EE1004 Tutorial 4 (Part 2)

1. Consider a trial in which a jury must decide between the hypothesis that the defendant is guilty and the hypothesis that he or she is innocent.

(a) In the framework of hypothesis testing and the Hong Kong legal system, which of the hypotheses should be the null hypothesis?

(b) What do you think would be an appropriate significance level in this situation?

Answer 1. (a) The null hypothesis should be the defendant is innocent.

(b) The significance level should be relatively small, say α = 0.01.

2. A colony of laboratory mice consists of several thousand mice. The average weight of all the mice is 32 grams with a standard deviation of 4 grams. A laboratory assistant was asked by a scientist to select 25 mice for an experiment. However, before performing the experiment the scientist decided to weigh the mice as an indicator of whether the assistant's selection constituted a random sample or whether it was made with some unconscious bias (perhaps the mice selected were the ones that were slowest in avoiding the assistant, which might indicate some inferiority about this group). If the sample mean of the 25 mice was 30.4, would this be significant evidence, at the 5 percent level of significance, against the hypothesis that the selection constituted a random sample?

2. If the selection was random, then the data would constitute a sample of size 25 from a normal population with mean 32 and standard deviation 4.

Define Hypothesis

- Null hypothesis H_0: e.g. the mean value = μ = 32
- Alternative hypothesis H_1: e.g. the mean value ≠ μ = 32

Then

- Calculate the z-score from the data set: $z = \left|\frac{\bar{x} - \mu}{\frac{\delta}{\sqrt{n}}}\right| = \left|\frac{30.4 - 32}{\frac{4}{\sqrt{25}}}\right| = 2$
- Use **Normal Distribution Calculator** to obtain the cumulative probability $P(Z < z)$ = 0.977.
- This is a **two-tailed** test so the *P*-value is obtained from

  $P = 2[1 - P(Z < z)] = 0.046$

  Since $P < \alpha = 0.05$, we reject the null hypothesis H_0.

3. A producer specifies that the mean lifetime of a certain type of battery is at least 240 hours. A sample of 18 such batteries yielded the following data.

237, 242, 232, 242, 248, 230, 244, 243, 254, 262, 234, 220, 225, 236, 232, 218, 228, 240

Assuming that the life of the batteries is approximately normally distributed, do the data indicate that the specifications are not being met at the $\alpha = 0.05$ level of significance?

3. The sample mean $\bar{x} = 237.0556$ and the sample standard deviation $s = 11.27972$

- Calculate the t statistic from the data set: $t = |\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}| = |\frac{237.0556 - 240}{\frac{11.27972}{\sqrt{18}}}| = 1.1075$
- Apply **Student's t-test** using the t statistic and degree of freedom 18 -1 = 17 to obtain the cumulative probability $P(T < t) = 0.8582$.
- This is a **one-tailed** test so the *P*-value is obtained from

  $P = 1 - P(T < t) = 0.1418$

  Since $P \geq \alpha$, we accept the null hypothesis.