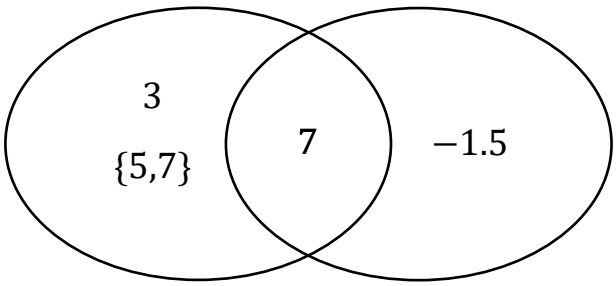
Tutorial – Week3

Q1. Let $A = \{3, 7, \{5, 7\}\}$

i. Find the power set $P(A)$.
ii. What is the cardinality of $P(A)$.
iii. $B = \{x | 2x^2 - 11x = 21\}$, use Venn diagram to show the relationship of $A \cap B$.
iv. $C = \{y | y^2 - 2y = 3\}$, determine $A \cap B \cup C$.

Answer:

i. $\{\{3\},\ \{7\},\ \{5, 7\}, \{3, 7\},\ \{3,\ \{5, 7\}\}, \{7,\ \{5, 7\}\},\ \{3, 7,\ \{5, 7\}\},\ \emptyset\}$
ii. $|P(A)| = 8$
iii. $2x^2 - 11\text{x} = 21 \Longrightarrow (2x + 3)(x - 7) = 0$
$B = \{-1.5, 7\}$

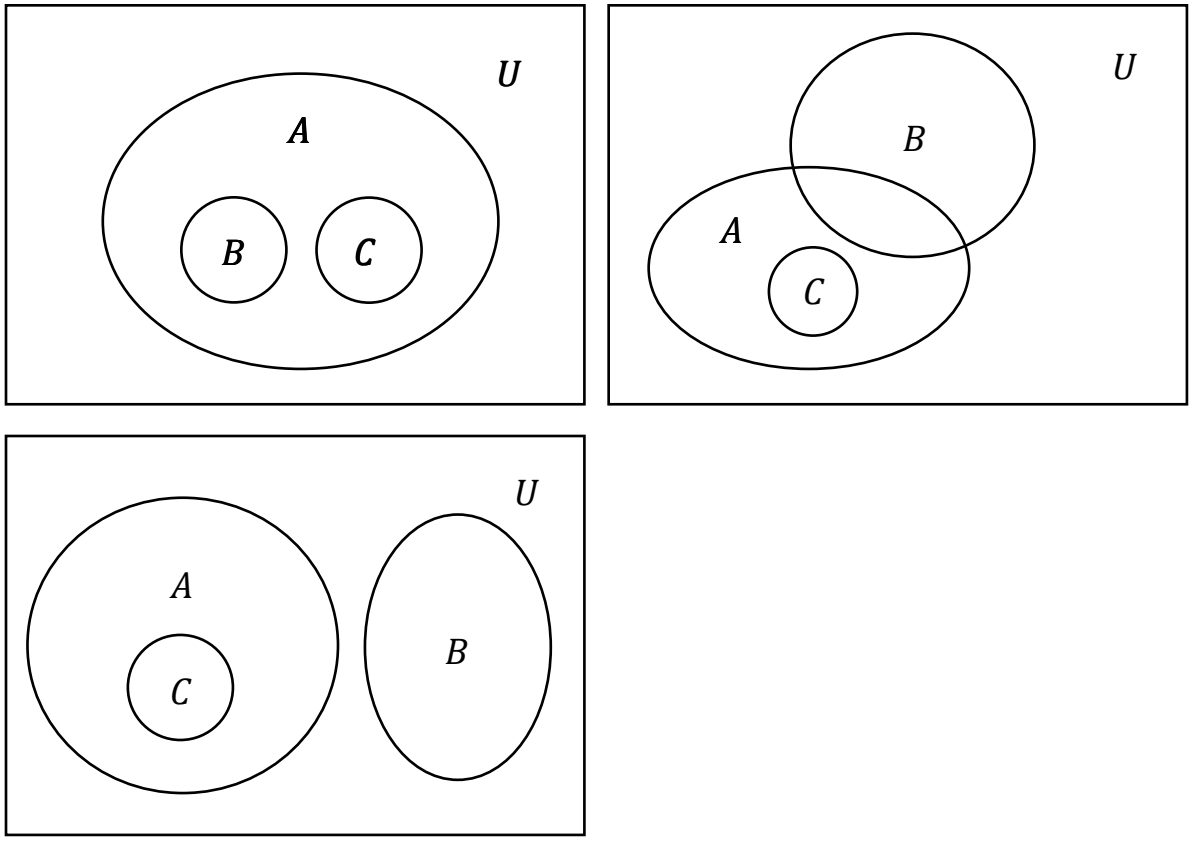


iv. $y^2 - 2y = 3 \Longrightarrow (y + 1)(y - 3) = 0$
$C = \{-1, 3\}$
$A \cap B \cup C = \{7,\ -1, 3\}$

Q2. Consider three sets A, B and C that satisfy the following 2 conditions:

$C \subseteq A$ and $\text{B} \cap \text{C} = \emptyset$. There are several possibilities regarding the relationship between these 3 sets. Draw the Venn diagram for each of these cases.

Answer:



Q3. Use IEEE 754 32-bit format. Find

1) $(256.875)_{10}$.
2) Find the second largest number in the 32-bit format.

Answer:

1) Step 1: observe the sign.
   $(256.875)_{10}$ is positive, which means the first bit will be 0.

Step 2: convert decimal to binary.

	Use 2^8
	256.875
−	256
	0.875

	Use 2^{-1}
	0.875
−	0.5
	0.375

	Use 2^{-2}
	0. 375
−	0.25
	0.125

	Use 2^{-3}
	0.125
−	0.125
	0.0

In a sum form: $2^8 + 2^{-1} + 2^{-2} + 2^{-3}$
In a binary form: $1\ 0000\ 0000.111$

Step 3: move the radix point just behind the leading bit, count how many shifts and that is the exponent. Get the significant from a scientific representation.
Move to left 8 times, $E = 8$.
Scientific representation: $1.0000\ 0000\ 111 \times 2^8$
If your significant is shorter than 23 bits add trailing zeros.
Significant = $0000\ 0000\ 1110\ 0000\ 0000\ 000$

Step 4: calculate exponent in binary.
The exponent is represented by 8 bits and is shifted by 127.
In our case, $E = 8$. So, we need to express 135 (from $8 + 127$) in binary.
In a sum form: $2^7 + 2^2 + 2^1 + 2^0 = 135$
In a binary form: $(1000\ 0111)_2$

$(256.875)_{10}$ in IEEE 754 32-bit is
$0\ 1000\ 0111\ 0000\ 0000\ 1110\ 0000\ 0000\ 000$

2) Largest Positive Number:

$$0\ 1111\ 1110\ 1111\ 1111\ 1111\ 1111\ 1111\ 111$$

Significand: 23 bits 1, $1 + (1 - 2^{-23}) = 2 - 2^{-23}$
Exponent: $(254 - 127) = 127$
Largest Number $= (2 - 2^{-23}) \times 2^{127} \cong 3.40282346 \times 10^{38}$

The second largest number:

$$0\ 1111\ 1110\ 1111\ 1111\ 1111\ 1111\ 1111\ 110$$

$$(2 - 2^{-22}) \times 2^{127} \cong 3.40282326 \times 10^{38}$$

Q4. Writing Set-Builder Notation

$$A = \{-7, -5, -3, -1, 1, 3\}$$

Answer:

$$A = \{2x + 1 | x \in Z, -4 \leq x \leq 1\}$$

Z is the set of integers.