Practice questions for EE1002

Multiple-choice question part:

Note: For all the practice questions below, reference answers are given. The answers are marked with nswer .

If $z = 3 - 2j$, please give the value of zz^* where z^* is the conjugate of z.

nswer
- ○ 13
- ○ 5
- ○ 5-12j
- ○ 13-12j

Express $z = -1 - j$ in polar form.

nswer
- ○ √2∠-135°
- ○ √2∠135°
- ○ √2∠45°
- ○ √2∠-45°

Find the rate of change of $y(x) = 4 + 2x - 3x^2$ at $x = 3$.

nswer
- ○ -16
- ○ 16
- ○ 12
- ○ -12

If $y = 2ln(\sqrt{x} + 6)$ please find the derivative of y .

nswer
- $\frac{1}{x+6\sqrt{x}}$
- $\frac{1}{\sqrt{x}}$
- $\frac{2}{\sqrt{x}+6}$
- $\frac{1}{x-6\sqrt{x}}$

If $y = e^{2x} sin3x - cosx$, please find $y\prime(\frac{\pi}{6})$.

Answer

$2e^{\frac{\pi}{3}} + \frac{\sqrt{3}}{2}$

$3e^{\frac{\pi}{6}} + \frac{\sqrt{3}}{2}$

$2e^{\frac{\pi}{3}} - \frac{\sqrt{3}}{2}$

$\frac{5\sqrt{2}}{2} e^{\frac{\pi}{3}} + \frac{\sqrt{3}}{2}$

If $y = \frac{cos(x^2)}{x^3}$, please find the derivative of y.

Answer

$-2x^{-2} sin(x^2) - 3x^{-4} cos(x^2)$

$2x^{-2} cos(x^2) + 3x^{-4} cos(x^2)$

$-2x^{-2} cos(x^2) + 3x^{-4} cos(x^2)$

$2x^{-2} sin(x^2) + 3x^{-4} cos(x^2)$

Evaluate $\int cos^3 x dx$.

Answer

$sinx - \frac{sin^3 x}{3} + c$

$\frac{cos^3 x}{3} + c$

$-sinx \frac{cos^3 x}{3} + c$

$sinx \frac{cos^3 x}{3} + c$

Solve the differential equation $\frac{dy}{dx} = x^3 e^{-y}$ with the initial condition of $y(0) = \frac{1}{2}$ for y.

nswer

$y = \ln\left(\frac{1}{4}x^4 + e^{\frac{1}{2}}\right)$

$y = \ln(x^3 + \frac{1}{2}e)$

$y = \ln\left(3x^2 + e^{\frac{1}{2}}\right)$

$y = \ln\left(\frac{1}{4}x^4 + \frac{1}{2}e\right)$

A sinusoidal function has an amplitude of $2\sqrt{2}$, a frequency of 3 and phase of $\frac{2\pi}{5}$. State a sinusoidal form of the function.

nswer

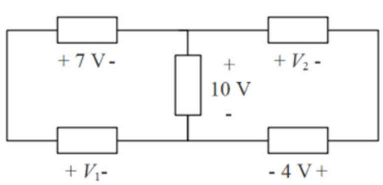
$2\sqrt{2}sin(6\pi t + \frac{2\pi}{5})$

$8sin(6\pi t + \frac{2\pi}{5})$

$2\sqrt{2}sin(\frac{2\pi}{3}t + \frac{2\pi}{5})$

$8sin(\frac{2\pi}{3}t + \frac{2\pi}{5})$

In the following circuit, calculate V_1 and V_2.



swer

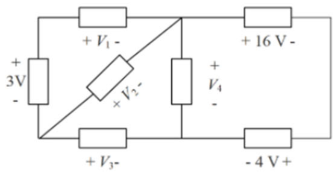
V_1 = 17 V, V_2 = 6 V

V_1 = 3 V, V_2 = 6 V

V_1 = 17 V, V_2 = -6 V

V_1 = - 17 V, V_2 = -6 V

Obtain v_1, v_2, v_3, and v_4 ($v_2 : v_3$ = 3:1) in the following circuit.



nswer

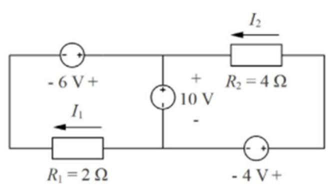
v_1 = -27 V, v_2 = -30 V, v_3 = -10 V, v_4 = *20 V*

v_1 = -15 V, v_2 = -18 V, v_3 = -6 V, v_4 = *12 V*

v_1 = 33 V, v_2 = 30 V, v_3 = 10 V, v_4 = *20 V*

v_1 = -15 V, v_2 = 18 V, v_3 = 6 V, v_4 = *12 V*

From the following circuit, find I_1, I_2, and the power dissipated by the resistor R_1, R_2.



nswer

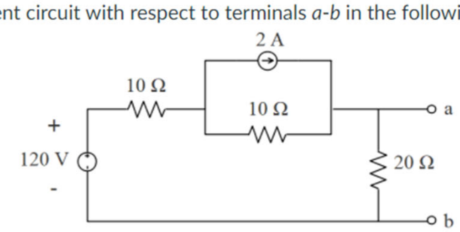
I_1 = -2 A, I_2 = *-1.5 A*, $P_1 : P_2$ = *8:9*

I_1 = 2 A, I_2 = *1.5 A*, $P_1 : P_2$ = *8:9*

I_1 = -8A, I_2 = *-3.5 A*, $P_1 : P_2$ = *128:49*

I_1 = 8A, I_2 = *3.5 A*, $P_1 : P_2$ = *128:49*

Find the Norton equivalent circuit with respect to terminals *a-b* in the following circuit.



nswer

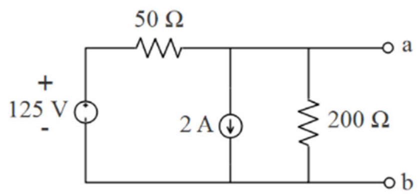
R_N = 10 Ω; I_N = 7 A

R_N = 10 Ω; I_N = 5 A

R_N = 40 Ω; I_N = 7 A

R_N = 40 Ω; I_N = 5 A

Determine the Thevenin equivalent circuit with respect to terminals *a-b* in the following circuit.



nswer

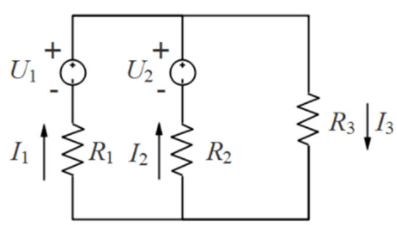
R_{th} = 40 Ω, V_{th} = 20 V

R_{th} = 40 Ω, V_{th} = 180 V

R_{th} = 250 Ω, V_{th} = 20 V

R_{th} = 250 Ω, V_{th} = 180 V

If U_1 = 40 V, U_2 = 20 V, R_1 = R_2 = 4 Ω, and R_3=13 Ω, apply the Thevenin's theorem to determine I_3 in the following circuit.



nswer

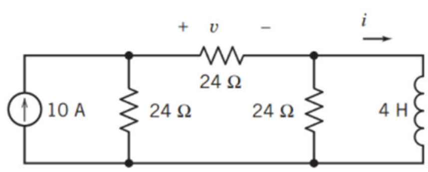
2 A

2.5 A

15 A

0.67 A

Under steady-state dc conditions, find i and v in the following circuit.



nswer

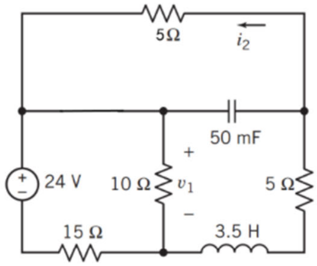
$i = 5\ A,\ v = 120\ V$

$i = 3.3\ A,\ v = -120\ V$

$i = 3.3\ A,\ v = 120\ V$

$i = 5\ A,\ v = -120\ V$

Under steady-state dc conditions, find v_1 and i_2 in the following circuit.



nswer

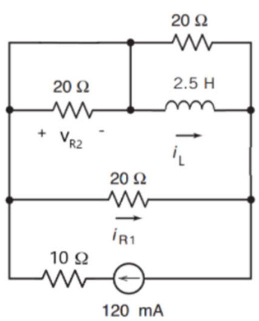
$v_1 = 6\,V,\ i_2 = -0.6\ A$

$v_1 = 6\,V,\ i_2 = 0.6\ A$

$v_1 = 9.6\,V,\ i_2 = -0.96\ A$

$v_1 = 9.6\,V,\ i_2 = 0.96\ A$

Under steady-state dc conditions, find i_{R1} , v_{R2} and i_L in the following circuit.



nswer

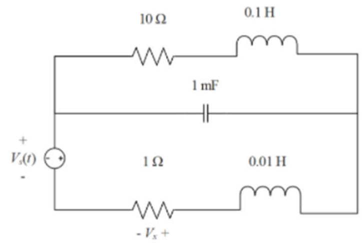
$i_{R1} = 0\,mA,\ v_{R2} = 0\,V,\ i_L = 120\,mA$

$i_{R1} = 60\,mA,\ v_{R2} = 0\,V,\ i_L = 0\,mA$

$i_{R1} = 60\,mA,\ v_{R2} = 1.2\,V,\ i_L = 60\,mA$

$i_{R1} = 80\,mA,\ v_{R2} = 0.8\,V,\ i_L = 0\,mA$

( ) Given that $V_s(t) = 10 \sin(100t + 90°)$, determine $V_x(t)$。



nswer

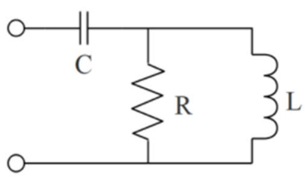
5∠0°

5∠90°

0.33∠0°

0.33∠90°

( ) Given that R = 5 Ω, L = 3 H, C = 1/3 F, if the net impedance is resistive, find the required frequency of the circuit?



nswer

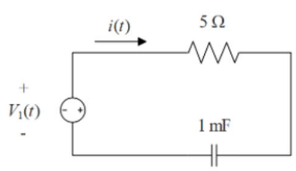
ω = 1.25 rad/s

ω = 1 rad/s

ω = 2 rad/s

ω = 0.5 rad/s

( ) If $V_1(t) = 10 \cos(200t)$, find $i(t)$.



nswer

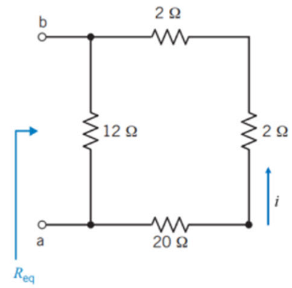
$$i\,(t) = \sqrt{2}\cos(200t + 45°)$$

$$i\,(t) = 10\cos(200t + 45°)$$

$$i\,(t) = 2\cos(200t)$$

$$i\,(t) = \sqrt{2}\cos(200t)$$

For the following circuit, find R_{eq} and i if V_{ab} = 40 V.



Answer

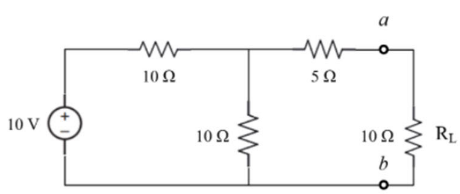
R_{eq} = 8 Ω, i = 5/3 A

R_{eq} = 12 Ω, i = 10/9 A

R_{eq} = 9 Ω, i = 40/27 A

R_{eq} =36 Ω, i = 20/27 A

For the following circuit, find the Thevenin equivalent circuit with respect to terminals *a*-*b*.



Answer

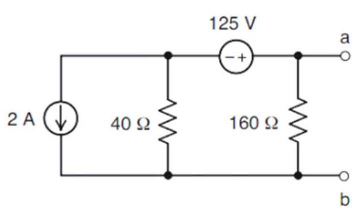
R_{th} = 10 Ω, V_{th} = 5 V

R_{th} = 16 Ω, V_{th} = 3.75 V

R_{th} = 20 Ω, V_{th} = 5 V

R_{th} = 25 Ω, V_{th} = 10 V

For the following circuit, find the Norton equivalent with respect to terminals *a*-*b*.



Answer

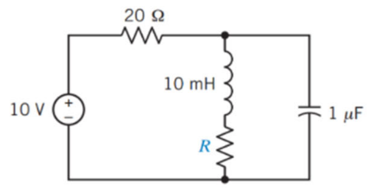
R_N = 32 Ω; I_N = 1.125 A

R_N = 40 Ω; I_N =2.5 A

R_N = 160 Ω; I_N = 1.5A

R_N = 200 Ω; I_N = 1A

For the following circuit, select a value of R so that the energy stored in the inductor is equal to that stored in the capacitor at its steady-state dc state.



nswer

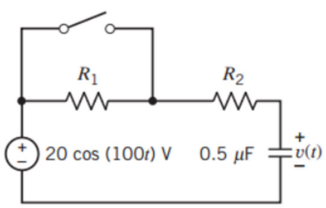
R = 100 Ω

R = 10000 Ω

R = 20 Ω

R = 10 Ω

When the switch in the following circuit is open, the time constant is 10 ms. After the switch is closed, the time constant becomes 5 ms. Determine the values of the resistances R_1 and R_2. (Hint: Time constant is given by $\tau = RC$)



nswer

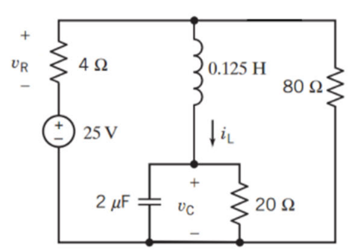
R_1=10 kΩ, and R_2=10 kΩ.

R_1=20 kΩ, and R_2=0 kΩ.

R_1=10 kΩ, and R_2=30 kΩ.

R_1=20 kΩ, and R_2=20 kΩ.

Under the steady-state dc condition, find i_L, v_C, and v_R in the following circuit.



nswer

i_L=1 A,v_C=20 V, and v_R=-5 V

i_L=1 A,v_C=20 V, and v_R=5 V

i_L=0.25 A,v_C=5 V, and v_R=-20 V

i_L=0.25 A,v_C=5 V, and v_R=20 V

Long question part:

(A) Q2(a) shows a dc circuit. The input voltage source is 9 V.

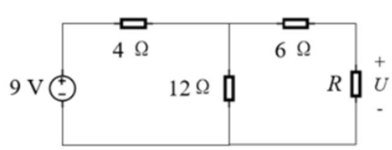


Fig. Q2(a)

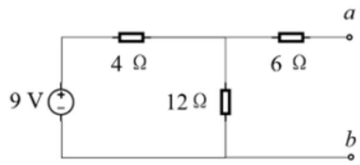


Fig. Q2(b)

Find

(a)the voltage *U* and power of *R* if *R* = 20 Ω; and (5 points)

(b)the resistance *R* by using mesh analysis if *U* = 4.5 V. (10 points)

(B) If the resistor *R* is removed as shown in Fig. Q2(b), find the Thevenin equivalent circuit at terminals *a*-*b*. (10 points)

Solution:

(a)

When $R = 20\ \Omega,\ R_{eq} = (R+6)||12 + 4 = 8.21 + 4 = 12.21\ \Omega$

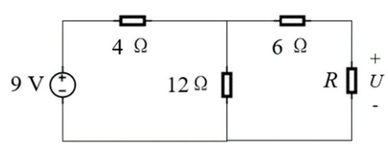
Based on voltage division method,

$$U = 9 \cdot \frac{8.21}{12.21} \cdot \frac{20}{20+6} = 4.66\ \mathrm{V}$$

Then

$$P = \frac{U^2}{R} = 1.08\ \mathrm{W}$$

(b)

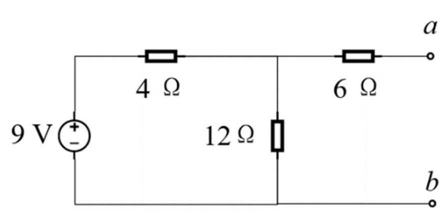


For left mesh, $9 = 4i_1 + 12(i_1 - i_2)$

For right mesh, $0 = 6i_2 + 12(i_2 - i_1) + 4.5$

Then, $i_1 = 0.75$ A and $i_2 = 0.25$ A. Given $i_2R = 4.5$ V, we can get $R = 18\ \Omega$

(c)



$$V_{th} = 9 \cdot \frac{12}{12+4} = 6.75\ \mathrm{V}$$

$$R_{th} = 4||12 + 6 = 9\ \Omega$$

(A) Fig. Q1(a) shows an ac circuit. The input voltage source is given by $v_{s1}(t)$=12*cos*(1000*t*+15°) V.

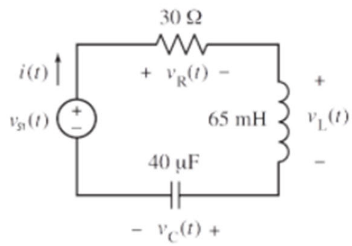


Fig.Q1(a)

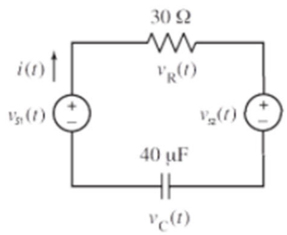


Fig.Q1(b)

Determine

(a) the impedances of the capacitor Z_C, Z_L, and Z_R; (3 points)

(b) the voltages V_R, V_L, and V_C; (6 points)

(c) the current *i*(*t*) and its rms value; and (4 points)

(d) the new current *i*(*t*) and its rms value if the inductor in Fig. Q1(a) is replaced by a second voltage source v_{s2}(t)=5*cos*(1000*t*) V, as shown in Fig. Q1(b). (6 points)

(B) If a current is given by *i*(*t*)=*A*+*Bcos*(*ωt*), where *A* and *B* are constants, drive the formula of the rms value of *i*(*t*). (6 points)

Solution:

(A)(a)

$$\omega = 1000\ rad/s$$
$$Z_C = \frac{1}{j\omega C} = \frac{1}{j1000(40 * 10^{-6})} = -j25\ \Omega$$
$$Z_L = j\omega L = j1000(65 * 10^{-3}) = j65\ \Omega$$
$$Z_R = R = 30\ \Omega$$

(A)(b)

$$V_s = 12\cos(1000t + 15°)\ \text{V} \quad \text{Phasor form: } V_s = 12\angle 15°\ \text{V and } \omega = 1000\ rad/s$$

Based on the voltage divider rule,

$$V_R = V_s \frac{Z_R}{Z_R + Z_C + Z_L} = Z_R \frac{30}{30 + j40} = \frac{12\angle 15° \cdot 30}{50\angle 53.13°} = 7.2\angle - 38.13°\ \text{V}$$
$$V_L = V_s \frac{Z_L}{Z_R + Z_C + Z_L} = Z_R \frac{j65}{30 + j40} = \frac{12\angle 15° \cdot 65\angle 90°}{50\angle 53.13°} = 15.6\angle 51.87°\ \text{V}$$
$$V_C = V_s \frac{Z_C}{Z_R + Z_C + Z_L} = Z_R \frac{-j25}{30 + j40} = \frac{12\angle 15° \cdot 25\angle - 90°}{50\angle 53.13°} = 6\angle - 128.13°\ \text{V}$$

Then,

$$V_R = 7.2\cos(1000t - 38.13°)\ \text{V}$$
$$V_L = 15.6\cos(1000t + 51.87°)\ \text{V}$$
$$V_C = 6\cos(1000t - 128.13°)\ \text{V}$$

(A)(c)

$$I = \frac{V_S}{Z} = \frac{V_S}{Z_R + Z_C + Z_L} = \frac{12\angle 15°}{30 + j40} = \frac{12\angle 15°}{50\angle 53.13°} = 0.24\angle -38.13°\ \text{A}$$
$$i(t) = 0.24\cos(1000t - 38.13°)\ \text{A}$$
$$i_{RMS} = \frac{0.24}{\sqrt{2}} = 0.17\ \text{A}$$

(A)(d)

$$V_{s1} = 12\cos(1000t + 15°)\ \text{V} \quad \text{Phasor form: } V_{s1} = 12\angle 15°\ \text{V and } \omega = 1000\ rad/s$$
$$V_{s2} = 5\cos(1000t)\ \text{V} \quad \text{Phasor form: } V_{s2} = 5\angle 0°\ \text{V and } \omega = 1000\ rad/s$$

$$Z_R = R = 30\ \Omega \qquad\qquad Z_C = -j25\ \Omega$$

Based on superposition,
For only V_{s1} source,

$$I_1 = \frac{V_{S1}}{Z} = \frac{V_{S1}}{Z_R + Z_C} = \frac{12\angle 15°}{30 - j25} = \frac{12\angle 15°}{39\angle -39.8°} = 0.31\angle 54.8°\ \text{A}$$

For only V_{s2} source,

$$I_2 = \frac{V_{S2}}{Z} = \frac{V_{S2}}{Z_R + Z_C} = \frac{5\angle 0°}{30 - j25} = \frac{5\angle 0°}{39\angle -39.8°} = 0.13\angle 39.8°\ \text{A}$$

V_{s1} and V_{s2} are in opposite directions, so

$$i(t) = 0.31\cos(1000t + 54.8°) - 0.13\cos(1000t + 39.8°)\ \text{A}$$
$$i_{RMS} = \sqrt{\frac{0.31^2}{\sqrt{2}^2} + \frac{0.13^2}{\sqrt{2}^2}} = 0.34\ \text{A}$$

(B)
Assuming a resistor R with the current $i(t) = A + B\cos(\omega t)$,

$$i(t) = i_{ac}(t) + i_{dc}(t) = B\cos(\omega t) + A$$

The power of the resistor $P = P_{ac} + P_{dc}$

$$P_{dc} = i_{dc}^2 R = A^2 R$$
$$P_{ac} = (\frac{i_{ac}}{\sqrt{2}})^2 R = \frac{B^2}{2} R$$

And

$$P = i_{RMS}^2 R$$

Then $i_{RMS}^2 = A^2 + \frac{B^2}{2}$ hence, $i_{RMS} = \sqrt{A^2 + \frac{B^2}{2}}$

(a) Find $\frac{dy}{dx}$ if $y = e^{3x+5}\sin^2(2x+1)$. **(5 points)**

(b) Evaluate $\int \frac{e^{3\sqrt{x}}}{\sqrt{x}}dx$. **(5 points)**

(c) Find cos $(\alpha + \beta)$ when

(1) tan α tan β = 2, and cos (α - β) = 3/5; **(4 points)**
(Hint: tan x = sin x / cos x)

(2) cos α + cos β=1/2, and sin α - sin β = 1/3. **(4 points)**
(Hint: $\sin^2 x + \cos^2 x = 1$)

(d) Fig. Q1(d) shows a periodic voltage $v_c(t)$ across a capacitor C = 1 mF.

(1) Find the rms value of $v_c(t)$. **(5 points)**

(2) Sketch the capacitor current $i_c(t)$ passing through the capacitor as a function of t. **(3 points)**

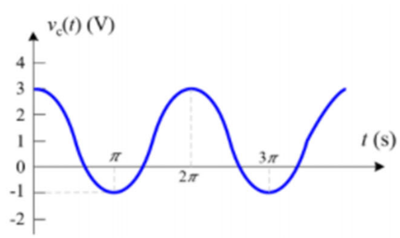


Fig. Q1(d)

Reference solution:

(a)

$$\frac{dy}{dx} = e^{3x+5}\,(2\sin(2x+1))(2\cos(2x+1)) + 3e^{3x+5}\sin^2(2x+1)$$

$$\frac{dy}{dx} = 4e^{3x+5}\sin(2x+1)\cos(2x+1) + 3e^{3x+5}\sin^2(2x+1)$$

$$\frac{dy}{dx} = e^{3x+5}\sin(2x+1)(4\cos(2x+1) + 3\sin(2x+1))$$

or

$$\frac{dy}{dx} = e^{3x+5}(3\sin^2(2x+1) + 2\sin(4x+2))$$

(b)

$$\because \left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$$

$$\therefore \int \frac{e^{3\sqrt{x}}}{\sqrt{x}}dx = 2\int e^{3\sqrt{x}}d\sqrt{x}$$

Set $t = \sqrt{x}$

$$\therefore \int \frac{e^{3\sqrt{x}}}{\sqrt{x}}dx = 2\int e^{3t}dt = \frac{2}{3}\int e^{3t}d3t = \frac{2}{3}e^{3t} + C$$

$$\therefore \int \frac{e^{3\sqrt{x}}}{\sqrt{x}}dx = \frac{2}{3}e^{3\sqrt{x}} + C$$

(c)(1)

$$tan\alpha \cdot tan\beta = 2 = \frac{sin\alpha \cdot sin\beta}{cos\alpha \cdot cos\beta} \Rightarrow sin\alpha \cdot sin\beta = 2cos\alpha \cdot cos\beta \qquad ①$$

$$\cos(\alpha - \beta) = cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta = \frac{3}{5} \qquad ②$$

From ① and ②, Obtain

$$cos\alpha \cdot cos\beta = \frac{1}{5} \qquad sin\alpha \cdot sin\beta = \frac{2}{5} \qquad ③$$

Then,

$$\cos(\alpha + \beta) = cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta = \frac{1}{5} - \frac{2}{5} = -\frac{1}{5}$$

(c)(2)

$$\cos\alpha + \cos\beta = \frac{1}{2} \Rightarrow (\cos\alpha + \cos\beta)^2 = cos^2\alpha + cos^2\beta + 2cos\alpha \cdot cos\beta = \frac{1}{2}^2 = \frac{1}{4} \qquad ④$$

$$\sin\alpha - \sin\beta = \frac{1}{3} \Rightarrow (sin\alpha - \sin\beta)^2 = sin^2\alpha + sin^2\beta - 2sin\alpha \cdot sin\beta = \frac{1}{3}^2 = \frac{1}{9} \qquad ⑤$$

Apply ④ + ⑤, obtain

$$cos^2\alpha + cos^2\beta + 2cos\alpha \cdot cos\beta + sin^2\alpha + sin^2\beta - 2sin\alpha \cdot sin\beta = \frac{1}{4} + \frac{1}{9} = \frac{13}{36}$$

$$2cos\alpha \cdot cos\beta - 2sin\alpha \cdot sin\beta = \frac{13}{36} - 2 = -\frac{59}{36}$$

Then,

$$\cos(\alpha + \beta) = cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta = -\frac{59}{72}$$

(d)(1)

Method A:

For $t \geq 0\ s$,

$$v_c(t) = 1 + 2\cos t = V_{dc}(t) + v_{ac}(t)$$

$$v_{RMS} = \sqrt{{V_{dc-RMS}}^2 + {v_{ac-RMS}}^2}$$

$$V_{dc-RMS} = 1\ V$$

$$v_{ac-RMS} = \frac{2}{\sqrt{2}}$$

$$v_{RMS} = \sqrt{1^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = \sqrt{3}\ V = 1.732\ V$$

Method B:

$$v_{RMS} = \sqrt{\frac{1}{T}\int_0^T v_c^2\,dt}$$

For the current in the following figure, $T = 2\pi\ s,\ \omega = 1$

$$V_c(t) = V_{dc}(t) + v_{ac}(t) = 1 + 2\cos t$$

$$\therefore v_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(4\cos^2 t + 4\cos t + 1)\,dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(2\cos 2t + 4\cos t + 3)\,dt}$$

$$= \sqrt{\frac{1}{2\pi}\cdot(sin2t + 4\sin t + 3t)|_0^{2\pi}}$$

$$= \sqrt{\frac{6\pi}{2\pi}} = \sqrt{3}\,V = 1.732\,V$$

(d)(2)

Voltage in one period is given by

$$v_c(t) = 1 + 2\cos(t) \quad (t \geq 0)\ \text{V}$$

Based on $i = C\,dv/dt$, where $C= 10^{-3}$ F, 0btain

$$i_c(t) = C\frac{dv}{dt} = -2\sin(t) \quad (t > 0)\ \ \text{mA}$$