EE 1001 Assignment 4

Q1. Write the first five terms of the sequences with nth term $a_n = (-1)^{n-1} 5^{n+1}$

Ans:

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$
$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$
$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$
$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$
$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$

Therefore, the required terms are 25, −125, 625, −3125, and 15625.

Q2.

(1) Find the number of words (with or without meaning) with a length of FIVE (5) that can be formed by using the letters of the word "apple".
(2) Two students are requested to generate one word (with or without meaning) with a length of FIVE (5), what is the probability that the two students generate exactly the same word?
(3) Two students are requested to generate one word (with or without meaning) with a length of THREE (3) letters, what is the probability that the two students generate exactly the same word?

Solution:

(1) The word 'apple' contains 5 letters, and the letters 'p' comes twice. Then, this is to find the distinguishable permutations. The number of words formed by 'accommodation' = 5!/(2!) = 60

(2) (a) For each student, he/she has 60 choices, thus, p=1/60*1/60*60=1/3600*60=1/60;

(b) If your assumption is any 5 letters from 26 letters.

(b) (i) 5 letters (non-repeat) from 26 letters (a-z):

For each student, he/she has ${}_{26}P_5$ = 7893600 choices,

Thus, p=1/7893600*1/7893600*7893600=1/7893600;

(b) (ii) 5 letters (may repeat) from 26 letters (a-z):

For each student, he/she has 26^5 = 11881376 choices,

Thus, p=1/11881376*1/11881376*11881376=1/11881376;

(3) (a) For the word contains 1 or 0 "p", the number of words are ${}_4P_3$=24;

For the word contains 2 "p", the number of words are : ${}_3C_1$ * 3!/(2!)=9

So there are total 24+9 possible words. Thus, the probability is p=1/33*1/33*33=1/33

(b) (i) 3 letters (non-repeat) from total 26 letters (a-z):

For each student, he/she has ${}_{26}P_3$ = 15600 choices,
Thus, p=1/15600*1/15600*15600=1/15600;

(b) (ii) 3 letters (may repeat) from 26 letters (a-z):
For each student, he/she has 26^3 = 17576 choices,
Thus, p=1/17576*1/17576*17576=1/17576;

Q3. (18 points) Given m, n are positive integers, $f(x) = (1+x)^m+(1+x)^n$. It is known that the coefficients of the terms x and x^2 are 7 and 9 respectively. Compute:

(1) The values of *m* and *n*; (6 points)
(2) The coefficient of the term x^3; (6 points)
(3) Use the binomial theorem to compute $(1.01)^4$ (6 points)

Solution:

(1) With the binomial theorem, we have: m+n=7; ${}_mC_2+{}_nC_2=(m^2+n^2-m-n)/2=9$;
It is computed: m=4, n=3, or m=3, n=4;

(2) No matter m=4, n=3, or m=3, n=4; $f(x) = (1+x)^3+(1+x)^4$;
Thus, the coefficient of the term x3 is ${}_3C_3+{}_4C_3 = 5$;

(3) $(1.01)^4=(1+0.01)^4={}_4C_0(1)^4(0.01)^0+{}_4C_1(1)^3(0.01)^1+{}_4C_2(1)^2(0.01)^2+{}_4C_3(1)^1(0.01)^3+{}_4C_1(1)^0(0.01)^4$
$= 1+0.04+0.0006+0.000004+0.00000001=1.04060401$

Q4. (16 points) Let A= {2, 3, 5, 6, 7, 9}; B = {3, 6, 9}, and C = {2, 4, 5, 6, 8}. Find each of the following:

(1) A ∪ B
(2) A ∩ B
(3) A ∪ C
(4) A ∩ C
(5) A - B
(6) B - A
(7) B ∪ C
(8) B ∩ C

Solution:

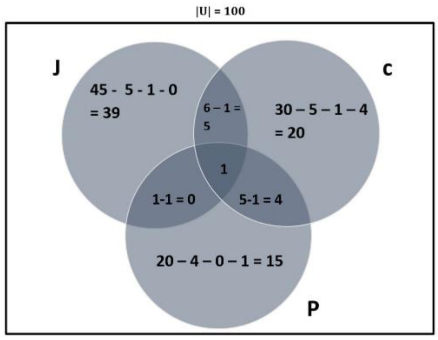
(1) {2, 3, 5, 6, 7, 9}
(2) {3, 6, 9}
(3) {2, 3, 4, 5, 6, 7, 8, 9}
(4) {2, 5, 6}
(5) {2 ,5, 7}
(6) ∅
(7) {2, 3, 4, 5, 6, 8, 9}
(8) {6}

Q5. (14 points) A large software development company employs 100 computer programmers. Amongst them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above.
Determine the number of computer programmers that are not proficient in any of these three languages.
Solution:
Let U denotes the set of all employed computer programmers and let J, C and P denotes respectively the set of programmers proficient in Java, C# and Python, respectively. Thus:

$$|U| = 100 \quad |J| = 45 \quad |C| = 30 \quad |P| = 20$$

$$|J \cap C| = 6 \quad |J \cap P| = 1 \quad |C \cap P| = 5 \quad |J \cap C \cap P| = 1$$

With Venn diagram, it is easy to obtain:



we need to determine the complement of the set $J \cup C \cup P$.
Calculate $|J \cup C \cup P|$ first before determining the complement value:
$|J \cup C \cup P| = 39 + 5 + 20 + 4 + 15 + 1 = 84$
Now calculate the complement: $|(J \cup C \cup P)'| = |U| - |J \cup C \cup P| = 100 - 84 = 16$
16 programmers are not proficient in any of the three languages.

Q6. (16 points) (1) A drawer contains 12 red and 12 blue socks, all unmatched. A person takes socks out at random in the dark. How many socks must be taken out to ensure that he has at least two blue socks? (4 points)
(2) Three students are running for a student government. There are 202 students voting, what is the minimum number of votes required to win the election? (4 points)
(3) Three students are running for a student government. There are 202 students voting, what is the minimum number of votes required to ensure the winning of the election? (6 points)
Solution:

(1) Given 12 red and 12 blue socks so, in order to take out at least 2 blue socks, first we need to take out 12 shocks (which might end up red in worst case) and then take out 2 socks (which would be definitely blue). Thus, we need to take out total 14 socks.
(2) By pigeonhole, there exists a person who has gotten at least set $\lceil 202/3 \rceil = 68$ votes. So, someone could win with a $67 - 67 - 68$ split.
(3) To ensure the wining, the one need more than 50% vote, which is 202*50% +1=102