CITY UNIVERSITY OF HONG KONG

Course code and title : MA1201 Calculus and Basic Linear Algebra II

Session : Semester B, 2012/2013

Time allowed : Two hours

This paper has **THREE** pages (including this cover page and the attached table).

Instructions to candidates:

1. This paper has two sections. Each section carries 50 marks.
2. Section A has two questions while Section B has three questions.
3. Attempt **ALL** questions in Section A and B.
4. Start each question on a new page.
5. Show all steps.

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*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable calculators

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorised materials or aids are found on them.

NOT TO BE TAKEN AWAY

Section A (50%)

Question 1

Evaluate the following integrals:

(a) $\int \tan 2x\,dx$ [3]

(b) $\int \frac{x^3}{3+x^2}\,dx$ [3]

(c) $\int e^{-3x}\sin(2x)\,dx$ [5]

(d) $\int \sqrt{9-16x^2}\,dx$ [6]

(e) $\int \frac{9x-7}{(x+2)(x^2-4x+13)}\,dx$ [9]

(f) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2\sin^5 x}{1+\cos^3 x}\,dx$ [4]

Question 2

(a) Calculate the volume of the solid generated by rotating the region bounded by the parabola $y=5-3x^2$ and the straight line $y=2$ about the horizontal line $y=1$. [10]

(b) Compute the arc length of the cycloid: $x=t-\sin t$, $y=1-\cos t$, $0\le t\le 2\pi$. [10]

Section B (50%)

Question 3

Let $A(3,-2,1)$, $B(1,-3,2)$ and $C(2,-1,-3)$ be three points in a plane Π.

(a) Evaluate the angle $\angle BAC$; [5]

(b) Determine a unit vector perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{AC}$; [5]

(c) Find the shortest distance from $D(-4,-1,2)$ to the plane Π. [5]

Question 4

(a) Rewrite the complex number $(2-\sqrt{2}\,i)^5$ in Euler form. [4]

(b) Solve the equation $z^3+1=\sqrt{3}i$ and express the solution in polar form. [6]

Question 5

(a) Find the inverse of the matrix $A=\begin{pmatrix}-1 & 1 & 2\\ 2 & 1 & -1\\ 3 & 2 & 0\end{pmatrix}$. [10]

(b) Use the matrix A in (a), compute the determinants: $|A|$, $|A^TA|$, $|A^5|$ and $|A^{-1}|$. [5]

(c) Use Gauss elimination to solve the system of linear equations: $Bx=\vec{b}$, where

$$B=\begin{pmatrix}1 & 3 & -2 & -1\\ 2 & 5 & -1 & 3\\ -1 & -1 & -3 & 2\end{pmatrix},\qquad \vec{b}=\begin{pmatrix}1\\ 2\\ -3\end{pmatrix}.$$ [10]

Brief Table of Derivatives and Integrals

Standard Derivatives	Standard Integrals
$\frac{d}{dx}(x^p) = px^{p-1}$	$\int x^p dx = \frac{x^{p+1}}{p+1} + C, \quad p \neq -1$
$\frac{d}{dx}(\ln x) = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x \, dx = \sin x + C$
$\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x \, dx = -\cos x + C$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x \, dx = \tan x + C$
$\frac{d}{dx}(\cot x) = -\csc^2 x$	$\int \csc^2 x \, dx = -\cot x + C$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x \, dx = \sec x + C$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$	$\int \csc x \cot x \, dx = -\csc x + C$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$	
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

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