Assignment Two (deadline 5 OCT)

Question 1

EXE 2 Question 8

(10 marks)

8(a)

$d_{CB} + d_{CD} + d_{CG} = d_{CE}$

For the left hand side

Let CB be u and CD be v and CG be z

$d_{CB} + d_{CD} + d_{CG}$

$= u + v + z$

Because this is parallelogram

Therefore AB = CD = v

AC = AB + BC

= u + v

EA = CG = z

For the right hand side

d_{CE}

=EA + AC

=z + u +v

=left hand size

Therefore $d_{CB} + d_{CD} + d_{CG} = d_{CE}$

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8(b)(i)

$d_{CB} = (7,12,1) - (4,6,0)$

$=(3,6,1)$

$d_{CD} = (7,12,1) - (5,8,3)$

$=(2,4,-2)$

d_{CG} = (7,12,1) – (11,12,0)

= (-4,0,1)

$d_{CB} + d_{CD} + d_{CG} = d_{CE}$

d_{CE} = (3,6,1) + (2,4,-2) + (-4,0,1)

= (1,10,0)

$\boxed{2}$

8(b)(ii)

d_{CE} = (1,10,0)

(7,12,1) – E = (1,10,0)

E = (7,12,1) – (1,10,0)

= (6,2,1)

Therefore the coordinates of E is (6,2,1).

Length of OE = sqrt(6^2 + 2^2 + 1^2)

= sqrt(41)

The unit vector of OE = (6(sprt(41))/41 , 2(sqrt(41))/41 , sqrt(41)/41)

$\boxed{4}$

Question 2

EXE 2 Question 14

(10 arks)

(i) $((a^Tb)c-(c^Ta)b)^T((a^Tb)c-(c^Ta)b)$

$=(a^Tb)^2+(c^Tb)^2-2(a^Tb)(c^Tb)b^Tc$

$=\cos^2\gamma+\cos^2\alpha-2\cos\alpha\cos\beta\cos\gamma$

$=(\cos^2\gamma+\cos^2\alpha-2\cos\alpha\cos\beta\cos\gamma)$^1/2

$u=\cos\gamma$

$v=\sin\gamma$

(ii) u^Ta

$=\|a\|(\cos\gamma)$

$=\|a\|\|u\|$

v^Ta

$=\|a\|(\sin\gamma)$

$=\|a\|\|v\|$

Therefore u and v are perpendicular to a.

(iii) $u^Tv=0$

$((a^Tb)c-(c^Ta)b)^T(b-(a^Tb)a)=0$

$(\cos\gamma)(\cos\alpha)-(\cos\beta)=0$

$\cos\beta=(\cos\gamma)(\cos\alpha)$

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Question 3

EXE 2 Question 21

(10 marks)

The total number of symptoms the patients is 1^Ts.

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Question 4

EXE 2 Question 22

(10 marks)

$W_1=(\frac{100}{10}\ \frac{25}{8})$, …$W_8=(\frac{100}{10}\ \frac{1}{8}\ \frac{25}{100})$,$W_9=(\frac{100}{120}\ \frac{35}{100})$, $W_{10}=(\frac{40}{100})$

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啲分數記得分開

Question 5

EXE 2 Question 31

(10 marks)

(a) $a_1 \ldots a_k$ is linearly independent, so it can't not be linearly dependent.

(b)No because $b_1 \ldots b_k$ can be linearly independent.

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Question 6

Use the Gram-Schmidt process to find the orthonormal vectors for

$$\boldsymbol{a_1}=\begin{pmatrix}\mathbf{-1}\\\mathbf{1}\\\mathbf{-1}\\\mathbf{-1}\end{pmatrix},\boldsymbol{a_2}=\begin{pmatrix}\mathbf{-1}\\\mathbf{4}\\\mathbf{-1}\\\mathbf{3}\end{pmatrix},\boldsymbol{a_3}=\begin{pmatrix}\mathbf{1}\\\mathbf{3}\\\mathbf{5}\\\mathbf{9}\end{pmatrix}$$

$$q_1 = \frac{a_1}{\|a_1\|} = \begin{pmatrix} -0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{pmatrix}$$

$q_1^T a_2$ = 3/2

$$\tilde{q}_2 = a_2 - (q_1^T a_2) q_1 = \begin{pmatrix} -1 \\ 4 \\ -1 \\ 3 \end{pmatrix} - 4 \begin{pmatrix} -0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} \\ \frac{13}{4} \\ -\frac{1}{4} \\ \frac{15}{4} \end{pmatrix}$$

$$q2 = \frac{\tilde{q}_2}{\|\tilde{q}_2\|} = \begin{pmatrix} -\frac{\sqrt{11}}{66} \\ \frac{13\sqrt{11}}{66} \\ -\frac{\sqrt{11}}{66} \\ \frac{5\sqrt{11}}{22} \end{pmatrix}$$

$$q_1^T a_3 = -6$$

$$q_2^T a_3 = \frac{28\sqrt{11}}{11}$$

$$\tilde{q}_3 = a_3 - (q_1^T a_3) q_1 - (q_2^T a_3) q_2$$

$$= \begin{pmatrix} 1 \\ 3 \\ 5 \\ 9 \end{pmatrix} - (-6) \begin{pmatrix} -0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{pmatrix} - \left(\frac{28\sqrt{11}}{11}\right) \begin{pmatrix} -\frac{\sqrt{11}}{66} \\ \frac{13\sqrt{11}}{66} \\ -\frac{\sqrt{11}}{66} \\ \frac{5\sqrt{11}}{22} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{52}{33} \\ \frac{16}{33} \\ \frac{80}{33} \\ -\frac{4}{11} \end{pmatrix}$$

$$q3 = \frac{\tilde{q}_3}{\|\tilde{q}_3\|} = \begin{pmatrix} -\frac{13\sqrt{66}}{198} \\ \frac{2\sqrt{66}}{99} \\ \frac{10\sqrt{66}}{99} \\ -\frac{\sqrt{66}}{66} \end{pmatrix}$$

$$q_1 = \begin{pmatrix} -0.5 \\ 0.5 \\ -0.5 \\ -0.5 \end{pmatrix} \; q2 = \begin{pmatrix} -\frac{\sqrt{11}}{66} \\ \frac{13\sqrt{11}}{66} \\ -\frac{\sqrt{11}}{66} \\ \frac{5\sqrt{11}}{22} \end{pmatrix} q3 = \begin{pmatrix} -\frac{13\sqrt{66}}{198} \\ \frac{2\sqrt{66}}{99} \\ \frac{10\sqrt{66}}{99} \\ -\frac{\sqrt{66}}{66} \end{pmatrix}$$

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