EE1001 Test 1 2020_21 Sem B

PART A: Short questions 2% each

Q1. 3 is the smallest prime number.

a) True
b) False

Answer: b) False, should be 2.

Q2. Difference of two distinct prime number is?

a) Odd and prime
b) Even and composite
c) None of the mentioned
d) All of the mentioned

Answer: c)
Explanation: $3 - 2 = 1$ is neither prime nor composite.

Q3. In 2's complement, what do all the positive numbers have in common?

Answer: The most significant bit (MSB) is 0.

Q4. Name 2 advantages that 2's complement have over 1's complement?

Answer: 1) just 1 "zero" 2) addition of positive to certain negative number gives accurate answer

Q5. Suppose we are using 13-bit floating-point representation where there is 1 sign-bit, 8 mantissa bits, and 4 exponent (in 2's complement) bits. What is $(48.75)_{10}$ in this floating-point representation?

Solution:

$$(48)_{10} = (11\ 0000)_2$$

$$(0.75)_{10} = (0.11)_2$$

$$(48.75)_{10} = (11\ 0000.11)_2$$

1 sign-bit $(0)_2$, 8 mantissa bits $(1100\ 0011)_2$,

Shift radix point left for 6 places, $6 = 2^2 + 2^1$, 4 exponent bits $(0110)_2$.

So, the answer is $0.1100\ 0011\ [0110]$.

Q6. Convert the following decimal numbers to binary using 6-bit 2's complement representation.

a) $(-31)_{10}$
b) $(26)_{10}$

Solution:

a) $(-31)_{10} = 100001$
b) $(26)_{10} = 011010$

Q7. What would the hexadecimal $(-3E)_{16}$ be as 1's complement?

Solution:

$(3E)_{16}$ is a positive number, $(0011\ 1110)_2$.

$(-3\mathrm{E})_{16} = 11000001$

Q8. Unsigned binary numbers addition: What would $(1010\ 0101)_2 + (0100\ 0100)_2$ be as a binary number?

Solution:

$$\begin{array}{rr} & 1010\ 0101 \\ + & 0100\ 0100 \\ \hline & 1110\ 1001 \end{array}$$

Q9. What is the resolution to cover a range of numbers $x_{max} - x_{min}$ with 'b' number of bits?

a) $(\mathrm{x}_{\mathrm{max}} + \mathrm{x}_{\mathrm{min}})/(2^{\mathrm{b}} - 1)$
b) $(x_{max} + x_{min})/(2^{\mathrm{b}} + 1)$
c) $(x_{max} - x_{min})/(2^{\mathrm{b}} - 1)$
d) $(x_{max} - x_{min})/(2^{\mathrm{b}} + 1)$
e) None of the above.

Answer:

c). A fixed-point representation of numbers allows us to cover a range of numbers, say, $x_{max} - x_{min}$ with a resolution $\Delta = (x_{max} - x_{min})/(\mathrm{m} - 1)$ where $\mathrm{m} = 2^{\mathrm{b}}$ is the number of levels and 'b' is the number of bits.

Q10. If the two numbers are to be multiplied, the mantissa are multiplied and the exponents are added.

a) True
b) False

Answer:

a) True: Let us consider two numbers $\mathrm{X} = \mathrm{M} \cdot 2^{\mathrm{E}}$ and $\mathrm{Y} = \mathrm{N} \cdot 2^{\mathrm{F}}$
If we multiply both X and Y, we get $\mathrm{X} \cdot \mathrm{Y} = (\mathrm{M} \cdot \mathrm{N}) \cdot 2^{\mathrm{E+F}}$
Thus if we multiply two numbers, the mantissa is multiplied and the exponents are added.

Q11. A is a set $\{0,1,\{1,2\},\{1,2,3\},\{1,3,6,7\},\{1,7,8,9\}\}$. What is the cardinality of the power set $P(A)$?

Solution:

The cardinality of the power set $P(A)$ is 2^n, where n is the cardinality of set A.

In our case, $n = 6$. $|P(A)| = 2^6 = 64$.

Q12. Which of the following is the correct set builder notation for the given set below?

$$\{1.1, 2.2, 3.3, 4.4, 5.5, 6.6, 7.7, 8.8, 9.9\}$$

a) $\{x \in \boldsymbol{R} \mid 1.1 \leq x \leq 9.9\}$
b) $\{x \in \boldsymbol{Z} \mid 1.1 \leq x \leq 9.9\}$
c) $\{(1.1x) \mid x \in \boldsymbol{R}, 1 \leq x \leq 9\}$
d) $\{(1.1x) \mid x \in \boldsymbol{Z}, 1 < x \leq 9\}$
e) $\{(1.1x) \mid x \in \boldsymbol{Z}, 1 < x < 9\}$
f) None of the above.

Answer:

f) none of the above, has to be $\{(1.1x) \mid x \in \boldsymbol{Z}, 1 \leq x \leq 9\}$ (be careful with the highlighted sign).

Q13. $A = \left\{\frac{p}{q} \,\middle|\, p \in Z, q \in Z, and\ q \neq 0\right\}$. What is A?

Answer:

Rational numbers.

Q14. To find the scientific notation of $(0.08888)_{10}$ in 16-bit floating point representation. Use 1 sign bit, 9 mantissa bits, and 6 exponent bits.

a) $(0.0110\ 1100\ 0\ [00\ 0100])_2$
b) $(0.0110\ 1100\ 0\ [11\ 1100])_2$
c) $(1.1110\ 1001\ 1\ [00\ 0000])_2$
d) $(0.1011\ 0110\ 0\ [11\ 1101])_2$
e) None of the answers.

Solution: d)

$(0.08888)_{10} \cong (0.0001\ 0110\ 1100\ 0)_2$ To move the radix point 3 places to the right so that it becomes 0.xxxxxxx that is 2 to power -3 or exp = [111101]

$= (0.1011\ 0110\ 0)_2 \times 2^{(11\ 1101)_2}$

Therefore, the answer is $(0.1011\ 0110\ 0\ [11\ 1101])_2$

Q15. $B = \{-9, -7 - 5, -3, -1,1,3,5\}$: which of the following is the correct set builder form to describe B?

a) $B = \{2x + 1 | x \in Z, -5 \leq x \leq 2\}$
b) $B = \{2x - 1 | x \in Z, -5 \leq x \leq 2\}$
c) $B = \{2x - 1 | x \in Z, -4 \leq x < 3\}$
d) $B = \{2x + 1 | x \in Z, -5 < x < 2\}$
e) None of the above

Answer: a)

Q16. If X is a real number with ‘r’ as the radix, A is the number of integer digits and B is the number of fraction digits, then

a) $X = \sum_{i=-A}^{B-1} b_i \, r^{-i}$
b) $X = \sum_{i=-A}^{B} b_{i-1} \, r^{-i}$
c) $X = \sum_{i=-A}^{B} b_i \, r^{-i+1}$
d) $X = \sum_{i=-A}^{B} b_i \, r^{-i}$
e) $X = \sum_{i=-A}^{B+1} b_i \, r^{-i}$

Answer: d)

Q17. Power set of empty set has exactly ____ subset.

a) Zero
b) One
c) Undefined
d) Two
e) None of the above

Answer: b) One.

Q18. What is the Cartesian product of A $= \{1, 2\}$ and B $= \{a, b\}$?

a) $\{(1, a), (1, b), (2, a), (b, b)\}$
b) $\{(1,1), (2,2), (a, a), (b, b)\}$
c) $\{(1, a), (2, a), (1, b), (2, b)\}$
d) $\{(1, 1), (a, a), (2, a), (1, b)\}$
e) $\{(1,1), (1, a), (1, b), (2, b)\}$

Answer: c)

Q19. Which of the following is true?

a) The function $f(x) = x^3$ is bijective from R to R.
b) The function $f(x) = x + 1$ from the set of integers to itself is onto.
c) Both a) and b)
d) None of the above

Answer: c)

Both a) and b) the following is true.

Q20. A function $f: N \to N$ defined by $f(x) = x^2$ is

a) one to one
b) not one to one
c) on to
d) bijective
e) cannot be defined

Answer: a)

PART B 10% each

QB1. (10%)

In a 32 bits floating-point representation (NOT IEEE 754 format), the mantissa is represented by 23 bits plus 1 bit sign bit. The exponent is represented by 8 bits in 2's complement. What is the 10^{th} smallest positive floating-point number that it can represent?

Solution: Let the mantissa be represented by 23 bits plus a sign bit and let the exponent be represented by 7 bits plus a sign bit.
Thus, the 10^{th} smallest positive floating point number that can be represented using the 32 bit number is

$$(0.0000\ 0000\ 0000\ 0000\ 0001\ 010\ [1000\ 0000])_2 = (2^{-20} + 2^{-22}) * 2^{-128}$$

$$= 3.5 * 10^{-45}$$

QB2. (10%)

A) Use IEEE 754 32-bit format. Find

$$(-16.5)_{10}$$

Solution:

1 1000 0011 0000 1000 0000 0000 0000 000

B) To find the scientific notation of $(0.0765)_{10}$ in 16-bit floating point representation. Use 1 sign bit, 9 mantissa bits, and 6 exponent bits.

Solution: $(0.1001\ 1100\ 1\ [11\ 1101])_2$

$(0.0765)_{10} \cong (0.0001\ 0011\ 1001\ 1)_2$ To move the radix point 3 places to the right so that it becomes 0.xxxxxxx. that is 2 to power -3 or exp = [111101]

$= (0.1001\ 1100\ 1)_2 \times 2^{(11\ 1101)_2}$

Therefore, the answer is $(0.1001\ 1100\ 1\ [11\ 1101])_2$

QB3. (10%)

Let $g: \{a, b, c\} \rightarrow \{a, b, c\}$, $g(a) = b$, $g(b) = c$, $g(c) = a$. Let $f: \{a, b, c\} \rightarrow \{1,2,3\}$, $f(a) = 3, f(b) = 2, f(c) = 1$. Determine what are the composition functions f of g, $f \circ g$, and what is the composition function $g \circ f$.

Solution:

$(f \circ g)(a) = f(g(a)) = f(b) = 2.$ (3%)

$(f \circ g)(b) = f(g(b)) = f(c) = 1.$ (3%)

$(f \circ g)(c) = f(g(c)) = f(a) = 3.$ (3%)

$g \circ f$ is undefined as the range of f is not a subset of the domain of g. (1%)

QB4. (10%)

Use contrapositive proof to prove: If $\forall y \in Z, 7y + 9$ is even, then y is odd.

Solution:

Contrapositive means to prove: If y is even, then $7y + 9$ is odd.

Proof:

$\forall y \ \in Z$, and y is even

$\Rightarrow y = 2n, n \in Z$

$\therefore 7y + 9 = 7(2n) + 9 = 14n + 9 = 2(7n + 4) + 1$

$\therefore 7y + 9 =$ even number $+1$

Must be odd. ■

QB5. (10%)

a. $g: R \rightarrow R, g(x) = x^2$. Determine if the function is surjective, injective, or bijective.

b. $f: R \to R, f(x) = 4x - 1$. Determine if the function is surjective, injective, or bijective.
c. The bank account numbers are domain and the bank account customers are co-domain. Determine if this mapping process is injective, surjective, or bijective.
d. Determine which of the above functions is/are invertible.

Solution: 2.5% each

a. Not Surjective and not injective.
b. bijective
c. Surjective.
d. B is invertible.

QB6. (10%)

a. Perform subtraction of the below and your answer must be in HEX:

$$DC8 - 79A$$

b. Find the 2's complement of $a = 75$, and $b = -102$. Find $a + b$ in 2's complement.

Solution:

a. $(DC8)_{16} = (1101\ 1100\ 1000)_2$ and $(79A)_{16} = (0111\ 1001\ 1010)_2$
Convert $(0111\ 1001\ 1010)_2$ to 2's complement: $(1000\ 0110\ 0110)_2$

$$\begin{array}{rr} & 1101\ 1100\ 1000 \\ + & 1000\ 0110\ 0110 \\ \hline 1 & 0110\ 0010\ 1110 \end{array}$$

MSB 1 is the overflow bit.

Convert $(0110\ 0010\ 1110)_2$ to equivalent hexadecimal number: $(62E)_{16}$

b. $a = (75)_{10}$, 2's complement in 8-bit form $(0100\ 1011)_2$.
$b = (-102)_{10}$, 2's complement in 8-bit form $(1001\ 1010)_2$.
$a + b$ is

$$\begin{array}{rr} & 0100\ 1011 \\ + & 1001\ 1010 \\ \hline & 1110\ 0101 \end{array}$$

$a + b$ in 2's complement is $(1110\ 0101)_2$.