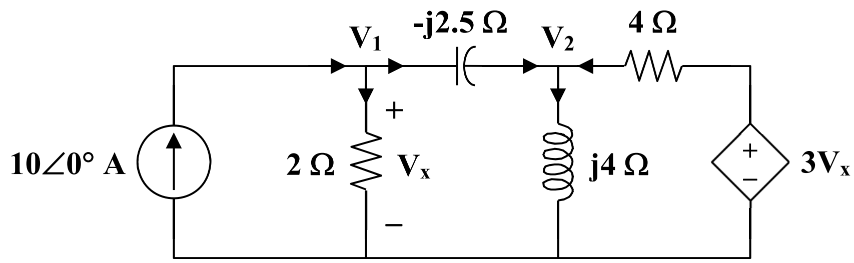
P.P.10.1 $10\cos(2t) \longrightarrow 10\angle 0°, \quad \omega = 2$

$2\text{ H} \longrightarrow j\omega L = j4$

$0.2\text{ F} \longrightarrow \dfrac{1}{j\omega C} = \text{-}j2.5$

Hence, the circuit in the frequency domain is as shown below.



At node 1, $$10 = \frac{\mathbf{V}_1}{2} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{\text{-}\,j2.5}$$

$$100 = (5 + j4)\,\mathbf{V}_1 - j4\mathbf{V}_2 \qquad (1)$$

At node 2, $$\frac{\mathbf{V}_2}{j4} = \frac{\mathbf{V}_1 - \mathbf{V}_2}{\text{-}\,j2.5} + \frac{3\mathbf{V}_x - \mathbf{V}_2}{4} \qquad \text{where } \mathbf{V}_x = \mathbf{V}_1$$

$$\text{-}\,j2.5\mathbf{V}_2 = j4(\mathbf{V}_1 - \mathbf{V}_2) + 2.5(3\mathbf{V}_1 - \mathbf{V}_2)$$

$$0 = \text{-}(7.5 + j4)\,\mathbf{V}_1 + (2.5 + j1.5)\,\mathbf{V}_2 \qquad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 5 + j4 & \text{-}\,j4 \\ \text{-}(7.5 + j4) & 2.5 + j1.5 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

where $\Delta = (5 + j4)(2.5 + j.15) - (\text{-}j4)(\text{-}(7.5 + j4)) = 22.5 - j12.5 = 25.74\angle\text{-}29.05°$

$$\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \frac{\begin{bmatrix} 2.5 + j1.5 & j4 \\ 7.5 + j4 & 5 + j4 \end{bmatrix}}{22.5 - j12.5} \begin{bmatrix} 100 \\ 0 \end{bmatrix}$$

$$\mathbf{V}_1 = \frac{2.5 + j1.5}{22.5 - j12.5}(100) = \frac{2.915\angle 30.96°}{25.74\angle\text{-}29.05°}(100) = 11.325\angle 60.01° V$$

$$\mathbf{V}_2 = \frac{7.5 + j4}{22.5 - j12.5}(100) = \frac{8.5\angle 28.07°}{25.74\angle\text{-}29.05°}(100) = 33.02\angle 57.12° V$$

In the time domain,

$v_1(t) =$ **11.325cos(2t + 60.01°) V**

$v_2(t) =$ **33.02cos(2t + 57.12°) V**

P.P.10.2 The only non-reference node is a supernode.

$$\frac{75-\mathbf{V}_1}{4} = \frac{\mathbf{V}_1}{j4} + \frac{\mathbf{V}_2}{-j} + \frac{\mathbf{V}_2}{2}$$

$$75 - \mathbf{V}_1 = -j\,\mathbf{V}_1 + j4\mathbf{V}_2 + 2\mathbf{V}_2$$

$$75 = (1-j)\,\mathbf{V}_1 + (2+j4)\,\mathbf{V}_2 \qquad (1)$$

The supernode gives the constraint of

$$\mathbf{V}_1 = \mathbf{V}_2 + 100\angle 60° \qquad (2)$$

Substituting (2) into (1) gives

$$75 = (1-j)(40\angle 60°) + (3+j3)\,\mathbf{V}_2$$

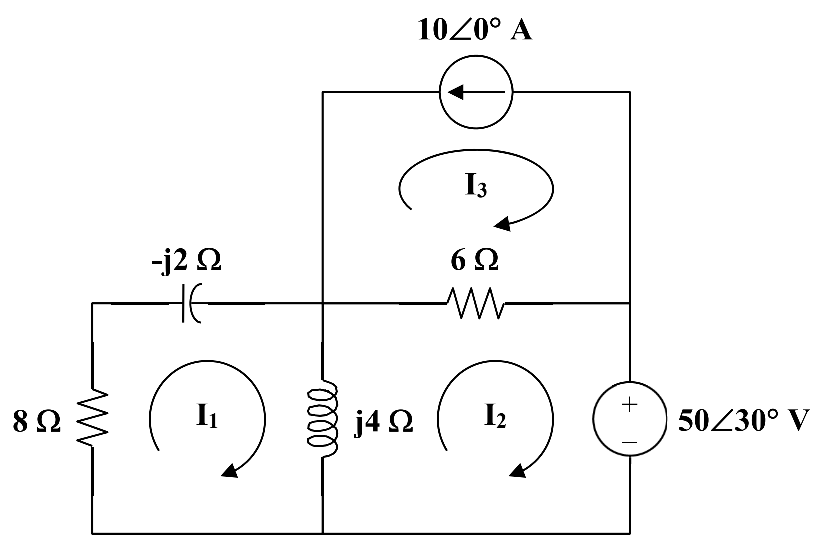
$$\mathbf{V}_2 = \frac{75-(1-j)(100\angle 60°)}{3+j3} = \frac{71.62\angle 210.72°}{4.243\angle 45°} = 16.881\angle 165.72°$$

$$\mathbf{V}_1 = \mathbf{V}_2 + 100\angle 60° = (-16.358 + j4.17) + (50 + j86.6)$$

$$\mathbf{V}_1 = 33.64 + j90.77$$

Therefore, $\mathbf{V}_1 = \mathbf{96.8\angle 69.66°\ V}$, $\mathbf{V}_2 = \mathbf{16.88\angle 165.72°\ V}$

P.P.10.3 Consider the circuit below.



For mesh 1, $(8 - \mathrm{j}2 + \mathrm{j}4)\,\mathbf{I}_1 - \mathrm{j}4\,\mathbf{I}_2 = 0$

$$(8 + \mathrm{j}2)\,\mathbf{I}_1 = \mathrm{j}4\,\mathbf{I}_2 \qquad (1)$$

For mesh 2, $(6 + j4)\mathbf{I}_2 - j4\mathbf{I}_1 - 6\mathbf{I}_3 + 50\angle 30° = 0$

For mesh 3, $\mathbf{I}_3 = -10$

Thus, the equation for mesh 2 becomes

$$(6+j4)\mathbf{I}_2 - j4\mathbf{I}_1 = -60 - 50\angle 30° \qquad (2)$$

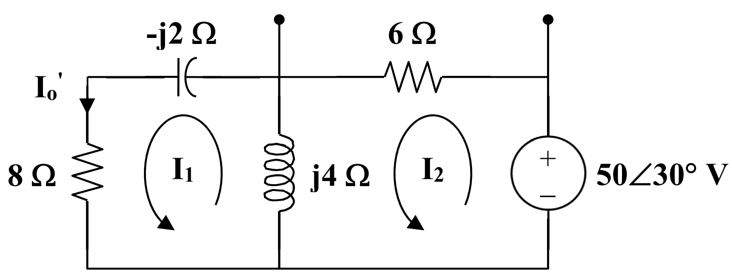
From (1), $$\mathbf{I}_2 = \frac{8+\text{j}2}{\text{j}4}\mathbf{I}_1 = (0.5-\text{j}2)\mathbf{I}_1 \qquad (3)$$

Substituting (3) into (2),

$$(6+j4)(0.5-j2)\mathbf{I}_1 - j4\mathbf{I}_1 = -60 - 50\angle 30°$$
$$(11-j14)\mathbf{I}_1 = -(103.3+j25)$$
$$\mathbf{I}_1 = \frac{-(103.3+j25)}{11-j14}$$

Hence, $$\mathbf{I}_o = -\mathbf{I}_1 = \frac{103.3+j25}{11-j14} = \frac{106.28\angle 13.605°}{17.804\angle -51.843°}$$
$$\mathbf{I}_o = \mathbf{5.969\angle 65.45° \text{ A}}$$

P.P.10.5 Let $\mathbf{I}_o = \mathbf{I}_o^{'} + \mathbf{I}_o^{''}$, where $\mathbf{I}_o^{'}$ and $\mathbf{I}_o^{''}$ are due to the voltage source and current source respectively. For $\mathbf{I}_o^{'}$ consider the circuit in Fig. (a).



(a)

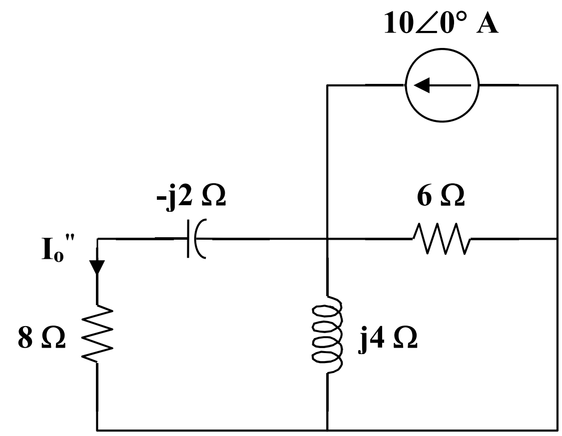
For mesh 1, $$(8+\text{j}2)\mathbf{I}_1 - \text{j}4\mathbf{I}_2 = 0$$
$$\mathbf{I}_2 = (0.5-\text{j}2)\mathbf{I}_1 \qquad (1)$$

For mesh 2, $$(6+j4)\mathbf{I}_2 - j4\mathbf{I}_1 - 50\angle 30° = 0 \qquad (2)$$

Substituting (1) into (2),

$$(6+j4)(0.5-j2)\mathbf{I}_1 - j4\mathbf{I}_1 = 50\angle 30°$$
$$\mathbf{I}_o^{'} = \mathbf{I}_1 = \frac{50\angle 30°}{11-j14} = 0.4 + j2.78$$

For $\mathbf{I}_o''$ consider the circuit in Fig. (b).



(b)

Let $\quad \mathbf{Z}_1 = 8 - \mathrm{j}2\,\Omega, \qquad \mathbf{Z}_2 = 6 \,||\, \mathrm{j}4 = \dfrac{\mathrm{j}24}{6+\mathrm{j}4} = 1.846 + \mathrm{j}2.769\,\Omega$

$$\mathbf{I}_o'' = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(10) = \frac{(10)(1.846 + j2.769)}{9.846 + j0.77} = 2.082 + j2.65$$

Therefore, $\quad \mathbf{I}_o = \mathbf{I}_o' + \mathbf{I}_o'' = 2.48 + j5.43$

$\mathbf{I}_o$ = **5.97∠65.45° A**

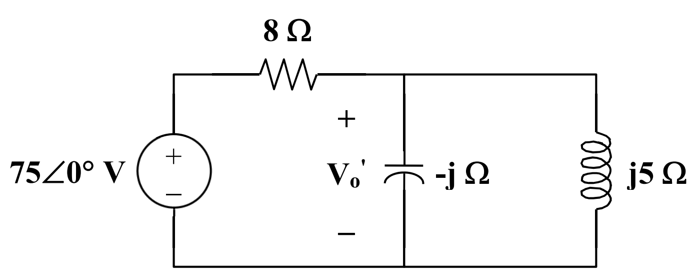
P.P.10.6 Let $\mathrm{v}_o = \mathrm{v}_o' + \mathrm{v}_o''$, where v_o' is due to the voltage source and v_o'' is due to the current source. For v_o', we remove the current source.

$$75\sin(5t) \longrightarrow 75\angle 0°, \quad \omega = 5$$

$$0.2\text{ F} \longrightarrow \frac{1}{\mathrm{j}\omega C} = \frac{1}{\mathrm{j}(5)(0.2)} = \text{-j}$$

$$1\text{ H} \longrightarrow \mathrm{j}\omega L = \mathrm{j}(5)(1) = \mathrm{j}5$$

The circuit in the frequency domain is shown in Fig. (a).



(a)

Note that $\quad$ $-j \parallel j5 = -j1.25$

By voltage division,

$$\mathbf{V}_o' = \frac{-j1.25}{8 - j1.25}(75) = 11.577\angle -81.12°$$

Thus, $\quad$ $v_o' = 11.577\sin(5t - 81.12°)V$

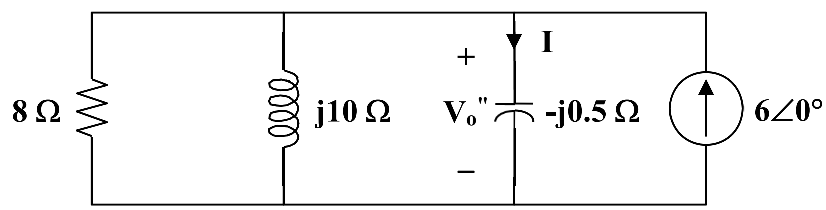
For v_o'', we remove the voltage source.

$$6\cos(10t) \longrightarrow 6\angle 0°, \quad \omega = 10$$

$$0.2\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(10)(0.2)} = -j0.5$$

$$1\text{ H} \longrightarrow j\omega L = j(10)(1) = j10$$

The corresponding circuit in the frequency domain is shown in Fig (b).



(b)

Let $\quad$ $\mathbf{Z}_1 = -j0.5$, $\quad$ $\mathbf{Z}_2 = 8 \parallel j10 = \dfrac{j80}{8 + j10} = 4.878 + j3.9$

By current division,

$$\mathbf{I} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(6)$$

$$\mathbf{V}_o'' = \mathbf{I}(-j0.5) = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}(6)(-j0.5) = \frac{-j(14.631 + j11.7)}{4.878 + j3.4}$$

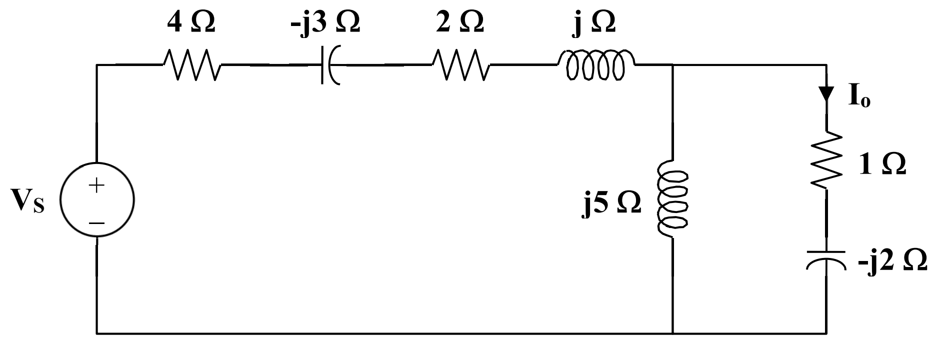
$$\mathbf{V}_o'' = \frac{18.735\angle -51.36°}{5.94\angle 34.88°} = 3.154\angle -86.24°$$

Thus, $\quad$ $v_o'' = 3.154\cos(10t - 86.24°)$

Therefore, $\quad$ $v_o = v_o' + v_o''$

$v_o =$ **[11.577sin(5t – 81.12°) + 3.154cos(10t – 86.24°)] V**

P.P.10.7 If we transform the current source to a voltage source, we obtain the circuit shown in Fig. (a).

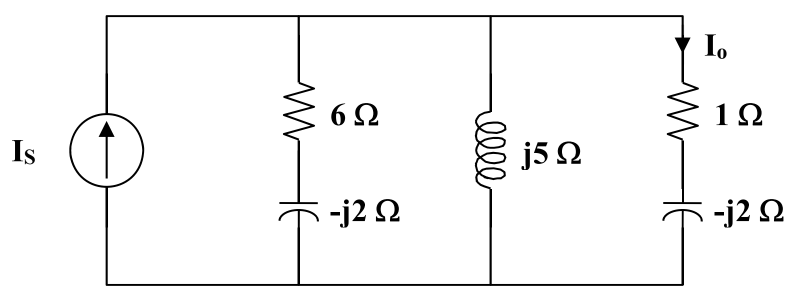


(a)

$$\mathbf{V}_s = \mathbf{I}_s\,\mathbf{Z}_s = (j12)(4 - j3) = 36 + j48$$

We transform the voltage source to a current source as shown in Fig. (b).

Let $\mathbf{Z} = 4 - \text{j}3 + 2 + \text{j} = 6 - \text{j}2$. Then, $\mathbf{I}_s = \dfrac{\mathbf{V}_s}{\mathbf{Z}} = \dfrac{36 + j48}{6 - j2} = 4.5 + j9$.



(b)

Note that $\mathbf{Z} \parallel \text{j}5 = \dfrac{(6 - \text{j}2)(\text{j}5)}{6 + \text{j}3} = \dfrac{10}{3}(1 + \text{j})$.

By current division,

$$\mathbf{I}_o = \frac{\frac{10}{3}(1 + j)}{\frac{10}{3}(1 + j) + (1 - j2)}(4.5 + j9)$$

$$\mathbf{I}_o = \frac{-60 + j120}{13 + j4} = \frac{134.16\angle 116.56^\circ}{13.602\angle 17.1^\circ}$$

$$\mathbf{I}_\text{o} = \mathbf{9.863\angle 99.46^\circ\ A}$$

P.P.10.8 When the voltage source is set equal to zero,

$$\mathbf{Z}_{th} = 10 + (\text{-j4}) \,||\, (6 + \text{j2})$$

$$\mathbf{Z}_{th} = 10 + \frac{(\text{-j4})(6+\text{j2})}{6 - \text{j2}}$$

$$\mathbf{Z}_{th} = 10 + 2.4 - \text{j3.2}$$

$$\mathbf{Z}_{th} = \mathbf{(12.4 - j3.2)\ \Omega}$$

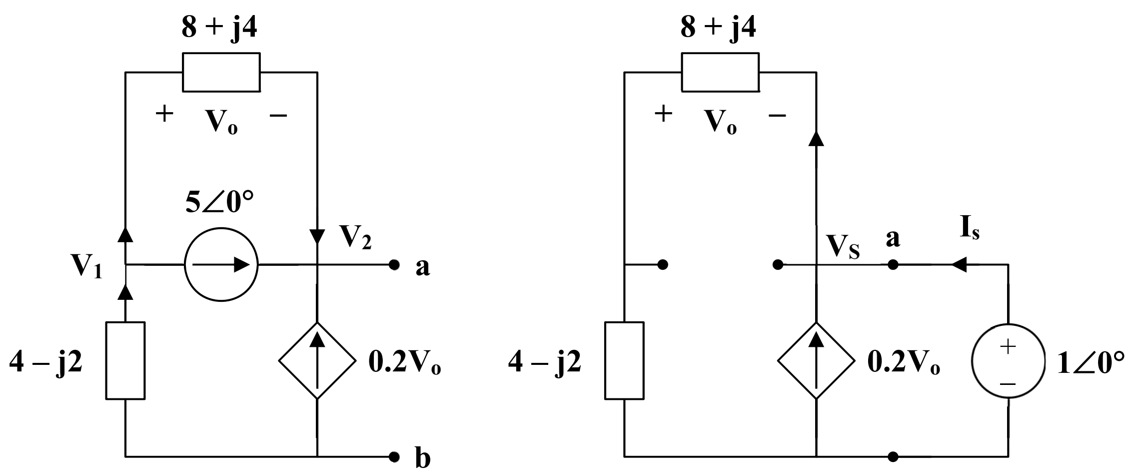
By voltage division,

$$\mathbf{V}_{th} = \frac{-j4}{6 + j2 - j4}(100\angle 20°) = \frac{(-j4)(100\angle 20°)}{6 - j2}$$

$$\mathbf{V}_{th} = \frac{(4\angle -90°)(100\angle 20°)}{6.325\angle -18.43°}$$

$$\mathbf{V}_{th} = \mathbf{63.24\angle{-51.57°}\ V}$$

P.P.10.9 To find $\mathbf{V}_{th}$, consider the circuit in Fig. (a).



(a) (b)

At node 1,
$$\frac{0 - \mathbf{V}_1}{4 - j2} = 5 + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4}$$

$$-(2 + j)\mathbf{V}_1 = 50 + (1 - j0.5)(\mathbf{V}_1 - \mathbf{V}_2)$$

$$50 = (1 - j0.5)\mathbf{V}_2 - (3 + j0.5)\mathbf{V}_1 \qquad (1)$$

At node 2,
$$5 + 0.2\mathbf{V}_o + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4} = 0,$$
where $\mathbf{V}_o = \mathbf{V}_1 - \mathbf{V}_2$.

Hence, the equation for node 2 becomes

$$5 + 0.2(\mathbf{V}_1 - \mathbf{V}_2) + \frac{\mathbf{V}_1 - \mathbf{V}_2}{8 + j4} = 0$$

$$\mathbf{V}_1 = \mathbf{V}_2 - \frac{50}{3 + j0.5} \qquad (2)$$

Substituting (2) into (1),

$$50 = (1 - j0.5)\mathbf{V}_2 - (3 + j0.5)\mathbf{V}_2 + (50)\frac{3 + j0.5}{3 - j0.5}$$

$$0 = -50 - (2 + j)\mathbf{V}_2 + \frac{50}{37}(35 + j12)$$

$$\mathbf{V}_2 = \frac{-2.702 + j16.22}{2 + j} = 7.35\angle 72.9°$$

$$\mathbf{V}_{th} = \mathbf{V}_2 = \mathbf{7.35\angle 72.9°\ V}$$

To find $\mathbf{Z}_{th}$, we remove the independent source and insert a 1-V voltage source between terminals a-b, as shown in Fig. (b).

At node a, $\quad \mathbf{I}_s = -0.2\mathbf{V}_o + \dfrac{\mathbf{V}_s}{8 + j4 + 4 - j2}$

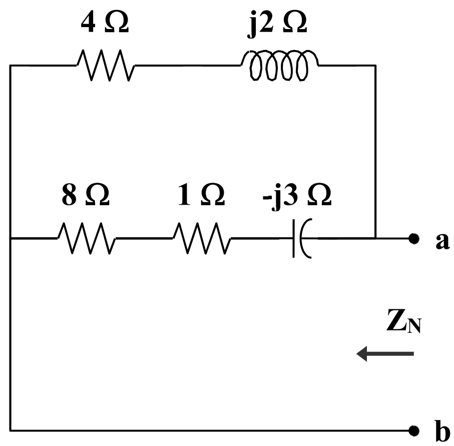
But, $\quad \mathbf{V}_s = 1 \quad$ and $\quad -\mathbf{V}_o = \dfrac{8 + j4}{8 + j4 + 4 - j2}\mathbf{V}_s$

So, $\quad \mathbf{I}_s = (0.2)\dfrac{8 + j4}{12 + j2} + \dfrac{\mathbf{1}}{12 + j2} = \dfrac{2.6 + j0.8}{12 + j2}$

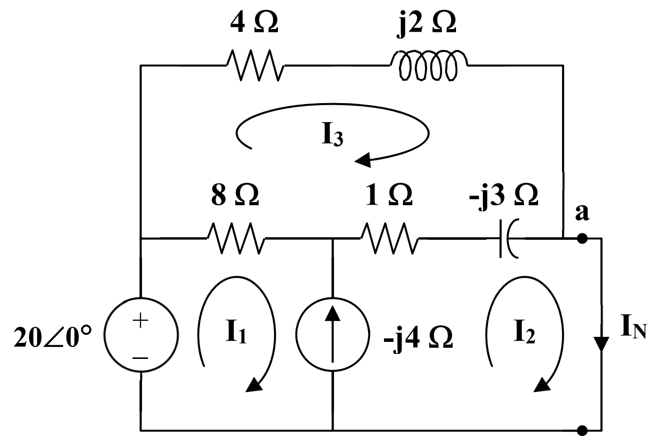
and $\quad \mathbf{Z}_{th} = \dfrac{\mathbf{V}_s}{\mathbf{I}_s} = \dfrac{1}{\mathbf{I}_s} = \dfrac{12 + j2}{2.6 + j0.8} = \dfrac{12.166\angle 9.46°}{2.72\angle 17.10°}$

$$\mathbf{Z}_{th} = \mathbf{4.473\angle{-7.64}°\ \Omega}$$

P.P.10.10 To find $\mathbf{Z}_N$, consider the circuit in Fig. (a).



(a)



(b)

$$\mathbf{Z}_N = (4 + j2) \,||\, (9 - j3) = \frac{(4 + j2)(9 - j3)}{13 - j}$$

$$\mathbf{Z}_N = \mathbf{(3.176 + j0.706)\ \Omega}$$

To find $\mathbf{I}_N$, short-circuit terminals a-b as shown in Fig. (b). Notice that meshes 1 and 2 form a supermesh.

For the supermesh, $$-20 + 8\mathbf{I}_1 + (1 - j3)\mathbf{I}_2 - (9 - j3)\mathbf{I}_3 = 0 \qquad (1)$$

Also, $$\mathbf{I}_1 = \mathbf{I}_2 + j4 \qquad (2)$$

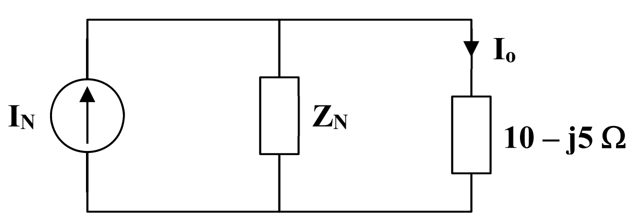
For mesh 3, $$(13 - j)\mathbf{I}_3 - 8\mathbf{I}_1 - (1 - j3)\mathbf{I}_2 = 0 \qquad (3)$$

Solving for $\mathbf{I}_2$, we obtain

$$\mathbf{I}_N = \mathbf{I}_2 = \frac{50 - j62}{9 - j3} = \frac{79.65\angle\text{-}51.11^\circ}{9.487\angle\text{-}18.43^\circ}$$

$$\mathbf{I}_N = \mathbf{8.396\angle{-32.68^\circ}\ A}$$

Using the Norton equivalent, we can find $\mathbf{I}_o$ as in Fig. (c).



(c)

By current division,

$$\mathbf{I}_o = \frac{\mathbf{Z}_N}{\mathbf{Z}_N + 10 - j5}\mathbf{I}_N = \frac{3.176 + j0.706}{13.176 - j4.294}(8.396\angle\text{-}32.68°)$$

$$\mathbf{I}_o = \frac{(3.254\angle 12.53°)(8.396\angle\text{-}32.68°)}{13.858\angle\text{-}18.05°}$$

$$\mathbf{I}_o = \mathbf{1.9714\angle{-2.10°}\ A}$$

P.P.11.7 $i(t) = \begin{cases} 16t & 0 < t < 1 \\ 32 - 16t & 1 < t < 2 \end{cases}$ T = 2

$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2\ dt = \frac{1}{2}\left[\int_0^1 (16t)^2\ dt + \int_1^2 (32 - 16t)^2\ dt\right]$$

$$I_{rms}^2 = \frac{256}{2}\left[\int_0^1 t^2\ dt + \int_1^2 (4 - 4t + t^2)\ dt\right]$$

$$I_{rms}^2 = 128\left[\frac{1}{3} + \left(4t - 2t^2 + \frac{t^3}{3}\right)\Big|_1^2\right] = \frac{256}{3}$$

$$I_{rms} = \sqrt{\frac{256}{3}} = \mathbf{9.238\ A}$$

$$P = I_{rms}^2 R = \left(9.238^2\right)(9) = \mathbf{768\ w}$$

P.P.11.8 T = π , $v(t) = 100\sin(t), \quad 0 < t < \pi$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\ dt = \frac{1}{\pi}\int_0^\pi (100\sin(t))^2\ dt$$

$$V_{rms}^2 = \frac{10^4}{\pi}\int_0^\pi \frac{1}{2}\left[1 - \cos(2t)\right]dt = 5000$$

V_{rms} = **70.71 V**

$$P = \frac{V_{rms}^2}{R} = \frac{5000}{6} = \mathbf{833.3\ W}$$