EE1002 Principles of Electrical Engineering   Assignment 1

1. If $z = \frac{a+3j}{1-2j}$, where $a \in R$ ("$\in$ " means "belong to", and "$a \in R$" means that a belongs to a real number.)

a) If z is a real number, then find a.

b) If z is an image number, then find a.

2.

a) If $\frac{\bar{z}}{1+j} = 2 + j$ where $\bar{z}$ is the complex conjugate of z. Find the complex number z.

b) If $z_1 = \frac{y+3j}{1+xj}$ $(x, y \in R)$ and $z_2 = (xy + j)^2$. Find the complex conjugate of z_2.

c) If $z = \frac{1+7j}{2-j} = a + jb$, find a, b, and $|z|$ (modulus of z).

3. Use Table 1 to find y', given

a) $y = t^2$

b) $y = t^{-3}$

c) $y = \frac{1}{\sqrt{t}}$

d) $y = \ln\ t$

e) $y = t^{1/2}$

f) $y = e^{3t}$

g) $y = \frac{1}{e^{5t}}$

h) $y = \sin(2t + 3)$

i) $y = \tan(\frac{t}{2} + 1)$

j) $y = \arcsin(t + \pi)$

4. Find the derivatives of the following functions:

a) $y = 4x^3 - 5x^2$

b) $y = 3\sin(5x) + 2e^{4x}$

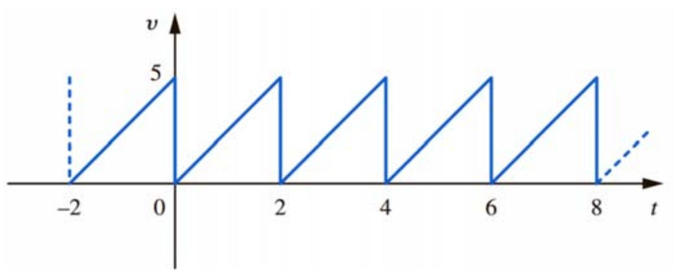
c) $y = 2e^{3x} + 17 - 4\sin(2x)$

d) $y = \frac{1}{x^3} + \frac{\cos 5x}{2}$

e) $y = \frac{2x^3}{3} + \frac{e^{4x}}{2}$

f) $y = \sqrt{x} + \ln(\sqrt{x})$ [Hint:$\sqrt{x} = x^{1/2}$, and use the formula $\ln a^b = b \ln a$]

5. Consider the saw-tooth waveform shown in the following figure. Calculate the average value of this waveform over a complete period.



6. Calculate the r.m.s. value of $f(t) = Asin(\omega t + \varphi)$. (Hint: The period is given by $T = 2\pi/\omega$.)

7. Evaluate the following integrals.

a) $\int_0^a x^2\sqrt{a^2 - x^2}\,dx$

b) $\int_{-1}^{1} \frac{xdx}{\sqrt{5-4x}}$

c) $\int_1^4 \frac{dx}{\sqrt{x}+1}$ (Hint: You may let $t = \sqrt{x}$)

d) $\int_{-\pi}^{\pi} x^4 \sin x dx$

8. Find the general solution of

(a) $\frac{dx}{dt} = 2x$

(b) $(1 + t)\frac{dx}{dt} = 3$

(c) $\frac{dy}{dx} = y^2 \cos x$

9. Find the general solution of

(a) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 0$

(b) $\frac{d^2y}{dx^2} + 4y = 0$

(c) $y'' + 8y' + 16y = 0$

10. Find the general solution of the following differential equation by using the integrating factor technique:

$$x\frac{dy}{dx} - \mathrm{y} = x^2\cos^3 x$$

11. Show that the following expression is equal to a constant. What is the value of this constant?

$$\frac{2\cos^2 a\text{-}1}{2\tan\left(\frac{\pi}{4} - a\right)\sin^2\left(\frac{\pi}{4} + a\right)}$$

12. A signal is given by $y(x) = A\sin(x + \phi)$ $(A > 0;\ 0 < \phi < \pi), x \in R$. It passes through the point $M(\frac{\pi}{3}, \frac{1}{2})$. The maximum value of $y(x)$ is equal to 1.

(1) Find A and ϕ.

(2) If $y(\alpha) = \frac{3}{5}, y(\beta) = \frac{12}{13},\ \alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, find $y(\alpha - \beta)$.