# Tutorial:
# Set Builder and IEEE 754

EE1001

# Set Builder – Test Q5

- $A = \{-377, -194, -83, -26, -5, -2, 1, 22, 79, 190, 373\}$. Use set builder to write it.

- Observing the trends.
- Finding potential symmetric.
- Start with the small numbers which are relatively easy for calculation.
- Example: $\{-26, -5, -2, 1, 22\}$.

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- Example: $\{-26, -5, -2, 1, 22\}$
  - Dramatic increase since 1 and $-5$. So considering $\{-5, -2, 1\}$ first.
  - $-5 + 3 = -2, -2 + 3 = 1$
  - $something + 3$, but need to consider $-5$
  - $-2 - 3 = -5, -2 - 3 + 3 = -2, -2 + 3 = 1$
  - $-2 + 3 * (-1), -2 + 3 * (0), -2 + 3 * (1)$
  - Maybe $3 * something - 2$
  - Explore $-26$ and 22, dramatic increase with absolute value but sign are kept, so maybe $n^3$.
  - $3n^3 - 2$, testing $-26 = 3 * (-2)^3 - 2$, correct; testing $22 = 3 * (2)^3 - 2$, correct.

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- Example: $\{-26, -5, -2, 1, 22\}$
  - $3n^3 - 2$, testing $-26 = 3 * (-2)^3 - 2$, correct; testing $22 = 3 * (2)^3 - 2$, correct.
- Back to check $\{-377, -194, -83, -26, -5, -2, 1, 22, 79, 190, 373\}$:
  - $3 * (-5)^3 - 2 = -377, 3 * (-4)^3 - 2 = -194, 3 * (-3)^3 - 2 = -83$;
  - $3 * (3)^3 - 2 = 79, 3 * (4)^3 - 2 = 190, 3 * (5)^3 - 2 = 373$.
- Then, the answer is $A = \{3x^3 - 2 \mid x \in \boldsymbol{Z}, -5 \le x \le 5\}$

# Set Builder – Question 1

- $S = \{5, 3, 5, 11, 21, 35\}$. Use set builder to write it.

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- $S = \{2x^2 + 3 \mid x \in \boldsymbol{Z}, -1 \le x \le 4\}$

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- Largest Positive Number:

$$0\ [1111\ 1110]\ [1111\ 1111\ 1111\ 1111\ 1111\ 111]$$

- Mantissa: 23 bits 1, $1 + (1 - 2^{-23}) = 2 - 2^{-23}$
- Exponent $[1111\ 1111]$ is reserved for infinity usage. So, the largest exponent is $[1111\ 1110]$.

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  - The largest and smallest number in the 32-bit format.

- Largest Positive Number:

$$0\ [1111\ 1110]\ [1111\ 1111\ 1111\ 1111\ 1111\ 111]$$

- Mantissa: 23 bits $1$, $1 + (1 - 2^{-23}) = 2 - 2^{-23}$
- Exponent: $(254 - 127) = 127$
- Notes:
  - Exponents range from −126 to +127 because exponents of −127 (all 0s) and +128 (all 1s) are reserved for special numbers.

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- Mantissa: 23 bits 1, $1 + (1 - 2^{-23}) = 2 - 2^{-23}$
- Exponent: $(254 - 127) = 127$
- Largest Number $= (2 - 2^{-23}) \times 2^{127} \cong 3.40282346 \times 10^{38}$

# Question 1

- Use IEEE 754 32-bit format. Find
  - The largest and smallest number in the 32-bit format.

- Smallest Number: (means smallest negative number)

  $1\ [1111\ 1110]\ [1111\ 1111\ 1111\ 1111\ 1111\ 111]$

- Smallest Number $= (2 - 2^{-23}) \times -2^{127} \cong -3.40282346 \times 10^{38}$

# Question 1

- Use IEEE 754 32-bit format. Find
  - The largest and smallest number in the 32-bit format.

- Smallest Positive Number: (means smallest number with the first digit being 0, +ve)

$$0\ [0000\ 0001]\ [0000\ 0000\ 0000\ 0000\ 0000\ 000]$$

- Smallest Positive Number $= (-1)^{(0)} \times 2^{(1-127)} \times (1+0) = 2^{-126} \cong 1.1755 \times 10^{-38}$

# Question 1

- Use IEEE 754 32-bit format. Find
  - The largest and smallest number in the 32-bit format.

- Largest negative number: (negative number that is closest to ZERO)

$$1\ [0000\ 0001]\ [0000\ 0000\ 0000\ 0000\ 0000\ 000]$$

- Largest negative number $= (-1)^{(1)} \times 2^{(1-127)} \times (1+0) = -\,2^{-126} \cong -1.1755 \times 10^{-38}$

# Question 1

- Use IEEE 754 32-bit format. Find
  - The second largest number in the 32-bit format.

- The second largest number:

0 [1111 1110] [1111 1111 1111 1111 1111 111] (largest number)

0 [1111 1110] [1111 1111 1111 1111 1111 110] (second largest number)

- $(-1)^{(0)} \times 2^{(254-127)} \times \left(1 + (1 - 2^{-22})\right)$

  $= 2^{127} \times (2 - 2^{-22}) \cong 3.40282326 \times 10^{38}$

# Question 2

- The following 2 numbers are in IEEE 754 floating point format:

$$A \quad [0][0011\ 1011][1011\ 0011\ 1000\ 0001\ 0000\ 000]$$
$$B \quad [0][0011\ 0101][1101\ 0111\ 0001\ 0100\ 0000\ 000]$$

- Find $A + B$ in IEEE 754 format.

# Question 2

- The following 2 numbers are in IEEE 754 floating point format:

$$A \quad [0][0011\ 1011][1011\ 0011\ 1000\ 0001\ 0000\ 000]$$
$$B \quad [0][0011\ 0101][1101\ 0111\ 0001\ 0100\ 0000\ 000]$$

- Find $A + B$ in IEEE 754 format.

- $B$ exponent is $0011\ 0101$, which is 6 less than $A$.
- $B$ exponent change to $0011\ 1011$.
- So, mantissa shift by 6 from $1.1101\ 0111\ 0001\ 0100\ 0000\ 000$ is

$$0.00\ 0001\ 1101\ 0111\ 0001\ 01\ \cdots$$

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$$A \quad [0][0011\ 1011][1011\ 0011\ 1000\ 0001\ 0000\ 000]$$
$$B \quad [0][0011\ 0101][1101\ 0111\ 0001\ 0100\ 0000\ 000]$$

- Find $A + B$ in IEEE 754 format.

- Focus on B because its exponent has a lower value with a difference of 6
- So, mantissa shift by 6 from $1.1101\ 0111\ 0001\ 0100\ 0000\ 000$ is

$$0.00\ 0001\ 1101\ 0111\ 0001\ 01\ \cdots$$

- $B'$ becomes $[0][0011\ 1011][0.0000\ 0111\ 0101\ 1100\ 0101 \cdots]$
- $A$ $\qquad [0][0011\ 1011][1.1011\ 0011\ 1000\ 0001\ 0000 \cdots]$

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$$A \quad [0][0011\ 1011][1011\ 0011\ 1000\ 0001\ 0000\ 000]$$
$$B \quad [0][0011\ 0101][1101\ 0111\ 0001\ 0100\ 0000\ 000]$$

- Find $A + B$ in IEEE 754 format.

$$B' \qquad [0][0011\ 1011][0.0000\ 0111\ 0101\ 1100\ 0101 \cdots]$$
$$A \qquad [0][0011\ 1011][1.1011\ 0011\ 1000\ 0001\ 0000 \cdots]$$
$$A + B' \qquad [0][0011\ 1011][1.1011\ 1010\ 1101\ 1101\ 0101 \cdots]$$

$$A + B = \quad [0][0011\ 1011][1011\ 1010\ 1101\ 1101\ 0101\ 000]$$