# EE3008 Test 1

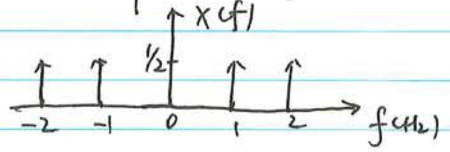
# (1:00-2:30pm, Feb. 24, 2020)

**Question 1 (33 marks)**

Solution:

1. (1) Both $x(t)$ and $y(t)$ are power-type signals because they are periodic signals.

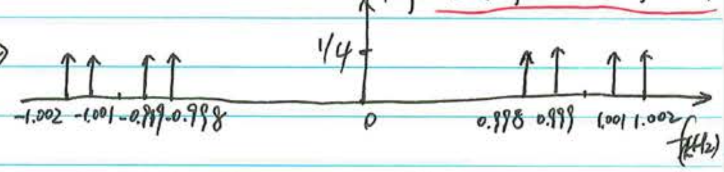
Fourier Spectrum of $x(t)$



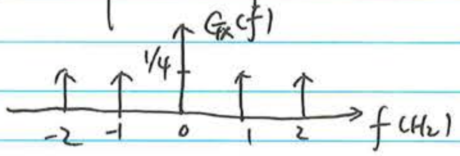
$\Rightarrow$

Fourier Spectrum of $y(t)$

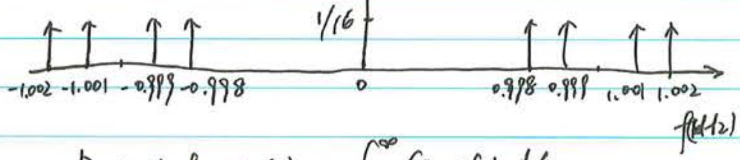
$Y(f) = \frac{1}{2}\left(X(f-1000) + X(f+1000)\right)$



Power Spectrum of $x(t)$



Power Spectrum of $y(t)$



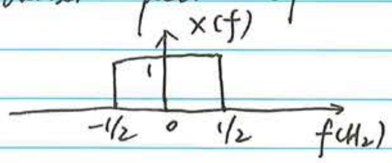
power of $x(t) = \int_{-\infty}^{\infty} G_x(f)\, df$

$= 1/4 \times 4 = 1$

Power of $y(t) = \int_{-\infty}^{\infty} G_Y(f)\, df$

$= 1/16 \times 8 = \frac{1}{2}$

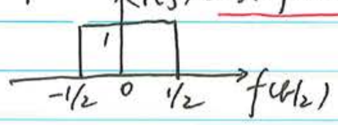
(2) Fourier Spectrum of $x(t)$



$\Rightarrow$

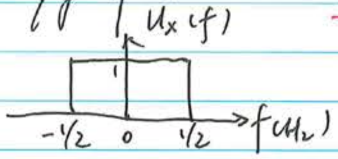
Fourier Spectrum of $y(t)$

$Y(f) = X(f) \cdot H(f) = X(f)$



Both $x(t)$ & $y(t)$ are energy-type signals because of finite energy.

Energy spectrum: $U_x(f) = U_Y(f) = |X(f)|^2 = |Y(f)|^2$

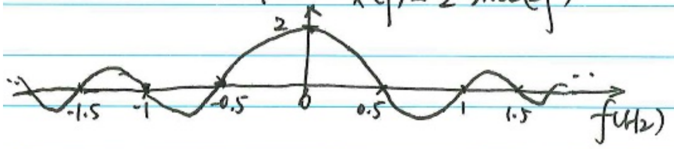


Energy: $\int_{-\infty}^{\infty} U_x(f)\, df = \int_{-\infty}^{\infty} U_Y(f)\, df = 1$

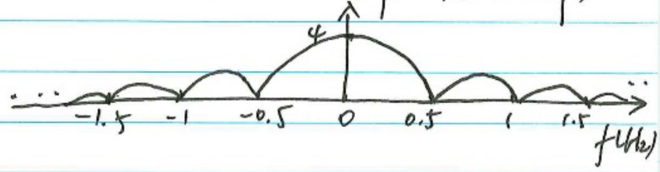
(3) $x(t)$ is an energy-type signal because its energy is finite:

$$\int_{-\infty}^{\infty} x^2(t)\,dt = 2.$$

Fourier Spectrum of $x(t)$ $\quad X(f) = 2\,\mathrm{sinc}(2f)$




Energy Spectrum of $x(t)$ $\quad \Psi_X(f) = 4\,\mathrm{sinc}^2(2f)$

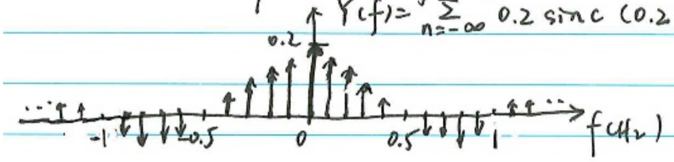



$y(t) = x(t) * h(t)$ is a rectangular pulse train with period 10 s.

It is a power-type signal because it is periodic.

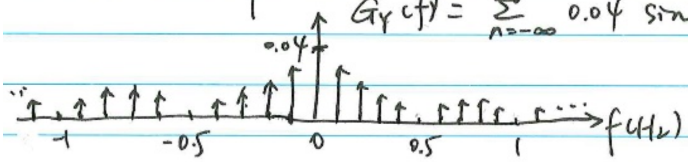
Fourier Spectrum of $y(t)$

$$Y(f) = \sum_{n=-\infty}^{\infty} 0.2\,\mathrm{sinc}(0.2n)\,\delta(f-0.1n)$$




Power Spectrum of $y(t)$

$$G_Y(f) = \sum_{n=-\infty}^{\infty} 0.04\,\mathrm{sinc}^2(0.2n)\,\delta(f-0.1n)$$




Power of $y(t)$: $\quad \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |y(t)|^2\,dt = 0.2$

(See Tutorial 1, Q2 for details)

2. (1) According to the power spectrum $G_Y(f)$, the bandwidth of $y(t)$ is $(1.002 - 0.998)$ kHz $= 4$ Hz.

(2) According to the energy spectrum $\Psi_Y(f)$, the bandwidth of $y(t)$ is 0.5 Hz.

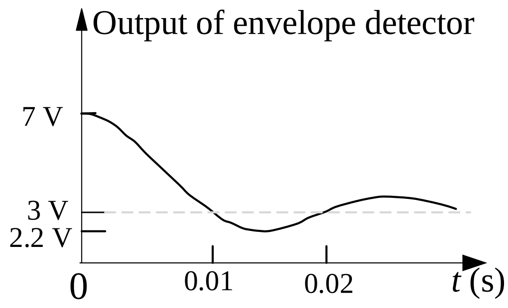
(3) According to the power spectrum $G_Y(f)$, the bandwidth of $y(t)$ is $\infty$.

**Question 2 (30 marks)**

Solution:

1. According to Fig. 2(a), we can see that the carrier frequency $f_c$=1MHz and the modulation index is $m$=4/3. As $m$=10/$c$, we can obtain that the DC offset $c$=7.5 V.
2. $y(t)$ cannot be properly detected by an envelope detector because the modulation index $m$>1. The minimum DC offset $c$ should be 10 V to ensure that $m$ does not exceed 1.

3a. According to Fig. 2b, the maximum and minimum amplitudes of $x(t)$ is 10V and -2V, respectively. With $c$=7.5 V, we can conclude that we can use the envelope detector because $c$+min$x(t)$>0. The output waveform of the envelope detector can be plotted as:




(Credit to HE Houbo: Note that according to Fig.2(a), we have $Acm/4=1$. By combining with $c$=7.5V and $m$=4/3, we can obtain the scaling factor $A$ as 0.4. The maximum and minimum amplitudes of the envelope detector should be 17.5*0.4=7 and 5.5*0.4=2.2, respectively.)

3b. According to $x(t)$=10sinc(100$t$), we can obtain that the bandwidth of $x(t)$ is 50 Hz. (See Tutorial 1, Q1.2 for details). The bandwidth of $y(t)$ is then 100 Hz.

3c. No. As the carrier is 1MHz, the required channel frequency range should be at least [999.95 kHz, 1000.05 kHz]. For [999 kHz, 1000 kHz], part of the frequency components of $y(t)$, i.e., [1000 kHz, 1000.05 kHz], cannot be included.

3d. Put $y(t)$ through a bandpass filter with the frequency range of [999.95 kHz, 1000 kHz] (for lower sideband) or [1000 kHz, 1000.05 kHz] (for upper sideband).

**Question 3 (37 marks)**

Solution:

1. The instantaneous frequency can be obtained as

$$f(t)=\frac{1}{2\pi}\cdot\frac{d\Psi(t)}{dt}=5\times10^{7}+500\cos(10^{3}\pi t)\,.$$

As the carrier frequency is $f_c$=50MHz, the peak frequency deviation is $\Delta f=\max_t|f(t)-f_c|=500\text{ Hz}$.

The input information signal is a sinusoidal signal with frequency $f_m$=500 Hz. The modulation index is then $\beta=\Delta f/f_m=1$.

The total output power is $P_t = 100^2/2 = 5000$ W.

2. The percentage of the output power at the second sidebands is $2\text{x}|J_2(1)|^2=2\text{x}\ 0.1149^2=2.64\%$.

3. 50 MHz is the carrier frequency $f_c$. The output power at 50 MHz is then given by $5000\text{x}|J_0(1)|^2= 5000\text{x}0.7652^2=2928$W.

49.998 MHz $=f_c-4f_m$. The output power at 49.998 MHz is then given by $5000\text{x}|J_4(1)|^2= 5000\text{x}0.0025^2=0.03$W.

50.0001 MHz$=f_c+0.2f_m$ As there is no frequency component at $f_c+0.2f_m$, the output power at 50.0001MHz is 0.

4. The percentage of the power at 1) the carrier is $|J_0(1)|^2=58.55\%$; 2) the first sidebands is $2\text{x}|J_1(1)|^2=38.74\%$; 3) the second sidebands is $2\text{x}|J_2(1)|^2=2.64\%$.

As $58.55\%+38.74\%<99.9\%<58.55\%+38.74\%+2.64\%$, to include 99.9% of the output power, all the second sidebands should be included. Therefore, the channel frequency range should be $[f_c-2f_m, f_c+2f_m]$, which is [49.999 MHz, 50.001 MHz].

5. 50.0005M=50M+1x500=$f_c+1f_m$. According to the table, as $\beta$ increases from 1, $J_1(\beta)$ first becomes zero when $\beta \approx 4$. Therefore, the effective bandwidth is $2(\beta+1)f_m = 5$ kHz.

(See Q3, Tutorial 3 for details.)