MA1200 MIDTERM EXAM FRIDAY 7:05 PM -8:05 PM, C61

Q1. (30 points) Write $25x^2+16y^2+100x-96y=156$ into the standard form, find foci, center, and vertices, (asymptotes if it is a hyperbola), and sketch the graph of it.

Q2. (15 points) Find the largest possible domains and the ranges of the following functions:

$$f(x)=\log_2(16-x^2) \quad \text{and} \quad g(x)=\log_2(8-x^3).$$

Q3. (20 points) Express $\dfrac{6x^3+5x^2+2x-10}{6x^2-x-2}$ as partial fractions.

Q4. (20 points) Simplify $\cos(\sin^{-1}(-\frac{4}{5})+\cos^{-1}(-\frac{5}{13}))$.
(Hint: $\cos(A+B)=\cos A\cos B-\sin A\sin B$, $3^2+4^2=5^2$, $5^2+12^2=13^2$)

Q5. (15 points) Solve $\cos(2x+\pi/2)=1/2$ in radians.

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Date

Q1 $25x^2+16y^2+100x-96y=156$.

$25(x^2+4x\ \ )+16(y^2-6y\ \ )=156$

$+4 \qquad +9 \qquad +100+154$

$\Rightarrow 25(x+2)^2+16(y-3)^2 = 400$

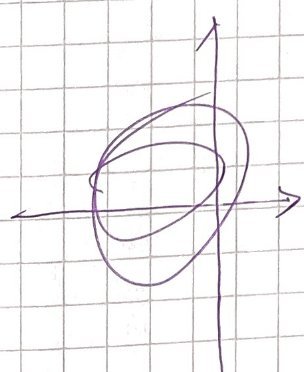
$\Rightarrow \dfrac{(x+2)^2}{16}+\dfrac{(y-3)^2}{25}=1.$ 10

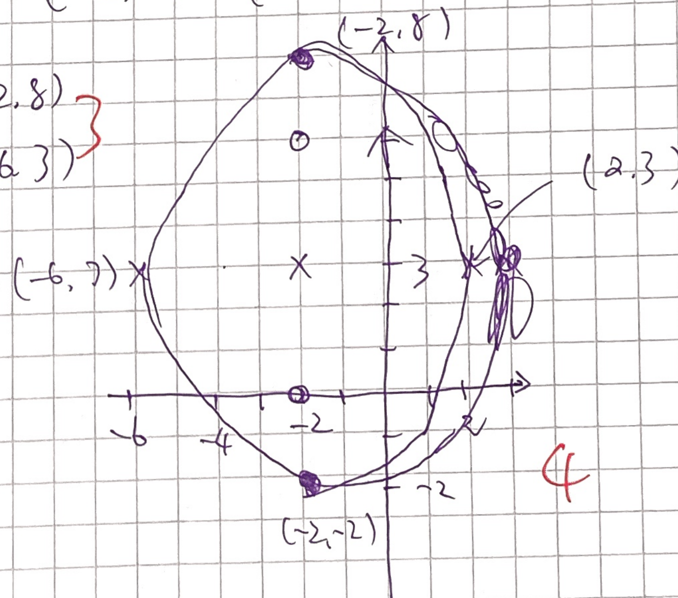
Center $(-2,3)$ 2

$c=\sqrt{25-16}=3$. foci $(-2,0)$ $(-2,6)$ 5

vertices $(-2,-2)$ $(-2,8)$ 3

$(2,3)$ $(-6,3)$





4

30

Date 3-2

Q2 $f(x) = \log_2(16-x^2)$

domain $16-x^2>0$, $\Rightarrow x \in (-4,4)$. 3

range $f(x) \le \log_2 16 = 4$. $(-\infty,4]$ 4

$g(x) = \log_2(8-x^3)$

domain $8-x^3>0 \Rightarrow x^3<8 \Rightarrow x<2$ $(-\infty,2)$ 3

range $(-\infty,\infty)$. Since $8-x^3$ can be any number >0 5

15

Q3 $\frac{5x^3-4x^2+21x-28}{x^4+10x^2+9} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+9}$

$x^4+10x^2+9 = (x^2+1)(x^2+9)$

$\Rightarrow 5x^3-4x^2+21x-28 = (Ax+B)(x^2+9) + (Cx+D)(x^2+1)$

3-3

Q3. $\dfrac{6x^3+5x^2+2x-10}{6x^2-x-2} = x+1+\dfrac{5x-8}{6x^2-x-2}.$

$\dfrac{5x-8}{6x^2-x-2} = \dfrac{A}{3x-2} + \dfrac{B}{2x+1}$ 1°

$10-24=6$

$10-24=$

$5x-8 = B(3x-2) + A(2x+1)$

$x=\frac{2}{3} \Rightarrow \frac{10}{3}-8 = A\cdot\frac{7}{3} \Rightarrow A=-2$ 4

$x=-\frac{1}{2} \Rightarrow -\frac{5}{2}-8 = B\left(-\frac{3}{2}-2\right) \Rightarrow B=3$ 4

$\Rightarrow \quad = x+1+\dfrac{-2}{3x-2}+\dfrac{3}{2x+1}$ 2

20

~~$\cos\left(\sin^{-1}\left(-\frac{4}{5}\right) + \tan^{-1}(\ldots)\right)$~~

~~$A=\sin^{-1}\left(-\frac{4}{5}\right)$~~

Q4 $\cos\left(\sin^{-1}\left(-\frac{4}{5}\right)+\cos^{-1}\left(-\frac{5}{13}\right)\right)$

$\sin A = -\frac{4}{5}$

$A=\sin^{-1}\left(-\frac{4}{5}\right) \quad A\in\left(-\frac{\pi}{2},0\right), \cos A = \frac{3}{5}$ 7

$B=\cos^{-1}\left(-\frac{5}{13}\right) \quad B\in\left(\frac{\pi}{2},\pi\right), \sin B=\frac{12}{13} \quad \cos B = -\frac{5}{13}$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ 6

$= \frac{3}{5}\cdot\left(-\frac{5}{13}\right) - \left(\frac{-4}{5}\cdot\frac{12}{13}\right) = \frac{-15+48}{65} = \frac{33}{65}.$ 20

Date 3-4

Q5 $\cos\left(2x+\pi/2\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

$\Rightarrow \quad 2x+\frac{\pi}{2} = 2n\pi \pm \frac{\pi}{3} . \quad n \in \mathbb{Z}$ 7

"+" $\quad 2x+\frac{\pi}{2} = 2n\pi + \pi/3$

$2x = 2n\pi - \frac{\pi}{6} \Rightarrow x = n\pi - \pi/12 , \; n \in \mathbb{Z}$

4

"−" $\quad 2x+\frac{\pi}{2} = 2n\pi - \frac{\pi}{3}$

4

$2x = 2n\pi - \frac{5}{6}\pi \Rightarrow x = n\pi - \frac{5}{12}\pi . \; n \in \mathbb{Z}$

15