EE1004 Tutorial 3 (Part 2)

1. An electric scale gives a reading equal to the true weight plus a random error that is normally distributed with mean 0 and standard deviation σ = 0.1 mg. Suppose that the results of five successive weighings of the same object are as follows: 3.142, 3.163, 3.155, 3.150, 3.141.

(a) Determine a 95 percent confidence interval estimate of the true weight.

(b) Determine a 99 percent confidence interval estimate of the true weight.

Answer 1. $E[X]$ = 3.1502. Use the **Normal Distribution Calculator** to find the critical z-score.

(a) For a 95 percent two-sided confidence interval:

critical probability = 1 – (1-0.95)/2 = 0.975

critical z-score = 1.960

confidence interval = $3.1502 \pm 1.960(0.1)/\sqrt{5} = 3.1502 \pm 0.08765$ = (3.06255, 3.23785)

(b) For a 99 percent two-sided confidence interval:

critical probability = 1 – (1-0.99)/2 = 0.995

critical z-score = 2.576

confidence interval = $3.1502 \pm 2.576(0.1)/\sqrt{5} = 3.1502 \pm 0.11520$ = (3.035, 3.2654)

2. The standard deviation of test scores on a certain achievement test is 11.3. If a random sample of 81 students had a sample mean score of 74.6, find a 90 percent confidence interval estimate for the average score of all students.

Answer 2. Use the **Normal Distribution Calculator** to find the critical z-score. $74.6 \pm 1.645(11.3)/9 = 74.6 \pm 2.065$ = (72.535, 76.665)

3. Each of 20 science students independently measured the melting point of lead. The sample mean and sample standard deviation of these measurements were (in degrees centigrade) 330.2 and 15.4, respectively. Construct (a) a 95 percent and (b) a 99 percent confidence interval estimate of the true melting point of lead.

Answer 3. Use the **t Distribution Calculator** to find the critical t statistic.

(a) For a 95 percent two-sided confidence interval:

critical probability = 1 – (1-0.95)/2 = 0.975

critical t statistic = 2.093

confidence interval = $330.2 \pm 2.093(15.4)/\sqrt{20} = 330.2 \pm 7.207339$ = (322.993, 337.407)

(b) For a 99 percent two-sided confidence interval:

critical probability = 1 – (1-0.99)/2 = 0.995

critical t statistic = 2.861

confidence interval = 330.2 ± 2.861(15.4)/$\sqrt{20}$ = 330.2 ± 9.85198 = (320.34802, 340.05198)

4. The following are the daily number of steps taken by a certain individual in 20 weekdays.

2,100 1,984 2,072 1,898

1,950 1,992 2,096 2,103

2,043 2,218 2,244 2,206

2,210 2,152 1,962 2,007

2,018 2,106 1,938 1,956

Assuming that the daily number of steps is normally distributed, construct (a) a 95 percent and (b) a 99 percent two-sided confidence interval for the mean number of steps.

Answer 4. Use the **t Distribution Calculator** to find the critical t statistic.

The sample standard deviation = 104.3435

The sample mean = 2062.75

degrees of freedom=20-1=19

(a) For a 95 percent two-sided confidence interval:

critical probability = 1 – (1-0.95)/2 = 0.975

critical t statistic = 2.093

confidence interval = 2062.75 ± 2.093(104.3435)/$\sqrt{20}$ = 2062.75 ± 48.83367 = (2013.91633, 2111.58367)

(b) For a 99 percent two-sided confidence interval:

critical probability = 1 – (1-0.99)/2 = 0.995

critical t statistic = 2.861

confidence interval = 2062.75 ± 2.861(104.3435)/$\sqrt{20}$ = 2062.75 ± 66.752611 = (1995.997, 2129.503)