MA1200 MIDTERM EXAM FRIDAY 9:05 AM -10:05 AM, E F G H

Q1. (30 points) Write $9x^2 - 16y^2 - 36x + 32y = 124$ into the standard form, find foci, center, and vertices, (asymptotes if it is a hyperbola), and sketch the graph of it.

Q2. (15 points) Find the largest possible domains and the ranges of the following functions:

$$f(x) = \log_2(4 - x^2) \quad \text{and} \quad g(x) = \log_2(8 - x^3).$$

Q3. (20 points) Express $\dfrac{-7x + 29}{(x+1)(x^2 - 4x + 13)}$ as partial fractions.

Q4. (20 points) Simplify $\cos(\sin^{-1}(-\frac{3}{5}) + \tan^{-1}(-\frac{5}{12}))$.
(Hint: $\cos(A + B) = \cos A \cos B - \sin A \sin B$, $3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$))

Q5. (15 points) Solve $\sin(2x + \pi/3) = 1/2$ in radians.

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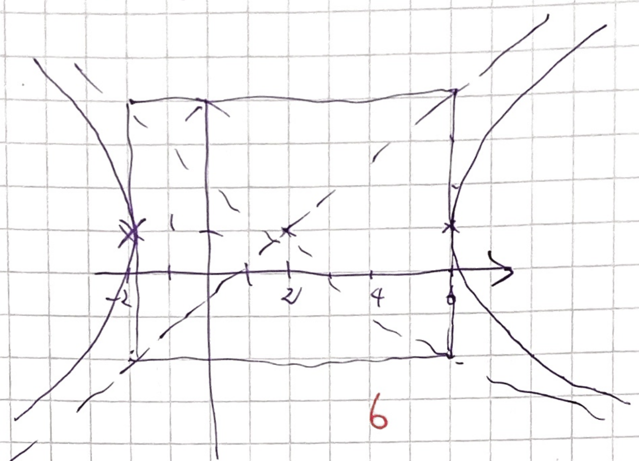
$9x^2 - 16y^2 - 36x + 32y = 124$

$9(x^2 - 4x) - 16(y^2 - 2y) = 124$

$+4 \qquad +1 \qquad +36 \; -16$

$9(x-2)^2 - 16(y-1)^2 = 144$

$\frac{(x-2)^2}{16} - \frac{(y-1)^2}{9} = 1$ 10



Center $(2,1)$ 2

$c = \sqrt{a^2+b^2} = 5$. Foci $(2 \pm 5, 1) = (7,1), (-3,1)$ 5

Vertices $(6,1), (-2,1)$ 3

asymptotes: $y - 1 = \pm \frac{3}{4}(x-2)$ 4

30

$g(x) = \log_2 (8 - x^3)$

domain

$8 - x^3 > 0 \Rightarrow x^3 < 8 \Rightarrow$

$x < 2$. 3

$\Rightarrow$ range is $(-\infty, \infty)$ since $8 - x^3$ can be any number > 0.

5

15

$f(x) = \log_2 (4 - x^2)$

domain $x \in (-2, 2)$ 3

range $(-\infty, 2]$ 4

Q3 Date 2-2

$$\frac{-7x+29}{(x+1)(x^2-4x+13)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-4x+13}$$

$x^2-4x+13$, $\Delta = 16-4\cdot 13 < 0$ 7

$-7x+29 = A(x^2-4x+13)+(x+1)(Bx+C)$

$x=-1 \Rightarrow 7+29 = A(1+4+13) \Rightarrow A=2$ 4

$x=0 \Rightarrow 29 = 13A+C = 13\cdot 2 + C \Rightarrow C=3$ 4

coeff of $x^2 \Rightarrow 0 = A+B \Rightarrow B=-2$ 4

thus $= \frac{2}{x+1} + \frac{-2x+3}{x^2-4x+13}$ 1

20

Q4. $\cos\left(\sin^{-1}\left(-\frac{3}{5}\right) + \tan^{-1}\left(-\frac{5}{12}\right)\right)$

$A = \sin^{-1}\left(-\frac{3}{5}\right)$

$\Rightarrow \sin A = -3/5$. $A \in \left(-\frac{\pi}{2}, 0\right)$, $\cos A = \frac{4}{5}$. 7

$B = \tan^{-1}\left(-\frac{5}{12}\right)$ $B \in (-\pi/2, 0)$, $\sin B = -\frac{5}{13}$ 7

$\cos B = \frac{12}{13}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ 6

$= \frac{4}{5}\cdot\left(\frac{12}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{-5}{13}\right) =$

$= \frac{48-15}{65} = 33/65$

20.

$\frac{5}{6} - \frac{3}{6}$
$= \frac{2}{6}$

Date

Q5. $\sin\left(2x+\frac{\pi}{3}\right) = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

$\Rightarrow 2x+\frac{\pi}{3} = n\pi + (-1)^n \frac{\pi}{6}$

$x = \frac{n}{2}\pi + (-1)^n \frac{\pi}{12} - \frac{\pi}{6}. \quad n \in \mathbb{Z}.$

7
8
> 15

or. $2x+\frac{\pi}{3} = 2n\pi + \frac{\pi}{6}$ or $2x+\frac{\pi}{3} = 2n\pi + \frac{5}{6}\pi$ 4

$\Rightarrow x = n\pi - \frac{\pi}{12}$ or $x = n\pi + \frac{\pi}{6} \quad n \in \mathbb{Z}.$ 4