# Block A Unit 2

- It is important to apply the 3 basic laws in circuit theory to analyze and design circuits
- This unit is organized into the following 2 sections

I. Nodal Voltage Analysis, Mesh Current Analysis, Superposition (Ch. 3, 4.3)

II. Thevenin Equivalent Circuit, Norton Equivalent Circuit, Source Transformation, (Ch. 4.4 - 4.6)

Dependent Source (Ch. 1.6)

Alexander & Sadiku, “Fundamentals of Electric Circuits”, 7th Edition, McGraw Hill

# I. Nodal Voltage Analysis (NVA)

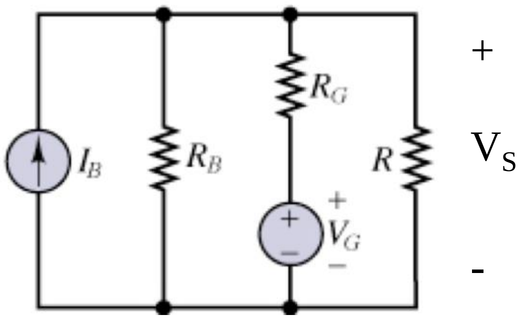
**KCL & Ohm's law in action**

- The method of nodal voltage analysis is *an application of KCL and Ohm's law together*
- The unknown variables that you will solve for are **node voltages**
- We will apply KCL at a mode and express the unknown currents as unknown node voltages

# Example on how to do NVA

**Find the voltage across the current source, $V_S$:**

Given: $I_B = 12A$, $V_G = 12V$, $R_G = 0.3\Omega$, $R_B = 1\Omega$, $R = 0.23\Omega$




**Apply KCL at $V_S$:**

$$I_B = \frac{V_S}{R_B} + \frac{V_S - V_G}{R_G} + \frac{V_S}{R}$$

$$12 = \frac{V_S}{1} + \frac{V_S - 12}{0.3} + \frac{V_S}{0.23}$$

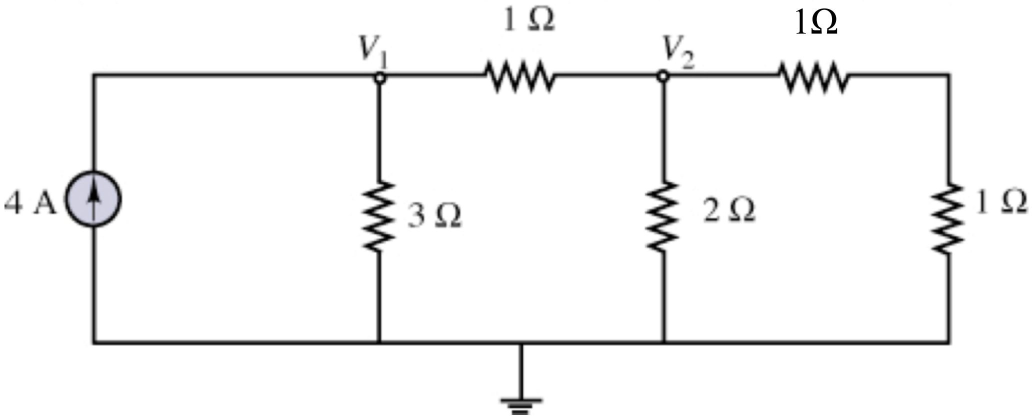
$$V_S = 5.99V$$

- Note that the negative terminal of $V_G$ is used as a reference (0V) in the example
- The value of $V_S$ is also referenced to this node
- The value of the reference node is not important since we are only interested in the voltage difference

# Worked Example on NVA 1

Use nodal voltage analysis to find $V_1$ and $V_2$. (Answer: $V_1 = 4.8V$, $V_2 = 2.4V$)






**First method:**

**-Apply KCL at $V_1$ (1)**

**-Apply KCL at $V_2$ (2)**

**-Solve equations (1) and (2)**

**Alternate method:**

**-Combine all the resistors to find $V_1$ first**

**-Then combine the 3 resistors on the right followed by voltage divider rule to find $V_2$**

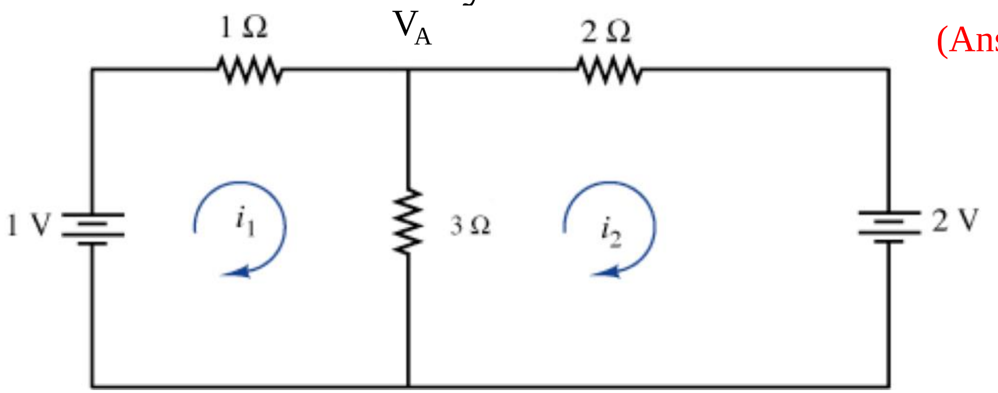
# Mesh Current Analysis (MCA)

**KVL & Ohm's law in action**

- The method of mesh current analysis is *an application of KVL and Ohm's law together*
- The unknown variables that you will solve for are **mesh currents**
- From KVL, the sum of voltage drops and rise must equal zero
- The voltage differences are expressed as the current going through each branch in the mesh

# Example on how to do MCA

Use mesh current analysis to find the current through the 3Ω resistor.




(Answer: $i_{3\Omega} = 4/11\text{A}$)

-There are 2 meshes and hence 2 mesh currents

-Apply KVL to each of these 2 meshes

**Apply KVL to mesh 1:**

$$1 = i_1(1+3) - 3i_2$$

$$4i_1 - 3i_2 = 1 \qquad (1)$$

**Apply KVL to mesh 2:**

Follow the defined direction of the mesh current

$$-2 = i_2(2+3) - 3i_1$$

$$5i_2 - 3i_1 = -2 \qquad (2)$$

**Now, solve equation (1) and (2):**

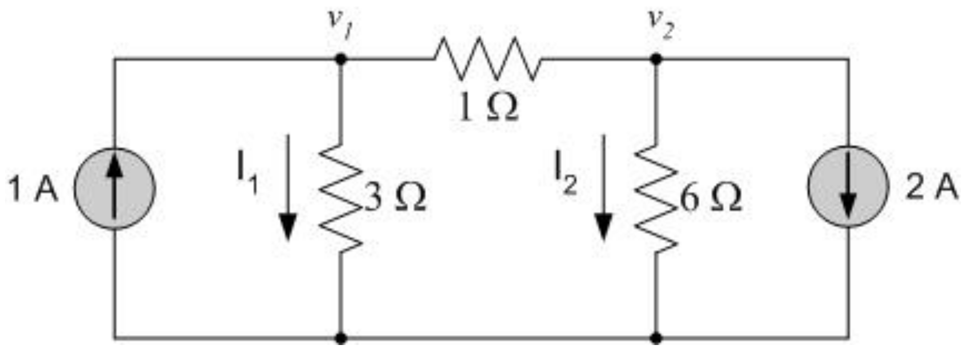
$$\mathbf{i_{3\Omega} = i_1 - i_2}$$

**Useful tip:**

**- keep voltages of sources on one side of the equation, and keep voltages of resistors on the other side**

# Worked Example on MCA 1

Use mesh current analysis to find currents $I_1$ and $I_2$. (Answer: $I_1 = I_2 = -0.5A$)



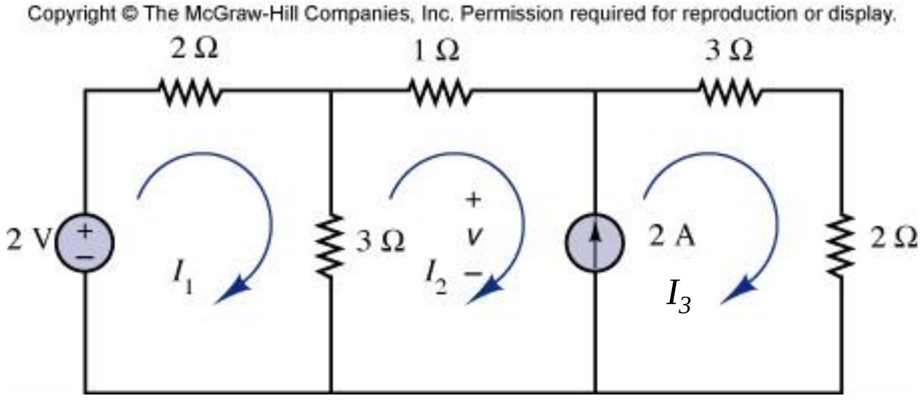

**-You see 3 meshes, but only the middle mesh is unknown, the other 2 are defined by the current sources.**

**-For simplicity, defined the direction of known mesh currents in the same direction as the sources.**

# Worked Example on MCA 2

Use mesh current analysis to find the voltage across the current source.

(Answer: V = 3.89 V)




**-In the process of deriving equations (2) and (3), a new variable V is introduced.**

**-Hence, we need one more equation, (4)**

**Apply KVL at mesh 1:**

$2 = I_1(2+3) - I_2(3)$

⇨ $5I_1 - 3I_2 = 2$ (1)

**Apply KVL at mesh 2:**

$-V = I_2(3+1) - I_1(3)$

⇨ $4I_2 - 3I_1 = -V$ (2)

**Apply KVL at mesh 3:**

$V = I_3(3+2)$

⇨ $5I_3 = V$ (3)

**One more equation:**

$I_3 – I_2 = 2A$ (4)

**Solve for V, verify by V = $(I_3)(3+2)$**

# Quick note on choosing NVA and MCA

How do you choose between NVA and MCA?

-Your choice should not be due to level of familiarity between the methods.

- One consideration is whichever is simpler to use for a given circuit.

- How then do you decide what is simpler?

- For a start, having fewer equations certainly makes solving easier.

# Superposition

## The method

- This method applies only to circuits that have **multiple sources**
- In such case, it can come in handy (or not)
- In a circuit with multiple sources, superposition considers the current or voltage associated with a given branch for one of the sources, while turning the rest off

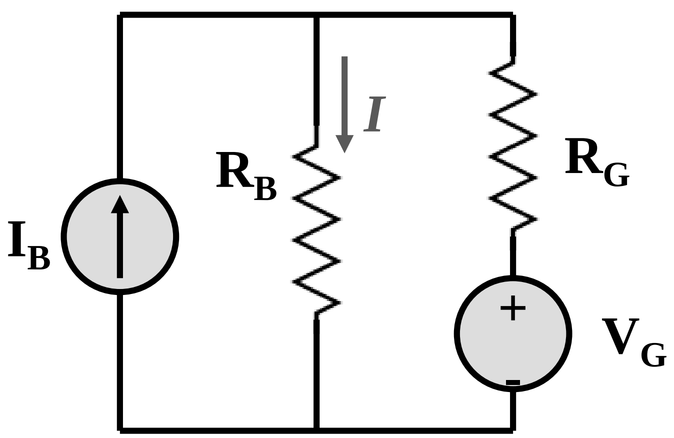



Fig 1: Basic circuit with 2 sources

**Aim: Find the current through $R_B$.**

1. **Find the current ($I_1$) through $R_B$ when only $I_B$ is present ($V_G$ is removed)**
2. **Find the current ($I_2$) through $R_B$ when only $V_G$ is present ($I_B$ is removed)**
3. **The net current $I = I_1 + I_2$**

# How do we “remove” a current source or voltage source?

**Disabling sources**

**Voltage source:** If the voltage source does not exist, the voltage across the terminals would be zero. It looks like a short circuit. (see Fig 14a)

**Current source:** If the current source does not exist, the current through it would be zero. It looks like an open circuit. (see Fig 14b)

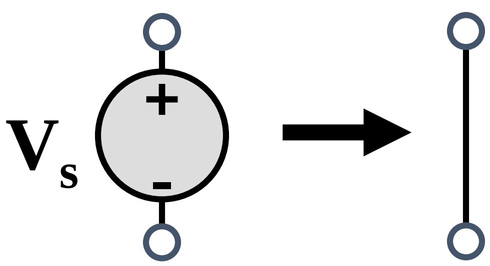



Fig 2a: Disable a voltage source by Replacing with a short circuit.

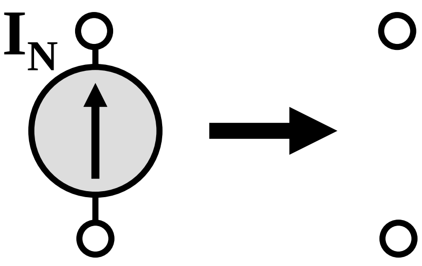



Fig 2b: Disable a current source by Replacing with an open circuit.

# Apply the tips to Fig 1

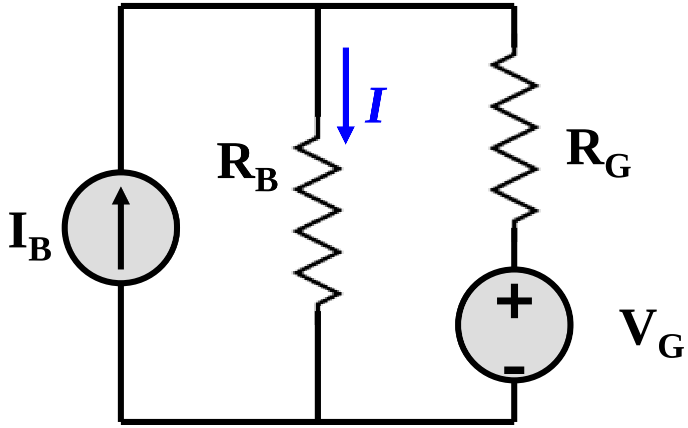
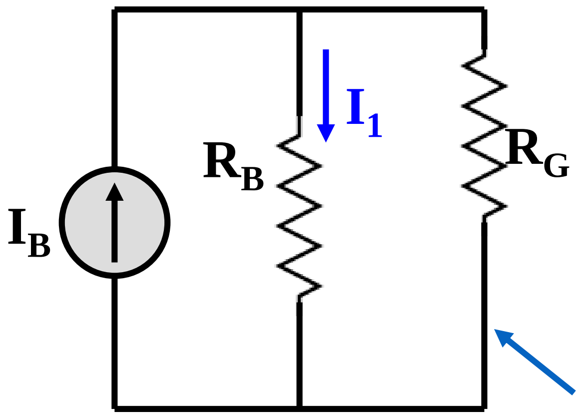


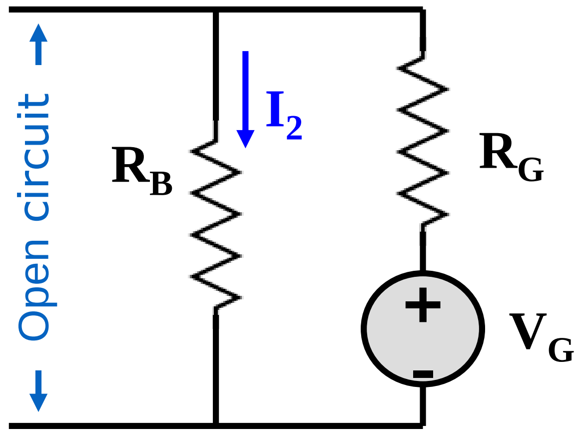

Fig 1: Basic circuit with 2 sources

## Disabling $V_G$, $V_G = 0$




$$\boxed{I = I_1 + I_2}$$

## Disabling $I_B$ , $I_B = 0$

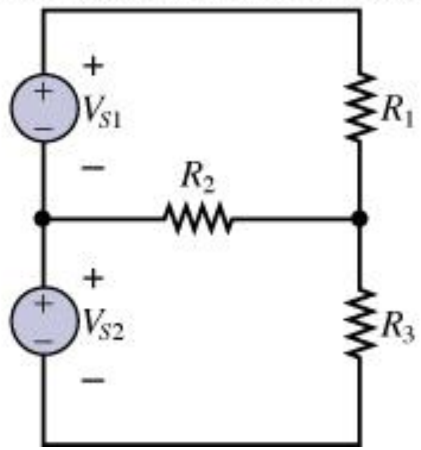


# Worked Example on Superposition

Determine the current through $R_1$ using superposition.

Given $R_1$ = 560Ω, $R_2$ = 3.5kΩ, $R_3$ = 810Ω, $V_{S2}$ = $V_{S1}$ = 90V






a. First, short $V_{S1}$
Voltage across $R_1$:

$$V_{R1a} = \frac{R_1 \| R_2}{\left(R_1 \| R_2\right) + R_3} V_{S2}$$

$$= \left( \frac{560 \| 3500}{\left(560 \| 3500\right) + 810} \right) 90$$

$$= 33.61V$$

Current through $R_1$:

$$I_{R1a} = V_{R1a} / R_{1a}$$

$$= 33.61 / 560$$

$$= 0.06A$$

# Worked Example on Superposition

b. Next, short $V_{S2}$

Current through $R_1$:

$$I_{R1b} = \frac{V_{S1}}{R_1 + (R_2 \parallel R_3)} = \frac{90}{560 + (3500 \parallel 810)}$$
$$= 0.074A$$

**Take careful note of the directions defined for each of these currents.**

Finally,

$I_{R1} = I_{R1a} + I_{R1b} = 0.134\text{A}$

# One-Port Network

**Introduction**

- A **one port network** is simply a **two terminal device** (See Fig below)

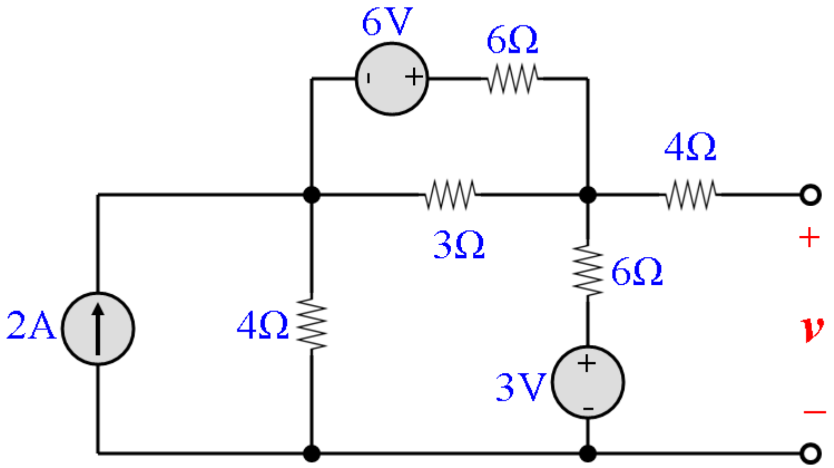


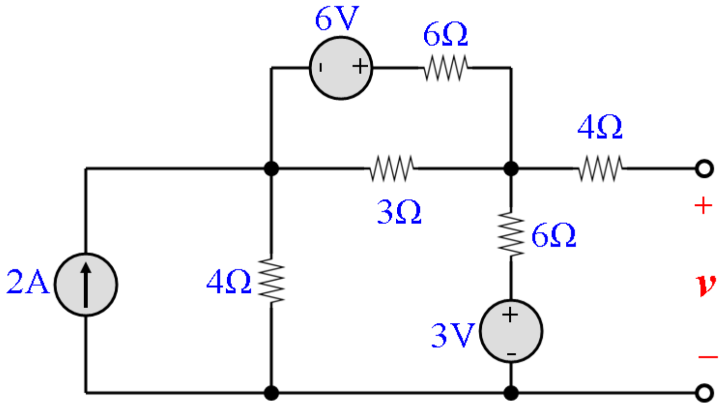

Fig 2: An example of a one port network

- Given a number of different resistor loads, if you were asked to find the current and voltage at the terminals for each resistor, you would have to recalculate the whole circuit
- A different load will give you different output current & voltage

# One-Port Network

**Simpler analysis**

- Transform any 2 terminal circuit into a circuit that is as simple as having 1 resistor and 1 source (see Fig 2)



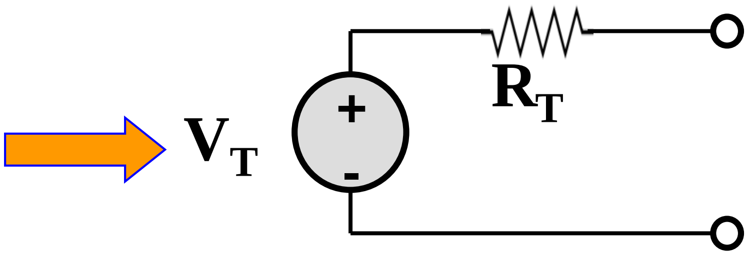




Fig 3: Transformed Thevenin equivalent circuit

The first part of this unit is organized into the following 3 sections:

- Thevenin equivalent
- Norton equivalent
- Source transformation

The last part of this unit will cover all concepts in Block A and apply them to dependent sources.

# II. Thevenin

**Definition**

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **voltage source** $V_T$ in **series** with an equivalent **resistance** $R_T$

**Deriving the Thevenin equivalent circuit**

**Step 1:** Remove the load from the rest of the one port network. This is the first most fundamental step.

**Step 2:** Find the equivalent resistance $R_T$. Disable all ideal sources then find the resistance across the terminals.

For voltage source – replace with short circuit

For current source – replace with open circuit

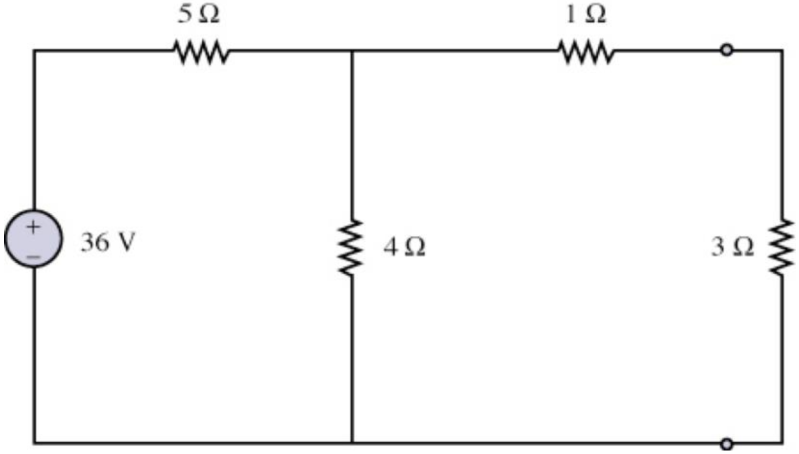
**Step 3:** Find the Thevenin voltage source. The Thevenin voltage source is equal to the open circuit voltage seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

# Worked Example on Thevenin 1

Derive the Thevenin equivalent circuit seen by the 3Ω load.

Answer: $R_{th}$ = 3.22 Ω, $V_{th}$ = 16 V






Steps:

1. Remove the 3Ω load
2. Find $R_T$ (disable the voltage source)
3. Find $V_T$ (with no load)
4. Draw the Thevenin circuit

# Norton

**Definition**

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **current source** $I_N$ in **parallel** with an equivalent **resistance** $R_N$

**Deriving the Norton equivalent circuit**

**Steps 1 and 2** for deriving the Norton equivalent circuit are exactly the same as that for Thevenin.

**Step 3:** Find the Norton current source. The Norton current source is equal to the short circuit current seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

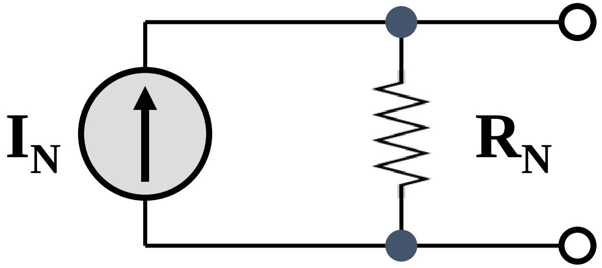


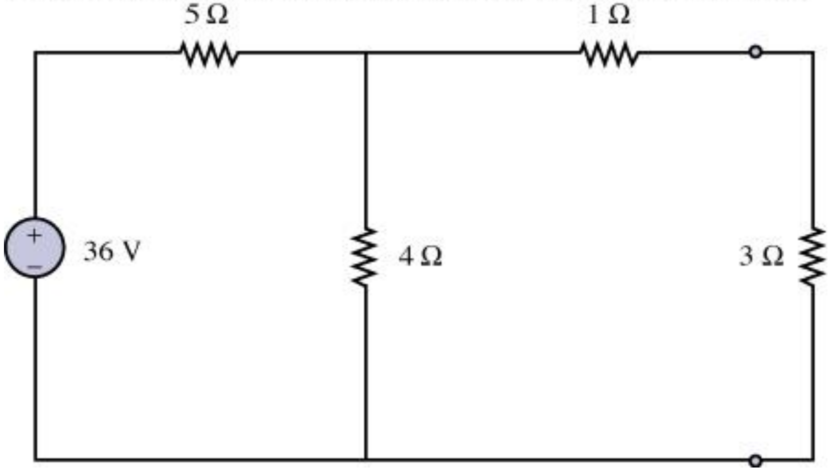

Fig 4: Transformed Norton equivalent circuit

# Worked Example on Norton 1

Derive the Norton equivalent circuit seen by the 3Ω load.

Answer: $R_N = 3.22\ \Omega$, $I_N = 4.96$ A





# Source Transformation

**Definition**

- This trick rests on the fact that the Thevenin and Norton forms are equivalent to each other and therefore are also interchangeable

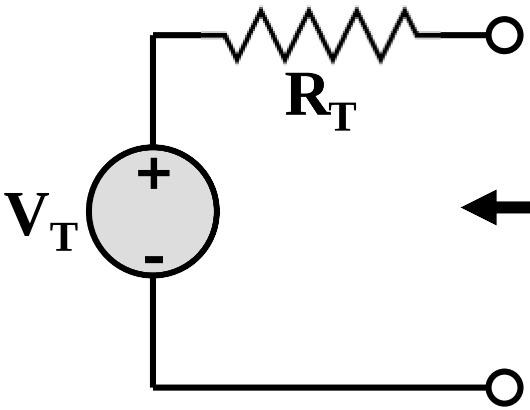



Fig 5a: Thevenin equivalent circuit

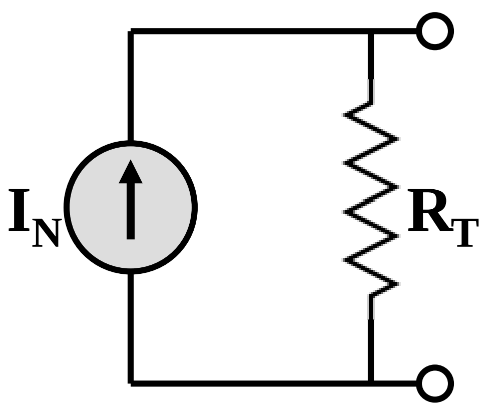



Fig 5b: Norton equivalent circuit

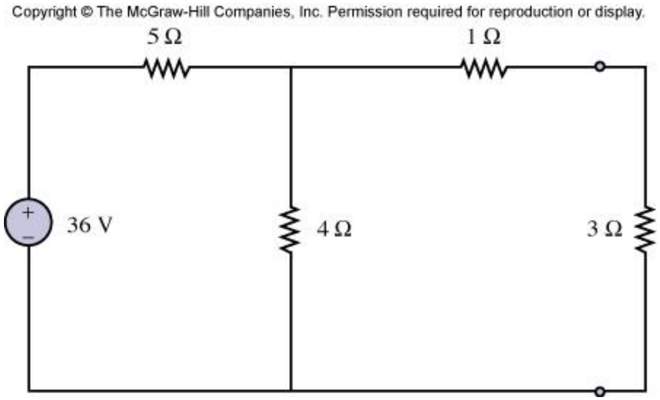
**We can transform between the two equivalent circuits, observing each time that:**

$$V_T = I_N R_T$$

- Rather than transform the whole circuit in one go to Thevenin or Norton equivalent forms, we can instead transform part of the circuit
- The process of merge transform and merge again can be repeated

# Worked Example on Source Transformation 1

Derive either of the equivalent circuit forms seen by the 3Ω load.



# Dependent Sources

- All the sources we have come across are independent source
  - For a voltage source, the voltage maintained across the source is fixed
  - For a current source, the current through the source is fixed
- There also exist dependent sources in circuit theory
- Unlike independent sources, the value of a dependent source is not predetermined, but is set by the current or voltage through a specific branch

**Symbol**

- The symbol for an independent source is a circle
- For a dependent source, the symbol is a diamond

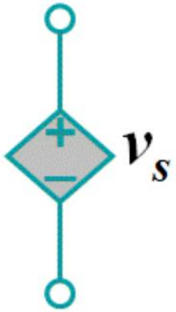



Fig 6a: Dependent voltage source

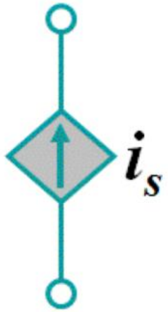
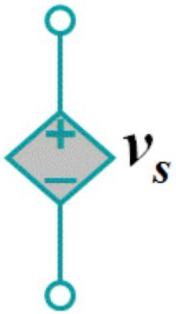



Fig 6b: Dependent current source

# Dependent Sources

## Dependent Voltage Source




$v_S$ is the voltage source across the terminals.

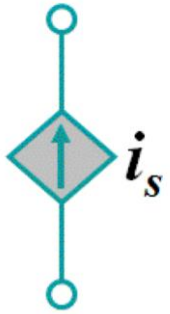
**Current Controlled Voltage Source**

For example, $v_S = 5i_X$. The current $i_X$ is not the current through the source but belongs to another branch.

**Voltage Controlled Voltage Source**

For example, $v_S = 7v_X$. The voltage $v_X$ is not the source voltage. As $v_X$ changes, so also $v_S$ according to the above relations.

## Dependent Current Source




$i_S$ is the current source through the terminals.

**Current Controlled Current Source**

For example, $i_S= 5i_X$. The current $i_X$ is not the current through the source but belongs to another branch.

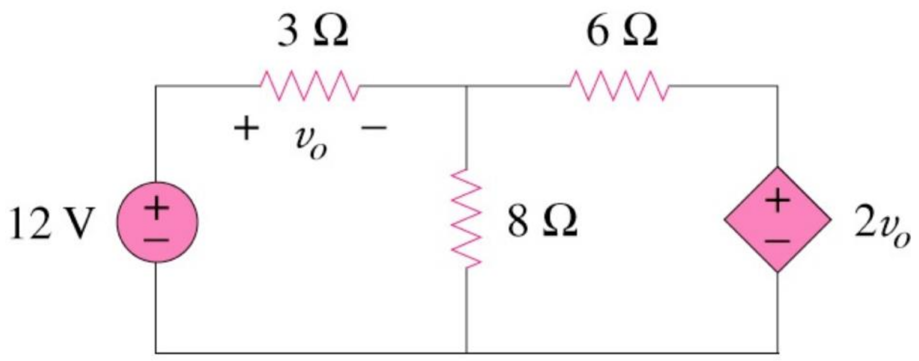
**Voltage Controlled Current Source**

For example, $i_S = 7v_X$. The voltage $v_X$ is not the source voltage. As $v_X$ changes, so also $i_S$ according to the above relations.

# Worked Example on Dependent Source 1

Calculate $v_o$ in the circuit.

Answer: $v_o = 3.65$ V




Hints:

1. The dependent source here is a voltage controlled voltage source
2. Apply KCL

# Summary on Block A

- This block covered the fundamental concepts to learn Blocks B, C, and D.

I. Kirchhoff's Current, Voltage Law, Ohm's Law, Power

II. Sources, Short Circuit, Open Circuit

III. Resistive Networks

IV. Measuring Instruments

V. Nodal Voltage Analysis, Mesh Current Analysis, Superposition

VI. Thevenin Equivalent Circuit, Norton Equivalent Circuit, Source Transformation, Dependent Source

- Students can refer to the textbook for more working examples

Alexander & Sadiku, "Fundamentals of Electric Circuits", 7th Edition, McGraw Hill