PHY1202

Assignment 1 - Solution

Due Date: 11:59 pm Tuesday, September 29th, 2020
Please submit your assignment:
1) To the Collection Box outside PHY GO Yeung G6702
2) Upload the softcopy of your assignment to Canvas

Lecture 01: Vectors

L01-01 (8 marks) What is the sum of the following four vectors in (a) unit-vector notation? For that sum, what are the (b) the magnitude, (c) the angle in degrees, and (d) the angle in radian.

$\vec{E}$: 6.00 *m at* $+0.900$ *rad*

$\vec{F}$: 5.00 *m at* -75.0°

$\vec{G}$: 4.00 *m at* $+1.20$ *rad*

$\vec{H}$: 6.00 *m at* -210°

Solution:

Angles are given in 'standard' fashion, so Eq. 3-5 applies directly. We use this to write the vectors in unit-vector notation before adding them. However, a very different-looking approach using the special capabilities of most graphical calculators can be imagined. Wherever the length unit is not displayed in the solution below, the unit meter should be understood.

(a) Allowing for the different angle units used in the problem statement, we arrive at

$$\vec{E} = 3.73\ \hat{\mathrm{i}} + 4.70\ \hat{\mathrm{j}}$$
$$\vec{F} = 1.29\ \hat{\mathrm{i}} - 4.83\ \hat{\mathrm{j}}$$
$$\vec{G} = 1.45\ \hat{\mathrm{i}} + 3.73\ \hat{\mathrm{j}}$$
$$\vec{H} = -5.20\ \hat{\mathrm{i}} + 3.00\ \hat{\mathrm{j}}$$
$$\vec{E} + \vec{F} + \vec{G} + \vec{H} = 1.28\ \hat{\mathrm{i}}\ + 6.60\ \hat{\mathrm{j}}.$$

(b) The magnitude of the vector sum found in part (a) is $\sqrt{(1.28\text{ m})^2 + (6.60\text{ m})^2} = 6.72\text{ m}$.

(c) Its angle measured counterclockwise from the $+x$ axis is $\tan^{-1}(6.60/1.28) = 79.0°$.

(d) Using the conversion factor π rad = 180°, 79.0° = 1.38 rad.

L01-02 (6 marks) If $\vec{B}$ is added to $\vec{A}$, the result is $6.0\hat{i} + 1.0\hat{j}$. If $\vec{B}$ is subtracted from $\vec{A}$, the result is $-4.0\hat{i} + 7.0\hat{j}$. Find $\vec{A}$ and $\vec{B}$.

Solution:

From the problem statement, we have

$$\vec{A} + \vec{B} = (6.0)\hat{i} + (1.0)\hat{j}$$
$$\vec{A} - \vec{B} = -(4.0)\hat{i} + (7.0)j$$

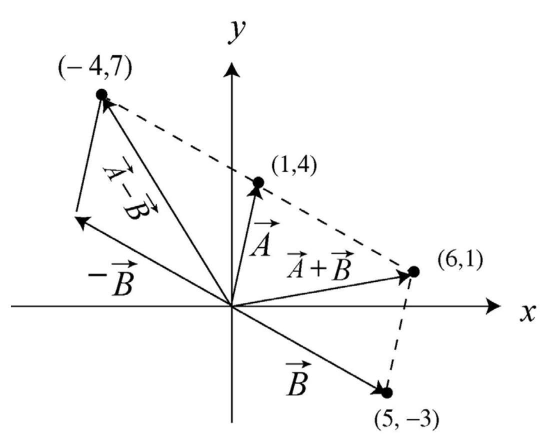
Adding the above equations and dividing by 2 leads to $\vec{A} = (1.0)\hat{i} + (4.0)\hat{j}$. Thus, the magnitude of $\vec{A}$ is

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(1.0)^2 + (4.0)^2} = 4.1$$

Similarly, the vector $\vec{B}$ is $\vec{B} = (5.0)\hat{i} + (-3.0)\hat{j}$, and its magnitude is

$$B = |\vec{B}| = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.0)^2 + (-3.0)^2} = 5.8.$$

The results are summarized in the figure below:



L01-03 (10 marks) Vector $\vec{A}$ and $\vec{B}$ lie in xy plane (with no z components), $\vec{A}$ has magnitude 8.00 and angle 130°, $\vec{B}$ has component $B_x = -7.72$ and $B_y = -9.20$.

a) What is $5\vec{A} \cdot \vec{B}$?
b) What is $4\vec{A} \times 3\vec{B}$ in unit-vector notation?

c) What is the angle between $\vec{A}$ and $4\vec{A}\times 3\vec{B}$?

d) What is $\vec{A}+3.00\,\hat{k}$ in unit-vector notation and in magnitude-angle notation with spherical coordinates R,θ,ϕ.

Solution:

The two vectors are given by

$$\vec{A} = 8.00(\cos 130°\,\hat{\mathrm{i}} + \sin 130°\,\hat{\mathrm{j}}) = -5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}}$$
$$\vec{B} = B_x\hat{\mathrm{i}} + B_y\hat{\mathrm{j}} = -7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}.$$

(a) The dot product of $5\vec{A}\cdot\vec{B}$ is

$$5\vec{A}\cdot\vec{B} = 5(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\cdot(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}) = 5[(-5.14)(-7.72) + (6.13)(-9.20)]$$
$$= -83.4.$$

(b) In unit vector notation

$$4\vec{A}\times 3\vec{B} = 12\vec{A}\times\vec{B} = 12(-5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}})\times(-7.72\,\hat{\mathrm{i}} - 9.20\,\hat{\mathrm{j}}) = 12(94.6\,\hat{\mathrm{k}}) = 1.14\times 10^3\,\hat{\mathrm{k}}$$

(c) Since $\vec{A}$ is in the *xy* plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90°.

(d) Clearly, $\vec{A} + 3.00\,\hat{\mathrm{k}} = -5.14\,\hat{\mathrm{i}} + 6.13\,\hat{\mathrm{j}} + 3.00\,\hat{\mathrm{k}}$.

From the Pythagorean theorem yields magnitude $A = \sqrt{(5.14)^2 + (6.13)^2 + (3.00)^2} = 8.54$. The azimuthal angle is θ = 130°, just as it was in the problem statement ($\vec{A}$ <u>is</u> the projection onto the *xy* plane of the new vector created in part (e)). The angle measured from the +*z* axis is

$$\phi = \cos^{-1}(3.00/8.54) = 69.4°.$$

Lecture 02: Electric Charge

L02-01 (10 marks) Point charges of $q_1 = +6.0\ \mu\text{C}$ and $q_2 = -4.0\ \mu\text{C}$ are placed on an x axis, at $x = 8\ \text{m}$ and $x = 16\ \text{m}$, respectively. What charge q_3 must be placed at $x = 24\ \text{m}$ so that any charge q placed at the origin would experience no electrostatic force?

Solution

We are looking for a charge q that, when placed at the origin, experiences $\vec{F}_{\text{net}} = 0$, where

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 .$$

The magnitude of these individual forces are given by Coulomb's law, Eq. 21-1, and without loss of generality we assume $q > 0$. The charges q_1 (+6 μC), q_2 (–4 μC), and q_3 (unknown), are located on the $+x$ axis, so that we know $\vec{F}_1$ points toward $-x$, $\vec{F}_2$ points toward $+x$, and $\vec{F}_3$ points toward $-x$ if $q_3 > 0$ and points toward $+x$ if $q_3 < 0$. Therefore, with $r_1 = 8$ m, $r_2 = 16$ m and $r_3 = 24$ m, we have

$$0 = -k\frac{q_1 q}{r_1^2} + k\frac{|q_2| q}{r_2^2} - k\frac{q_3 q}{r_3^2}.$$

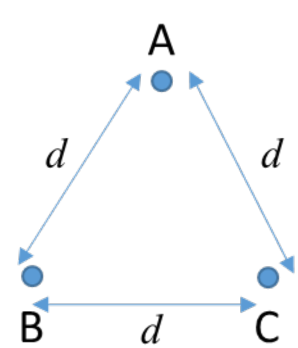
Simplifying, this becomes

$$0 = -\frac{6}{8^2} + \frac{4}{16^2} - \frac{q_3}{24^2}$$

where q_3 is now understood to be in μC. Thus, we obtain $q_3 = -45\ \mu$C.

L02-02 (12 marks) Three identical conducting spheres as shown in the diagram form an equilateral triangle of side length $d = 30.0\ cm$. The sphere radii are much smaller than d, so that they can be considered as point charges with $q_A = -2.00\ \text{nC}$, $q_B = -4.00\ \text{nC}$, and $q_C = +8.00\ \text{nC}$. The following steps are then taken:

i. A and B are connected by a thin wire and then disconnected
ii. B is then grounded
iii. B and C are connected by a thin wire and then disconnected.



a) What was the electrostatic force between spheres A and C before step (i), (before A and B were connected by the thin wire)?

b) What are the new charges on A, B and C, after steps (i), (ii) and (iii)?

c) What is the magnitude of the electrostatic force between A and C after step (iii)?

d) What is the magnitude of the electrostatic force between A and B after step (iii)?

Solution:

(a) Since $q_A = -2.00$ nC and $q_C = +8.00$ nC,

$$|\vec{F}_{AC}| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{|(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(-2.00\times10^{-9}\ \mathrm{C})(8.00\times10^{-9}\ \mathrm{C})|}{(0.300\ \mathrm{m})^2} = 1.60 \text{ x } 10^{-6}\ \mathrm{N}$$

(b) After making contact with each other in step (i), both *A* and *B* have a charge of

$$\frac{q_A + q_B}{2} = \left(\frac{-2.00 + (-4.00)}{2}\right)\ \mathrm{nC} = -3.00\ \mathrm{nC}.$$

While C have the same charge as before q_C=+8.00 nC.

When *B* is grounded in step (ii) its charge is zero, while A and C did not change.

After *B* making contact with *C* in step (iii), which has a charge of +8.00 nC, *B* acquires a charge of [0 + (–8.00 nC)]/2 = –4.00 nC, which charge *C* has as well.

Finally, after step (iii) we have $Q_A = -3.00$ nC and $Q_B = Q_C = -4.00$ nC.

(c)

$$\left|\vec{F}_{AC}\right| = \frac{|q_A q_C|}{4\pi\varepsilon_0 d^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(-3.00\times10^{-9}\mathrm{C})(-4.00\times10^{-9}\mathrm{C})}{(0.300\ \mathrm{m})^2} = 1.20\times10^{-6}\ \mathrm{N}.$$

(d) We also obtain

$$\left|\vec{F}_{AB}\right| = \frac{|q_A q_B|}{4\pi\varepsilon_0 d^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(-3.00\times10^{-9}\mathrm{C})(-4.00\times10^{-9}\mathrm{C})}{(0.300\ \mathrm{m})^2} = 1.20\times10^{-6}\ \mathrm{N}.$$

L02-03 (10 marks) We know that the negative charge on the electron and the positive charge on the proton are equal. Suppose, however, that these magnitudes differ from each other by 0.00010%. With what force would two copper coins, placed 1.0 m apart, repel each other? Assume that each coin contains 3×10^{22} copper atoms. (*Hint:* A neutral copper atom contains 29 protons and 29 electrons.) What do you conclude?

Solution:

If the relative difference between the proton and electron charges (in absolute value) were

$$\frac{q_p - |q_e|}{e} = 0.0000010$$

then the actual difference would be $q_p - |q_e| = 0.0000010e = 1.6\times10^{-25}$ C Amplified by a factor of 29 × 3 × 10^{22} as indicated in the problem, this amounts to a deviation from perfect neutrality of

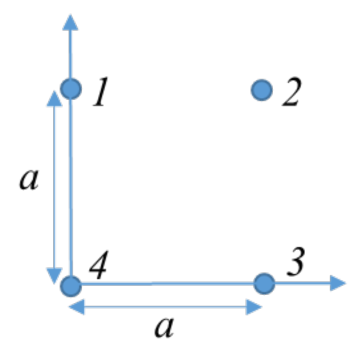
$$\Delta q = 29\times3\times10^{22}\times1.6\times10^{-25}\,\mathrm{C} = 0.14\,\mathrm{C}$$

in a copper penny. Two such pennies, at r = 1.0 m, would therefore experience a very large force. Equation 21-1 gives

$$F = k\frac{\Delta q^2}{r^2} = 1.74\times10^{8}\ \mathrm{N}.$$

Lecture 03: Electric Field

L03-01 (10 marks) Four particles form a square of edge length $a = 5.00\,\mathrm{cm}$ and have charges $q_1 = +10.0\,\mathrm{nC}$, $q_2 = -20.0\,\mathrm{nC}$, $q_3 = +20.0\,\mathrm{nC}$, and $q_4 = -10.0\,\mathrm{nC}$. In unit-vector notation, what net electric field do the particles produce at the square's center?



Solution:

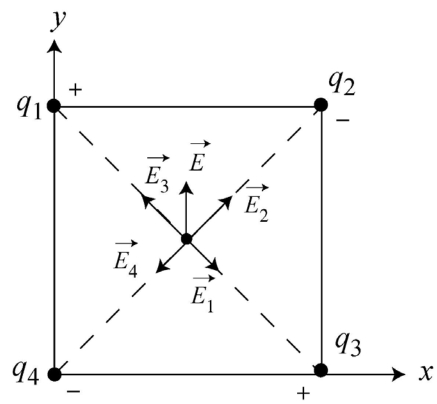
The x component of the electric field at the center of the square is given by

$$\begin{aligned}
E_x &= \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}-\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45^\circ \\
&= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(|q_1|+|q_2|-|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\
&= 0.
\end{aligned}$$

Similarly, the y component of the electric field is

$$\begin{aligned}E_y &= \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2}+\frac{|q_2|}{(a/\sqrt{2})^2}+\frac{|q_3|}{(a/\sqrt{2})^2}-\frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45° \\ &= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(-|q_1|+|q_2|+|q_3|-|q_4|\right)\frac{1}{\sqrt{2}} \\ &= \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(2.0\times10^{-8}\ \mathrm{C})}{(0.050\ \mathrm{m})^2/2}\frac{1}{\sqrt{2}} = 1.02\times10^5\ \mathrm{N/C}.\end{aligned}$$

Thus, the electric field at the center of the square is $\vec{E}=E_y\hat{\mathrm{j}}=(1.02\times10^5\ \mathrm{N/C})\hat{\mathrm{j}}$. The net electric field is depicted in the figure below (not to scale). The field, pointing to the $+y$ direction, is the vector sum of the electric fields of individual charges.



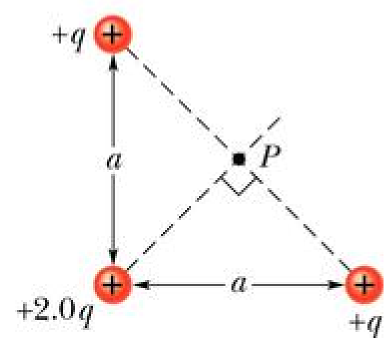
L03-02 (12 marks) Calculate the direction and magnitude of the electric field at point P in the figure, due to the three point charges.

Solution:

By symmetry we see the contributions from the $+q$ changes cancel each other at point P, and the field at P is only due to the $+2.0q$ change, this field points at 45º where

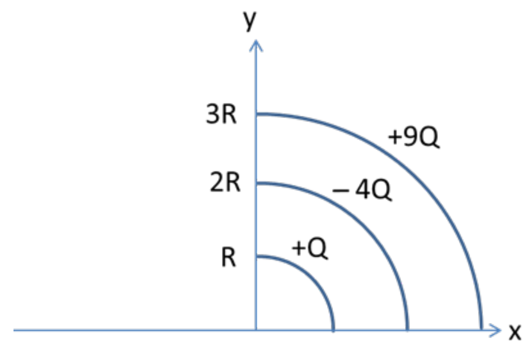
$$\left|\vec{E}_{net}\right| = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$$

With $r=a/\sqrt{2}$, we have $\left|\vec{E}_{net}\right| = q/\pi\varepsilon_0 a^2$.



L03-03 (12 marks) The Figure below shows three circular arcs centered on the origin of a coordinate system. On each arc, the uniformly distributed charge is given in terms of $Q = 2.00\ \mu\text{C}$. The radii are given in terms of $R = 100\ \text{cm}$. What are a) magnitude and b) direction (relative to the positive x direction) of the net electric field at the origin due to the arcs?

Solution:



The smallest arc is of length $L_1 = \pi r_1/2 = \pi R/2$; the middle-sized arc has length $L_2 = \pi r_2/2 = \pi(2R)/2 = \pi R$; and, the largest arc has $L_3 = \pi(3R)/2$.

The charge per unit length for each arc is $\lambda = q/L$ where each charge q is specified in the figure.

Please note that the equation given on the first page $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$ *is for a long straight line of charge.*

This situation is different as the line charge is shaped into an arc. To solve this problem, we will need to use the method described in the example in L03.

If we rotate the arcs clockwise by 45°, the y-component of the field will be cancelled out because of symmetry. Then following the example in L03 slide 17 except for a positive line charge density, we have

$$E = \int dE_x = -\frac{1}{4\pi\varepsilon_0}\int_{-45}^{45}\frac{\lambda}{r^2}\cos\theta r d\theta = -\frac{\lambda}{4\pi\varepsilon_0 r}\int_{-45}^{45}\cos\theta d\theta = -\frac{\lambda}{4\pi\varepsilon_0 r}[\sin\theta]_{-45}^{45}$$

$$= -\frac{\lambda}{4\pi\varepsilon_0 r}\left[\sin(45^\circ) - \sin(-45^\circ)\right] = -\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r}$$

$$E_x = -\frac{\sqrt{2}}{4\pi\varepsilon_0}\left[\frac{\lambda_1}{R} + \frac{\lambda_2}{2R} + \frac{\lambda_3}{3R}\right] = -\frac{\sqrt{2}}{4\pi\varepsilon_0}\left[\frac{2Q_1}{\pi R^2} - \frac{(2)(4Q)}{\pi(2R)^2} + \frac{(2)(9Q)}{\pi(3R)^2}\right]$$

$$= -\frac{\sqrt{2}}{4\pi\varepsilon_0}\left[\frac{2Q}{\pi R^2} - \frac{2Q}{\pi R^2} + \frac{2Q}{\pi R^2}\right] = -\frac{Q}{\sqrt{2}\varepsilon_0(\pi R)^2}$$

which yields $E_{net} = 1.62 \times 10^4$ N/C .

(b) The direction is 135°, measured clockwise from the $+x$ axis or is 225°, measured counterclockwise from the $+x$ axis.