

**EE 2000 Logic Circuit
Design Semester A 2021/22A**

Tutorial 1

1. Simplify the following expressions using boolean algebra

$$\begin{aligned}
 \text{(i)} \quad f(x, y, z) &= xz + yz + x'yz + xy \\
 &= x'z(1+y') + yz' + xy \\
 &= x'z(1) + (x + x')yz' + xy \\
 &= x'z + xyz' + x'yz' + xy \\
 &= x'(z + yz') + xy(1 + z') \\
 &= x'(z + y) + xy \\
 &= x'z + x'y + xy \\
 &= x'z + y(x + x') \\
 &= x'z + y
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad F(A, B, C) &= \overline{AB}(\overline{B+C})(A+C) \\
 &= \overline{AB}(AB' + AC' + B'C + CC') \\
 &= \overline{AB} + \overline{AB}C' + \overline{AB}C \\
 &= \overline{AB}
 \end{aligned}$$

2. Simplify the following expressions using k-map.

$$\begin{aligned}
 \text{(a)} \quad f &= \overline{abc} + \overline{b}cd + \overline{bcd} + \overline{acd} + \overline{a}bcd + \overline{abcd} \\
 \text{(b)} \quad f &= wxy + yz + xyz + xy \\
 \text{(c)} \quad f(a, b, c, d) &= \Sigma m(4, 6, 7, 15) \\
 \text{(d)} \quad f(a, b, c, d) &= \Sigma m(3, 7, 11, 13, 14, 15) \\
 \text{(e)} \quad f(a, b, c, d) &= \Sigma m(0, 6, 8, 13, 14) + \Sigma d(2, 4, 10) \\
 \text{(f)} \quad f(a, b, c, d) &= \Sigma m(1, 3, 5, 7, 9, 15) + \Sigma d(4, 6, 12, 13)
 \end{aligned}$$

Answer:

$$\begin{aligned}
 \text{(a)} \quad &ac + b'd' + a'bd + b'c \text{ or } ac + b'd' + a'bd + cd \\
 \text{(b)} \quad &xz + wy + x'y \\
 \text{(c)} \quad &BCD + A'BD' \\
 \text{(d)} \quad &CD + ABC + ABD \\
 \text{(e)} \quad &B'D' + CD' + ABC'D \\
 \text{(f)} \quad &A'D + BD + C'D
 \end{aligned}$$

3. Map the following SOP expression on a K-map.

$$BC + AB + ABC + ABCD + ABCD + ABCD$$

Ans:

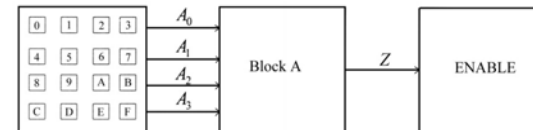
		CD			
AB	00	00	01	11	10
	00	1	1		
	01				
	11	1	1		
	10	1	1	1	1

4. A simple locker system that has a output signal $Z = 1$ when the hexadecimal keypad input is either 0, 1, 2, 3, 4, 5, 8, 9, or A; otherwise $Z = 0$. Assume that $A_0 A_1 A_2 A_3$ represent a 4-digit binary number output from the keypad. Block A decodes these signals and outputs the signal Z.

- (i) Write down the truth table of Block A.

- (ii) Find the SOP and POS expression of Block A.

- (iii) Design the circuit of Block A using minimum number of AND, OR and NOT gates.



(The answer can be different if $A_0 A_1 A_2 A_3$ represent by Gray Code, and others.)

- (i) Assume a conventional BCD for $A_0 A_1 A_2 A_3$.

Hexadecimal	A_0	A_1	A_2	A_3	Z
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
A	1	0	1	0	1
B	1	0	1	1	0

C	1	1	0	0	0
D	1	1	0	1	0
E	1	1	1	0	0
F	1	1	1	1	0

(ii)

		$A_0 A_1$			
		00	01	11	10
$A_2 A_3$	00	1	1		1
	01	1	1		1
	11	1			
	10	1			1

SOP: $f = A'_0 A'_1 + A'_0 A'_2 + A'_1 A'_2 + A'_1 A'_3$

		$A_0 A_1$			
		00	01	11	10
$A_2 A_3$	00			0	
	01			0	
	11		0	0	0
	10		0	0	

$f' = A_0 A_1 + A_1 A_2 + A_0 A_2 A_3$
 POS: $f = (A'_0 + A'_1)(A'_1 + A'_2)(A'_0 + A'_2 + A'_3)$
 (By DeMorgan)

(iii) Simplify the answer from SOP:

$$\begin{aligned}
 f &= A'_0 A'_1 + A'_0 A'_2 + A'_1 A'_2 + A'_1 A'_3 \\
 &= A'_0 (A'_1 + A'_2) + A'_1 (A'_2 + A'_3) \\
 &= A'_0 (A_1 A_2)' + A'_1 (A_2 A_3)'
 \end{aligned}$$

(Please draw the circuit.)

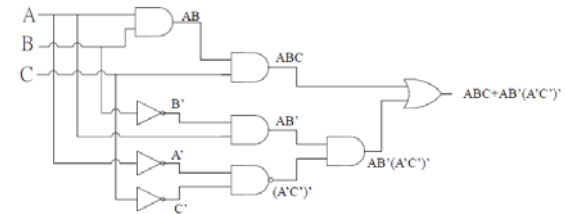
5. (a) Draw the logic circuit of the following Boolean function without simplification:

$$Y(A, B, C) = ABC + AB'(A'C')'$$

(b) Simplify the logic function in (a) and, hence, re-draw the logic circuit.

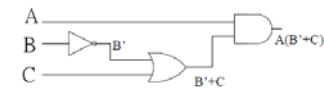
Ans:

(a)

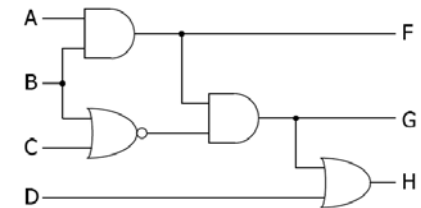


(b) $Y(A, B, C)$

$$\begin{aligned}
 &= ABC + AB'(A'C')' = ABC + AB'(A + C) = ABC + AB' + AB'C = A(B + B')C + AB' \\
 &= AC + AB' = A(B' + C)
 \end{aligned}$$



6. (a) Derive Boolean functions to describe the operations of the logic circuit as follow:



(b) Hence, simply and draw a new logic circuit.

7. (a) Use a K-map to convert the following canonical SOP expression into a minimum POS expression.

$$f(a, b, c, d) = \sum m(0, 1, 2, 5, 7, 8, 10, 13, 14, 15)$$

		ab			
		00	01	11	10
cd	0	0	0	1	1
	1	0	1	1	0

00	0	0
01		0
11	0	0
10	0	

$$f(a,b,c,d) = (b' + c + d)(a' + b + d')(a + b' + d)(b + c' + d')$$

(b) Use a K-map to convert the following canonical POS expression into a minimum SOP expression.

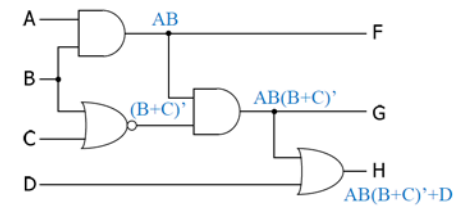
$$f(a,b,c,d) = \prod M(0,2,5,7,8,9,13)$$

	<i>ab</i>			
<i>cd</i>	0	0	1	1
	0	1	1	0
00		1	1	
01	1			
11	1		1	1
10		1	1	1

$$f(a,b,c,d) = ac + bd' + a'b'd$$

Ans:

(a)



(b)

Therefore,

$$F = AB$$

$$G = AB(B+C)' = ABB'C' = 0$$

$$H = AB(B+C)' + D = D$$

