

## CITY UNIVERSITY OF HONG KONG

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Course code & title : EE1001 Fundamental of Digital Techniques

Session : Semester B 2020/21

Time allowed : 2 hours OPEN BOOK EXAMINATION  
(Online exam have additional 15 minutes for uploading)

- This paper consists of 2 parts, PART A and PART B.
- PART A consist of 5 questions and carries 50 marks. PART B consist of 9 questions and carries 50 marks.
- You MUST separate (**Do NOT mix them**) “PART A answer script” with “PART B answer script”
- Write your SID and full name on each page top corner for both PART A and PART B
- Answer PART A and Part B on separate answering books.

### Online examination

- Take photo of your answering scripts.
- Upload “PART A answer script” onto Canvas folder “**EXAM PART A RETURN**”
- Answer PART B in another set of answer script and upload “PART B answer script” onto Canvas folder “**EXAM PART B RETURN**”
- This paper consists of SEVEN pages (including 2 cover pages).

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### Instructions:

Please make sure you follow all instructions from the University, ARRO, and EE. Please note the following:

1. This paper consists of 2 PARTS **15** questions. The questions are ALL compulsory. Make sure that you attempt all of them. The total score is 100 marks.
2. For online examination, you MUST answer and upload **PART A** and **PART B separately**.

3. This is an **open-book** exam. Students can read the lecture notes but **cannot search** materials online.
4. For online examination, you are responsible for receiving the questions on Canvas, hand-write all answers on blank answer sheets, compile or take photo of the answers into a single PDF file. Upload the files before the end of the examination.
5. Stay on ZOOM until the examiner allows you to leave.

Answering this exam paper implies your acknowledgment of the Pledge for following the Rules on Academic Honesty:

“I pledge that the answers in this examination are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

1. I will not plagiarize (copy without citation) from any source;
2. I will not communicate or attempt to communicate with any other person during the examination; neither will I give or attempt to give assistance to another student taking the examination; and
3. I will use only approved devices (e.g., calculators) and/or approved device models.
4. I understand that any act of academic dishonesty can lead to disciplinary action.”

On the first page of your answer sheets, copy the following sentence and sign it: *I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties.*

(Signature) \_\_\_\_\_ (Date) \_\_\_\_\_

#### Contact Information

- Should you have any technical problem during the online exam, contact your course lecturers or invigilator via Zoom private chat.
- If you are not able to contact course leader/invigilator, you can reach the department via:
  - (a) Departmental hotline at (+852) 3442-7740
  - (b) Department Whatsapp phone: 9269-4066
  - (c) Department WeChat ID: wxid\_lly7yf5fzozj722 or scan the following QR Code



**PART A (50 marks)**

5 Questions

ANSWER ALL QUESTIONS

Online examination: upload PART A Answers to Canvas “**EXAM PART A RETURN**”

**QA1. (10 marks)**

Below  $A$  and  $B$  are Hexadecimal numbers. Perform  $A - B$  using binary subtraction and show the steps.

$$A = 6D9, B = 8AC.$$

- Write down the 12 bits representation for the 2 Hexadecimal numbers,  $A$  and  $B$ .  
(4%)
- Find the 2's complement of  $A - B$  in a 12 bits representation.  
(4%)
- Find the 2's complement of  $A - B$  in a 16 bits representation.  
(2%)

**QA2. (10 marks)**

$A$  and  $B$  are two binary numbers written in the IEEE 754 32 bits floating point format. Find  $A - B$  in the format of IEEE 754 32 bits representation. Perform your operation in binary string and **show your steps with comments on your steps** for clarity.

$A$ : 0 1000 0000 001 0111 0110 0110 1110 0110

$B$ : 0 0111 1101 100 0001 0000 1000 0000 0000

The IEEE 754 32 bits format can be referred to the below.

1 bit	8 bits	23 bits
Sign bit	Exponent bits	Mantissa

**QA3. (10 marks)**

Suppose  $H = \{\text{parrot}, \text{cat}, \text{dog}, \text{rabbit}, \text{mouse}\}$ ,  $F = \{\text{dog}, \text{cow}, \text{duck}, \text{pig}, \text{rabbit}\}$ ,  $W = \{\text{duck}, \text{rabbit}, \text{deer}, \text{frog}, \text{mouse}\}$ . Draw a Venn diagram to illustrate  $(H \cap F)^c \cap W$ .

**QA4. (10 marks)**

Use contradiction proof to prove that  $(A - B) \cap (B - A) = \emptyset$ , where  $A, B$  are sets, and  $\emptyset$  is the empty set. Advise to add statement to elaborate each step of the proof.

**QA5. (10%)**

- a. Which of the following functions is NOT injective?
- $f: R \rightarrow R, f(x) = x^3 + 4$
  - $f: N \rightarrow N, f(x) = x^3 + 4$
  - $f: R \rightarrow R, f(x) = x^2 + 4$
  - $f: N \rightarrow N, f(x) = x^2 + 4$
- b. Which of the following functions is NOT injective?
- $f: [0^\circ, 180^\circ] \rightarrow R, f(x) = \cos(x)$
  - $f: [0^\circ, 180^\circ] \rightarrow [-1, 1], f(x) = \cos(x)$
  - $f: [0^\circ, 360^\circ] \rightarrow R, f(x) = \cos(x)$
  - $f: [0^\circ, 90^\circ] \rightarrow R, f(x) = \tan(x)$
- c. A function  $f: A \rightarrow B$  is \_\_\_\_\_ if the image of  $f$  equals its range.
- Injective
  - Surjective
  - Invertible
  - NOT surjective
- d. If a function is both surjective and injective, then the function is known as
- Composite
  - Universal
  - Bijjective
  - Associative
  - None of the above
- e. Which of the following is true?
- The function  $g: R \rightarrow R, g(x) = x^2$  is surjective.
  - The function  $g: R \rightarrow R, g(x) = x^2$  is NOT surjective.
  - The function  $g: R \rightarrow R, g(x) = x^2$  is NOT injective.
  - Both i and ii.
  - Both ii and iii.

**PART B (50 marks)**

**9 Questions**

**ANSWER ALL QUESTIONS**

Online examination: upload PART B Answers to Canvas “**EXAM PART B RETURN**”

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**QB1. (6 marks)**

Multiple choice question. Select one answer for each question.

Q1(i). Find the number of words, with or without meaning, that can be formed with the letters of the following word “HONGKONG”. (1 mark)

- A. 5020      B. 5030      C. 5040      D. 5050

Q1(ii). In a class, there are 16 students. Each student needs to shake hands with one another. How many handshakes are in the class? (1 mark)

- A. 120      B. 150      C. 180      D. 60

Q1(iii). Assume the set  $I = \{1, 2, 3, 4, 5\}$ . **A** and **B** are two non-empty subsets of **I**. It is required that the minimal element in **B** should be larger than the maximal element in **A**. Find the number of ways to form **A** and **B**. (2 marks)

- A. 50      B. 49      C. 48      D. 47

Q1(iv). You need to randomly pick several distinct positive integers chosen between 1 and 1000, inclusively. To ensure that at least two integers have a difference that is a multiple of 7, how many integers are needed? (2 marks)

- A. 127      B. 45      C. 19      D. 8

**QB2. (5 marks)**

Determine the following statements whether they are true or false. (5 marks)

- (i) “Can you tell me if this statement is true?” is a proposition.
- (ii) “All human beings are animals.” is a valid and sound argument.
- (iii) Invalid argument never goes from true premises to a true conclusion.
- (iv) If an argument is sound, its conclusion must be true.
- (v)  $\sim[\forall x \in C \forall y \in R, H(x) \rightarrow K(y)] \equiv \exists y \in R \exists x \in C, \sim[H(x) \rightarrow K(y)]$

**QB3. (8 marks)**

Given the following statements:

- 1) If Alice does not study computer science or engineering, then she would not be an engineer.
- 2) If Alice wants to get the scholarship, then she must study engineering.
- 3) If Ben wants to get the scholarship, then he must study psychology.
- 4) Alice does not get the scholarship.
- 5) Either Alice or Ben gets the scholarship.

- 6) Alice would be an engineer.  
 7) If Alice studies computer science, then Ben would not study psychology.

Use inference rules and logical equivalence relation to determine which course(s) Alice studies. Justify the reason for each step. (8 marks)

**QB4. (6 marks)**

- (i) Without using truth table, show that

$$\sim[(a \vee \sim b) \wedge (\sim c \vee a)] \vee (d \wedge \sim a) \equiv \sim a \wedge (b \vee c \vee d)$$

Justify the reason for each step.

(2 marks)

- (ii) Given  $d = \sim b \wedge \sim c$ , determine whether the following statement is a tautology:

$$\sim\{[(a \vee \sim b) \wedge (\sim c \vee a)] \vee (d \wedge \sim a)\} \rightarrow (\sim a \vee \sim b \vee \sim c)$$

Justify the reason for each step.

(4 marks)

**QB5. (6 marks)**

Given  $a_1 = 5$ , the sum  $S_m$  of the first  $m$  terms of an arithmetic series is 63. The sum of the next 7 terms (i.e., the sum of  $(m+1)^{\text{th}}$  term to  $(m+7)^{\text{th}}$  term) is 77. Determine the values of  $m$  and the common difference  $d$ . (6 marks)

**QB6. (4 marks)**

Fill in the blanks.

Q6(i)  $A = \{0, 1, 2\}$ ,  $B = \{x - y \mid x \in A, y \in A\}$ ,  $B = \underline{\hspace{2cm}}$

(1 mark)

Q6(ii)  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{1, 2, 3\}$ , the number of subsets of  $A \cap B$  is  $\underline{\hspace{2cm}}$

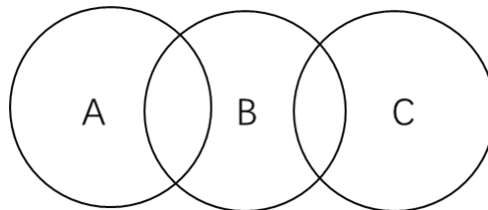
(1 mark)

Q6(iii)  $A = \{x \mid 0.5 < x - 1 < 4\}$ ,  $B = \{x \mid 0.3 < x^2 < 10\}$ ,  $C = \{x \mid x \text{ is an integer}\}$ .  $A \cap C - B \cap C = \underline{\hspace{2cm}}$

(1 mark)

Q6(iv) Identify  $A \cup B \cup C - B$  in the Venn figure.

(1 mark)



**QB7. (5 marks)**

Given:

$$(1+mx)^n = a_0 + a_1x + a_2x^2 + \dots + a_nx^n \quad (m > 0, n > 0)$$

- (i) Find the value of  $a_0$ . (1 mark)
- (ii) If  $m=1$ ,  $a_3=7a_1$ . Find the value of  $n$ . (2 mark)
- (iii) If  $n=7$ ,  $a_2, a_3, a_4$  becomes an arithmetic sequence. Find the value of  $m$ . (2 mark)

**QB8. (6 marks)**

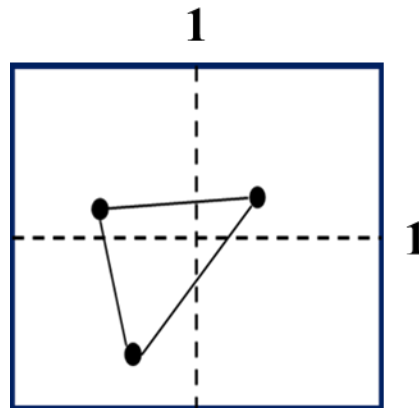
100 students stand in a line facing the teacher. In the beginning, the teacher asks students, from left to right, to report the number as 1,2,3... in turn. Then, the teacher asks students who have reported a multiple of 4 to turn backwards (180-degree turn). Afterward, the teacher asks students who have reported a multiple of 6 to turn backwards.

- (i) How many students are still facing the teacher? (3 marks)
- (ii) For the third time, the teacher asks students who have reported a multiple of 9 to turn backwards. How many students are still facing the teacher? (3 marks)

**QB9. (4 marks)**

Points (more than 3) are placed at random into a square with length 1. Any three points indeed form a triangle. It is obvious that the more points are inserted, more triangles can be constructed. The newly constructed triangles have different areas which depend on the position of points.

You are requested to guarantee that there is at least one triangle to be constructed with an area equal to or small than  $1/8$ . What is the minimal number of points required?



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