Tutorial 2

1. Consider a 4-variable Boolean function. Using K-map, list the cells adjacent to cell m_{13}.

Ans:

	00	01	11	10
00	m0	m4	m12	m8
01	m1	m5	m13	m9
11	m3	m7	m15	m11
10	m2	m6	m14	m10

$A + \bar{A} = 1$

Adjacent cells to cell 13 is cell 5, 9, 12, 15

2. Plot the following functions on the K-map.
(a) $f(x, y, z) = \Sigma\, m\,(0, 1, 3, 5)$
(b) $f(a, b, c, d) = \Sigma\, m\,(2, 4, 6, 7, 15)$
(c) Identify the prime implicants and the essential prime implicants for the answers (a) and (b).
(d) Find the simplest SOP from of the above functions from the K-maps.
(e) Find the simplest POS from of the above functions from the K-maps.

Ans:
(a), (b) and (c):
$f(x, y, z) = \Sigma\, m\,(0, 1, 3, 5)$

z \ xy	00	01	11	10
0	1*			
1	1	1*		1*

3 prime implicants ($x'y'$, $y'z$, $x'z$)
3 essential prime implicants ($x'y'$, $y'z$, $x'z$)

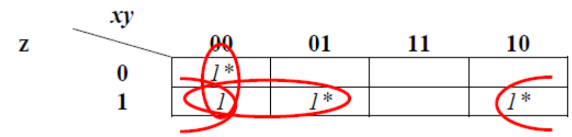
$f(a, b, c, d) = \Sigma\, m\,(2, 4, 6, 7, 15)$

cd \ ab	00	01	11	10
00		1*		
01				
11		1	1*	
10	1*	1		

	00	01	11	10
00	m0	m4	m12	m8
01	m1	m5	m13	m9
11	m3	m7	m15	m11
10	m2	m6	m14	m10

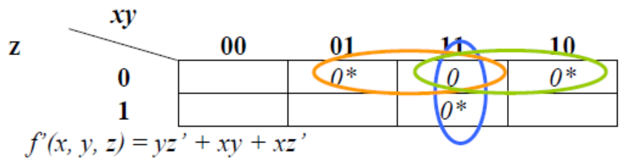
4 prime implicants ($a'bd'$, bcd, $a'cd'$, $a'bc$)
3 essential prime implicants ($a'bd'$, bcd, $a'cd'$)

(d)
$f(x, y, z) = x'y' + y'z + x'z$
$f(a, b, c, d) = a'bd' + bcd + a'cd'$



(e)

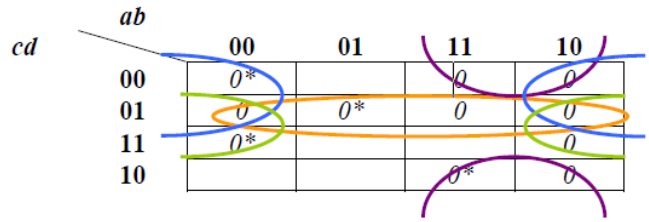
For (a)



$f'(x, y, z) = yz' + xy + xz'$

$f(x, y, z) = (y' + z)(x' + y')(x' + z)$

For (b)

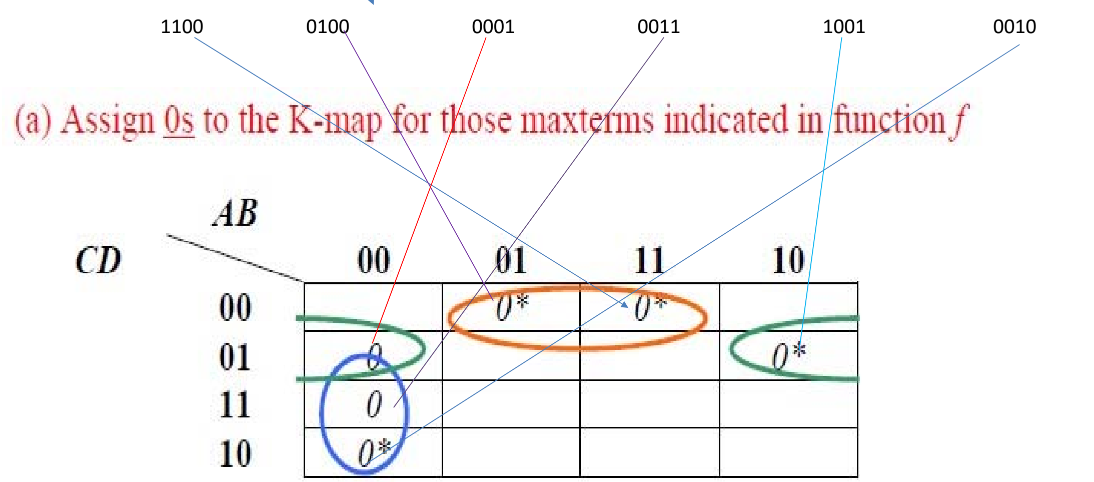


$f'(a, b, c, d) = b'c' + b'd + c'd + ad'$

$f(a, b, c, d) = (b + c)(b + d')(c + d')(a' + d)$

3. (a) Plot the following function on the K-map.

$f(A, B, C, D) = (A'+B'+C+D)(A+B'+C+D)(A+B+C+D')(A+B+C'+D')(A'+B+C+D')(A+B+C'+D)$



(b) Convert the standard POS expression in part (a) into
(i) Minimum POS expression.
(ii) Standard SOP expression.
(iii) Minimum SOP expression.

CD \ AB	00	01	11	10
00		0*	0*	
01	0			0*
11	0			
10	0*			

(b)

(i) Group the 0s to produce the complement of f in SOP form

$$f'(A, B, C, D) = A'B'C + BC'D' + B'C'D$$

Obtain f by applying De Morgan's theorem to f'

$$f(A, B, C, D) = (A + B + C')(B' + C + D)(B + C + D')$$

CD \ AB	00	01	11	10
00	1*			1*
01		1*	1*	
11		1	1	1*
10		1*	1	1*

(ii) Assign 1s to the K-map, we obtain

$$f(A, B, C, D) = \Sigma m(0, 5, 6, 7, 8, 10, 11, 13, 14, 15)$$

(iii) By grouping the 1s, we have

$$f(A, B, C, D) = AC + BC + BD + B'C'D'$$

4. Simplify the following function to SOP form using Q-M method:
$f(a,b,c,d) = \Sigma\, m(4, 5, 6, 8, 11, 13, 15)$

Ans:

List the minterms

minterms	*a b c d*
m_4	0 1 0 0
m_5	0 1 0 1
m_6	0 1 1 0
m_8	1 0 0 0
m_{11}	1 0 1 1
m_{13}	1 1 0 1
m_{15}	1 1 1 1

$A + \bar{A} = 1$

	00	01	11	10
00	m0	1 m4	m12	1 m8
01	m1	1 m5	1 m13	m9
11	m3	m7	1 m15	1 m11
10	m2	1 m6	m14	m10

Partition and Combine the minterms from neigboring group. Find the PI.

	minterms		*a b c d*	minterms		*a b c d*
One 1	m_4	✔	0 1 0 0	m_4, m_5	PI_1	0 1 0 –
	m_8	PI_6	1 0 0 0	m_4, m_6	PI_2	0 1 – 0
Two 1s	m_5	✔	0 1 0 1	m_5, m_{13}	PI_3	– 1 0 1
	m_6	✔	0 1 1 0	m_{11}, m_{15}	PI_4	1 – 1 1
Three 1s	m_{11}	✔	1 0 1 1	m_{13}, m_{15}	PI_5	1 1 – 1
	m_{13}	✔	1 1 0 1			
Four 1s	m_{15}	✔	1 1 1 1			

minterms		$a\ b\ c\ d$
m_4	✓	0 1 0 0
m_8	PI_6	1 0 0 0
m_5	✓	0 1 0 1
m_6	✓	0 1 1 0
m_{11}	✓	1 0 1 1
m_{13}	✓	1 1 0 1
m_{15}	✓	1 1 1 1

minterms		$a\ b\ c\ d$
m_4, m_5	PI_1	0 1 0 –
m_4, m_6	PI_2	0 1 – 0
m_5, m_{13}	PI_3	– 1 0 1
m_{11}, m_{15}	PI_4	1 – 1 1
m_{13}, m_{15}	PI_5	1 1 – 1

Create PI chart. Find the EPI. Reduce the chart.

	PIs	$a\ b\ c\ d$	✓ m_4	m_5	✓ m_6	✓ m_8	✓ m_{11}	m_{13}	✓ m_{15}
	PI_1	0 1 0 –	x	x					
✓	PI_2	0 1 – 0	x		x				
	PI_3	– 1 0 1		x				x	
✓	PI_4	1 – 1 1					x		x
	PI_5	1 1 – 1						x	x
✓	PI_6	1 0 0 0				x			

Further reduce the chart by covering minterms.

	PIs	$a\ b\ c\ d$	m_5	m_{13}
	PI_1	0 1 0 –	x	
✓	PI_3	– 1 0 1	x	x
	PI_5	1 1 – 1		x

PI_3 covers PI_1 and PI_5.
PI_1 and PI_5 can be eliminated

Write the final answer.

EPIs are PI_2, PI_4 and PI_6.
Select PI_3 to cover the remaining minterms m_5 and m_{13}.
So the selected PIs are PI_2, PI_3, PI_4 and PI_6.
f(a, b, c, d) = a'bd' + bc'd + acd + ab'c'd'