EE2302 Foundations of Information Engineering

Assignment 8 (Solution)

1. Consider two arbitrary matrices in the subset, $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$, where $a_{12} = -a_{21}$ and $b_{12} = -b_{21}$.

   First, addition is closed, since $A + B = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{pmatrix}$ and $a_{12} + b_{12} = -(a_{21} + b_{21})$.
   Second, scalar multiplication is closed, since $cA = \begin{pmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{pmatrix}$ and $ca_{12} = -ca_{21}$.
   Hence, the subset is a subspace of all 2×2 real matrices.

2.
   a) No. Choose $x = (1,0)$, $y = (0,1)$, and $\alpha = \beta = 1$. Then, $f(\alpha x + \beta y) = 1$ but $\alpha f(x) + \beta f(y) = 2$, which shows that superposition fails.

   b) Yes. a is the vector whose first component is –1, the last component is 1, and all other components are 0, i.e., $a = (-1, 0, 0, \ldots, 0, 1)$.

3.
   a) A vector (a, b, c) reflecting through the x-y plane becomes the vector $(a, b, -c)$. That means, the x-coordinate and the y-coordinate remain unchanged while the z-coordinate is multiplied by -1. The corresponding matrix is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

   It can be checked that

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ b \\ -c \end{bmatrix}.$$

   b) Since only the x- and y-coordinates are rotated while the z-axis remain unchanged, the matrix must be of the form

$$\begin{bmatrix} ? & ? & 0 \\ ? & ? & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where the 2×2 submatrix in the upper left corner is the rotation matrix with angle $= 90^o$. By the formula for the rotation matrix (given in the lecture notes), we obtain the transformation matrix as follows:

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

4.

a) $a = (10, 10)$, so the slope is 1. Consider a right-angled triangle with base 1, height 1, and hypotenuse $\sqrt{2}$. Therefore, $\cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$.

The projection matrix is

$$P = \begin{bmatrix} c^2 & cs \\ cs & s^2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

b) The vector after projection is

$$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}.$$