Tutorial – Week5

Q1. Simple direct proof:

Prove "If $5|2a$ for $a \in \boldsymbol{Z}$, then $5|a$." "Read as 5 divides a"

Note: Remind you 5|20. It means $\exists q \in \mathbf{Z}, (5)(q) = 20$ or $\frac{20}{5} = q$.

Proof:

If $5|2a \Rightarrow \exists x \in \mathbf{Z}$, $(5)(x) = 2a$,

we need prove $5|a \Rightarrow \exists k \in \mathbf{Z}, (5)(k) = a$.

$\because a \in Z \ \therefore 2a$ is even.

Frim the above, we know $(5)(x)$ must be even

And $\because$ 5 is odd. $\therefore x$ must be even.

It means $x = 2k$ for some $k \in \boldsymbol{Z}$, by the definition of an even integer.

Then $\exists k \in \boldsymbol{Z}, (5)(2k) = 2a \ \Rightarrow \exists k \in \boldsymbol{Z}, (5)(k) = a$

We proved $5|a$.

Q2. Example 1 on contrapositive proof.

Prove "If n is an integer and $3n + 2$ is odd, then n is odd."

Proof:

Contrapositive flip the statement becomes $n \in \boldsymbol{Z}$,

"If n is NOT odd, then $3n + 2$ is NOT odd."

Or "If n is EVEN, then $3n + 2$ is EVEN."

Now assume n is even $\Rightarrow n = 2x$ for some $x \in Z$.

$3n + 2 = 3(2x) + 2 = 6x + 2 = 2(3x + 1)$ - we like this form $2(\cdots)$, it shows it is even.

$\because x \in Z \Rightarrow 2(y)$, where $y = 3x + 1$

$\therefore 3n + 2$ must be even.

There, it shows $3n + 2$ is EVEN, so "If n is EVEN, then $3n + 2$ is EVEN."

Or we proved "If n is an integer and $3n + 2$ is odd, then n is odd."

Q3. Example 2 on contrapositive proof.

Prove "If n is an integer, and $3n + 2$ is even, then n is even."

Proof:

Flip $\Rightarrow$ If n is NOT EVEN, then $3n + 2$ is NOT EVEN.

Or to prove If n is odd, then $3n + 2$ is odd.

Assume n is odd, so we rewrite $n = 2x + 1$ for some $x \in \boldsymbol{Z}$, by the definition of an odd integer.

$3n + 2 = 3(2x + 1) + 2 = 6x + 3 + 2 = 6x + 5 = 2(3x + 2) + 1 = 2(y) + 1$

y is integer.

$\therefore$ $3n + 2$ must be odd.

We proved "If n is odd, then $3n + 2$ is odd."

Or "If $3n + 2$ is even, n is even."

Q4. Proof by counter example.

"If $x, y \in R$, $\forall x\ \exists y\ x^2 > y^2$."

Is the above statement TRUE or FALSE?

Solution:

The statement is False.

Counter example:

Let $x = 0, x^2 = 0$

$\forall x\ \exists y\ x^2 > y^2$ is False as $x^2 = 0$ in the counter example.

Q5. If and only if (iff) proof.

Let $a \in Z$, prove that $33|a$ (33 divides a) iff $11|a$ and $3|a$.

Proof: (2 step2 or 2 directions proof for iff proof)

Step 1: Show $33|a \Rightarrow 11|a$ and $3|a$.

Assume $33|a \Rightarrow a = 33k$ for some k, and $k \in Z$

$a = 11(3k) \Rightarrow 11|a$ ($33|a \Rightarrow 11|a$ is proved)

Because 11 is a factor of $(11)(3k)$

Now $a = 11(3k) \Rightarrow a = 3(11k) \Rightarrow 3|a$, (because $11k$ must be integer so it means 3 divides a)

We proved the first direction: If $33|a$, then $11|a$ and $3|a$.

Step 2: We want to show $11|a$ and $3|a$, then $33|a$.

Assume $11|a, 3|a \Rightarrow a = 3k$ for some k condition (i)

$a = 11\omega$ for some ω condition (ii)

Use $a = 11\omega$ and a =3k condition (i) and (ii) to prove then $33|a$

$\Rightarrow \omega = \frac{a}{11}$ and $a = 3k \Rightarrow \omega = \frac{3k}{11}$

Since 3 cannot be divided by 11 to give integer ω.

$\omega = \frac{3k}{11} \Rightarrow \frac{k}{11}$ must give an integer, or 11 divides k.

$\Rightarrow 11|k$ (11 divides k) or $k = 11n$ for some n.

$\Rightarrow 3(11)|(3)k$ or $33|\mathrm{a}$, where $\mathrm{a} = (33)(\mathrm{k})$

Thus If $11|a$ and $3|a$, then $33|a$ is proved.

We proved "If $11|a$ and $3|a$, then $33|a$."

Based on step 1 necessary, step 2 sufficient, we prove $33|a$ iff $11|a$ and $3|a$.