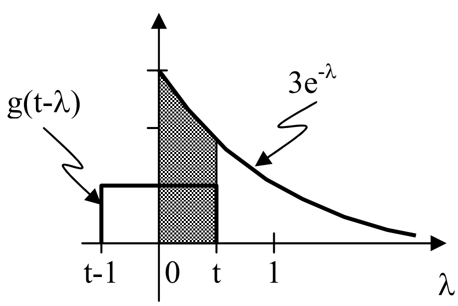
$$y(t) = \begin{cases} \mathbf{t} & \mathbf{0 < t < 2} \\ \mathbf{6 - 2t} & \mathbf{2 < t < 3} \\ \mathbf{0} & \mathbf{otherwise} \end{cases}$$



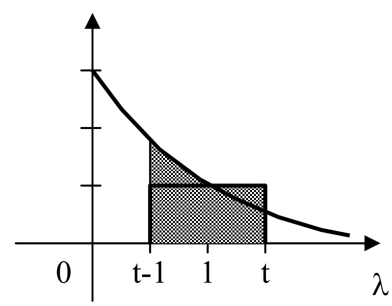
(d)

The result of the convolution is shown in Fig. (d).

P.P.15.13



(a)



(b)

For $0 < t < 1$, consider Fig. (a).

$$y(t) = \int_{0}^{t} (1)\, 3e^{-\lambda}\, d\lambda = -3e^{-\lambda} \Big|_{0}^{t} = 3(1 - e^{-t})$$

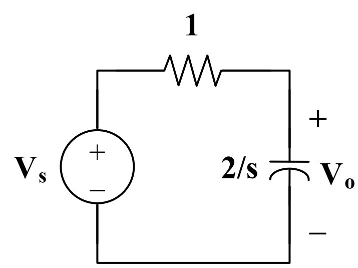
For $t > 1$, consider Fig. (b).

$$y(t) = \int_{t-1}^{t} (1)\,3\,e^{-\lambda}\,d\lambda = -3\,e^{-\lambda}\Big|_{t-1}^{t} = 3\,e^{-t}\,(e-1)$$

Thus,

$$y(t) = \begin{cases} \mathbf{3\,(1-e^{-t})} & \mathbf{0 \le t \le 1} \\ \mathbf{3\,e^{-t}\,(e-1)} & \mathbf{t \ge 1} \\ \mathbf{0} & \mathbf{elsewhere} \end{cases}$$

P.P.15.14 The circuit in the s-domain is shown below.



$$V_o = \frac{2/s}{1+2/s} V_s$$

$$H(s) = \frac{V_o}{V_s} = \frac{2}{s+2} \quad \longrightarrow \quad h(t) = 2\,e^{-2t}$$

$$\begin{aligned} v_o(t) = h(t) * v_s(t) &= \int_0^t h(\lambda)\,v_s(t-\lambda)\,d\lambda \\ &= \int_0^t 2\,e^{-2\lambda}\,10 e^{-(t-\lambda)}\,d\lambda \\ &= 20 e^{-t} \int_0^t e^{-2\lambda}\,e^{\lambda}\,d\lambda = 20 e^{-t}(-e^{-\lambda})\Big|_0^t \\ &= \mathbf{20\,(e^{-t} - e^{-2t})u(t)\ V} \end{aligned}$$

P.P.15.15 Taking the Laplace transform of each term gives

$$\left[s^2V(s) - sv(0) - v'(0)\right] + 4\left[sV(s) - v(0)\right] + 4\,V(s) = \frac{2}{s+1}$$

$$(s^2+4s+4)\,V(s) = 2s + 10 + \frac{2}{s+1} = \frac{2s^2+12s+12}{s+1}$$

$$V(s) = \frac{2s^2+12s+12}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$2s^2+12s+12 = A\,(s^2+4s+4) + B\,(s^2+3s+2) + C\,(s+1)$$

Equating coefficients :

$s^2:$ $\quad 2 = A + B \longrightarrow B = 2 - A \quad \text{or} \quad A = 2 - B$

$s^1:$ $\quad 12 = 4A + 3B + C \longrightarrow 12 = A + 6 + C \quad \text{or} \quad C = 6 - A$

$s^0:$ $\quad 12 = 4A + 2B + C \longrightarrow 12 = 12 - B \quad \text{or} \quad B = 0$

Thus,

$$A = 2, \qquad B = 0, \qquad C = 4$$

and

$$V(s) = \frac{2}{s+1} + \frac{4}{(s+2)^2}$$

Therefore,

$v(t) = \mathbf{(2e^{-t} + 4\,t\,e^{-2t})\,u(t)}$ **Note, there were no units given for v(t).**

P.P.15.16 Taking the Laplace transform of each term gives

$$sY(s) - y(0) + 3Y(s) + \frac{2}{s}Y(s) = \frac{2}{s+3}$$

$$\left[s^2 + 3s + 2\right]Y(s) = \frac{2s}{s+3}$$

$$Y(s) = \frac{2s}{(s+1)(s+2)(s+3)} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$A = Y(s)\,(s+1)\Big|_{s=-1} = -1$$

$$B = Y(s)\,(s+2)\Big|_{s=-2} = 4$$

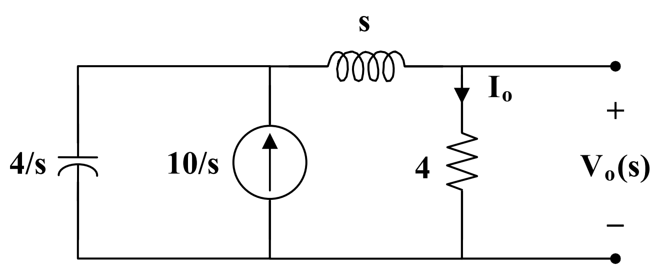
$$C = Y(s)\,(s+3)\Big|_{s=-3} = -3$$

$$Y(s) = \frac{-1}{s+1} + \frac{4}{s+2} - \frac{3}{s+3}$$

$y(t) = \mathbf{(-e^{-t} + 4\,e^{-2t} - 3\,e^{-3t})\,u(t)}$

CHAPTER 16

P.P.16.1 Consider the circuit shown below.



Using current division,

$$I_o = \frac{\frac{4}{s}}{\frac{4}{s} + s + 4} \cdot \frac{10}{s} = \frac{40}{s(s^2 + 4s + 4)}$$

$$V_o(s) = 4 I_o = \frac{160}{s(s+2)^2}$$

$$\frac{160}{s(s+2)^2} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$160 = A(s^2 + 4s + 4) + B(s^2 + 2s) + Cs$$

Equating coefficients :

s^0: $80 = 4A \longrightarrow A = 40$

s^1: $0 = 4A + 2B + C$

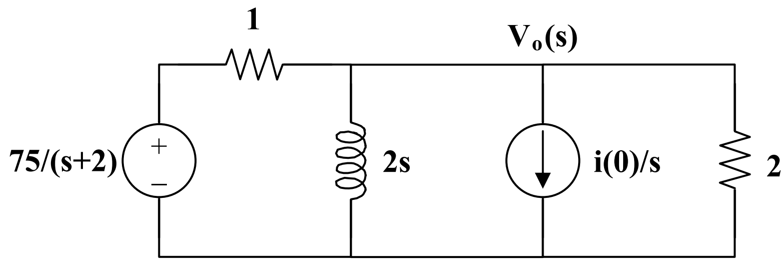
s^2: $0 = A + B \longrightarrow B = -A = -40$

Hence, $0 = 4A + 2B + C \longrightarrow C = -80$

$$V_o(s) = \frac{40}{s} - \frac{40}{s+2} - \frac{80}{(s+2)^2}$$

$$v_o(t) = \mathbf{40(1 - e^{-2t} - 2t\,e^{-2t})\,u(t)\,V}$$

P.P.16.2 The circuit in the s-domain is shown below.



At node o,

$$\frac{V_o - \frac{75}{s+2}}{1} + \frac{V_o}{2s} + \frac{V_o}{2} + \frac{i(0)}{s} = 0 \qquad \text{where} \quad i(0) = 0\text{A}$$

$$\left(1 + \frac{1}{2} + \frac{1}{2s}\right)V_o = \frac{75}{s+2}$$

$$V_o = \frac{150s}{(s+2)(3s+1)} = \frac{50s}{(s+2)(s+1/3)} = \frac{A}{s+2} + \frac{B}{s+1/3}$$

Solving for A and B we get,

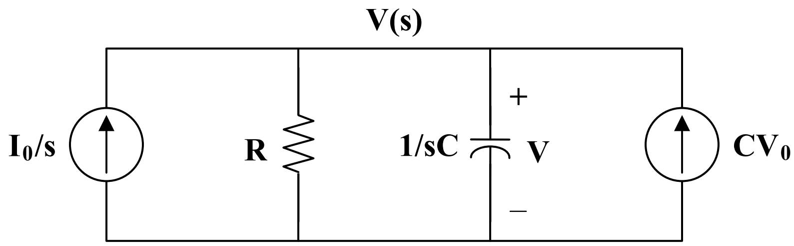
A = [50(–2)]/(–2+1/3) = 300/5 = 60, B = [50(–1/3)]/[(–1/3)+2] = –150/15 = –10

$$V_o = \frac{60}{s+2} - \frac{10}{s+1/3}$$

Hence,

$$v_o(t) = \left(\mathbf{60}\,e^{-2t} - \mathbf{10}e^{-t/3}\right)u(t)V$$

P.P.16.3 $v(0) = V_0$ is incorporated as shown below.



We apply KCL to the top node.

$$\frac{I_0}{s} + CV_0 = \frac{V}{R} + sCV = \left(sC + \frac{1}{R}\right)V$$

$$V = \frac{I_0}{s(sC + 1/R)} + \frac{CV_0}{sC + 1/R}$$

$$V = \frac{V_0}{s + 1/RC} + \frac{I_0/C}{s(s + 1/RC)}$$

$$V = \frac{V_0}{s + 1/RC} + \frac{A}{s} + \frac{B}{s + 1/RC}$$

where $A = \dfrac{I_0/C}{1/RC} = I_0 R$, $\quad B = \dfrac{I_0/C}{-1/RC} = -I_0 R$

$$V(s) = \frac{V_0}{s + 1/RC} + \frac{I_0 R}{s} - \frac{I_0 R}{s + 1/RC}$$

$$v(t) = \boldsymbol{((V_0 - I_0 R)\, e^{-t/\tau} + I_0 R), \quad t > 0, \quad \textit{where}\ \tau = RC}$$

P.P.16.4 We solve this problem the same as we did in Example 16.4 up to the point where we find V_1. Once we have V_1, all we need to do is to divide V_1 by 5s to and add in the contribution from i(0)/s to find I_L.

$I_L = V_1/5s – i(0)/s = 7/(s(s+1)) – 6/(s(s+2)) – 1/s$
$\quad = 7/s – 7/(s+1) – 3/s + 3/(s+2) – 1/s = 3/s – 7/(s+1) + 3/(s+2)$

Which leads to $i_L(t) =$ $\mathbf{(3 - 7e^{-t} + 3e^{-2t})u(t)A}$

P.P.16.5 We can use the same solution as found in Example 16.5 to find i_L.

All we need to do is divide each voltage by 5s and then add in the contribution from i(0). Start by letting $i_L = i_1 + i_2 + i_3$.

$I_1 = V_1/5s – 0/s = 6/(s(s+1)) – 6/(s(s+2)) = 6/s – 6/(s+1) – 3/s + 3/(s+2)$

or $i_1 = (3 – 6e^{–t} + 3e^{–2t})u(t)A$

$I_2 = V_2/5s – 1/s = 2/(s(s+1)) – 2/(s(s+2)) – 1/s = 2/s – 2/(s+1) – 1/s + 1/(s+2) –1/s$

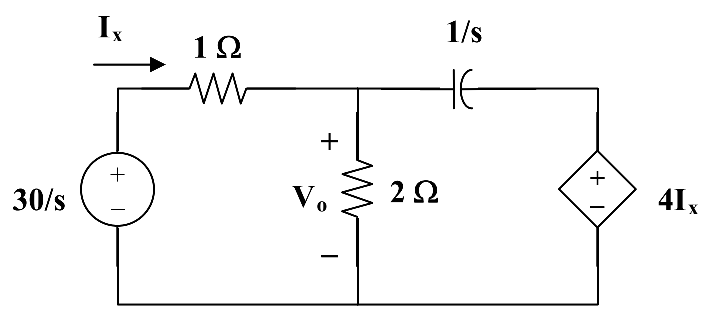
or $i_2 = (–2e^{–t} + e^{–2t})u(t)A$

$I_3 = V_3/5s – 0/s = –1/(s(s+1)) + 2/(s(s+2)) = –1/s + 1/(s+1) + 1/s – 1/(s+2)$

or $i_3 = (e^{–t} – e^{–2t})u(t)A$

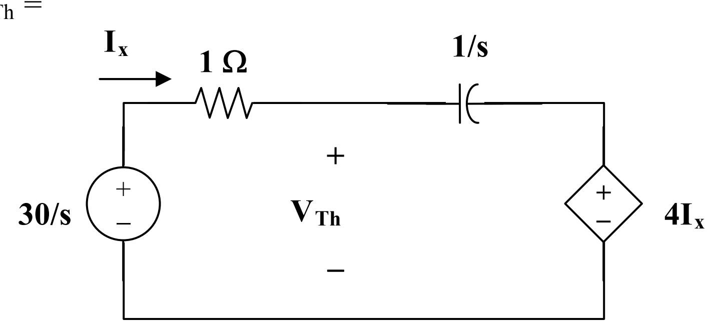
This leads to $i_L(t) = i_1 + i_2 + i_3 =$ $\mathbf{(3 – 7e^{–t} + 3e^{–2t})u(t)\ A}$

P.P.16.6



(a) Take out the 2 Ω and find the Thevenin equivalent circuit.

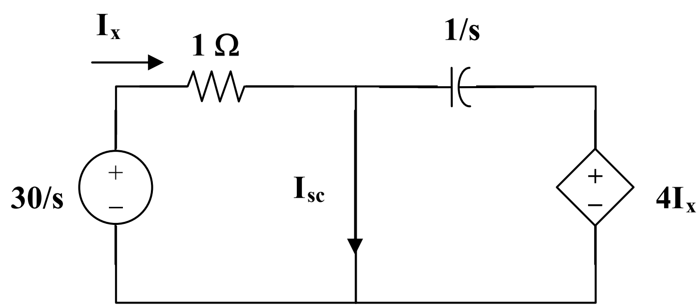
$V_{Th} =$



Using mesh analysis we get,

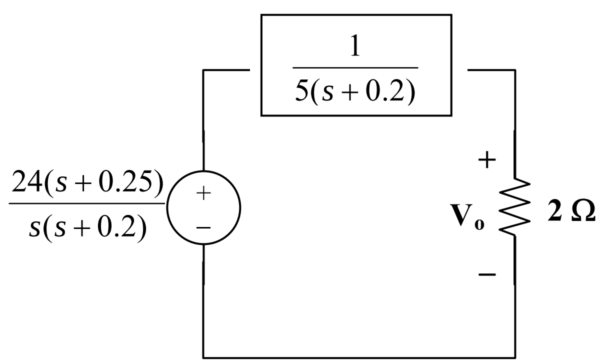
$-30/s + 1I_x + I_x/s + 4I_x = 0$ or $(1 + 1/s + 4)I_x = 30/s$ or $I_x = 30/(5s+1)$

$V_{Th} = 30/s - 30/(5s+1) = (150s+30-30s)/(s(5s+1))$
$= 30(4s+1)/(s(5s+1)) = \mathbf{24(s+0.25)/(s(s+0.2))}$



$I_x = (30/s)/1 = 30/s$ $I_{sc} = 30/s + 4(30/s)/(1/s) = 30/s + 120 = (120s+30)/s = 120(s+0.25)/s$

$Z_{Th} = V_{Th}/I_{sc} = \{24(s+0.25)/(s(s+0.2))\}/\{120(s+0.25)/s\} = \mathbf{1/(5(s+0.2))}$



$$V_o = \frac{\frac{24(s+0.25)}{s(s+0.2)}}{\frac{1}{5(s+0.2)}+2}2 = \frac{24(s+0.25)}{s(0.2+2s+0.4)}2 = \mathbf{\frac{24(s+0.25)}{s(s+0.3)}} \;\; \boldsymbol{or} \;\; \mathbf{\frac{60(4s+1)}{s(10s+3)}}$$

(b) Initial value: $v_o(0^+) = \lim_{s\to\infty} sV_o = \mathbf{24\ V}$

Final value: $v_o(\infty) = \lim_{s\to 0} sV_o = 24(0+0.25)/(0+0.3) = \mathbf{20\ V}$

(c) Partial fraction expansion leads to $V_o = 20/s + 4/(s+0.3)$

Taking the inverse Laplace transform we get,

$$v_o(t) = \mathbf{(20 + 4e^{-0.3t})u(t)V}$$

P.P.16.7 If $x(t) = 10e^{-3t}\,u(t)$, then $X(s) = \dfrac{10}{s+3}$.

$$Y(s) = H(s)\,X(s) = \frac{20s}{(s+3)(s+6)} = \frac{A}{s+3} + \frac{B}{s+6}$$

$$A = Y(s)\,(s+3)\big|_{s=-3} = -20$$
$$B = Y(s)\,(s+6)\big|_{s=-6} = 40$$

$$Y(s) = \frac{-20}{s+3} + \frac{40}{s+6}$$

$$y(t) = \mathbf{(-20e^{-3t} + 40\,e^{-6t})u(t)}$$

$$H(s) = \frac{2s}{(s+6)} = \frac{2(s+6-6)}{s+6} = 2 - \frac{12}{s+6}$$

$$h(t) = \mathbf{2\delta(t) - 12e^{-6t}\,u(t)}$$

P.P.16.8 By current division,

$$I_1 = \frac{2 + 1/2s}{s + 4 + 2 + 1/2s} I_0$$

$$H(s) = \frac{I_1}{I_0} = \frac{2 + 1/2s}{s + 4 + 2 + 1/2s} = \mathbf{\frac{4s+1}{2s^2 + 12s + 1}}$$

P.P.16.9

(a) $$\frac{V_o}{V_i} = \frac{1 \,||\, 2/s}{1 + 1 \,||\, 2/s} = \frac{\dfrac{2/s}{1+2/s}}{1 + \dfrac{2/s}{1+2/s}} = \frac{2}{s+4}$$

$$H(s) = \frac{V_o}{V_i} = \frac{\mathbf{2}}{\mathbf{s+4}}$$

(b) $h(t) = \mathbf{2e^{-4t}\,u(t)}$

(c) $$V_o(s) = H(s)\,V_i(s) = \frac{2}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

$$A = s\,V_o(s)\Big|_{s=0} = \frac{1}{2}, \qquad B = (s+4)\,V_o(s)\Big|_{s=-4} = \frac{-1}{2}$$

$$V_o(s) = \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+4}\right)$$

$$v_o(t) = \mathbf{\frac{1}{2}(1-e^{-4t})\,u(t)\ V}$$

(d) $$v_i(t) = 8\cos(2t) \longrightarrow V_i(s) = \frac{8s}{s^2+4}$$

$$V_o(s) = H(s)\,V_i(s) = \frac{16s}{(s+4)(s^2+4)} = \frac{A}{(s+4)} + \frac{Bs+C}{(s^2+4)}$$

$$A = (s+4)\,V_o(s)\Big|_{s=-4} = \frac{-16}{5}$$

Multiplying both sides by $(s+4)(s^2+4)$ gives

$$16s = A(s+4) + B(s^2+4s) + C(s+4)$$

Equating coefficients :

$$s^2: \quad 0 = A + B \longrightarrow B = -A = \frac{16}{5} \qquad (1)$$

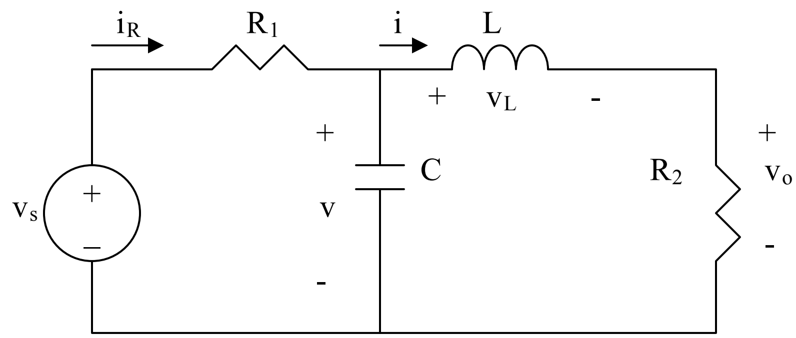
$$s^1: \quad 16 = 4B + C \longrightarrow C = \frac{16}{5} \qquad (2)$$

$$s^0: \quad 0 = 4A + 4C \longrightarrow C = -A \qquad (3)$$

$$V_o(s) = \frac{16}{5}\left(\frac{-1}{s+4} + \frac{s+1}{s^2+4}\right) = \frac{16}{5}\left(\frac{-1}{s+4} + \frac{s}{s^2+4} + \frac{1}{2}\cdot\frac{2}{s^2+4}\right)$$

$$v_o(t) = \mathbf{3.2\left[-e^{-4t} + \cos(2t) + 0.5\sin(2t)\right]\,u(t)\ V}$$

P.P. 16.10 Consider the circuit below.



$$i_R = i + C\frac{dv}{dt}$$

$$v_o = R_2 i \qquad (1)$$

But $\quad i_R = \dfrac{v_s - v}{R_1}$

Hence,

$$\frac{v_s - v}{R_1} = i + C\frac{dv}{dt}$$

or

$$\dot{v} = -\frac{v}{R_1 C} - \frac{i}{C} + \frac{v_s}{R_1 C} \qquad (2)$$

Also,

$$-\ v + v_L + v_o = 0$$

$$v_L = L\frac{di}{dt} = v - v_o$$

But $v_o = iR_2$. Hence

$$\dot{i} = v/L - v_o/L = \frac{v}{L} - \frac{iR_2}{L} \qquad (3)$$

Putting (1) to (3) into the standard form

$$\begin{bmatrix} \dot{v} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1C} & -\frac{1}{C} \\ \frac{I}{L} & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1C} \\ 0 \end{bmatrix} v_s$$

$$v_o = \begin{bmatrix} 0 & R_2 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix}$$

If we let $R_1 = 1$, $R_2 = 2$, $C = ½$, $L = 1/5$, then

$$A = \begin{bmatrix} -2 & -2 \\ 5 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s+2 & 2 \\ -5 & s+10 \end{bmatrix}$$

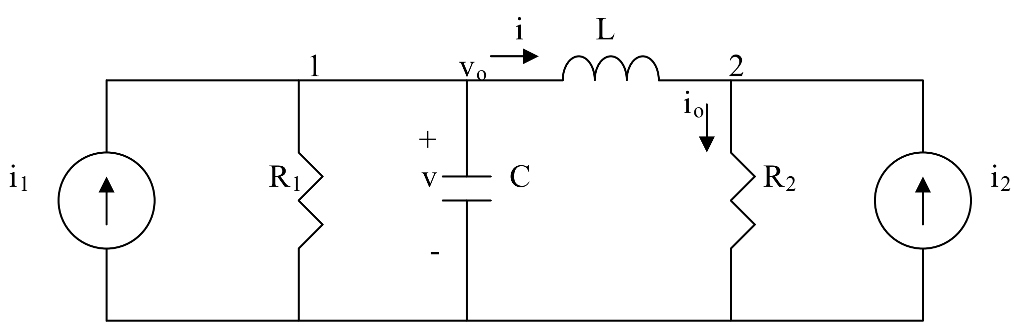
$$(sI - A)^{-1} = \frac{\begin{bmatrix} s+10 & -2 \\ 5 & s+2 \end{bmatrix}}{s^2 + 12s + 30}$$

$$\mathrm{H(s) = C(sI - A)^{-1}B} = \frac{\begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} \mathrm{s+10} & -2 \\ 5 & \mathrm{s+2} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}}{\mathrm{s^2 + 12s + 30}}$$

$$= \frac{20}{\mathrm{s^2 12s + 30}}$$

$$= \mathbf{\frac{20}{s^2 + 12s + 30}}$$

P.P. 16.11 Consider the circuit below.



At node 1,

$$i_1 = \frac{v}{R_1} + C\dot{v} + i$$

or $$\dot{v} = -\frac{1}{R_1 C}v - \frac{1}{C}i + \frac{i_1}{C} \qquad (1)$$

This is one state equation.
At node 2,

$$i_o = i + i_2 \qquad (2)$$

Applying KVL around the loop containing C, L, and R_2, we get

$$-v + L\dot{i} + i_o R_2 = 0$$

or

$$\dot{i} = \frac{v}{L} - \frac{R_2}{L}i_o \qquad (3)$$

Substituting (2) into (3) gives

$$\dot{i} = \frac{v}{L} - \frac{R_2}{L}i - \frac{R_2}{L}i_2 \qquad (4)$$

$$v_o = v \qquad (5)$$

From (1), (3), (4), and (5), we obtain the state model as

$$\begin{bmatrix} \dot{v} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1 C} & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_o \\ i_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

Substituting $R_1 = 1$, $R_2 = 2$, $C = ½$, $L = ¼$ yields

$$\begin{bmatrix} \dot{v} \\ \dot{i} \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 4 & -8 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & -8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_o \\ i_o \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ i \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

P.P. 16.12

Let $x_1 = y \qquad (1)$
so that

$$\dot{x}_1 = \dot{y} \qquad (2)$$

Let
$$x_2 = \dot{x}_1 = \dot{y} \qquad (3)$$

Finally, let
$$x_3 = \dot{x}_2 = \ddot{y} \qquad (4)$$

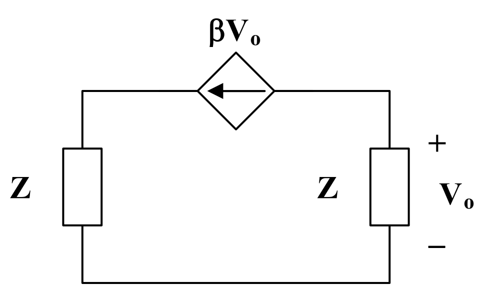
then
$$\dot{x}_3 = \dddot{y} = \quad 10\ddot{y} \quad 20\dot{y} \quad 5y + z \qquad (5)$$

From (1) to (5), we obtain,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -20 & -18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} z(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

P.P.16.13 The circuit in the s-domain is equivalent to the one shown below.



$$-V_o = (\beta V_o)\,Z \longrightarrow -1 = \beta Z, \qquad \text{where}$$

$$Z = R \,\|\, 1/sC = \frac{R}{1 + sRC}$$

Thus, $-1 = \dfrac{\beta R}{1 + sRC}$ or $-(1 + sRC) = \beta R$

For stability,

$$\beta R > -1 \qquad \text{or} \qquad \beta > \frac{\mathbf{-1}}{\mathbf{R}}$$

From another viewpoint,

$$V_o = -(\beta V_o)\,Z \longrightarrow (1 + \beta Z)\,V_o = 0$$

$$\left(1+\frac{\beta R}{1+sRC}\right)V_o = 0$$

$$(sRC+\beta R+1)\,V_o = 0$$

$$\left(s+\frac{\beta R+1}{RC}\right)V_o = 0$$

For stability $\frac{\beta R+1}{RC}$ must be positive, i.e.

$$\beta R+1>0 \qquad \text{or} \qquad \boldsymbol{\beta > \frac{-1}{R}}$$

P.P.16.14

(a) Following Example 15.24, the circuit is stable when

$$25+\alpha>0 \qquad \text{or} \qquad \boldsymbol{\alpha > -25}$$

(b) For oscillation,

$$25+\alpha=0 \qquad \text{or} \qquad \boldsymbol{\alpha = -25}$$

P.P.16.15

$$\frac{V_o}{V_i}=\frac{R}{R+sL+\frac{1}{sC}}=\frac{s\cdot\frac{R}{L}}{s^2+s\cdot\frac{R}{L}+\frac{1}{LC}}$$

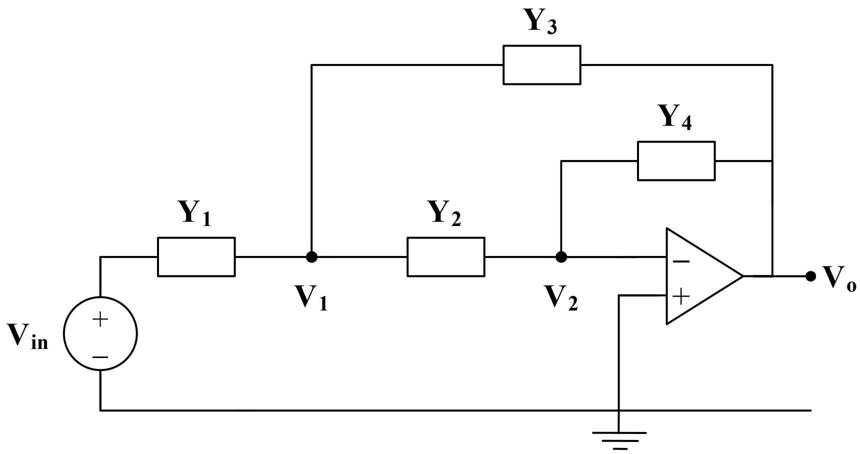
Comparing this with the given transfer function,

$$\frac{R}{L}=4 \qquad \text{and} \qquad \frac{1}{LC}=20$$

If we select $R=2$, then

$$L=\frac{2}{4}=\mathbf{500}\ \boldsymbol{mH} \qquad \text{and} \qquad C=\frac{1}{20L}=\frac{1}{10}=\mathbf{100}\ \boldsymbol{mF}$$

P.P.16.16 Consider the circuit shown below.



Clearly, $V_2 = 0$

At node 1,

$$(V_{in} - V_1)\,Y_1 = (V_1 - V_o)\,Y_3 + (V_1 - 0)\,Y_2$$

or $$V_{in}\,Y_1 = V_1\,(Y_1 + Y_2 + Y_3) - V_o\,Y_3 \qquad (1)$$

At node 2,

$$(V_1 - 0)\,Y_2 = (0 - V_o)\,Y_4$$

or $$V_1 = \frac{-Y_4}{Y_2}\,V_o \qquad (2)$$

Substituting (2) into (1),

$$V_{in}\,Y_1 = \frac{-Y_4}{Y_2}\,V_o\,(Y_1 + Y_2 + Y_3) - V_o\,Y_3$$

or $$\frac{V_o}{V_{in}} = \frac{-Y_1 Y_2}{Y_4\,(Y_1 + Y_2 + Y_3) + Y_2 Y_3}$$

If we select $Y_1 = \frac{1}{R_1}$, $Y_2 = sC_1$, $Y_3 = sC_2$, and $Y_4 = \frac{1}{R_2}$, then

$$\frac{V_o}{V_{in}} = \frac{-s \cdot \dfrac{C_1}{R_1}}{\dfrac{1}{R_2}\left(\dfrac{1}{R_1} + sC_1 + sC_2\right) + s^2 C_1 C_2}$$

$$\frac{V_o}{V_{in}} = \frac{-s \cdot \frac{1}{R_1 C_2}}{s^2 + s \cdot \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

Comparing this with the given transfer function shows that

$$\frac{1}{R_1 C_2} = 2, \qquad \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 6, \qquad \frac{1}{R_1 R_2 C_1 C_2} = 10$$

If $R_1 = 10\ \text{k}\Omega$, then

$$C_2 = \frac{1}{2 \times 10^3} = 0.5\ \text{mF}$$

$$\frac{1}{R_2 C_1} = 5 \longrightarrow \frac{1}{R_2} = 5C_1$$

$$\frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 6 \longrightarrow 5\left(1 + \frac{C_1}{C_2}\right) = 6 \longrightarrow C_1 = \frac{C_2}{5} = 0.1\ \text{mF}$$

$$R_2 = \frac{1}{5C_1} = \frac{1}{(5)(0.1 \times 10^{-3})} = 2\ \text{k}\Omega$$

Therefore,

C_1 = **100 μF**, C_2 = **500 μF**, R_2 = **2 kΩ**.

CHAPTER 17

P.P.17.1 $T = 2,\ \omega_o = 2\pi/T = \pi$

$$f(t) = \begin{cases} 1, & 0 < t < 1 \\ -1, & 1 < t < 2 \end{cases}$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\left[\int_0^1 (1)dt + \int_1^2 (-1)dt\right] = 0.5(1 - 1) = 0$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o dt = \frac{2}{2}\left[\int_0^1 1\cos n\pi t dt + \int_1^2 (-1)\cos n\pi t dt\right]$$

$$= \frac{1}{n\pi}[\sin n\pi t]_0^1 - \frac{1}{n\pi}[\sin n\pi t]_1^2 = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o dt = \frac{2}{2}\left[\int_0^1 1\sin n\pi t dt + \int_1^2 (-1)\sin n\pi t dt\right]$$

$$= \frac{-1}{n\pi}[\cos n\pi t]_0^1 + \frac{1}{n\pi}[\cos n\pi t]_1^2 = \frac{2}{n\pi}[1 - \cos n\pi]$$

$$b_n = \begin{cases} 4/(n\pi), & \text{for } n = \text{odd} \\ 0, & \text{for } n = \text{even} \end{cases}$$

$$f(t) = \boldsymbol{\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin n\pi t, \qquad n = 2k - 1}$$

See Fig. 17.6 for the spectra.

P.P.17.2 $T = 1,\ \omega_o = 2\pi/T = 2\pi,\ f(t) = 6t,\ 0 < t < 1.$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \left[\int_0^1 (6t)dt\right] = \frac{3t^2}{1}\Big|_0^1 = 3$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o dt = \frac{2}{1}\left[\int_0^1 6t\cos n\pi t dt\right]$$

$$= 2\left[\frac{6}{(2n\pi)^2}[\cos 2n\pi t] + \frac{6t}{2n\pi}[\sin 2n\pi t]\right]_0^1$$

$$= \frac{12}{4n^2\pi^2}[[\cos 2n\pi 1] - 1] = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o dt = \frac{2}{1}\left[\int_0^1 6t\sin 2n\pi t dt\right]$$

$$= 2\left[\frac{6}{4n^2\pi^2}[\sin 2n\pi t] - \frac{6t}{2n\pi}[\cos 2n\pi t]\right]_0^1 = \frac{-12}{2n\pi}[\cos 2n\pi] = -6/(n\pi)$$

$$f(t) = \mathbf{3 - \frac{6}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin 2n\pi t}$$

P.P.17.3

$$f(t) = \begin{cases} 8, & -\pi < t < 0 \\ -8, & 0 < t < \pi \end{cases}$$

f(t) is an odd function, $a_o = 0 = a_n$

$T = 2\pi$, $\omega_o = 2\pi/T = 1$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_o dt = \frac{4}{2\pi}\left[\int_0^{\pi}(-8)\sin ntdt\right]$$

$$= \left[\frac{16}{n\pi}[\cos nt]\right]_0^{\pi} = \frac{16}{n\pi}[\cos n\pi - 1]$$

$$= \begin{cases} \mathbf{-32/(n\pi)}, & \mathbf{n = odd} \\ \mathbf{0}, & \mathbf{n = even} \end{cases}$$

$$f(t) = \mathbf{\frac{-32}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin nt}, \quad \mathbf{n = 2k - 1}$$

P.P.17.4 $f(t) = 8t/\pi, \; 0 < t < \pi, \; T = 2\pi, \; \omega_o = 1$

This is an even function, $b_n = 0$.

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{2\pi}\left[\int_0^{\pi} (8t/\pi)dt\right] = \left(\frac{1}{\pi^2}\right)\left(\frac{8t^2}{2}\right)\Big|_0^{\pi} = \mathbf{4}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{4}{2\pi}\left[\int_0^{\pi} \frac{8t}{\pi}\cos ntdt\right]$$

$$= \frac{2}{\pi^2}\left[\left[(8t/n)\sin nt\right]_0^{\pi} - \frac{8}{n}\int_0^{\pi}\sin ntdt\right]$$

$$= \frac{(-2)}{n\pi^2}\frac{(-8)}{n}\cos nt\Big|_0^{\pi} = \frac{16}{n^2\pi^2}(\cos n\pi - 1)$$

$$= \begin{cases} \mathbf{-32/(n^2\pi^2),} & \mathbf{n = odd} \\ \mathbf{0,} & \mathbf{n = even} \end{cases}$$

$$f(t) = \mathbf{4 - \frac{32}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{n^2}\cos nt}, \qquad \mathbf{n = 2k - 1}$$

P.P.17.5 $f(t) = 5t/\pi, \; 0 < t < \pi, \; \omega_o = 2\pi/T = 1$

This is half-wave symmetric. For odd n,

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{4}{2\pi}\left[\int_0^{\pi} \frac{5t}{\pi}\cos ntdt\right]$$

$$= \frac{2}{\pi^2}\left[\left[(5t/n)\sin nt\right]_0^{\pi} - \frac{1}{n}\int_0^{\pi}5\sin ntdt\right]$$

$$= \frac{(-2)}{n\pi^2}\frac{(-5)}{n}\cos nt\Big|_0^{\pi} = \frac{10}{n^2\pi^2}(\cos n\pi - 1)$$

$$= \begin{cases} \mathbf{-20/(n^2\pi^2),} & \mathbf{n = odd} \\ \mathbf{0,} & \mathbf{n = even} \end{cases}$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_o dt = \frac{4}{2\pi}\left[\int_0^{\pi}\frac{5t}{\pi}\sin ntdt\right]$$

$$= \left[\frac{10}{n^2\pi^2}[\sin nt - nt\cos nt]\right]_0^{\pi} = \frac{10}{n\pi}, \qquad n = \text{odd}$$

Thus, $$f(t) = \boldsymbol{\frac{10}{\pi}\sum_{k=1}^{\infty}\left(\frac{-2}{n^2\pi}\cos nt + \frac{1}{n}\sin nt\right)}, \qquad \mathbf{n = 2k - 1}$$

P.P.17.6

$$v_s(t) = 1.5 - (3/\pi)\sum_{n=1}^{\infty}\frac{1}{n}\sin 2\pi nt, \quad \omega = 2\pi n$$

$v_o(\omega) = (1/(j\omega C))v_s/(R + (1/j\omega C)) = v_s/(1 + j\omega RC) = v_s/(1 + j2\omega)$, $RC = 2$

For the DC component ($\omega = 0$, or $n = 0$), $v_s = 1.5$ and $v_o = 1.5$

For the nth harmonic, $v_s = -(3/(n\pi))$ or

$$v_o = -(3/(n\pi))/\sqrt{1+4\omega^2}\ \angle\tan^{-1}2\omega$$

$$\text{or} \quad v_o = -3\angle(-\tan^{-1}2\omega)/(n\pi\sqrt{1+4\omega^2})$$

Hence, $$v_o(t) = 1.5 - (3/\pi)\sum_{n=1}^{\infty}\frac{1}{n\sqrt{1+4\omega^2}}\sin(2\pi nt - \tan^{-}1(2\omega))$$

$$= \boldsymbol{\frac{3}{2} - \frac{3}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2\pi nt - \tan^{-1}4\pi n)}{n\sqrt{1+16\pi^2 n^2}}\,V}$$

P.P.17.7

$$v(t) = (1/3) + (1/\pi^2)\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\cos nt - \frac{\pi}{n}\sin nt\right)$$

$$= (1/3) + (1/\pi^2)\sum_{n=1}^{\infty}A_n\cos(nt - \phi_n)$$

where $$A_n = \frac{1}{n}\sqrt{\frac{1}{n^2}+\pi^2} = \frac{1}{n^2}\sqrt{1+n^2\pi^2}$$

$$\phi_n = \tan^{-1}(b_n/a_n) = \tan^{-1}(-n\pi)$$

$$v(t) = (1/3) + \frac{1}{\pi^2}\sum\frac{1}{n^2}\sqrt{1+n^2\pi^2}\cos[nt - \tan^{-1}(-n\pi)]$$

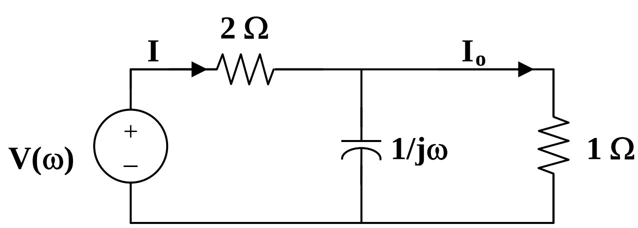
$$Z = 2 + 1\|(1/(j\omega)), \quad \omega = n$$

$$= 2 + (1/(j\omega))/(1 + (1/(j\omega))) = 2 + (1/(1 + (j\omega))$$

$$= (3 + 2j\omega)/(1 + j\omega), \quad \omega = n$$

$$= (3 + j2n)/(1 + jn)$$

$$I = V/Z = [(1 + jn)/(3 + j2n)]\ V$$



By current division, $I_o = (1/j\omega)I/[1 + (1/j\omega)] = I/(1 + j\omega) = V/(3 + j2n)$

For the DC component ($n = 0$), $V = 1/3$ and $I_o = V/3 = 1/9$

For the nth harmonic, $V = [1/(n^2\pi^2)]\sqrt{1+n^2\pi^2}\ \angle -\tan^{-1}(-n\pi)$

$$I_o = V/[\sqrt{9+4n^2}\ \angle -\tan^{-1}(2n/3)]$$

$$= \sqrt{1+n^2\pi^2}\ \angle[-\tan^{-1}(-n\pi)-\tan^{-1}(2n/3)]/[n^2\pi^2\sqrt{9+4n^2}\ ]$$

But, $\tan^{-1}(-n\pi) = -\tan^{-1}(n\pi)$

In the time domain,

$$i_o(t) = \mathbf{\{(1/9) + \sum_{n=1}^{\infty}\frac{\sqrt{1+n^2\pi^2}}{n^2\pi^2\sqrt{9+4n^2}}\cos[nt - \tan^{-1}(2n/3) + \tan^{-1}(n\pi)]\}A}$$

P.P.17.8 $P = V_{DC}\, I_{DC} + 0.5\sum_{n=0}^{\infty} V_n\, I_n \cos(\phi_n - \theta_n)$

$= 128(0) + 0.5(192)(4)\cos(10°) + 0.5(96)(1.6)\cos(30°)$

$= 378.2 + 66.51$

$=$ **444.7 watts**

P.P.17.9 $I^2_{rms} = 8^2 + 0.5[30^2 + 20^2 + 15^2 + 10^2]$

$= 64 + 0.5x1625 = 876.5$

$=$ **29.61 A**

P.P.17.10 $f(t) = 1, \quad 0 < t < 1$

$= 0, \quad 1 < t < 2$

$T = 2, \quad \omega_o = 2\pi/T = \pi$

$C_n = (1/T)\int_0^T f(t)e^{-jn\omega_o t}dt = 0.5[\int_0^1 1e^{-jn\pi t}dt + 0]$

$= 0.5[1/(-jn\pi)]e^{-jn\pi t}\Big|_0^1 = [j/(2n\pi)](e^{-jn\pi} - 1)$

But $e^{-jn\pi} = \cos(n\pi) - j\sin(n\pi) = \cos(n\pi) = (-1)^n$

$C_n = [j/(2\pi n)][(-1)^n - 1] \quad = 0, \quad n = \text{even}$

$= [-j/(n\pi)], \quad n = \text{odd} \neq 0$

For $n = 0$, $C_o = 0.5\int_0^1 1dt = 0.5$

Hence, $$\mathbf{f(t) = \frac{1}{2} - \sum_{\substack{n=-\infty \\ n\neq 0 \\ n=odd}}^{\infty} \frac{j}{n\pi} e^{jn\pi t}}$$

P.P.17.11 $f(t) = t, \ -1 < t < 1, \ T = 2, \ \omega_o = 2\pi/T = \pi$

$$C_n = (1/T)\int_{-T/2}^{T/2} f(t)e^{-jn\omega_o t}dt = 0.5\int_{-1}^{1} te^{-jn\pi t}dt$$

$$= 0.5[e^{-jn\pi t}/(-jn\pi)^2](-jn\pi t - 1)\Big|_{-1}^{1}$$

$$= [-1/(2n^2\pi^2)][e^{-jn\pi}(-jn\pi - 1) - e^{jn\pi}(jn\pi - 1)]$$

$$= [-1/(2n^2\pi^2)][(\cos n\pi - j\sin n\pi)(-jn\pi - 1) - (\cos n\pi + j\sin n\pi)(jn\pi - 1)]$$

$$= [j\cos n\pi]/(n\pi)$$

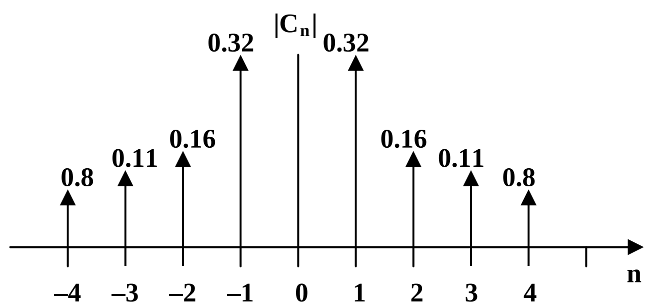
$$C_n = j(-1)^n/n\pi, \quad n \neq 0$$

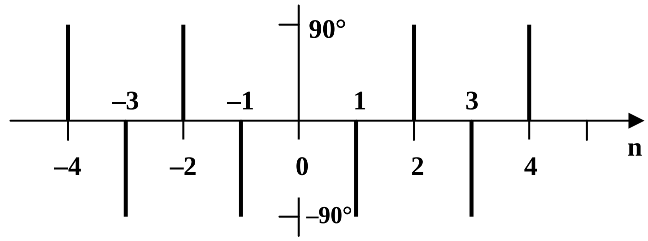
For $n = 0$, $$C_o = (1/T)\int_{-T/2}^{T/2} f(t)dt = 0$$

Thus, $$f(t) = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j(-1)^n}{n\pi} e^{jn\pi t}$$

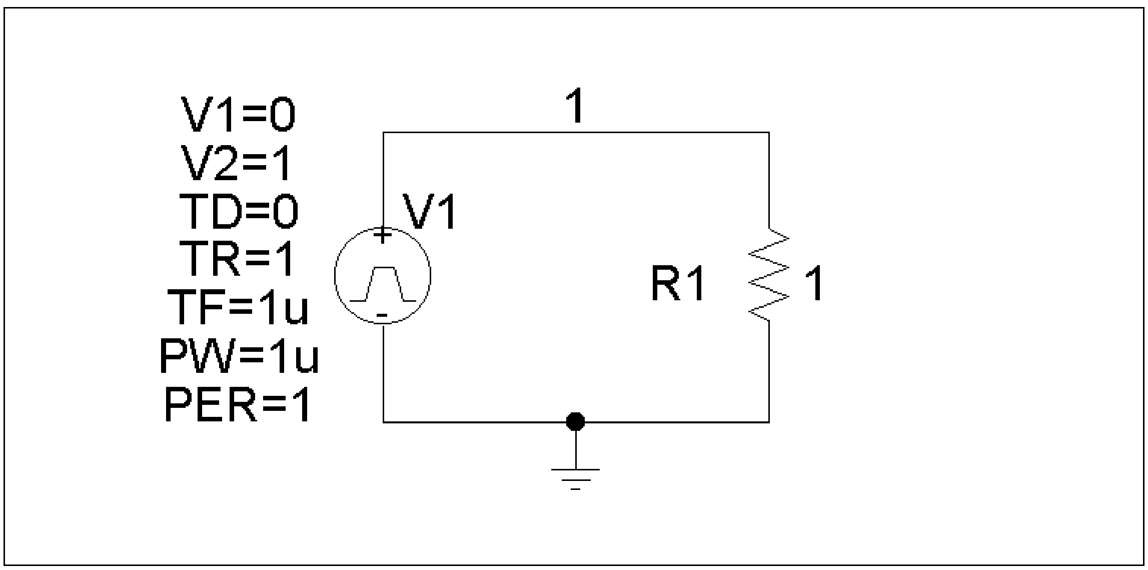
$$|C_n| = 1/(n\pi), \quad n \neq 0, \quad \theta_n = (-1)^n 90°, \ n \neq 0$$

The amplitude and phase spectra are shown below.





P.P.17.12 The schematic is shown below. The attributes of the voltage source is entered as shown. After entering the final time (5 or 6T), the Print Step, the Step Ceiling, and the Center Frequency in the transient dialog box, the circuit is saved. Once the PSpice is run, the output contains the following Fourier coefficients.



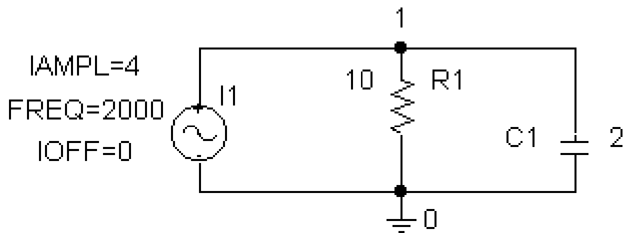
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = **4.950000E-01**

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	**1.000E+00**	**3.184E-01**	**1.000E+00**	**-1.782E+02**	**0.000E+00**
2	**2.000E+00**	**1.593E-01**	**5.002E-01**	**-1.764E+02**	**1.800E+00**
3	**3.000E+00**	**1.063E-01**	**3.338E-01**	**-1.746E+02**	**3.600E+00**
4	**4.000E+00**	**7.979E-02**	**2.506E-01**	**-1.728E+02**	**5.400E+00**
5	**5.000E+00**	**6.392E-02**	**2.008E-01**	**-1.710E+02**	**7.200E+00**
6	**6.000E+00**	**5.337E-02**	**1.676E-01**	**-1.692E+02**	**9.000E+00**
7	**7.000E+00**	**4.584E-02**	**1.440E-01**	**-1.674E+02**	**1.080E+01**
8	**8.000E+00**	**4.021E-02**	**1.263E-01**	**-1.656E+02**	**1.260E+01**
9	**9.000E+00**	**3.584E-02**	**1.126E-01**	**-1.638E+02**	**1.440E+01**

TOTAL HARMONIC DISTORTION = 7.363360E+01 PERCENT

P.P.17.13 The schematic is shown below. Since T = 1/f = 0.55 ms, in the transient dialog box, we set Print Step = 0.01 ms, Final Time = 4 ms, Center Frequency = 2,000 Hz, Number of Harmonics = 5, and Output Vars = V(R1:1). Once the circuit is saved, we simulate it and obtain the following results.



DC COMPONENT = -1.507149E-04

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	2.000E+03	1.455E-04	1.000E+00	9.006E+01	0.000E+00
2	4.000E+03	1.851E-06	1.273E-02	9.597E+01	5.910E+00
3	6.000E+03	1.406E-06	9.662E-03	9.323E+01	3.167E+00
4	8.000E+03	1.010E-06	6.946E-03	8.077E+01	-9.292E+00

TOTAL HARMONIC DISTORTION = 1.830344E+00 PERCENT

From this, we use the amplitude and phase of the Fourier components to get

$$v(t) = \mathbf{\{-150.72 + 145.5\sin(4\pi 10^3 t + 90°) + 1.845\sin(8\pi 10^3 t + 96.24°) + \cdots\}\mu V}$$

P.P.17.14 From Example 16.14,

$$\begin{aligned}
2\omega_o &= 4\pi = 12.566 \text{ rad/s} \\
3\omega_o &= 6\pi = 18.84 \text{ rad/s} \\
4\omega_o &= 8\pi = 25.13 \text{ rad/s} \\
5\omega_o &= 10\pi = 31.41 \text{ rad/s} \\
6\omega_o &= 12\pi = 37.7 \text{ rad/s}
\end{aligned}$$

Since the ideal bandpass filter passes only $15 < \omega < 25$, it means that only the 3rd, 4th, and 5th harmonics will be passed. Hence,

$$y(t) = \mathbf{(-1/3\pi)\sin(3\omega_o t) - (1/(4\pi))\sin(4\omega_o t) - (/(5\pi))\sin(5\omega_o t), \quad \omega_o = 2\pi}$$

CHAPTER 18

P.P.18.1 (a) $g(t) = 4u(t+1) - 4u(t-2) = \begin{cases} 4, & 1 < t < 2 \\ 0, & \text{otherwise} \end{cases}$

$$G(\omega) = \int_1^2 4 \cdot e^{-j\omega t} dt = -\frac{4}{j\omega} e^{-j\omega t} \Big|_1^2$$

$$= \mathbf{\frac{4(e^{-j\omega} - e^{-j2\omega})}{j\omega}}$$

(b) $F(t) = 4\delta(t+2)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt = \int_{-\infty}^{\infty} 4\delta(t+2)e^{-j\omega t} dt$$

$$= 4e^{j\omega t}\Big|_{t=2} = \mathbf{4e^{j2\omega}}$$

(c) $F(t) = 10\sin(\omega_o t)$

$$F(\omega) = F\left[\frac{-10e^{j\omega_o t}}{2j}\right] = \frac{10}{j2}\left[F\left(e^{j\omega_o t}\right) - F\left(e^{-j\omega_o}\right)\right]$$

$$= \mathbf{-j10\pi\left[\delta(\omega - \omega_o) - \delta(\omega + \omega_o)\right]} \text{ or } \mathbf{j10\pi[\delta(\omega+\omega_o) - \delta(\omega - \omega_o)]}$$

P.P.18.2 $$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} = \int_{-1}^{0} 10e^{-j\omega t} dt + \int_0^1 (-10)e^{-j\omega t} dt$$

$$= \frac{10e^{-j\omega t}}{-j\omega}\Bigg|_{-1}^{0} - \frac{10e^{-j\omega t}}{-j\omega}\Bigg|_0^1 = \frac{j10}{\omega}\left[1 - e^{j\omega} - e^{-j\omega} + 1\right]$$

$$= \mathbf{\frac{20(\cos\omega - 1)}{j\omega}}$$

P.P.18.3 $$f(t) = \begin{vmatrix} 10e^{at}, & t < 0 \\ 0, & t > 0 \end{vmatrix}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{j\omega t} dt = \int_{-\infty}^{\infty} 10e^{at}e^{-j\omega t} dt$$

Let x = -t, then dt = -dx

$$F(\omega) = \int_{\infty}^{0} 10e^{-ax}e^{j\omega x}(-dx) = -10\int_{\infty}^{0} e^{-(a-j\omega)x} dx$$

$$= \frac{10}{a - j\omega} e^{a-j\omega)x} = \mathbf{\frac{10}{a - j\omega}}$$

P.P.18.4 (a) $g(t) = u(t) - u(t - 1)$

$F(\omega) = u(\omega) - e^{-j\omega}u(\omega) = (1 - e^{-j\omega})u(\omega)$

$= \mathbf{(1 - e^{-j\omega})(\pi\delta(\omega) + 1/(j\omega))}$

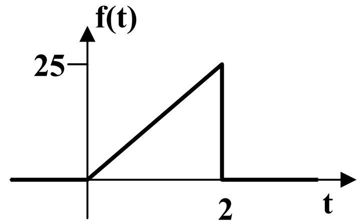
(b) $f(t) = te^{-2t}u(t)$

Let $g(t) = e^{-2t}u(t) \longrightarrow G(\omega) = 1/(2 + j\omega)$

$f(t) = tg(t) \longrightarrow j\dfrac{dG}{d\omega} = j(-1)\,(2 + j\omega)^{-2}(j)$

$$F(\omega) = \mathbf{\frac{1}{(2 + j\omega)^2}}$$

(c) f(t) is sketched below.



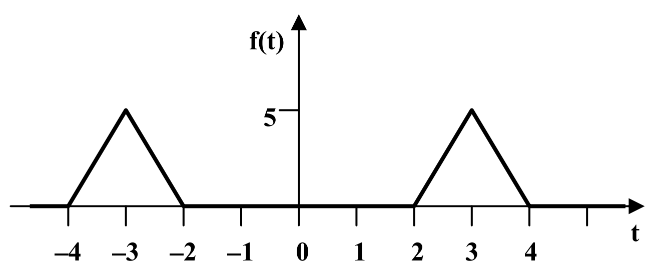
$f'(t) = -50\delta(t-2) - 100\delta(t-2)$

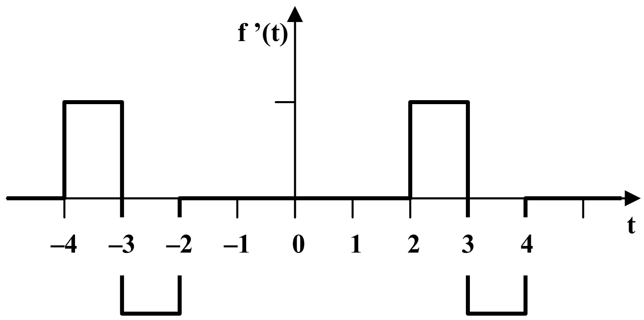
$f''(t) = 50\delta(t) - 50\delta(t - 2) - 100\delta'(t - 2)$

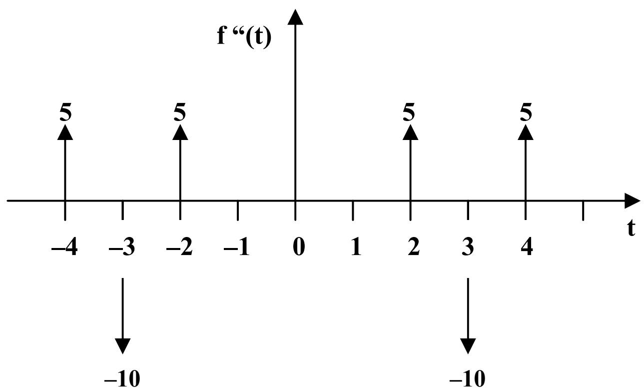
$(j\omega)^2F(\omega) = 50(1 - e^{-j\omega 2}) - 100j\omega e^{-j\omega 2}$

$$F(\omega) = \frac{50\left(e^{-j\omega 2} - 1\right)}{\omega^2} + \frac{100je^{-j\omega 2}}{\omega}$$

P.P.18.5 Given f(t), f '(t) and f "(t) are sketched below:







f "(t) = $5\delta(t + 4) - 10\delta(t + 3) + 5\delta(t + 2) + 5\delta(t - 2) - 10\delta(t + 3) + 5\delta(t - 4)$

We take the Fourier transform of each term.

$$(j\omega)^2 F(\omega) = 5(e^{j4\omega} + e^{-j4\omega}) - 10(e^{j3\omega} + e^{-j3\omega}) + 5(e^{j2\omega} + e^{-j2\omega})$$
$$= 4\cos 4\omega - 8\cos 3\omega + 4\cos 2\omega$$
$F(\omega)$ = $\mathbf{[1/(\omega^2)](20\cos 3\omega - 10\cos 4\omega - 10\cos 2\omega)}$

P.P.18.6 (a) $H(\omega) = \dfrac{6(2j\omega + 3)}{(j\omega+1)(j\omega+4)(j\omega+2)}$

$$= \frac{2}{j\omega+1} + \frac{3}{j\omega+2} - \frac{5}{j\omega+4}$$

$h(t)$ = $\mathbf{(2e^{-t} + 3e^{-2t} - 5e^{-4t})u(t)}$

(b) $y(t) = u(t) + 2e^{-t}\cos 4t\, u(t)$
$= \mathbf{(1 + 2e^{-t}\cos 4t)\, u(t)}$

P.P.18.7 $v_i = 5\,\text{sgn}(t) \longrightarrow V_i(\omega) = 10/(j\omega)$
$H(\omega) = 4/(4 + j\omega)$

$$V_o(\omega) = H(\omega)V_i(\omega) = \frac{40}{j\omega(4+j\omega)} = \frac{A}{j\omega} + \frac{B}{4+j\omega}$$

$$= \frac{10}{j\omega} - \frac{10}{4 + j\omega}$$

$v_o(t) = 5 \text{ sgn } (t) – 10e^{-4t}u(t) = 5[–1 + u(t)] – 10e^{-4t}u(t)$

$= \mathbf{–5 + 10\ [1 – e^{-4t}]u(t)\ V}$

P.P.18.8 $I_s(\omega) = 20\pi[\delta(\omega + 4) + \delta(\omega - 4)]$

$$H(\omega) = \frac{6 + j\omega 2}{10 + 6 + j2\omega} = \frac{3 + j\omega}{8 + j\omega}$$

$$I_0(\omega) = H(\omega)I_s(\omega) = \left(\frac{3 + j\omega}{8 + j\omega}\right)(20\pi)[\delta(\omega + 4) + \delta(\omega - 4)]$$

$$i_o(t) = \mathcal{F}^{-1}I_o(\omega) = \frac{20\pi}{2\pi}\int_{-\infty}^{\infty}\left(\frac{3 + j\omega}{8 + j\omega}\right)[\delta(\omega + 4) + \delta(\omega - 4)e^{j\omega t}d\omega]$$

$$= 10\left[\frac{3 - j4}{8 - j\omega}e^{-j4t} + \frac{3 + j4}{8 + j4}e^{j4t}\right]$$

But

$$\frac{3 + j4}{8 + j4} = \frac{5\angle 53.13^\circ}{\sqrt{80}\angle 26.56^\circ} = 0.559\angle 26.57^\circ$$

$$i_o(t) = 5.59\left(e^{-j(4t + 26.57^\circ)} + e^{j(4t + 26.57^\circ)}\right)$$

$i_o(t) = \mathbf{11.18\ cos\ (4t + 26.57°)\ A}$

P.P.18.9 (a) $W_{1\Omega} = \int_{-\infty}^{\infty} 100e^{-4|t|}\,dt = 200\int_0^{\infty} e^{-4t}\,dt$

since $|t|$ is even.

$$W_{1\Omega} = \frac{200e^{-4t}}{-4}\Big|_0^{\infty} = \mathbf{50\ J}$$

(b) $H(\omega) = \dfrac{40}{4 + \omega^2}$

$$W_{1\Omega} = \frac{1}{\pi}\int_0^{\infty}\frac{1600}{(4 + \omega^2)^2}d\omega = \frac{1600}{\pi}\cdot\frac{1}{8}\left(\frac{\omega}{\omega^2 + 4} + \frac{1}{2}\tan^{-1}\frac{\omega}{2}\right)\Big|_0^{\infty}$$

$$W_{1\Omega} = \frac{200}{\pi}\left(0 + \frac{\pi}{4} - 0 - 0\right) = \mathbf{50\ J}$$

P.P.18.10 $F(\omega) = \dfrac{2}{1 + j\omega} \longrightarrow |F(\omega)|^2 = \dfrac{4}{1 + \omega^2}$

$$W_{2\Omega} = \frac{8}{\pi}\int_0^{\infty}\frac{d\omega}{1 + \omega^2} = \frac{8}{\pi}\tan^{-1}\omega\Big|_0^{\infty} = \frac{8}{\pi}\cdot\frac{\pi}{2} = 4\,J$$

for -4 < ω < 4,

$$W = \frac{8}{\pi}\int_0^4 \frac{d\omega}{1+\omega^2} = \frac{8}{\pi}\tan^{-1}\omega\Big|_0^4 = \frac{8}{\pi}\cdot\frac{76}{180}\pi = 3.378\,\text{J}$$

Percentage energy = (3.378/4)100 = **84.4%** of the total energy.

P.P.18.11 If f_c = 2 MHz, f_m = 4 kHz

upper sideband = 2,000,000 + 4,000 = **2,004,000 Hz**
Carrier = **2,000,000 Hz**
Lower sideband = 2,000,000 -4,000 = **1,996,000 Hz**

P.P.18.12 W = 12.5 kHz, f_s = 2W = 25 kHz

$$T_s = \frac{1}{f_s} = \frac{1}{25x10^3} = \mathbf{40\ \mu s}$$

CHAPTER 19

P.P.19.1 Comparing the network with that in Fig. 19.5, we observe that

$\mathbf{z}_{12} = \mathbf{z}_{21} = \mathbf{3\ \Omega}$
$\mathbf{z}_{22} - \mathbf{z}_{12} = 0$ or $\mathbf{z}_{22} = \mathbf{z}_{12} = \mathbf{3\ \Omega}$
$z_{11} - z_{12} = 4$ or $z_{11} = 4 + 3 = \mathbf{7\ \Omega}$

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{7} & \mathbf{3} \\ \mathbf{3} & \mathbf{3} \end{bmatrix} \Omega$$

P.P.19.2
$$\mathbf{V}_1 = 6\mathbf{I}_1 - j4\mathbf{I}_2 \qquad (1)$$
$$\mathbf{V}_2 = -j4\mathbf{I}_1 + 8\mathbf{I}_2 \qquad (2)$$

But $\mathbf{V}_2 = 0$

and $2\angle 30° = \mathbf{V}_1 + 2\mathbf{I}_1 \longrightarrow \mathbf{V}_1 = 2\angle 30° - 2\mathbf{I}_1$

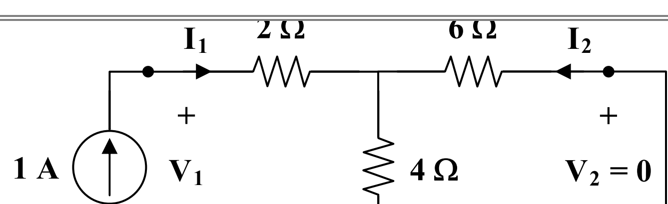
Substituting these into (1) and (2),
$$2\angle 30° - 2\mathbf{I}_1 = 6\mathbf{I}_1 - j4\mathbf{I}_2$$
$$2\angle 30° = 8\mathbf{I}_1 - j4\mathbf{I}_2 \qquad (3)$$

$$0 = -j4\mathbf{I}_1 + 8\mathbf{I}_2$$
$$\mathbf{I}_1 = -j2\mathbf{I}_2 \qquad (4)$$

Substituting (4) into (3),
$$2\angle 30° = -j16\mathbf{I}_2 - j4\mathbf{I}_2 = -j20\mathbf{I}_2$$
$$\mathbf{I}_2 = \frac{2\angle 30°}{20\angle -90°} = \mathbf{100\angle 120^\circ\ mA}$$

$$\mathbf{I}_1 = -j2\mathbf{I}_2 = \mathbf{200\angle 30^\circ\ mA}$$

P.P.19.3 Consider the circuit in Fig. (a) for $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$.



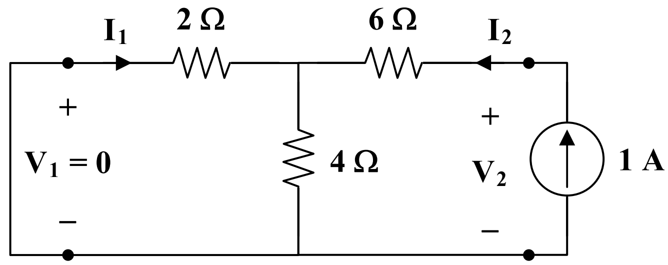
$\mathbf{V_1}$ = [2+(4x6/(4+6))]$\mathbf{I_1}$ = 4.4 $\mathbf{I_1}$
$\mathbf{y_{11}}$ = ($\mathbf{I_1}$/$\mathbf{V_1}$) = 1/4.4 = **0.2273 S**

By current division,

$$\mathbf{I}_2 = \frac{-4}{10}\mathbf{I}_1 = -0.4\mathbf{I}_1$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = -0.4\mathbf{I}_1/(4.4\mathbf{I}_1) = \mathbf{-0.09091\ S}$$

For $\mathbf{y}_{12}$ and $\mathbf{y}_{22}$, consider the circuit in Fig. (b).



(b)

$\mathbf{V_2}$ = [6 +(4x2/(4+2))]$\mathbf{I_2}$ = 7.333$\mathbf{I_2}$

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = 1/7.333 = \mathbf{0.13636\ S}$$

By current division,

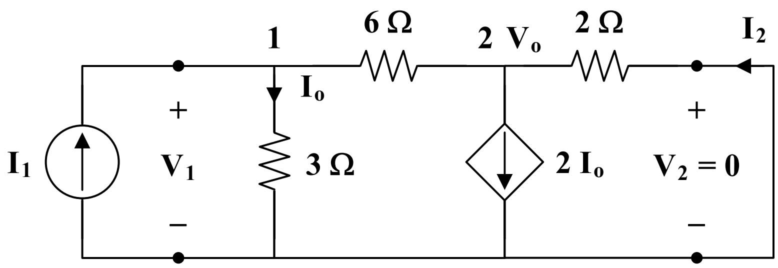
$\mathbf{I_1}$ = [–4/(4+2)]$\mathbf{I_2}$ = –(2/3)$\mathbf{I_2}$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = -(2/3)\mathbf{I}_2/7.333\mathbf{I}_2 = \mathbf{-0.09091\ S}$$

Therefore,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{227.3} & \mathbf{-90.91} \\ \mathbf{-90.91} & \mathbf{136.36} \end{bmatrix} mS$$

P.P.19.4 Consider the circuit in Fig (a).



(a)

At node 1,

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{3} + \frac{\mathbf{V}_1 - \mathbf{V}_o}{6}$$

$$6\mathbf{I}_1 = 3\mathbf{V}_1 - \mathbf{V}_o \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_o}{6} + \mathbf{I}_2 = 2\mathbf{I}_o = \frac{2}{3}\mathbf{V}_1$$

But $\mathbf{I}_2 = \frac{0 - \mathbf{V}_o}{2} = \frac{-\mathbf{V}_o}{2}$

Hence, $\frac{\mathbf{V}_1 - \mathbf{V}_o}{6} - \frac{\mathbf{V}_o}{2} = \frac{2}{3}\mathbf{V}_1 \longrightarrow \mathbf{V}_1 = \frac{-4}{3}\mathbf{V}_o \qquad (2)$

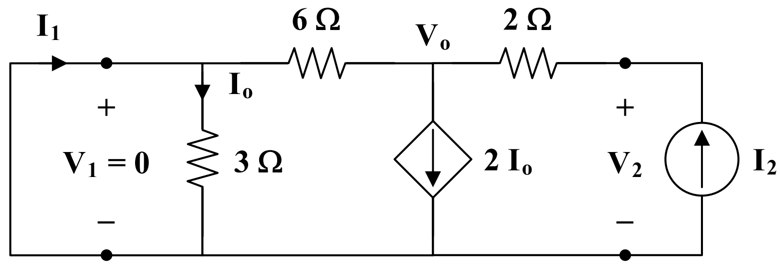
From (1) and (2),

$$6\mathbf{I}_1 = -4\mathbf{V}_o - \mathbf{V}_o = -5\mathbf{V}_o \longrightarrow \mathbf{I}_1 = \frac{-5}{6}\mathbf{V}_o$$

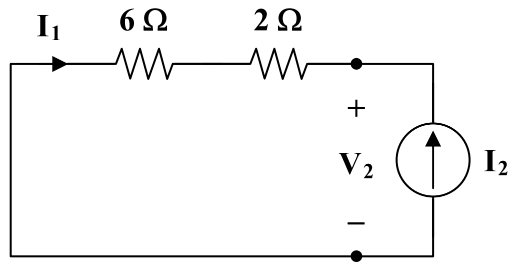
Thus, $\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{(-5/6)\mathbf{V}_o}{(-4/3)\mathbf{V}_o} = \frac{5}{8} = 0.625\ \text{S}$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{(-1/2)\mathbf{V}_o}{(-4/3)\mathbf{V}_o} = \frac{3}{8} = 0.375\ \text{S}$$

Consider the circuit in Fig. (b). The 3-Ω resistor is short-circuited so that $\mathbf{I}_o = 0$. Consequently, the circuit is equivalent to that shown in Fig.(c).



(b)



(c)

$$\mathbf{V}_2 = 8\mathbf{I}_2, \qquad \mathbf{I}_1 = -\mathbf{I}_2$$

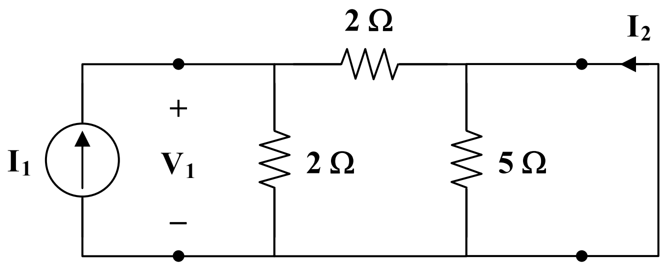
$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{8} = 0.125\ \text{S}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-\mathbf{I}_2}{8\mathbf{I}_2} = -0.125\ \text{S}$$

Therefore,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.625} & \mathbf{-0.125} \\ \mathbf{0.375} & \mathbf{0.125} \end{bmatrix} \mathbf{S} \ \ \mathbf{or} \ \begin{bmatrix} \mathbf{625} & \mathbf{-125} \\ \mathbf{375} & \mathbf{125} \end{bmatrix} \mathbf{mS}$$

P.P.19.5 To find $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, we use the circuit in Fig. (a). Note that the 5-Ω resistor is short-circuited.



(a)

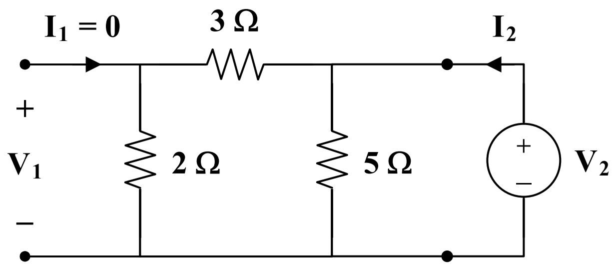
$\mathbf{V_1}$ = [2x3/(2+3)]$\mathbf{I_1}$ = 1.2$\mathbf{I_1}$

$$\mathbf{h_{11}} = \frac{\mathbf{V_1}}{\mathbf{I_1}} = \mathbf{1.2\ \Omega}$$

$$-\mathbf{I}_2 = \frac{2}{2+3}\mathbf{I}_1 = 0.4\mathbf{I}_1 = (4/(4+6))\mathbf{I_1} = 0.4\mathbf{I_1}$$

$$\mathbf{h_{21}} = \frac{\mathbf{I_2}}{\mathbf{I_1}} = \mathbf{-0.4}$$

To get $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, we use the circuit in Fig. (b).



(b)

$\mathbf{V_1}$ = [2/(2+3)]$\mathbf{V_1}$ = 0.4$\mathbf{V_1}$

$$\mathbf{h_{12}} = \frac{\mathbf{V_1}}{\mathbf{V_2}} = \mathbf{0.4}$$

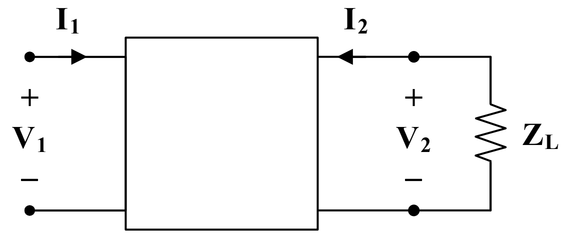
$\mathbf{V_2}$ = [5x5/(5+5)]$\mathbf{I_2}$ = 2.5$\mathbf{I_2}$

$$\mathbf{h}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{2.5} = 0.4\ S$$

Therefore,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{1.2\ \Omega} & \mathbf{0.4} \\ \mathbf{-0.4} & \mathbf{400\ \mathit{mS}} \end{bmatrix}$$

P.P.19.6 Our goal is to get $\mathbf{Z}_{in} = \mathbf{V}_1/\mathbf{I}_1$.



$$\mathbf{V}_1 = \mathbf{h}_{11}\,\mathbf{I}_1 + \mathbf{h}_{12}\,\mathbf{V}_2 \qquad (1)$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\,\mathbf{I}_1 + \mathbf{h}_{22}\,\mathbf{V}_2 \qquad (2)$$

But $\mathbf{V}_2 = \text{-}\mathbf{Z}_L\,\mathbf{I}_2$

Substituting this equation into (1) and (2),

$$\mathbf{I}_2 = \mathbf{h}_{21}\,\mathbf{I}_1 - \mathbf{Z}_L\,\mathbf{h}_{22}\,\mathbf{I}_2 \longrightarrow \mathbf{I}_2 = \frac{\mathbf{h}_{21}\,\mathbf{I}_1}{1+\mathbf{Z}_L\,\mathbf{h}_{22}}$$

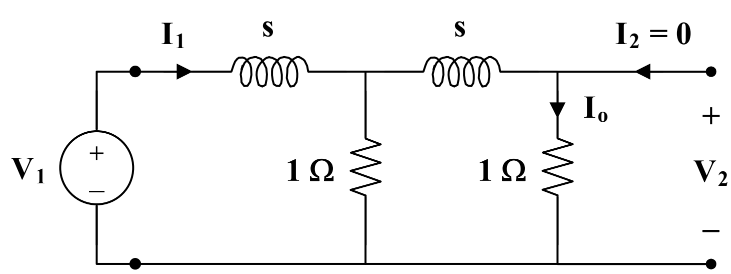
$$\mathbf{V}_2 = \text{-}\mathbf{Z}_L\,\mathbf{I}_2 = \frac{\text{-}\mathbf{Z}_L\,\mathbf{h}_{21}\,\mathbf{I}_1}{1+\mathbf{Z}_L\,\mathbf{h}_{22}}$$

$$\mathbf{V}_1 = \mathbf{h}_{11}\,\mathbf{I}_1 - \frac{\mathbf{Z}_L\,\mathbf{h}_{12}\,\mathbf{h}_{21}\,\mathbf{I}_1}{1+\mathbf{Z}_L\,\mathbf{h}_{22}}$$

$$\mathbf{Z}_{in} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \mathbf{h}_{11} - \frac{\mathbf{Z}_L\,\mathbf{h}_{12}\,\mathbf{h}_{21}}{1+\mathbf{Z}_L\,\mathbf{h}_{22}}$$

$$\mathbf{Z}_{in} = 2000 - \frac{(50\times10^3)(10^{-4})(100)}{1+(50\times10^3)(10^{-5})} = \mathbf{1.6667\ k\Omega}$$

P.P.19.7 We get $\mathbf{g}_{11}$ and $\mathbf{g}_{21}$ using the circuit in Fig. (a).



(a)

$$\mathbf{V}_1 = [\,s + 1\,\|\,(s+1)\,]\;\mathbf{I}_1$$

$$\frac{\mathbf{V}_1}{\mathbf{I}_1} = s + \frac{s+1}{s+2} = \frac{s^2+3s+1}{s+2}$$

$$\mathbf{g}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{s+2}{s^2+3s+1}$$

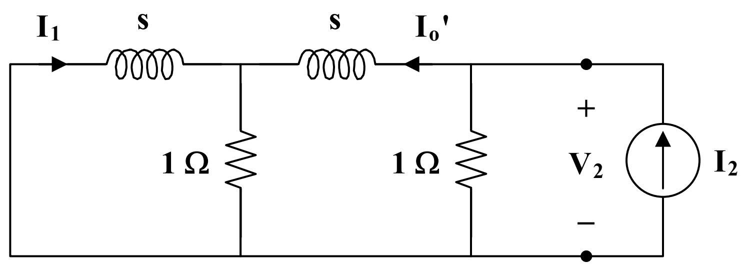
By current division,

$$\mathbf{I}_o = \frac{1}{s+2}\mathbf{I}_1 = \frac{\mathbf{V}_1}{s^2+3s+1}$$

$$\mathbf{V}_2 = \mathbf{I}_o = \frac{\mathbf{V}_1}{s^2+3s+1}$$

$$\mathbf{g}_{21} = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{1}{s^2+3s+1}$$

We obtain $\mathbf{g}_{12}$ and $\mathbf{g}_{22}$ using the circuit in Fig. (b).



(b)

$$\mathbf{V}_2 = [1 \,\|\, (s+1 \,\|\, s)]\,\mathbf{I}_2 = \left[1 \,\|\left(s + \frac{s}{s+1}\right)\right]\mathbf{I}_2$$

$$\frac{\mathbf{V}_2}{\mathbf{I}_2} = 1 \,\|\, \frac{s^2+2s}{s+1} = \frac{\dfrac{s^2+2s}{s+1}}{1+\dfrac{s^2+2s}{s+1}} = \frac{s(s+2)}{s^2+3s+1}$$

$$\mathbf{g}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2} = \frac{s(s+2)}{s^2+3s+1}$$

By current division,

$$\mathbf{I}_1 = \frac{-1}{s+1}\mathbf{I}_o{}'$$

and

$$\mathbf{I}_o{}' = \frac{1}{1+\dfrac{s^2+2s}{s+1}}\mathbf{I}_2 = \frac{s+1}{s^2+3s+1}\mathbf{I}_2$$

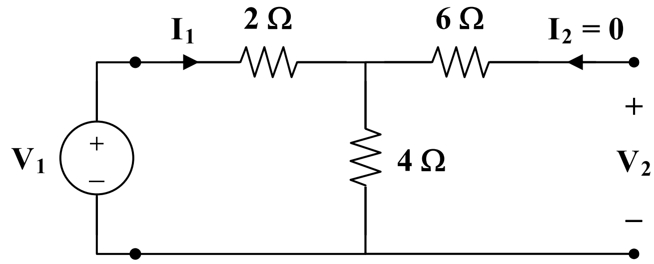
$$\mathbf{I}_1 = \frac{-1}{s^2+3s+1}\mathbf{I}_2$$

$$\mathbf{g}_{12} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{-1}{s^2 + 3s + 1}$$

Therefore,

$$[\mathbf{g}] = \begin{bmatrix} \dfrac{s+2}{s^2+3s+1} & \dfrac{-1}{s^2+3s+1} \\ \dfrac{1}{s^2+3s+1} & \dfrac{s(s+2)}{s^2+3s+1} \end{bmatrix}$$

P.P.19.8 To get **A** and **C**, we use the circuit in Fig. (a).

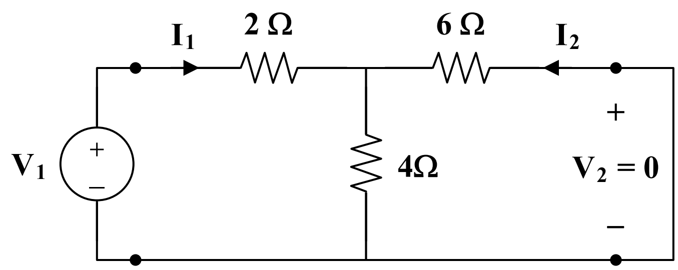


(a)

$\mathbf{A} = \mathbf{V}_1/\mathbf{V}_2 = (2+4)\mathbf{I}_1/(4\mathbf{I}_1) = \mathbf{1.5}$

$\mathbf{C} = \mathbf{I}_1/\mathbf{V}_2 = \mathbf{I}_1/(4\mathbf{I}_1) = \mathbf{0.25\ S}$

To get **B** and **D**, we use the circuit in Fig. (b).



(b)

$$\mathbf{I}_2 = \frac{-4}{10}\mathbf{I}_1 \longrightarrow \mathbf{D} = \frac{-\mathbf{I}_1}{\mathbf{I}_2} = \frac{10}{4} = 2.5$$

Also, $\mathbf{V}_1 = [2+(4\text{x}6/(4+6))]\mathbf{I}_1 = 4.4\mathbf{I}_1$
$\mathbf{B} = -\mathbf{V}_1/\mathbf{I}_2 = -4.4\mathbf{I}_1/(-0.4\mathbf{I}_1) = \mathbf{11\ \Omega}$

Therefore,

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{1.5} & \mathbf{11\,\Omega} \\ \mathbf{250}\ \boldsymbol{mS} & \mathbf{2.5} \end{bmatrix}$$

P.P.19.9 From Eq. (19.22),

$$\mathbf{V}_1 = 5\mathbf{V}_2 - 10\mathbf{I}_2 \qquad (1)$$

$$\mathbf{I}_1 = 0.4\mathbf{V}_2 - \mathbf{I}_2 \qquad (2)$$

At the output port, $\mathbf{V}_2 = \text{-}10\mathbf{I}_2$. At the input port, $\mathbf{V}_1 = 14 - 2\mathbf{I}_1$.
Substituting these into (1) and (2),

$$14 - 2\mathbf{I}_1 = \text{-}50\mathbf{I}_2 - 10\mathbf{I}_2$$

$$14 = 2\mathbf{I}_1 - 60\mathbf{I}_2 \qquad (3)$$

$$\mathbf{I}_1 = \text{-}4\mathbf{I}_2 - \mathbf{I}_2$$

$$\mathbf{I}_1 = \text{-}5\mathbf{I}_2 \qquad (4)$$

Substituting (4) into (3),

$$14 = \text{-}10\mathbf{I}_2 - 60\mathbf{I}_2$$

$$\mathbf{I}_2 = \frac{\text{-}14}{70} = \mathbf{-\,0.2\ A}$$

$$\mathbf{I}_1 = (\text{-}5)(\text{-}0.2) = \mathbf{1\ A}$$

P.P.19.10

$$[\mathbf{z}] = \begin{bmatrix} 6 & 4 \\ 4 & 6 \end{bmatrix} \qquad \Delta_z = 36 - 16 = 20$$

$$\mathbf{z}_{11} = 6 = \mathbf{z}_{22} \qquad \mathbf{z}_{12} = \mathbf{z}_{21} = 4$$

From Table 19.1,

$$\mathbf{y}_{11} = \frac{\mathbf{z}_{22}}{\Delta_z} = \frac{6}{20} = 0.3\text{ S} \qquad \mathbf{y}_{12} = \frac{\text{-}\,\mathbf{z}_{12}}{\Delta_z} = \frac{\text{-}\,4}{20} = \text{-}0.2\text{ S}$$

$$\mathbf{y}_{21} = \frac{\text{-}\,\mathbf{z}_{21}}{\Delta_z} = \text{-}0.2\text{ S} \qquad \mathbf{y}_{22} = \frac{\mathbf{z}_{11}}{\Delta_z} = 0.3\text{ S}$$

$$\mathbf{A} = \frac{\mathbf{z}_{11}}{\mathbf{z}_{21}} = \frac{6}{4} = 1.5 \qquad \mathbf{B} = \frac{\Delta_z}{\mathbf{z}_{21}} = \frac{20}{4} = 5\,\Omega$$

$$\mathbf{C} = \frac{1}{\mathbf{z}_{21}} = \frac{1}{4} = 0.25\text{ S} \qquad \mathbf{D} = \frac{\mathbf{z}_{22}}{\mathbf{z}_{21}} = \frac{6}{4} = 1.5$$

Therefore,

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{0.3} & \mathbf{-0.2} \\ \mathbf{-0.2} & \mathbf{0.3} \end{bmatrix} \boldsymbol{S} \qquad [\mathbf{T}] = \begin{bmatrix} \mathbf{1.5} & \mathbf{5\ \Omega} \\ \mathbf{0.25\ S} & \mathbf{1.5} \end{bmatrix}$$

P.P.19.11

$$\mathbf{I}_1 = \frac{\mathbf{V}_1 - 0}{R_1} \longrightarrow \mathbf{V}_1 = \mathbf{I}_1 R_1$$

Also, $$\mathbf{I}_1 = \frac{0 - \mathbf{V}_2}{R_2} \longrightarrow \mathbf{V}_2 = -\mathbf{I}_1 R_2$$

Comparing these with

$$\mathbf{V}_1 = \mathbf{z}_{11}\,\mathbf{I}_1 + \mathbf{z}_{12}\,\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{z}_{21}\,\mathbf{I}_1 + \mathbf{z}_{22}\,\mathbf{I}_2$$

shows that

$$\mathbf{z}_{11} = R_1, \qquad \mathbf{z}_{21} = -R_2, \qquad \mathbf{z}_{12} = \mathbf{z}_{21} = 0$$

Hence,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{R_1} & \mathbf{0} \\ \mathbf{-R_2} & \mathbf{0} \end{bmatrix}$$

Since $\Delta_z = \mathbf{z}_{11}\,\mathbf{z}_{22} - \mathbf{z}_{12}\,\mathbf{z}_{21} = 0$, $\mathbf{[z]^{-1}}$ **does not exist**. Consequently, **[y] does not exist**.

P.P.19.12 This is a series connection of two two-ports.

For N_a, $\mathbf{z}_{12a} = \mathbf{z}_{21a} = 20$, $\mathbf{z}_{11a} = 20 - j15$, $\mathbf{z}_{22a} = 20 + j10$

For N_b, $\mathbf{z}_{12b} = \mathbf{z}_{21b} = 50$, $\mathbf{z}_{11b} = 50 + j40$, $\mathbf{z}_{22b} = 50 - j20$

Thus, $$[\mathbf{z}] = [\mathbf{z}_a] + [\mathbf{z}_b]$$

$$[\mathbf{z}] = \begin{bmatrix} 20 - j15 & 20 \\ 20 & 20 + j10 \end{bmatrix} + \begin{bmatrix} 50 + j40 & 50 \\ 50 & 50 - j20 \end{bmatrix}$$

$$[\mathbf{z}] = \begin{bmatrix} 70 + j25 & 70 \\ 70 & 70 - j10 \end{bmatrix}$$

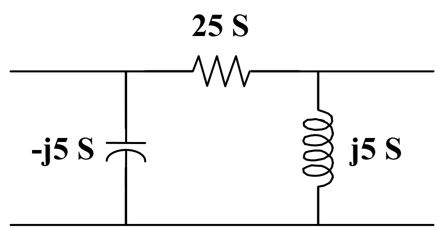
$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{\mathbf{z}_{12}\,\mathbf{Z}_L}{(\mathbf{z}_{11} + \mathbf{Z}_s)(\mathbf{z}_{22} + \mathbf{Z}_L) - \mathbf{z}_{12}\,\mathbf{z}_{21}}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{(70)(40)}{(70 + j25 + 5)(70 - j10 + 40) - 4900}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{2800}{8250 - j750 + j2750 + 250 - 4900}$$

$$\frac{\mathbf{V}_2}{\mathbf{V}_s} = \frac{2800}{3600 + j2000} = \mathbf{0.6799\angle -29.05^\circ}$$

P.P.19.13 We convert the upper T network N_a to a Π network, as shown below.



$$\mathbf{y}_a = \frac{\mathbf{y}_1\mathbf{y}_2 + \mathbf{y}_2\mathbf{y}_3 + \mathbf{y}_3\mathbf{y}_1}{\mathbf{y}_2} = \frac{(\text{-j5})(\text{j5}) + (\text{j5})(1) + (1)(\text{-j5})}{\text{j5}} = \text{-j5}$$

$$\mathbf{y}_b = 5, \qquad \mathbf{y}_c = 25$$

For N_a,

$$\mathbf{y}_{12a} = \text{-}25 = \mathbf{y}_{21a}, \qquad \mathbf{y}_{11a} = 25 - \text{j5}, \qquad \mathbf{y}_{22a} = 25 + \text{j5}$$

$$[\mathbf{y}_a] = \begin{bmatrix} 25 - \text{j5} & -25 \\ -25 & 25 + \text{j5} \end{bmatrix}$$

For N_b,

$$\mathbf{y}_{12b} = \text{j10} = \mathbf{y}_{21b}, \qquad \mathbf{y}_{11b} = 2 - \text{j10} = \mathbf{y}_{22b}$$

$$[\mathbf{y}_b] = \begin{bmatrix} 2 - \text{j10} & \text{j10} \\ \text{j10} & 2 - \text{j10} \end{bmatrix}$$

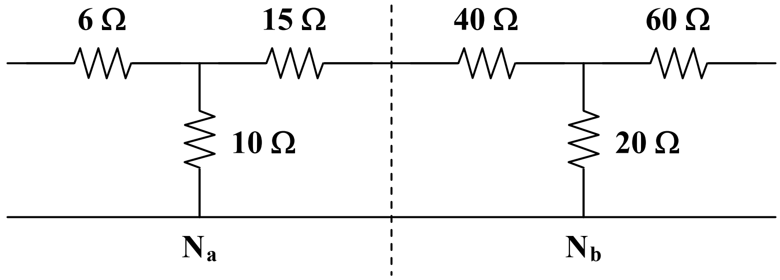
Since N_a and N_b are in parallel, $[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b]$

$$[\mathbf{y}] = \begin{bmatrix} \mathbf{27 - j15} & \mathbf{-25 + j10} \\ \mathbf{-25 + j10} & \mathbf{27 - j5} \end{bmatrix} \mathbf{S}$$

P.P.19.14 Convert the left Π network to a T network.

$$R_1 = \frac{(20)(30)}{20 + 30 + 50} = 6, \qquad R_2 = \frac{(20)(50)}{100} = 10, \qquad R_3 = \frac{(30)(50)}{100} = 15$$

Putting this network into the given network produces the network shown below. This may be regarded as a cascaded connection of T two-port networks.



For N_a,

$$\mathbf{A}_a = 1 + \frac{6}{10} = 1.6, \qquad \mathbf{B}_a = 15 + \left(\frac{6}{10}\right)(25) = 30$$

$$\mathbf{C}_a = \frac{1}{10} = 0.1, \qquad \mathbf{D}_a = 1 + \frac{15}{10} = 2.5$$

$$[\mathbf{T}_a] = \begin{bmatrix} 1.6 & 30 \\ 0.1 & 2.5 \end{bmatrix}$$

For N_b,

$$\mathbf{A}_b = 1 + \frac{40}{20} = 3, \qquad \mathbf{B}_b = 60 + \left(\frac{40}{20}\right)(80) = 220$$

$$\mathbf{C}_b = \frac{1}{20} = 0.05, \qquad \mathbf{D}_b = 1 + \frac{60}{20} = 4$$

$$[\mathbf{T}_b] = \begin{bmatrix} 3 & 220 \\ 0.05 & 4 \end{bmatrix}$$

Hence,

$$[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b] = \begin{bmatrix} 1.6 & 30 \\ 0.1 & 2.5 \end{bmatrix}\begin{bmatrix} 3 & 220 \\ 0.05 & 4 \end{bmatrix}$$

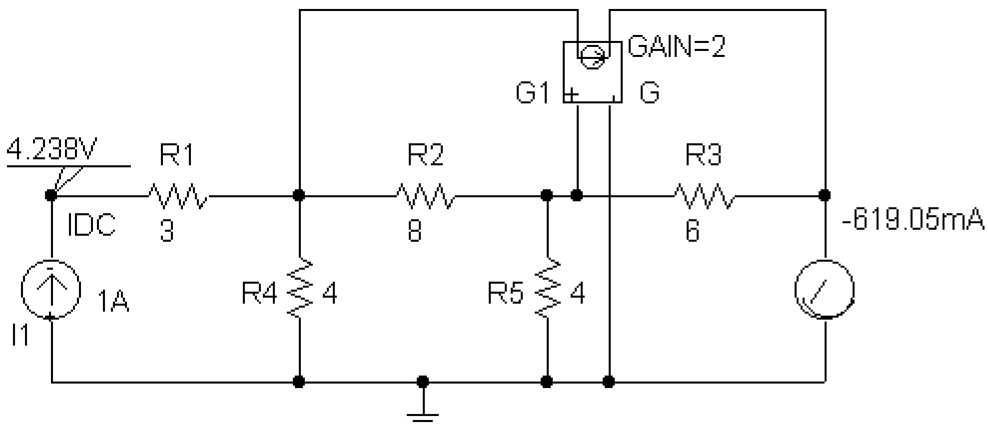
We can now use MATLAB to obtain T.

```
>> Ta=[1.6,30;0.1,2.5]
Ta =
   1.6000   30.0000
   0.1000    2.5000
>> Tb=[3,220;0.05,4]
Tb =
   3.0000  220.0000
```

0.0500   4.0000
>> T=Ta*Tb
T =
6.3000 472.0000
0.4250   32.0000

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{6.3} & \mathbf{472\ \Omega} \\ \mathbf{0.425\ S} & \mathbf{32} \end{bmatrix}$$

P.P.19.15 To obtain $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$, simulate the schematic in Fig. (a) using PSpice.



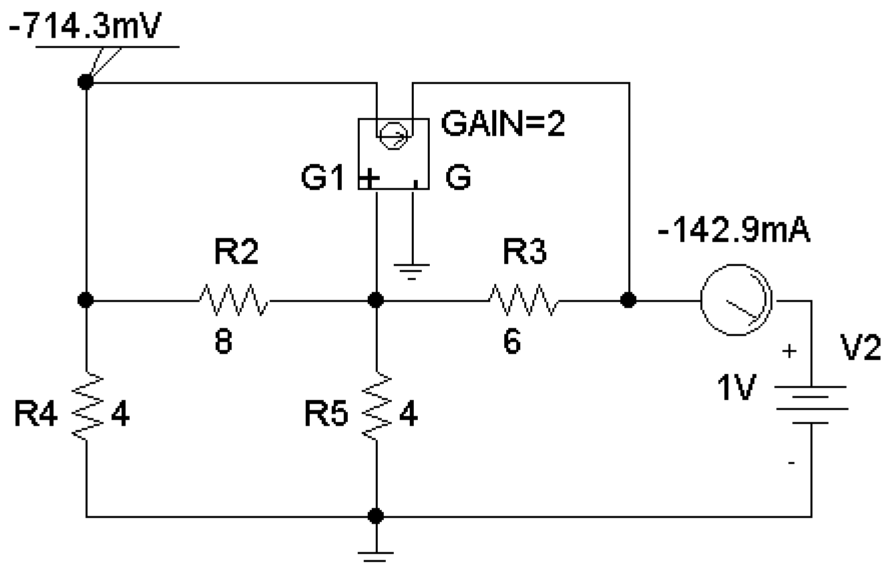
(a)

Insert a 1-A dc current source to account for $\mathbf{I}_1 = 1\ \text{A}$. Also, include pseudocomponents VIEWPOINT and IPROBE to display $\mathbf{V}_1$ and $\mathbf{I}_2$ respectively. When the circuit is saved and run, the values of $\mathbf{V}_1$ and $\mathbf{I}_2$ are displayed on the pseudocomponents as shown in Fig. (a). Thus,

$$\mathbf{h}_{11} = \frac{\mathbf{V}_1}{1} = 4.238\ \Omega, \qquad \mathbf{h}_{21} = \frac{\mathbf{I}_2}{1} = \text{-0.6190}$$

To obtain $\mathbf{h}_{12}$ and $\mathbf{h}_{22}$, insert a 1-V dc voltage source at the output port to account for $\mathbf{V}_2 = 1\ \text{V}$. The pseudocomponents VIEWPOINT and IPROBE are included to display $\mathbf{V}_1$ and $\mathbf{I}_2$ respectively. After simulation, the schematic displays the results as shown in Fig. (b).

$$\mathbf{h}_{12} = \frac{\mathbf{V}_1}{1} = \text{-0.7143}, \qquad \mathbf{h}_{22} = \frac{\mathbf{I}_2}{1} = \text{-0.1429 S}$$

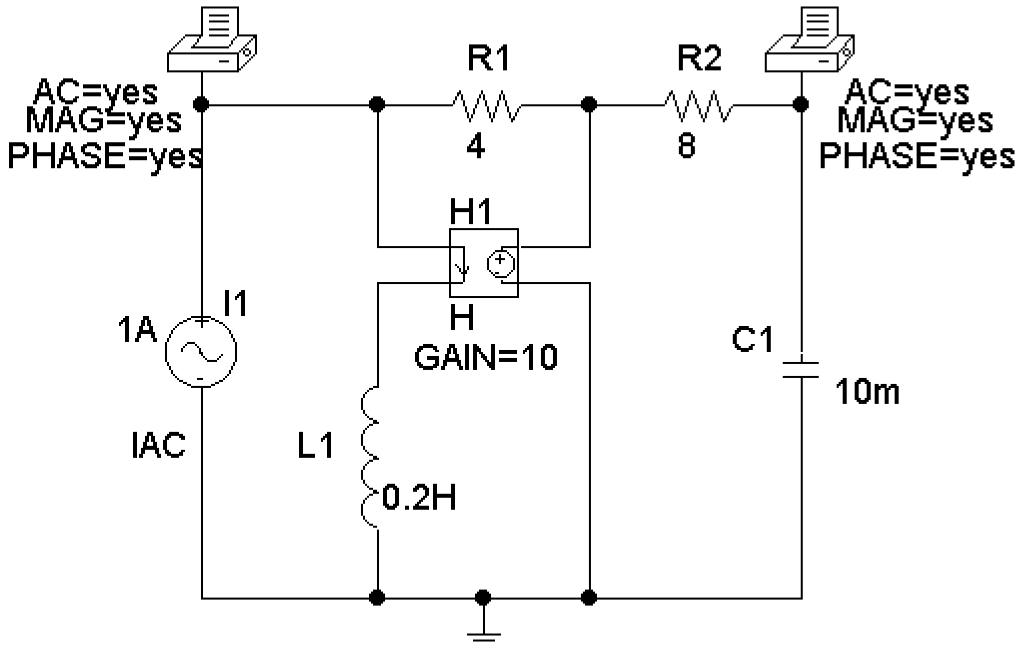


(b)

Thus,

$$[\mathbf{h}] = \begin{bmatrix} \mathbf{4.238\,\Omega} & \mathbf{-0.7143} \\ \mathbf{-0.6190} & \mathbf{-0.1429\,S} \end{bmatrix}$$

P.P.19.16 Insert a 1-A ac current source at the output terminals to account for $\mathbf{I}_1 = 1\text{ A}$. Include two VPRINT1 pseudocomponents to output $\mathbf{V}_1$ and $\mathbf{V}_2$. For each VPRINT1, set the attributes to AC = yes, PHASE = yes, and MAG = yes. In the AC Sweep and Noise Analysis dialog box, set Total pt : 1, Start Freq : 60, and End Freq : 60. The schematic is shown in Fig. (a).



(a)

Once the schematic is saved and run, the output results include :

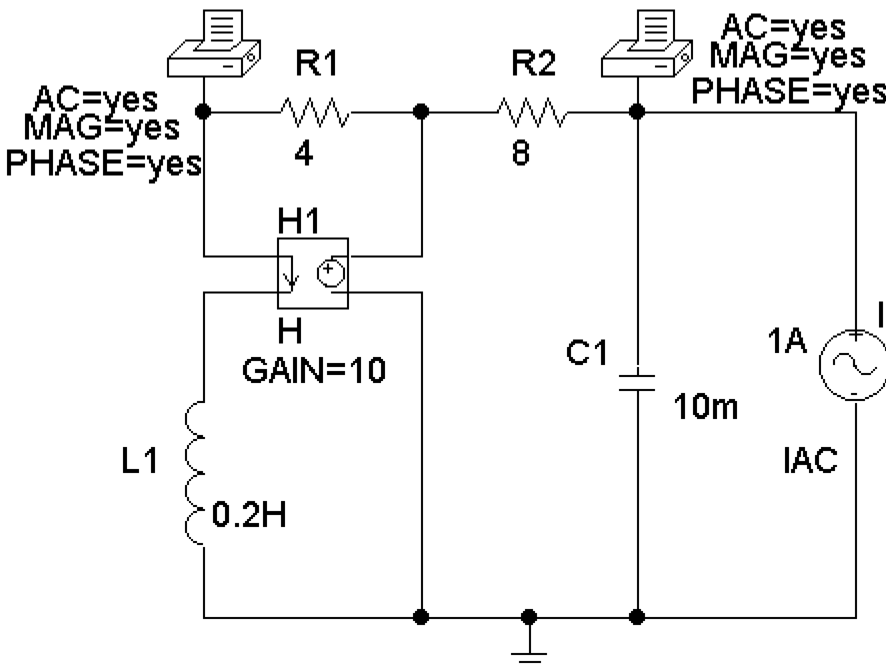
FREQ	VM($N_0002)	VP($N_0002)
6.000E+01	3.987E+00	1.755E+02

FREQ	VM($N_0003)	VP($N_0003)
6.000E+01	1.752E-02	-2.651E+00

From this table,

$$\mathbf{z}_{11} = \frac{\mathbf{V}_1}{1} = 3.987\angle 175.5°, \qquad \mathbf{z}_{21} = 0.0175\angle -2.65°$$

Similarly, insert a 1-A ac source at the output port with the two pseudocomponents in place as in Fig. (a). The result is the schematic in Fig. (b).



(b)

When the schematic is saved and run, the output results include :

FREQ	VM($N_0002)	VP($N_0002)
6.000E+01	1.000E-30	0.000E+00

FREQ	VM($N_0003)	VP($N_0003)
6.000E+01	2.651E-01	9.190E+01

From this table,

$$\mathbf{z}_{12} = \frac{\mathbf{V}_1}{1} \cong 0 \qquad\qquad \mathbf{z}_{22} = 0.265\angle 91.9°$$

Thus,

$$[\mathbf{z}] = \begin{bmatrix} \mathbf{3.987\angle 175.5°} & \mathbf{0} \\ \mathbf{0.0175\angle -2.65°} & \mathbf{0.2651\angle 91.9°} \end{bmatrix} \Omega$$

P.P.19.17 In this case, $R_s = 150\ k\Omega$, $R_L = 3.75\ k\Omega$.

$$h_{ie}\, h_{oe} - h_{re}\, h_{fe} = (6\times 10^{3})(8\times 10^{-6}) - (1.5\times 10^{-4})(200) = 18\times 10^{-3}$$

The gain for the transistor is given as,

$$A_V = \frac{-(200)(3750)}{6000 + (18\times 10^{-3})(3.75\times 10^{3})} = V_o/V_b = \mathbf{-123.61}$$

To calculate the gain of the circuit we need to use,

$-V_s + 150kI_b + V_b = 0$ or $0.002 = 150k(0.002/156k) - V_c/123.61$

$V_c = -9.506$ mV which leads to the gain = –9.506/2 = **–4.753**

$$A_i = \frac{200}{1 + (8\times 10^{-6})(3.75\times 10^{3})} = \mathbf{194.17}$$

The input resistance for the transistor is equal to h_{ie} = **6 kΩ**.

The input resistance for the circuit is equal to,

$$Z_{in} = 150{,}000 + 6000 - (1.5\times 10^{-4})(194.17) \cong \mathbf{156\ k\Omega}$$

$$Z_{out} = \frac{150\times 10^{3} + 6\times 10^{3}}{(150\times 10^{3})(8\times 10^{-6}) - (1.5\times 10^{-4})(200)}$$

$$= \frac{156}{1.248 - 0.03}\ k\Omega = \mathbf{128.08\ k\Omega}$$

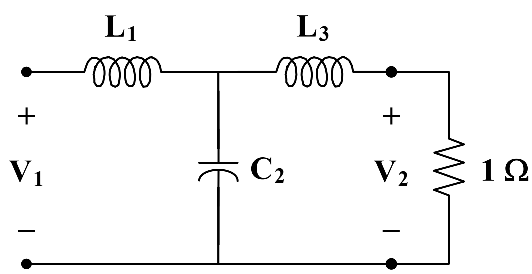
P.P.19.18 Let $\mathbf{D}(s) = (s^3 + 4s) + (s^2 + 2)$

Dividing both numerator and denominator by $s^3 + 4s$ gives

$$\mathbf{H}(s) = \frac{\dfrac{2}{s^3 + 4s}}{1 + \dfrac{s^2 + 2}{s^3 + 4s}}$$

i.e. $\quad \mathbf{y}_{21} = \dfrac{-2}{s^3 + 4s} \qquad \mathbf{y}_{22} = \dfrac{s^2 + 2}{s^3 + 4s}$

As a third order function, we can realize $\mathbf{H}(s)$ by the **LC network shown in Fig. (a)**.

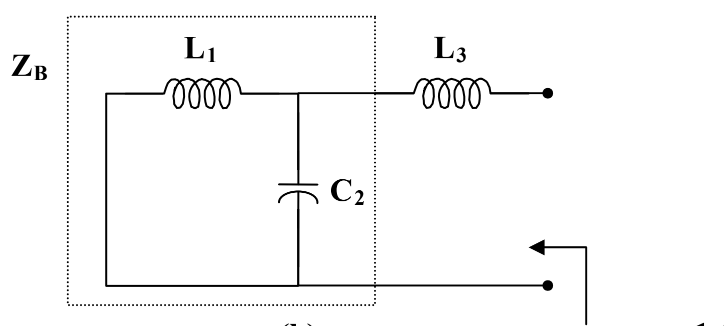


(a)

$$\mathbf{Z}_A = \frac{1}{\mathbf{y}_{22}} = \frac{s^3+4s}{s^2+2} = s + \frac{2s}{s^2+2} = sL_3 + \mathbf{Z}_B$$

$$L_3 = 1\,\text{H}$$

$$\mathbf{Z}_B = \frac{2s}{s^2+2}$$



(b)

$$\mathbf{Y}_B = \frac{1}{\mathbf{Z}_B} = \frac{s^2+2}{2s} = 0.5s + \frac{1}{s} = sC_2 + \frac{1}{\mathbf{Y}_C}$$

$$C_2 = 0.5\,\text{F}$$

$$\mathbf{Y}_C = \frac{1}{sL_1} = \frac{1}{s} \longrightarrow L_1 = 1\,\text{H}$$

Hence,

L_1 = **1 H**, C_2 = **500 mF**, L_3 = **1 H**.