Lecture 6

Capacitance

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Lecture 05 Review

- In Lecture 05, we learnt that potential energy is defined as the stored energy of potential.
- The potential energy of a system composes of objects in a force field is defined by the positions of the objects. By rearranging their positions, one can adjust the amount of the stored energy in the physical system.
- Potential energy is represented by the symbol U.
- One very important concept regarding potential energy is to account for how the work gets done.
- The easy way to determine this concept is by comparing the potential energy stored in the system before and after the adjustment/rearrangement.

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Lecture 05 Review

- If there are more stored energy after the adjustment, then energy is added to the system. In contrast, if the system has less potential energy after the adjustment, the system lost or released energy during the arrangement.
- We learnt the differences between conservative and non-conservative forces.
- The conservative forces are path independence. All the Electromagnetic forces considered in this course are conservative forces.
- We also introduced the term Electric Potential which is represented by the symbol V.

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Lecture 05 Review

- Electric Potential (V) is closely related to Electric Potential Energy (U), however there are some important differences:
  - V is defined as the potential energy per unit charge $V = U/q$.
  - Unlike the electric potential energy U where a minimum of two charges are needed to define its value, a single charge can create a electric potential distribution.
  - This is because the electric potential is characterized by the electric field created by the electric field distribution.
  - The potential difference V_f-V_i between any two points *i* and *f* in an electric field is equal to the negative of the line integral from *i* to *f* $V = -\int_i^f \vec{E} \cdot ds$.
  - The SI unit of the electric potential is V Volt.

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Lecture 05 Review

- Electric potential is a scalar field, or a scalar function that is defined in space V(x,y,z).
- From V(x,y,z), we can identify surfaces that have the same V values, we call these equipotential surfaces.
- There are two important properties about equipotential surfaces:
  - The electric field is perpendicular to these surfaces;
  - A test charge moving along the equipotential surface does not result in any work done.
- Finally, we can determine the positive or negative work done on a charge moving between the different potential surfaces from the product of the charge and the potential difference

$$-W = \Delta U = q\Delta V$$

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Lecture Outline

- **Chapter 25**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Capacitance
  - Capacitance and Unit
  - Parallel plate capacitors
  - Calculating capacitance of capacitors in series and in parallel
  - Energy stored in a capacitor
  - Potential energy and energy density of an electric field.

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Capacitors

A **Capacitor** is a common passive electrical component uses to store electric charges, energy and can be used to control variation time scales in a circuit.

You can find capacitors in virtually all electronic circuits and appliances.

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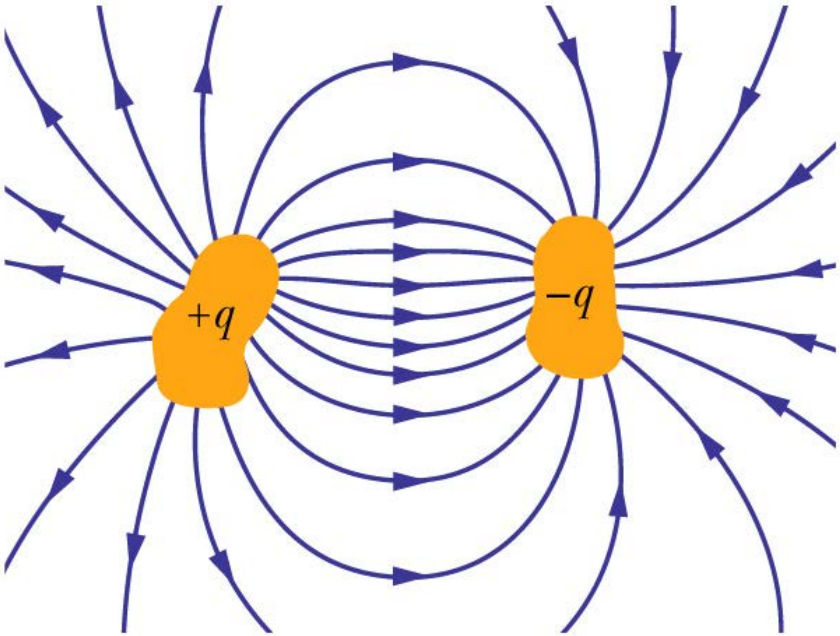
25.2: Capacitance:

A capacitor can be constructed by simply placing two conductors of any shape in isolation. No matter what their geometry, these conductors are usually called conductor ‘plates’.

Two conductors, isolated electrically from each other and from their surroundings, form a *capacitor*. When the capacitor is charged, the conductors, or *plates* as they are called, have equal but opposite charges of magnitude q.



25.2: Capacitance:

A parallel-plate capacitor, is constructed by separating two conductive plates of area *A* by a distance *d*.

Dielectric (insulator) is placed between the plates to prevent any flow of electrons between the plates.

If these plates contain no charge, their electric potentials are the same.

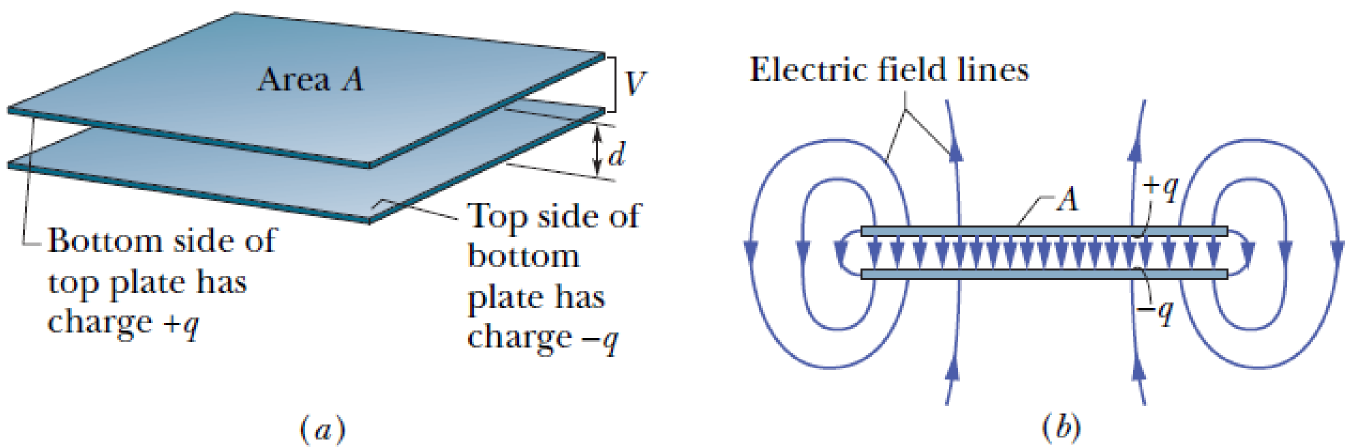


Fig. 25-3 (*a*) A parallel-plate capacitor, made up of two plates of area *A* separated by a distance *d*. The charges on the facing plate surfaces have the same magnitude *q* but opposite signs. (*b*) As the field lines show, the electric field due to the charged plates is uniform in the central region between the plates. The field is not uniform at the edges of the plates, as indicated by the "fringing" of the field lines there.

When a capacitor is charged, its plates have charges of equal magnitudes but opposite signs: $q+$ and $q-$. However, we refer to the charge of a capacitor as being q, the absolute value of these charges on the plates.
The charge q and the potential difference V for a capacitor are proportional to each other:

$$q = CV.$$

The proportionality constant *C is called the **capacitance*** of the capacitor. Its value depends only on the geometry of the plates and not on their charge or potential difference.

Capacitance is a measure of the capacitor's ability to store electric charge.

The SI unit is called the *farad (F):* **1 farad (1 F)= 1 coulomb per volt =1 C/V.**

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25.2: Charging a Capacitor:

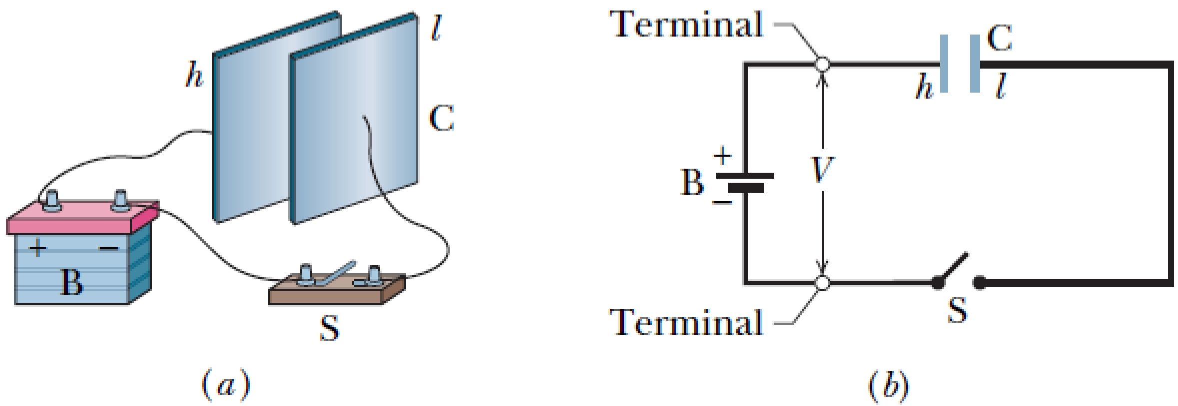

Fig. 25-4 (*a*) Battery B, switch S, and plates *h* and *l* of capacitor C, connected in a circuit. (*b*) A schematic diagram with the *circuit elements* represented by their symbols.

The circuit shown is incomplete because switch S is open; that is, the switch does not electrically connect the wires attached to it. When the switch is closed, electrically connecting those wires, the circuit is complete and charge can then flow through the switch and the wires.

As the plates become oppositely charged, that potential difference increases until it equals the potential difference V between the terminals of the battery. With the electric field zero, there is no further drive of electrons. The capacitor is then said to be fully charged, with a potential difference V and charge q.

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25.3: Calculating the Capacitance (parallel plate capacitor):

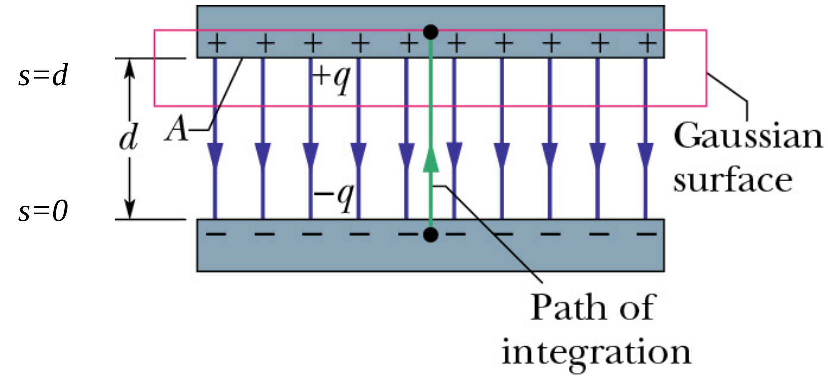
Gauss' Law

$$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q. \qquad q = \varepsilon_0 E A$$

Integrate E to get the potential

Eq. 25-25

$$V = V_f - V_i = -\int_{-}^{+} \vec{E} \cdot d\vec{s} = E \int_0^d ds = Ed$$



$q=\varepsilon_0 AV/d$

Relates q and V

$$q = CV.$$

Integrate E from the –ve plate to the +ve plate.

The E field is in opposite direction with the integration path ds

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{(parallel-plate capacitor).}$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}.$$

For the air filled parallel plate capacitor, if the region between the plate is filled with dielectric with permittivity constant ε*, then* $C = \dfrac{\varepsilon A}{d}$

25.4: Capacitors in Parallel:

$$q_1 = C_1 V, \quad q_2 = C_2 V, \quad \text{and} \quad q_3 = C_3 V.$$

$$\Downarrow$$

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$\Downarrow$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

$$\Downarrow$$

$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j \qquad (n \text{ capacitors in parallel}).}$$

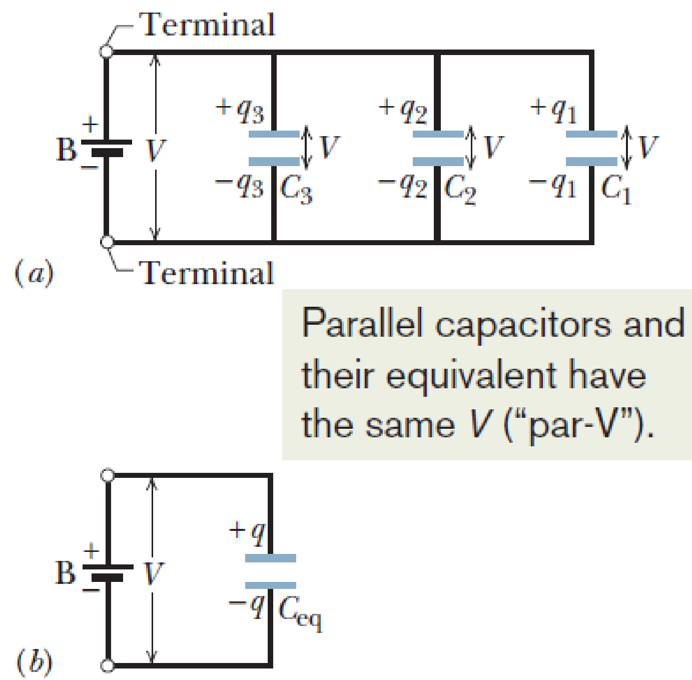


Fig. 25-8 (*a*) Three capacitors connected in parallel to battery B. The battery maintains potential difference V across its terminals and thus across *each* capacitor. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the parallel combination.

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25.4: Capacitors in Series:

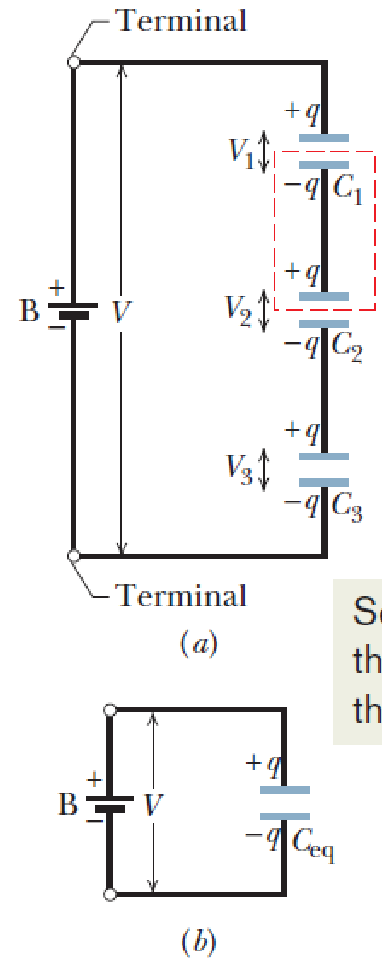
$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \quad (n \text{ capacitors in series}).$$



Series capacitors and their equivalent have the same *q* ("seri-q").

Fig. 25-9 (*a*) Three capacitors connected in series to battery B. The battery maintains potential difference *V* between the top and bottom plates of the series combination. (*b*) The equivalent capacitor, with capacitance C_{eq}, replaces the series combination.

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Example, Capacitors in Parallel and in Series:

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10*a*, across which potential difference V is applied. Assume

$$C_1 = 12.0\ \mu\text{F},\quad C_2 = 5.30\ \mu\text{F},\quad \text{and}\quad C_3 = 4.50\ \mu\text{F}.$$

⇨

$$C_{12} = C_1 + C_2 = 12.0\ \mu\text{F} + 5.30\ \mu\text{F} = 17.3\ \mu\text{F}.$$

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$= \frac{1}{17.3\ \mu\text{F}} + \frac{1}{4.50\ \mu\text{F}} = 0.280\ \mu\text{F}^{-1},$$

$$C_{123} = \frac{1}{0.280\ \mu\text{F}^{-1}} = 3.57\ \mu\text{F}. \qquad \text{(Answer)}$$

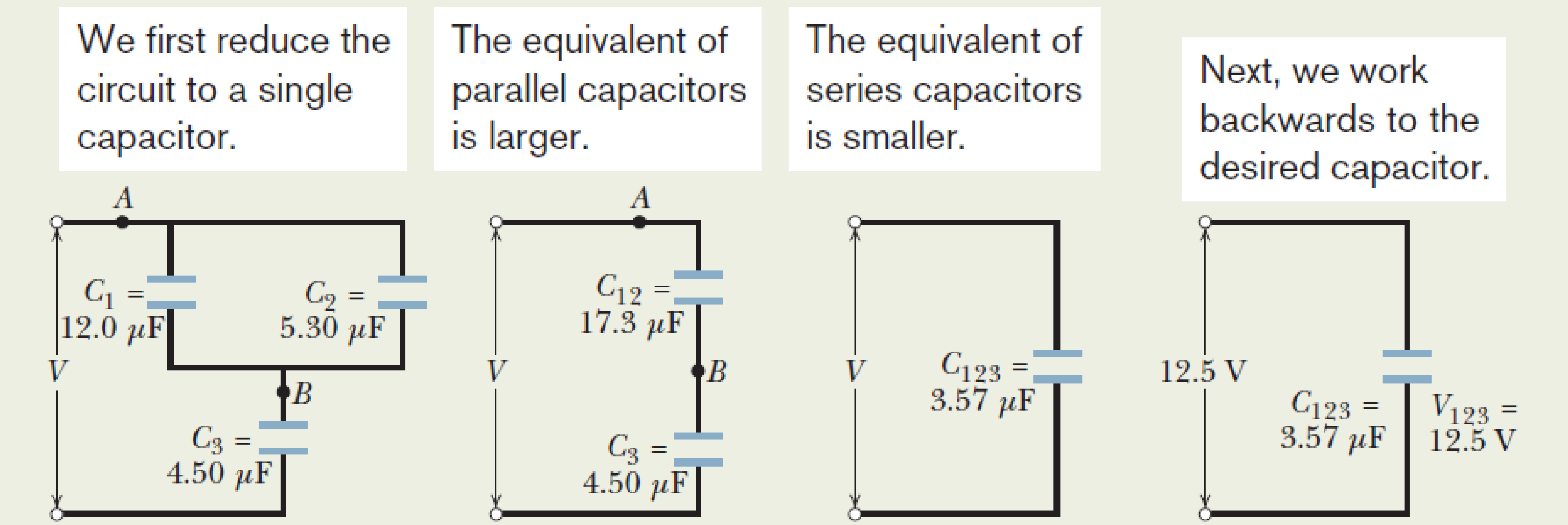

Fig. 25-10 (*a*) (*b*) (*c*) (*d*)

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Example, One Capacitor Charging up Another Capacitor:

Capacitor 1, with $C_1 = 3.55\ \mu F$, is charged to a potential difference $V_0 = 6.30$ V, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95\ \mu F$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

After the switch is closed, charge is transferred until the potential differences match.

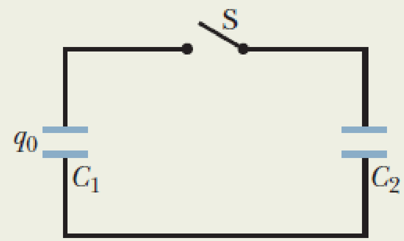

Fig. 25-11 A potential difference V_0 is applied to capacitor 1 and the charging battery is removed. Switch S is then closed so that the charge on capacitor 1 is shared with capacitor 2.

Calculations: Initially, when capacitor 1 is connected to the battery, the charge it acquires is, from Eq. 25-1,

$$q_0 = C_1 V_0 = (3.55 \times 10^{-6}\ \text{F})\ (6.30\ \text{V})$$
$$= 22.365 \times 10^{-6}\ \text{C}.$$

When switch S in Fig. 25-11 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until

$$V_1 = V_2 \qquad \text{(equilibrium)}.$$

From Eq. 25-1, we can rewrite this as

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \qquad \text{(equilibrium)}.$$

Because the total charge cannot magically change, the total after the transfer must be

$$q_1 + q_2 = q_0 \qquad \text{(charge conservation)};$$

thus

$$q_2 = q_0 - q_1.$$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}.$$

Solving this for q_1 and substituting given data, we find

$$q_1 = 6.35\ \mu C. \qquad \text{(Answer)}$$

The rest of the initial charge ($q_0 = 22.365\ \mu C$) must be on capacitor 2:

$$q_2 = 16.0\ \mu C. \qquad \text{(Answer)}$$

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25.5: Energy Stored in an Electric Field:

Consider that we are charging up a capacitor by moving a small amount of charge dq from one plate to the other plate. The amount of work dW we do moving the change dq across the plates with potential V', is

$$dW = V'\,dq' = \frac{q'}{C}\,dq'.$$

The total W needed to move all the charge q is

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

This is the energy that is stored in the charged capacitor

$$U = \frac{q^2}{2C} \qquad \text{(potential energy).}$$

$$U = \tfrac{1}{2}CV^2 \qquad \text{(potential energy).}$$

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

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25.5: Energy Density:

We define the energy density u (J/m^3) in the parallel plate capacitor by dividing the potential energy U with the volume of the capacitor.

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{(parallel-plate capac}$$

$$u = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2.$$

From slide 12

$$V = Ed$$

Eq. 25-25

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density).}}$$

Note that the energy density u is a function of the electric field E only.

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Dielectric Filled Capacitors

- We have so far studied the properties of parallel plate capacitors where the plates are separated by air.
- We will like to ask what will happen if we fill the space between the plates with materials other than air.
- We can consider two cases:
  - If we fill the space with a conductor, the charges on the plates cannot be isolated, thus C=0, V=0.
  - To maintain capacitance and keeping the plates isolated, the filled material must be an insulator, dielectric is an insulating material that can perform this function.

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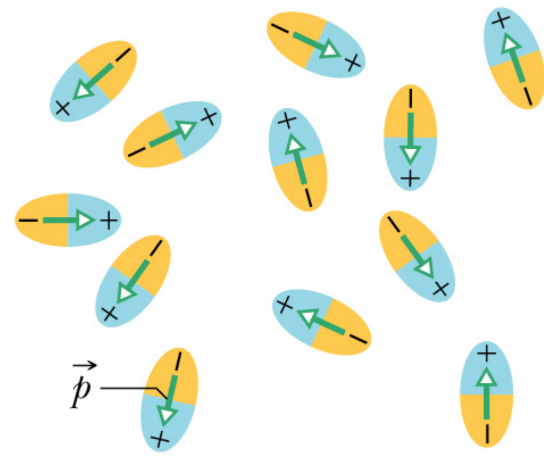
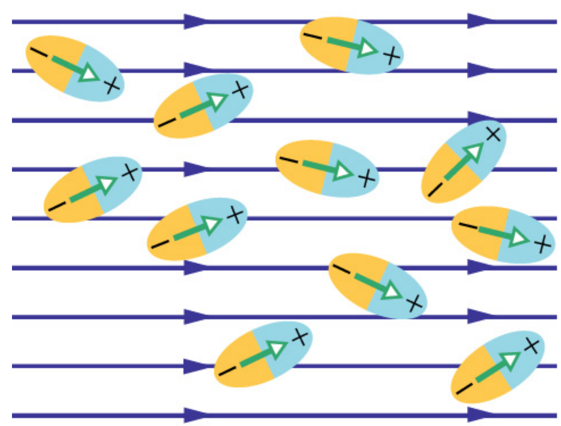
Polar Dielectrics

(a) Molecules with a permanent electric dipole moment, such as water molecules, showing their random orientation in the absence of an external electric field.



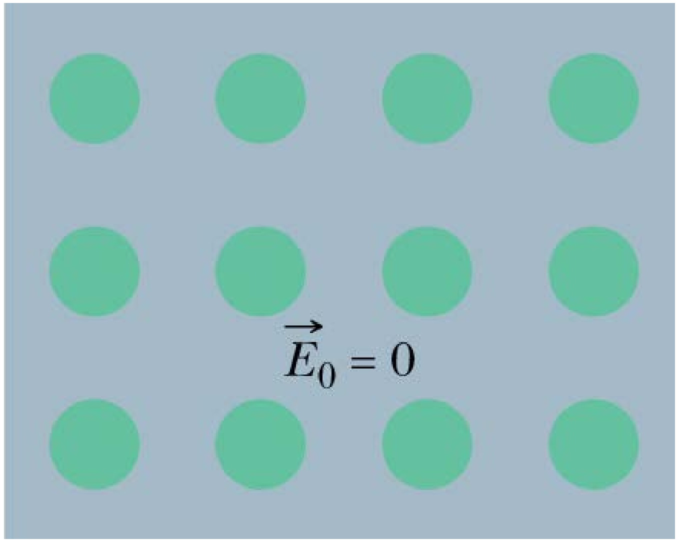
(a)

(b) Once an electric field is applied, producing partial alignment of the dipoles. Thermal agitation prevents complete alignment.

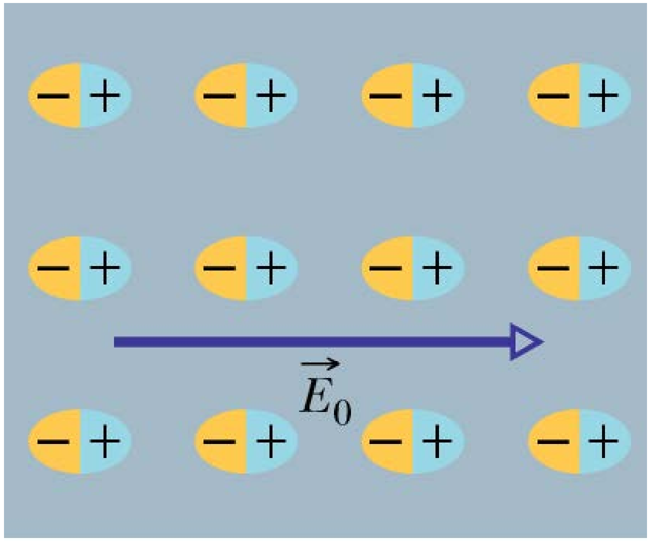


(b)

Induced Electric Dipoles



Symmetrical atoms or molecules, e.g. Ar, CO_2



An electric field distorts the electron cloud of the atoms or molecules, causing induced electric dipoles

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Dielectric

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$

e.g. Electric field due to a point charge inside a dielectric will be modified by

$$E = \frac{1}{4\pi\kappa\varepsilon_0}\frac{q}{r^2}$$

For a fixed distribution of charges in the dielectric, the electric field will be weaken by a factor of κ compares to in free space (air).

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25.8: Dielectrics and Gauss' Law:

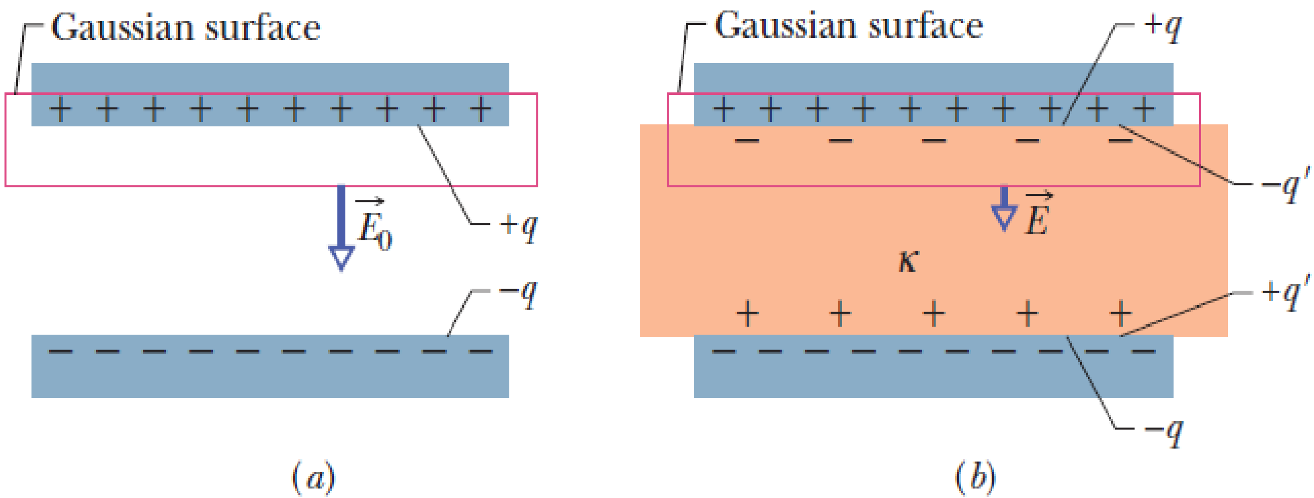

Fig. 25-16 A parallel-plate capacitor (*a*) without and (*b*) with a dielectric slab inserted. The charge q on the plates is assumed to be the same in both cases.

A *dielectric*, is an insulating material such as mineral oil or plastic, and is characterized by a numerical factor κ, called the *dielectric constant of the material.*

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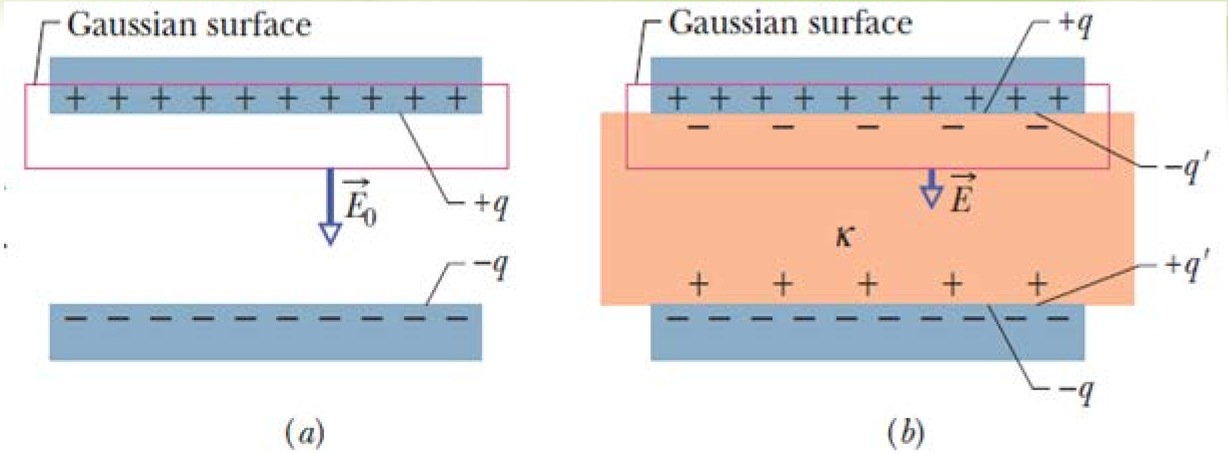
25.8: Dielectrics and Gauss' Law:



$$\varepsilon_0 \oint \vec{E}_0 \cdot d\vec{A} = \varepsilon_0 E_0 A = q \qquad\qquad \varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 E A = q - q'$$

$$E_0 = \frac{q}{\varepsilon_0 A} \qquad\qquad E = \frac{q - q'}{\varepsilon_0 A}$$

Let $q - q' = \frac{q}{\kappa}$, then $E = \frac{q}{\kappa \varepsilon_0 A} = \frac{q}{\varepsilon A}$ where $\varepsilon = \kappa \varepsilon_0$ ε *dielectric permittivity*

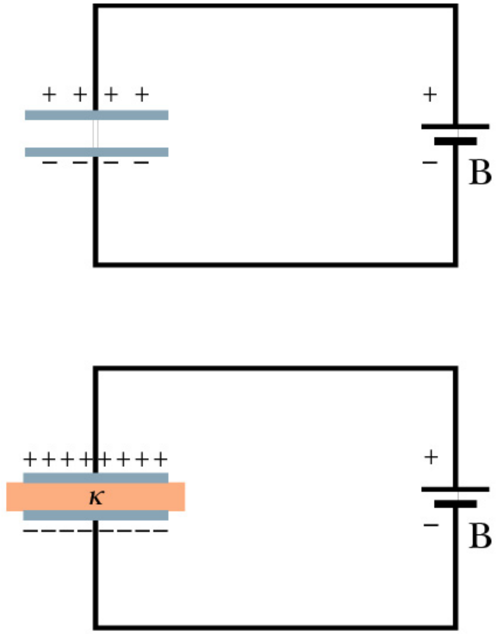
$$\varepsilon_0 \oint \kappa \vec{E} \cdot d\vec{A} = \varepsilon \oint \vec{E} \cdot d\vec{A} = q \quad \text{(Gauss's Law with dielectric)}$$ Eq. 25-36

Capacitor filled with Dielectric with constant κ

If the potential difference between the plates of a capacitor is maintained, as by battery B, the effect of a dielectric is to increase the charge on the plates.

The presence of a dielectric increases the capacitance of a capacitor by a factor κ called the dielectric constant.

When a dielectric slab is inserted between the plates, the charge q on the plates increases by a factor of κ; the additional charge is delivered to the capacitor plates by the battery.



V = a constant

$$q = \kappa C_{air} V = CV$$

$$C = \kappa C_{air}$$

25.6: Capacitor with a Dielectric:

The introduction of a dielectric limits the potential difference that can be applied between the plates to a certain value V_{max}, called the breakdown potential. Every dielectric material has a characteristic *dielectric strength*, which is the maximum value of the electric field that it can tolerate without breakdown.

It actually can increase the capacitance of the device. Recall that

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{(parallel-plate capacitor).}$$

$$u = \frac{1}{2}\varepsilon_0 E^2 \qquad \text{(energy density).}$$

$$\varepsilon_0 \Rightarrow \varepsilon = \kappa\varepsilon_0$$

Table 25-1

Some Properties of Dielectrics[a]

Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

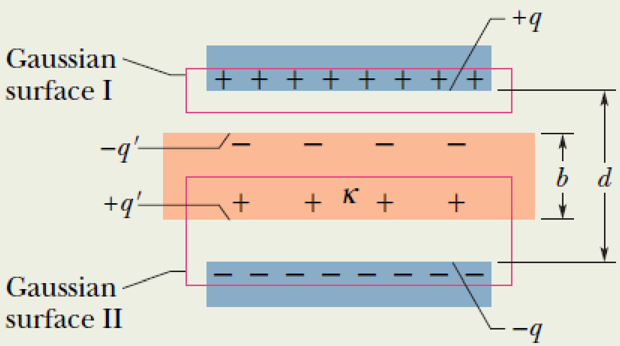
[a]Measured at room temperature, except for the water.

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$.

Example, Dielectric Partially Filling a Gap in a Capacitor:

Figure 25-17 shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference V_0 is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume $A = 115\ \text{cm}^2$, $d = 1.24\ \text{cm}$, $V_0 = 85.5\ \text{V}$, $b = 0.780\ \text{cm}$, and $\kappa = 2.61$.

Fig. 25-17 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.



(a) What is the capacitance C_0 before the dielectric slab is inserted?

Calculation: From Eq. 25-9 we have

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(115 \times 10^{-4}\ \text{m}^2)}{1.24 \times 10^{-2}\ \text{m}}$$
$$= 8.21 \times 10^{-12}\ \text{F} = 8.21\ \text{pF}. \qquad \text{(Answer)}$$

(b) What free charge appears on the plates?

Calculation: From Eq. 25-1,

$$q = C_0 V_0 = (8.21 \times 10^{-12}\ \text{F})(85.5\ \text{V})$$
$$= 7.02 \times 10^{-10}\ \text{C} = 702\ \text{pC}. \qquad \text{(Answer)}$$

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

Calculations: That surface passes through the gap, and so it encloses *only* the free charge on the upper capacitor plate. Electric field pierces only the bottom of the Gaussian surface. Because there the area vector $d\vec{A}$ and the field vector $\vec{E}_0$ are both directed downward, the dot product in Eq. 25-36 becomes

$$\vec{E}_0 \cdot d\vec{A} = E_0\, dA \cos 0° = E_0\, dA.$$

Equation 25-36 then becomes

$$\varepsilon_0 \kappa E_0 \oint dA = q.$$

The integration now simply gives the surface area A of the plate. Thus, we obtain

$$\varepsilon_0 \kappa E_0 A = q,$$

or

$$E_0 = \frac{q}{\varepsilon_0 \kappa A}.$$

We must put $\kappa = 1$ here because Gaussian surface I does not pass through the dielectric. Thus, we have

$$E_0 = \frac{q}{\varepsilon_0 \kappa A} = \frac{7.02 \times 10^{-10}\ \text{C}}{(8.85 \times 10^{-12}\ \text{F/m})(1)(115 \times 10^{-4}\ \text{m}^2)}$$
$$= 6900\ \text{V/m} = 6.90\ \text{kV/m}. \qquad \text{(Answer)}$$

Note that the value of E_0 does not change when the slab is introduced because the amount of charge enclosed by Gaussian surface I in Fig. 25-17 does not change.

Example, Dielectric Partially Filling a Gap in a Capacitor, cont.:

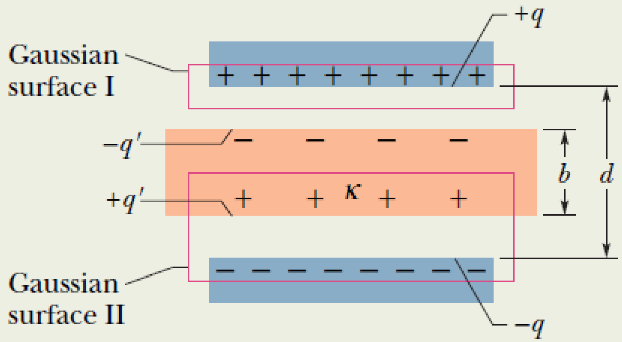


Fig. 25-17 A parallel-plate capacitor containing a dielectric slab that only partially fills the space between the plates.

(d) What is the electric field E_1 in the dielectric slab?

Calculations: That surface encloses free charge $-q$ and induced charge $+q'$, but we ignore the latter when we use Eq. 25-36. We find

$$\varepsilon_0 \oint \kappa \vec{E}_1 \cdot d\vec{A} = -\varepsilon_0 \kappa E_1 A = -q. \qquad (25\text{-}37)$$

The first minus sign in this equation comes from the dot product $\vec{E}_1 \cdot d\vec{A}$ along the top of the Gaussian surface because now the field vector $\vec{E}_1$ is directed downward and the area vector $d\vec{A}$ (which, as always, points outward from the interior of a closed Gaussian surface) is directed upward. With 180° between the vectors, the dot product is negative. Now $\kappa = 2.61$. Thus, Eq. 25-37 gives us

$$E_1 = \frac{q}{\varepsilon_0 \kappa A} = \frac{E_0}{\kappa} = \frac{6.90 \text{ kV/m}}{2.61}$$

$$= 2.64 \text{ kV/m}. \qquad \text{(Answer)}$$

(e) What is the potential difference V between the plates after the slab has been introduced?

Calculation: Within the dielectric, the path length is b and the electric field is E_1. Within the two gaps above and below the dielectric, the total path length is $d - b$ and the electric field is E_0. Equation 25-6 then yields

$$V = \int_-^+ E\,ds = E_0(d-b) + E_1 b$$

$$= (6900 \text{ V/m})(0.0124 \text{ m} - 0.00780 \text{ m})$$

$$+ (2640 \text{ V/m})(0.00780 \text{ m})$$

$$= 52.3 \text{ V}. \qquad \text{(Answer)}$$

This is less than the original potential difference of 85.5 V.

(f) What is the capacitance with the slab in place between the plates of the capacitor?

Calculation: Taking q from (b) and V from (e), we have

$$C = \frac{q}{V} = \frac{7.02 \times 10^{-10} \text{ C}}{52.3 \text{ V}}$$

$$= 1.34 \times 10^{-11} \text{ F} = 13.4 \text{ pF}. \qquad \text{(Answer)}$$

This is greater than the original capacitance of 8.21 pF.

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Example, Work and Energy when a Dielectric is inserted inside a Capacitor:

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference $V = 12.5$ V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

KEY IDEA

We can relate the potential energy U_i of the capacitor to the capacitance C and either the potential V (with Eq. 25-22) or the charge q (with Eq. 25-21):

$$U_i = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}.$$

Calculation: Because we are given the initial potential V (= 12.5 V), we use Eq. 25-22 to find the initial stored energy:

$$U_i = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(13.5 \times 10^{-12}\ \text{F})(12.5\ \text{V})^2$$
$$= 1.055 \times 10^{-9}\ \text{J} = 1055\ \text{pJ} \approx 1100\ \text{pJ}. \quad \text{(Answer)}$$

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

KEY IDEA

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change.

Calculations: Thus, we must now use Eq. 25-21 to write the final potential energy U_f, but now that the slab is within the capacitor, the capacitance is κC. We then have

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055\ \text{pJ}}{6.50}$$
$$= 162\ \text{pJ} \approx 160\ \text{pJ}. \quad \text{(Answer)}$$

When the slab is introduced, the potential energy decreases by a factor of κ.

The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162)\ \text{pJ} = 893\ \text{pJ}.$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.