Lecture 4

Gauss's Law

Lecture 03 – Review

- In Lecture 3 we studied the properties of electric field
- Similar to the electrostatic force, the electric field is a vector field.
- The electric field is the property of a charged particle q with SI unit newton per coulomb (N/C).
- It is defined as the electrostatic force between the charged particle q with a test charge q_0 at a displacement r divided by the q_0.

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{q_0} \times k \frac{q q_0}{r^2} \hat{r} = k \frac{q}{r^2} \hat{r}$$

- One can visualize that the electric field radiates from a positive charge and collapses into the negative charge.

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Lecture 03 – Review

- To determine the electric field at a point, one needs to sum up all the electric field vectors due to each of the individual charged bodies around that point.
- An electric dipole is a system of two charged particles of opposite signs that are separated by a displacement.
- Electric dipole $\vec{p}$ is characterized by its dipole moment, which is a vector quantity.
- For a dipole consists of two point charges $\pm q$ that are separated by a displacement d, the dipole moment is defined as $\vec{p} = q\vec{d}$, where the magnitude of the dipole vector is the product of the charge q and the displacement d, with the direction of the dipole vector points from $-q$ to $+q$.
- The magnitude of dipole electric field is proportional to $\mathrm{E} \propto 1/r^3$, this is different than for a point source where $E \propto 1/r^2$.

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \qquad \text{(electric dipole).}$$

Lecture 03 – Review

- For a continuous charge distribution, one can specify them according to their geometries. These symbols are used to define such distributions: $\lambda\ C/m$ as line charge density, $\sigma\ C/m^2$ as surface charge density and $\rho\ C/m^3$ as volume charge density.
- To calculate the electric field due to a charge distribution, one needs to integrate the contribution from each segment of the distribution.
- When a dipole is placed in an electric field, the field will produce a torque on the dipole, the magnitude and the direction of the torque depends on the orientation and dipole with the electric field where

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}$$

- Potential energy U can be associated with the orientation of an electric dipole in an electric field.
- If we choose zero potential reference is when the dipole is perpendicular to the electric field, then we have

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}$$

Lecture Outline

- **Chapter 23**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Gauss's Law
  - Electric flux
  - Gauss's law and Coulomb's law
  - Applications of Gauss' law

23.1 Physics of the Electric Field Distribution:

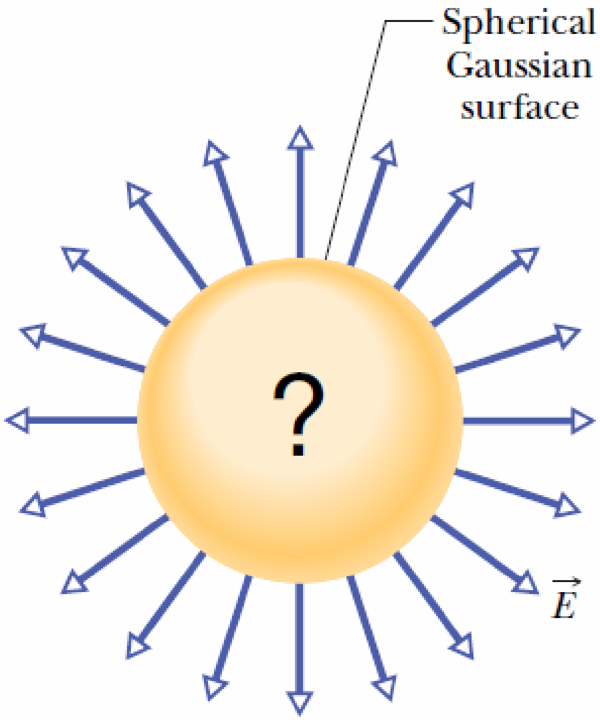

Fig. 23-1 A spherical Gaussian surface. If the electric field vectors are of uniform magnitude and point radially outward at all surface points, you can conclude that a net positive distribution of charge must lie within the surface and have spherical symmetry.

Gauss' law relates the electric fields at points on a (closed) Gaussian surface to the net charge enclosed by that surface.

Consider the flux passing through a closed surface and the amount of charge inside.

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23.2: Flux

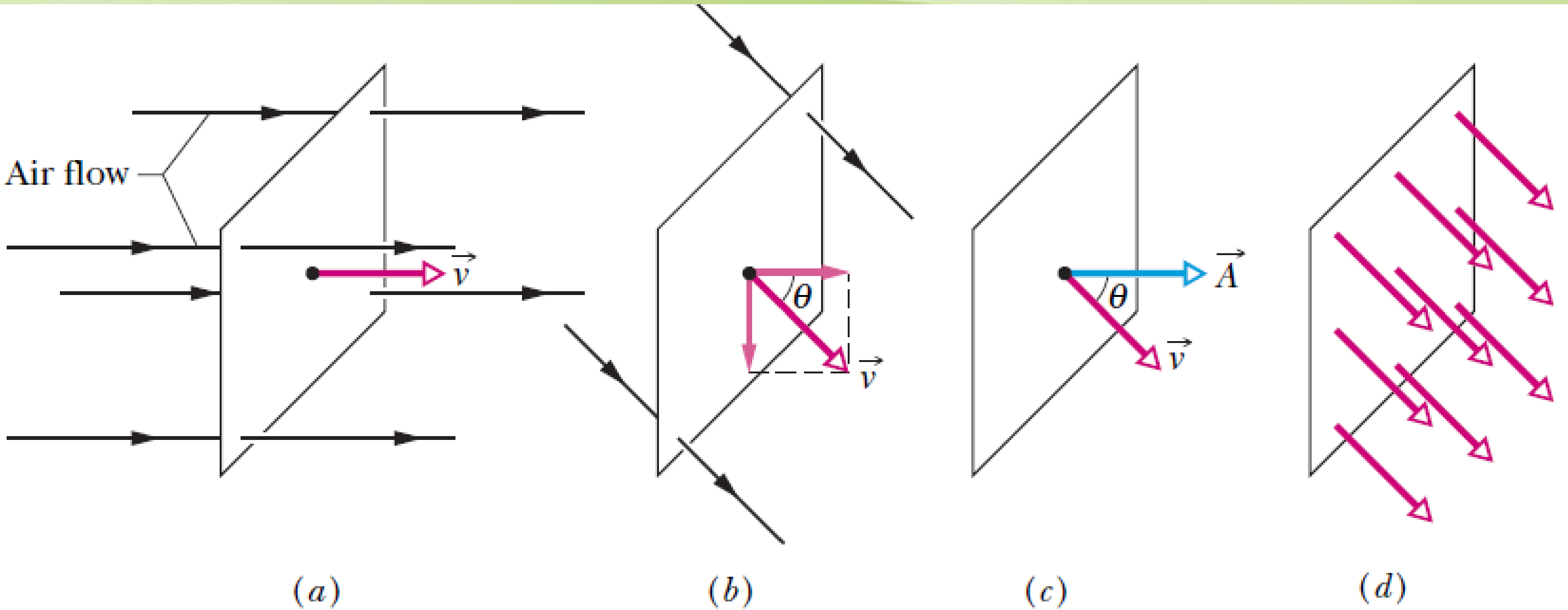

Fig. 23-2 (a) A uniform airstream of velocity is perpendicular to the plane of a square loop of area ***A***. (b) The component of perpendicular to the plane of the loop is *v cos θ,* where *θ* is the angle between ***v*** and a normal to the plane. (c) The area vector ***A*** is perpendicular to the plane of the loop and makes an angle *θ* with ***v***. (d) The velocity field intercepted by the area of the loop. The **rate of volume flow** through the loop is Φ= (*v cos θ*)*A.*

This rate of flow through an area is an example of a flux—a *volume flux* in this situation.

$$\Phi = vA\cos\theta = \vec{v}\cdot\vec{A},$$

#The actual physical quantity referred to by the term 'flux' can be different in different occasions.

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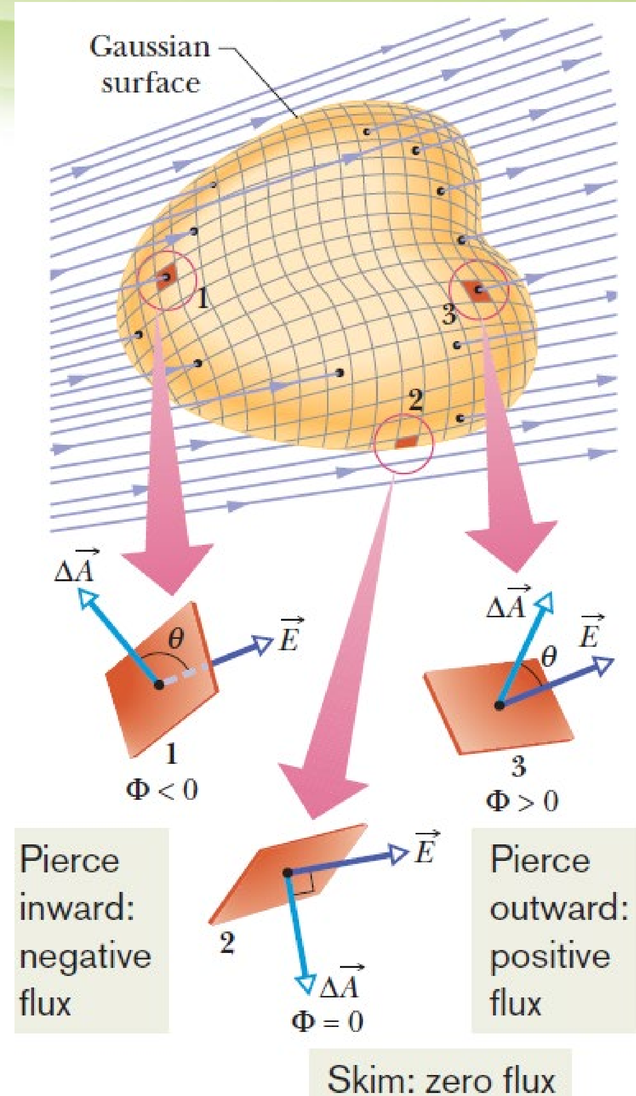
The electric flux through a surface is defined to be the inner product of the electric field and the surface vector:

$$\Phi_E = \vec{E} \bullet \Delta\vec{A}$$

For a closed surface, it is

$$\boxed{\Phi = \sum \vec{E} \cdot \Delta\vec{A}.}$$

$$\boxed{\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface)}}$$



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Example, Flux through a closed cylinder, uniform field:

Figure 23-4 shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field $\vec{E}$, with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?

KEY IDEA

We can find the flux Φ through the Gaussian surface by integrating the scalar product $\vec{E} \cdot d\vec{A}$ over that surface.

Calculations: We can do the integration by writing the flux as the sum of three terms: integrals over the left cylinder cap a, the cylindrical surface b, and the right cap c. Thus, from Eq. 23-4,

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}. \qquad (23\text{-}5)$$

For all points on the left cap, the angle θ between $\vec{E}$ and $d\vec{A}$ is 180° and the magnitude E of the field is uniform. Thus,

$$\int_a \vec{E} \cdot d\vec{A} = \int E(\cos 180^\circ)\, dA = -E \int dA = -EA,$$

where $\int dA$ gives the cap's area $A\ (= \pi R^2)$. Similarly, for the right cap, where $\theta = 0$ for all points,

$$\int_c \vec{E} \cdot d\vec{A} = \int E(\cos 0)\, dA = EA.$$

Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E} \cdot d\vec{A} = \int E(\cos 90^\circ)\, dA = 0.$$

Substituting these results into Eq. 23-5 leads us to

$$\Phi = -EA + 0 + EA = 0. \qquad \text{(Answer)}$$

The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

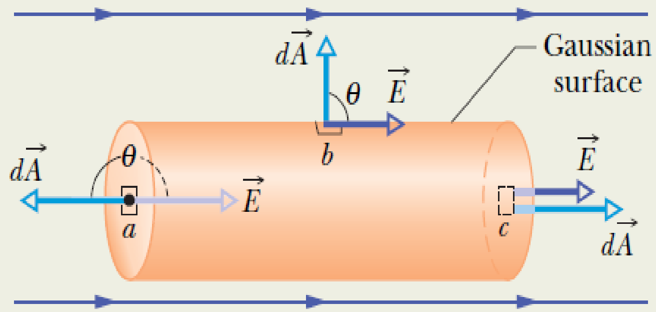
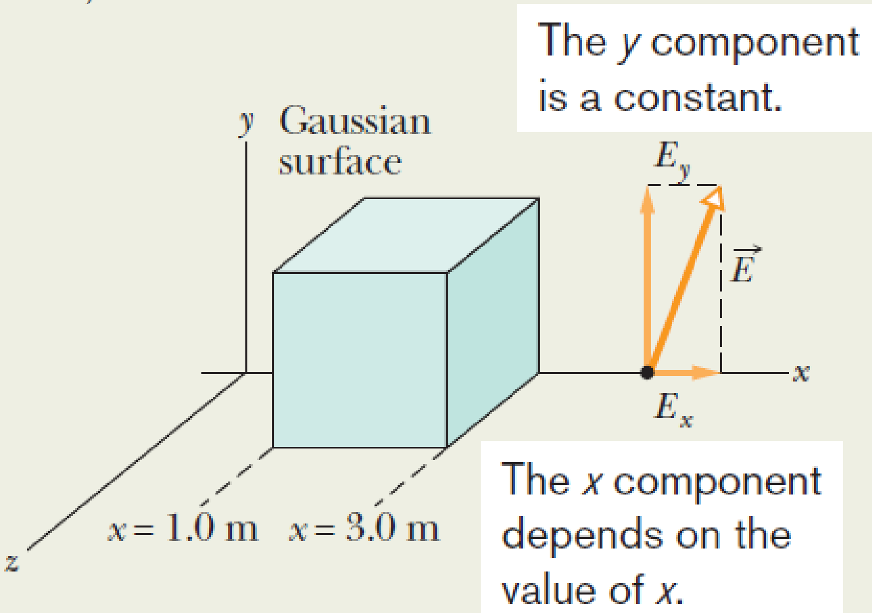


Fig. 23-4 A cylindrical Gaussian surface, closed by end caps, is immersed in a uniform electric field. The cylinder axis is parallel to the field direction.

Example, Flux through a closed cube, Non-uniform field:

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{\imath} + 4.0\hat{\jmath}$ pierces the Gaussian cube shown in Fig. 23-5*a*. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)



(*a*)

Right face: An area vector **A** is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector for any area element **dA** (small section) on the right face of the cube must point in the positive direction of the x axis. The most convenient way to express the vector is in unit-vector notation,

$$d\vec{A} = dA\hat{\imath}.$$

$$\Phi_r = \int \vec{E}\cdot d\vec{A} = \int (3.0x\hat{\imath} + 4.0\hat{\jmath})\cdot(dA\hat{\imath})$$

$$= \int [(3.0x)(dA)\hat{\imath}\cdot\hat{\imath} + (4.0)(dA)\hat{\jmath}\cdot\hat{\imath}]$$

$$= \int (3.0x\, dA + 0) = 3.0\int x\, dA.$$

Although x is certainly a variable as we move left to right across the figure, because the right face is perpendicular to the x *axis,* every point on the face has the same x coordinate. (The y and z coordinates do not matter in our integral.) Thus, we have

$$\Phi_r = 3.0\int (3.0)\, dA = 9.0\int dA. = (9.0\ \text{N/C})(4.0\ \text{m}^2) = 36\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

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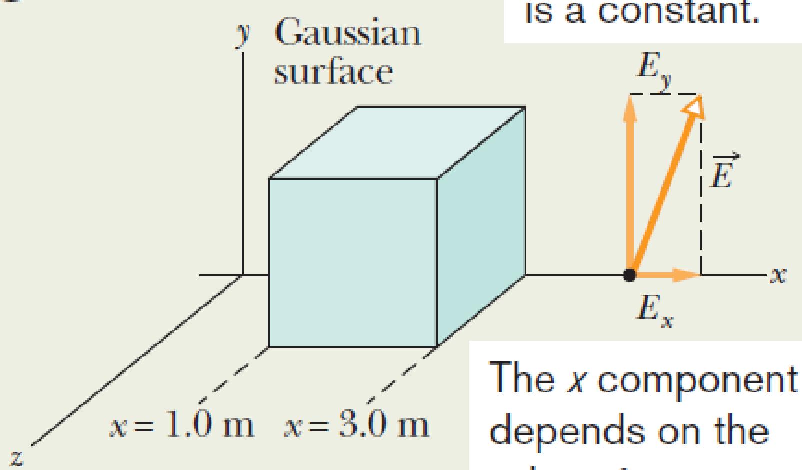
Example, Flux through a closed cube, Non-uniform field:

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-5*a*. (E is in newtons per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)

Fig. 23-5



The y component is a constant.

The x component depends on the value of x.

Left face: The procedure for finding the flux through the left face is the same as that for the right face. However, two factors change. (1) The differential area vector $d\vec{A}$ points in the negative direction of the x axis, and thus $d\vec{A} = -dA\hat{i}$ (Fig. 23-5*d*). (2) The term x again appears in our integration, and it is again constant over the face being considered. However, on the left face, $x = 1.0$ m. With these two changes, we find that the flux Φ_l through the left face is

$$\Phi_l = -12\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

Top face: The differential area vector $d\vec{A}$ points in the positive direction of the y axis, and thus $d\vec{A} = dA\hat{j}$ (Fig. 23-5*e*). The flux Φ_t through the top face is then

$$\begin{aligned}\Phi_t &= \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{j}) \\ &= \int [(3.0x)(dA)\hat{i}\cdot\hat{j} + (4.0)(dA)\hat{j}\cdot\hat{j}] \\ &= \int (0 + 4.0\, dA) = 4.0 \int dA \\ &= 16\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}\end{aligned}$$

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23.4 Gauss' Law:

$$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q_{enc} \qquad \text{(Gauss' Law)}$$

$$\varepsilon_0 \Phi = q_{\mathrm{enc}} \qquad \text{(Gauss' law).}$$

The net charge q_{enc} is the algebraic sum of all the enclosed positive and negative charges, and it can be positive, negative, or zero.

The electric field at the surface is due to all the charge distribution, including both that inside and outside the surface.

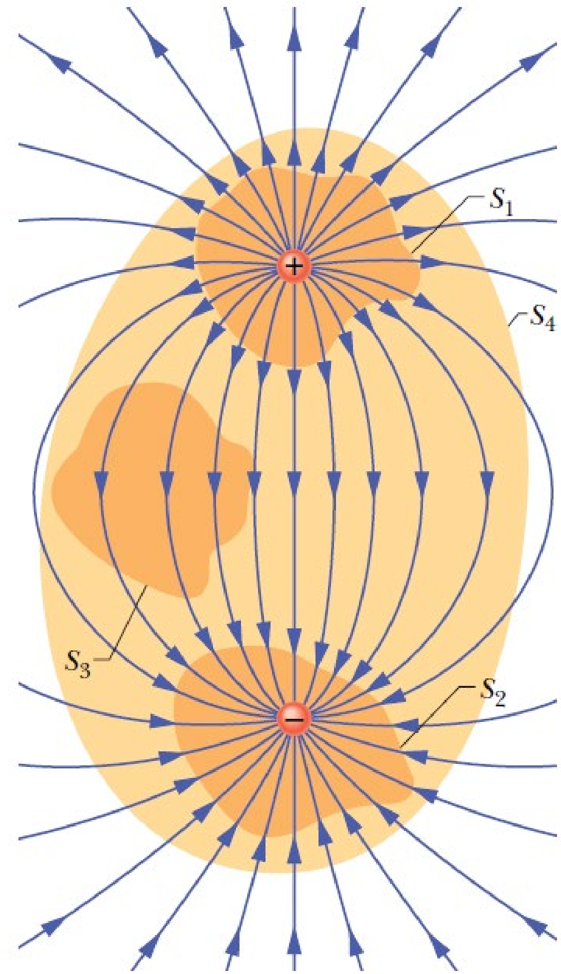

Fig. 23-6 Two point charges, equal in magnitude but opposite in sign, and the field lines that represent their net electric field. Four Gaussian surfaces are shown in cross section. Surface S_1 encloses the positive charge. Surface S_2 encloses the negative charge. Surface S_3 encloses no charge. Surface S_4 encloses both charges and thus no net charge.

23.5 Gauss' Law and Coulomb's Law:

$$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oiint E\, dA = q_{enc}$$

$$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q$$

$$\varepsilon_0 E(4\pi r^2) = q$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}.$$

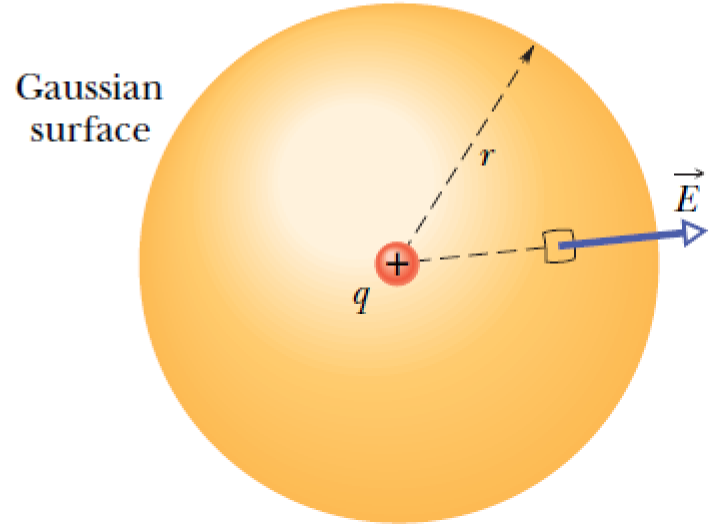

Fig. 23-8 A spherical Gaussian surface centered on a point charge q.

One may also derive Gauss' law from Coulomb's law.
These two laws are equivalent.

Example, Relating the net enclosed charge and the net flux:

Fig. 23-7 Five plastic objects, each with an electric charge, and a coin, which has no net charge. A Gaussian surface, shown in cross section, encloses three of the plastic objects and the coin.

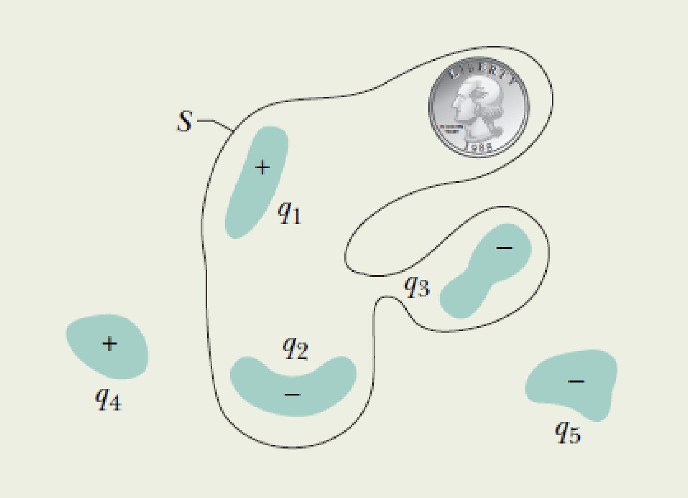

Figure 23-7 shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1$ nC, $q_2 = q_5 = -5.9$ nC, and $q_3 = -3.1$ nC?

KEY IDEA

The *net* flux Φ through the surface depends on the *net* charge q_{enc} enclosed by surface S.

Calculation: The coin does not contribute to Φ because it is neutral and thus contains equal amounts of positive and negative charge. We could include those equal amounts, but they would simply sum to be zero when we calculate the *net* charge enclosed by the surface. So, let's not bother. Charges q_4 and q_5 do not contribute because they are outside surface S. They certainly send electric field lines through the surface, but as much enters as leaves and no net flux is contributed. Thus, q_{enc} is only the sum $q_1 + q_2 + q_3$ and Eq. 23-6 gives us

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{+3.1 \times 10^{-9}\,\text{C} - 5.9 \times 10^{-9}\,\text{C} - 3.1 \times 10^{-9}\,\text{C}}{8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}$$

$$= -670\ \text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

The minus sign shows that the net flux through the surface is inward and thus that the net charge within the surface is negative.

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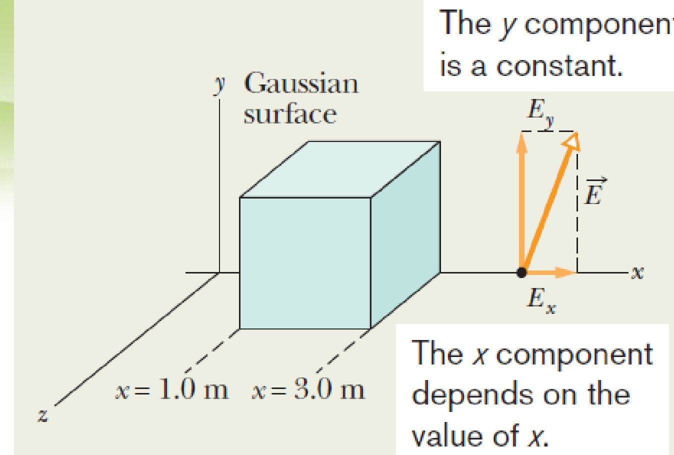
Example, Enclosed charge in a non-uniform field:

Fig. 23-5



What is the net charge enclosed by the Gaussian cube of Fig. 23-5, which lies in the electric field $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$? ($E$ is in newtons per coulomb and x is in meters.)

KEY IDEA

The net charge enclosed by a (real or mathematical) closed surface is related to the total electric flux through the surface by Gauss' law as given by Eq. 23-6 ($\varepsilon_0\Phi = q_{enc}$).

Flux: To use Eq. 23-6, we need to know the flux through all six faces of the cube. We already know the flux through the right face ($\Phi_r = 36\ \mathrm{N\cdot m^2/C}$), the left face ($\Phi_l = -12\ \mathrm{N\cdot m^2/C}$), and the top face ($\Phi_t = 16\ \mathrm{N\cdot m^2/C}$).

For the bottom face, our calculation is just like that for the top face *except* that the differential area vector $d\vec{A}$ is now directed downward along the y axis (recall, it must be *outward* from the Gaussian enclosure). Thus, we have $d\vec{A} = -dA\hat{j}$, and we find

$$\Phi_b = -16\ \mathrm{N\cdot m^2/C}.$$

For the front face we have $d\vec{A} = dA\hat{k}$, and for the back face, $d\vec{A} = -dA\hat{k}$. When we take the dot product of the given electric field $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ with either of these expressions for $d\vec{A}$, we get 0 and thus there is no flux through those faces. We can now find the total flux through the six sides of the cube:

$$\Phi = (36 - 12 + 16 - 16 + 0 + 0)\ \mathrm{N\cdot m^2/C} = 24\ \mathrm{N\cdot m^2/C}.$$

Enclosed charge: Next, we use Gauss' law to find the charge q_{enc} enclosed by the cube:

$$q_{enc} = \varepsilon_0\Phi = (8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(24\ \mathrm{N\cdot m^2/C}) = 2.1\times10^{-10}\ \mathrm{C}. \quad \text{(Answer)}$$

Thus, the cube encloses a *net* positive charge.

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23.6 A Charged Isolated Conductor:

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

Figure 23-9*a* shows, in cross section, an isolated lump of copper hanging from an insulating thread and having an excess charge *q*. The Gaussian surface is placed just inside the actual surface of the conductor. The electric field inside this conductor must be zero. Since the excess charge is not inside the Gaussian surface, it must be outside that surface, which means it must lie on the actual surface of the conductor.

Figure 23-9b shows the same hanging conductor, but now with a cavity that is totally within the conductor. A Gaussian surface is drawn surrounding the cavity, close to its surface but inside the conducting body. Inside the conductor, there can be no flux through this new Gaussian surface. Therefore, there is no net charge on the cavity walls; all the excess charge remains on the outer surface of the conductor.

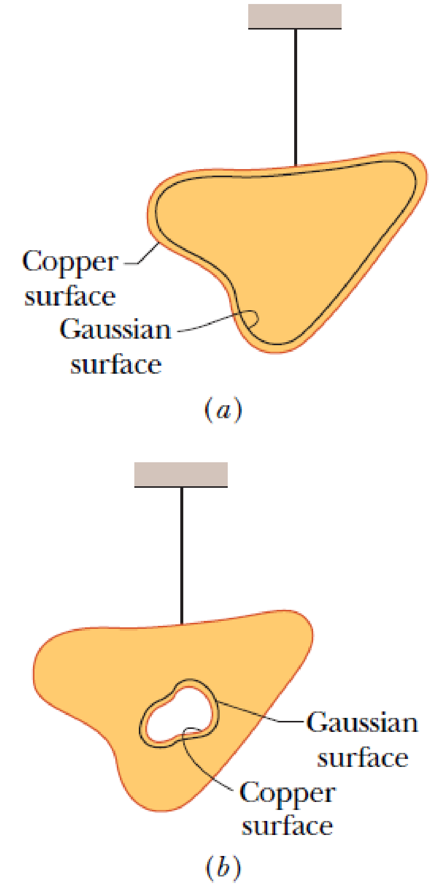

Fig. 23-9 (*a*) A lump of copper with a charge *q* hangs from an insulating thread. A Gaussian surface is placed within the metal, just inside the actual surface. (*b*) The lump of copper now has a cavity within it. A Gaussian surface lies within the metal, close to the cavity surface.

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23.6 A Charged Isolated Conductor; The External Electric Field:

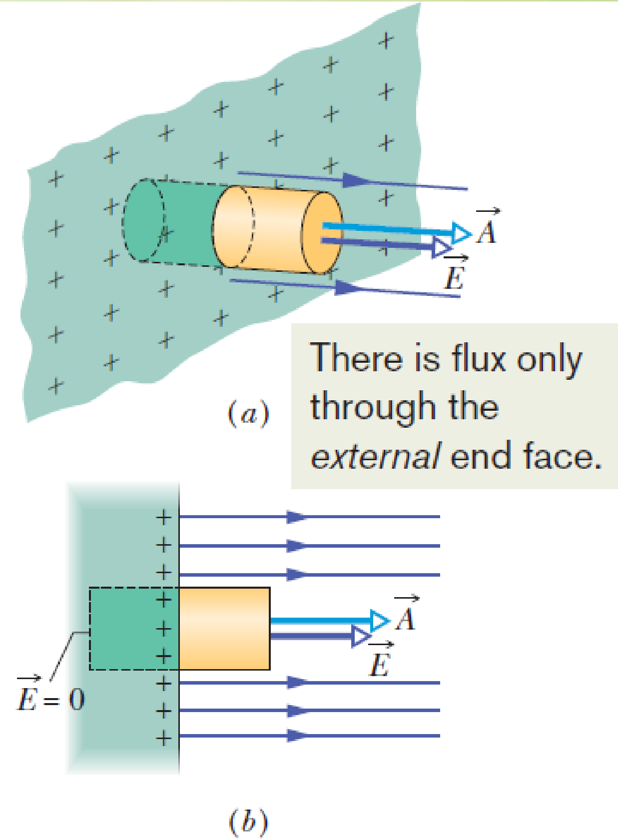

Fig. 23-10 (*a*) Perspective view and (*b*) side view of a tiny portion of a large, isolated conductor with excess positive charge on its surface. A (closed) cylindrical Gaussian surface, embedded perpendicularly in the conductor, encloses some of the charge. Electric field lines pierce the external end cap of the cylinder, but not the internal end cap. The external end cap has area A and area vector $\vec{A}$.

σ is the charge per unit area.

q_{enc} is equal to σA.

$$\varepsilon_0 EA = \sigma A,$$

$$E = \frac{\sigma}{\varepsilon_0} \qquad \text{(conducting surface).}$$

Example, Spherical Metal Shell, Electric Field, and Enclosed Charge:

Figure 23-11*a* shows a cross section of a spherical metal shell of inner radius *R*. A point charge of $-5.0\ \mu C$ is located at a distance *R*/2 from the center of the shell. If the shell is electrically neutral, what are the (induced) charges on its inner and outer surfaces? Are those charges uniformly distributed? What is the field pattern inside and outside the shell?

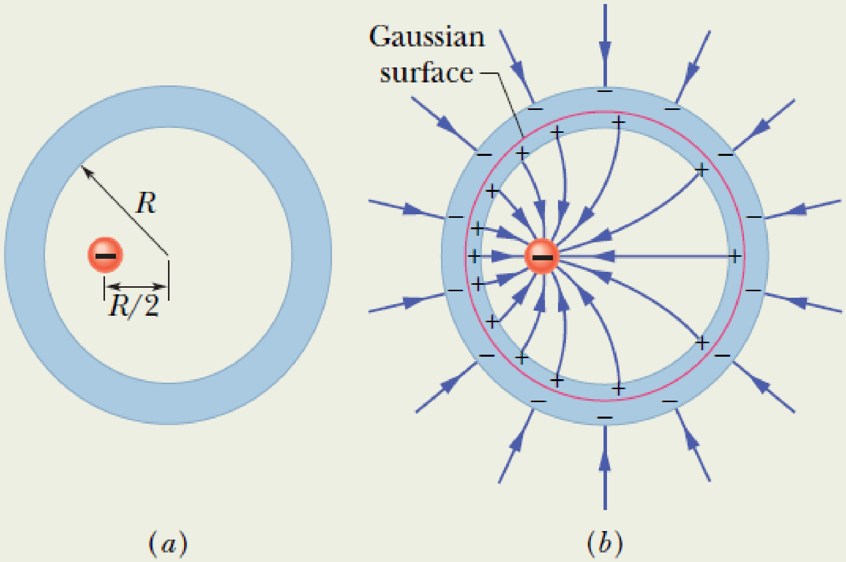

Fig. 23-11 (*a*) A negative point charge is located within a spherical metal shell that is electrically neutral. (*b*) As a result, positive charge is nonuniformly distributed on the inner wall of the shell, and an equal amount of negative charge is uniformly distributed on the outer wall.

Reasoning: With a point charge of $-5.0\ \mu C$ within the shell, a charge of $+5.0\ \mu C$ must lie on the inner wall of the shell in order that the net enclosed charge be zero. If the point charge were centered, this positive charge would be uniformly distributed along the inner wall. However, since the point charge is off-center, the distribution of positive charge is skewed, as suggested by Fig. 23-11*b*, because the positive charge tends to collect on the section of the inner wall nearest the (negative) point charge.

Because the shell is electrically neutral, its inner wall can have a charge of $+5.0\ \mu C$ only if electrons, with a total charge of $-5.0\ \mu C$, leave the inner wall and move to the outer wall. There they spread out uniformly, as is also suggested by Fig. 23-11*b*. This distribution of negative charge is uniform because the shell is spherical and because the skewed distribution of positive charge on the inner wall cannot produce an electric field in the shell to affect the distribution of charge on the outer wall. Furthermore, these negative charges repel one another.

The field lines inside and outside the shell are shown approximately in Fig. 23-11*b*. All the field lines intersect the shell and the point charge perpendicularly. Inside the shell the pattern of field lines is skewed because of the skew of the positive charge distribution. Outside the shell the pattern is the same as if the point charge were centered and the shell were missing. In fact, this would be true no matter where inside the shell the point charge happened to be located.

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23.7 Applying Gauss' Law and Cylindrical Symmetry:

$$\varepsilon_0 \Phi = q_{enc},$$

$$\varepsilon_0 E(2\pi r h) = \lambda h,$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \text{(line of charge).}$$

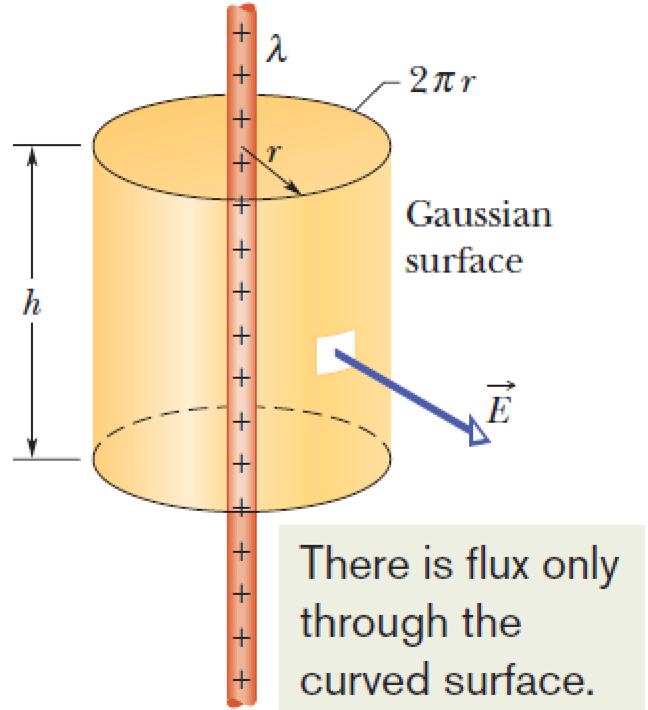

Fig. 23-12 A Gaussian surface in the form of a closed cylinder surrounds a section of a very long, uniformly charged, cylindrical plastic rod.

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Example, Gauss' Law and an upward streamer in a lightning storm:

Upward streamer in a lightning storm. The woman in Fig. 23-13 was standing on a lookout platform in the Sequoia National Park when a large storm cloud moved overhead. Some of the conduction electrons in her body were driven into the ground by the cloud's negatively charged base (Fig. 23-14*a*), leaving her positively charged. You can tell she was highly charged because her hair strands repelled one another and extended away from her along the electric field lines produced by the charge on her.

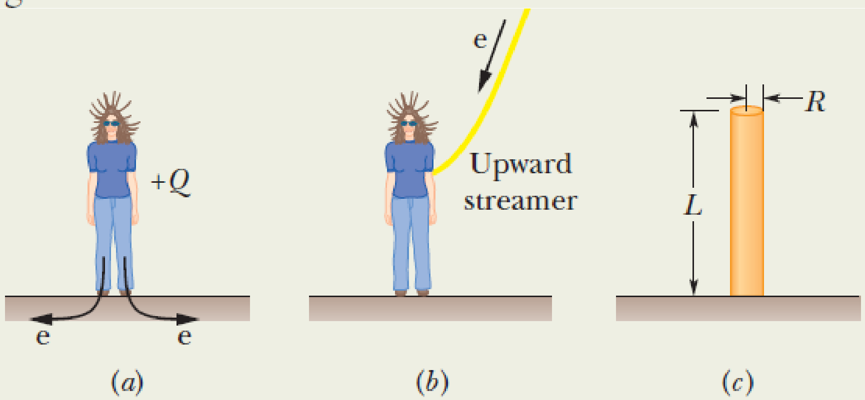


Fig. 23-14 (*a*) Some of the conduction electrons in the woman's body are driven into the ground, leaving her positively charged. (*b*) An upward streamer develops if the air undergoes electrical breakdown, which provides a path for electrons freed from molecules in the air to move to the woman. (*c*) A cylinder represents the woman.

Let's model her body as a narrow vertical cylinder of height $L = 1.8$ m and radius $R = 0.10$ m (Fig. 23-14*c*). Assume that charge Q was uniformly distributed along the cylinder and that electrical breakdown would have occurred if the electric field magnitude along her body had exceeded the critical value $E_c = 2.4$ MN/C. What value of Q would have put the air along her body on the verge of breakdown?

KEY IDEA

Because $R \ll L$, we can approximate the charge distribution as a long line of charge. Further, because we assume that the charge is uniformly distributed along this line, we can approximate the magnitude of the electric field along the side of her body with Eq. 23-12 ($E = \lambda/2\pi\varepsilon_0 r$).

Calculations: Substituting the critical value E_c for E, the cylinder radius R for radial distance r, and the ratio Q/L for linear charge density λ, we have

$$E_c = \frac{Q/L}{2\pi\varepsilon_0 R},$$

or

$$Q = 2\pi\varepsilon_0 R L E_c.$$

Substituting given data then gives us

$$Q = (2\pi)(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.10\ \mathrm{m}) \times (1.8\ \mathrm{m})(2.4 \times 10^6\ \mathrm{N/C})$$
$$= 2.402 \times 10^{-5}\ \mathrm{C} \approx 24\ \mu\mathrm{C}. \quad \text{(Answer)}$$

23.8 Applying Gauss' Law, Planar Symmetry
Non-conducting Sheet:

$$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q_{enc}$$

$$\varepsilon_0(EA + EA) = \sigma A$$

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(sheet of charge).}$$

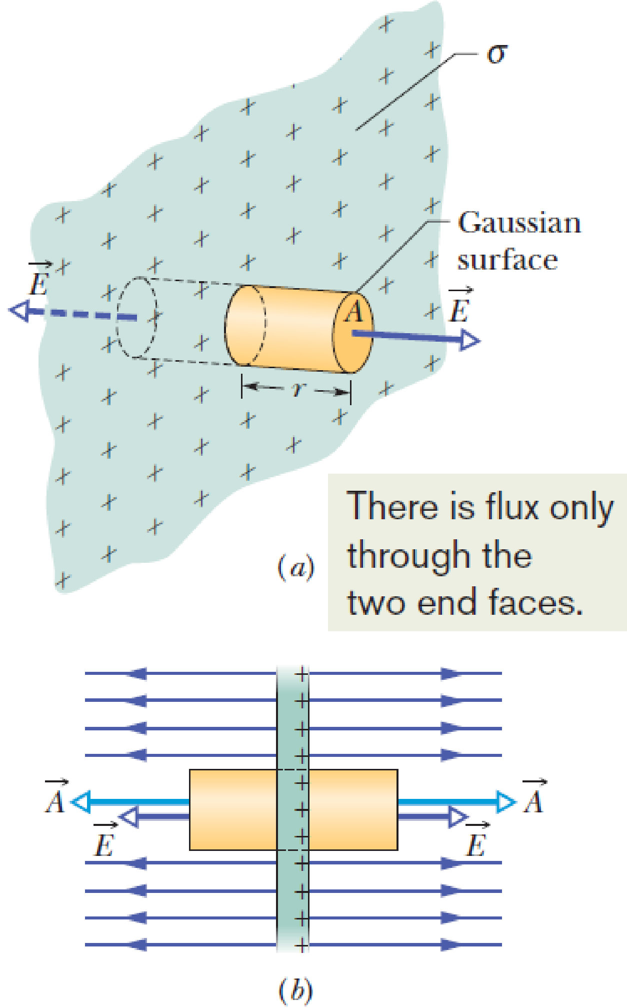


Fig. 23-15 (*a*) Perspective view and (*b*) side view of a portion of a very large, thin plastic sheet, uniformly charged on one side to surface charge density σ. A closed cylindrical Gaussian surface passes through the sheet and is perpendicular to it.

23.8 Applying Gauss' Law, Planar Symmetry

Two Conducting Plates:

$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}.$$

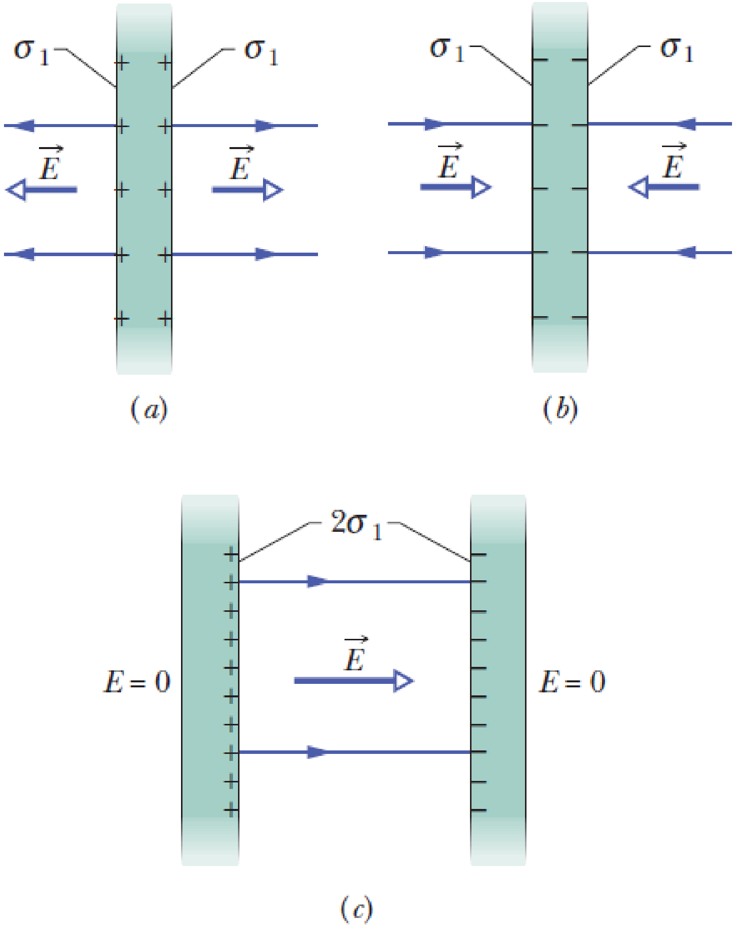


Fig. 23-16 (*a*) A thin, very large conducting plate with excess positive charge. (*b*) An identical plate with excess negative charge. (*c*) The two plates arranged so they are parallel and close.

Example, Electric Field:

Figure 23-17*a* shows portions of two large, parallel, nonconducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8\ \mu\text{C/m}^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3\ \mu\text{C/m}^2$ for the negatively charged sheet.

Find the electric field $\vec{E}$ (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.

KEY IDEA

With the charges fixed in place (they are on nonconductors), we can find the electric field of the sheets in Fig. 23-17*a* by (1) finding the field of each sheet as if that sheet were isolated and (2) algebraically adding the fields of the isolated sheets via the superposition principle. (We can add the fields algebraically because they are parallel to each other.)

Calculations: At any point, the electric field $\vec{E}_{(+)}$ due to the positive sheet is directed *away* from the sheet and, from Eq. 23-13, has the magnitude

$$E_{(+)} = \frac{\sigma_{(+)}}{2\varepsilon_0} = \frac{6.8 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.84 \times 10^5\ \text{N/C}.$$

Similarly, at any point, the electric field $\vec{E}_{(-)}$ due to the negative sheet is directed *toward* that sheet and has the magnitude

$$E_{(-)} = \frac{\sigma_{(-)}}{2\varepsilon_0} = \frac{4.3 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.43 \times 10^5\ \text{N/C}.$$

Figure 23-17*b* shows the fields set up by the sheets to the left of the sheets (L), between them (B), and to their right (R).

The resultant fields in these three regions follow from the superposition principle. To the left, the field magnitude is

$$\begin{aligned} E_L &= E_{(+)} - E_{(-)} \\ &= 3.84 \times 10^5\ \text{N/C} - 2.43 \times 10^5\ \text{N/C} \\ &= 1.4 \times 10^5\ \text{N/C}. \qquad \text{(Answer)} \end{aligned}$$

Because $E_{(+)}$ is larger than $E_{(-)}$, the net electric field $\vec{E}_L$ in this region is directed to the left, as Fig. 23-17*c* shows. To the right of the sheets, the electric field has the same magnitude but is directed to the right, as Fig. 23-17*c* shows.

Between the sheets, the two fields add and we have

$$\begin{aligned} E_B &= E_{(+)} + E_{(-)} \\ &= 3.84 \times 10^5\ \text{N/C} + 2.43 \times 10^5\ \text{N/C} \\ &= 6.3 \times 10^5\ \text{N/C}. \qquad \text{(Answer)} \end{aligned}$$

The electric field $\vec{E}_B$ is directed to the right.

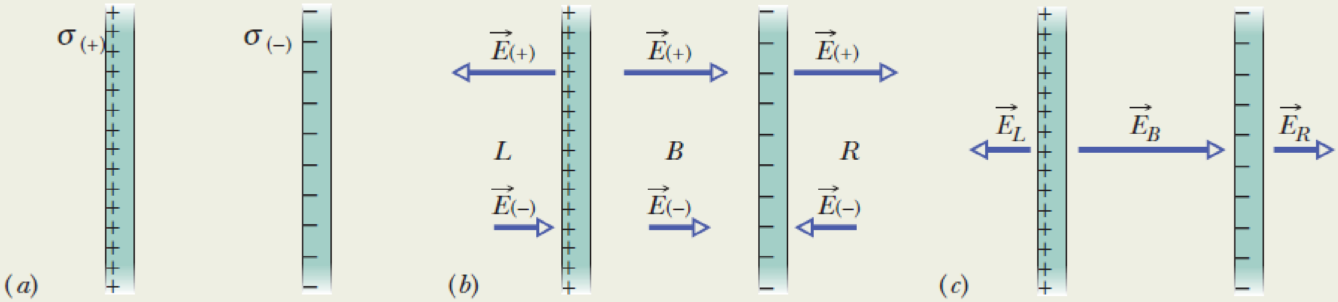


Fig. 23-17 (*a*) Two large, parallel sheets, uniformly charged on one side. (*b*) The individual electric fields resulting from the two charged sheets. (*c*) The net field due to both charged sheets, found by superposition.

23.9 Applying Gauss' Law, Spherical Symmetry:

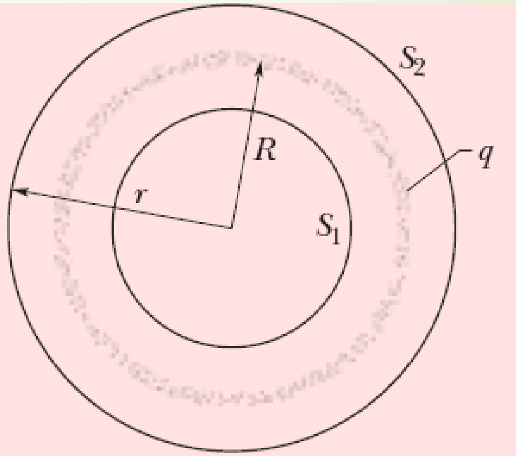
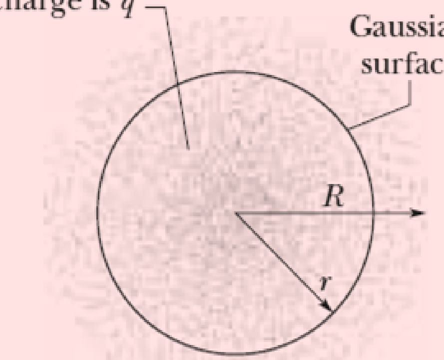


Fig. 23-18 A thin, uniformly charged, spherical shell with total charge q, in cross section. Two Gaussian surfaces S_1 and S_2 are also shown in cross section. Surface S_2 encloses the shell, and S_1 encloses only the empty interior of the shell.

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{(spherical shell, field at } r \geq R\text{).}$$

$$E = 0 \qquad \text{(spherical shell, field at } r < R\text{),}$$



$$r \leq R;\ q' = \frac{4\pi r^3}{3}\rho$$

$$r > R;\ q' = \frac{4\pi R^3}{3}\rho$$

Fig. 23-19 The dots represent a spherically symmetric distribution of charge of radius R, whose volume charge density ρ is a function only of distance from the center. The charged object is not a conductor, and therefore the charge is assumed to be fixed in position. A concentric spherical Gaussian surface with $r < R$ is shown.

$$r \leq R;\ E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{4\pi r^3}{3}\rho = \frac{\rho}{3\varepsilon_0}r$$

$$r > R;\ E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{4\pi R^3}{3}\rho = \frac{R^3\rho}{3\varepsilon_0 r^2}$$

Recall the case for gravitation.

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