Assignment 1

Q1. (10%) Simplify $\frac{2\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$ such that the denominator consists of an integer only.

Solution:

$$\frac{2\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{(2\sqrt{7}-\sqrt{5})(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} = \frac{14-3\sqrt{35}+5}{7-5} = \frac{19-3\sqrt{35}}{2}$$

Q2. (30%) $A = \{red, green, blue\}$, $B = \{red, yellow, orange\}$,

$C = \{red, orange, yellow, green, blue, purple\}$. Find the following:
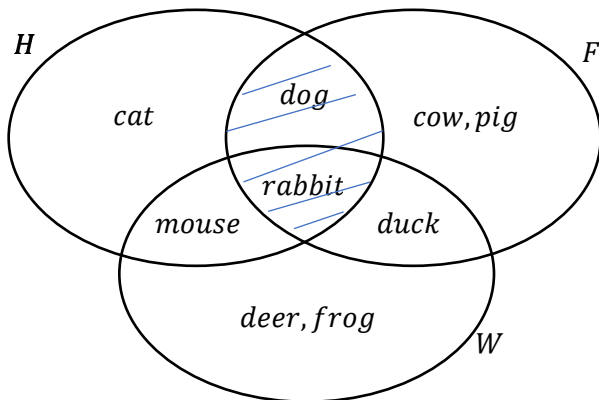
    a. (10%) $A \cup B$
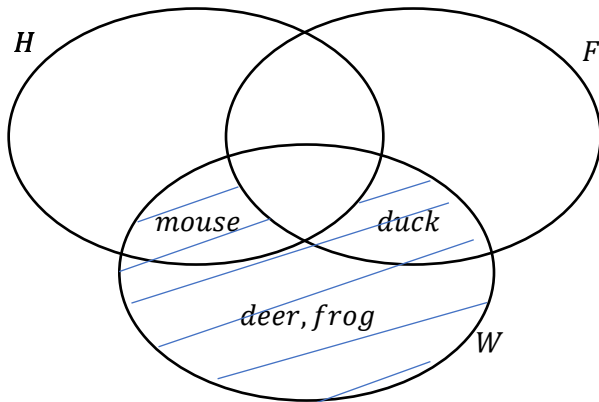   b. (10%) $A \cap B$
    c. (10%) $A^C \cap C$

Solution:

    a. $A \cup B = \{red, green, blue, yellow, orange\}$
    b. $A \cap B = \{red\}$
    c. $A^C \cap C = \{orange, yellow, purple\}$

Q3. (10%) Suppose $H = \{cat, dog, rabbit, mouse\}$, $F = \{dog, cow, duck, pig, rabbit\}$, $W = \{duck, rabbit, deer, frog, mouse\}$. Use Venn diagram to illustrate $(H \cap F)^C \cap W$.

Solution:

$(H \cap F)^C \cap W$

Q4. (10%) Set cartesian product: $A = \{cow, horse\}$, $B = \{egg, juice\}$. Find $A \times B$.

Solution:

	egg	jucie
cow	(cow, egg)	(cow, jucie)
horse	(horse, egg)	(horse, juice)

$$A \times B = \{(cow, egg), (horse, juice), (cow, jucie), (horse, egg)\}$$

Q5.

    a. (10%) Write the following sets in the set-builder form:

$$A = \{3,15,35,63,99,143,195,255\}$$

Solution:

$$A = \{(2x)^2 - 1 | x \in \mathbf{Z}, 1 \le x \le 8\}$$

$\mathbf{Z}$ is the set of Integer

    b. (10%) Find the set A, $A = \{x \in \mathbf{R} | x = x^2\}$.

Solution:

$$A = \{0,1\}$$

c. (10%) Write the following sets in the set-builder form:
$$A = \{\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{5}{6},\frac{6}{7},\frac{7}{8},\frac{8}{9},\frac{9}{10}\}$$

Solution:
$$A = \{\frac{x}{x+1} | x \in \mathbf{Z}, 1 \le x \le 9\}$$

Z is the set of integers.

d. (10%) Write the following sets in the set-builder form:
$$A = \{-21, -19, \cdots, -1, 1, 3, \cdots, 21\}$$

Solution:
$$A = \{2x + 1 | x \in \mathbf{Z}, -11 \le x \le 10\}$$

Z is the set of integers.