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1) Analysis with single source

- Methodology [Section 10.1]
- Two examples

Alexander & Sadiku,
"Fundamentals of Electric Circuits"
7th Edition Chapters 9 &10

2) Analysis with multiple sources (single frequency)

- Nodal analysis [Section 10.2] (1 example)
- Mesh analysis [Section 10.3] (1 example)

3) Analysis by superposition (multiple frequencies)

- Circuits containing DC sources and AC sources of a single frequency (1 example)
- Circuits containing multiple AC sources with different frequencies (1 example)

4) AC Power: Instantaneous vs. Average

General methodology

1) Transform the circuit to the frequency domain

- Transform AC sources from time domain sinusoids to phasor form
- Work out impedances for a given frequency ω

2) Solve the problem using circuit techniques
3) Transform phasor form solutions back to time domain

$$v(t) = V_m \cos(\omega t + \emptyset) \Leftrightarrow \boldsymbol{V} = V_m \angle \emptyset$$

(Time-domain representation) (Phasor-domain representation)

TABLE 9.3

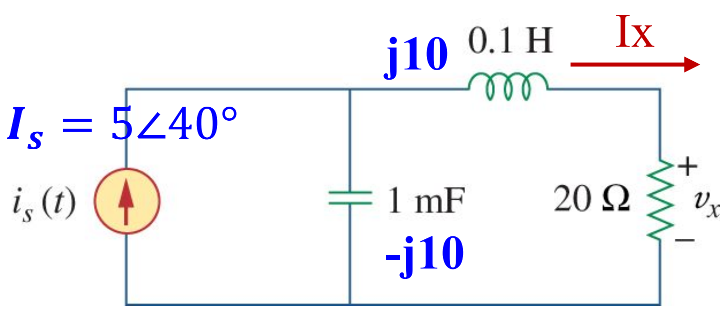
Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \dfrac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \dfrac{1}{j\omega L}$
C	$\mathbf{Z} = \dfrac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

1A) Single source analysis example 1

Alexander 9.50

Given that $i_s(t) = 5\cos(100t + 40°)$, determine $v_x(t)$



Step 1a: Transform source to phasor

$i_s(t) = 5\cos(100t + 40°) \rightarrow \boldsymbol{I_s} = 5\angle 40°, \omega = 100\,\frac{\text{rad}}{\text{s}}$

Step 1b: Work out the impedance values

1 mF → -j/(100*0.001) = -j10

0.1 H → j100*0.1 = j10

Step 2: Solve using circuit analysis methods

Using current divider rule,

$$\boldsymbol{I_x} = \left(\frac{-j10}{-j10 + j10 + 20}\right)(5\angle 40°) = (-j0.5)(5\angle 40°) = 2.5\angle -50°$$

$$\boldsymbol{V_x} = 20\boldsymbol{I_x} = 50\angle -50°$$

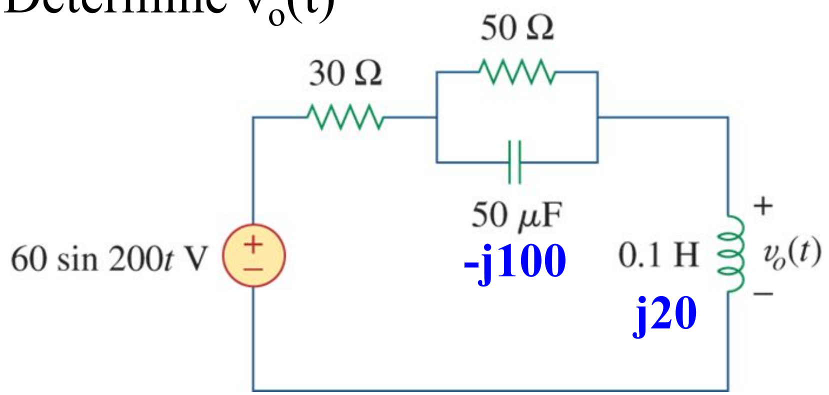
Step 3: Transform back to time domain

$$\boldsymbol{V_x} = 50\angle -50°, \omega = 100\,\frac{rad}{s} \rightarrow v_x(t) = 50\cos(100t - 50°)V$$

1B) Single source analysis example 2

Alexander 9.42

Determine $v_o(t)$



Step 1a: Transform source to phasor

$60\sin(200t) \rightarrow 60\angle-90°, \omega = 200\,\frac{\text{rad}}{\text{s}}$

$60\cos(200t - 90°)$

Step 1b: Work out the impedance values

50 μF → $-j/(200*50*10^{-6}) = -j100$

0.1 H → $j200*0.1 = j20$

Step 2: Solve using circuit analysis methods

$$50 \parallel -j100 = \frac{(50)(-j100)}{(50-j100)} = 40 - j20$$

Using voltage divider role,

$$\boldsymbol{V_o} = \left(\frac{j20}{40 - j20 + 30 + j20}\right)(60\angle-90°\,) = 17.14\angle 0°$$

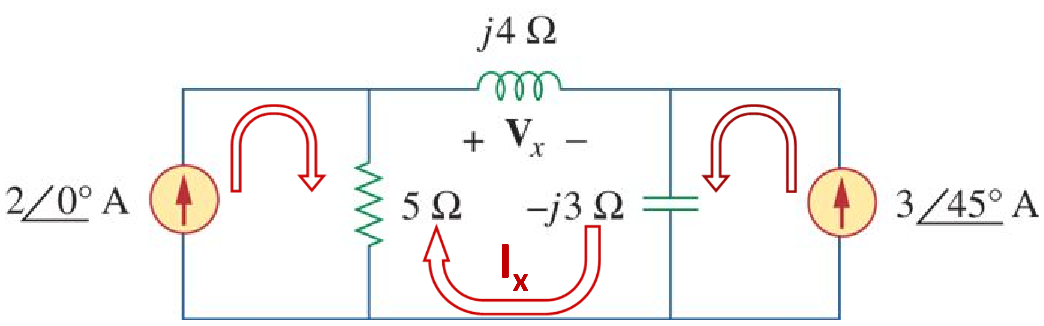
Step 3: Transform back to time domain

$$\boldsymbol{V_o} = 17.14\angle 0°, \omega = 200\,\frac{rad}{s} \rightarrow v_x(t) = 17.14\cos(200t)V$$

2A) Single freq multiple sources example 1

Focus only on STEP 2 (solving by circuit techniques) in this example

Alexander 10.16
Use **MESH CURRENT ANALYSIS** to find $\mathbf{V_x}$



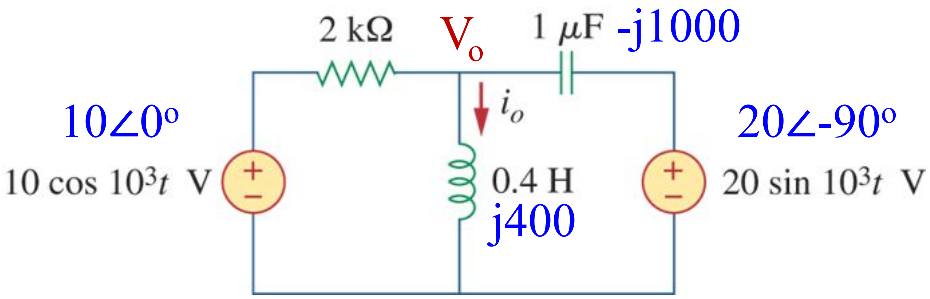
Applying KVL around mesh $\mathbf{I_x}$:
$\mathbf{I_x}$*(j4 - j3 + 5) – (2∠0°)*(5) + (3∠45°)*(-j3) = 0
⇒ $\mathbf{I_x}$ = 1.437∠48.94° A

$\mathbf{V_x}$ = (1.437∠48.94°)*(j4) = 5.749∠138.94° V

2B) Single freq multiple sources example 2

Alexander 10.26

Use nodal voltage analysis to find $i_o(t)$



Step 1a: Transform source to phasor

$10\cos(10^3 t) \rightarrow 10\angle 0^o$, $\omega = 10^3$ rad/s

$20\sin(10^3 t) \rightarrow 20\angle{-90^o}$, $\omega = 10^3$ rad/s

Step 1b: Work out the impedance values

$1\ \mu F \rightarrow -j/(10^3 * 10^{-6}) = -j1000$

$0.4\ H \rightarrow j10^3 * 0.4 = j400$

Step 2: Solve using circuit analysis methods

Applying KCL at node $\mathbf{V_o}$:

$$\frac{(10\angle 0°) - \boldsymbol{V_o}}{2000} + \frac{(20\angle - 90°) - \boldsymbol{V_o}}{-j1000} = \frac{\boldsymbol{V_o}}{j400}$$

$\Rightarrow \mathbf{V_o} = 15.8\angle 71.57^o$ **V**

$\mathbf{I_o}$ = **(**$15.8\angle 71.57^o$**)**/j400 = $39.5\angle{-18.43^o}$ mA

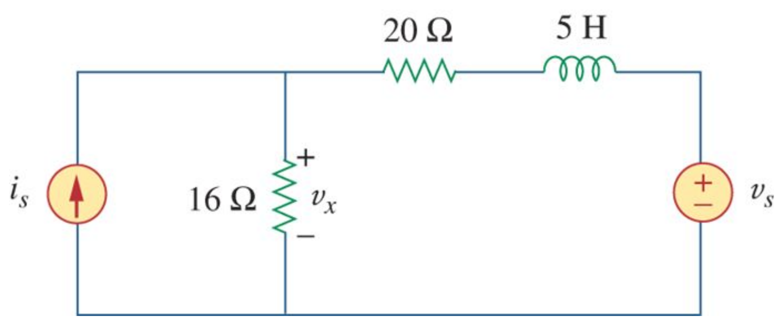
Step 3: Transform back to time domain

$\mathbf{I_o} = 39.5\angle{-18.43^o}$ mA, $\omega = 10^3$ rad/s $\rightarrow i_o(t) = 39.5\cos(10^3 t - 18.43^o)$ mA

3A) Superposition (multiple freq) example 1

Alexander 10.44 (Modified)

Given $v_s(t) = 50\cos(2t)$ and $i_s = 0.9$ A, find $v_x(t)$



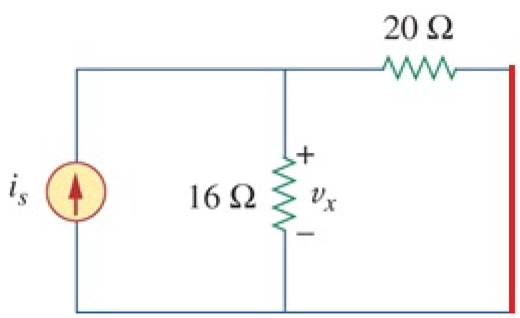
Method: Analyze circuit one frequency at a time

In the case, first analyze at DC then at 2 rad/s and then add up the two solutions **(superposition)**

At DC, redraw circuit without $v_s(t)$:

Replace voltage source with short circuit

Replace inductor with short circuit



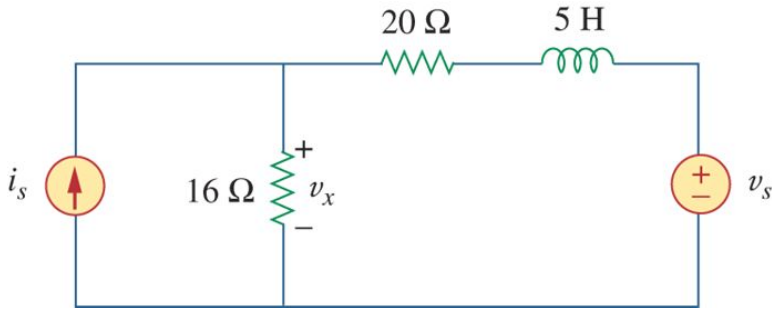
Resistance across current source = 20 ‖ 16 = 80/9 Ω

$V_x = 0.9*80/9 = 8$ V (DC)

3A) Superposition (multiple freq) example 1

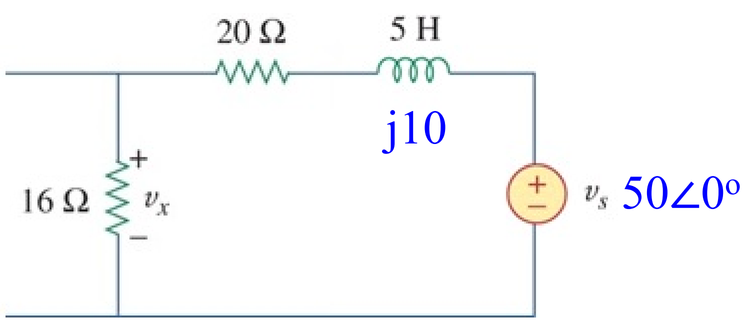
Alexander 10.44 (Modified)

Given $v_s(t) = 50\cos(2t)$ and $i_s = 0.9$ A, find $v_x(t)$



At 2 rad/s, redraw circuit without i_s:

Replace current source with open circuit



Final step: Add up both solutions
$v_x(t) = 21.41\cos(2t - 15.52^o) + 8$ V

Step 1a: Transform source to phasor

$50\cos(2t) \rightarrow 50\angle 0^o$, $\omega = 2$ rad/s

Step 1b: Work out the impedance values

5 H → $j2*5 = j10$

Step 2: Solve using circuit analysis methods

Using voltage divider rule,

$$\mathbf{V_x} = \left(\frac{16}{16+20+j10}\right)(50\angle 0^o) = 21.41\angle\text{-}15.52^o$$

Step 3: Transform back to time domain

$\mathbf{V_x} = 21.41\angle\text{-}15.52^o$, $\omega = 2$ rad/s

$\rightarrow v_x(t) = 21.41\cos(2t - 15.52^o)$ V

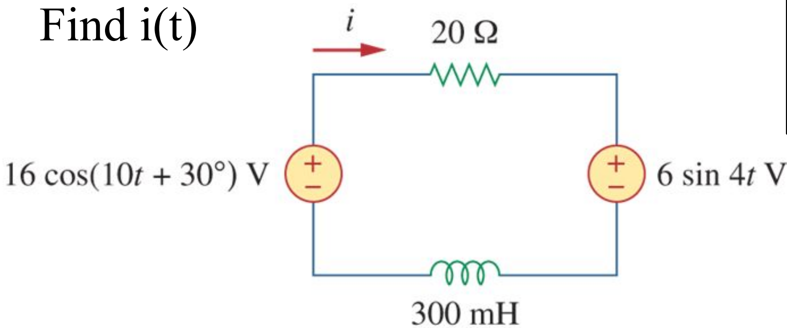
(NEVER SKIP THIS STEP)

Note that there are 2 terms in the final form (one at DC and another at 2 rad/s)

3B) Superposition (multiple freq) example 2

Alexander 10.45

Find i(t)



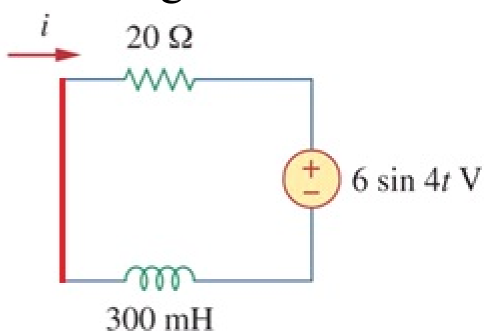
Method: Analyze circuit one frequency at a time

In the case, first analyze at 4 rad/s then 10 rad/s.

Finally, add up the two solutions **(superposition)**

At 4 rad/s, redraw circuit without 10 rad/s source:

Replace 10 rad/s voltage source with short circuit



Step 1a: Transform source to phasor

6sin(4t) → $6\angle\text{-}90^o$, ω = 4 rad/s

Step 1b: Work out the impedance values

300 mH → j4*0.3 = j1.2

Step 2: Solve using circuit analysis methods

I*(20 + j1.2) + $6\angle\text{-}90^o$ = 0

I = $0.2995\angle 86.57^o$ A

Step 3: Transform back to time domain

I = $0.2995\angle 86.57^o$, ω = 4 rad/s

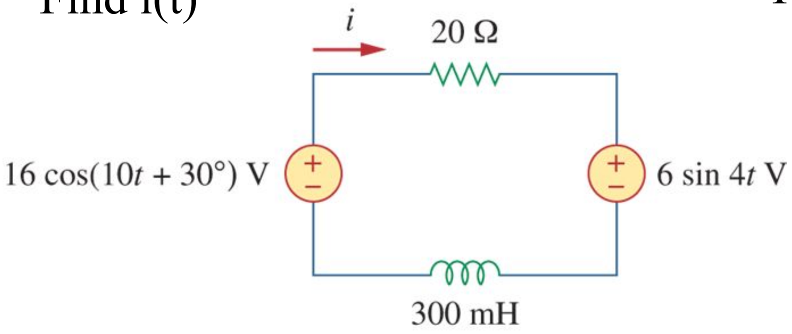
→ i(t) = 0.2995 cos(4t + 86.57^o) A

NEVER SKIP THIS STEP

3B) Superposition (multiple freq) example 2

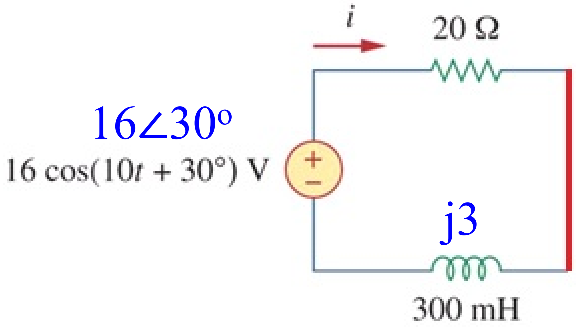
Alexander 10.45

Find i(t)



At 10 rad/s, redraw circuit without 4 rad/s source:

Replace 4 rad/s voltage source with short circuit



Step 1a: Transform source to phasor

16 cos(10t + 30°) → 16∠30°, ω = 10 rad/s

Step 1b: Work out the impedance values

300 mH → j10*0.3 = j3

Step 2: Solve using circuit analysis methods

I*(20 + j3) = 16∠30°

I = 0.7911∠21.47° A

Step 3: Transform back to time domain

I = 0.7911∠21.47° , ω = 10 rad/s

→ i(t) = 0.7911 cos(10t + 21.47°) A ← **NEVER SKIP THIS STEP**

Final step: Add up both solutions

i(t) = 0.2995 cos(4t + 86.57°) + 0.7911 cos(10t + 21.47°) A

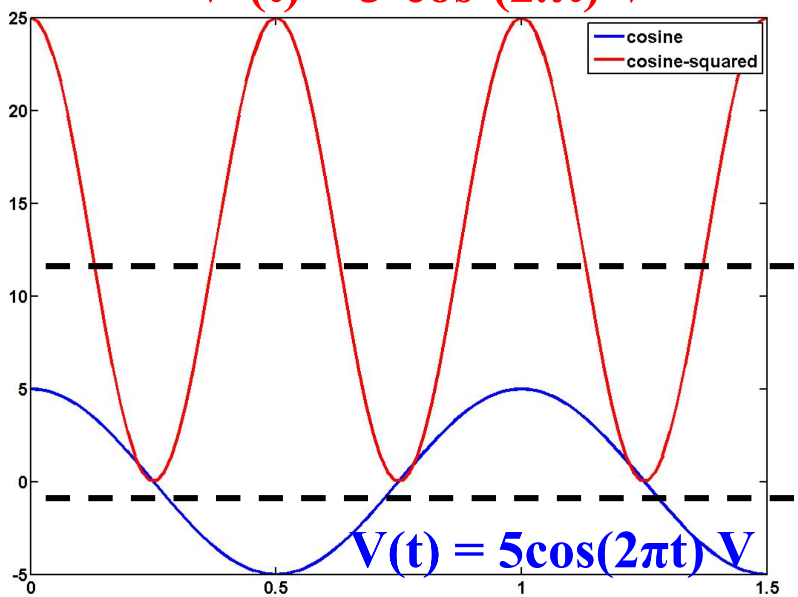
Note that there are 2 terms in the final form
(one at 4 rad/s and another at 10 rad/s)

AC Power: Instantaneous vs. Average

Although the average voltage across a resistor with a sinusoidal AC voltage across it is zero, note that the average power dissipated is not zero. Since $P = V^2/R$, we should consider the **square of the voltage**.

The following graph shows the voltage across a 1Ω resistor in blue and the corresponding square of this voltage in red. The red curve therefore shows the **INSTANTANEOUS POWER**.

$V^2(t) = 5^2\cos^2(2\pi t)$ V



The instantaneous power is clearly sinusoidal with a DC offset:

$$V^2(t) = 5^2 * 0.5 * (1 + \cos(4\pi t))$$
$$= V_{peak}^2 * 0.5 * \cos(4\pi t) + V_{peak}^2 * 0.5$$
$$= 12.5\cos(4\pi t) + 12.5$$

Mean of V^2 = 12.5 (DC offset)

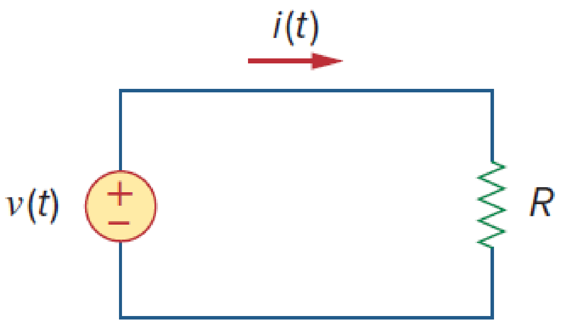
Average power of $V^2(t)$ is 12.5 W
Average power of $V^2(t) = 0.5 * V_{peak}^2$

Mean of V = 0

Effective or RMS Value

The average power absorbed by the resistor in the ac circuit of Fig. (a) is

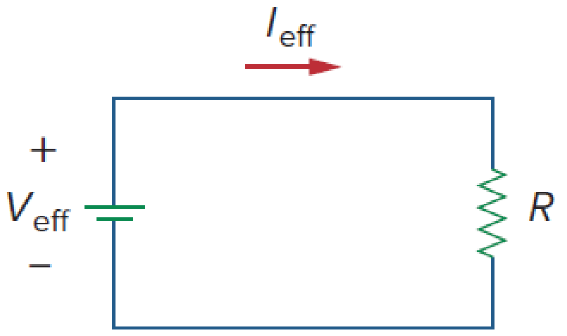
$$P = \frac{1}{T}\int_0^T i^2 R\, dt = \frac{R}{T}\int_0^T i^2\, dt$$



(a)

To find the dc effective current I_{eff} (Fig. (b)) that will transfer the same power to resistor R as the sinusoid $i(t)$:

$$P = I_{\text{eff}}^2 R$$



(b)

Equating these two expressions of P and solving for I_{eff}, we obtain

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt}$$

Effective or RMS Value

The effective value of the voltage is found in the same way as current, i.e.,

$$V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt}$$

This indicates that the effective value is the square root of the mean (or average) of the square of the periodic signal. Therefore, the effective value is known as the root-mean-square (rms) value. We write

$$I_{\text{eff}} = \mathrm{I}_{\text{rms}}, \qquad V_{\text{eff}} = V_{\text{rms}}$$

For the sinusoid $i(t) = I_m \cos \omega t$, the effective or rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t\,dt}$$

$$= \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\,dt} = \frac{I_m}{\sqrt{2}}$$

Effective or RMS Value

Similarly, for $v(t) = V_m \cos \omega t$:

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}}$$

Note: The simple formulas for V_{rms} and I_{rms} are only valid for sinusoidal signals.

The average power absorbed by the resistor R in the previous simple circuit (Fig.(a) and (b)) can be written as

$$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$