Appendix Mathematical formulas

Derivatives

1. $\frac{d}{dx}[cu] = cu'$	**2.** $\frac{d}{dx}[u \pm v] = u' \pm v'$	**3.** $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$	**5.** $\frac{d}{dx}[c] = 0$	**6.** $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$	**8.** $\frac{d}{dx}[\lvert u\rvert] = \frac{u}{\lvert u\rvert}(u'), \quad u \neq 0$	**9.** $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$	**11.** $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$	**12.** $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$	**14.** $\frac{d}{dx}[\cos u] = -(\sin u)u'$	**15.** $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$	**17.** $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$	**18.** $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1 - u^2}}$	**20.** $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$	**21.** $\frac{d}{dx}[\arctan u] = \frac{u'}{1 + u^2}$
22. $\frac{d}{dx}[\text{arccot}\, u] = \frac{-u'}{1 + u^2}$	**23.** $\frac{d}{dx}[\text{arcsec}\, u] = \frac{u'}{\lvert u\rvert\sqrt{u^2 - 1}}$	**24.** $\frac{d}{dx}[\text{arccsc}\, u] = \frac{-u'}{\lvert u\rvert\sqrt{u^2 - 1}}$
25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$	**26.** $\frac{d}{dx}[\cosh u] = (\sinh u)u'$	**27.** $\frac{d}{dx}[\tanh u] = (\text{sech}^2\, u)u'$
28. $\frac{d}{dx}[\coth u] = -(\text{csch}^2\, u)u'$	**29.** $\frac{d}{dx}[\text{sech}\, u] = -(\text{sech}\, u \tanh u)u'$	**30.** $\frac{d}{dx}[\text{csch}\, u] = -(\text{csch}\, u \coth u)u'$
31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2 + 1}}$	**32.** $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2 - 1}}$	**33.** $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1 - u^2}$
34. $\frac{d}{dx}[\coth^{-1} u] = \frac{u'}{1 - u^2}$	**35.** $\frac{d}{dx}[\text{sech}^{-1} u] = \frac{-u'}{u\sqrt{1 - u^2}}$	**36.** $\frac{d}{dx}[\text{csch}^{-1} u] = \frac{-u'}{\lvert u\rvert\sqrt{1 + u^2}}$

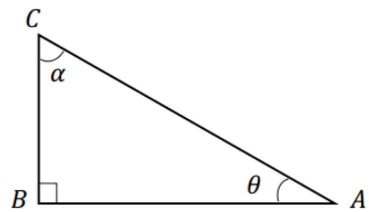
u means $u(x)$

Integrals

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
k, constant	$kx + c$	$\cos(ax+b)$	$\frac{\sin(ax+b)}{a} + c$
x^n	$\frac{x^{n+1}}{n+1} + c \quad n \neq -1$	$\tan x$	$\ln\lvert\sec x\rvert + c$
$x^{-1} = \frac{1}{x}$	$\ln\lvert x\rvert + c$	$\tan ax$	$\frac{\ln\lvert\sec ax\rvert}{a} + c$
e^x	$e^x + c$	$\tan(ax+b)$	$\frac{\ln\lvert\sec(ax+b)\rvert}{a} + c$
e^{-x}	$-e^{-x} + c$	$\text{cosec}(ax+b)$	$\frac{1}{a}\{\ln\lvert\text{cosec}(ax+b) - \cot(ax+b)\rvert\} + c$
e^{ax}	$\frac{e^{ax}}{a} + c$		
$\sin x$	$-\cos x + c$	$\sec(ax+b)$	$\frac{1}{a}\{\ln\lvert\sec(ax+b) + \tan(ax+b)\rvert\} + c$
$\sin ax$	$\frac{-\cos ax}{a} + c$		
$\sin(ax+b)$	$\frac{-\cos(ax+b)}{a} + c$	$\cot(ax+b)$	$\frac{1}{a}\{\ln\lvert\sin(ax+b)\rvert\} + c$
$\cos x$	$\sin x + c$	$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\frac{x}{a} + c$
$\cos ax$	$\frac{\sin ax}{a} + c$	$\frac{1}{a^2 + x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + c$

Note that a, b, n and c are constants. When integrating trigonometric functions, angles must be in radians.

Trigonometric functions



A right-angled triangle, ABC

Degrees and radians:

$$1° = \frac{2\pi}{360} = \frac{\pi}{180} \text{ radians}$$

Trigonometric Ratios:

$$\sin\theta = \frac{\text{side opposite to angle}}{\text{hypotenuse}} = \frac{BC}{AC} = \cos\alpha$$
$$\cos\theta = \frac{\text{side adjacent to angle}}{\text{hypotenuse}} = \frac{AB}{AC} = \sin\alpha$$
$$\tan\theta = \frac{\text{side opposite to angle}}{\text{side adjacent to angle}} = \frac{BC}{AB} = \frac{1}{\tan\alpha}$$

$-\sin x = \sin(-x)$

$\cos x = \cos(-x)$

$-\tan x = \tan(-x)$

$\tan A = \dfrac{\sin A}{\cos A}$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin^2 A + \cos^2 A = 1$

$\cos 2A = 1 - 2\sin^2 A = 2\cos^2 A - 1 = \cos^2 A - \sin^2 A$

$\sin 2A = 2 \sin A \cos A$