Solutions to EE3210 Tutorial 4 Problems

Problem 1:

(a) Consider

$$\xrightarrow{x[n]} \boxed{S_1} \xrightarrow{v[n]} \boxed{S_2} \xrightarrow{y[n]}$$

where the output of system S_1 is denoted by $v[n]$. We have

$$\begin{cases} v[n] = 2x[n] + 4x[n-1] \\ y[n] = v[n-2] + \frac{1}{2}v[n-3]. \end{cases}$$

Thus, we obtain

$$\begin{aligned} y[n] &= v[n-2] + \tfrac{1}{2}v[n-3] \\ &= 2x[n-2] + 4x[n-3] + x[n-3] + 2x[n-4] \qquad (1) \\ &= 2x[n-2] + 5x[n-3] + 2x[n-4]. \end{aligned}$$

(b) Consider

$$\xrightarrow{x[n]} \boxed{S_2} \xrightarrow{v[n]} \boxed{S_1} \xrightarrow{y[n]}$$

where the output of system S_2 is denoted by $v[n]$. We have

$$\begin{cases} v[n] = x[n-2] + \frac{1}{2}x[n-3] \\ y[n] = 2v[n] + 4v[n-1]. \end{cases}$$

Thus, we obtain

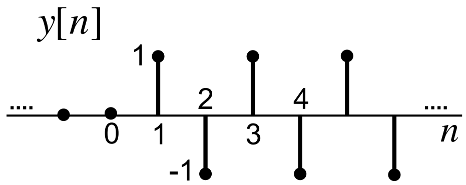
$$\begin{aligned} y[n] &= 2v[n] + 4v[n-1] \\ &= 2x[n-2] + x[n-3] + 4x[n-3] + 2x[n-4] \\ &= 2x[n-2] + 5x[n-3] + 2x[n-4] \end{aligned}$$

which is equivalent to (1). Therefore, the input-output relationship of system S does not change if the order in which S_1 and S_2 are connected in series is reversed.

Problem 2: Since $e[n] = x[n] - y[n]$, we have

$$y[n] = e[n-1] = x[n-1] - y[n-1].$$

The output $y[n]$ is sketched in the figure below.



Problem 3: Note that $x_2(t) = x_1(t) - x_1(t-2)$. Therefore, using linearity and time invariance, we get $y_2(t) = y_1(t) - y_1(t-2)$, which is shown in the figure below.