Lecture 9

Magnetic Field

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Lecture 08 Review

- Electromotive force (emf) is a device that produces a steady flow of charge between a pair of terminals through a resistor.
- Electric generator, battery, solar cells and fuel cells are all emf devices that produce the steady flow of charges by maintaining a potential difference between a pair of terminals.
- An **ideal emf device** is one that has no internal resistance to the internal movement of charge from terminal to terminal.
- A **real emf device,** such as any real battery, has internal resistance to the internal movement of charge. When a real emf device is not connected to a circuit, and thus does not have current through it, the potential difference between its terminals is equal to its emf. However, when that device has current through it, the potential difference between its terminals differs from its emf.

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- We define the emf of the emf device in terms of this work:

$$\mathscr{E} = \frac{dW}{dq} \qquad \text{(definition of } \mathscr{E}\text{).}$$

- The amount of work *dW* on the charge *dq* to force it to move from the lower potential terminal to the higher potential terminal.
- For a single-loop circuit with a emf connected to a certain amount of resistance, the current passing through the loop is simply the ratio between the emf and the resistance.

$$i = \frac{\mathscr{E}}{R}.$$

- We can also calculate the current in a single-loop circuit using the potential method, where we sum up the potential drop and gain over the closed circuit and equate that to zero.

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- To identify the potential drop or gain at each circuit component, we can follow the following two rules:

RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$.

- For the real emf devices, we need to account for the internal resistance in the analysis.
- When a potential difference V is applied across resistances connected in series, all the resistances experience the same current i. The sum of the potential differences across all the resistances is equal to the applied potential difference V.

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- To find the potential between any two points in a circuit, we starts from one point and sum up the potential drop and gain of each elements to the other point. We can choose any path that is connected the two points. The result will be identical.
- Grounding a circuit means to define a zero potential point within the circuit. The point of that circuit is supposedly to be connected to the earth's surface, so the grounds of all circuits have the same potential.
- We are provided the expressions of power, potential and emf in a circuit.
- The Kirchhoff's junction rule specifies that the sum of the currents entering any junction must be equal to the sum of the currents leaving that junction. It is a very useful method for solving multi-loop circuits.

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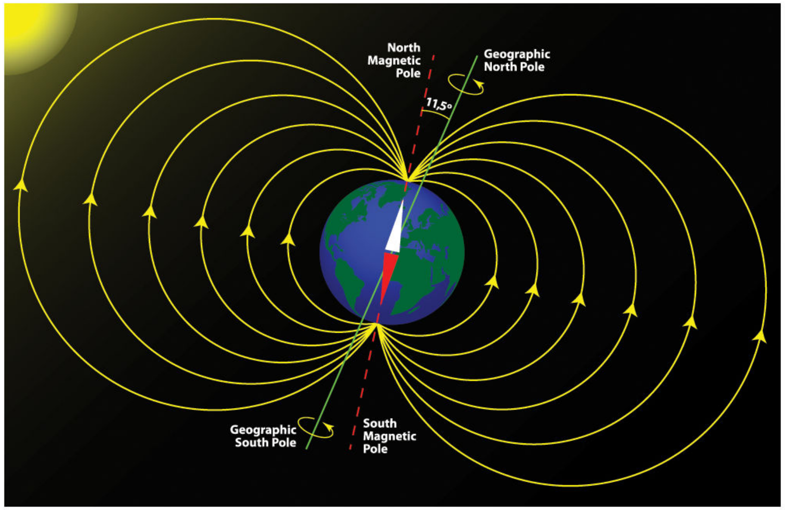
Lecture Outline

- **Chapter 28**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Magnetic Field
  - Definition of **B**
  - Magnetic field lines
  - The Hall effect
  - Magnetic force on a current carrying wire
  - Torque on a current loop

Magnetism

- Properties of magnets:
  - All magnets have two poles: North and South.
  - Like poles repel each other.
  - Unlike poles attract each other.
  - Magnetic poles always found in pairs.
  - Isolated magnetic poles never been found.

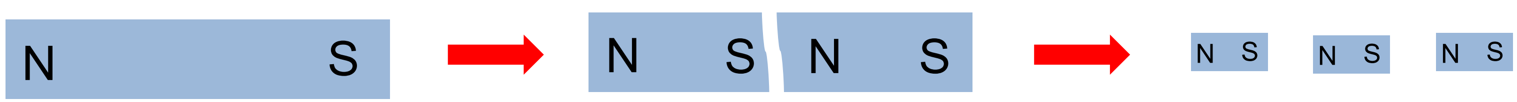


Magnetic field of Earth

https://www.sott.net/article/248315-Scientists-Offer-Theory-For-Why-Earths-Magnetic-Field-Is-Wonky

Magnetism

- The (non)existence of magnetic monopoles:
  - In the study of theoretical physics, such as in grand unified theories and quantum gravity, one predicts that there exist magnetic charges, just like electric charges.
  - Such an entity would be called a magnetic monopole, having +ve or –ve magnetic charge.
  - However, if we break a piece of magnet into many small pieces, each individual piece still contains both N & S poles.



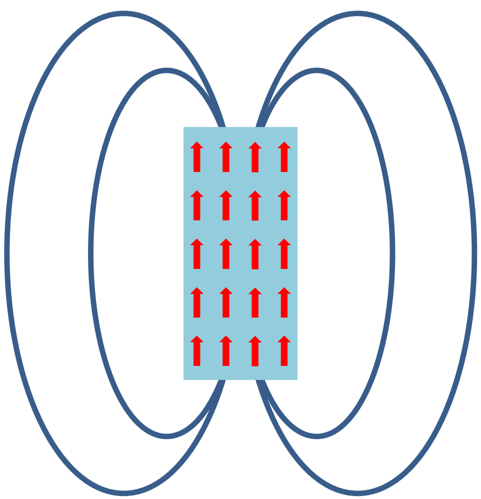
  - As a matter of fact, even a single individual electron has a magnetic 'dipole'.
  - Although some experimental results said to have demonstrated the existence of magnetic monopoles, all were dismissed as inconclusive.
  - So far no monopoles have ever been found and their existence remains to be discovered.

Source of the Magnetic Field

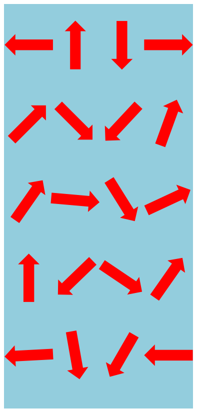
- If there are no magnetic charges, then what is the source of the magnetic field?
  - The answer is **electric charges in motion**.
  - One way that magnetic fields are produced is to use moving electrically charged particles, such as a current in a wire, to make an **electromagnet.** The current produces a magnetic field that is utilizable.
  - Another way to produce a magnetic field is by means of elementary particles such as electrons, because these particles have an *intrinsic magnetic field* around them.

Permanent Magnet

- The magnetic fields of the electrons/molecules in certain materials add together to give a net magnetic field around the material. Such addition is the reason why a **permanent magnet**, has a permanent magnetic field.



- In other materials, the magnetic fields of the electrons/molecules cancel out, giving no net magnetic field surrounding the material.



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The definition of Magnetic Field **B**

- Recall that if we want to define the magnitude and direction of an unknown electric field distribution, we can place a stationary test charge q in the field and measure the force $\boldsymbol{F}_E$ encountered by the charge due to the field and extract $\boldsymbol{E}$ from

$$\mathbf{F_E} = q\mathbf{E} \longrightarrow \mathbf{E} = \mathbf{F_E}/q$$

- Unfortunately we cannot define the magnitude and direction of the magnetic field $\mathbf{B}$ using the same stationary test charge because there is no equivalent magnetic charge q_m.
- In order to define the magnetic field **B**, we will need a moving test charge. Because the force from the **B** field $\mathbf{F}_\mathrm{B}$ is proportional to both the velocity and charge of the test charge.

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The definition of Magnetic Field **B**

- Experimentally we find that when a charged particle moving with velocity **v** in a magnetic field, it encounters a force $\mathbf{F_B}$ due to the field that acts on the particle.

$$\boxed{\mathbf{F_B} = q\mathbf{v} \times \mathbf{B}}$$

Experimental result

- Since $\mathbf{F_B}$, **v** can be measured, this proves to be a convenient way to define and measure **B**.
- Here q is the charge of the particle, ***v*** is its velocity, and **B** the magnetic field in the region. The magnitude of this force is then:

$$\boxed{F_B = |q| vB \sin\phi}$$

- Here ϕ is the angle between vectors **v** and **B**.

Magnetic Force: Observations

- The magnitude of the magnetic force is proportional to the charge q and the speed v of the particle.
- The direction of the force depend on the velocity of the particle and on the direction of the magnetic field.
- When the charged particle moves *parallel* to the magnetic field the magnetic force on the charge is *zero.*
- When the charged particle moves in a direction not parallel to the magnetic field the magnetic force is *perpendicular to both the velocity of the charge and the magnetic field*.
- Positive and negative charges moving in the same direction experience magnetic forces in opposite directions.
- If the velocity vector makes an angle of ϕ with the magnetic field vector, the magnetic force is proportional to $\sin\phi$.

28.3: Finding the Magnetic Force on a Particle:

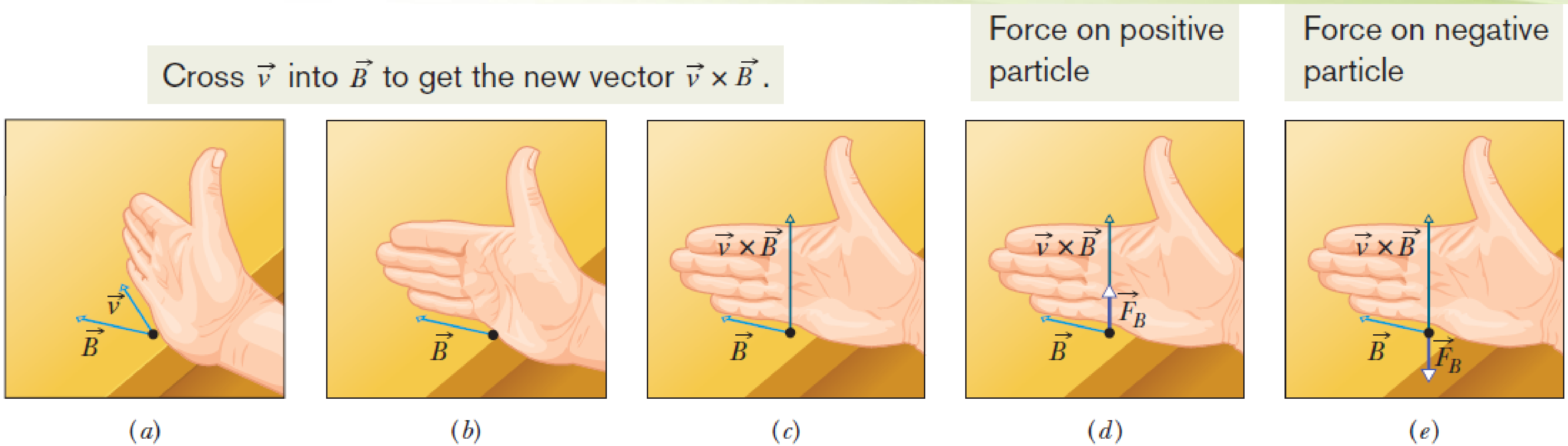

Fig. 28-2 (*a*) – (*c*) The right-hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle ϕ between them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb. (*d*) If q is positive, then the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$. (*e*) If q is negative, then the direction of $\vec{F}_B$ is opposite that of $\vec{v} \times \vec{B}$.

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

Table 28-1

Some Approximate Magnetic Fields

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

The SI unit for ***B*** that follows is newton per coulomb-meter per second. For convenience, this is called the **tesla (T):**

$$1 \text{ tesla} = 1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})}$$

$$= 1 \frac{\text{newton}}{(\text{coulomb/second})(\text{meter})} = 1 \frac{\text{N}}{\text{A} \cdot \text{m}}$$

An earlier (non-SI) unit for ***B*** is the *gauss (G)*, and

$$1 \text{ tesla} = 10^4 \text{ gauss.}$$

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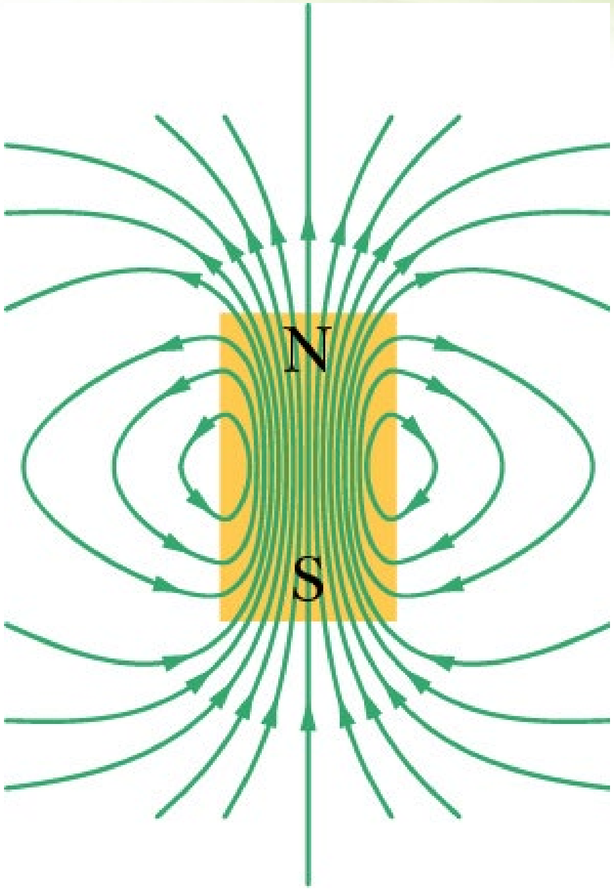
28.3: Magnetic Field Lines:

We can represent magnetic fields with field lines, as we did for electric fields. Similar rules apply; that is,

1. the direction of the tangent to a magnetic field line at any point gives the direction of **B** at that point, and

2. the spacing of the lines represents the magnitude of **B** — the magnetic field is stronger where the lines are closer together, and conversely.



Opposite magnetic poles attract each other, and like magnetic poles repel each other.

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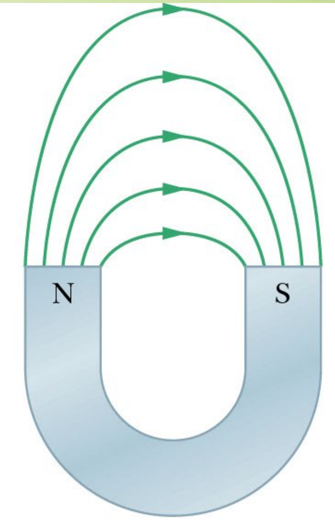
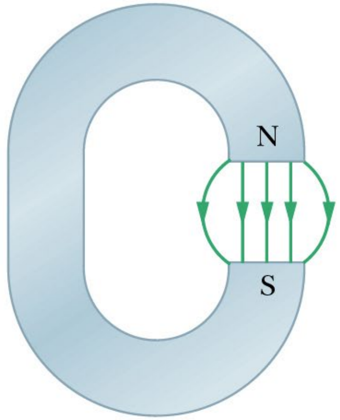
28.3: Magnetic Field Lines:

- The end of a magnet from which the field lines emerge is called the *north pole* of the magnet; the other end, where field lines enter the magnet, is called the *south pole.*

- Opposite magnetic poles attract each other, and like magnetic poles repel each other.



(a)



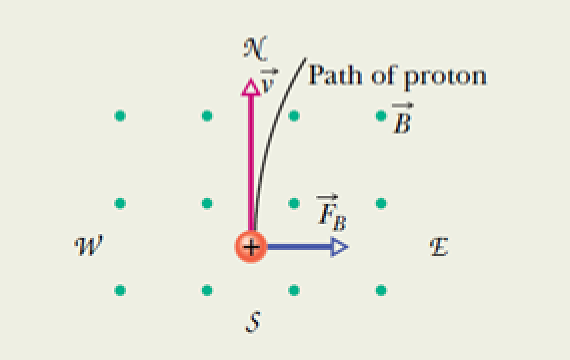
(b)

Example, Magnetic Force on a Moving Charged Particle :

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)

KEY IDEAS

Because the proton is charged and moving through a magnetic field, a magnetic force $\vec{F}_B$ can act on it. Because the initial direction of the proton's velocity is not along a magnetic field line, $\vec{F}_B$ is not simply zero.



Example, Magnetic Force on a Moving Charged Particle :

Magnitude: To find the magnitude of $\vec{F}_B$, we can use Eq. 28-3 ($F_B = |q|vB \sin\phi$) provided we first find the proton's speed v. We can find v from the given kinetic energy because $K = \frac{1}{2}mv^2$. Solving for v, we obtain

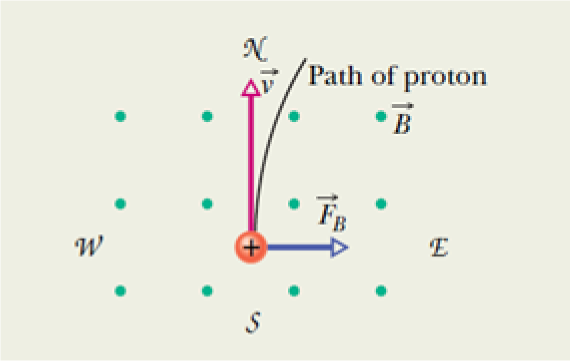
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3\text{ MeV})(1.60 \times 10^{-13}\text{ J/MeV})}{1.67 \times 10^{-27}\text{ kg}}}$$
$$= 3.2 \times 10^7\text{ m/s}.$$

Equation 28-3 then yields

$$\begin{aligned} F_B &= |q|vB \sin\phi \\ &= (1.60 \times 10^{-19}\text{ C})(3.2 \times 10^7\text{ m/s}) \\ &\quad \times (1.2 \times 10^{-3}\text{ T})(\sin 90°) \\ &= 6.1 \times 10^{-15}\text{ N}. \qquad \text{(Answer)} \end{aligned}$$

This may seem like a small force, but it acts on a particle of small mass, producing a large acceleration; namely,

$$a = \frac{F_B}{m} = \frac{6.1 \times 10^{-15}\text{ N}}{1.67 \times 10^{-27}\text{ kg}} = 3.7 \times 10^{12}\text{ m/s}^2.$$



28.4: Crossed Fields, Discovery of an Electron:

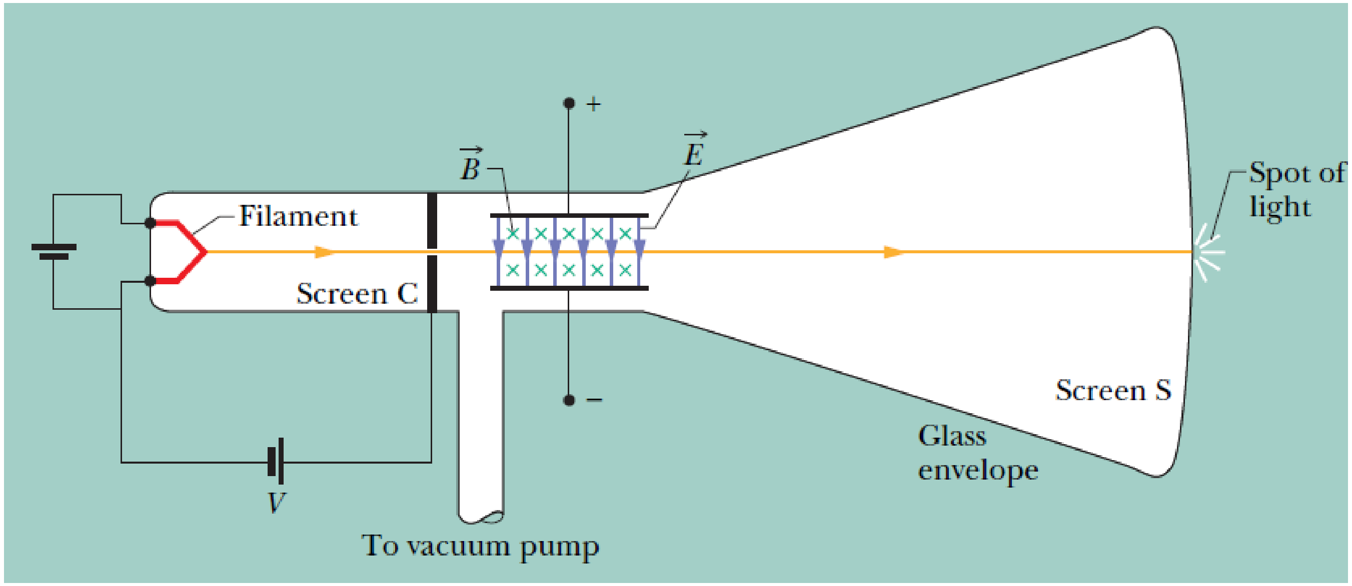


Fig. 28-7 A modern version of J.J. Thomson's apparatus for measuring the ratio of mass to charge for the electron. An electric field **E** is established by connecting a battery across the deflecting-plate terminals. The magnetic field **B** is set up by mean of a current in a system of coils. The procedure first determine the velocity of the particle by adjusting the **E** and **B** field ratio, then the B field is turned off and the e/m ratio can then be determined from the amount of deflection measured on the screen.

When the two fields in Fig. 28-7 are adjusted so that the two deflecting forces acting on the charged particle cancel, we have

$$|q|E = |q|vB\sin(90°) = |q|vB \Rightarrow v = \frac{E}{B}.$$

Thus, the crossed fields allow us to measure the speed of the charged particles passing through them.

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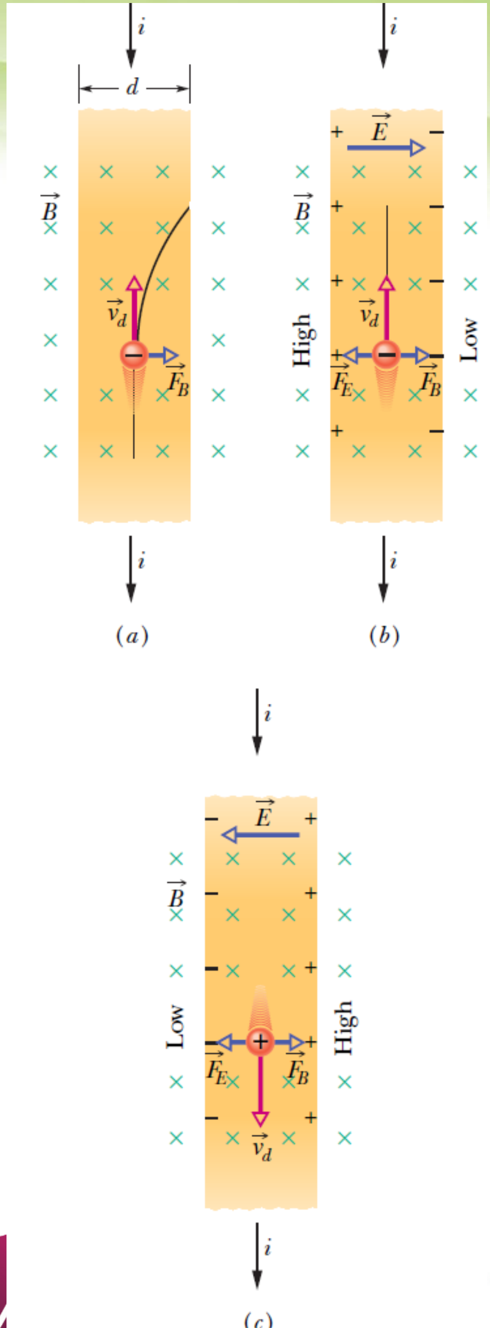


Fig. 28-8 A strip of copper carrying a current *i is immersed in a magnetic field* .

(*a*)*The situation immediately after the* magnetic field is turned on. The curved path that will then be taken by ***an electron*** is shown.

(*b*) *The situation at equilibrium,* which quickly follows. Note that negative charges pile up on the right side of the strip, leaving uncompensated positive charges on the left. Thus, the left side is at a higher potential than the right side.

(*c*) *For the same* current direction, if the charge carriers were positively charged, *they would pile up* on the right side, and the right side would be at the higher potential.

28.5: Crossed Fields, The Hall Effect:

A Hall potential difference V is associated with the electric field across strip width d, and the magnitude of that potential difference is $V = Ed$. When the electric and magnetic forces are in balance (Fig. 28-8b),

$$F_E = F_B \Rightarrow eE = ev_d B$$

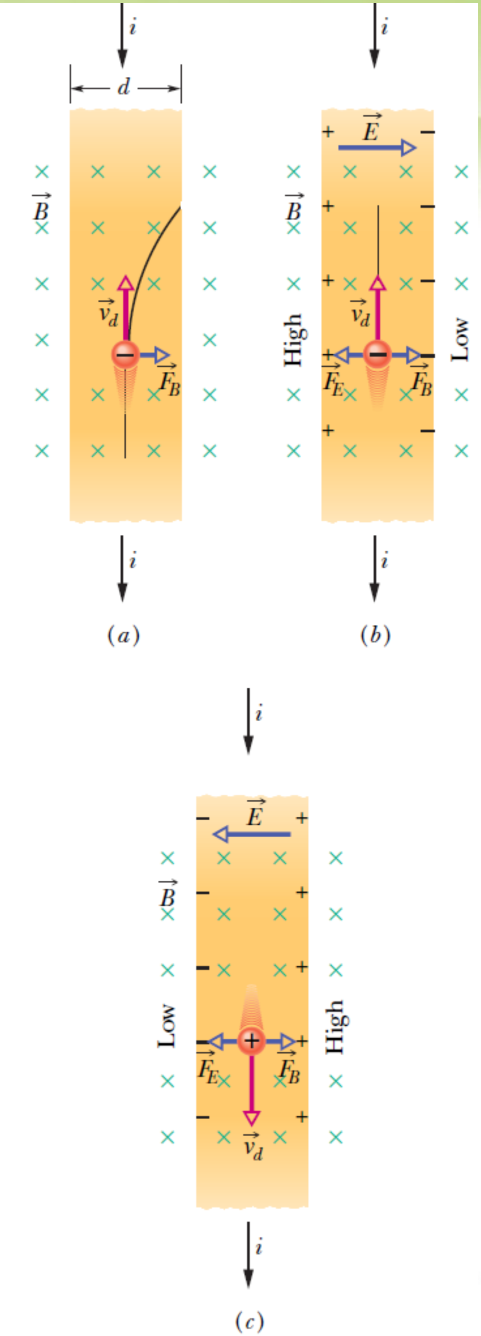
where v_d is the drift speed. But,

$$v_d = \frac{J}{ne} = \frac{i}{neA}$$

Where J is the current density, A the cross-sectional area, e the electronic charge, and n the number of charges per unit volume, or commonly known as the carrier density.

Therefore, $$n = \frac{Bi}{Vle},$$

Here, $l=(A/d)$, the thickness of the strip.



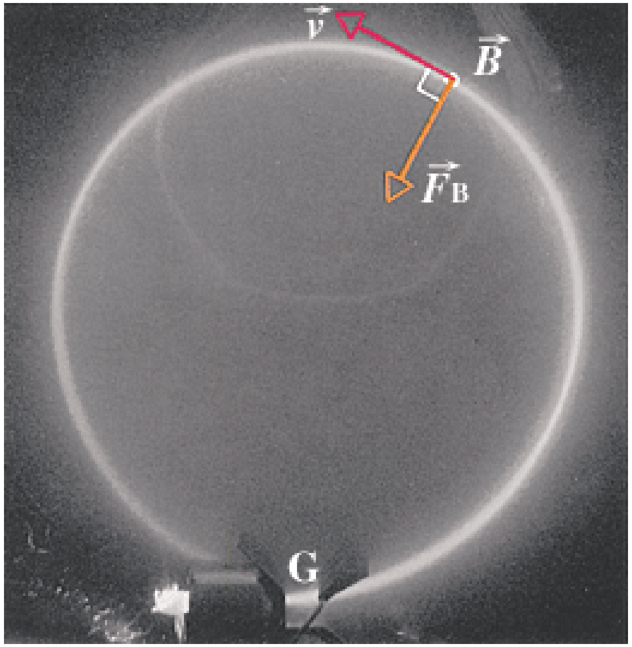
28.6: A Circulating Charged Particle:

Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field **B**, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force F_B; for circular motion to occur, F_B *must* point toward the centre of the circle. Use the right-hand rule for cross products to confirm that $F_B = q\mathbf{v} \times \mathbf{B}$ gives F_B the proper direction. (Don't forget the sign of q.)

The magnetic force continuously deflects the particle, and since ***B*** and ***v*** are always perpendicular to each other, this deflection causes the particle to follow **a circular path.**

The magnetic force acting on the particle has a magnitude of $|q|vB$.

For uniform circular motion $F = m\dfrac{v^2}{r}$, $\Rightarrow$ $\boxed{|q|vB = \dfrac{mv^2}{r}.}$



$$r = \frac{mv}{|q|B} \qquad \text{(radius).}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \qquad \text{(period).} \quad \text{Eq. 28-17}$$

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \qquad \text{(frequency).}$$

$$\omega = 2\pi f = \frac{|q|B}{m} \qquad \text{(angular frequency).}$$

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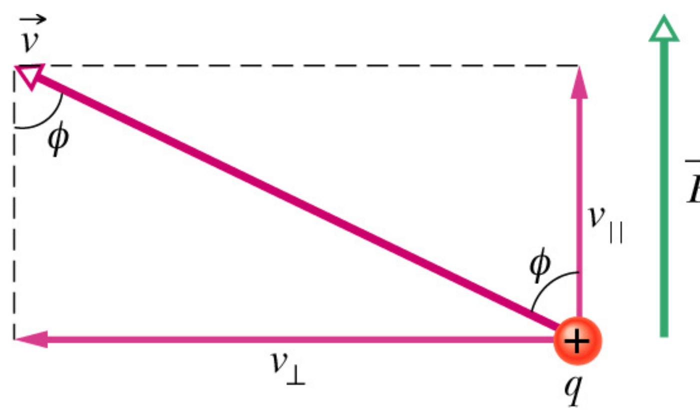
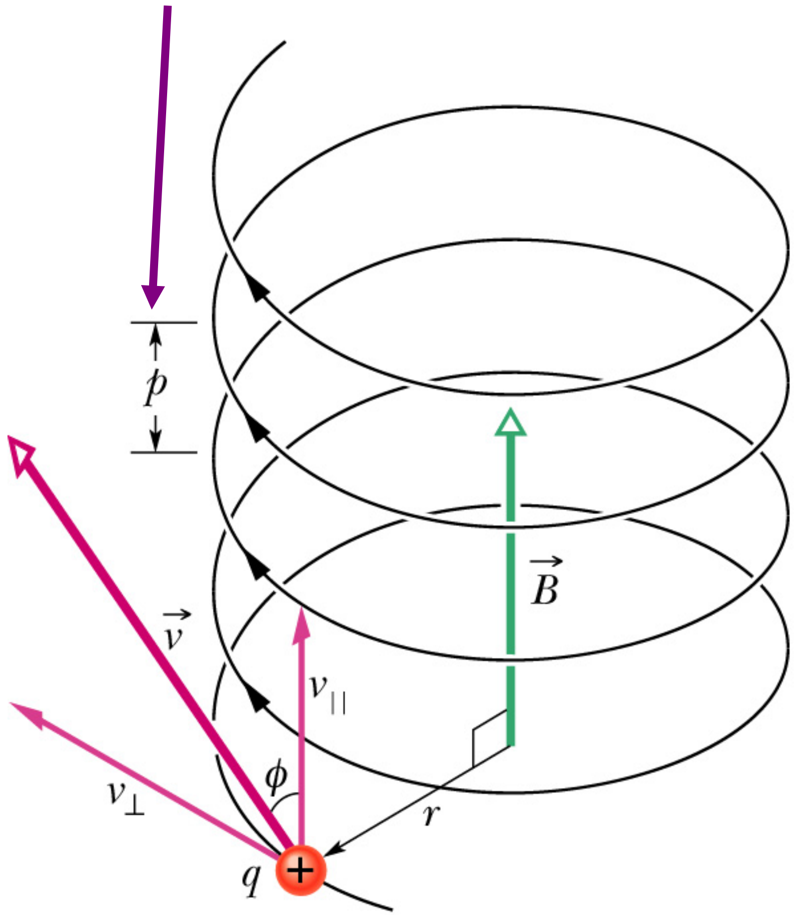
28.6: Helical Paths:

- In the previous example, we see that a moving charged particle moves in a circulating path if it is in a uniform **B** field.
- If the velocity of a charged particle has a component parallel to the (uniform) magnetic field, the particle will move in a helical (corkscrew) path about the direction of the field vector.
- Note that the **B** field only acts on the component of the velocity that is perpendicular to the **B** field to generate the force to move the particle in the circular motion.

The parallel component determines the *pitch p* of the helix—that is, the distance between adjacent turns





$$v_{II} = v\cos\phi \qquad v_\perp = v\sin\phi$$

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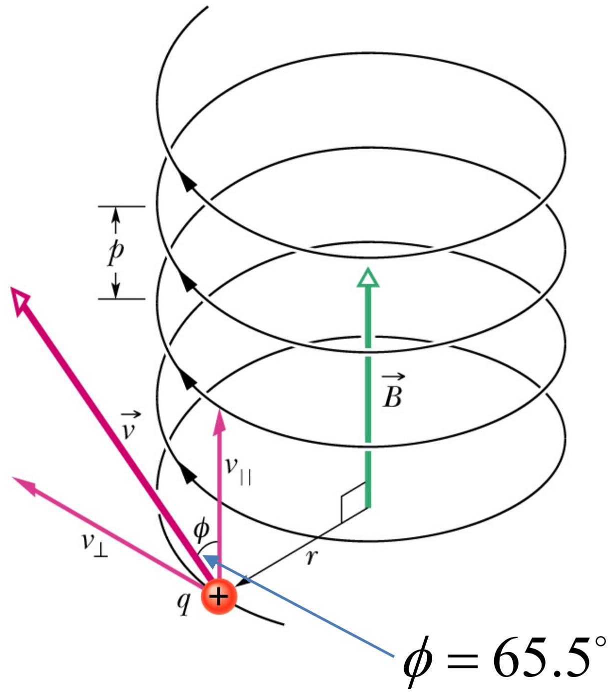
Example, Helical Motion of a Charged Particle in a Magnetic Field:

An electron with a kinetic energy of 22.5 eV moves into a region of uniform magnetic field $\vec{B}$ of magnitude 4.55×10^{-4} T. The angle between the directions of $\vec{B}$ and the electron's velocity $\vec{v}$ is 65.5°. What is the pitch of the helical path taken by the electron?

KEY IDEAS

(1) The pitch p is the distance the electron travels parallel to the magnetic field $\vec{B}$ during one period T of circulation. (2) The period T is given by Eq. 28-17 regardless of the angle between the directions of $\vec{v}$ and $\vec{B}$ (provided the angle is not zero, for which there is no circulation of the electron).



Example, Helical Motion of a Charged Particle in a Magnetic Field:

Calculations: Using Eqs. 28-20 and 28-17, we find

$$p = v_{\parallel} T = (v \cos \phi) \frac{2\pi m}{|q|B}. \qquad (28\text{-}21)$$

Calculating the electron's speed v from its kinetic energy, find that $v = 2.81 \times 10^6$ m/s. Substituting this and known data in Eq. 28-21 gives us

$$\begin{aligned} p &= (2.81 \times 10^6 \text{ m/s})(\cos 65.5°) \\ &\quad \times \frac{2\pi(9.11 \times 10^{-31} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(4.55 \times 10^{-4} \text{ T})} \\ &= 9.16 \text{ cm.} \qquad \text{(Answer)} \end{aligned}$$

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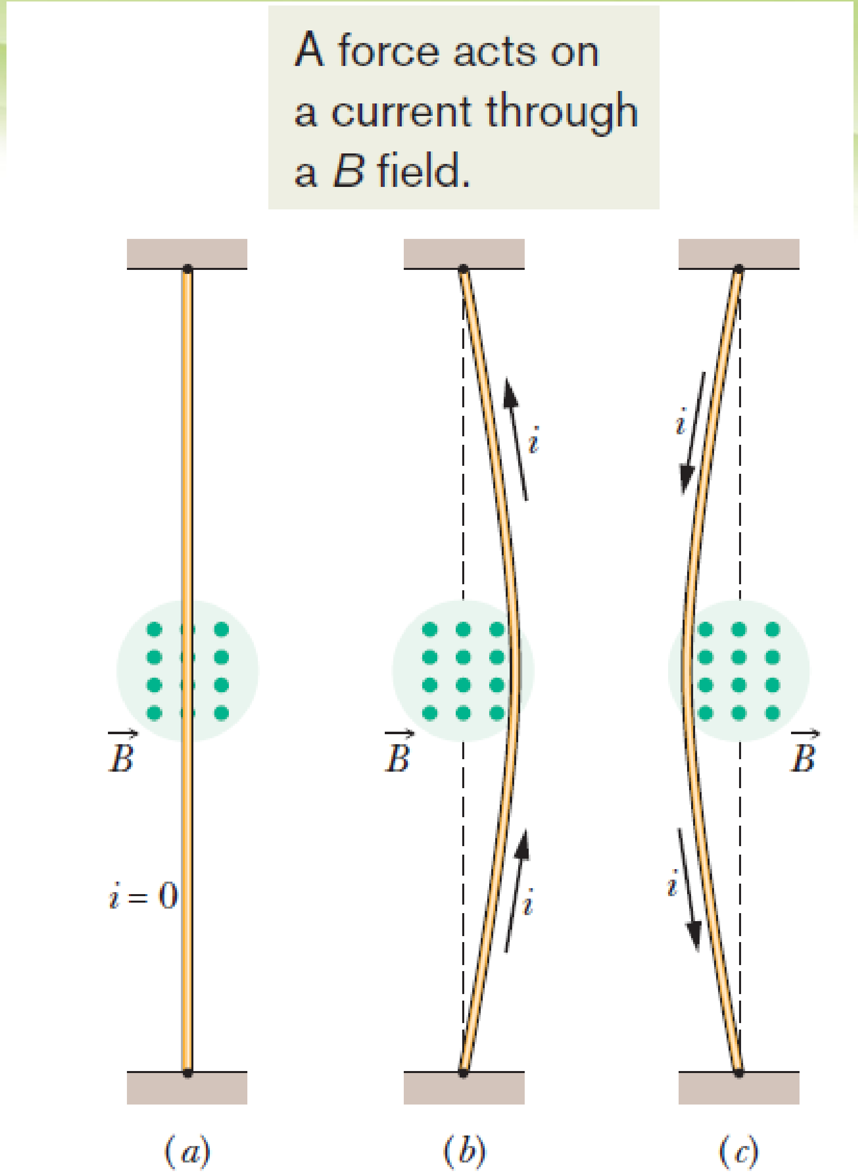
28.8: Magnetic Force on a Current-Carrying Wire:

Fig. 28-14 A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (*a*) Without current in the wire, the wire is straight. (*b*) With upward current, the wire is deflected rightward. (*c*) With downward current, the deflection is leftward. The connections for getting the current into the wire at one end and out of it at the other end are not shown.



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28.8: Magnetic Force on a Current-Carrying Wire:

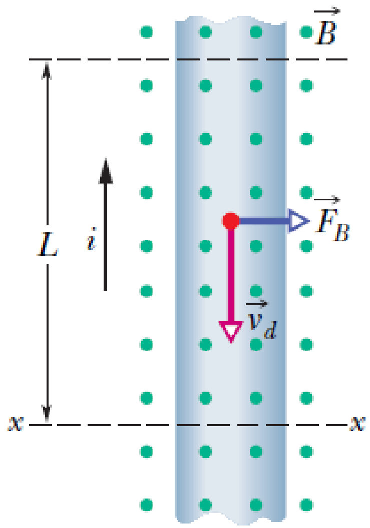

Fig. 28-15 A close-up view of a section of the wire of Fig. 28-14*b*. The current direction is upward, which means that electrons drift downward. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

Consider a length L of the wire in the figure. All the conduction electrons in this section of wire will drift past plane xx in a time $t = L/v_d$.

Thus, in that time a charge will pass through that plane that is given by

$$q = it = i\frac{L}{v_d}$$

$$F_B = qv_dB\sin\phi = \frac{iL}{v_d}v_dB\sin 90°$$

$$F_B = iLB.$$

$$\vec{F}_B = i\vec{L}\times\vec{B} \qquad \text{(force on a current).}$$

Here $\boldsymbol{L}$ is a length vector that has magnitude L and is directed along the wire segment in the direction of the (conventional) current.

If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments . The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write $d\vec{F}_B = i\,d\vec{L}\times\vec{B}$, and we can find the resultant force on any given arrangement of currents by integrating Eq. 28-28 over that arrangement.

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Example, Magnetic Force on a Wire Carrying Current:

A straight, horizontal length of copper wire has a current $i = 28$ A through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

KEY IDEAS

(1) Because the wire carries a current, a magnetic force $\vec{F}_B$ can act on the wire if we place it in a magnetic field $\vec{B}$. To balance the downward gravitational force $\vec{F}_g$ on the wire, we want $\vec{F}_B$ to be directed upward (Fig. 28-17). (2) The direction of $\vec{F}_B$ is related to the directions of $\vec{B}$ and the wire's length vector $\vec{L}$ by Eq. 28-26 ($\vec{F}_B = i\vec{L} \times \vec{B}$).

Calculations: Because $\vec{L}$ is directed horizontally (and the current is taken to be positive), Eq. 28-26 and the right-hand rule for cross products tell us that $\vec{B}$ must be horizontal and rightward (in Fig. 28-17) to give the required upward $\vec{F}_B$.

The magnitude of $\vec{F}_B$ is $F_B = iLB \sin \phi$ (Eq. 28-27). Because we want $\vec{F}_B$ to balance $\vec{F}_g$, we want

$$iLB \sin \phi = mg, \tag{28-29}$$

where mg is the magnitude of $\vec{F}_g$ and m is the mass of the wire.

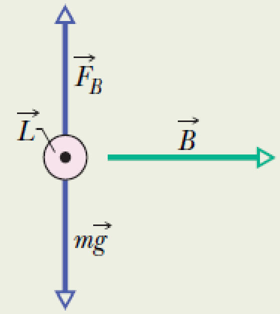

Fig. 28-17 A wire (shown in cross section) carrying current out of the page.

We also want the minimal field magnitude B for $\vec{F}_B$ to balance $\vec{F}_g$. Thus, we need to maximize $\sin \phi$ in Eq. 28-29. To do so, we set $\phi = 90°$, thereby arranging for $\vec{B}$ to be perpendicular to the wire. We then have $\sin \phi = 1$, so Eq. 28-29 yields

$$B = \frac{mg}{iL \sin \phi} = \frac{(m/L)g}{i}. \tag{28-30}$$

We write the result this way because we know m/L, the linear density of the wire. Substituting known data then gives us

$$B = \frac{(46.6 \times 10^{-3}\ \text{kg/m})(9.8\ \text{m/s}^2)}{28\ \text{A}}$$

$$= 1.6 \times 10^{-2}\ \text{T}. \qquad \text{(Answer)}$$

This is about 160 times the strength of Earth's magnetic field.

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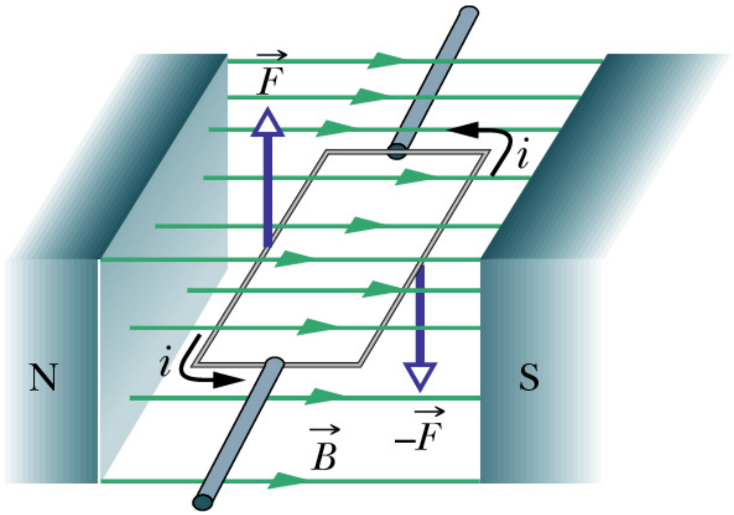
The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.



The two magnetic forces $\boldsymbol{F}$ and $-\boldsymbol{F}$ produce a torque on the loop, tending to rotate it about its central axis.

28.9: Torque on a Current Loop:

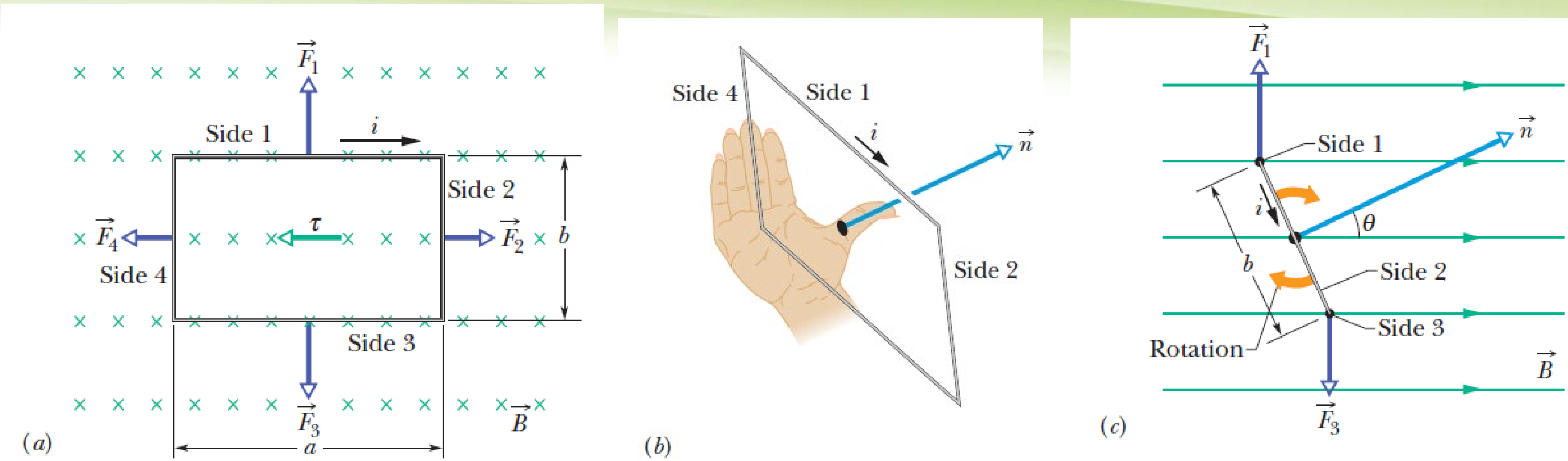


Figure 28-19

To define the orientation of the loop in the magnetic field, we use a normal vector ***n*** that is perpendicular to the plane of the loop. Along with the current i and the area A plane we can define a magnetic dipole moment for the single wire loop, similar to the electric dipole moment,

$$\mu = iA$$

Figure 28-19*b* shows a right-hand rule for finding the direction of ***n***. In Fig. 28-19*c*, the normal vector of the loop is shown at an arbitrary angle θ to the direction of the magnetic field.

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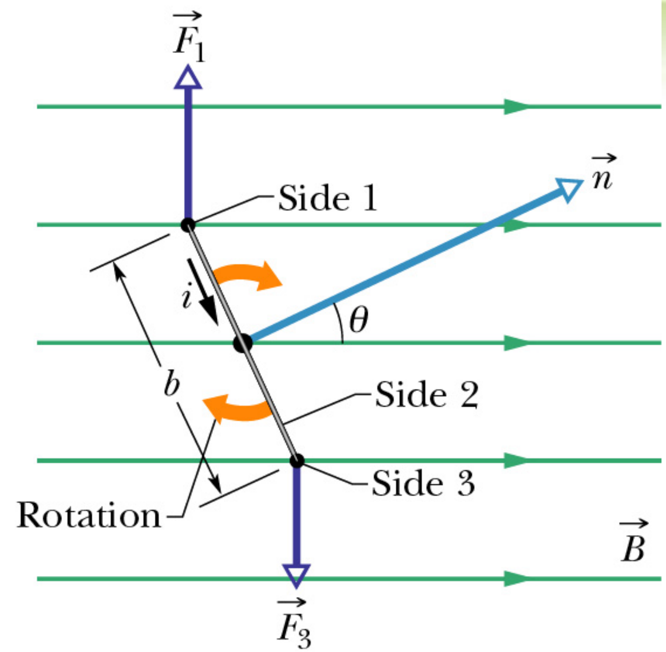
For side 2 the magnitude of the force acting on this side is

F_2=ibB sin(90°-θ)=ibB cosθ =F_4.

F_2 and ***F_4*** cancel out exactly.

Forces ***F_1*** and ***F_3*** have the common magnitude *iaB.* As Fig. 28-19*c* shows, these two forces do not share the same line of action; so they produce a net torque.

$$\tau' = \left(iaB\frac{b}{2}\sin\theta\right) + \left(iaB\frac{b}{2}\sin\theta\right) = iabB\sin\theta.$$



a = length of sides 1 and 3

b = length of sides 2 and 4

A = a x b = area enclosed by the coil

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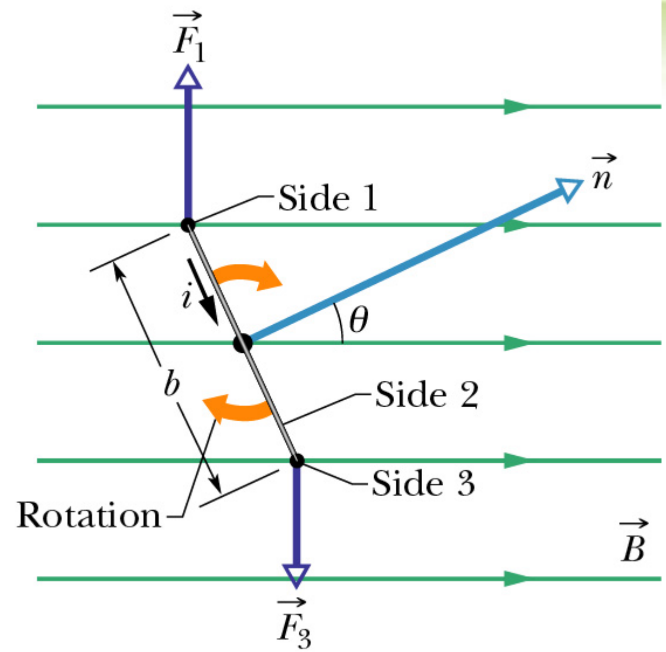
For *N* loops, when *A*=*ab*, the area of the loop, the total torque is:

$$\tau = N\tau' = NiabB\sin\theta = (NiA)B\sin\theta,$$



a = length of sides 1 and 3

b = length of sides 2 and 4

A = a x b = area enclosed by the coil

Magnetic dipole moment of a coil with N loop:

$$\mu = NiA$$

The torque on a magnetic dipole moment due to a magnetic field:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \tau = \mu B \sin\theta$$

Similar to $\vec{\tau} = \vec{p} \times \vec{E}$ for an electric dipole

Potential Energy of the Magnetic Dipole in a Magnetic Field $U = -\vec{\mu} \cdot \vec{B}$

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