EE2000 Logic Circuit Design

Chapter 1 – Review of Boolean Algebra and Logic Function

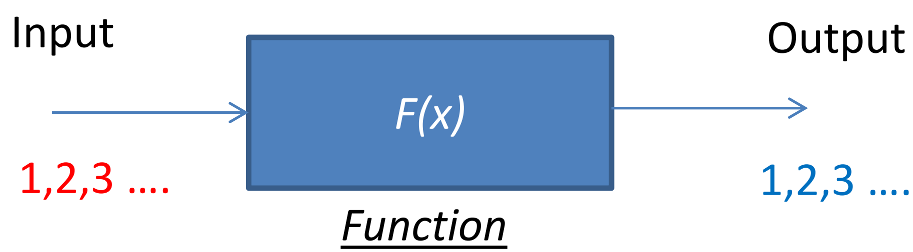
Outline

- 1.1 Basic logic gates
- 1.2 Boolean algebra
- 1.3 Logic Circuit and Boolean Expression
- 1.4 Simplification using Boolean Algebra
  - SOP, POS, minterm, maxterm, canonical form

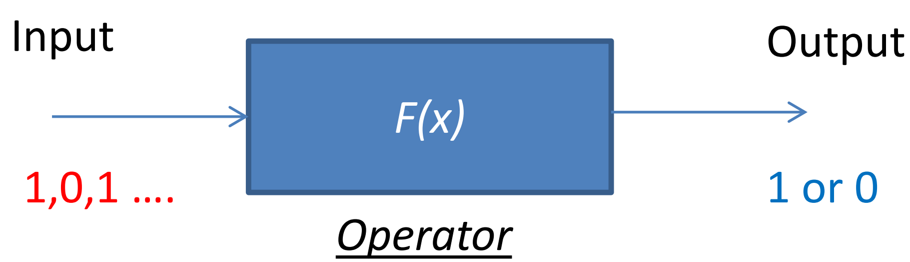
1.1 Logic Gate

- The term **gate** describes a circuit that performs a basic logic operation.

Conventional numerical / mathematical function

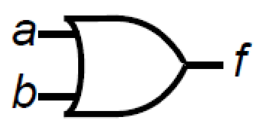
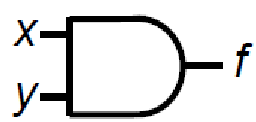
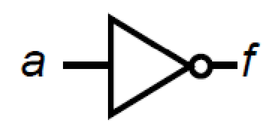


Logic operator/ function



Binary decision output e.g, Yes/No ; True/False and 1/0 .

Logic Operator

	OR	AND	NOT
Binary / Unary operator?	Binary	Binary	Unary
Symbols	1: + 2: V	1: · 2: Λ 3: absence of an operator	1: ’ 2: ~ 3: $\overline{}$
Examples	1: $a + b$ 2: $a \vee b$	1: $x \cdot y$ 2: $x \wedge y$ 3: xy	1: a' 2: $\sim a$ 3: $\bar{a}$
Logic Gate Symbol			

a	b	$a + b$
0	0	0
0	1	1
1	0	1
1	1	1

Input Output

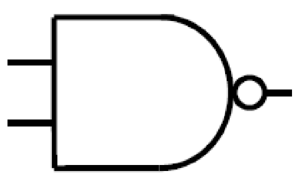
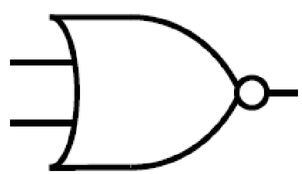
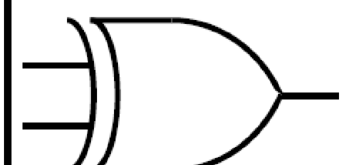
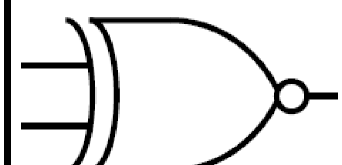
x	y	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

Input Output

a	a'
0	1
1	0

Input Output

Logic Operator

Operation		*NAND*	*NOR*	*XOR*	*XNOR*
a	***b***	$(ab)'$	$(a+b)'$	$a \oplus b$	$a \otimes b$
0	0	1	1	0	1
0	1	1	0	1	0
1	0	1	0	1	0
1	1	0	0	0	1
Symbol					

1.2 Boolean Algebra

- A set of element ***S*** with at least two different elements (***x* , *y***) satisfying binary operations (**+**) and (**•**) .

- For Boolean algebra in which ***S***= {0,1}, the formulation is referred as switching function.

Basic Postulates

If $x,y \in S$,

$$x + y = y + x$$

$$x \cdot y = y \cdot x$$

commutative

If $x,y,z \in S$,

$$x + (y + z) = (x + y) + z$$

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

associative

If $x,y,z \in S$,

$$x + (y \cdot z) = (x + y) \cdot (x + z)$$

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$

distributive

Distributive Law

- Proof $$x + (y \cdot z) = (x + y) \cdot (x + z)$$

x	y	z	$y \cdot z$	$x + y \cdot z$	$x + y$	$x + z$	$(x + y)(x + z)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Identical

Duality

- If an expression is valid in Boolean algebra, the dual of the expression is also valid.

- Principle of duality:

$$0 \cdot x = 0 \qquad 1 + x = 1$$

$$1 \cdot x = x \qquad 0 + x = x$$

$$x \cdot x = x \qquad x + x = x$$

$$x \cdot x' = 0 \qquad x + x' = 1$$

The expressions are interchangeable by replacing “0” by “1” and “+” by “ · ” .

Theorems

Idempotent

$$x + x = x \qquad x \cdot x = x$$

Involution

$$(x')' = x$$

Absorption

$$x + xy = x \qquad x(x + y) = x$$

Logical adjacency

$$xy + xy' = x$$

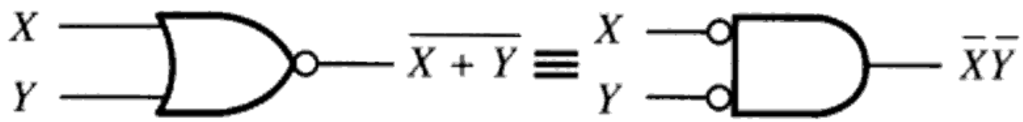
DeMorgan

$$(x + y)' = x'y' \qquad (xy)' = x' + y'$$

- The complement of sum is equal to the product of the complement
- The product of complement is equal to the sum of the complement

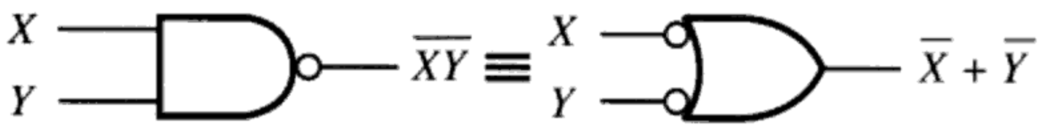
DeMorgan

$(x + y)' = x'y'$



X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

$(xy)' = x' + y'$



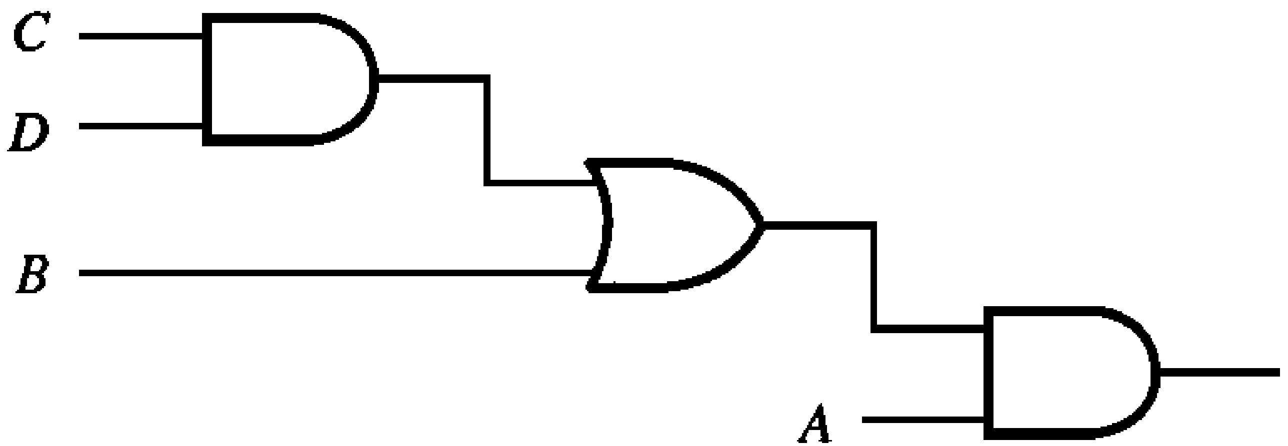
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

Consensus

- $at_1 + a't_2 + t_1t_2 = at_1 + a't_2$
- $(a + t_1)(a' + t_2)(t_1 + t_2) = (a + t_1)(a' + t_2)$

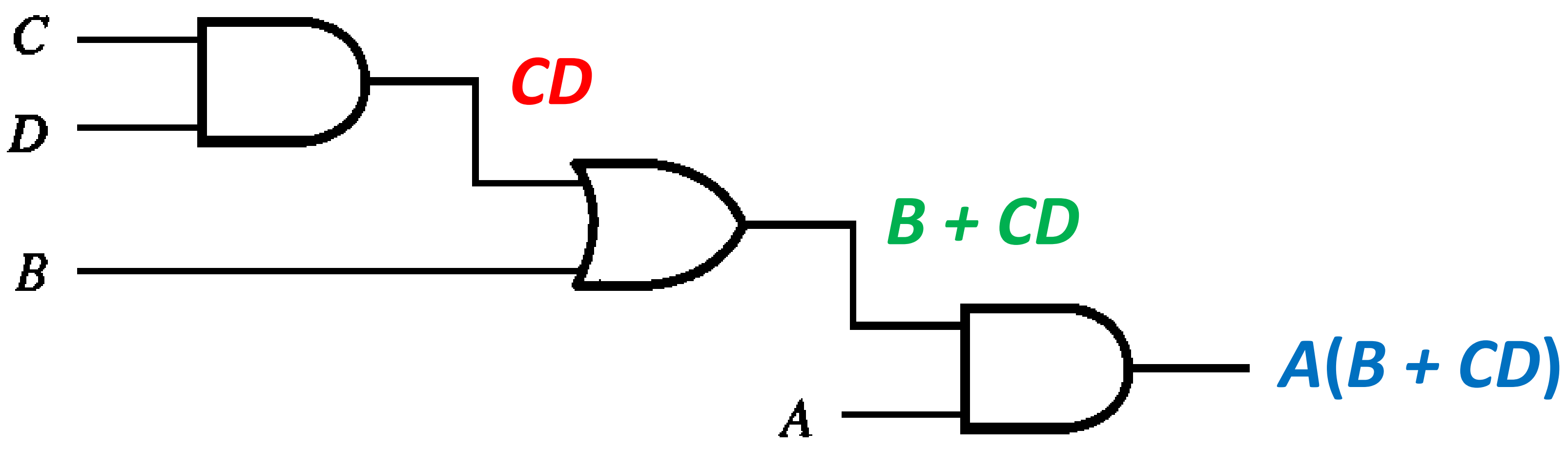
- The theorem shows that the consensus term, t_1t_2, is redundant and can be eliminated
- Proof: $at_1 + a't_2 + t_1t_2$
  - $= at_1 + a't_2 + t_1t_2 \cdot 1$ (identity)
  - $= at_1 + a't_2 + t_1t_2 \cdot (a + a')$ (complementation)
  - $= at_1 + a't_2 + at_1t_2 + a't_1t_2$ (distributivity)
  - $= at_1 + a't_2$ (absorption)

1.3 Logic Circuit and Boolean Expression



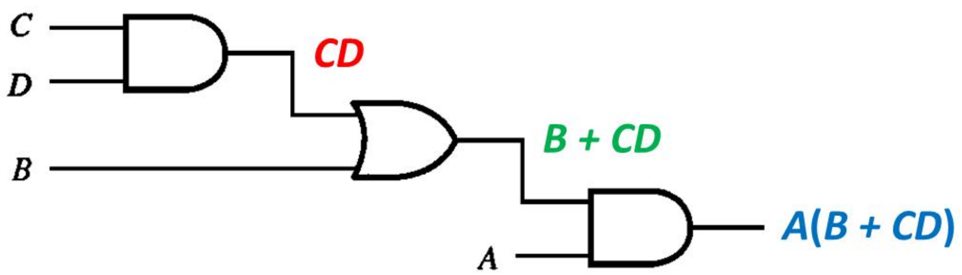
- To derive the Boolean expression of a given logic circuit, begin at the left-most inputs and work towards the final output, writing the expression for each gate.

Boolean expression from a logic circuit



- Write down the output expression from all logic operators
- The Boolean function of this circuit is *A (B + CD)*
- Construct a truth table for above logic circuit

Truth table for a logic circuit



Examples of numbering systems		Inputs				Output
Decimal	Hexadecimal	A	B	C	D	A(B+CD)
0	0	0	0	0	0	0
1	1	0	0	0	1	0
2	2	0	0	1	0	0
3	3	0	0	1	1	0
4	4	0	1	0	0	0
5	5	0	1	0	1	0
6	6	0	1	1	0	0
7	7	0	1	1	1	0
8	8	1	0	0	0	0
9	9	1	0	0	1	0
10	A	1	0	1	0	0
11	B	1	0	1	1	1
12	C	1	1	0	0	1
13	D	1	1	0	1	1
14	E	1	1	1	0	1
15	F	1	1	1	1	1

Completed solution for a logic circuit design must include:

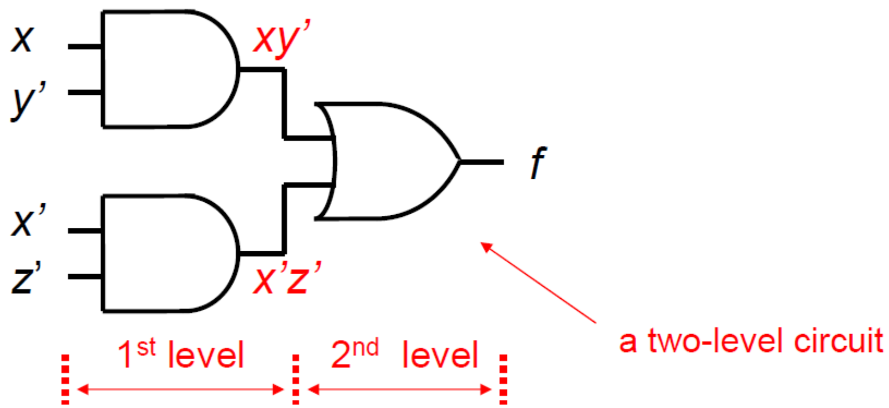
1. Boolean Algebra
2. Circuit schematic
3. Truth table
4. Table Assignment

For the truth table, find the output as a following:

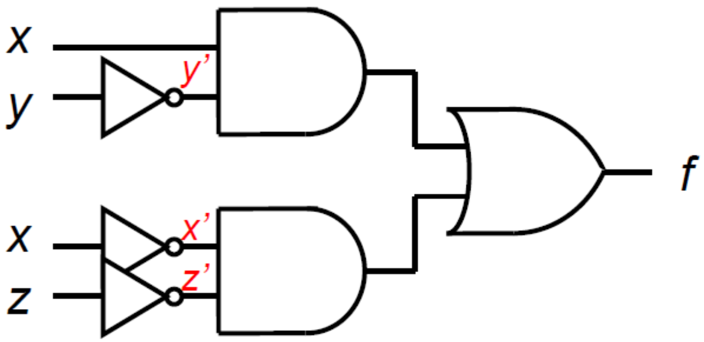
1. Write down all input possibility
2. Write down the stage output (i.e. *CD*, *B + CD*)
3. Write down the final stage output (i.e. *A(B + CD)*)

Logic circuit from a Boolean expression

Provided that a Boolean function $f(x,y,z)=xy'+x'z'$, then the logic circuit can be formed as:



or



Boolean function → Truth Table

- e.g. $f(x, y, z) = xy' + x'z'$

Input(s)

Output

The number of rows is 2^n (n is the number of variables)

x	y	z	xy'	$x'z'$	$xy' + x'z'$
0	0	0	0	1	1
0	0	1	0	0	0
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	0	0
1	1	1	0	0	0

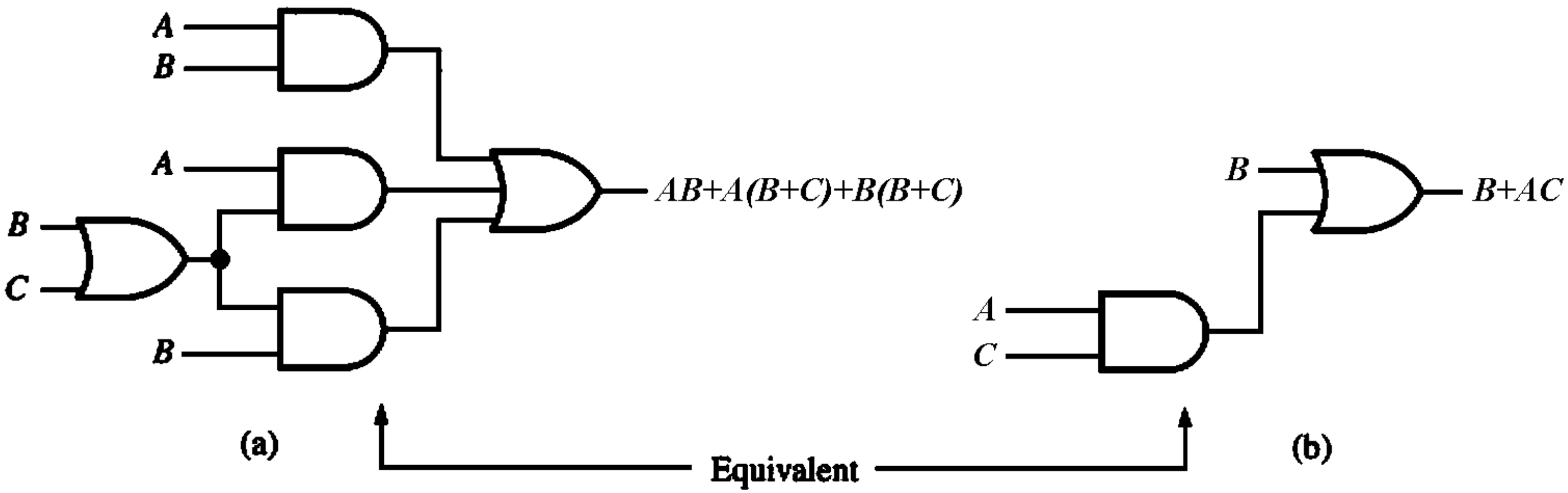
Truth Table → Boolean function

Inputs			Output
a	*b*	*c*	*f*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

f is 1 if (*a* = 0 AND *b* = 0 AND *c* = 1) OR
(*a* = 1 AND *b* = 0 AND *c* = 0) OR
(*a* = 1 AND *b* = 0 AND *c* = 1)

$$f(a, b, c) = \underbrace{a'b'c + ab'c' + ab'c}_{\text{Is it the simplest form?}}$$

1.4 Simplification using Boolean Algebra



- Prove that the above Circuit (a) is equivalent to Circuit (b).

Solution by Boolean Algebra Simplification

$AB + A(B + C) + B(B + C)$		
$AB + AB + AC + BB + BC$		
$AB + AB + AC + B + BC$	$BB=B$	*Idempotent*
$AB + AC + B + BC$	$AB+AB=AB$	*Idempotent*
$AB + AC + B$	$B+BC=B$	*Absorption*
$B + AC$	$AB+B=B$	*Absorption*

Boolean Algebra Simplification

Example 1

Simplify $f = \overline{AB+AC} + \overline{A}\,\overline{B}\,C$

$$= (\overline{AB})(\overline{AC}) + \overline{A}\,\overline{B}\,C$$

$$= (\overline{A}+\overline{B})(\overline{A}+\overline{C}) + \overline{A}\,\overline{B}\,C$$

$$= \overline{A}\,\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C$$

$$= \overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

$$= \overline{A} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

Example 2

Simplify $f = \overline{(A+B)\,\overline{C}\,\overline{D} + E + \overline{F}}$

$$= [\overline{(A+B)\,\overline{C}\,\overline{D}}][\overline{E+\overline{F}}]$$

$$= [\overline{(A+B)} + \overline{\overline{C}\;\overline{D}}][\overline{E}\;\overline{\overline{F}}]$$

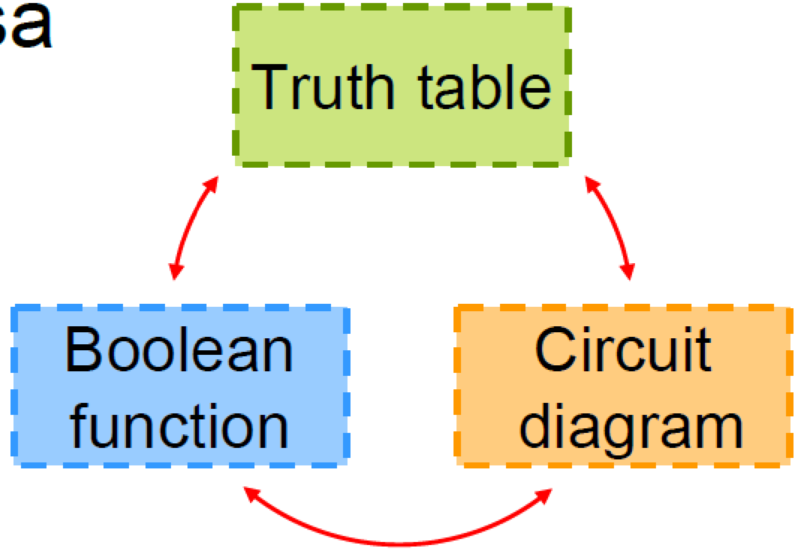
$$= (\overline{A}\,\overline{B} + \overline{\overline{C}} + \overline{\overline{D}})\;\overline{E}F$$

$$= (\overline{A}\,\overline{B} + C + D)\;\overline{E}F$$

- **Please write the properties of switching algebra for every steps**

Relationship

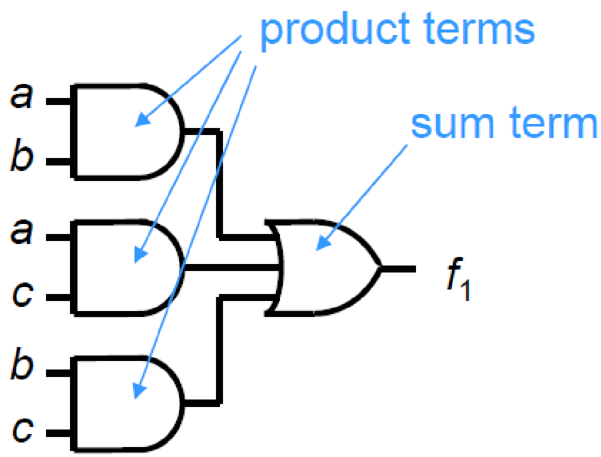
- Relation between Boolean function, truth table and logic circuit diagram
- A Boolean function can be represented by truth table and logic circuit diagram, and vice versa



SOP & POS Implementation

Sum of products

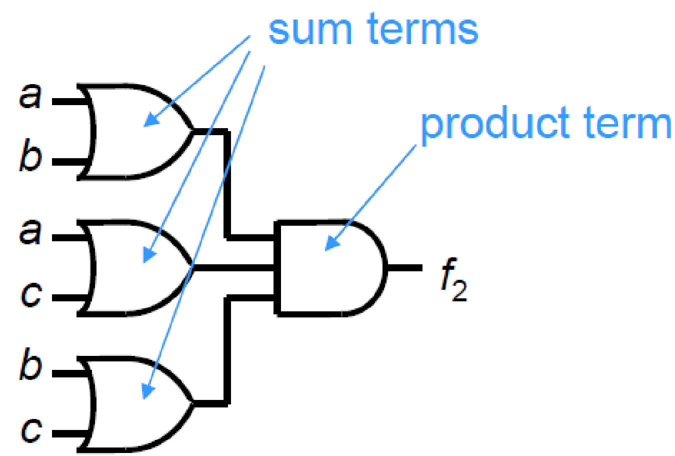
A group of AND gates followed by a single OR gate



$f_1(a, b, c) = ab + ac + bc$

Product of sums

A group of OR gates followed by a single AND gate



$f_2(a, b, c) = (a + b)(a + c)(b + c)$

Description for minterms and maxterms for 3 variables logic function

			Minterms		Maxterms	
x	y	z	Term	designation	term	designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'y\,z'$	m_2	$x + y' + z$	M_2
0	1	1	$x'y\,z$	m_3	$x + y' + z'$	M_3
1	0	0	$x\,y'z'$	m_4	$x' + y + z$	M_4
1	0	1	$x\,y'z$	m_5	$x' + y + z'$	M_5
1	1	0	$x\,y\,z'$	m_6	$x' + y' + z$	M_6
1	1	1	$x\,y\,z$	m_7	$x' + y' + z'$	M_7

Minterm and Maxterm

[Minterm] For a function of *n* variables, if a product term contains each of the *n* variables **exactly one time** in complemented or uncomplemented form, the product term is called *minterm.* Complement = 0 and Uncomplement =1.

Function	Minterm	Not minterm	Not minterm
f(*A, B, C*)	*A' B' C*	*(A B)' C*	*A'B'*

If the function is represented as a sum of minterms only, the function is in *standard sum of product (SOP)* form.

$$f(A,B,C) = \overline{A}B\overline{C} + AB\overline{C} + \overline{A}BC + ABC$$

Minterm	Code	Number
A'BC'	010	m_2
ABC'	110	m_6
A'BC	011	m_3
ABC	111	m_7

$$f(A,B,C) = m_2 + m_3 + m_6 + m_7$$

Canonical form $$f(A,B,C) = \sum m(2,3,6,7) = \sum(2,3,6,7)$$

Minterm and Maxterm

[Maxterm] If a sum term of a function of *n* variables contains each of the *n* variables **exactly one time** in complemented or uncomplemented form, the sum term is called a *maxterm.* Complement = 1 and Uncomplement =0.

Function	Maxterm	Not maxterm	Not maxterm
$f(A, B, C)$	$A' + B' + C$	$(A + B)' C$	$A' + B'$

If a function is represented as a product of maxterms only, the function is in *standard* product of sum (*POS)* form.

$$f(A,B,C) = (A+B+C)(A+B+\overline{C})(\overline{A}+B+C)(\overline{A}+B+\overline{C})$$

Minterm	Code	Number
$A + B + C$	000	M_0
$A + B + C'$	001	M_1
$A' + B + C$	100	M_4
$A' + B + C'$	101	M_5

$$f(A,B,C) = M_0 M_1 M_4 M_5$$

Canonical form $$f(A,B,C) = \prod M(0,1,4,5) = \prod(0,1,4,5)$$

Relationship

- $\overline{maxterm_i} = minterm_i$ (i.e. $\overline{M_i} = m_i$)
- $\overline{minterm_i} = maxterm_i$ (i.e. $\overline{m_i} = M_i$)

- e.g. $M_3' = (a + b' + c')'$
- $= a'bc$ (De Morgan's Theorem)
- $= m_3$

Example: Find *f*, *f'* in SSP form

	Inputs		Outputs	
a	*b*	*c*	*f*	*f'*
0	0	0	0	1
0	0	1	$\boxed{1}$	0
0	1	0	0	1
0	1	1	0	1
1	0	0	$\boxed{1}$	0
1	0	1	$\boxed{1}$	0
1	1	0	0	1
1	1	1	0	1

f is 1 if (a = 0 AND b = 0 AND c = 1) OR
(a = 1 AND b = 0 AND c = 0) OR
(a = 1 AND b = 0 AND c = 1)

$$f(a, b, c) = a'b'c + ab'c' + ab'c$$
$$= m_1 + m_4 + m_5$$
$$= \Sigma m(1, 4, 5)$$

$$f'(a, b, c) = a'b'c' + a'bc' + a'bc + abc' + abc$$
$$= m_0 + m_2 + m_3 + m_6 + m_7$$
$$= \Sigma m(0, 2, 3, 6, 7)$$

Abbreviated form:
Σ = logical sum (Boolean OR)

Example: Find *f*, *f'* in SPS form

Take the complement of f' to obtain f:

$$f(a, b, c) = (f')'$$

$$= (m_0 + m_2 + m_3 + m_6 + m_7)'$$

$$= m_0' \cdot m_2' \cdot m_3' \cdot m_6' \cdot m_7'$$

$$= M_0 \cdot M_2 \cdot M_3 \cdot M_6 \cdot M_7$$

$$= \Pi M(0, 2, 3, 6, 7) \qquad \text{or} = (a+b+c)(a+b'+c)(a+b'+c')(a'+b'+c)(a'+b'+c')$$

Following the same idea, we can obtain f' by:

$$f'(a, b, c) = (f)' = (m_1 + m_4 + m_5)'$$

$$= m_1' \cdot m_4' \cdot m_5'$$

$$= M_1 \cdot M_4 \cdot M_5$$

$$= \Pi M(1, 4, 5) \qquad \text{or} = (a+b+c')(a'+b+c)(a'+b+c')$$

Abbreviated form: Π = logical product (Boolean AND)