EE1004 Tutorial 2 (Part 2)

Basic Questions

1. Five men and five women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let X denote the highest ranking achieved by a woman (for instance, X = 2 if the top-ranked person was male and the next-ranked person was female). Find P{$X = i$}, i = 1, 2, 3, . . ., 8, 9, 10.

Answer 1. $P_1 = 5/10$, $P_2 = 5/10 \times 5/9 = .2778$, $P_3 = 5/10 \times 4/9 \times 5/8 = .1389$.
$P_4 = 5/10 \times 4/9 \times 3/8 \times 5/7 = .0595$, $P_5 = 5/10 \times 4/9 \times 3/8 \times 2/7 \times 5/6 = .0198$,
$P_6 = 5/10 \times 4/9 \times 3/8 \times 2/7 \times 1/6 = .0040$, where $P_i = P(X = i)$.
$P_7 = P_8 = P_9 = P_{10} = 0$.

2. If $E[X] = 2$ and $E[X^2] = 8$, calculate (a) $E[(2+4X)^2]$ and (b) $E[X^2+(X+1)^2]$.

Answer 2. (a) $E[4 + 16X + 16X^2] = 164$

(b) $E[X^2 + X^2 + 2X + 1] = 21$

3. An insurance company writes a policy to the effect that an amount of money A must be paid if some event E occurs within a year. If the company estimates that E will occur within a year with probability p, what should it charge the customer so that its expected profit will be 10 percent of A?

Answer 3. If the company charges c, then

$E[\text{profit}] = c - Ap$

Therefore, $E[\text{profit}] = 0.1A$ when $c = A(p + 0.1)$.

4. Find the probability of finding 2 busy subscribers out of a total of 20 subscribers, where the probability that a given subscriber is busy is $P = 0.1$.

(a) Find the mean and variance of the number of busy subscribers.

(b) Find the probability that the number of busy subscribers is more than 2.

Answer 4. For Binomial distribution

$P(X=2) = 20!/18!/2!\ 0.1^2 \times 0.9^{18} = 0.2852$

(a)

Mean = 20x0.1 =2

Var = 2X0.9 = 1.8

(b)

P(X=0) = 20!/20!/0! 0.9^{20} = 0.1216

P(X=1) = 20!/19!/1! 0.1 x 0.9^{19} = 0.2702

P(X=2) = 20!/18!/2! 0.1^{2} x 0.9^{18} = 0.2852

P(X≤2)= 0.6770

P(X>2) = 0.323

5. Calculate the values of $P(X = r)$ on the same table (for comparison) for r values where r = 0, 1, 2,…, 10 in the following two distributions.

(a) binomial distribution with n = 10, P = 0.1,

(b) Poisson distribution with μ = 1.0.

Answer 5.

r	0	5	10
Binomial distribution	0.3487	$1.488x10^{-3}$	10^{-10}
Poisson distribution	0.3679	$3.0658x10^{-3}$	$1.0148x10^{-7}$

6. Earthquakes in a certain city occur according to a Poisson process with a rate of 1 per 30 years. Find the probability that NO earthquake will occur during a randomly-chosen future 100-year period.

Answer 6. μ =100/30

P(X=0)=e^(-10/3)=0.03568

7. Patients arrive at an Intensive Care Unit (ICU) according to a Poisson process with a rate of 2 per 10 hours. Find the probability that 3 or more patients will arrive at the ICU during a 60-minute period.

Answer 7. μ =0.2

$P(X \leq 2) = (1+0.2+0.02)e^{(-0.2)} = 0.9989$

$P(X>2) = 1- P(X \leq 2) = 0.0011$

Advanced Questions

1. Suppose that an insurance company would like to sell an investment fund product. She has the probability table.

	B	B^C
A	0.2	0.1
A^C	0.1	0.6

A: the event that a customer buys an investment fund

B: the event that a customer buys a life insurance.

Based on the table, the company would like to sell the investment fund by phone call.

(a) If the company calls a customer, what is the probability that the customer buys the investment fund product?

(b) If the company calls the customers who have a life insurance, what is the probability that the

customer buys the fund product?

(c) Assume that

The company has 1000 customers.

The manpower cost to make a phone $100.

The profit of successful selling the investment fund product is $1,000.

Describe the best way ("call all customers", "call the customers who have a life insurance", or "call the customers who do not have a life insurance") to sell the fund product, such that the net profit is better.

Answer (1a)

P(A)=0.3;

Answer (1b)

$P(A| B)=P(A \cap B)/P(B) = 0.2/0.3=2/3$

Answer (1c)

Call all customers

The expected profit per call = (0.3*(900)-0.7*100)=200;

Net profit = 1000*200 = 200,000

Call the customers who have a life insurance

The expected profit per call = 900*(2/3)-(1/3)*100=566.67

Next profit = 300*566.67=170,000

Call the customers who do not have a life insurance

$P(A| B^C)=P(A\cap B^C)/P(B^C)=0.1/0.7=1/7$

The expected profit per call = 900*(1/7)-(6/7)*100=42.86

Next profit = 700*42.86=30,002

Therefore, “call all customers” is the best way.