

### Problem Set 1c Hint Sheet

Please note this document is a hint sheet. The information contained herein is meant to guide you up to the main equations required to solve a given circuit. If you are able to get up to this point, then this document has served its chief purpose. The main focus of this course is on the concepts behind these equations. Therefore, the details on how to solve these equations lies outside of this course and therefore omitted from this document. The details contained in this document are meant to supplement the numerical answers given at the end of the problem set.

#### Q2

##### Equivalent resistance across terminals a-b

Replace voltage source with short circuit  $\Rightarrow$  The two resistors appear in parallel

##### Thevenin voltage

This is simply the voltage across terminals a-b, which can be found by applying voltage divider rule given that the two resistors are in series to the voltage source.

##### Norton current

Short terminals a-b and find the current through the short circuit. Note that the  $20\ \Omega$  is bypassed by the short circuit. Hence the voltage from the source is dropped entirely over the  $5\ \Omega$  resistor.

#### Q3

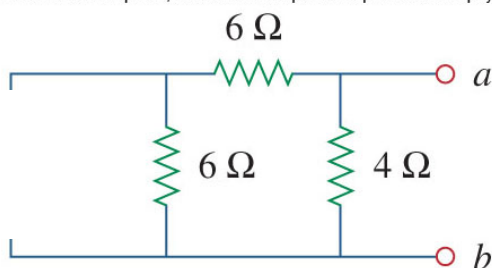
Let the unknown load be denoted by  $R_L$ . Note that  $R_L$  and  $R_{Th}$  are in series with a total voltage drop of  $V_{Th}$ .

#### Q4

##### Norton resistance

Replace the current source with open circuit: The objective is to find  $R_{ab}$

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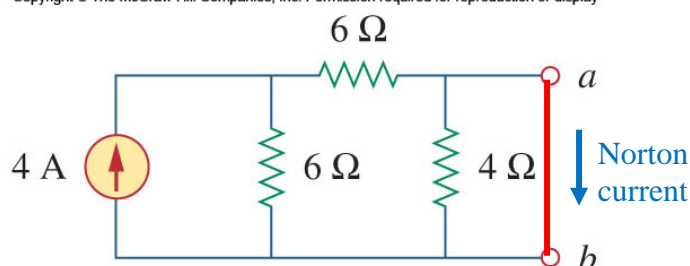
The two  $6\ \Omega$  resistors are now in series  $\Rightarrow 12\ \Omega$

This  $12\ \Omega$  lies in parallel with the  $4\ \Omega$  resistor  $\Rightarrow R_N = 3\ \Omega$

##### Norton current

Short terminals a-b and find the current through the short circuit:

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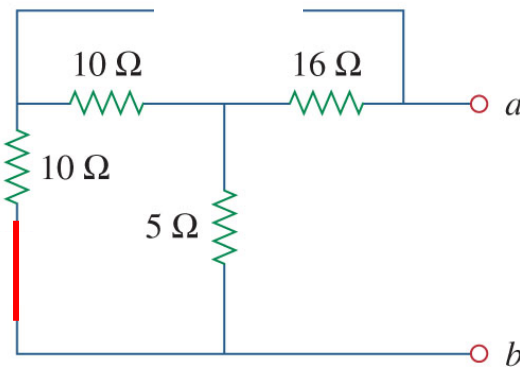


Now the  $4\ \Omega$  is bypassed by the short circuit so that the two  $6\ \Omega$  resistors are now in parallel. Thus the  $4\ \text{A}$  from the current source will divide equally between these two resistors. Apply current divider rule  $\Rightarrow I_N = 4/2 = 2\ \text{A}$

**Q5**Thevenin resistance

Replace the current source with an open circuit and replace the voltage source with a short circuit: The objective is to find  $R_{ab}$

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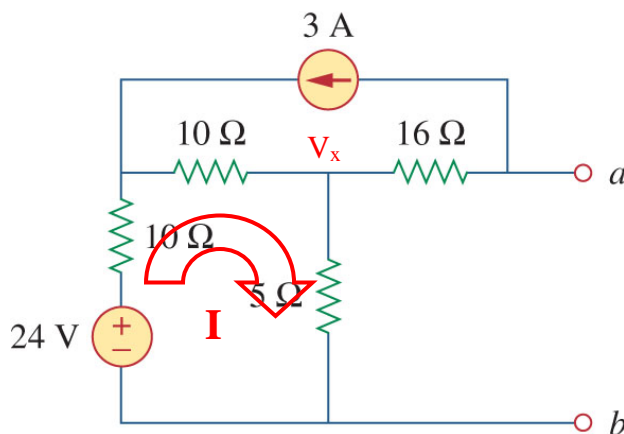
The two  $10\ \Omega$  resistors are now in series  $\Rightarrow 20\ \Omega$

This  $20\ \Omega$  is parallel to the  $5\ \Omega$  resistor  $\Rightarrow 4\ \Omega$

This  $4\ \Omega$  is in series with the  $16\ \Omega$  resistor  $\Rightarrow R_{Th} = 20\ \Omega$

Thevenin voltage

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Objective: Find voltage across a-b with the least effort

Strategy: Obtain  $V_{ab}$  by using  $V_x$  and the voltage drop across the  $16\ \Omega$  resistor (we know that the current through it is  $3\ \text{A}$ )  $\leftarrow$  find  $V_x$  by using MCA around mesh I.

Apply KVL mesh I:  $24 = I(10 + 5) + (I + 3)*10$

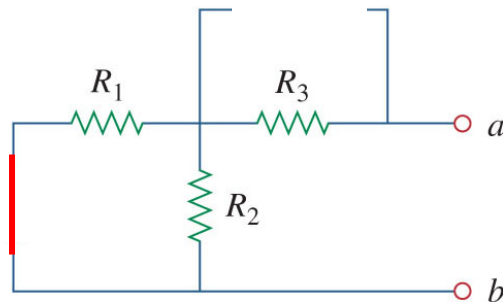
Next:  $V_x = I*5$

Finally:  $V_{Th} = V_{ab} = V_x - 3*16$  (note that voltage drops from  $V_x$  to  $V_a$ )

**Q6**Thevenin resistance

Replace the current source with an open circuit and replace the voltage source with a short circuit: The objective is to find  $R_{ab}$

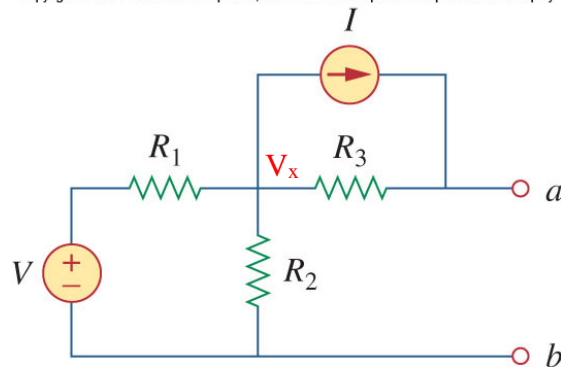
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$$R_{ab} = R_1 \parallel R_2 + R_3$$

Thevenin voltage

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Objective: Find voltage across a-b with the least effort

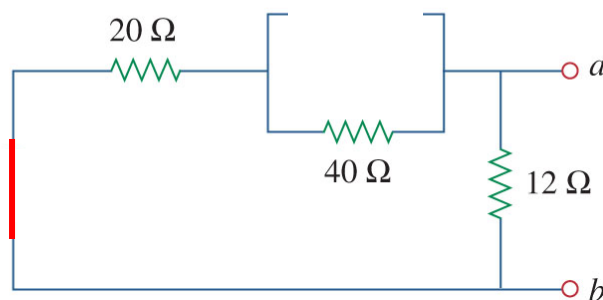
Strategy: Obtain  $V_{ab}$  by using  $V_x$  and the voltage drop across  $R_3$  (we know that the current through it is  $I$ ) ←  $V_x$  can be found by voltage divider rule since  $R_1$  and  $R_2$  are in series (note that  $I$  does not make any contribution to the currents in  $R_1$  and  $R_2$ ).

$$\Rightarrow V_x = 6 \text{ V, voltage drop across } R_3 = 6 \text{ V}$$

**Q7**Norton resistance

Replace the current source with an open circuit and replace the voltage source with a short circuit: The objective is to find  $R_{ab}$

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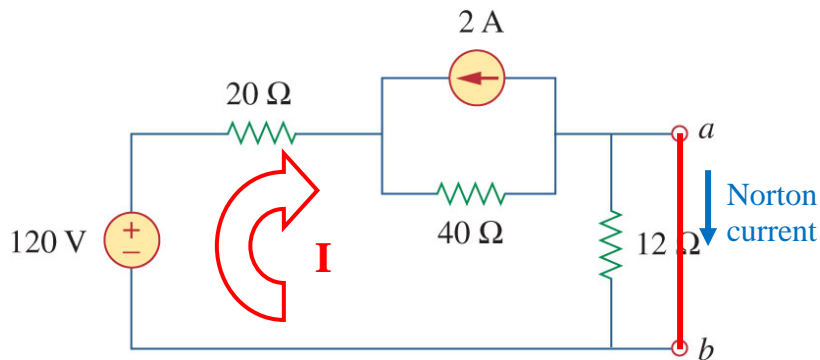


The  $20 \Omega$  and  $40 \Omega$  resistors are in series  $\Rightarrow 60 \Omega$ ; this  $60 \Omega$  is in parallel with the  $12 \Omega$

Norton current

Short terminals a-b and find the current through the short circuit:

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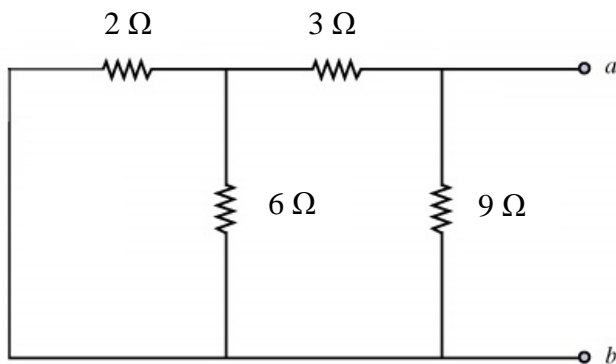


Now the  $12\ \Omega$  is bypassed by the short circuit. We find the Norton current by applying mesh current analysis around the loop:

$$120 = I \cdot 20 + (I + 2) \cdot 40 \quad (12\ \Omega \text{ is bypassed so no voltage drop})$$

**Q8**Norton resistance

Replace the voltage source with a short circuit: The objective is to find  $R_{ab}$



After replacing the 18 V source with a short circuit,

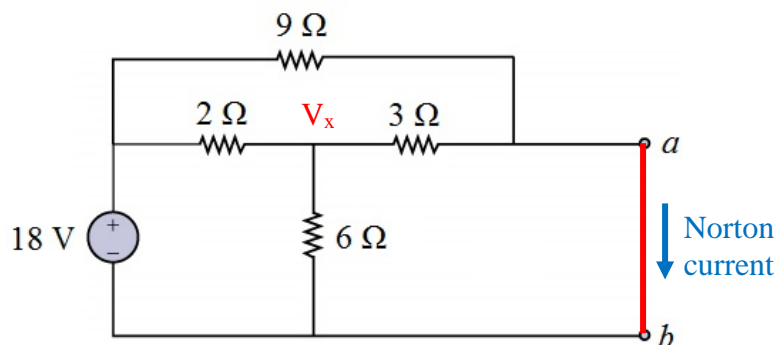
$2\ \Omega$  resistor appears in parallel with the  $6\ \Omega$  resistor  $\Rightarrow 1.5\ \Omega$

This  $1.5\ \Omega$  lies in series with the  $3\ \Omega$  resistor  $\Rightarrow 4.5\ \Omega$

This  $4.5\ \Omega$  lies in parallel with the  $9\ \Omega$  resistor

Norton current

Short terminals a-b and find the current through the short circuit:



Strategy: Find the Norton current by adding up the current in the  $3\ \Omega$  resistor and the current in the  $9\ \Omega$  resistor  $\leftarrow$  voltage across  $9\ \Omega$  set by source  $\leftarrow$  find current in  $3\ \Omega$  using  $V_x$   $\leftarrow$  find  $V_x$  by voltage divider rule.

After terminals a-b have been shorted together, the  $3\ \Omega$  and  $6\ \Omega$  resistors are in parallel ( $2\ \Omega$ ). Together, they appear in series with the  $2\ \Omega$  resistor with a total voltage drop of  $18\ \text{V}$ .

By voltage divider rule:  $V_x = 9\ \text{V}$

Current through  $9\ \Omega = 2\ \text{A}$

Current through  $3\ \Omega = 3\ \text{A}$

### Q9

Transform the two sets of voltage sources with series resistors into Norton equivalents to obtain a parallel arrangement of 3 current sources and 3 resistors.

Series combination of  $12\ \text{V}$  source and  $20\ \Omega$  resistor  $\rightarrow 12/20\ \text{A}$  and  $20\ \Omega$  in parallel

Series combination of  $16\ \text{V}$  source and  $40\ \Omega$  resistor  $\rightarrow 16/40\ \text{A}$  and  $40\ \Omega$  in parallel

Parallel current sources combine by adding up  $\Rightarrow$  total of  $4\ \text{A}$  to give  $I_N$

$R_N = 10\ \Omega \parallel 20\ \Omega \parallel 40\ \Omega$

### Q10

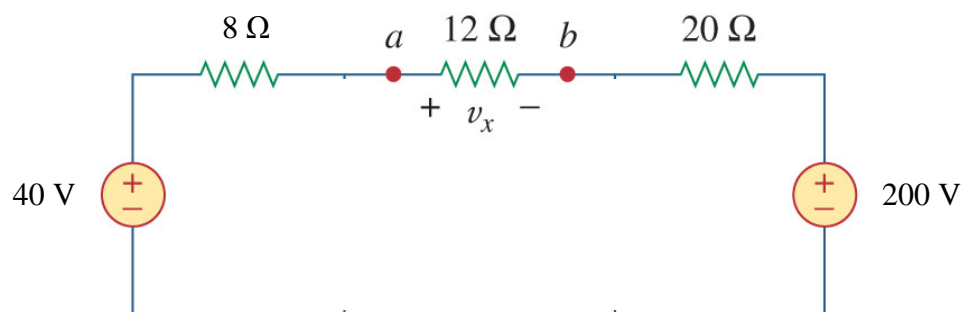
Transform the networks on the left of terminal a as well as to the right of terminal b separately by removing the  $12\ \Omega$  resistor first.

Network on left hand side:  $50\ \text{V}$ ,  $10\ \Omega$ ,  $40\ \Omega \rightarrow 40\ \text{V}$  in series with  $8\ \Omega$

Network on right hand side:  $40\ \text{V}$ ,  $8\ \text{A}$ ,  $20\ \Omega \rightarrow 200\ \text{V}$  in series with  $20\ \Omega$

Insert the  $12\ \Omega$  resistor back into the reduced networks:

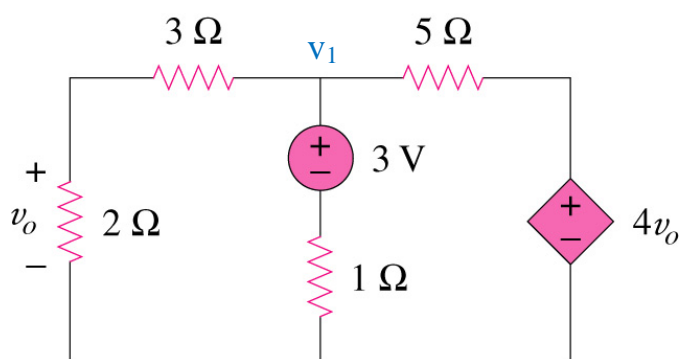
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### Q11

Using NVA is most convenient in this case requiring only one equation applied at node  $v_1$ .

Note that  $v_1$  represents the voltage difference across the  $3\ \text{V}$  source and  $1\ \Omega$  resistor



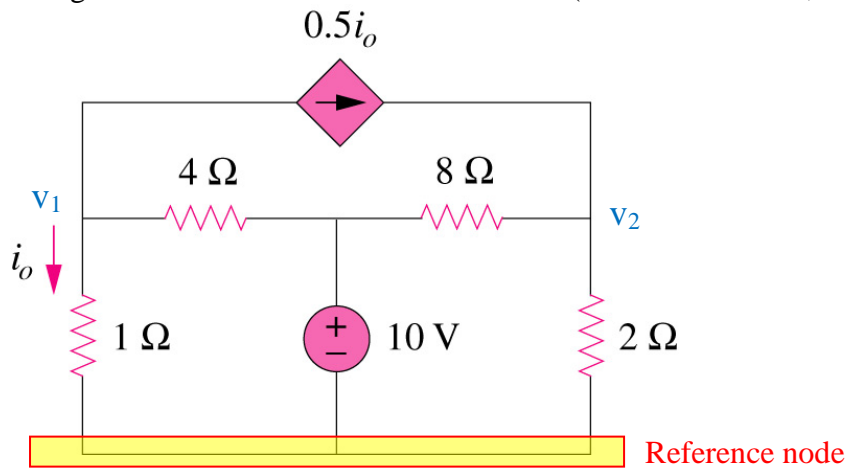
KCL at node  $v_1$ :  $\frac{v_1}{5} + \frac{v_1 - 3}{1} + \frac{v_1 - 4v_0}{5} = 0$

Along the left mesh, can you see that:  $v_0 = \frac{2}{5}v_1$  (voltage divider rule)?

Sub the above relation into the nodal voltage equation and solve for  $v_1$ , then use  $v_1$  to find  $v_0$ .

### Q12

Using NVA is most convenient in this case (2 unknown nodes, 1 on each side)



Note that  $i_o = v_1$  (why?)

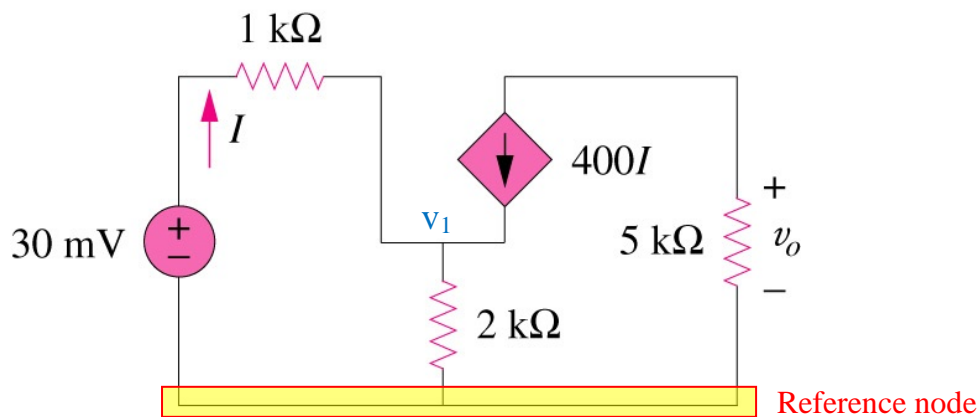
KCL at node  $v_1$ :  $(v_1/1) + (0.5v_1/1) = (10 - v_1)/4 \Rightarrow v_1 = 10/7 \text{ V}$

KCL at node  $v_2$ :  $(0.5v_1/1) + ((10 - v_2)/8) = v_2/2 \Rightarrow v_2 = 22/7 \text{ V}$

Use  $v_1$  to find  $i_o$

### Q13

Use NVA (apply KCL at node above the 2kΩ resistor, denoted by  $v_1$ )



$$[(0.03 - v_1)/1k] + 400I = v_1/2k$$

Get rid of  $I$  in the above equation by considering the current through 1kΩ:  $I = (0.03 - v_1)/1k$

$$\Rightarrow v_1 = 29.963 \text{ mV}, I = 37.4 \text{ nA}$$

Now the current source value is known ( $400I$ )  $\Rightarrow$  use this to find  $v_o$  (pay attention to the direction of the current relative to the sign convention of the voltage across 5kΩ)