Tutorial 6

Capacitance

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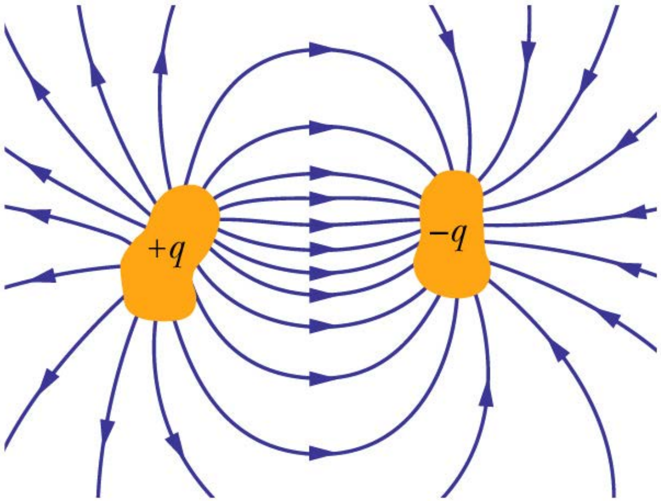
Lecture Outline

- **Chapter 25**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Capacitance
  - Capacitance and Unit
  - Parallel plate capacitors
  - Calculating capacitance of capacitors in series and in parallel
  - Energy stored in a capacitor
  - Potential energy and energy density of an electric field.

Lecture 06 Review

- In Lecture 6, we learnt that the common passive electrical component Capacitor is used to store static electric potential energy.
- One can construct a capacitor by simply placing two conductors (plates) in isolation.
- A charged capacitor is a capacitor with equal magnitude but opposite charges, $+q$ and $-q$, on its plates.



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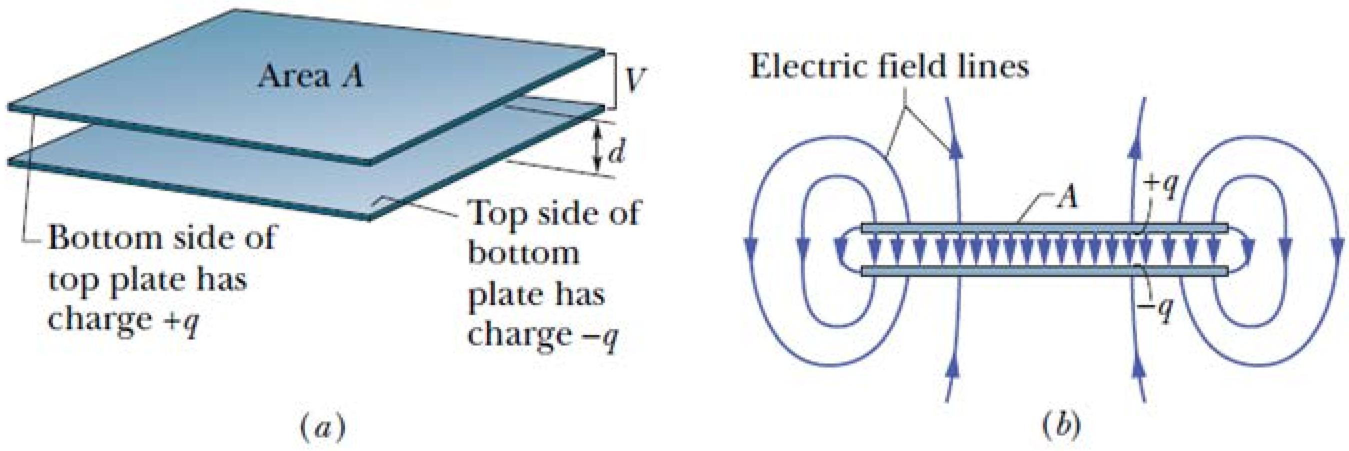
Lecture 06 Review

- The capacitor stores electric potential via the charges that it accumulate on its conductors (plates).
- From Lecture 5, we realize that the two plates of the capacitor are at different electric potentials, so there is a potential difference across the capacitor when they are charged.



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Lecture 06 Review

- The potential difference between the two plates V is a function of the amount of charges q on the plates and the geometry of the plates.
- In fact, there is linear relationship between q and V, where $q = CV$.
- The proportionality constant C is called the capacitance of the capacitor.

$$\boxed{q = CV.}$$

Capacitance is a measure of the capacitor's ability to store electric charge.

The SI unit is called the *farad (F):* **1 farad (1 F)= 1 coulomb per volt =1 C/V.**

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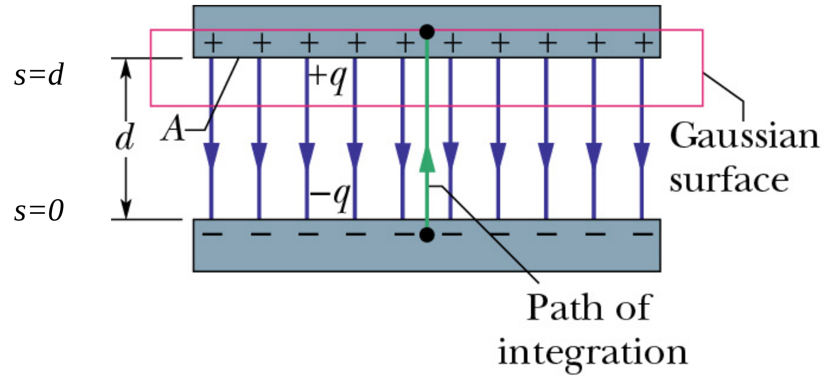
Lecture 06 Review

- For the parallel capacitor, with $q = CV$, we need to find q and V to determine C.
- Use Gauss's law to find q:

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q. \qquad q = \varepsilon_0 EA$$



- Determine V between the bottom and top plates by from the path integral $V = -\int_i^f \vec{E} \cdot ds$.

$$V = V_f - V_i = -\int_{-}^{+} \vec{E} \cdot d\vec{s} = E\int_0^d ds = Ed$$

$$C = \frac{\varepsilon_0 A}{d}$$

(parallel-plate capacitor).

- Note that C depends only on the plates geometry.

Lecture 06 Review

- From the relationship of charge and potential, one can see that the higher the capacitance, the more charges that the capacitor can store for the same amount of potential difference.
- The amount of energy stored by the capacitor is the same as the amount of energy needs to separate these charges. Thus, more charges means more energy store.
- The potential energy in a capacitor

$$U = \frac{q^2}{2C} = \frac{1}{2}CV^2$$

Lecture 06 Review

- We also studied the parallel plates capacitor in depth, which is assumed to has a uniform electric field across the two plates because the dimension of the plates is much larger than their separation.
- We derived the formulations for capacitors in series and in parallel.

(n capacitors in parallel).

$$C_{eq} = \sum_{j=1}^{n} C_j$$

Same V across all capacitors

(n capacitors in series).

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

Same $|q|$ on all capacitor plates

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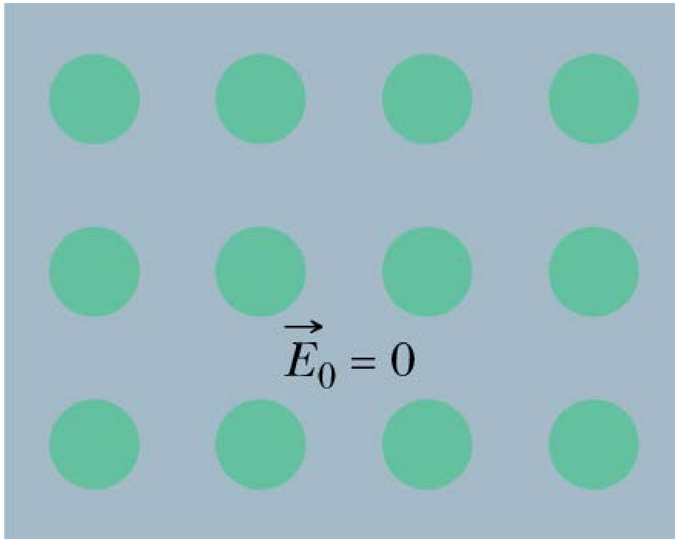
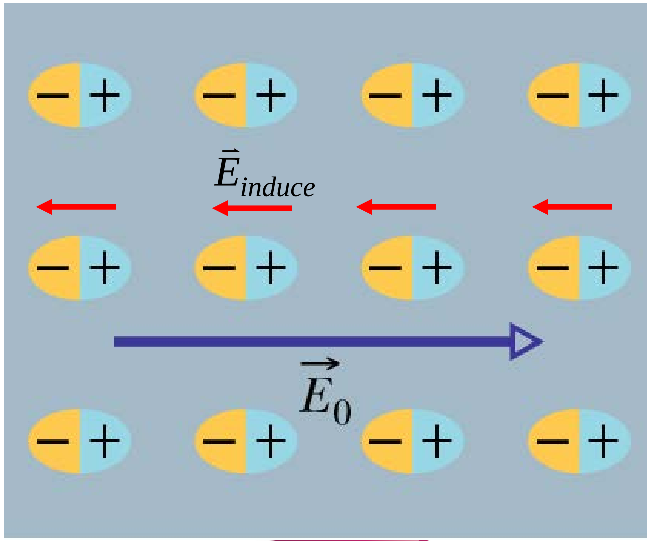
Lecture 06 Review

- When one inserts a dielectric between the plates of the capacitor, it increases the capacitance of the capacitor due to the induced dielectric dipoles.
- The induced dipoles create a microscopic electric field that is in the opposite direction of the initial field. The weaker E field means more charges are needed to create the same potential difference across the plate.





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Lecture 06 Review

- The ratio between the amplitude of the E field in vacuum and in the dielectric is proportional to the dielectric constant κ.
- The modified Gauss's law in a dielectric becomes

$$\varepsilon_0 \oiint \kappa \vec{E} \cdot d\vec{A} = \varepsilon \oiint \vec{E} \cdot d\vec{A} = q$$

- The addition of the dielectric in between a parallel plate capacitor increases its capacitance by a factor of κ

$$C = \frac{\kappa \varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$$

- The addition of the dielectric allows the capacitor to store more charges or more energy.
- The potential energy in a capacitor

$$U = \frac{q^2}{2C} = \frac{1}{2} C V^2 \qquad C = \kappa\, C_{air}$$

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Halliday/Resnick/Walker
Fundamentals of Physics 8th edition

Classroom Response System Questions

Chapter 25 Capacitance

Interactive Lecture Questions

PHY 1202

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25.4.1. Capacitor B has one-half the capacitance of capacitor A. How does the charge on capacitor A compare to that on B when the two are connected in series to a battery for a long time?

a) The charge on capacitor A is one-fourth the charge on capacitor B.

b) The charge on capacitor A is one-half the charge on capacitor B.

c) The charge on capacitor A is the same as the charge on capacitor B.

d) The charge on capacitor A is twice the charge on capacitor B.

e) The charge on capacitor A is four times the charge on capacitor B.

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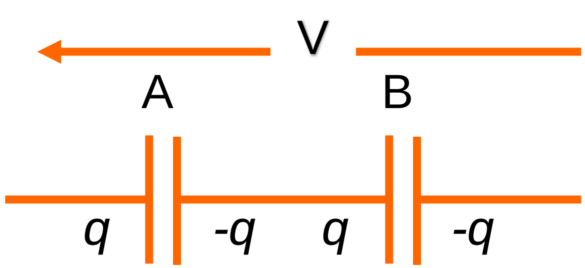

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25.4.4. Two parallel conducting plates are connected to a battery for a long time and become fully-charged. How does the potential difference across the plates change, if at all, when a conducting slab is inserted in between the plates without touching either plate?

a) The potential difference will increase.

b) The potential difference will decrease.

c) The potential difference will remain unchanged.

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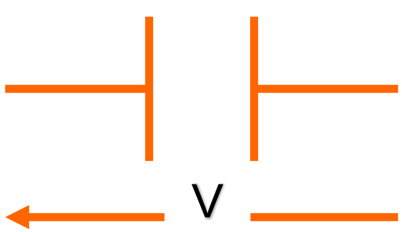
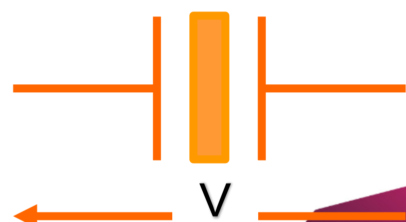


$$V_1 + V_2 = V$$



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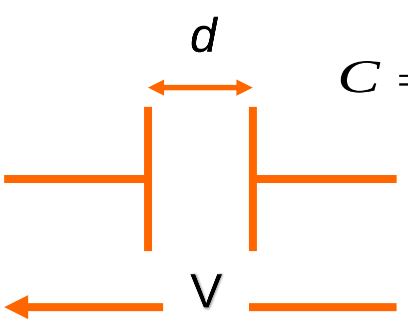
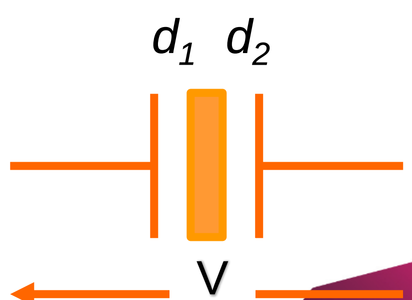


$$C = \frac{\varepsilon_0 A}{d}$$



$C=\varepsilon_0 A/(d_1+d_2)$

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25.4.7. Three parallel plate capacitors, each having a capacitance of 1.0 μF are connected in series. The potential difference across the combination is 100 V. What is the charge on any one of the capacitors?

a) 33 μC

b) 330 μC

c) 3300 μC

d) 100 μC

e) 1000 μC

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25.4.7. Three parallel plate capacitors, each having a capacitance of 1.0 μF are connected in series. The potential difference across the combination is 100 V. What is the charge on any one of the capacitors?

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d) 100 μC

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$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} = 3 \Rightarrow C_{eq} = \frac{1}{3}\mu\text{F}$$

$$q = CV = \frac{1}{3}\mu\text{F} \times 100\text{V} = 33\mu\text{C}$$

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25.5.1. The plates of an isolated parallel plate capacitor are separated by a distance d and carry charge of magnitude q. The distance between the plates is then reduced to $d/2$. How is the energy stored in the capacitor affected by this change?

a) The energy increases to twice its initial value.

b) The energy increases to four times its initial value.

c) The energy is not affected by this change.

d) The energy decreases to one fourth of its initial value.

e) The energy decreases to one half of its initial value.

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b) The energy increases to four times

c) The energy is not affected by this c

d) The energy decreases to one fourth

e) The energy decreases to one half of its initial value.

Since the capacitor in isolation, the plates maintain the same amount of charges when the distance is reduced by a half. With the capacitance inversely proportional to the separation, the capacitance is doubled after the distance is reduced by half. However, the increase of the capacitance decreases the energy by half it's initial value. $U = q^2/2C$

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25.6.2. A parallel plate capacitor is connected to a battery that maintains a constant potential difference across the plates. Initially, the space between the plates contains only air. Then, a Teflon (κ = 2.1) sheet is inserted between, but not touching, the plates. How does the stored energy of the capacitor change as a result of inserting the Teflon sheet?

a) The energy will decrease.

b) The energy will not be affected.

c) The energy will increase.

d) The energy will be zero joules.

$$\boxed{U = \frac{q^2}{2C} = \frac{1}{2}CV^2}$$

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The key point here is the capacitor is connected to a battery, so V is constant before and after the insertion of the Teflon. Since the capacitance of the capacitor increase by 2.1 with the dielectric compares to the air-filled capacitor. Since *q*=*CV*, the charge on the dielectric filled capacitor also increased by 2.1. Therefore, the energy will increase because of $U = \frac{1}{2}CV^2$

If this is an isolated capacitor, then the result will not be the same.

25.6.4. Which one of the following changes will necessarily increase the capacitance of a capacitor?

a) decreasing the charge on the plates

b) increasing the charge on the plates

c) placing a dielectric between the plates

d) increasing the potential difference between the plates

e) decreasing the potential difference between the plates

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25.6.5. Complete the following statement: When a dielectric with constant κ is inserted between the plates of a charged *isolated* capacitor

a) the capacitance is reduced by a factor κ.

b) the charge on the plates is reduced by a factor of κ.

c) the charge on the plates is increased by a factor of κ.

d) the electric field between the plates is reduced by a factor of κ.

e) the potential difference between the plates is increased by a factor of κ

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