EE1001, Test 2, 28 October 2020, online, 9:05am – 10:20am (1hr 15min)

EE1001 Test 2 Q&A

Q1)
Determine the following statements whether they are proposition, true or false.

(18 marks)

(i) The Sun is a star.
(ii) May God bless you!
(iii) Every real number is a complex number.
(iv) Every quadratic equation has two real roots.
(v) $\sin 2\theta = 2\sin\theta\cos\theta$ for all $\theta \in R$.
(vi) "$\sim q \leftrightarrow \sim p$" is logically equivalent to "$q \leftrightarrow p$"
(vii) An invalid argument can go from false premises to a true conclusion.
(viii) An argument with all true premises must be sound.
(ix) $\sim[\forall h \in \mathbf{C}\ \exists k \in \mathbf{E}, F(h,k)] \equiv \exists h \in \boldsymbol{C}\ \forall k \in \boldsymbol{E}, F(h,k)$.

Q2)
Consider the interpretation u where $u(p) = F$, $u(q) = T$, $u(r) = T$, does u satisfy the following propositional formulas?

(4 marks)

1. $(p \rightarrow \sim q) \vee \sim(r \wedge q)$
2. $(\sim p \vee \sim q) \rightarrow (p \vee \sim r)$
3. $\sim(\sim p \rightarrow \sim q) \wedge r$
4. $\sim(\sim p \rightarrow q \wedge \sim r)$

Q3)
If A = {3, 4, 6, 8}, determine the truth value of each of the following:

i. $\exists\, x \in A$, such that $x + 4 = 7$
ii. $\forall\, x \in A$, $x + 4 < 10$.
iii. $\forall\, x \in A$, $x + 5 \geq 13$

(6 marks)

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Q4)

Considering the addition between two 1-bit binary (0 or 1) number A and B.

(1) Compute the truth table of A XOR B ($A \oplus B$) , A and B (A.B);

(2) Compute the truth table of A+B ;

(3) Observing the result of A+B. The first column is called carry and the second column is called sum.

a.) Find the relationship between A, B and carry.

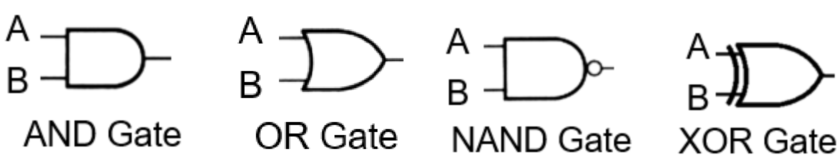
b.) Find the relationship between A, B and sum.

(4) Draw the logic circuit of A+B with appropriate logic gate

A	**B**	**A.B**
0	**0**	
0	**1**	
1	**0**	
1	**1**	

A	**B**	**A.B**
0	**0**	
0	**1**	
1	**0**	
1	**1**	

A	**B**	**A+B**	
0	**0**		
0	**1**		
1	**0**		
1	**1**		



(12 marks)

Q5)

Express the following in symbolic form.

i. I like playing but not singing.

ii. Anand neither likes cricket nor tennis.

iii. Rekha and Rama are twins.

iv. It is not true that ‘i’ is a real number.

v. Either 25 is a perfect square or 41 is divisible by 7.

(6 marks)

Q6)

Determine whether $((A \rightarrow (B \vee C)) \vee (A \rightarrow B))$ is a tautology.

(5 marks)

Q7)

Let P be $((A1 \wedge A2) \rightarrow A1)$, let P1 be $(B \vee C)$ and let P2 be $(C \wedge D)$. Determine J be $(((B \vee C) \wedge (C \wedge D)) \rightarrow (B \vee C))$ is T or F.

(5 marks)

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Q8)

There are two types of computers: normal and abnormal computers. The normal computers always deliver real message, whereas the abnormal computers always deliver fake message. There are three computers *A*, *B*, and *C*. Their messages are delivered as follows.

A: "Either *B* is normal or *C* is abnormal";
B: "At least one of us is normal";
C: "*B* is normal"

Determine whether the computers *A*, *B*, and *C* are normal or abnormal. Use truth table to justify your answer.

(10 marks)

Q9)

Without using a truth table, determine whether $(\sim q \wedge \sim r \wedge p) \vee (p \wedge q) \vee (r \wedge p) \rightarrow (p \vee s)$ is a tautology or a contradiction. State the reason of each step. Then use a truth table to verify.

(20 marks)

Q10)

Given the following statements:

1) I read the book in the office or I read the book in the living room.
2) If I read the book in the office, then I would drink coffee.
3) If I read the book in the office and drink coffee, I would put my phone on the work desk.
4) If I read the book in my living room, then I would put my phone on the bedside table.
5) I can't find my phone on the bedside table.

Use inference rules and logical equivalence relation to determine the location of the phone. Justify the reason of each step.

(10 marks)

Q11)

For the network shown in Fig Q11, determine, using truth table, the condition when the lamp grows.

(4 marks)

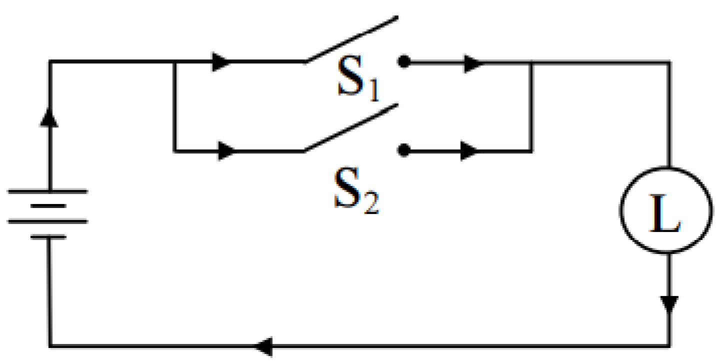

Fig. Q11

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<u>Solution to EE1001 Test 1</u>

<u>Q1) Solution:</u>

i. It is a proposition; its truth value is 'T'.
ii. It is an exclamatory sentence, hence, it is not a proposition; its truth value is undetermined.
iii. It is a proposition; its truth value is 'T'.
iv. It is a proposition; its truth value is 'F'.
v. It is a proposition; its truth value is 'T'.
vi. It is a proposition; its truth value is 'T'.
vii. It is a proposition; its truth value is 'T'.
viii. It is a proposition; its truth value is 'F'.
ix. It is a proposition; its truth value is 'F'. **(18 marks)**

<u>Q2) Solution:</u>

By substituting $u(p)$ = F, $u(q)$ = T, $u(r)$ = T into formulas 1, 2, 3, and 4, the results of 1, 3, and 4 are true, whereas the result of 2 is false.
Therefore, u satisfies 1, 3 and 4, and doesn't satisfy 2. **(4 marks)**

<u>Q3) Solution:</u>

i. Since x = 3 ∈ A, satisfies x + 4 = 7 ∴ the given statement is true. ∴ Its truth value is 'T'.
ii. Since x = 6, 8 ∈ A, do not satisfy x + 4 < 10, ∴ the given statement is false. ∴ Its truth value is 'F'
iii. Since x = 3, 4, 6 ∈ A, do not satisfy x + 5 ≥ 13, ∴ the given statement is false. ∴ Its truth value is 'F'.

(6 marks)

<u>Q4) Solution:</u>

(1)

A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

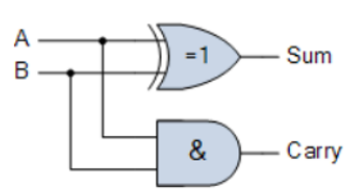
A	B	A⊕B
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	A+B	
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

(3) Comparing the results in question (1) and (2), it can be observed that:
   a.) CARRY = A AND B = A.B
   b.) SUM = A XOR B = A ⊕ B

(4) Based on the conclusion in (3), one AND gate and one XOR gate should be adopted:



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(12 marks)

Q5) Solution:

i. Let p: I like playing, q: I like singing, ∴ The symbolic form of the given statement is $p \wedge \sim q$.
ii. Let p: Anand likes cricket, q: Anand likes tennis. ∴ The symbolic form of the given statement is $\sim p \wedge \sim q$.
iii. In this statement 'and' is combining two nouns and not two simple statements. Hence, it is not used as a connective, so given statement is a simple statement which can be symbolically expressed as p itself.
iv. Let p: 'i' is a real number. ∴ The symbolic form of the given statement is $\sim p$.
v. Let p: 25 is a perfect square, q: 41 is divisible by 7. ∴ The symbolic form of the given statement is "$p \vee q$" (XOR: "$p \oplus q$" is also acceptable). **(6 marks)**

Q6) Solution:

((A → (B ∨ C)) ∨ (A→ B))

A	B	C	B ∨ C	A→(B ∨ C)	A→ B	((A → (B ∨ C)) ∨ (A→ B))
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	F	T
T	F	F	F	F	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T
F	F	F	F	T	T	T

It's not a tautology **(5 marks)**

Q7) Solution:

((A1 ∧ A2) → A1)
= ~(A1 ∧ A2) ∨ A1 (Definition of →)
= ~A1 ∨ ~A2 ∨ A1 (De Morgan's Laws)
= ~A1 ∨ A1 ∨ ~A2 (Commentative laws)
= **t** ∨ ~A2 (Negation laws)
= **t** (Universal bound laws)
i.e., P is a tautology

Assume that P is a tautology. For any assignment of truth values to the statement letters in J, the forms P1, ... , Pn have truth values x1, ... , xn (where each xn is T or F). If we assign the values x1, ... , xn to A1, ... , An, respectively, then the resulting truth value of P is the truth value of J for the given assignment of truth values. Since P is a tautology, this truth value must be T. Thus, the truth value of J is always T.

Alternative Answer:
((A1 ∧ A2) → A1)
= ~(A1 ∧ A2) ∨ A1 (Definition of →)
= ~A1 ∨ ~A2 ∨ A1 (De Morgan's Laws)
= ~A1 ∨ A1 ∨ ~A2 (Commentative laws)
= **t** ∨ ~A2 (Negation laws)
= **t** (Universal bound laws)
i.e., P is a tautology

By substituting "A1 = B ∨ C" and "A2 = C ∧ D", J is equal to P. As a result, J is a tautology, and its truth value is always T.

(5marks)

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Q8) Solution:

Let
a = "A is normal computer"
b = "B is normal computer"
c = "C is normal computer"
"$b \oplus \sim c$" = A's message = "Either B is normal or C is abnormal"
"$a \vee b \vee c$" = B's message = "At least one of us is normal"
"b" = C's message = "B is normal"
The condition is satisfied only when the truth values of the computers and the truth values of the computers' messages are the same, i.e.,
$[a \leftrightarrow (b \oplus \sim c)] \wedge [b \leftrightarrow (a \vee b \vee c)] \wedge (c \leftrightarrow b)$ = true

a	b	c	$b \oplus \sim c$	$a \vee b \vee c$	$a \leftrightarrow (b \oplus \sim c)$	$b \leftrightarrow (a \vee b \vee c)$	$c \leftrightarrow b$	$[a \leftrightarrow (b \oplus \sim c)] \wedge$ $[b \leftrightarrow (a \vee b \vee c)] \wedge$ $(c \leftrightarrow b)$
T	T	T	T	T	T	T	T	**T**
T	T	F	F	T	F	T	F	F
T	F	T	F	T	F	F	F	F
T	F	F	T	T	T	F	T	F
F	T	T	T	T	F	T	T	F
F	T	F	F	T	T	T	F	F
F	F	T	F	T	T	F	F	F
F	F	F	T	F	F	T	T	F

Therefore, computers A, B, and C are all normal computers. **(10 marks)**

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Q9) Solution:

$(\sim q \wedge \sim r \wedge p) \vee (p \wedge q) \vee (r \wedge p) \rightarrow (p \vee s)$

$\equiv (\sim q \wedge \sim r \wedge p) \vee (p \wedge q) \vee (p \wedge r) \rightarrow (p \vee s)$	(Commutative laws)
$\equiv (\sim q \wedge \sim r \wedge p) \vee [p \wedge (q \vee r)] \rightarrow (p \vee s)$	(Distributive laws)
$\equiv (p \wedge \sim q \wedge \sim r) \vee [p \wedge (q \vee r)] \rightarrow (p \vee s)$	(Commutative laws)
$\equiv [p \wedge \sim(q \vee r)] \vee [p \wedge (q \vee r)] \rightarrow (p \vee s)$	(De Morgan's laws)
$\equiv p \wedge [\sim(q \vee r) \vee (q \vee r)] \rightarrow (p \vee s)$	(Distributive laws)
$\equiv p \wedge \mathbf{t} \rightarrow (p \vee s)$	(Negation Laws)
$\equiv p \rightarrow (p \vee s)$	(Identity laws)
$\equiv \sim p \vee (p \vee s)$	(Definition of $\rightarrow$)
$\equiv (\sim p \vee p) \vee s$	(Associative Laws)
$\equiv \mathbf{t} \vee s$	(Negation Laws)
$\equiv \mathbf{t}$	(Universal Bound Laws)

It's a tautology.

p	q	r	s	$\sim q \wedge \sim r \wedge p$	$(p \wedge q)$	$(r \wedge p)$	$(\sim q \wedge \sim r \wedge p) \vee (p \wedge q) \vee (r \wedge p)$	$p \vee s$	$(\sim q \wedge \sim r \wedge p) \vee (p \wedge q) \vee (r \wedge p) \rightarrow (p \vee s)$
T	T	T	T	F	T	T	T	T	**T**
T	T	T	F	F	T	T	T	T	**T**
T	T	F	T	F	T	F	T	T	**T**
T	T	F	F	F	T	F	T	T	**T**
T	F	T	T	F	F	T	T	T	**T**
T	F	T	F	F	F	T	T	T	**T**
T	F	F	T	T	F	F	T	T	**T**
T	F	F	F	T	F	F	T	T	**T**
F	T	T	T	F	F	F	F	T	**T**
F	T	T	F	F	F	F	F	F	**T**
F	T	F	T	F	F	F	F	T	**T**
F	T	F	F	F	F	F	F	F	**T**
F	F	T	T	F	F	F	F	T	**T**
F	F	T	F	F	F	F	F	F	**T**
F	F	F	T	F	F	F	F	T	**T**
F	F	F	F	F	F	F	F	F	**T**

(20 marks)

(Q10) Solution:

Let
o = office
r = living room
c = coffee
w = work desk
b = bedside table

1.	$o \vee r$	(premise)
2.	$o \rightarrow c$	(premise)
3.	$o \wedge c \rightarrow w$	(premise)
4.	$r \rightarrow b$	(premise)
5.	$\sim b$	(premise)
6.	$o \rightarrow o \wedge c$	(Absorption 2)
7.	$o \rightarrow w$	(HS 6,3)
8.	$\sim r$	(MT 4,5)
9.	o	(DS 1,8)
10.	w	(MP 7,9)

The phone is on the work desk. **(10 marks)**

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<u>Q11) Solution:</u>

Let

p = "S1 is closed"

q = "S2 is closed"

The lamp L grows when $p \vee q$ is true

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

When two switches are opened, the lamp doesn't grow.

When two switches are closed or either one switch is closed, the lamp grows. (4 marks)

-- END --