1. **There are ELEVEN (11) questions (total: 100 marks). Answer ALL questions.**
2. **Observe rules demanded by CityU.**
3. **EE1001 Test 2 on 13 April 2021 Tuesday (week 12), starts 10:15am, duration: 10:15am - 11:15 am**
4. **Please write your FULL English name, SID, convert final answers to a pdf file and upload Test 2 answer scripts onto CANVAS "TEST 2_KF Tsang_RETURN" on or before 11:25am on 13 April 2021.**

EE1001 Test 2

Q1) (10 marks)

Determine the validity of the following arguments and brief the reasons.

(i) If it rains today, we would not go swimming. We go swimming today. Therefore, it did not rain today.

(ii) Alex is a history major. Thus, Alex is either an engineering major or a history major.

(iii) If Smith plays football, then he will stay outside too long. If Smith stays outside too long, then he will get heatstroke. Therefore, if Smith plays football, he will get heatstroke.

(iv) If I am tired and if I does not spend time on the mobile phone, then I will sleep early. I didn't sleep early. Therefore, I was not tired and I spent time on the mobile phone.

(v) If Kate doesn't work hard, then she will fail in this course. If Kate doesn't fail in this course, then she will get the scholarship. Kate gets the scholarship. Therefore, Kate worked hard.

Q2) (10 marks)

Without using a truth table, determine whether $\sim(a \vee \sim b) \wedge (\sim b \vee c) \rightarrow (a \vee c)$ is a tautology. State the reason for each step.

Q3) (10 marks)

Annie, Ben, and Charis are suspected of a crime. Their statements are as follows:
Annie: "At least one of us is guilty."
Ben: "I'm innocent, and at least one of the others is innocent."
Charis: "If Ben is guilty, then Annie is also guilty."

Let A, B, and C be
A = "Annie is innocent"
B = "Ben is innocent"
C = "Charis is innocent"

(i) Formulate the statements of Annie, Ben, and Charis.

(ii) Given that the innocent told the truth and the guilty lied, who is innocent and who is guilty?

Q4) (10 marks)

Given the initial condition of $a_1 = -5$ and the general formula of $a_n = -2a_{n-1} - 3$

(i) Express the general formula using an explicit formula.

(ii) Determine the value of the 13^{th} term, i.e., a_{13}.

Q5) (10 marks)

Given the following sequence of positive integers:

{1, 3, 5, …, 2017, 2019, 2021}

(i) Determine the sum of the integers.

(ii) The powers of 3 (i.e., 3^0, 3^1, 3^2, …) are now removed from the above sequence. Determine the sum of the remaining integers.

Q6 (8 marks)

Consider the word *BUILDN*. With or without meaning, to form words with length of 6, considering each of the following circumstance:

(i) If the letters can be reused (repeat), how many arrangements can be made using the letters?
(ii) If the letters cannot be reused, how many arrangements can be made using all the letters?
(iii) If the letters cannot be reused, how many arrangements begin with the letter N?
(iv) If the letters cannot be reused, how many arrangements begin with the letter N and end with D?
(v) If the letters cannot be reused, how many arrangements with DIN?
(vi) If the letters cannot be reused, how many arrangements end with DIN?
(vii) If the letters cannot be reused, how many arrangements begin with N or end in D?

Q7 (10 marks)

(i) Find $(2x-1)^6-(2x+1)^6$ with binomial theorem.
(ii) Compute $(\sqrt{3}-1)^6-(\sqrt{3}+1)^6$.

Q8 (8 marks)

In a standard deck of cards, there are 52 cards.
- There are 4 of each card (4 Aces, 4 Kings, 4 Queens, etc.)
- There are 4 suits (Clubs, Hearts, Diamonds, and Spades) and there are 13 cards in each suit (Clubs/Spades are black, Hearts/Diamonds are red)

You are playing a game with one of your friends.

(i) You and your friend are required to pick 5 cards in total. In how many ways can 5 cards be picked so that 3 are with you and 2 are with your friend?

For instance:
you pick 2 cards → your friend pick 2 cards → you pick 1 card again
you pick 3 cards → your friend pick 2 cards

(ii) You need to pick two cards from the 52 cards. What is the probability that two cards you have picked are a pair (such as heart 9 and diamond 9)?

(iii) Both of you and your friend need to pick two cards from the 40 cards with only numbers (No King, queen…). Your friend first picks a "8" and a "7" and you need to pick 2 cards from the remaining cards. What is the probability that the sum of your cards is larger than your friend?

Q9 (4 marks)

Let A={x|x<12}, B={x|x≤8}, C={x|x=2k, k=1,2,3,4,5…}. Identify the following sets with A, B, and C.

(i) {10};
(ii) {x| x is odd integer that are larger than 12};

Q10 (10 marks)

Suppose there is a wooden stick with a length of 200cm.

i) Leo draws marks on the stick every 3cm from the left end to the right end. Leo then passes the stick to Bob.
ii) Bob draws marks on the stick every 4cm from the left end to the right end.
iii) Tom draws marks on the stick every 5cm from the left end to the right end. (Noticed that marks from different students can be overlapped).
iv) Finally, Jack obtains this stick (short sticks) by cutting the stick conforming to these marks. How many short sticks will Jack obtain?

Q11 (10 marks)

In the plane, there are *n* non-parallel straight lines. These lines will form several angles (Fig. Q11 a). To ensure there is at least one angle less than 26 degrees, what is the minimal number of lines? Why?
(Tip: The translation of the straight line does not change the angle, shown in Fig. Q11 b)

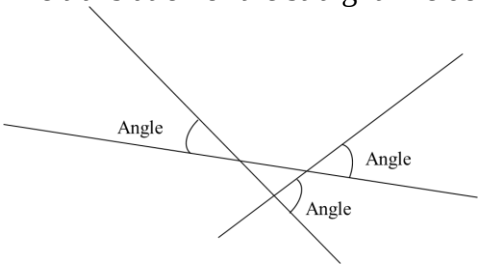

Fig. Q11 a

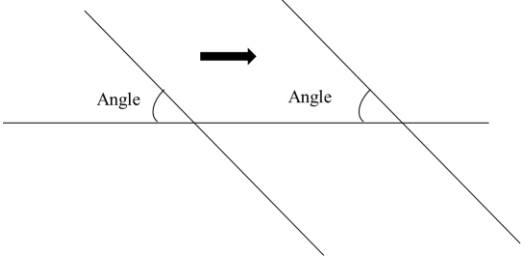

Fig. Q11 b

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