# Lecture 5

# Electric Potential



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# Lecture Outline

- **Chapter 24**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Electric Potential
  - Conservative force and potential energy
  - Potential energy and potential
  - Potential due to different charge distribution
  - Field and potential (versus force and potential energy)
  - Conductors

# Definition of Potential Energy

- Potential energy is defined as the *stored energy of position*.
- It can be viewed as energy that is 'stored' by the physical system, (e.g. arrangement of objects in a force field).
- It is called *potential* because there is no work being done at its current form, it does not cause any change in its surroundings.
- However, it has the *potential* to be converted to other forms of energy, such as kinetic energy.
- The standard unit for measuring potential energy is joule.



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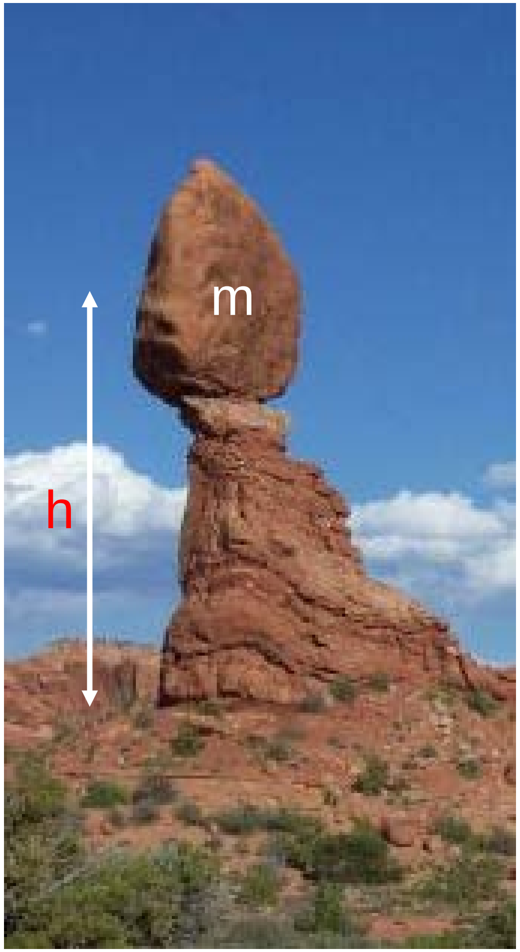
# Gravitational Potential Energy

- The amount of potential energy stored by the rock is a function of its mass and height:

$$P.E._{grav} = mgh \qquad g = 9.8\,\mathrm{m/s^2}$$

- The higher the rock is placed, the higher the potential.
- The bigger the mass, the higher the potential.
- Because these will create larger kinetic energy if the rock falls to the ground.



# Work and Potential Energy

- Let’s take a closer look at the gravitational potential energy formulation (P.E. is often represented by $U$):

$$P.E._{grav} = U_{grav} = \underbrace{mg}_{\text{Gravitational Force } F_{grav}}\underbrace{h}_{\text{Displacement from ground } d}$$

- We can express the potential energy as

$$U_{grav} = \vec{F}_{grav} \cdot \vec{d}$$

<u>Potential energy has the same SI unit *J* Joule as Work Done *W*</u>

# Account for the Work Done

- Consider the potential energy of the rock on slide 4 when it is located on the ground and when it is at a distance *h* above ground, the corresponding potential energies are:

$$U_{d=0} = mg \cdot 0 = 0 \qquad U_{d=h} = mg \cdot h = mgh$$

- Case 1: moving the rock from d=0 to d=h

$$U_{gain/loss} = U_{after} - U_{before}$$

$$\Delta U = U_{d=h} - U_{d=0} = mgh - 0 = mgh = \boxed{W_{you}}$$

***<u>You contributed work</u>* (energy) into the system.**

- Case 2: dropping the rock from d=h to d=0

$$\Delta U = U_{d=0} - U_{d=h} = 0 - mgh = -mgh = \boxed{-W_{you} = W_{system}}$$

*<u>The system did the work</u> and released the energy.*

# Conservative and Non-Conservative Forces

- Consider a system consists of multiple objects where there are forces acting between any object with all other objects in the system.
- If one of the objects in the system is moved from A to B, work done is required to apply the force on the object to make it move, lets call this work done $W_1$. Here the kinetic energy used to move the object is conversed into some other type of energy.
- When the configuration is reversed, where the object is moved from B to A. The force reverses the energy transfer, doing work $W_2$ in the process.

*When $W_1 = -W_2$ is always true and the other type of energy is a potential energy, then the force is said to be a <u>conservative force.</u>*

*A force that is not conservative is called a non-conservative force. For example, the kinetic frictional force and drag force are <u>non-conservative.</u>*



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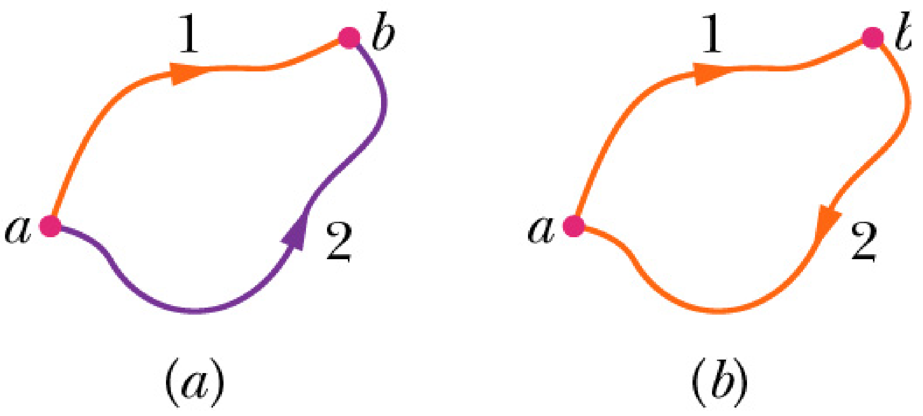
# Path Independence of Conservative Forces

(a) As a conservative force acts on it, a particle can move from point *a* to point *b* along either path 1 or path 2.

(b) The particle moves in a round trip, from point *a* to point *b* along path 1 and then back to point *a* along path 2.




a) The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

b) The net work done by a conservative force on a particle moving around every closed path is zero.



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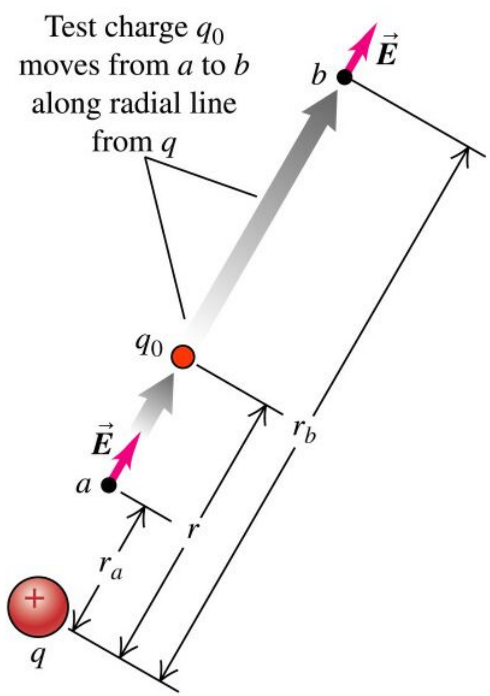
# Newton's Law for Gravitation and Coulomb's Law

$$F = G\frac{m_1 m_2}{r^2} \qquad\qquad F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Both forces are conservative forces, i.e. work done is path independent

Gravitational force is always attractive. Electrostatic force can be attractive or repulsive depending on the sign of the charges.

# Potential Energy of Two Point Charges




Suppose we have two charges $q$ and $q_0$ that are separated by a distance $r$.

The force between them is given by Coulomb's Law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$

We now displace the charge $q_0$ along a radial line from point $a$ to point $b$.

Since the force is not constant and is a function of the displacement, we need to integrate

$$W_{a\to b} = \int_{r_a}^{r_b} F_r\,dr = \int_{r_a}^{r_b} \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\,dr = -\frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$



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# Electric Potential Energy

Let's rearrange the result and we can see that the right hand side represent the same function at points *a* and *b* that we are taking the difference.

$$W_{a\to b} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = \frac{qq_0}{4\pi\varepsilon_0 r_a} - \frac{qq_0}{4\pi\varepsilon_0 r_b}$$

We define this function to be the potential energy

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r}$$

$$W_{a\to b} = U(r_a) - U(r_b)$$

$$\Delta U = U(r_b) - U(r_a) = -W_{a\to b}$$

The usual convention is when $r = \infty \quad U(r) = 0$

The potential energy is negative when the charges are of opposite sign (attractive force). This means that the system did the work to bring the electron at infinity next to the electron that has the opposite charge.



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# 24.2: Electric Potential Energy - Summary



When an electrostatic force acts between two or more charged particles within a system of particles, we can assign an **electric potential energy** ***U*** to the system. If the system changes its configuration from an initial state *i* to a different final state *f*, the electrostatic force does work *W* on the particles. If the resulting change is $\Delta U$, then

$$\boxed{\Delta U = U_f - U_i = -W.}$$

As with other conservative forces, the work done by the electrostatic force is *path independent*.

Usually the reference configuration of a system of charged particles is taken to be that in which the particles are all infinitely separated from one another. The corresponding reference potential energy is usually set be zero $U_i = 0$. Therefore,

$$U_f = U = -W_\infty$$

*Potential energy equals to the negative of the work done by the system required to bring the particle from infinity to f.*

Potential Energy *increases* if the particle moves in the direction opposite to the force on it and *decreases* if the particle moves in the same direction as the force on it.



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# Example, Work and potential energy in an electric field:

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electrostatic force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150$ N/C and is directed downward. What is the change $\Delta U$ in the electric potential energy of a released electron when the electrostatic force causes it to move vertically upward through a distance $d = 520$ m (Fig. 24-1)?

**KEY IDEAS**

(1) The change $\Delta U$ in the electric potential energy of the electron is related to the work $W$ done on the electron by the electric field. Equation 24-1 ($\Delta U = -W$) gives the relation.

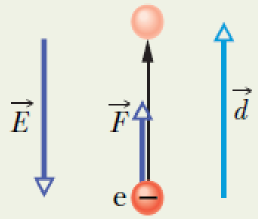


**Fig. 24-1** An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electrostatic force $\vec{F}$ due to an electric field $\vec{E}$.

(2) The work done by a constant force $\vec{F}$ on a particle undergoing a displacement $\vec{d}$ is

$$W = \vec{F} \cdot \vec{d}. \qquad (24\text{-}3)$$

(3) The electrostatic force and the electric field are related by the force equation $\vec{F} = q\vec{E}$, where here $q$ is the charge of an electron ($= -1.6 \times 10^{-19}$ C).

***Calculations:*** Substituting for $\vec{F}$ in Eq. 24-3 and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos\theta, \qquad (24\text{-}4)$$

where $\theta$ is the angle between the directions of $\vec{E}$ and $\vec{d}$. The field $\vec{E}$ is directed downward and the displacement $\vec{d}$ is directed upward; so $\theta = 180°$. Substituting this and other data into Eq. 24-4, we find

$$W = (-1.6 \times 10^{-19}\ \text{C})(150\ \text{N/C})(520\ \text{m}) \cos 180°$$
$$= 1.2 \times 10^{-14}\ \text{J}.$$

Equation 24-1 then yields

$$\Delta U = -W = -1.2 \times 10^{-14}\ \text{J}. \qquad \text{(Answer)}$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by $1.2 \times 10^{-14}$ J.



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# 24.3 Electric Potential:

***Electric potential V*** (or simply the *potential*) is the potential energy per unit charge at a point. This is a scalar quantity that is a function of the location in an electric field. Thus,

$$V = \frac{\text{Electric Potential Energy}}{\text{Unit Charge}} = \frac{U}{q}$$

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}.$$

$$= -\frac{W}{q} \qquad \text{(potential difference defined).}$$

If we set $U_i = 0$ at infinity as our reference potential energy, then the electric potential $V$ must also be zero there. Therefore, the electric potential at any point in an electric field can be defined to be

$$V = -\frac{W_\infty}{q} \qquad \text{(potential defined)}$$

Here $W_\infty$ is the work done by the electric field on a charged particle as that particle moves in from infinity to point $f$.

The SI unit for electric potential is the joule per coulomb. This combination is called the *volt (abbreviated V)*.

$$1 \text{ volt} = 1 \text{ joule per coulomb.}$$



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# 24.3 Electric Potential: Units:

This unit of volt allows us to adopt a more conventional unit for the electric field, *E*, which is expressed in Newton per Coulomb.

*P.E. (J) = Electric Potential (V) x Charge (C)*

$$1\ \mathrm{N/C} = \left(1\frac{\mathrm{N}}{\mathrm{C}}\right)\left(\frac{1\ \mathrm{V\cdot C}}{1\ \mathrm{J}}\right)\left(\frac{1\ \mathrm{J}}{1\ \mathrm{N\cdot m}}\right)$$
$$= 1\ \mathrm{V/m}.$$

*Work (J) = Force (N) x Displacement (m)*

We can now define an energy unit that is a convenient one for energy measurements in the atomic/subatomic domain: One *electron-volt (eV)* is the energy equal to the work required to move a single elementary charge *e*, such as that of the electron or the proton, through a potential difference of exactly one volt. The magnitude of this work is $q\Delta V$, and

$$1\ \mathrm{eV} = e(1\ \mathrm{V})$$
$$= (1.60 \times 10^{-19}\ \mathrm{C})(1\ \mathrm{J/C}) = 1.60 \times 10^{-19}\ \mathrm{J}.$$



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# 24.4 Equipotential Surfaces:

Adjacent points that have the same electric potential form an equipotential surface, which can be either an imaginary surface or a real, physical surface.
No net work *W* is done on a charged particle by an electric field when the particle moves between two points *i* and *f* on the same equipotential surface.

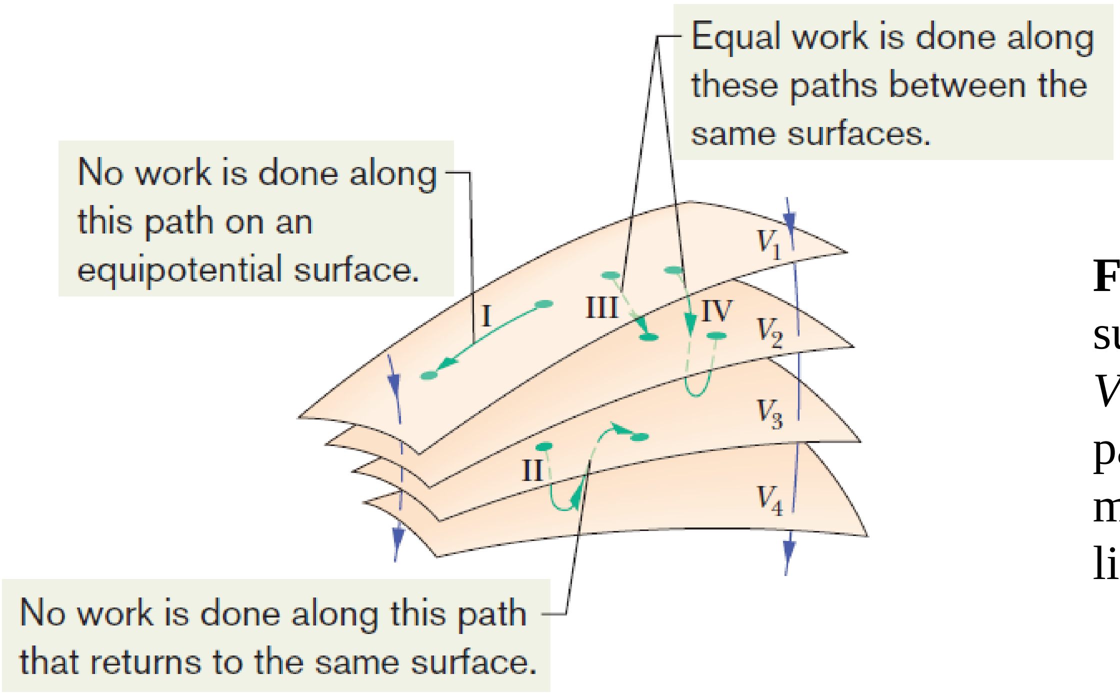


**Fig. 24-2** Portions of four equipotential surfaces at electric potentials $V_1 = 100$ V, $V_2 = 80$ V, $V_3 = 60$ V, and $V_4 = 40$ V. Four paths along which a test charge may move are shown. Two electric field lines are also indicated.



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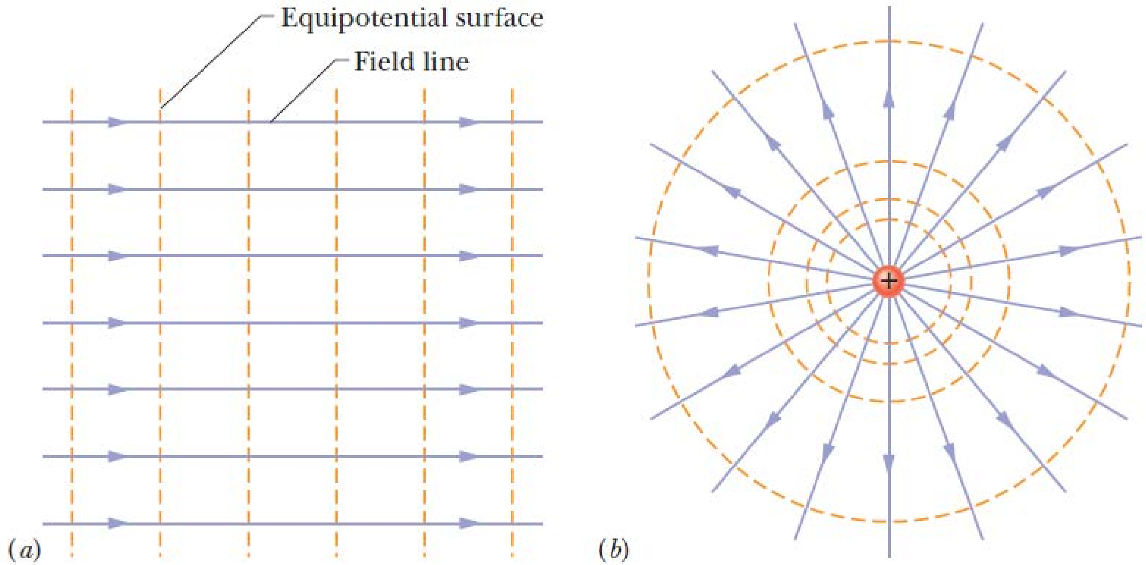
# 24.4 Equipotential Surfaces:



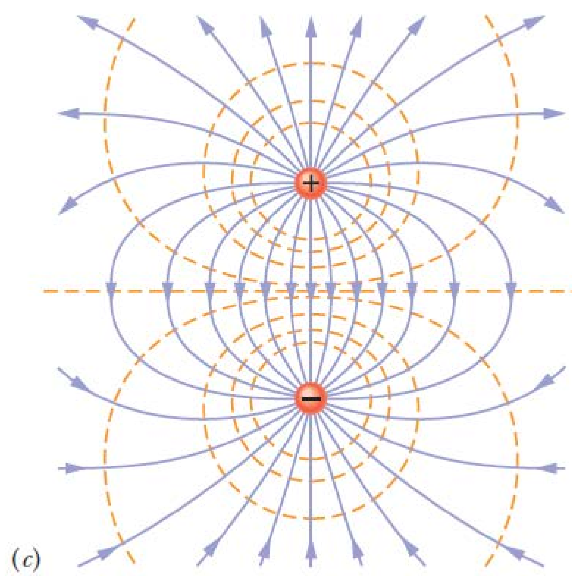




**Fig. 24-3** Electric field lines (purple) and cross sections of equipotential surfaces (gold) for *(a)* a uniform electric field, *(b)* the field due to a point charge, and *(c)* the field due to an electric dipole.



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# 24.5 Calculating the Potential from the Field:

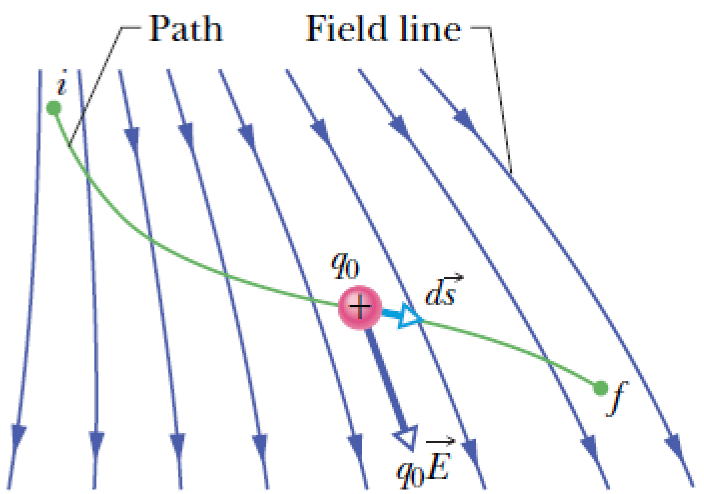


**Fig. 24-4** A test charge $q_0$ moves from point $i$ to point $f$ along the path shown in a nonuniform electric field. During a displacement $d\vec{s}$, an electrostatic force $q_0\vec{E}$ acts on the test charge. This force points in the direction of the field line at the location of the test charge.

$$dW = \vec{F} \cdot d\vec{s}.$$

$$dW = q_0\vec{E} \cdot d\vec{s}.$$

$$W = q_0 \int_i^f \vec{E} \cdot d\vec{s}.$$

$$\boxed{\Delta U = U_f - U_i = -W.}$$

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

If we set potential $V_i = 0$, then

$$V = -\int_i^f \vec{E} \cdot d\vec{s},$$



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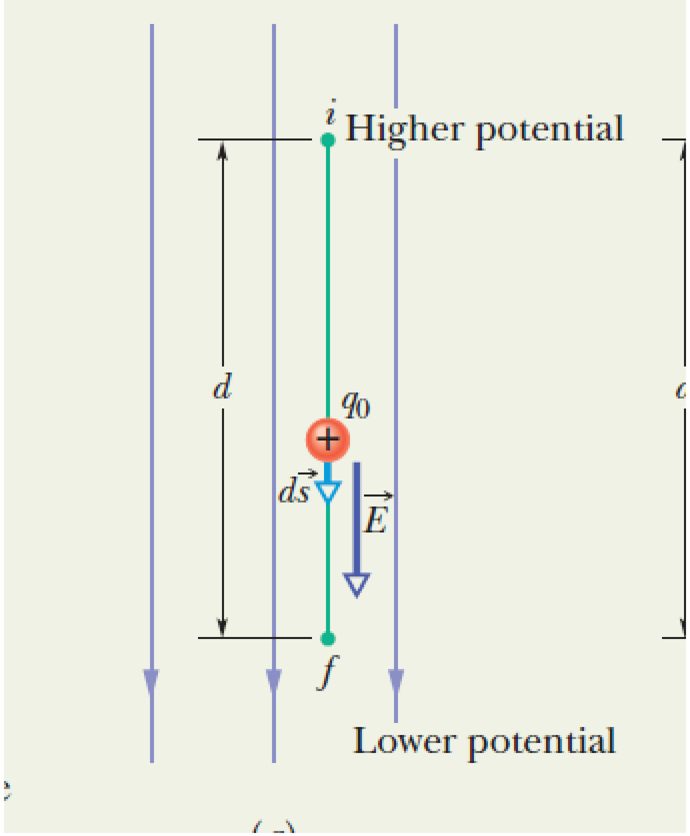
# Example, Finding the Potential change from the Electric Field:

(a) Figure 24-5*a* shows two points *i* and *f* in a uniform electric field $\vec{E}$. The points lie on the same electric field line (not shown) and are separated by a distance *d*. Find the potential difference $V_f - V_i$ by moving a positive test charge $q_0$ from *i* to *f* along the path shown, which is parallel to the field direction.

The electric field points *from* higher potential *to* lower potential.




***Calculations:*** We begin by mentally moving a test charge $q_0$ along that path, from initial point *i* to final point *f*. As we move such a test charge along the path in Fig. 24-5*a*, its differential displacement $d\vec{s}$ always has the same direction as $\vec{E}$. Thus, the angle $\theta$ between $\vec{E}$ and $d\vec{s}$ is zero and the dot product in Eq. 24-18 is

$$\vec{E}\cdot d\vec{s} = E\,ds\cos\theta = E\,ds. \qquad (24\text{-}20)$$

Equations 24-18 and 24-20 then give us

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s} = -\int_i^f E\,ds. \qquad (24\text{-}21)$$

Since the field is uniform, *E* is constant over the path and can be moved outside the integral, giving us

$$V_f - V_i = -E\int_i^f ds = -Ed, \qquad \text{(Answer)}$$



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# Example, Finding the Potential change from the Electric Field:



(b) Now find the potential difference $V_f - V_i$ by moving the positive test charge $q_0$ from $i$ to $f$ along the path $icf$ shown in Fig. 24-5$b$.

***Calculations:*** The Key Idea of (a) applies here too, except now we move the test charge along a path that consists of two lines: $ic$ and $cf$. At all points along line $ic$, the displacement $d\vec{s}$ of the test charge is perpendicular to $\vec{E}$. Thus, the angle $\theta$ between $\vec{E}$ and $d\vec{s}$ is 90°, and the dot product $\vec{E}\cdot d\vec{s}$ is 0. Equation 24-18 then tells us that points $i$ and $c$ are at the same potential: $V_c - V_i = 0$.

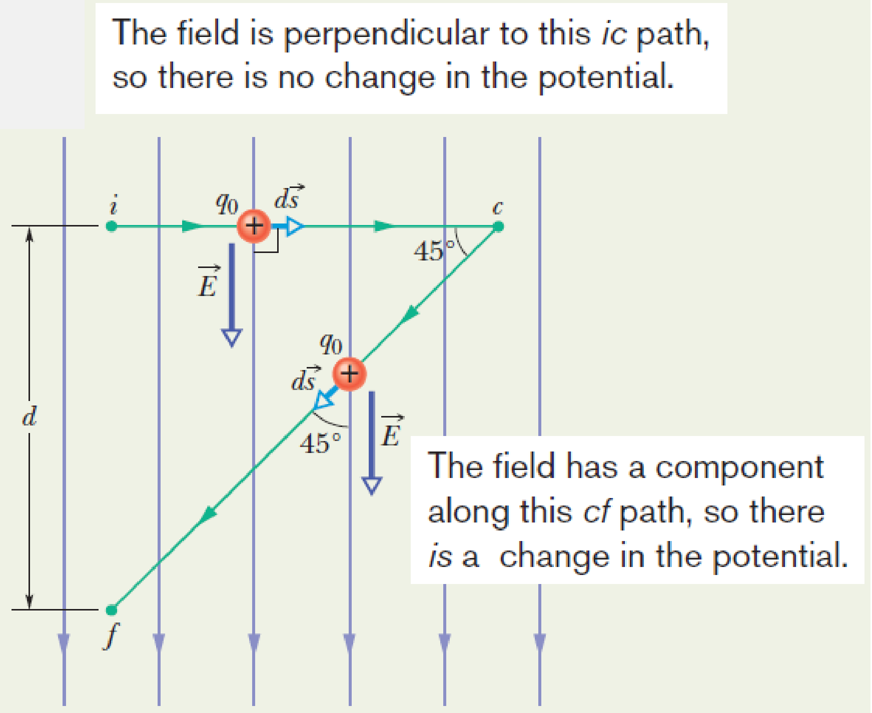
For line $cf$ we have $\theta = 45°$ and, from Eq. 24-18,

$$V_f - V_i = -\int_c^f \vec{E}\cdot d\vec{s} = -\int_c^f E(\cos 45°)\, ds$$

$$= -E(\cos 45°)\int_c^f ds.$$

The integral in this equation is just the length of line $cf$; from Fig. 24-5$b$, that length is $d/\cos 45°$. Thus,

$$V_f - V_i = -E(\cos 45°)\frac{d}{\cos 45°} = -Ed. \quad \text{(Answer)}$$




(b)



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# 24.6 Potential Due to a Point Charge:



*The work done by the electric force in moving a test charge $q_0$ from point a to point b is given by*

$$W_{a\to b} = \int_a^b \vec{F}\cdot dl = \int_a^b q_0\vec{E}\cdot dl$$

*Dividing through by the test charge $q_0$ and using the relationships between U, V, and W, we have*

$$-\frac{W_{a\to b}}{q_0} = V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$$

*Rearranging so the order of the subscript is the same on both sides*

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l}$$

*Substituting the electric field of the point charge*

$$V_b - V_a = -\int_a^b \frac{q}{4\pi\varepsilon_0 r^2}dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_b} - \frac{1}{r_a}\right]$$

$$\boxed{V(r) = \frac{q}{4\pi\varepsilon_0 r} = \frac{U}{q_0}}$$

To find the potential of the charged particle, we move this test charge out to infinity.

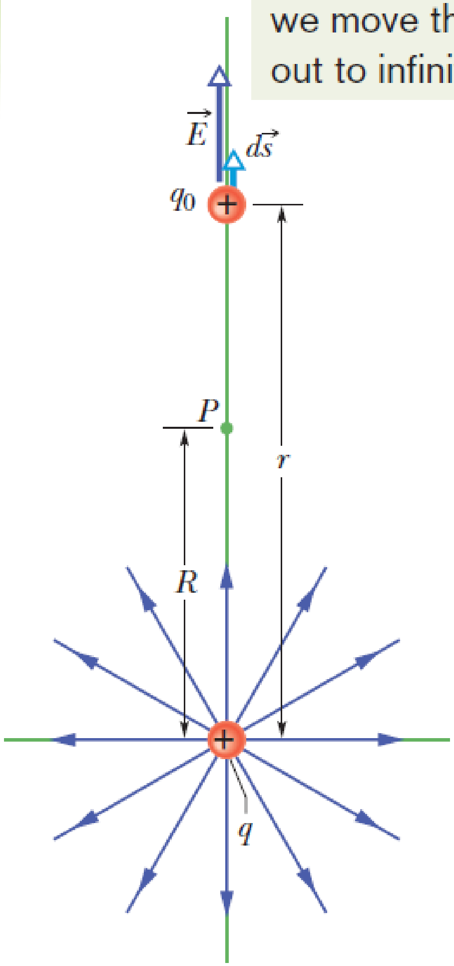



**Fig. 24-6** The positive point charge $q$ produces an electric field $\vec{E}$ and an electric potential $V$ at point $P$. We find the potential by moving a test charge $q_0$ from $P$ to infinity. The test charge is shown at distance $r$ from the point charge, during differential displacement $d\vec{s}$.

# 24.7 Potential Due to a Group of Point Charges:

The net potential at a point due to a group of point charges can be found with the help of the superposition principle. First the individual potential resulting from each charge is considered at the given point. Then we sum the potentials.

For *n* charges, the net potential is

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \qquad (n \text{ point charges}).$$

We see that the electric field points in the direction of *decreasing* potential

# Example, Net Potential of Several Charged Particles:

What is the electric potential at point $P$, located at the center of the square of point charges shown in Fig. 24-8$a$? The distance $d$ is 1.3 m, and the charges are

$$q_1 = +12\text{ nC}, \qquad q_3 = +31\text{ nC},$$
$$q_2 = -24\text{ nC}, \qquad q_4 = +17\text{ nC}.$$

**KEY IDEA**

The electric potential $V$ at point $P$ is the algebraic sum of the electric potentials contributed by the four point charges.

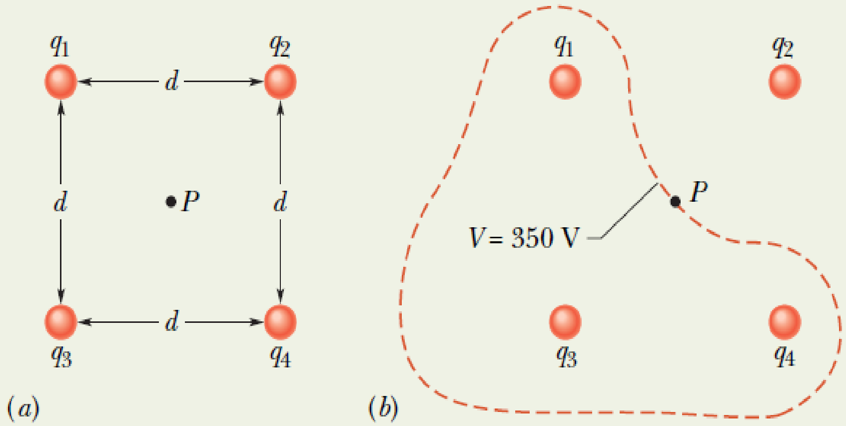



**Fig. 24-8** ($a$) Four point charges are held fixed at the corners of a square. ($b$) The closed curve is a cross section, in the plane of the figure, of the equipotential surface that contains point $P$. (The curve is drawn only roughly.)

(Because electric potential is a scalar, the orientations of the point charges do not matter.)

***Calculations:*** From Eq. 24-27, we have

$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

The distance $r$ is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9}\text{ C}$$
$$= 36 \times 10^{-9}\text{ C}.$$

Thus, $$V = \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-9}\text{ C})}{0.919\text{ m}}$$
$$\approx 350\text{ V}. \qquad \text{(Answer)}$$

Close to any of the three positive charges in Fig. 24-8$a$, the potential has very large positive values. Close to the single negative charge, the potential has very large negative values. Therefore, there must be points within the square that have the same intermediate potential as that at point $P$. The curve in Fig. 24-8$b$ shows the intersection of the plane of the figure with the equipotential surface that contains point $P$. Any point along that curve has the same potential as point $P$.

# Example, Potential is not a Vector:



(a) In Fig. 24-9*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius $R$. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center $C$ of the circle due to these electrons?

**KEY IDEAS**

(1) The electric potential $V$ at $C$ is the algebraic sum of the electric potentials contributed by all the electrons. (Because electric potential is a scalar, the orientations of the electrons do not matter.) (2) The electric field at $C$ is a vector quantity and thus the orientation of the electrons *is* important.

***Calculations:*** Because the electrons all have the same negative charge $-e$ and are all the same distance $R$ from $C$, Eq. 24-27 gives us

$$V = -12 \frac{1}{4\pi\varepsilon_0} \frac{e}{R}. \qquad \text{(Answer)} \qquad (24\text{-}28)$$

Because of the symmetry of the arrangement in Fig. 24-9*a*, the electric field vector at $C$ due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at $C$,

$$\vec{E} = 0. \qquad \text{(Answer)}$$

Potential is a scalar and orientation is irrelevant.

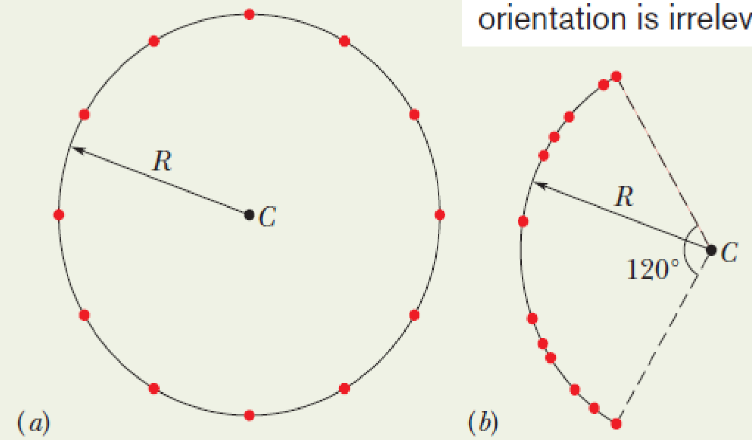


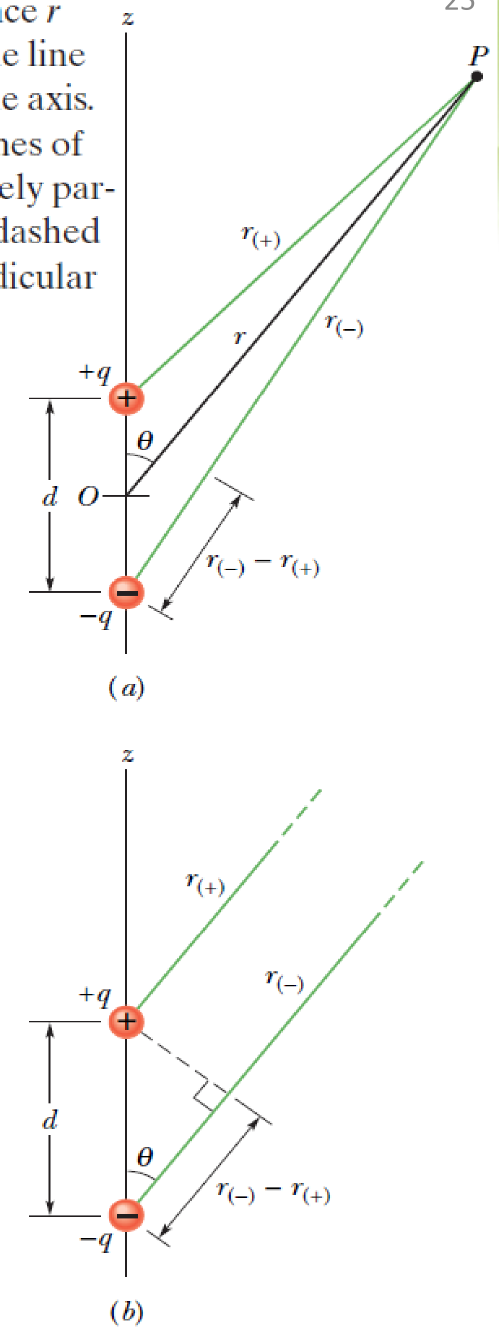

**Fig. 24-9** (*a*) Twelve electrons uniformly spaced around a circle. (*b*) The electrons nonuniformly spaced along an arc of the original circle.

(b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 24-9*b*), what then is the potential at $C$? How does the electric field at $C$ change (if at all)?

***Reasoning:*** The potential is still given by Eq. 24-28, because the distance between $C$ and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.

# 24.8 Potential Due to an Electric Dipole:

**Fig. 24-10** (*a*) Point $P$ is a distance $r$ from the midpoint $O$ of a dipole. The line $OP$ makes an angle $\theta$ with the dipole axis. (*b*) If $P$ is far from the dipole, the lines of lengths $r_{(+)}$ and $r_{(-)}$ are approximately parallel to the line of length $r$, and the dashed black line is approximately perpendicular to the line of length $r_{(-)}$.




$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

$$r_{(-)} - r_{(+)} \approx d\cos\theta \quad \text{and} \quad r_{(-)}r_{(+)} \approx r^2.$$

$$V = \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\,\frac{p\cos\theta}{r^2} \qquad \text{(electric dipole)},$$

# Example, Potential Energy of a System of Three Charged Particles:

Figure 24-16 shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy $U$ of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

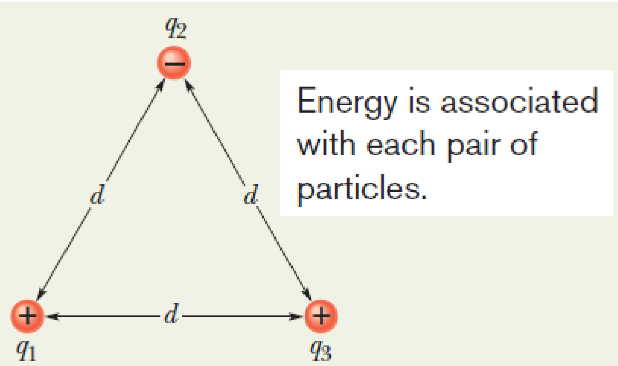



**Fig. 24-16** Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

***Calculations:*** Let's mentally build the system of Fig. 24-16, starting with one of the point charges, say $q_1$, in place and the others at infinity. Then we bring another one, say $q_2$, in from infinity and put it in place. From Eq. 24-43 with $d$ substituted for $r$, the potential energy $U_{12}$ associated with the pair of point charges $q_1$ and $q_2$ is

$$U_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d}.$$

We then bring the last point charge $q_3$ in from infinity and put it in place. The work that we must do in this last step is equal to the sum of the work we must do to bring $q_3$ near $q_1$ and the work we must do to bring it near $q_2$. From Eq. 24-43, with $d$ substituted for $r$, that sum is

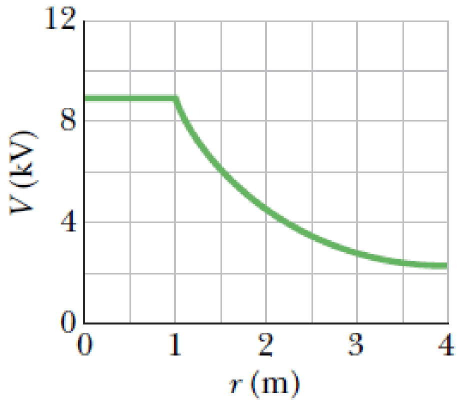
charges. This sum (which is actually independent of the order in which the charges are brought together) is

$$\begin{aligned}
U &= U_{12} + U_{13} + U_{23} \\
&= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right) \\
&= -\frac{10q^2}{4\pi\varepsilon_0 d} \\
&= -\frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10)(150 \times 10^{-9}\ \text{C})^2}{0.12\ \text{m}} \\
&= -1.7 \times 10^{-2}\ \text{J} = -17\ \text{mJ}. \qquad \text{(Answer)}
\end{aligned}$$

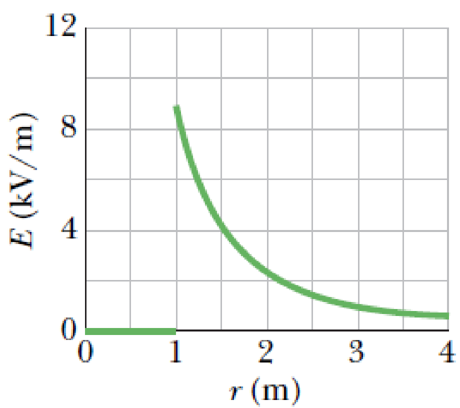
# 24.12 Potential of a Charged, Isolated Conductor:



An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.




(a)




(b)

**Fig. 24-18** (*a*) A plot of $V(r)$ both inside and outside a charged spherical shell of radius 1.0 m. (*b*) A plot of $E(r)$ for the same shell.

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$



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# 24.12 Isolated Conductor in an External Electric Field:

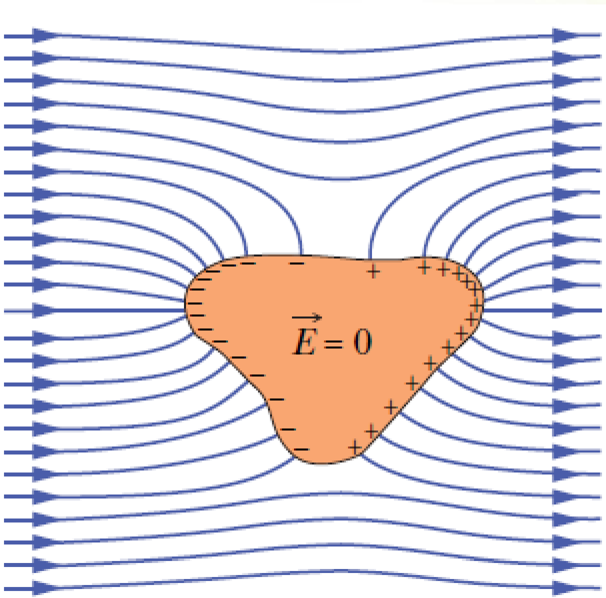


**Fig. 24-20** An uncharged conductor is suspended in an external electric field. The free electrons in the conductor distribute themselves on the surface as shown, so as to reduce the net electric field inside the conductor to zero and make the net field at the surface perpendicular to the surface.

If an isolated conductor is placed in an external electric field, all points of the conductor still come to a single potential regardless of whether the conductor has an excess charge.

The free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field that would otherwise be there.

Furthermore, the electron distribution causes the net electric field at all points on the surface to be perpendicular to the surface. If the conductor in Fig. 24-20 could be somehow removed, leaving the surface charges frozen in place, the internal and external electric field would remain absolutely unchanged.

# Note the stronger field near sharper tips of the conductor.

# 24.12 Spark Discharge from a Charged Conductor:

**Fig. 24-19** A large spark jumps to a car's body and then exits by moving across the insulating left front tire (note the flash there), leaving the person inside unharmed. *(Courtesy Westinghouse Electric Corporation)*

On nonspherical conductors, a surface charge does not distribute itself uniformly over the surface of the conductor. At sharp points or edges, the surface charge density—and thus the external electric field, —may reach very high values. The air around such sharp points or edges may become ionized, producing the corona discharge that golfers and mountaineers see on the tips of bushes, golf clubs, and rock hammers when thunderstorms threaten. Such corona discharges are often the precursors of lightning strikes. In such circumstances, it is wise to enclose yourself in a cavity inside a conducting shell, where the electric field is guaranteed to be zero. A car (unless it is a convertible or made with a plastic body) is almost ideal