Trigonometric identities

$$\sin^2\theta+\cos^2\theta=1 \qquad 1+\tan^2\theta=\sec^2\theta \qquad 1+\cot^2\theta=\csc^2\theta$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta} \qquad \cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$\csc\theta=\frac{1}{\sin\theta} \qquad \sec\theta=\frac{1}{\cos\theta} \qquad \cot\theta=\frac{1}{\tan\theta}$$

$$\sin(-\theta)=-\sin\theta \qquad \cos(-\theta)=\cos\theta \qquad \tan(-\theta)=-\tan\theta$$

$$\pi \text{ rad}=180^\circ$$

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x \qquad \sin\left(\frac{\pi}{2}+x\right)=\cos x$$

$$\cos\left(\frac{\pi}{2}-x\right)=\sin x \qquad \cos\left(\frac{\pi}{2}+x\right)=-\sin x$$

Compound Angle Formulae

$$\sin(A-B)=\sin A\cos B-\cos A\sin B$$
$$\sin(A+B)=\sin A\cos B+\cos A\sin B$$

$$\cos(A-B)=\cos A\cos B+\sin A\sin B$$
$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$$

Double angle formulae

Using the compound angle formulae for $\sin(A+A)$ and $\cos(A+A)$, we have the following:

$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A$$

Half angle formulae

Using the double angle formula $\cos 2A=\cos^2 A-\sin^2 A$ and the identity $\sin^2 A+\cos^2 A=1$, we have the following:

$$\cos^2 A=\frac{1}{2}(1+\cos 2A)$$

$$\sin^2 A=\frac{1}{2}(1-\cos 2A)$$

Sum-to-product and product-to-sum formulae

Using the formulae for $\sin(A \pm B)$ and $\cos(A \pm B)$, we have the following:

$$\sin A\cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$$

$$\cos A\sin B = \frac{1}{2}[\sin(A+B)-\sin(A-B)]$$

$$\cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$$

$$\sin A\sin B = -\frac{1}{2}[\cos(A+B)-\cos(A-B)]$$

These are called the **Product-to-Sum Formulae**.

Putting $A = \frac{x+y}{2}$ and $B = \frac{x-y}{2}$ in the above formulae, we obtain the following:

$$\sin x+\sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x-\sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x+\cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x-\cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

These are called the **Sum-to-Product Formulae**.