2.25. Let the signal

$$y[n] = x[n] * h[n],$$

where

$$x[n] = 3^n u[-n-1] + \left(\frac{1}{3}\right)^n u[n]$$

and

$$h[n] = \left(\frac{1}{4}\right)^n u[n+3].$$

(a) Determine $y[n]$ *without* utilizing the distributive property of convolution.
(b) Determine $y[n]$ *utilizing* the distributive property of convolution.

2.26. Consider the evaluation of

$$y[n] = x_1[n] * x_2[n] * x_3[n],$$

where $x_1[n] = (0.5)^n u[n]$, $x_2[n] = u[n+3]$, and $x_3[n] = \delta[n] - \delta[n-1]$.
(a) Evaluate the convolution $x_1[n] * x_2[n]$.
(b) Convolve the result of part (a) with $x_3[n]$ in order to evaluate $y[n]$.
(c) Evaluate the convolution $x_2[n] * x_3[n]$.
(d) Convolve the result of part (c) with $x_1[n]$ in order to evaluate $y[n]$.

2.27. We define the area under a continuous-time signal $v(t)$ as

$$A_v = \int_{-\infty}^{+\infty} v(t)\,dt.$$

Show that if $y(t) = x(t) * h(t)$, then

$$A_y = A_x A_h.$$

2.28. The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.
(a) $h[n] = (\frac{1}{5})^n u[n]$
(b) $h[n] = (0.8)^n u[n+2]$
(c) $h[n] = (\frac{1}{2})^n u[-n]$
(d) $h[n] = (5)^n u[3-n]$
(e) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[n-1]$
(f) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[1-n]$
(g) $h[n] = n(\frac{1}{3})^n u[n-1]$

2.29. The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.
(a) $h(t) = e^{-4t}u(t-2)$
(b) $h(t) = e^{-6t}u(3-t)$
(c) $h(t) = e^{-2t}u(t+50)$
(d) $h(t) = e^{2t}u(-1-t)$

2.37. Consider a system whose input and output are related by the first-order differential equation (P2.33–1). Assume that the system satisfies the condition of final rest [i. e., if $x(t) = 0$ for $t > t_0$, then $y(t) = 0$ for $t > t_0$]. Show that this system is *not* causal. [*Hint:* Consider two inputs to the system, $x_1(t) = 0$ and $x_2(t) = e^t(u(t) - u(t-1))$, which result in outputs $y_1(t)$ and $y_2(t)$, respectively. Then show that $y_1(t) \neq y_2(t)$ for $t < 0$.]

2.38. Draw block diagram representations for causal LTI systems described by the following difference equations:
(a) $y[n] = \frac{1}{3}y[n-1] + \frac{1}{2}x[n]$
(b) $y[n] = \frac{1}{3}y[n-1] + x[n-1]$

2.39. Draw block diagram representations for causal LTI systems described by the following differential equations:
(a) $y(t) = -(\frac{1}{2})\,dy(t)/dt + 4x(t)$
(b) $dy(t)/dt + 3y(t) = x(t)$

ADVANCED PROBLEMS

2.40. **(a)** Consider an LTI system with input and output related through the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)}x(\tau - 2)\,d\tau.$$

What is the impulse response $h(t)$ for this system?

(b) Determine the response of the system when the input $x(t)$ is as shown in Figure P2.40.

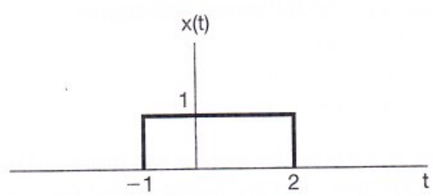

Figure P2.40

2.41. Consider the signal

$$x[n] = \alpha^n u[n].$$

(a) Sketch the signal $g[n] = x[n] - \alpha x[n-1]$.
(b) Use the result of part (a) in conjunction with properties of convolution in order to determine a sequence $h[n]$ such that

$$x[n] * h[n] = \left(\frac{1}{2}\right)^n \{u[n+2] - u[n-2]\}.$$

2.42. Suppose that the signal

$$x(t) = u(t + 0.5) - u(t - 0.5)$$

(i) Compute $y[n]$ by first computing $w[n] = x[n] * h_1[n]$ and then computing $y[n] = w[n] * h_2[n]$; that is, $y[n] = [x[n] * h_1[n]] * h_2[n]$.

(ii) Now find $y[n]$ by first convolving $h_1[n]$ and $h_2[n]$ to obtain $g[n] = h_1[n] * h_2[n]$ and then convolving $x[n]$ with $g[n]$ to obtain $y[n] = x[n] * [h_1[n] * h_2[n]]$.

The answers to (i) and (ii) should be identical, illustrating the associativity property of discrete-time convolution.

(c) Consider the cascade of two LTI systems as in Figure P2.43(b), where in this case

$$h_1[n] = \sin 8n$$

and

$$h_2[n] = a^n u[n], \quad |a| < 1,$$

and where the input is

$$x[n] = \delta[n] - a\delta[n-1].$$

Determine the output $y[n]$. (*Hint:* The use of the associative and commutative properties of convolution should greatly facilitate the solution.)

2.44. (a) If

$$x(t) = 0, \ |t| > T_1,$$

and

$$h(t) = 0, \ |t| > T_2,$$

then

$$x(t) * h(t) = 0, \ |t| > T_3$$

for some positive number T_3. Express T_3 in terms of T_1 and T_2.

(b) A discrete-time LTI system has input $x[n]$, impulse response $h[n]$, and output $y[n]$. If $h[n]$ is known to be zero everywhere outside the interval $N_0 \leq n \leq N_1$ and $x[n]$ is known to be zero everywhere outside the interval $N_2 \leq n \leq N_3$, then the output $y[n]$ is constrained to be zero everywhere, except on some interval $N_4 \leq n \leq N_5$.

(i) Determine N_4 and N_5 in terms of N_0, N_1, N_2, and N_3.

(ii) If the interval $N_0 \leq n \leq N_1$ is of length M_h, $N_2 \leq n \leq N_3$ is of length M_x, and $N_4 \leq n \leq N_5$ is of length M_y, express M_y in terms of M_h and M_x.

(c) Consider a discrete-time LTI system with the property that if the input $x[n] = 0$ for all $n \geq 10$, then the output $y[n] = 0$ for all $n \geq 15$. What condition must $h[n]$, the impulse response of the system, satisfy for this to be true?

(d) Consider an LTI system with impulse response in Figure P2.44. Over what interval must we know $x(t)$ in order to determine $y(0)$?

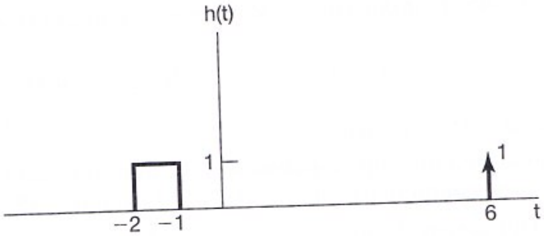

Figure P2.44

2.45. **(a)** Show that if the response of an LTI system to $x(t)$ is the output $y(t)$, then the response of the system to

$$x'(t) = \frac{dx(t)}{dt}$$

is $y'(t)$. Do this problem in three different ways:

(i) Directly from the properties of linearity and time invariance and the fact that

$$x'(t) = \lim_{h \to 0} \frac{x(t) - x(t-h)}{h}.$$

(ii) By differentiating the convolution integral.

(iii) By examining the system in Figure P2.45.

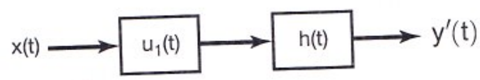

Figure P2.45

(b) Demonstrate the validity of the following relationships:

(i) $y'(t) = x(t) * h'(t)$

(ii) $y(t) = \left(\int_{-\infty}^{t} x(\tau)\,d\tau\right) * h'(t) = \int_{-\infty}^{t}[x'(\tau) * h(\tau)]d\tau = x'(t) * \left(\int_{-\infty}^{t} h(\tau)d\tau\right)$

[*Hint:* These are easily done using block diagrams as in (iii) of part (a) and the fact that $u_1(t) * u_{-1}(t) = \delta(t)$.]

(c) An LTI system has the response $y(t) = \sin \omega_0 t$ to input $x(t) = e^{-5t}u(t)$. Use the result of part (a) to aid in determining the impulse response of this system.

(d) Let $s(t)$ be the unit step response of a continuous-time LTI system. Use part (b) to deduce that the response $y(t)$ to the input $x(t)$ is

$$y(t) = \int_{-\infty}^{+\infty} x'(\tau)s(t-\tau)d\tau. \qquad \text{(P2.45–1)}$$

Show also that

$$x(t) = \int_{-\infty}^{+\infty} x'(\tau)u(t-\tau)\,d\tau. \qquad \text{(P2.45–2)}$$

(e) Use eq. (P2.45–1) to determine the response of an LTI system with step response

$$s(t) = (e^{-3t} - 2e^{-2t} + 1)u(t)$$

to the input $x(t) = e^{t}u(t)$.

(f) Let $s[n]$ be the unit step response of a discrete-time LTI system. What are the discrete-time counterparts of eqs. (P2.45–1) and (P2.45–2)?

2.46. Consider an LTI system S and a signal $x(t) = 2e^{-3t}u(t-1)$. If

$$x(t) \longrightarrow y(t)$$

and

$$\frac{dx(t)}{dt} \longrightarrow -3y(t) + e^{-2t}u(t),$$

determine the impulse response $h(t)$ of S.

2.47. We are given a certain linear time-invariant system with impulse response $h_0(t)$. We are told that when the input is $x_0(t)$ the output is $y_0(t)$, which is sketched in Figure P2.47. We are then given the following set of inputs to linear time-invariant systems with the indicated impulse responses:

	Input $x(t)$	*Impulse response $h(t)$*
(a)	$x(t) = 2x_0(t)$	$h(t) = h_0(t)$
(b)	$x(t) = x_0(t) - x_0(t-2)$	$h(t) = h_0(t)$
(c)	$x(t) = x_0(t-2)$	$h(t) = h_0(t+1)$
(d)	$x(t) = x_0(-t)$	$h(t) = h_0(t)$
(e)	$x(t) = x_0(-t)$	$h(t) = h_0(-t)$
(f)	$x(t) = x_0'(t)$	$h(t) = h_0'(t)$

[Here $x_0'(t)$ and $h_0'(t)$ denote the first derivatives of $x_0(t)$ and $h_0(t)$, respectively.]

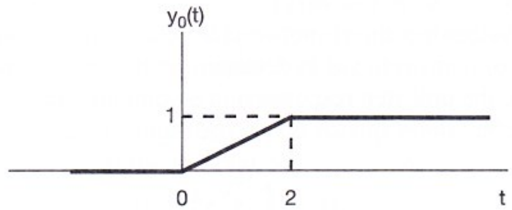

Figure P2.47

In each of these cases, determine whether or not we have enough information to determine the output $y(t)$ when the input is $x(t)$ and the system has impulse response $h(t)$. If it is possible to determine $y(t)$, provide an accurate sketch of it with numerical values clearly indicated on the graph.