EE2302 Foundations of Information Engineering

Assignment 10 (Solution)

1. Totally, there are **four** possible Cayley's tables, as shown below:

*	**e**	**a**	**b**	**c**
e	*e*	*a*	*b*	*c*
a	*a*	*e*	*c*	*b*
b	*b*	*c*	*e*	*a*
c	*c*	*b*	*a*	*e*

*	**e**	**a**	**b**	**c**
e	*e*	*a*	*b*	*c*
a	*a*	*e*	*c*	*b*
b	*b*	*c*	*a*	*e*
c	*c*	*b*	*e*	*a*

*	**e**	**a**	**b**	**c**
e	*e*	*a*	*b*	*c*
a	*a*	*b*	*c*	*e*
b	*b*	*c*	*e*	*a*
c	*c*	*e*	*a*	*b*

*	**e**	**a**	**b**	**c**
e	*e*	*a*	*b*	*c*
a	*a*	*c*	*e*	*b*
b	*b*	*e*	*c*	*a*
c	*c*	*b*	*a*	*e*

There are **two** distinct groups, since the last three tables are the same.

- In the third table, if we swap the roles of *a* and *b*, then we obtain the second table.
- In the fourth table, if we swap the roles of *a* and *c*, then we obtain the second table.

2. There are four subgroups, namely, $\langle\{0\},+\rangle$, $\langle\{0,3\},+\rangle$, $\langle\{0,2,4\},+\rangle$, $\mathbb{Z}_6$, where the operation + is addition modulo 6.

   Remark: $\langle\{0,1\},+\rangle$ is ***not*** a subgroup because $1 + 1 = 2$, which does not belong to {0, 1}, violating the closure property. Intuitively, a group (or subgroup) must have some kind of symmetry. You should be able to see that {0, 3} is somewhat symmetric in

   {**0**, 1, 2, **3**, 4, 5}.

   The elements 0 and 3 are marked in bold to highlight the "symmetry". The same applies to {0, 2, 4} as follows:

   {**0**, 1, **2**, 3, **4**, 5}.

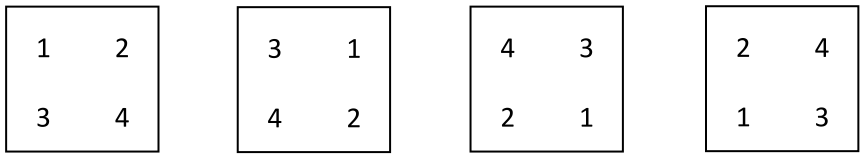
3. a) Multiplication table:

$\circ$	$\boldsymbol{e}$	$\boldsymbol{r}$	$\boldsymbol{r^2}$	$\boldsymbol{f}$	$\boldsymbol{rf}$	$\boldsymbol{r^2f}$
$\boldsymbol{e}$	e	r	r^2	f	rf	r^2f
$\boldsymbol{r}$	r	r^2	e	rf	r^2f	f
$\boldsymbol{r^2}$	r^2	e	r	r^2f	f	rf
$\boldsymbol{f}$	f	r^2f	rf	e	r^2	r
$\boldsymbol{rf}$	rf	f	r^2f	r	e	r^2
$\boldsymbol{r^2f}$	r^2f	rf	f	r^2	r	e

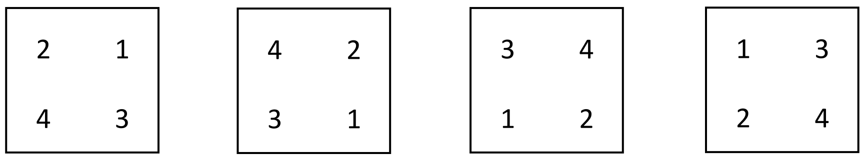
   b) No, it is not an Abelian group. It is because the multiplication table is not symmetric across the diagonal, i.e., the operation is not commutative. For example, $r \circ f = rf$ but $f \circ r = r^2f$, which are not equal.

4.

a) There are eight of them:



e r rr rrr



f rf rrf = frr rrrf = fr

b) The order of rf is 2. It can be seen that $rfrf = e$

c) No. It is easy to check that $fr = rrrf \neq rf$.

d) Ten subgroups: $\{e\}, \{e, rr\}, \{e, f\}$, $\{e, rf\}$, $\{e, rrrf\}, \{e, rrf\}$,
$\{e, r, rr, rrr\}, \{e, rr, rf, rrrf\}, \{e, f, rr, rrf\}$, $\{e, r, rr, rrr, f, rf, rrf, rrrf\}$.