EE1004 Tutorial 3 (Part 2)

1. An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation σ = 0.1 mg. Suppose that the results of five successive weighings of the same object are as follows: 3.142, 3.163, 3.155, 3.150, 3.141.

(a) Determine a 95 percent confidence interval estimate of the true weight.

(b) Determine a 99 percent confidence interval estimate of the true weight.

Answer 1. $E[X]$ = 3.1502. Use the **Normal Distribution Calculator** to find the critical z-score.

(a) For a 95 percent two-sided confidence interval:

critical probability = 1 – (1-0.95)/2 = 0.975

critical z-score = 1.960

confidence interval = 3.1502 $\pm$ 1.960(0.1)/$\sqrt{5}$ = 3.1502 $\pm$ 0.08765 = (3.06255, 3.23785)

(b) For a 99 percent two-sided confidence interval:

critical probability = 1 – (1-0.99)/2 = 0.995

critical z-score = 2.576

confidence interval = 3.1502 $\pm$ 2.576(0.1)/$\sqrt{5}$ = 3.1502 $\pm$ 0.11520 = (3.035, 3.2654)

2. The PCB (Polychlorinated Biphenyl - 多氯聯苯, which is highly toxic industrial compounds) concentration of a fish caught in Lake Michigan was measured by a technique that is known to result in an error of measurement that is normally distributed with a standard deviation of 0.08 ppm (parts per million). Suppose the results of 10 independent measurements of this fish are 11.2, 12.4, 10.8, 11.6, 12.5, 10.1, 11.0, 12.2, 12.4, 10.6.

(a) Give a 95 percent confidence interval for the PCB level of this fish.

(b) Give a 97 percent confidence interval for the PCB level of this fish.

Answer 2. E[X] = 11.48. Use the **Normal Distribution Calculator** to find the critical z-score.

(a) 11.48 ± 1.96(0.08)/$\sqrt{10}$ = 11.48 ± 0.0496 = (11.4304, 11.5296)

(b) 11.48 ± 2.17(0.08)/$\sqrt{10}$ = 11.48 ± 0.0549 = (11.4251, 11.5349)

3. The standard deviation of test scores on a certain achievement test is 11.3. If a random sample of 81 students had a sample mean score of 74.6, find a 90 percent confidence interval estimate for the average score of all students.

Answer 3. Use the **Normal Distribution Calculator** to find the critical z-score. 74.6 ± 1.645(11.3)/9 = 74.6 ± 2.065 = (72.535, 76.665)

4. The following are scores on IQ tests of a random sample of 18 students at a large university.

130, 122, 119, 142, 136, 127, 120, 152, 141,

132, 127, 118, 150, 141, 133, 137, 129, 142

Construct a 95 percent confidence interval estimate of the average IQ score of all students at the university.

Answer 4. Use the **t Distribution Calculator** to find the critical t statistic.

The sample standard deviation = STDEV.S(130, 122, 119, 142, 136, 127, 120, 152, 141, 132, 127, 118, 150, 141, 133, 137, 129, 142)= 10.21277

The sample mean = AVERAGE(130, 122, 119, 142, 136, 127, 120, 152, 141, 132, 127, 118, 150, 141, 133, 137, 129, 142)= 133.2222

degrees of freedom=18-1=17

For a 95 percent two-sided confidence interval:

critical probability = 1 – (1-0.95)/2 = 0.975

critical t statistic = 2.11

confidence interval = 133.2222 ± 2.11(10.21277)/$\sqrt{}$18 = 133.2222 ± 5.07913 = (128.14307, 138.30133)

5. Each of 20 science students independently measured the melting point of lead. The sample mean and sample standard deviation of these measurements were (in degrees centigrade) 330.2 and 15.4, respectively. Construct (a) a 95 percent and (b) a 99 percent confidence interval estimate of the true melting point of lead.

Use the **t Distribution Calculator** to find the critical t statistic.

(a) For a 95 percent two-sided confidence interval:

critical probability = 1 – (1-0.95)/2 = 0.975

critical t statistic = 2.093

confidence interval = 330.2 ± 2.093(15.4)/$\sqrt{}$20 = 330.2 ± 7.207339 = (322.993, 337.407)

(b) For a 99 percent two-sided confidence interval:

critical probability = 1 – (1-0.99)/2 = 0.995

critical t statistic = 2.861

confidence interval = 330.2 ± 2.861(15.4)/$\sqrt{}$20 = 330.2 ± 9.85198 = (320.34802, 340.05198)

[**Assignment 3**] 6. The following are the daily number of steps taken by a certain individual in 20 weekdays.

2,100 1,984 2,072 1,898

1,950 1,992 2,096 2,103

2,043 2,218 2,244 2,206

2,210 2,152 1,962 2,007

2,018 2,106 1,938 1,956

Assuming that the daily number of steps is normally distributed, construct (a) a 95 percent and (b) a 99 percent two-sided confidence interval for the mean number of steps.