City University of Hong Kong

Course code & title:	MA 1201 Calculus and Basic Linear Algebra II
Session:	Semester B, 2020–2021
Time allowed:	Three hours

This exam has FIVE pages (including this cover page and the attached table).

1. This exam consists of SIX problems (100 points in total).
2. Attempt ALL problems.
3. Start each problem on a NEW page.
4. Show your work clearly to receive full credits.
5. The use of pencil is not permitted.

This is a CLOSED-BOOK examination.

Candidates are allowed to use the following materials/aids:

- Non-programmable portable battery operated calculator.

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

If assistance is needed, contact your instructor/TA in Zoom or call department hotline (3442 8646).

1. (15 points) Evaluate the following integrals.

   (a) (5 points) $\int (e^x + e^{-x})^2 \, dx.$

   (b) (5 points) $\int \sqrt{x\sqrt{x}} \, dx.$

   (c) (5 points) $\int_0^1 \frac{x}{\sqrt{1+x}} \, dx.$

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2. (20 points) Evaluate the following integrals.

   (a) (7 points) $\int \frac{xe^{\tan^{-1} x}}{(1+x^2)^{\frac{3}{2}}} \, dx.$

   (b) (5 points) $\int \frac{1}{1+\sin x} \, dx.$

   (c) (8 points) $\int \frac{10x^2+14x+13}{(2x+1)(2x^2+4x+10)} \, dx.$

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3. (15 points)

   (a) (8 points) Find the volume of the solid generated by revolving the region bounded by the upper branch of the curve $y = \cos x$, where $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, and the x-axis about the straight line $x = \pi$.

   (b) (7 points) Find the area of the surface generated by revolving the curve $x = \cos^3 t$ and $y = \sin^3 t$, where $0 \leq t \leq \frac{\pi}{2}$, about the x-axis.

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4. (15 points) Let $A(1,1,0)$, $B(2,1,1)$, $C(0,2,0)$, and $D(1,2,-2)$ be four points in $\mathbb{R}^3$. Using vector method:

   (a) (5 points) Find the volume of the parallelepiped with adjacent edges AB, AC, and AD.

   (b) (5 points) Find the equation of the plane that contains A, B, and C.

   (c) (5 points) Find the distance from the line CD to the line AB.

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5. (15 points)

   (a) (8 points) Compute $(i + e^{i\frac{\pi}{4}})^{\frac{2}{3}}$ and express your answer in Euler's form.

   (b) (7 points) By considering $(\cos\theta + i\sin\theta)^3$, show that $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$.

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6. (20 points)

(a) (10 points) Solve the linear system

$$\begin{aligned} y + z &= 2 \\ 2x + 3y + 2z &= 5 \\ 3x + y - z &= -1 \end{aligned}$$

by Gaussian elimination.

(b) (10 points) Find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 2 \\ 1 & 1 & 1 & -1 \\ 1 & 0 & -2 & -6 \end{pmatrix}$$

— THE END —

Useful Elementary Integrals

Constant and powers

1. $\int k\,dx = kx + C.$

2. $\int x^n\,dx = \begin{cases} \dfrac{x^{n+1}}{n+1} + C, & n \neq -1 \\ \ln|x| + C, & n = -1 \end{cases}.$

Exponentials

3. $\int \mathrm{e}^x\,dx = \mathrm{e}^x + C.$

4. $\int a^x\,dx = \dfrac{a^x}{\ln a} + C,\ a \neq 1,\ a > 0.$

Trigonometric functions

5. $\int \sin x\,dx = -\cos x + C.$

6. $\int \cos x\,dx = \sin x + C.$

7. $\int \sec^2 x\,dx = \tan x + C.$

8. $\int \csc^2 x\,dx = -\cot x + C.$

9. $\int \sec x \tan x\,dx = \sec x + C.$

10. $\int \csc x \cot x\,dx = -\csc x + C.$

11. $\int \tan x\,dx = \ln|\sec x| + C.$

12. $\int \cot x\,dx = \ln|\sin x| + C.$

13. $\int \sec x\,dx = \ln|\sec x + \tan x| + C.$

14. $\int \csc x\,dx = \ln|\csc x - \cot x| + C.$

Algebraic functions

15. $\int \dfrac{1}{1+x^2}\,dx = \tan^{-1} x + C.$

16. $\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C.$

Hyperbolic functions

17. $\int \sinh x\,dx = \cosh x + C.$

18. $\int \cosh x\,dx = \sinh x + C.$

Useful Trigonometric Identities

Pythagorean identities

1. $\sin^2\theta + \cos^2\theta = 1$.

2. $1 + \tan^2\theta = \sec^2\theta$.

3. $1 + \cot^2\theta = \csc^2\theta$.

Double-angle formulas

4. $\sin 2\theta = 2\sin\theta\cos\theta$.

5. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$.

Half-angle formulas

6. $\sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta)$.

7. $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta)$.

Compound-angle formulas

8. $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$.

9. $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$.

10. $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$.

Sum-to-product formulas

11. $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$.

12. $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$.

13. $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$.

14. $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$.

Product-to-sum formulas

15. $\sin A\cos B = \dfrac{1}{2}\big[\sin(A+B) + \sin(A-B)\big]$.

16. $\cos A\sin B = \dfrac{1}{2}\big[\sin(A+B) - \sin(A-B)\big]$.

17. $\cos A\cos B = \dfrac{1}{2}\big[\cos(A+B) + \cos(A-B)\big]$.

18. $\sin A\sin B = -\dfrac{1}{2}\big[\cos(A+B) - \cos(A-B)\big]$.

Euler's formulas

19. $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$.

20. $e^{i\theta} + e^{-i\theta} = 2\cos\theta$, $\cos\theta = \dfrac{1}{2}(e^{i\theta} + e^{-i\theta})$.

21. $e^{i\theta} - e^{-i\theta} = 2i\sin\theta$, $\sin\theta = \dfrac{1}{2i}(e^{i\theta} - e^{-i\theta})$.

Remark. Formulas of the form $A \pm B = C \pm D$ contain two separate formulas

$$A + B = C + D, \qquad \text{and} \qquad A - B = C - D.$$

Likewise, formulas of the form $A \pm B = C \mp D$ contain two separate formulas

$$A + B = C - D, \qquad \text{and} \qquad A - B = C + D.$$