CITY UNIVERSITY OF HONG KONG

Course code & title	:	EE1002 Principles of Electrical Engineering
Session	:	Semester A 2021/22
Time allowed	:	One and a half hours (ninety minutes)

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1. Multiple Choice questions (10 MC questions carry 30 marks)
   Written questions (3 questions carry a total of 70 marks)

2. Start a new page for each written question.

3. It is an open-book examination

Materials, aids & instruments permitted to be used during the examination:

Non-programmable portable battery-operated calculator

Academic Honesty

"I pledge that the answers in this exam are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- I will not plagiarize (copy without citation) from any source;
- I will not communicate or attempt to communicate with any other person during the exam; neither will I give or attempt to give assistance to another student taking the exam; and
- I will use only approved devices (e.g., calculators) and/or approved device models.
- I understand that any act of academic dishonesty can lead to disciplinary action."

Please reaffirm this honesty pledge by writing "I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties" onto the "Answer Sheet" attached to this test paper.

Part I: Multiple choice (Total 30 minutes; 10 questions, 3 points per question)
Please provide your answers in the attached "Answer Sheet".

1. If $z = \frac{1}{j^3 - 3j}$ and z^* is the complex conjugate of z, where $j = \sqrt{-1}$, find the real and imaginary parts of z^*.
   A. real part: 0; imaginary part: $\frac{1}{4}$.
   B. real part: 0; imaginary part: $-\frac{1}{4}$.
   C. real part: $-\frac{1}{10}$; imaginary part: $-\frac{3}{10}$.
   D. real part: $-\frac{1}{10}$; imaginary part: $\frac{3}{10}$.

Solution: (B)

2. Let $v_1 = 3 + 3j$, $v_2^* = -2 + 2j$, and $v_3 = -3 - j$ where * denotes a complex conjugation, find $v = v_1 - v_2 + v_3^*$.
   A. $v = 2\angle 45°$
   B. $v = 2\sqrt{2}\angle 45°$
   C. $v = 4\angle 45°$
   D. $v = 2\angle 135°$

Solution: As mentioned during the test, there were mistakes in the provided answers. The correct should be $\sqrt{40}\ tan^{-1}3 = \sqrt{40}\angle 71.56°$.

3. A light bulb draws a current of 1.1 A current from an input voltage source of 220 V. Determine the resistance of the light bulb and the dissipated power.

   A. $200\ \Omega, 220\ W$
   B. $400\ \Omega, 1.1\ W$
   C. $400\ \Omega, 0.55\ W$
   D. $200\ \Omega, 242\ W$

Solution: (D)

4. Find $z \cdot z^*$ if $z = 8e^{-j60^\circ}$, where z^* is the complex conjugate of z.
   A. -32
   B. 32
   C. 64
   D. 48

Solution: (C)

5. If $y = \frac{7}{(3x)^{-2}}$, find the derivative of y.
   A. $-\frac{7}{6}$
   B. $-\frac{7}{6x}$
   C. $63x$
   D. $126x$

Solution: (D)

6. If $y = \frac{5x-2}{x^2+1}$, find the derivative of y.

   A. $\frac{-5x^2+4x+5}{(x^2+1)^2}$

   B. $\frac{15x^2-4x+5}{(x^2+1)^2}$

   C. $\frac{-5x^2+4x+5}{(x^2+1)^{-2}}$

   D. $\frac{15x^2-4x+5}{(x^2+1)^{-2}}$

Solution: (A)

7. Find the general solution of $\frac{dy}{dx} = 2x\, e^{x^2}$.

   A. $y = e^{x^2} + C$ where C is a constant.

   B. $y = x^2 e^{x^2} + C$ where C is a constant.

   C. $y = 2xe^{x^2} + C$ where C is a constant.

   D. $y = 2e^{x^2} + C$ where C is a constant.

Solution: (A)

8. Find $y = \int_0^2 xe^x dx$.

   (Hint: $\int_a^b u\left(\frac{dv}{dx}\right) dx = [uv]_a^b - \int_a^b v\left(\frac{du}{dx}\right) dx$)

   A. e^2

   B. $3e^2 - 1$

   C. $e^2 + 1$

   D. $e^2 - 1$

Solution: (C)

9. If cos $\theta > 0$ and tan $\theta < 0$, which quadrant does θ lie in?

   A. First quadrant

   B. Second quadrant

   C. Third quadrant

   D. Forth quadrant

Solution: (D)

10. A sinusoidal function has an amplitude of 2, a frequency of 3 Hz, and a phase angle of $\frac{\pi}{3}$, what is the sinusoidal function?

    A. $2 \sin (6\pi t + \frac{\pi}{3})$

    B. $4 \sin (6\pi t + \frac{\pi}{3})$

    C. $2 \sin [6\pi\left(t + \frac{\pi}{3}\right)]$

    D. $2 \sin (\frac{2\pi}{3} t + \frac{\pi}{3})$

Solution: (A)

Part II: Written Questions (Total 60 minutes; 3 questions)

Question 1 **(20 points)**

(a)

(i) Find the amplitude, angular frequency, period and phase angle of the signal $y_1(x) = 3\sin(2x + \frac{\pi}{3})$. **(6 points)**

(ii) Another signal is given by $y_2(x) = A\sin(2(x + \varphi))$ $(A > 0; 0 < \varphi < \pi)$, $x \in R$. If the phase angle of this signal is $\frac{\pi}{6}$ and the minimum value of $y_2(x)$ is -3, find A and φ. **(6 points)**

(iii)Determine whether the two signals $y_1(x)$ and $y_2(x)$ pass through the point $M(\frac{\pi}{3}, 0)$ or not. **(2 points)**

(b) Let $y_3(x) = \sin 2x + k\cos 2x$ where k is a constant. If $y_3(x)$ can be expressed as a single sine function with an amplitude of $\sqrt{10}$, find k. **(6 points)**

Solution:

(a)

(i) amplitude: $A = 3$; angular frequency: $w = 2$; period: $T = \pi$; and phase angle $\varphi = \frac{\pi}{3}$.

(ii) $y_2(x) = 3\sin(2x + \frac{\pi}{6})$, $A = 3$, and $\varphi = \frac{\pi}{12}$.

(iii) $y_1\left(\frac{\pi}{3}\right) = 3\sin\pi = 0$, $y_1(x)$ passes through the point M;

$y_2\left(\frac{\pi}{3}\right) = 3\sin\frac{5\pi}{6} \neq 0$, $y_2(x)$ doesn't pass through the point M.

(b)Let $\sin 2x + k\cos 2x = R \cdot \sin(2x + \varphi) = R \cdot \cos\varphi \sin 2x + R \cdot \sin\varphi \cos 2x$

Hence $R \cdot \cos\varphi = 1$, $R \cdot \sin\varphi = k$

by squaring and adding we obtain

$$(R \cdot \cos\varphi)^2 + (R \cdot \sin\varphi)^2 = R^2 = k^2 + 1 = 10$$

$$k = \pm 3$$

Question 2 **(25 points)**

The speed $v(t)$ of a particle is a function of time t given by $v(t) = 2 - e^{-t}$.

(a) Calculate the speeds of the particle at $t = 0$ and $t = 3$. What is the average speed over this time interval? **(4 points)**

(b) Calculate the distance s travelled by the particle between $t = 0$ and $t = 3$. (**Hint:** $s = \int v(t)dt$) **(9 points)**

(c) Acceleration is the rate of change of velocity with respect to t. Determine the acceleration a as a function of time and find its value at $t = 4$. **(6 points)**

(d) When is the maximum acceleration obtained? What is the maximum acceleration? **(6 points)**

Solution:

(a) $v(0) = 2 - e^{-0} = 2 - 1 = 1$, $v(3) = 2 - e^{-3}$

$$v_a = \frac{s}{3-0} = \frac{\int_0^3 (2 - e^{-t})dt}{3-0} = \frac{5 + e^{-3}}{3}$$

(b) $s = \int v(t)dt = \int (2 - e^{-t})dt = 2t + e^{-t} + C$
$s = s(3) - s(0) = 5 + e^{-3} = 5.0498$

(c) $a = \frac{dv}{dt} = e^{-t}, a(4) = e^{-4}$

(d) $\frac{da}{dt} = -e^{-t} < 0$, when t=0, maximum acceleration obtained.

$a_{max} = a(0) = e^0 = 1$

Question 3 **(25 points)**

Solve the following equations.

(a) Solve the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$. Find the solution if the initial condition is given by $y(0) = 2$. **(11 points)**

(b) Solve the differential equation $\frac{dy}{dx} = x^2 y$. **(6 points)**

(c) Solve the differential equation $\frac{dy}{dx} + y = e^x$. **(8 points)**

Solution:

(a)

$$y^2 dy = x^2 dx$$

$$\int y^2 dy = \int x^2 dx$$

$$\frac{1}{3}y^3 = \frac{1}{3}x^3 + C$$

$$y = (x^3 + K)^{\frac{1}{3}}$$

where $K = 3C$ (Since *C* is an arbitrary constant, so is *K*).
Since

$$y(0) = (0 + K)^{\frac{1}{3}} = 2$$

$\therefore K = 8$

$\therefore$ the solution of the initial-value problem is

$y = (x^3 + 8)^{\frac{1}{3}}$

(b)

$$\frac{dy}{dx} = x^2 y$$

$$\int \frac{1}{y} dy = \int x^2 dx$$

$$\ln|y| = \frac{x^3}{3} + C$$

$\therefore y = \pm e^C e^{\frac{x^3}{3}}$

In addition, $y = 0$ is also a solution of the given differential equation.

$\therefore y = Ae^{\frac{x^3}{3}}$

where $A = \pm e^C$, or $A = 0$

(c)

$$P(x) = 1, Q(x) = e^x$$

$$u(x) = e^{\int P(x)dx} = e^x$$

$$\therefore y = \frac{1}{u(x)} \int Q(x)u(x)dx$$

$$y = e^{-x}\left(\frac{1}{2}e^{2x} + C\right)$$

$$= \frac{1}{2}e^x + Ce^{-x}$$