Lecture 3

Electric Fields

Department of Physics
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Lecture 02 – Review

- In Lecture 2 we introduced the properties of electric charges.
- Charge is not a substance rather it is a property of the particles.
- Electric charge has two different signs, positive or negative.
- A charged particle will encounter a force when it is placed near another charged particle.
- The charges repel each other if they have the same sign and attract each other when they have opposite signs.
- A conductor consists of mobile electrons that can move freely within the conductor, we call the mobile electrons conduction or free electrons.
- Insulators also have electrons, however these electrons are bounded to the atom and cannot move freely.

Lecture 02 – Review

- The force between two charged particles is called the electrostatic force, it is governed by the Coulomb's law.
- The magnitude of the force is proportional to the product of the charges and inversely proportional to the square of the displacement between the charges.

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} \qquad \text{(Coulomb's law)},$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\,\mathrm{N\cdot m^2/C^2}.$$

- The constant k is called the *electrostatic constant* and the quantity ε is called the *permittivity*.
- The quantity ε is the property of the medium between the two charged particles and ε_0 is the *vacuum permittivity.*
- We also defined current as the rate at which charge moves past a point, a surface, or a region.

Lecture 02 – Review

- The total force encountered by a charged particle when it is placed near a group of charged particles is given by the vector sum of the particle with each individual particles.
- When two conducting objects are connected by a conducting wire, the charges on the objects will be shared between them. As a result, the two objects will have the same amount of charges.
- A ground is a huge conductor and can be used to neutralize a charged object when connecting it with a wire.
- Charge is quantized $e=1.602 \times 10^{-19}$ C
- Conservation of charge means the total electric charge of an isolated system is constant and cannot be changed.

Lecture Outline

- **Chapter 22**, D Halliday, R Resnick, and J Walker, “Fundamentals of Physics” 9th Edition, Wiley (2005).

- Electric Field
  – Forces and fields
  – The electric field due to different charge distributions
  – A point charge in an electric field
  – A dipole in an electric field

22.2 The Electric Field:

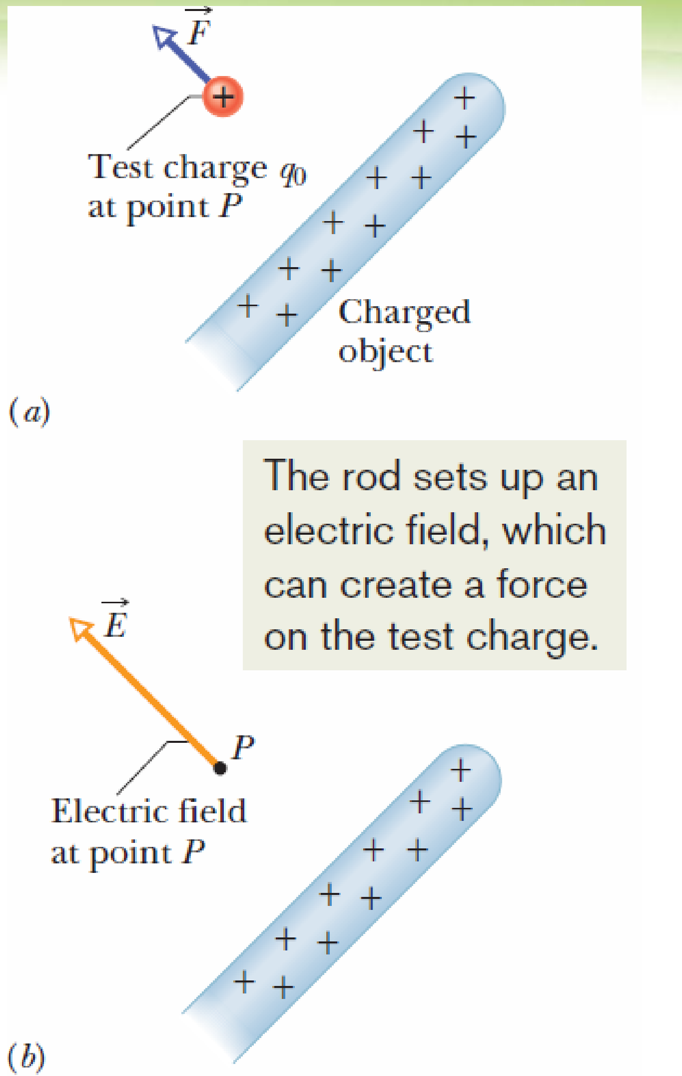


Fig. 22-1 (*a*) A positive test charge q_0 placed at point P near a charged object. An electrostatic force $\vec{F}$ acts on the test charge. (*b*) The electric field $\vec{E}$ at point P produced by the charged object.

The electric field is a *vector field*.

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{(electric field).}$$

The SI unit for the electric field is the newton per coulomb (N/C).

Table 22-1

Some Electric Fields

Field Location or Situation	Value (N/C)
At the surface of a uranium nucleus	3×10^{21}
Within a hydrogen atom, at a radius of 5.29×10^{-11} m	5×10^{11}
Electric breakdown occurs in air	3×10^{6}
Near the charged drum of a photocopier	10^{5}
Near a charged comb	10^{3}
In the lower atmosphere	10^{2}
Inside the copper wire of household circuits	10^{-2}

22.3 Electric Field Lines:

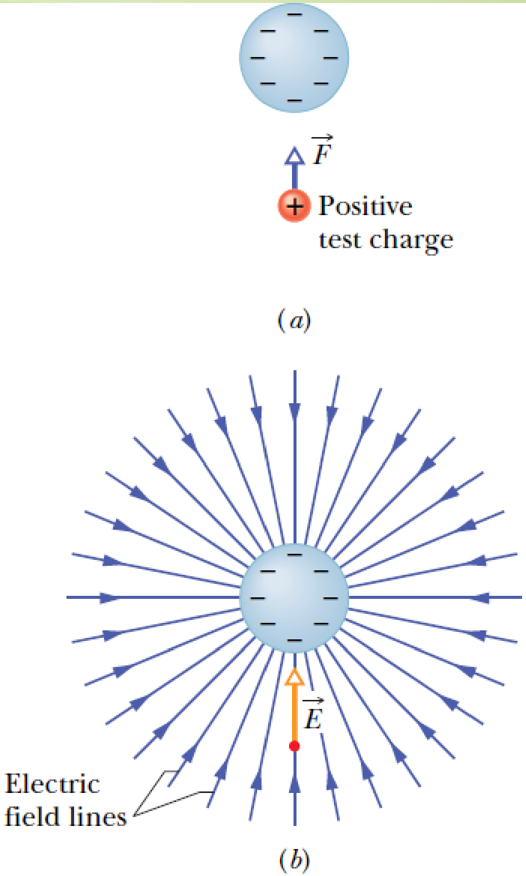


Fig. 22-2 (*a*) The electrostatic force $\vec{F}$ acting on a positive test charge near a sphere of uniform negative charge. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend *toward* the negatively charged sphere. (They originate on distant positive charges.)

- Electric field lines are imaginary lines which extend away from positive charge (where they originate) and toward negative charge (where they terminate).
- At any point, the direction of the tangent to a curved field line gives the direction of the electric field at that point.
- The field lines are drawn so that the number of lines per unit area, measured in a plane that is perpendicular to the lines, is proportional to the magnitude of ***E***. Thus, ***E*** is large where field lines are close together and small where they are far apart.

22.3 Electric Field Lines:

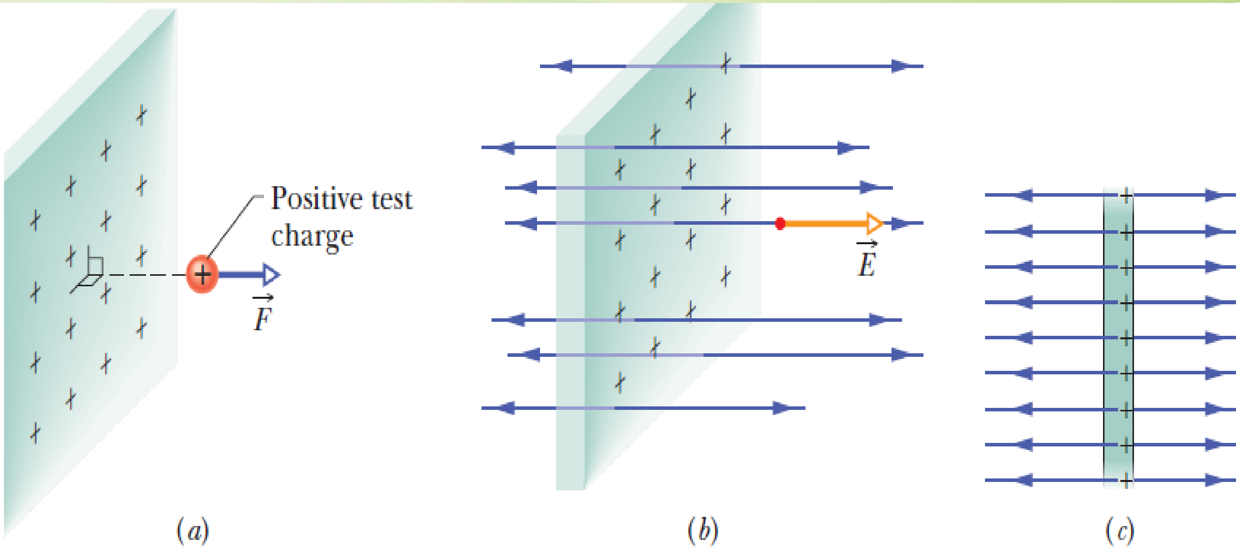
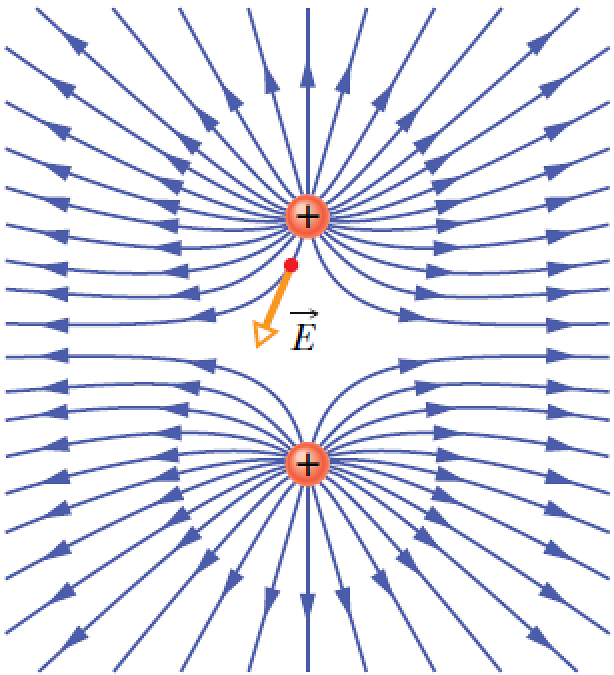


Fig. 22-3 (*a*) The electrostatic force $\vec{F}$ on a positive test charge near a very large, nonconducting sheet with uniformly distributed positive charge on one side. (*b*) The electric field vector $\vec{E}$ at the location of the test charge, and the electric field lines in the space near the sheet. The field lines extend *away from* the positively charged sheet. (*c*) Side view of (*b*).

Fig. 22-4 Field lines for two equal positive point charges. The charges repel each other. (The lines terminate on distant negative charges.) To "see" the actual three-dimensional pattern of field lines, mentally rotate the pattern shown here about an axis passing through both charges in the plane of the page. The three-dimensional pattern and the electric field it represents are said to have *rotational symmetry* about that axis. The electric field vector at one point is shown; note that it is tangent to the field line through that point.



22.4 The Electric Field due to a Point:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{\mathrm{r}} \qquad \text{(point charge).}$$

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

$$\begin{aligned}\vec{E} = \frac{\vec{F}_0}{q_0} &= \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} \\ &= \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.\end{aligned}$$

Example, The net electric field due to three charges:

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

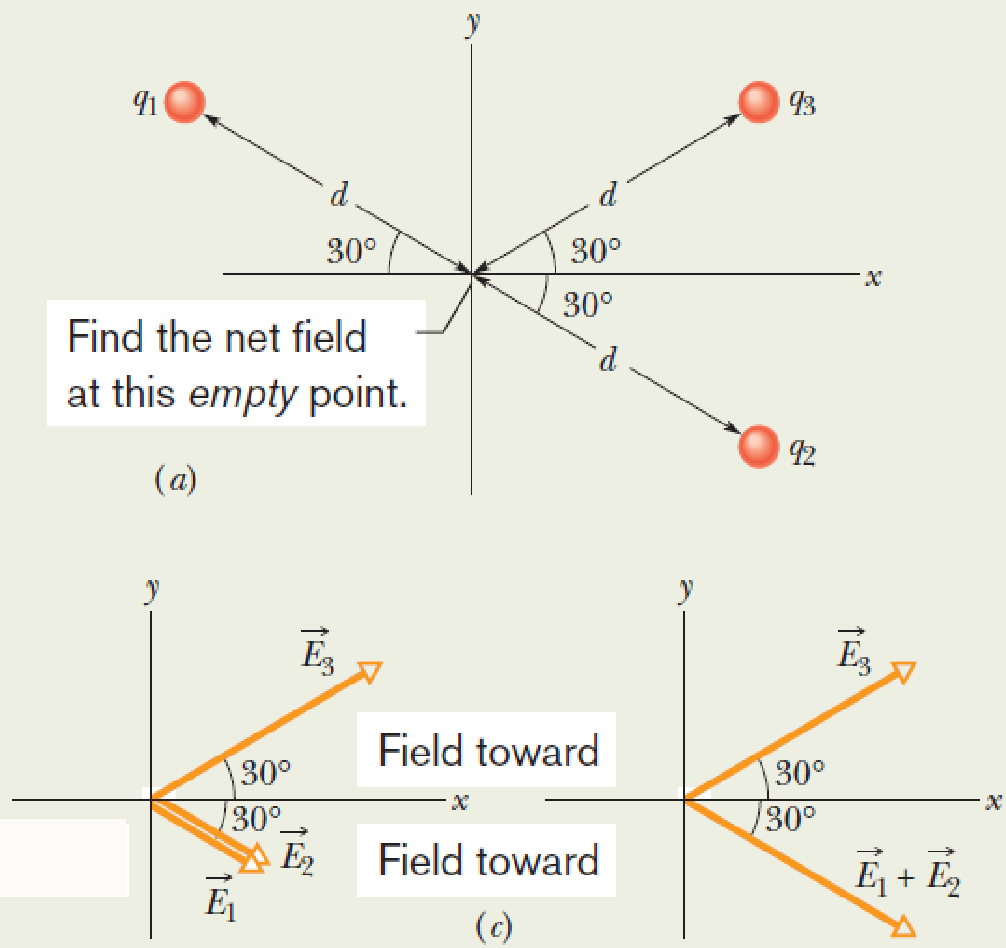


Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

From the symmetry of Fig. 22-7c, we realize that the equal y components of our two vectors cancel and the equal x components add.

Thus, the net electric field at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3\cos 30^\circ$$

$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}.$$

22.5 The Electric Field due to an Electric Dipole:

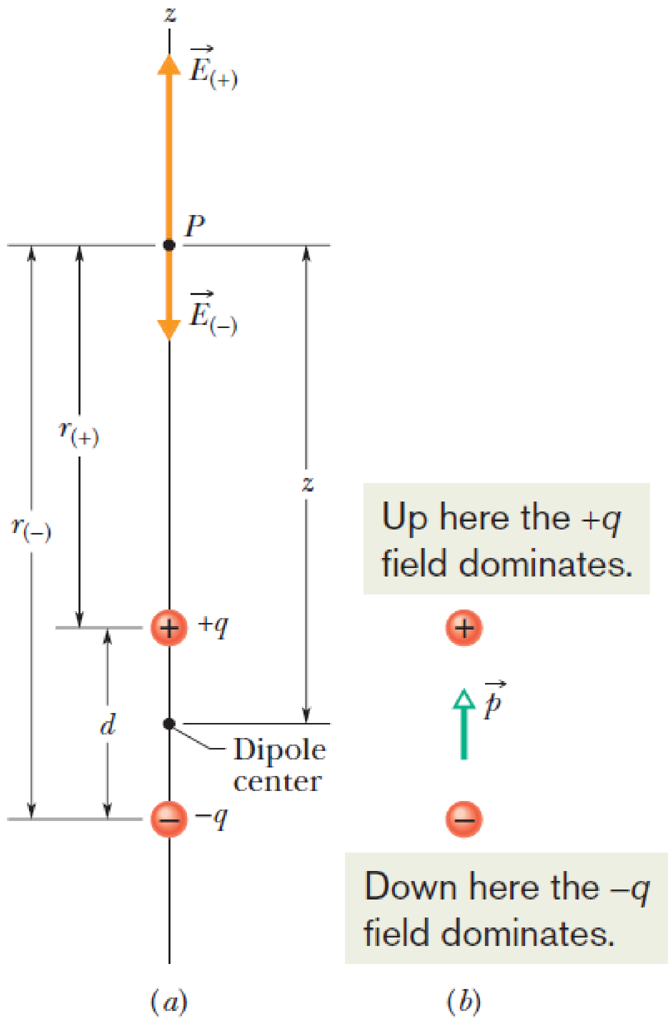
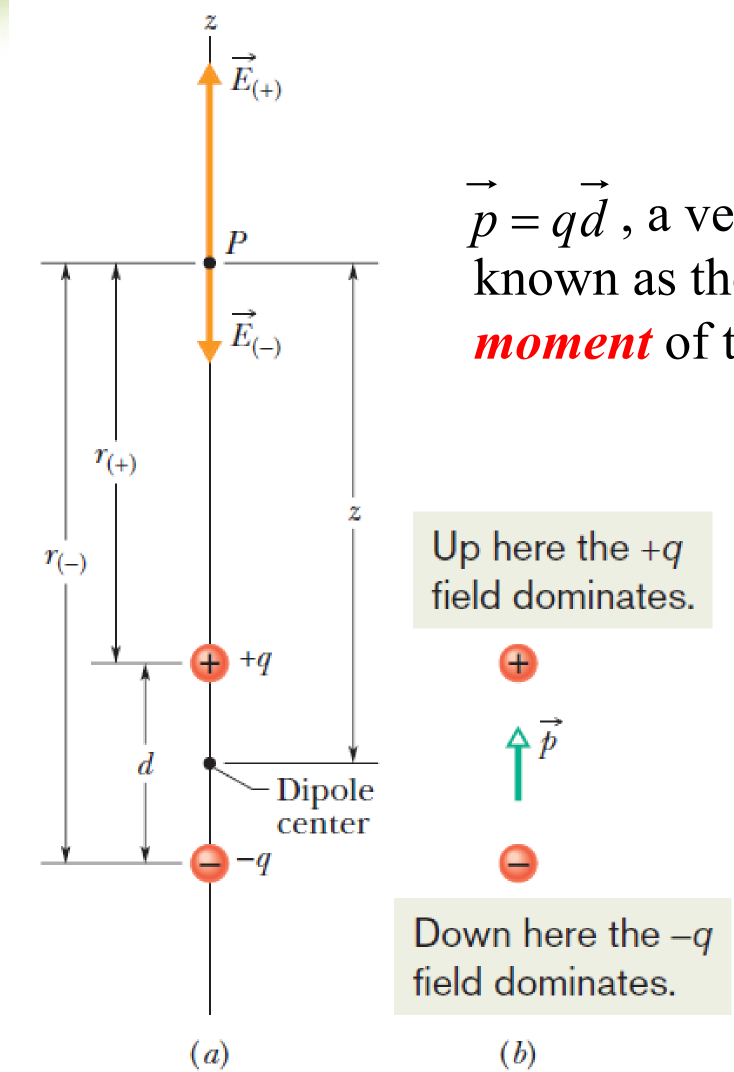


Fig. 22-8 (*a*) An electric dipole. The electric field vectors $\vec{E}_{(+)}$ and $\vec{E}_{(-)}$ at point P on the dipole axis result from the dipole's two charges. Point P is at distances $r_{(+)}$ and $r_{(-)}$ from the individual charges that make up the dipole. (*b*) The dipole moment $\vec{p}$ of the dipole points from the negative charge to the positive charge.

22.5 The Electric Field due to an Electric Dipole:



$\vec{p} = q\vec{d}$, a vector quantity known as the ***electric dipole moment*** of the dipole

$$E = E_{(+)} - E_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2}.$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2}\right).$$

See Supplement

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2} = \frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left(1 - \left(\frac{d}{2z}\right)^2\right)^2}.$$

$d/2z \ll 1$ $\qquad E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \quad \text{(electric dipole).}$$

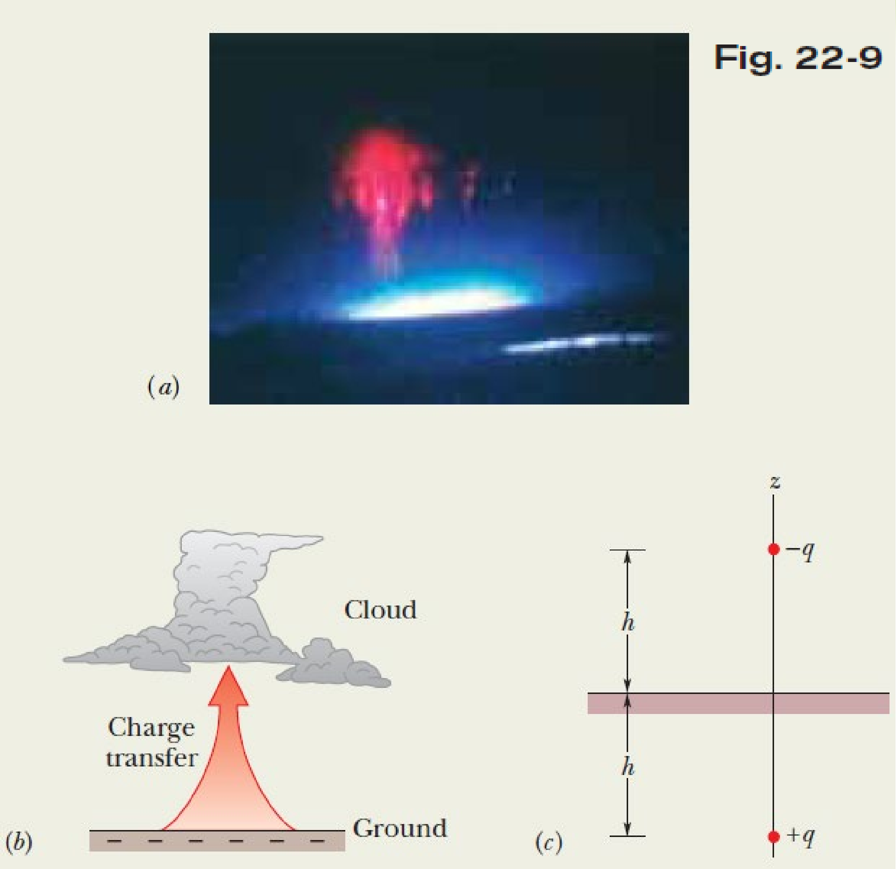
Supplement

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1-\frac{d}{2z}\right)^2}-\frac{1}{\left(1+\frac{d}{2z}\right)^2}\right).$$

$$E = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{\left[1+\frac{d}{2z}\right]^2-\left[1-\frac{d}{2z}\right]^2}{\left[1-\frac{d}{2z}\right]^2\left[1+\frac{d}{2z}\right]^2}\right)=\frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{\frac{2d}{2z}+\frac{2d}{2z}}{\left[\left(1-\frac{d}{2z}\right)\left(1+\frac{d}{2z}\right)\right]^2}\right)$$

$$=\frac{q}{4\pi\varepsilon_0 z^2}\frac{2d/z}{\left[\left(1-\left(\frac{d}{2z}\right)^2\right)\right]^2}=\frac{q}{2\pi\varepsilon_0 z^3}\frac{d}{\left[\left(1-\left(\frac{d}{2z}\right)^2\right)\right]^2}$$

Example, Electric Dipole and Atmospheric Sprites:



Sprites (Fig. 22-9*a)* are huge flashes that occur far above a large thunderstorm. They are still not well understood but are believed to be produced when especially powerful lightning occurs between the ground and storm clouds, particularly when the lightning transfers a huge amount of negative charge -*q* from the ground to the base of the clouds (Fig. 22-9*b)*.

We can model the electric field due to the charges in the clouds and the ground by assuming a vertical electric dipole that has charge -*q* at cloud height *h* and charge +*q* at below-ground depth *h (Fig.* 22-9*c).* If *q =200 C* and *h =6.0* km, what is the magnitude of the dipole's electric field at altitude z_1 *=30 km* somewhat above the clouds and altitude z_2 *=60* km somewhat above the stratosphere?

$$E = \frac{1}{2\pi\varepsilon_0}\frac{q(2h)}{z^3},$$

where $2h$ is the separation between $-q$ and $+q$ in Fig. 22-9*c*. For the electric field at altitude $z_1 = 30$ km, we find

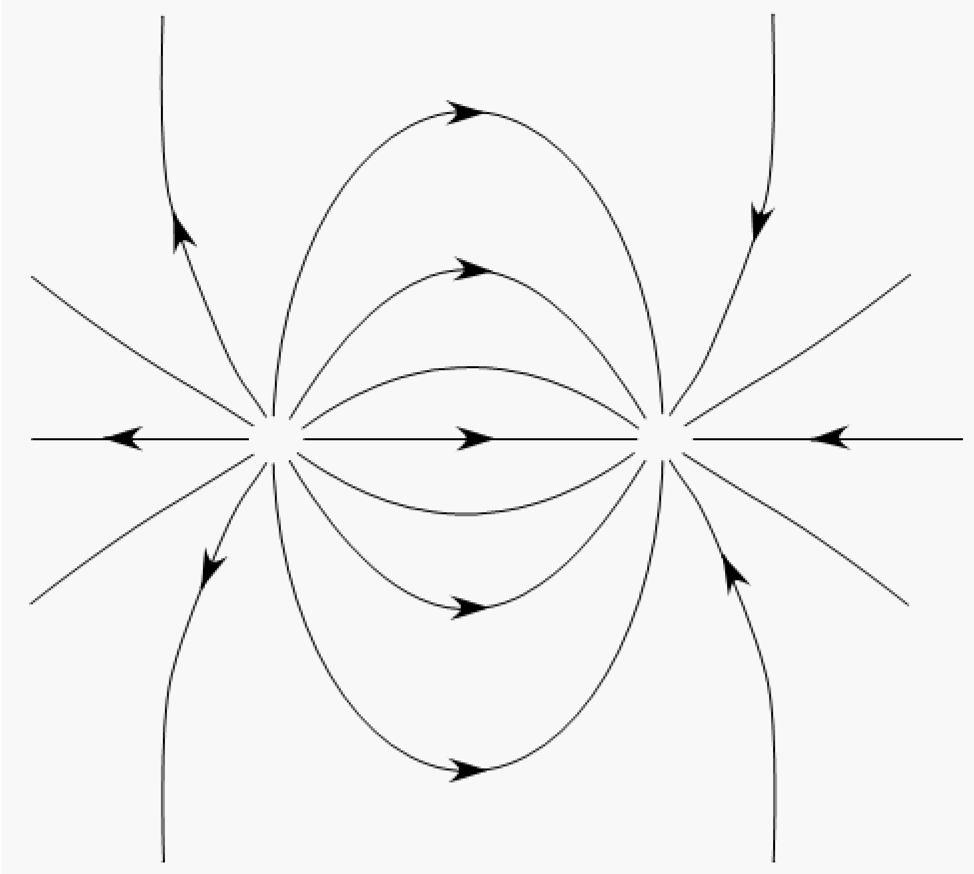
$$E = \frac{1}{2\pi\varepsilon_0}\frac{(200\text{ C})(2)(6.0\times10^3\text{ m})}{(30\times10^3\text{ m})^3}$$

$$= 1.6\times10^3\text{ N/C.} \qquad \text{(Answer)}$$

Similarly, for altitude $z_2 = 60$ km, we find

$$E = 2.0\times10^2\text{ N/C.} \qquad \text{(Answer)}$$

22.4.2. Consider the field lines shown in the drawing. Which one of the following statements concerning this situation is true?



a) These field lines are those for a positively charged particle.

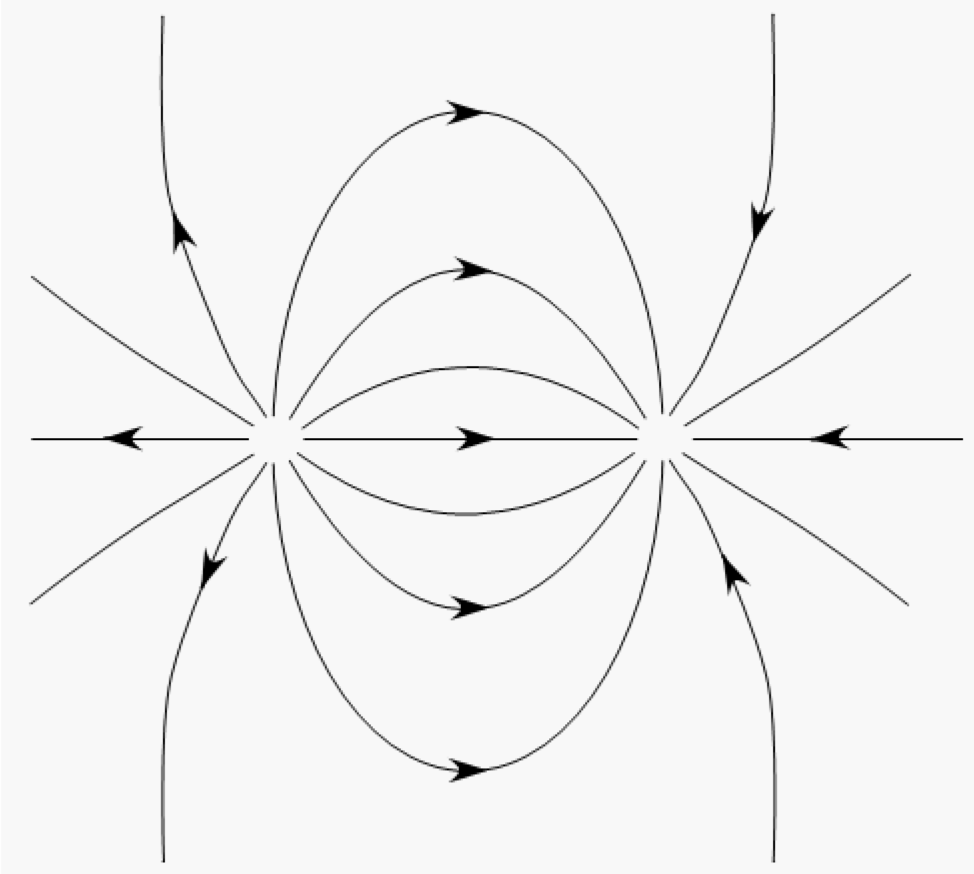
b) These field lines are those for a negatively charged particle.

c) These field lines are those for a positively charged particle and a negatively charged particle.

d) These field lines are those for two positively charged particles.

e) These field lines are those for two negatively charged particles.

22.4.2. Consider the field lines shown in the drawing. Which one of the following statements concerning this situation is true?



a) These field lines are those for a positively charged particle.

b) These field lines are those for a negatively charged particle.

c) These field lines are those for a positively charged particle and a negatively charged particle.

d) These field lines are those for two positively charged particles.

e) These field lines are those for two negatively charged particles.

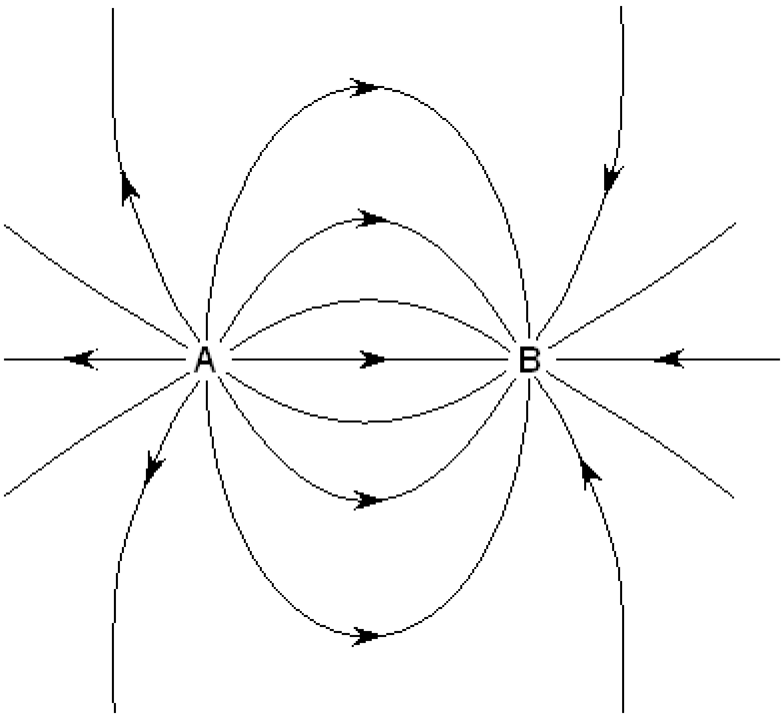
22.4.3. Consider the field lines shown in the drawing. Which one of the following statements concerning this situation is true?

a) A is a positively charged particle and B is negatively charged.

b) B is a positively charged particle and A is negatively charged.

c) A and B are both positively charged.

d) A and B are both negatively charged.



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a) A is a positively charged particle and B is negatively charged.

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c) A and B are both positively charged.

d) A and B are both negatively charged.

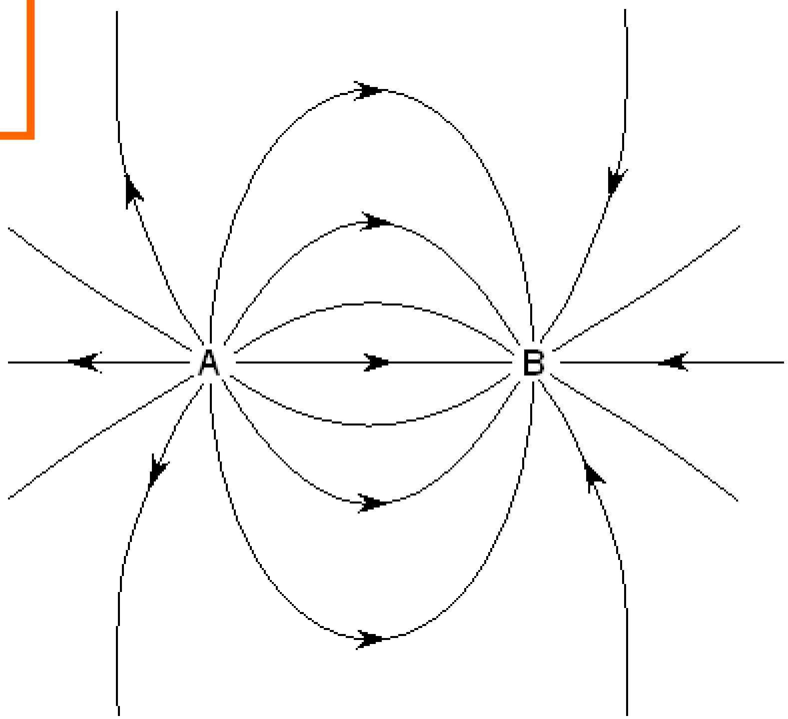
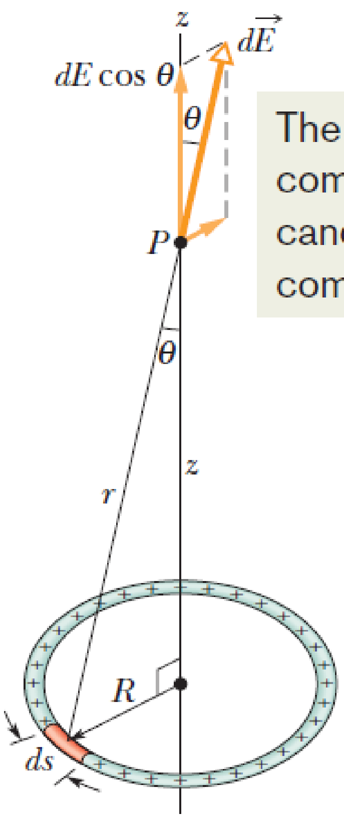


Table 22-2

Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

22.6 The Electric Field due to a Line Charge:



The perpendicular components just cancel but the parallel components add.

Fig. 22-10 A ring of uniform positive charge. A differential element of charge occupies a length ds (greatly exaggerated for clarity). This element sets up an electric field $d\vec{E}$ at point P. The component of $d\vec{E}$ along the central axis of the ring is $dE\cos\theta$.

$$dq = \lambda\, ds.$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}.$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}.$$

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\, ds.$$

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds$$

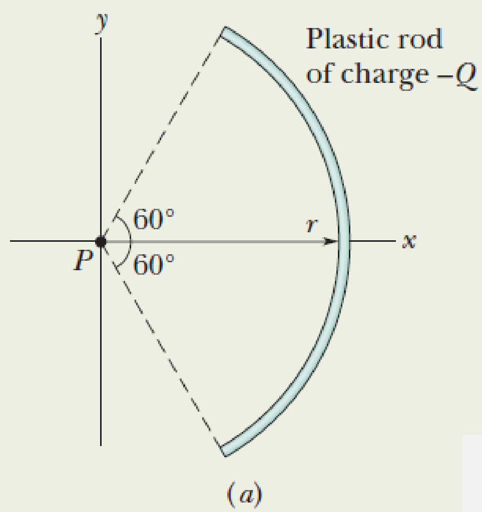
$$= \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}.$$

Example, Electric Field of a Charged Circular Rod

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}$$

Figure 22-11a shows a plastic rod having a uniformly distributed charge $-Q$. The rod has been bent in a 120° circular arc of radius r. We place coordinate axes such that the axis of symmetry of the rod lies along the x axis and the origin is at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ due to the rod at point P?

This negatively charged rod is obviously not a particle.



These x components add. Our job is to add all such components.

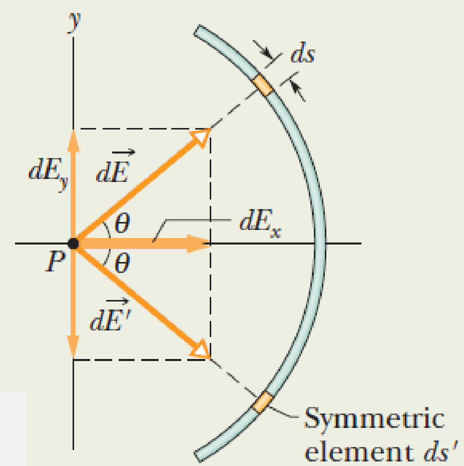


Fig. 22-11 (a) A plastic rod of charge Q is a circular section of radius r and central angle 120°; point P is the center of curvature of the rod. (b) The field components from symmetric elements from the rod.

$$\lambda = \frac{\text{charge}}{\text{length}} = \frac{Q}{2\pi r/3} = \frac{0.477Q}{r}.$$

$$dq = \lambda\, ds.$$

Our element has a symmetrically located (mirror image) element ds in the bottom half of the rod.

If we resolve the electric field vectors of ds and ds' into x and y components as shown in we see that their y components cancel (because they have equal magnitudes and are in opposite directions). We also see that their x components have equal magnitudes and are in the same direction.

$$E = \int dE_x = \int_{-60^\circ}^{60^\circ} \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r^2}\cos\theta\, r\, d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}\int_{-60^\circ}^{60^\circ}\cos\theta\, d\theta = \frac{\lambda}{4\pi\varepsilon_0 r}\Big[\sin\theta\Big]_{-60^\circ}^{60^\circ}$$

$$= \frac{\lambda}{4\pi\varepsilon_0 r}[\sin 60^\circ - \sin(-60^\circ)]$$

$$= \frac{1.73\lambda}{4\pi\varepsilon_0 r}. = \frac{0.83Q}{4\pi\varepsilon_0 r^2}.$$

22.8: A Point Charge in an Electric Field

$$\boxed{\vec{F} = q\vec{E},}$$

When a charged particle, of charge *q,* is in an electric field, ***E*,** set up by other stationary or slowly moving charges, an electrostatic force, ***F*,** acts on the charged particle as given by the above equation.

Measuring the Elementary Charge

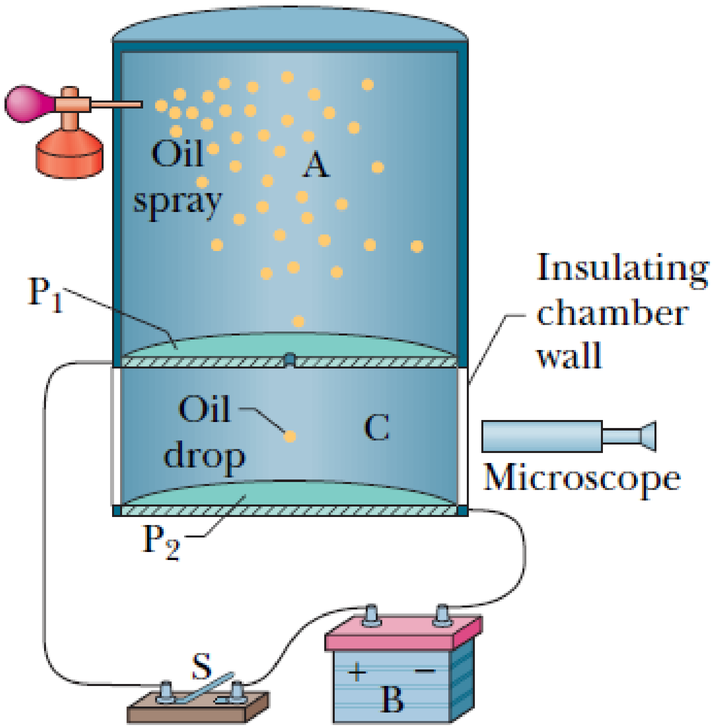


Fig. 22-14 The Millikan oil-drop apparatus for measuring the elementary charge e. When a charged oil drop drifted into chamber C through the hole in plate P_1, its motion could be controlled by closing and opening switch S and thereby setting up or eliminating an electric field in chamber C. The microscope was used to view the drop, to permit timing of its motion.

Ink-Jet Printing

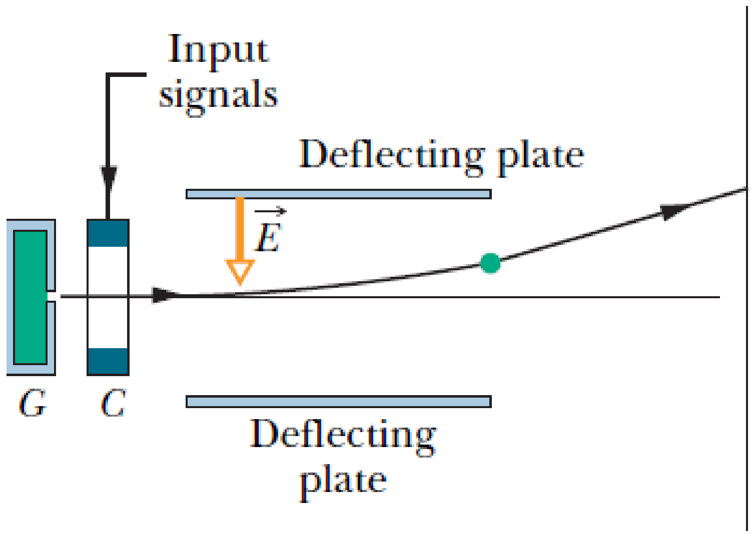


Fig. 22-15 Ink-jet printer. Drops shot from generator G receive a charge in charging unit C. An input signal from a computer controls the charge and thus the effect of field $\vec{E}$ where the drop lands on the paper.

Example, Motion of a Charged Particle in an Electric Field

Figure 22-17 shows the deflecting plates of an ink-jet printer, with superimposed coordinate axes. An ink drop with a mass m of 1.3×10^{-10} kg and a negative charge of magnitude $Q = 1.5 \times 10^{-13}$ C enters the region between the plates, initially moving along the x axis with speed $v_x = 18$ m/s. The length L of each plate is 1.6 cm. The plates are charged and thus produce an electric field at all points between them. Assume that field $\vec{E}$ is downward directed, is uniform, and has a magnitude of 1.4×10^6 N/C. What is the vertical deflection of the drop at the far edge of the plates? (The gravitational force on the drop is small relative to the electrostatic force acting on the drop and can be neglected.)

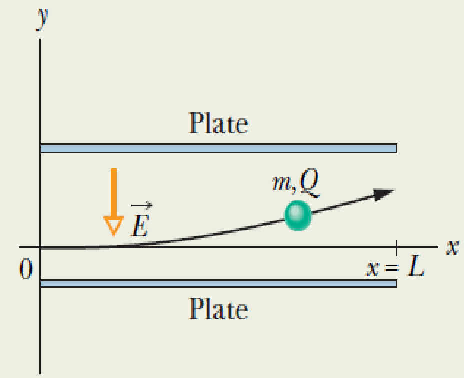


Fig. 22-17 An ink drop of mass m and charge magnitude Q is deflected in the electric field of an ink-jet printer.

KEY IDEA

The drop is negatively charged and the electric field is directed *downward*. From Eq. 22-28, a constant electrostatic force of magnitude QE acts *upward* on the charged drop. Thus, as the drop travels parallel to the x axis at constant speed v_x, it accelerates upward with some constant acceleration a_y.

Calculations: Applying Newton's second law ($F = ma$) for components along the y axis, we find that

$$a_y = \frac{F}{m} = \frac{QE}{m}. \qquad (22\text{-}30)$$

Let t represent the time required for the drop to pass through the region between the plates. During t the vertical and horizontal displacements of the drop are

$$y = \tfrac{1}{2}a_y t^2 \quad \text{and} \quad L = v_x t, \qquad (22\text{-}31)$$

respectively. Eliminating t between these two equations and substituting Eq. 22-30 for a_y, we find

$$y = \frac{QEL^2}{2mv_x^2}$$

$$= \frac{(1.5 \times 10^{-13}\ \text{C})(1.4 \times 10^6\ \text{N/C})(1.6 \times 10^{-2}\ \text{m})^2}{(2)(1.3 \times 10^{-10}\ \text{kg})(18\ \text{m/s})^2}$$

$$= 6.4 \times 10^{-4}\ \text{m}$$

$$= 0.64\ \text{mm}. \qquad \text{(Answer)}$$

22.9: A Dipole in an Electric Field

Although the net force on the dipole from the field is zero, and the center of mass of the dipole does not move, the forces on the charged ends do produce a net torque $\boldsymbol{\tau}$ on the dipole about its center of mass.

The center of mass lies on the line connecting the charged ends, at some distance x from one end and a distance d -x from the other end.

The net torque is:

$$\tau = Fx \sin\theta + F(d - x)\sin\theta = Fd\sin\theta.$$

$$= pE\sin\theta.$$

$$\vec{\tau} = \vec{p} \times \vec{E} \quad \text{(torque on a dipole).}$$

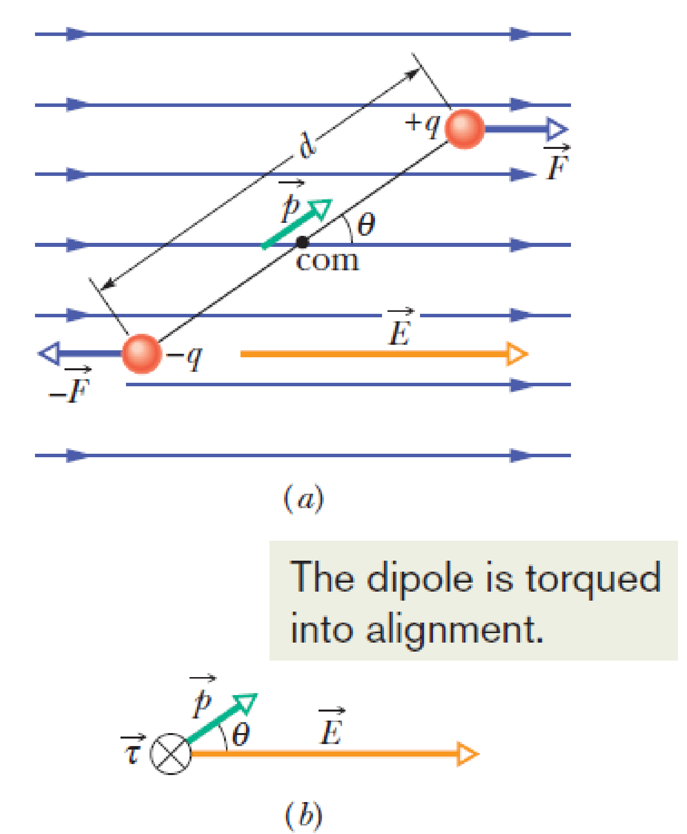


Fig. 22-19 (*a*) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (*b*) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

22.9: A Dipole in an Electric Field: Potential Energy

Potential energy can be associated with the orientation of an electric dipole in an electric field.

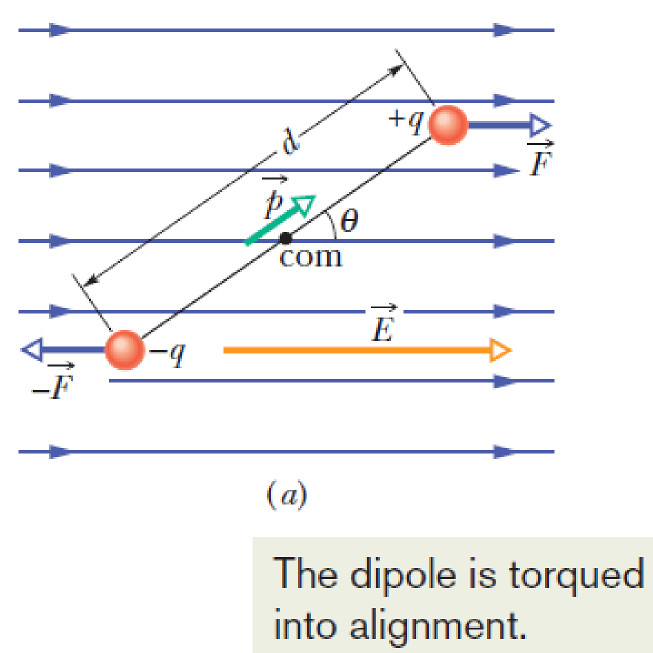
The dipole has its least potential energy when it is in its equilibrium orientation, which is when its moment ***p*** is lined up with the field ***E***.

The expression for the potential energy of an electric dipole in an external electric field is simplest if we choose the potential energy to be zero when the angle θ (Fig.22-19) is 90°.

The potential energy U of the dipole at any other value of θ can be found by calculating the work W done by the field on the dipole when the dipole is rotated to that value of θ from 90°.

$$U = -W = -\int_{90^\circ}^{\theta} \tau \, d\theta = \int_{90^\circ}^{\theta} pE \sin\theta \, d\theta = -pE\cos\theta.$$

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}$$



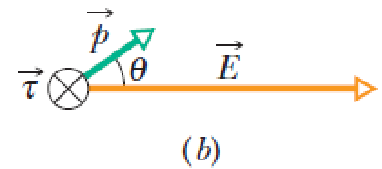


Fig. 22-19 (*a*) An electric dipole in a uniform external electric field $\vec{E}$. Two centers of equal but opposite charge are separated by distance d. The line between them represents their rigid connection. (*b*) Field $\vec{E}$ causes a torque $\vec{\tau}$ on the dipole. The direction of $\vec{\tau}$ is into the page, as represented by the symbol ⊗.

Example, Torque, Energy of an Electric Dipole in an Electric Field

A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude 6.2×10^{-30} C·m.

(a) How far apart are the molecule's centers of positive and negative charge?

KEY IDEA

A molecule's dipole moment depends on the magnitude q of the molecule's positive or negative charge and the charge separation d.

Calculations: There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which d is the separation we are seeking and e is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \text{C}\cdot\text{m}}{(10)(1.60 \times 10^{-19}\ \text{C})}$$
$$= 3.9 \times 10^{-12}\ \text{m} = 3.9\ \text{pm}. \qquad \text{(Answer)}$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

Microwave cooking, solubility in water

(b) If the molecule is placed in an electric field of 1.5×10^4 N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

KEY IDEA

The torque on a dipole is maximum when the angle θ between $\vec{p}$ and $\vec{E}$ is 90°.

Calculation: Substituting $\theta = 90°$ in Eq. 22-33 yields

$$\tau = pE \sin\theta$$
$$= (6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})(\sin 90°)$$
$$= 9.3 \times 10^{-26}\ \text{N}\cdot\text{m}. \qquad \text{(Answer)}$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

KEY IDEA

The work done by an external agent (by means of a torque applied to the molecule) is equal to the change in the molecule's potential energy due to the change in orientation.

Calculation: From Eq. 22-40, we find

$$W_a = U_{180°} - U_0$$
$$= (-pE\cos 180°) - (-pE\cos 0)$$
$$= 2pE = (2)(6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})$$
$$= 1.9 \times 10^{-25}\ \text{J}. \qquad \text{(Answer)}$$

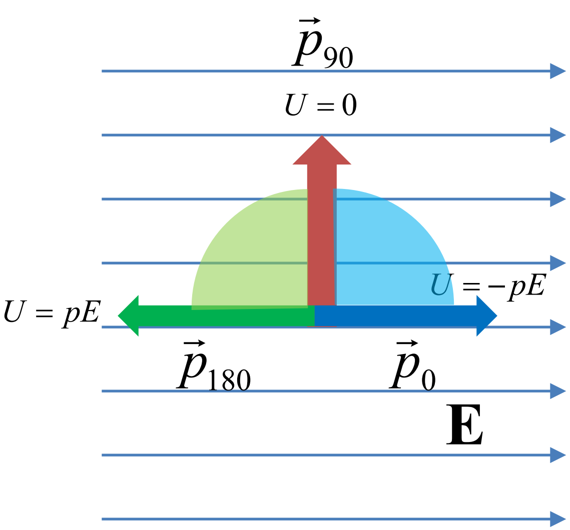
Supplement to Torque and Energy of an Dipole

From Figure 22-19, taking $\theta = 90°$ as the reference point where $U = 0$ we arrived to the following expression for the potential energy of a dipole.

$$U = -W = -\int_{90°}^{\theta} \tau \, d\theta = \int_{90°}^{\theta} pE \sin\theta \, d\theta = -pE\cos\theta.$$

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}$$

Consider the following situation where the three dipoles, indicated by their dipole moment $\vec{p}$, are in a uniform **E** field pointing to the right.



By definition, *U=0* when the dipole is perpendicular to **E,** *U=pE* when **p** is in opposite direction of **E**, *U=-pE* when **p** is in the direction of **E.**
The sign of *U* provides the information on who is doing the work, that is how much work does one need to apply to the system to move the dipole from its reference position (the red arrow) to a given θ.
If we move the dipole from $\theta = 90°$ into the green shaded region, we found that *U > 0.* In this region, the electric field is pushing the dipole back in the clockwise direction, so have to do work to counter this force, thus *U > 0.*
If we move the dipole from $\theta = 90°$ to the blue shaded region, we found that *U < 0.* In this situation, the electric field is helping us to move the dipole in the clockwise direction, so the system is doing the work instead of us.