EE1004 Tutorial 2 (Part 2)

1. Five men and five women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let X denote the highest ranking achieved by a woman (for instance, X = 2 if the top-ranked person was male and the next-ranked person was female). Find P{$X = i$}, i = 1, 2, 3, . . ., 8, 9, 10.

Answer 1. $P_1 = 5/10$, $P_2 = 5/10 \times 5/9 = .2778$, $P_3 = 5/10 \times 4/9 \times 5/8 = .1389$.
$P_4 = 5/10 \times 4/9 \times 3/8 \times 5/7 = .0595$, $P_5 = 5/10 \times 4/9 \times 3/8 \times 2/7 \times 5/6 = .0198$,
$P_6 = 5/10 \times 4/9 \times 3/8 \times 2/7 \times 1/6 = .0040$, where $P_i = P(X = i)$.
$P_7 = P_8 = P_9 = P_{10} = 0$.

2. If $E[X] = 2$ and $E[X^2] = 8$, calculate (a) $E[(2+4X)^2]$ and (b) $E[X^2+(X+1)^2]$.

Answer 2. (a) $E[4 + 16X + 16X^2] = 164$

(b) $E[X^2 + X^2 + 2X + 1] = 21$

3. An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability p, what should it charge the customer so that its expected profit will be 10 percent of A?

Answer 3. If the company charges c, then

$$E[\text{profit}] = c - Ap$$

Therefore, $E[\text{profit}] = 0.1A$ when $c = A(p + 0.1)$.