EE1001
Foundations of Digital Techniques

Logic

Tutorial 1 (with answer)

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Validity and Soundness of Argument

Propositional Logic

Conditionals

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Question 1

- Determine the validity and soundness of the following arguments:

Argument 1

CityU is in Hong Kong.
Hong Kong is in Europe.
Therefore, CityU is in Europe.

Ans: Valid and Unsound

Argument 2

All CityU students are smart.
Albert is smart.
Therefore, Albert is a CityU student.

Ans: Invalid and Unsound

Argument 3

All lions are mammals.
No mammals are creatures with scales.
Therefore, no lions are creatures with scales.

Ans: Valid and Sound

Argument 4

No monkeys are animals.
All blossom trees are monkeys.
Therefore, no blossom trees are animals.

Ans: Valid and Unsound

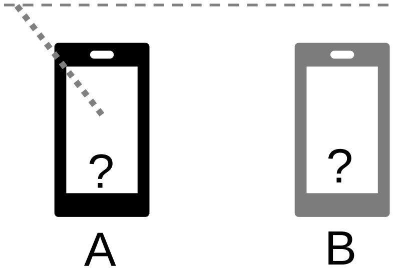
Question 2

There are two types of phones:

Always gives real messages	Always gives fake messages
Ordinary phone	Hijacked phone

A message given by A:

Either A is ordinary or B is hijacked



Question:
Are A and B ordinary or hijacked? Use truth table to justify.

Ans:
Let *p* = "A is an ordinary phone", and *q* = "B is an ordinary phone".
Then $p \vee \sim q$ = "either A is ordinary or B is hijacked"

p	q	$p \vee \sim q$	A's Message	
T	T	T	Fulfill	
T	F	T	Not fulfill	"A=ordinary & B= hijacked" does not fulfill the A's message
F	T	F	Fulfill	"A=hijacked & B=ordinary", the A's message must be false
F	F	T	Not fulfill	Hijacked phone muse give false messages

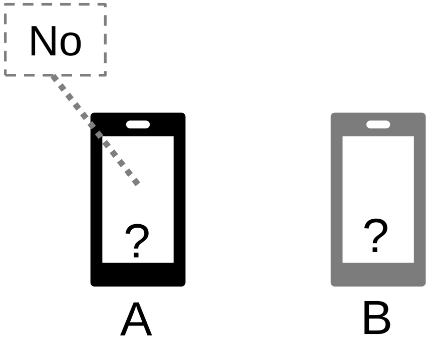
∴ Two possible solutions: (1) "A and B are ordinary phone"; (2) "A is hijacked and B is ordinary."

Question 3

There are two types of phones:

Always gives real messages	Always gives fake messages
Ordinary phone	Hijacked phone

I asked A a question: Is either of you an ordinary phone?
Then A replies "No"



Question:
Are A and B ordinary or hijacked? Use truth table to justify.

Ans:
Let p = "A is an ordinary phone", and q = "B is an ordinary phone".
Then $p \vee q$ = "either A or B is ordinary"

p	q	$p \vee q$	A's reply
T	T	T	Yes
T	F	T	Yes
F	T	T	No
F	F	F	Yes

This is the only possible solution that satisfies the given condition

∴ A is a hijacked phone and B is an ordinary phone.

Question 4

Use the Theorem of Logical Equivalences in lecture notes to verify that

$$\sim(p \wedge (\sim p \vee q)) \equiv \sim(p \wedge q)$$

State the reason of each step.

Ans:

$\sim(p \wedge (\sim p \vee q))$	$\equiv \sim p \vee \sim(\sim p \vee q)$	(De Morgan's law)
	$\equiv \sim p \vee (p \wedge \sim q)$	(De Morgan's law & double negative law)
	$\equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$	(Distributive law)
	$\equiv \mathbf{t} \wedge (\sim p \vee \sim q)$	(Negation law)
	$\equiv (\sim p \vee \sim q)$	(Identity law)
	$\equiv \sim(p \wedge q)$	(De Morgan's law)

You can verify it using the truth table, but the computing complexity grows <u>exponentially</u> with the number of variables

$\sim(p \wedge (\sim p \vee q))$ and $\sim(p \wedge q)$ always have the same truth values, so they are logically equivalent

p	q	$(\sim p \vee q)$	$\sim(p \wedge (\sim p \vee q))$	$\sim(p \wedge q)$
T	T	T	F	F
T	F	F	T	T
F	T	T	T	T
F	F	T	T	T

Question 5

a) Rewrite “I say what I mean” in if-then form.

b) Rewrite “I mean what I say” in if-then form.

c) Are they logically equivalent? Explain the logical relation between them

Ans:

a) If I mean something, then I say it.

b) If I say something, then I mean it.

c) Let p = “I mean something”, and q = “I say something”.

Then, $p \rightarrow q$ = “If I mean something, then I say it.”

$q \rightarrow p$ = “I say something, then I mean it.”

∴ They are not logically equivalent. They are the converse of each other.

Question 6

Without using a truth table, determine whether $\sim(\sim p \lor (p \lor q)) \to q$ is a tautology or contradiction.

Ans:

$\sim(\sim p \lor (p \lor q)) \to q$	$\equiv \boldsymbol{\sim}(\sim(\sim p \lor (p \lor q))) \lor q$	(Definition of $\to$)
	$\equiv (\sim p \lor (p \lor q)) \lor q$	(Double negative law)
	$\equiv ((\sim p \lor p) \lor q) \lor q$	(Associative law)
	$\equiv (\mathbf{t} \lor q) \lor q$	(Negation law)
	$\equiv \mathbf{t} \lor q$	(Universal bound law)
	$\equiv \mathbf{t}$	(Universal bound law)