EE1001
Foundations of Digital Techniques

Logic

Tutorial 2

From Proposition to Predicate

Predicate Logic

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Question 1

- Is the following argument valid?

$$\sqrt{2} > \frac{3}{2} \rightarrow (\sqrt{2})^2 > \left(\frac{3}{2}\right)^2$$

$$\sqrt{2} > \frac{3}{2}$$

$$\overline{\quad 2 > \left(\frac{3}{2}\right)^2 \quad}$$

Ans:
Valid, the argument is constructed using Modus Ponens.

$$p \rightarrow q$$

$$p$$

$$\overline{\quad q \quad}$$

A valid argument can lead to an incorrect conclusion if one of its premises is wrong.
i.e., valid and unsound argument

Question 2

1) If the 5G is adopted, then we can connect to the network on the rooftop.
2) The network covers either the area inside the campus or the basement.
3) If the network covers the area inside the campus, then 5G is used.
4) If the network covers the basement, then Wifi is adopted.
5) We cannot connect to the network on the rooftop.

Question: What is wireless technology of the network?

Ans:

Let

g = "5G"

c = "connect to the network on the rooftop"

a = "area inside the campus"

b = "basement"

w = "Wifi"

1) $g \to c$
2) $a \lor b$
3) $a \to g$
4) $b \to w$
5) $\sim c$
6) $a \to c$ (HS 3,1)
7) $\sim a$ (MT 6,5)
8) b (DS 2,7)
9) w (MP 4,8)

* 1) to 5) are the premises

<u>Inference Rules</u>

HS: Hypothetical Syllogism

MT: Modus Tollens

DS: Disjunctive Syllogism

MP: Modus Ponens

Therefore, Wifi is used.

Question 3

The Logical Problem of Evil, by Epicurus, an ancient Greek philosopher and the founder of the school of philosophy called Epicureanism

1) If ***G***od exists, then God is omni***p***otent (all-powerful) and omni***b***enevolent (perfectly-good).
2) If God is omnipotent, then He would be ***a***ble to prevent evil.
3) If God is omnibenevolent, then He would be ***w***illing to prevent evil.
4) If God is able to and willing to prevent evil, then there would be no evil.
5) There is ***e***vil.

Conclusion: God does not exist.

Use inference rules and logical equivalence relation to determine the validity of the argument above.

Ans:

1) $G \rightarrow (p \land b)$
2) $p \rightarrow a$
3) $b \rightarrow w$
4) $(a \land w) \rightarrow \sim e$
5) e
6) $\sim(a \land w)$ (MT 4,5)
7) $\sim a \lor \sim w$ (De Morgan 6)
8) $\sim a \rightarrow \sim p$ (contrapositive 2)
9) $\sim w \rightarrow \sim b$ (contrapositive 3)
10) $(\sim a \rightarrow \sim p) \land (\sim w \rightarrow \sim b)$ (C 8,9)
11) $\sim p \lor \sim b$ (CD 10,7)
12) $\sim(p \land b)$ (De Morgan 11)
13) $\sim G$ (MT 1,12)

<u>Therefore, the argument is valid</u>

Caution: A valid argument may not be sound.

It is widely accepted that this argument has be defeated by the “free-will defence”. You can read books on philosophy or religion if interested.

Question 4

Consider "$\forall x \in \boldsymbol{R}$, if $x^2 > 4$, then $x > 2$".

i) What is its negation?

ii) Is its negation true?

Ans:

i) Let $P(x)$ = "$x^2 > 4$" and Q(x) = "$x > 2$"

$$
\begin{aligned}
\sim(\forall x \in \boldsymbol{R}, P(x) \rightarrow Q(x)) &\equiv \exists x, \sim(P(x) \rightarrow Q(x)) & \\
&\equiv \exists x, \sim(\sim P(x) \vee Q(x)) && \text{(Definition of } \rightarrow\text{)} \\
&\equiv \exists x, \sim\sim P(x) \wedge \sim Q(x) && \text{(De Morgan law)} \\
&\equiv \exists x, P(x) \wedge \sim Q(x) && \text{(Double negative law)}
\end{aligned}
$$

$\exists x \in \boldsymbol{R}$, such that $x^2 > 4$ and $x \leq 2$

i) YES

Question 5

1) There is a course which every student scores an A
2) There is a student scoring A in all courses

i) Are these two arguments the same?
ii) Find out their negations.

Ans:
Domains: S = {Students}, C = {Courses}
Predicates $A(x,y)$: The student *scores* an A in the course y.

1) $\exists y \in C, \forall x \in S, A(x,y)$
2) $\exists x \in S, \forall y \in C, A(x,y)$

i) No
ii) Statement 1):
$\sim(\exists y \in C, \forall x \in S, A(x,y)) \equiv \forall y \in C, \exists x \in S, \sim A(x,y)$
Statement 2):
$\sim(\exists x \in S, \forall y \in C, A(x,y)) \equiv \forall x \in S, \exists y \in C, \sim A(x,y)$

In all courses, there is a student who does not scores an A.

There is a course the student does not score an A.