EE1004 Assignment 4

1. Suppose that an insurance company would like to sell an investment fund product. She has the probability table. [40 marks]

	B	B^C
A	0.2	0.1
A^C	0.1	0.6

A: the event that a customer buys an investment fund
B: the event that a customer buys a life insurance.

Based on the table, the company would like to sell the investment fund by phone call.

(a) If the company calls a customer, what is the probability that the customer buys the investment fund product?

(10 marks)

(b) If the company calls the customers who have a life insurance, what is the probability that the customer buys the fund product

(10 marks)

(c) Assume that

The company has 1000 customers.
The manpower cost to make a phone $100.
The profit of successful selling the investment fund product is $1,000.

Describe the best way ("call all customers", "call the customers who have a life insurance", or "call the customers who do not have a life insurance") to sell the fund product, such that the net profit is better.

(20 marks)

Answer (1a)

P(A)=0.3;

Answer (1b)

$P(A|\ B)=P(A\cap B)/P(B) = 0.2/0.3=2/3$

Answer (1c)

Call all customers

The expected profit per call = (0.3*(900)-0.7*100)=200;

Net profit = 1000*200 = 200,000

Call the customers who have a life insurance

The expected profit per call = 900*(2/3)-(1/3)*100=566.67

Next profit = 300*566.67=170,000

Call the customers who do not have a life insurance

$P(A| B^C)=P(A\cap B^C)/P(B^C)=0.1/0.7=1/7$

The expected profit per call = 900*(1/7)-(6/7)*100=42.86

Next profit = 700*42.86=30,002

Therefore, "call all customers" is the best way.

2. The following are the daily number of steps taken by a certain individual in 16 weekdays.

[30 marks]

2,100 1,984 2,072 1,898

1,950 1,992 2,096 2,103

2,043 2,218 2,244 2,206

2,210 2,152 1,962 2,007

Assuming that the daily number of steps is normally distributed, construct (a) a 95 percent and (b) a 99 percent two-sided confidence interval for the mean number of steps.

Answer 2. Use the **t Distribution Calculator** to find the critical t statistic.

The sample standard deviation = 107.2814

The sample mean = 2077.3125

degrees of freedom=16-1=15

(a) For a 95 percent two-sided confidence interval:

critical probability = 1 – (1-0.95)/2 = 0.975

critical t statistic = 2.131

confidence interval = $2077.3125 \pm 2.131(107.2814)/\sqrt{16} = 2077.3125 \pm 57.1542 = (2020.1583, 2134.4667)$

(b) For a 99 percent two-sided confidence interval:

critical probability = 1 – (1-0.99)/2 = 0.995

critical t statistic = 2.947

confidence interval = 2077.3125 ± 2.947(107.2814)/√16 = 2077.3125 ± 79.0396 = (1998.2724, 2156.3521)

3. A producer specifies that the mean lifetime of a certain type of battery is at least 240 hours. A sample of 16 such batteries yielded the following data. [30 marks]

237, 242, 232, 242, 248, 230, 244, 243, 254, 262, 234, 220, 225, 236, 232, 218

Assuming that the life of the batteries is approximately normally distributed, do the data indicate that the specifications are not being met at the $\alpha = 0.05$ level of significance?

Answer 3. The sample mean $\bar{x} = 237.4375$ and the sample standard deviation $s = 11.7472$

- Calculate the t statistic from the data set: $t = \left|\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}\right| = \left|\frac{237.4375 - 240}{\frac{11.7472}{\sqrt{16}}}\right| = 0.8725$
- Apply **Student's t-test** using the t statistic and degree of freedom 16 -1 = 15 to obtain the cumulative probability $P(T < t) = 0.8017$.
- This is a **one-tailed** test so the P-value is obtained from

$P = 1 - P(Z < z) = 0.1983$

Since $P \geq \alpha$, we accept the null hypothesis.