EE3008 Assignment 1

Solution

Question 1 (36 marks)

For **each** of the following three cases shown in Fig. 1:

1. Plot the Fourier spectrum of $y(t)$;
2. Determine whether the signal $y(t)$ is a power-type or energy-type signal. State your reason;
3. If $y(t)$ is an energy-type signal, determine its signal energy and plot the energy spectrum. If $y(t)$ is a power-type signal, determine its signal power and plot the power spectrum;
4. Determine the bandwidth of $x(t)$.

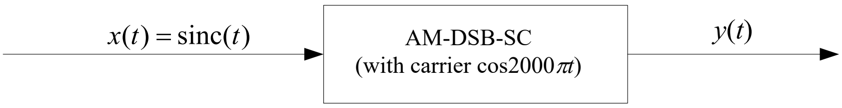


Fig. 1a

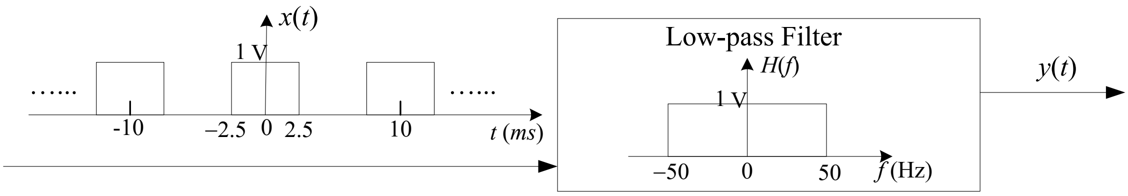
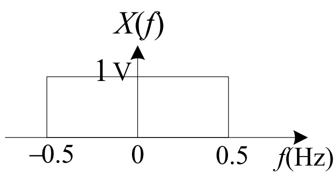


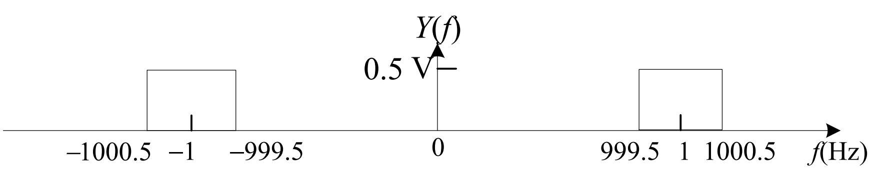
Fig. 1b

Solution:

a) Because the spectrum of $x(t)$ is (see Tutorial 1, Q1.2 for details)

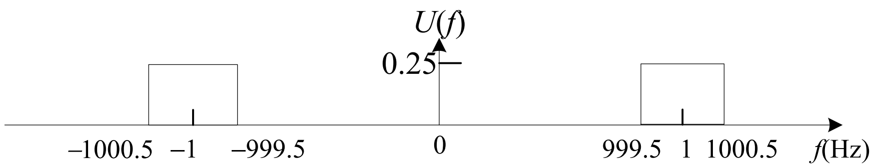


according to $Y(f)=\frac{1}{2}(X(f-f_c)+X(f+f_c))$, f_c=1000Hz, we can plot $Y(f)$ as



We can see from $Y(f)$ that $y(t)$ is an energy-type signal because its energy is finite.

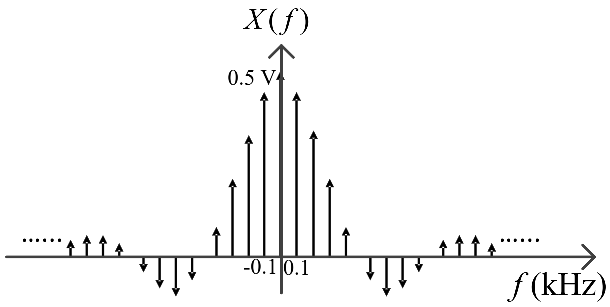
Its energy spectrum is $U(f) = |Y(f)|^2$, which can be plotted as



and its energy is $E_y = \int_{-\infty}^{\infty} U(f)df = 0.5\,\text{J}$.

The bandwidth of $x(t)$ is 0.5Hz according to $X(f)$.

b) The spectrum of $X(t)$ is given by $X(f) = 0.5\sum_{n=-\infty}^{\infty} \text{sinc}(n/2)\delta\left(f - 100n\right)$ (See Tutorial 1, Q1.3 for details).



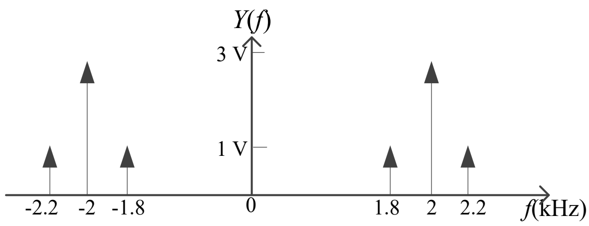
After $X(t)$ passes through the low-pass filter shown in Fig. 1, $Y(f) = X(f)H(f) = 0.5\delta\left(f\right)$, i.e., only the DC frequency component is left. We can see from $Y(f)$ that $y(t)$ is a power-type signal because it is a dc (constant) in the time domain.

The power spectrum $G(f) = |Y(f)|^2 = 0.25\delta\left(f\right)$ and the power is $P_y = \int_{-\infty}^{\infty} G(f)df = 0.25\,\text{W}$.
The bandwidth of $x(t)$ is infinite.

Question 2 (35 marks)

An input signal $x(t)$ is applied to an AM-DSB-C modulator, and the Fourier spectrum $Y(f)$ of the output signal $y(t)$ is given in Fig. 2.

1. Determine the minimum required channel bandwidth such that all the frequency components of the modulated signal $y(t)$ can pass through;
2. Determine the modulation index;
3. Determine the expression of $y(t)$;
4. Determine whether $y(t)$ is a power-type signal or an energy-type signal. For energy-type signals, determine the energy spectrum and the signal energy. For power-type signals, determine the power spectrum and the signal power;
5. Specify whether the modulated signal $y(t)$ can be properly detected by an envelope detector. If yes, sketch and label the output waveform of the envelope detector. If no, determine the minimum DC offset for the envelope detector to properly work;
6. If the dc offset of the AM-DSB-C modulator is cut in half, determine whether the modulated signal $y(t)$ can be properly detected by an envelope detector. State your reason.



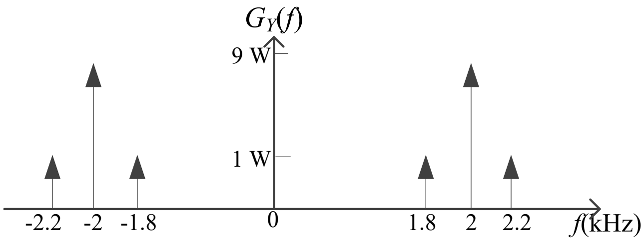
Solution:

1. According to Fig. 2, the minimum required channel bandwidth is equal to the modulated signal bandwidth 400 Hz.

2. For $Y(f)$, the magnitude of the carrier components is $Ac/2$ and the magnitude of the sidebands is $Acm/4$ (See Q3, Tutorial 2 for details). According to $\frac{Acm/4}{Ac/2} = \frac{1}{3}$, we have $m = 2/3$.

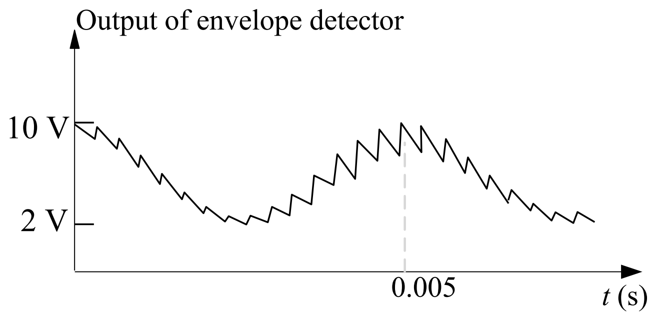
3. We can see from Fig. 2 that the input signal is a sinusoidal signal with frequency 200Hz, and the carrier frequency is 2000 Hz. Moreover, according to $Ac/2 = 3$, we have $Ac = 6$. Therefore, the expression of $y(t)$ can be obtained as $y(t) = 6(\frac{2}{3}\cos(400\pi t) + 1)\cos(4000\pi t)$.

4. $y(t)$ is a power-type signal with the power spectrum



The output power is $P = 2\times 9 + 4\times 1 = 22$ W .

5. We can use the envelope detector because the modulation index $m \le 1$. According to $Ac(1+m)$=10 V, $Ac(1-m)$=2 V and f_m=200 Hz, the output waveform of the envelope detector is



6. If the dc offset c of the AM-DSB-C modulator is cut in half, then according to $m = \frac{x}{c}$ (where x is the peak amplitude of the input signal), we know that the modulation index m would be doubled, and exceed 1. In that case, over-modulation would occur and the envelop detector cannot properly work.

Question 3 (32 marks)

The output signal of an FM system is given by:

$$s_{FM}(t) = 20\cos[10^8\pi t + 1000\pi\int_{-\infty}^{t}\cos(10^3\pi\tau)d\tau].$$

1. Determine the peak frequency deviation; (4 marks)
2. Determine the modulation index; (4 marks)
3. Determine the output power at the second sidebands; (4 marks)
4. Determine the output power at 49.9999 MHz and 49.9995 MHz, respectively; (6 marks)
5. Determine the minimum channel bandwidth required for transmitting those sidebands whose magnitudes are larger than 5% of the magnitude of the carrier component; (6 marks)
6. Suppose that the amplitude of the input signal is carefully increased until the output signal at 50.0005MHz is zero. Determine the effective bandwidth of the output signal according to Carson's rule. (8 marks)

Solution:

1. The instantaneous frequency can be obtained as

$$f(t) = \frac{1}{2\pi}\cdot\frac{d\Psi(t)}{dt} = 5\times10^7 + 500\cos(10^3\pi t).$$

As the carrier frequency is f_c=50 MHz, the peak frequency deviation is $\Delta f = \max_t |f(t) - f_c| = 500\text{Hz}$.

2. The input message signal is a sinusoidal signal with frequency f_m=500Hz. The modulation index is then $\beta=\Delta f/f_m=1$.

3. The total power is $P_t = 20^2/2 = 200$ W. The output power at the second sidebands can be therefore obtained as $2\times P_t|J_2(1)|^2 = 400\times0.1149^2 = 5.28$ W.

4. As there is no frequency component at 49.9999M=f_c-0.2f_m, the output power at 49.9999MHz is 0. On the other hand, 49.9995M=f_c-1f_m. The output power at 49.9995MHz is then given by $P_t|J_1(1)|^2 = 200\times0.44^2 = 38.72$ W.

5. From the table we can see that when $n > 2$, $|J_n(1)| < |J_0(1)|\cdot5\% = 0.03826$. Therefore, the sidebands from f_c-2f_m to f_c+2f_m will be transmitted, and the required channel bandwidth is 4f_m=2kHz.

6. 50.0005M=f_c+1f_m. According to the table, as β increases from 1, $J_1(\beta)$ first becomes zero when $\beta \approx 4$. Therefore, the effective bandwidth is $2(\beta+1)f_m = 5$ kHz.