

Question 1.

- (a) Big data has the 4V properties. Use the operation of a big supermarket company to explain the terms: Volume and Velocity in Big Data.

(6 marks)

- (b) Suppose that \mathbf{S} is a $D \times n$ transaction matrix of a supermarket company, that gives the sales amount (in HKD) of n products over D days. What are the meanings of $\mathbf{S}\mathbf{1}$ and $\mathbf{S}^T\mathbf{1}$? Your answer should be in English.

(6 marks)

Question 2.

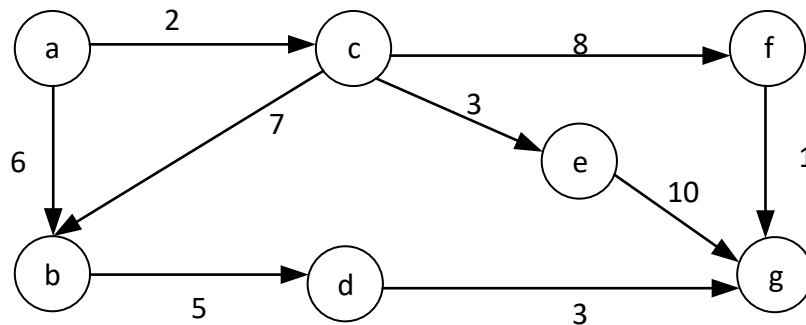


Figure Q1a

- (a) Figure Q1 show a weighted digraph.

- (a1) What is the adjacency matrix of the weight digraph?

(3 marks)

- (a2) Use the Dijkstra's Algorithm to find the shortest path from node-a to node-g. Show your steps.

(8 marks)

- (b) Given an $n \times n$ matrix adjacency matrix \mathbf{A} of a graph (not a weighted graph).

- (b1) Based on multiplication of matrix and vector, discuss the way to get the degree of each node.

(3 marks)

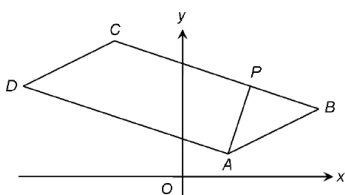
- (b2) Based on multiplication of matrix and vector, discuss the way to check if there is a path from node-i to node-j. Hint: consider the standard unit vector \mathbf{e}_i .

(8 marks)

Question 3

In Figure Q3, $ABCD$ is a parallelogram in 2 dimensional space. The coordinates of A , B and C are $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$, and $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

- Find the displacement vector: \mathbf{d}_{BC} (from B to C). (2 marks)
- Find the coordinates of D (2 marks)
- Express D as a linear combination of B and C . (4 marks)
- Let P be a point on \mathbf{d}_{BC} such that $\|\mathbf{d}_{BP}\| : \|\mathbf{d}_{PC}\| = 1 : m$. Find \mathbf{d}_{AP} in terms of m , where \mathbf{d}_{BP} is the displacement vector from B to P and \mathbf{d}_{PC} is the displacement vector from P to C . (4 marks)
- If \mathbf{d}_{AP} and \mathbf{d}_{BC} are orthogonal, what is the value of m . (4 marks)



Question Q3

Question 4

- Let P be a 3×3 matrix.
 - Show that $(P - 2I)(P - I) = P^2 - 3P + 2I$. Show your steps. (3 marks)
 - Given that $M = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, show that $M^2 - 3M + 2I = \mathbf{0}$, where $\mathbf{0}$ is a zero matrix. (3 marks)
 - What is the determinant of M ? Show your steps. (3 marks)
 - If $P^2 - 3P + 2I = \mathbf{0}$, can you conclude that either $P = 2I$ or $P = I$? Explain. (3 marks)
- Let Q be block matrix, given by $Q = \begin{pmatrix} I & C \\ B & A \end{pmatrix}$, where A is a 5×3 matrix and I is 4×4 identity matrix. What are the sizes of B , C , and Q ? Explain your answer. (4 marks)
- Let $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$. Describe in words how x and $y = Ax$ are related. (4 marks)