P.P.9.7 Given that

$$2\frac{dv}{dt} + 5v + 10\int v\,dt = 50\cos(5t - 30°)$$

we take the phasor of each term to get

$$2j\omega\,\mathbf{V} + 5\,\mathbf{V} + \frac{10}{j\omega}\mathbf{V} = 50\angle\text{-}30°, \qquad \omega = 5$$

$$\mathbf{V}\,[j10 + 5 - j(10/5)] = \mathbf{V}\,(5 + j8) = 50\angle\text{-}30°$$

$$\mathbf{V} = \frac{50\angle\text{-}30°}{5 + j8} = \frac{50\angle\text{-}30°}{9.434\angle 58°}$$

$$\mathbf{V} = 5.3\angle\text{-}88°$$

Converting **V** to the time domain

$$v(t) = \mathbf{5.3\cos(5t - 88°)V}$$

Another approach by Ben Leung: You may first differential both sides of the original integrodifferential equation with respect to *t* to remove the integration term, and then use $d/dt \leftrightarrow j\omega$ and $d^2/dt^2 \leftrightarrow (j\omega)^2$.

P.P.9.8 For the capacitor,

$\mathbf{V} = \mathbf{I}/(j\omega C)$, where $\mathbf{V} = 10\angle 30°$, $\omega = 100$

$\mathbf{I} = j\omega C\,\mathbf{V} = (j100)(50\text{x}10^{-6})(10\angle 30°)$

$\mathbf{I} = 50\angle 120°$ mA

$i(t) = \mathbf{50\cos(100t + 120°)\ mA}$

P.P.9.9 $\mathbf{V}_s = 20\angle 30°, \omega = 10$

$\mathbf{Z} = 4 + j\omega L = 4 + j2$

$$\mathbf{I} = \mathbf{V}_s / \mathbf{Z} = \frac{20\angle 30°}{4 + j2} = \frac{20\angle 30°(4 - j2)}{16 + 4} = 4.472\angle 3.43°$$

$$\mathbf{V} = j\omega L\,\mathbf{I} = j2\,\mathbf{I} = (2\angle 90°)(4.472\angle 3.43°) = 8.944\angle 93.43°$$

Therefore, $v(t) = \mathbf{8.944\sin(10t + 93.43°)\ V}$

$i(t) = \mathbf{4.472\sin(10t + 3.43°)\ A}$

P.P.9.10

Let $\mathbf{Z}_1$ = impedance of the 1-mF capacitor in series with the 100-Ω resistor

$\mathbf{Z}_2$ = impedance of the 1-mF capacitor

$\mathbf{Z}_3$ = impedance of the 8-H inductor in series with the 200-Ω resistor

$$\mathbf{Z}_1 = 100 + \frac{1}{j\omega C} = 100 + \frac{1}{j(10)(1\times 10^{-3})} = 80 - j100$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C} = \frac{1}{j(10)(1\times 10^{-3})} = -j100$$

$$\mathbf{Z}_3 = 200 + j\omega L = 200 + j(10)(8) = 200 + j80$$

$$\mathbf{Z}_{in} = \mathbf{Z}_1 + \mathbf{Z}_2 \,||\, \mathbf{Z}_3 = \mathbf{Z}_1 + \mathbf{Z}_2\,\mathbf{Z}_3 \,/\, (\mathbf{Z}_2 + \mathbf{Z}_3)$$

$$\mathbf{Z}_{in} = 100 - j100 + \frac{-j100x(200 + j80)}{-j100 + 200 + j80}$$

$$\mathbf{Z}_{in} = 100 - j100 + 49.52 - j95.04$$

$$\mathbf{Z}_{in} = \mathbf{[149.52 - j195]\ \Omega}$$

P.P.9.11 In the frequency domain,

the voltage source is $\mathbf{V}_s = 20\angle 100°$

the 0.5-H inductor is $j\omega L = j\,(10)(0.5) = j5$

the $\frac{1}{20}$-F capacitor is $\frac{1}{j\omega C} = \frac{1}{j(10)(1/20)} = -j2$

Let $\mathbf{Z}_1$ = impedance of the 0.5-H inductor in parallel with the 10-Ω resistor
and $\mathbf{Z}_2$ = impedance of the (1/20)-F capacitor

$$\mathbf{Z}_1 = 10 \,||\, j5 = \frac{(10)(j5)}{10 + j5} = 2 + j4 \qquad \text{and} \qquad \mathbf{Z}_2 = -j2$$

$$\mathbf{V}_o = \mathbf{Z}_2 \,/\, (\mathbf{Z}_1 + \mathbf{Z}_2)\,\mathbf{V}_s$$

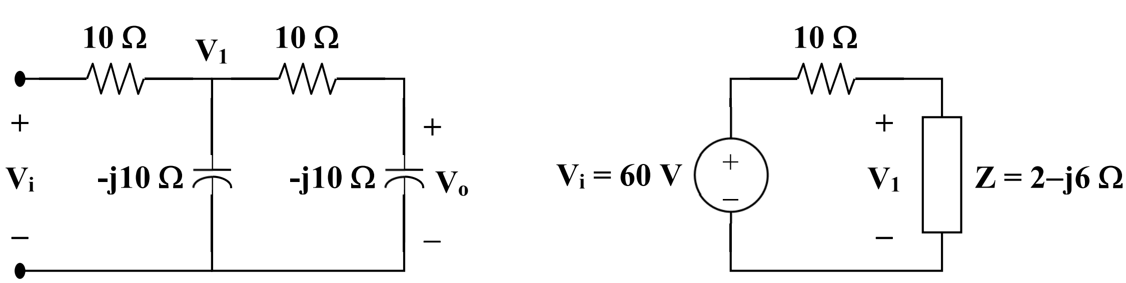
$$\mathbf{V}_o = \frac{-j2}{2 + j4 - j2}(50\angle 30°) = \frac{-j(50\angle 30°)}{1 + j} = \frac{50\angle(30° - 90°)}{\sqrt{2}\angle 45°}$$

$$\mathbf{V}_o = 35.36\angle -105°$$

$v_o(t)$ = **35.36 cos(10t – 105°) V**

P.P.9.13 To show that the circuit in Fig. (a) meets the requirement, consider the equivalent circuit in Fig. (b).

$$\mathbf{Z} = -j10 \,||\, (10 - j10) = \frac{-j10(10 - j10)}{10 - j20} = \frac{-j(10 - j10)}{1 - j2} = 2 - j6\ \Omega$$



(a) **(b)**

$$\mathbf{V}_1 = \frac{2 - j6}{10 + 2 - j6}(60) = \frac{60}{3}(1 - j)$$

$$\mathbf{V}_o = \frac{-\mathrm{j}10}{10 - \mathrm{j}10}\mathbf{V}_1 = \left(\frac{-j}{1-j}\right)\left(\frac{60}{3}\right)(1 - j) = -\mathrm{j}20$$

$$\mathbf{V}_o = 20\angle{-90^\circ}$$

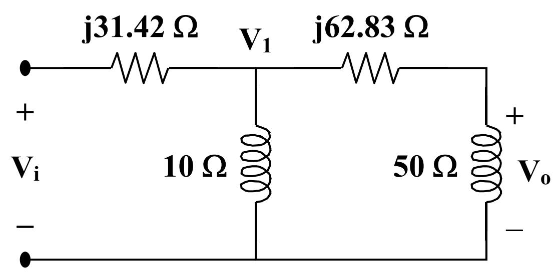
This implies that the RC circuit provides a 90° lagging phase shift.
The output voltage is = **20 V**

P.P.9.14

the 1-mH inductor is $\mathrm{j}\omega\mathrm{L} = \mathrm{j}(2\pi)(5\times10^{3})(1\times10^{-3}) = \mathrm{j}31.42$

the 2-mH inductor is $\mathrm{j}\omega\mathrm{L} = \mathrm{j}(2\pi)(5\times10^{3})(2\times10^{-3}) = \mathrm{j}62.83$

Consider the circuit shown below.



$$\mathbf{Z} = 10 \,||\, (50 + \mathrm{j}62.83) = \frac{(10)(50 + \mathrm{j}62.83)}{60 + \mathrm{j}62.83}$$

$$\mathbf{Z} = 9.205 + \mathrm{j}0.833 = 9.243\angle 5.17^\circ$$

$$\mathbf{V}_1 = \mathbf{Z} / (\mathbf{Z} + \mathrm{j}31.42)\,\mathbf{V}_i = \frac{9.243\angle 5.17^\circ}{9.205 + j32.253}(10)$$

$$= [(9.243\angle 5.17^\circ)/(33.54\angle 74.07^\circ)]10 = 2.756\angle{-68.9^\circ}$$

$$\mathbf{V}_o = \frac{50}{50 + \mathrm{j}62.83}\mathbf{V}_1 = \frac{50(2.756\angle -68.9^\circ)}{80.297\angle 51.49^\circ} = 1.7161\angle{-120.39^\circ}$$

Therefore,

magnitude = **1.7161 V**
phase = **120.39°**
phase shift is **lagging**