Tutorial 7 Digital Demodulation

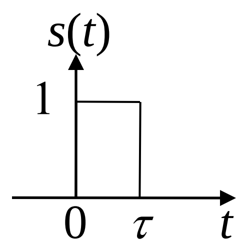
Problem 1 (AWGN)

AWGN $n(t)$ passes through a filter $h(t)$ and is sampled at t_0. Suppose that the two-sided power spectral density of $n(t)$ is $N_0/2$ and the sampled noise $n_o(t_0) \sim N\ (0, \sigma_0^2)$. Show that

$$\sigma_0^2 = \tfrac{N_0}{2}\int_0^{t_0} h^2(t_0 - x)dx.$$

Problem 2 (Matched Filter)

Suppose $s(t)$ is a positive rectangular pulse given below.



1) Draw the time-domain response of the corresponding matched filter;
2) Draw the output of the matched filter with an input positive rectangular pulse. Sample the output at $t = \tau$.
3) What is the output of the correlator? Compare it with the sampled output of matched filter at $t = \tau$.

Problem 3 (BER of Binary PAM)

A binary PAM system is transmitting signals at 10k bits/sec, and the optimal detector is employed at the receiver. Suppose the two-sided noise power spectral density at the receiver is $N_0/2=10^{-6}$ W/Hz. What is the minimum magnitude of the binary PAM signal to guarantee a BER less than or equal to 10^{-3}?

Tabulation of the Q-function

z	$Q(z)$	z	$Q(z)$
0.0	0.50000	2.0	0.02275
0.1	0.46017	2.1	0.01786
0.2	0.42074	2.2	0.01390
0.3	0.38209	2.3	0.01072
0.4	0.34458	2.4	0.00820
0.5	0.30854	2.5	0.00621
0.6	0.27425	2.6	0.00466
0.7	0.24196	2.7	0.00347
0.8	0.21186	2.8	0.00256
0.9	0.18406	2.9	0.00187
1.0	0.15866	3.0	0.00135
1.1	0.13567	3.1	0.00097
1.2	0.11507	3.2	0.00069
1.3	0.09680	3.3	0.00048
1.4	0.08076	3.4	0.00034
1.5	0.06681	3.5	0.00023
1.6	0.05480	3.6	0.00016
1.7	0.04457	3.7	0.00011
1.8	0.03593	3.8	0.00007
1.9	0.02872	3.9	0.00005

Problem 4 (Binary Baseband)

A binary signaling system uses digital waveforms $s_1(t)$ and $s_2(t)$ to represent bits “1” and “0”, respectively, where

$$s_1(t) = A, \quad 0 \le t < \tau$$

and

$$s_2(t) = \begin{cases} A, & 0 \le t < \tau / 2 \\ -A, & \tau / 2 \le t < \tau \end{cases}$$

Suppose the signal is transmitted over an AWGN channel with bit rate R_b=1Mbps and A=1V. The optimal receiver is adopted and the two-sided spectral density of noise at the receiver is 10^{-7} W/Hz. Determine the BER performance.

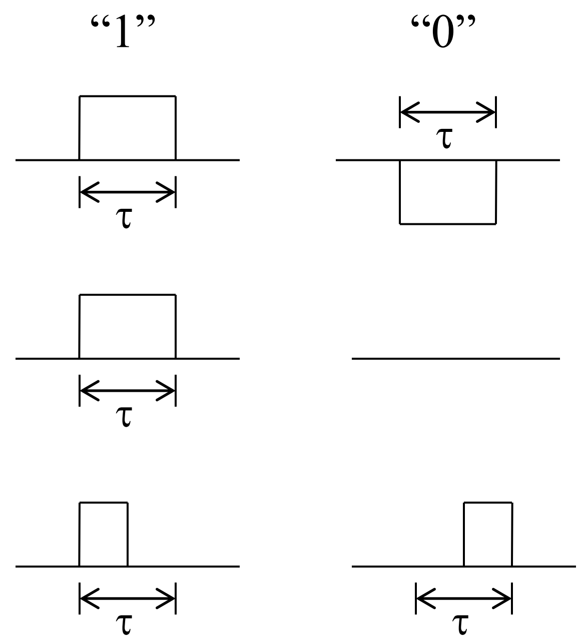
Problem 5 (Binary Baseband)

Three kinds of binary signaling are used for transmission:

1) Binary PAM:

2) Binary ON-off Keying:

3) Binary Pulse Position
Modulation (PPM):



Compare their BER performance under the conditions of:
1) Equal energy per bit E_b; 2) Equal magnitude of pulses.

Problem 6 (BASK and BPSK)

If a system's main performance criterion is BER performance, which of the following two modulation schemes should be selected for an AWGN channel?

- Coherent BASK with E_b/N_0=10dB;
- Coherent BPSK with E_b/N_0=8dB.

Problem 7 (BPSK)

Suppose a coherent BPSK system has a bit rate of 500 kbits/sec and the single-sided noise power spectral density at the receiver is $N_0=10^{-8}$ W/Hz. Determine the signal power required for a probability of bit error of 10^{-6}.

$Q(a)$	a
10^{-4}	3.73
5×10^{-5}	3.90
10^{-5}	4.27
5×10^{-6}	4.43
10^{-6}	4.76
10^{-7}	5.20
10^{-8}	5.61

Problem 8 (BFSK)

Suppose a BFSK system is transmitting data through a channel with bandwidth 1MHz. Coherent receiver is adopted and the two-sided noise power spectral density is 0.5×10^{-8} W/Hz.

1) Determine the maximum bit rate.

2) Determine the signal power required for a probability of bit error of 10^{-6}.

Problem 9 (Binary Bandpass)

Consider a digital communication system using binary signaling shown by

$$s(t) = \begin{cases} A\cos(2\pi f_c t), & 0 \le t < \tau \quad \text{if } b = 1 \\ A\sin(2\pi f_c t + \theta), & 0 \le t < \tau \quad \text{if } b = 0 \end{cases}$$

where θ is a fixed number locating between $-\pi/2$ and $\pi/2$, and τ is the bit interval chosen such that $f_c\tau$ is an integer. The system operates on an AWGN channel.

1) Suppose that for $\theta=\pi/6$, we find the probability of bit error to be 0.0125. Determine the probability of bit error when θ is chosen to be -0.3726 radian.

2) Find θ that minimizes the probability of bit error.

Problem 10 (SER of 4-ary PAM)

Consider a 4-ary PAM system has a bit rate of 60,000 bits/sec and the signal power is 15W.

1) Suppose the two-sided noise power spectral density at the receiver is 10×10^{-6} W/Hz. Determine the SER and BER of the correlation receiver (suppose each symbol error corresponds to a one-bit error).

2) Suppose a binary PAM system has the same bit rate and uses the same power. Determine the corresponding BER.

3) Compare the required channel bandwidth (90% in-band power) of the above two systems.

Problem 11.1 (SER of QPSK)

A 1Mbps bit stream is to be transmitted using QPSK.

1) What is the symbol rate?
2) What is the required channel bandwidth if only the main lobe of the signal spectrum is to be passed?
3) What is the probability of bit error and the probability of symbol error if the coherent demodulator is used? Suppose the received signal power is 8mW and N_0=10^{-9} W/Hz.

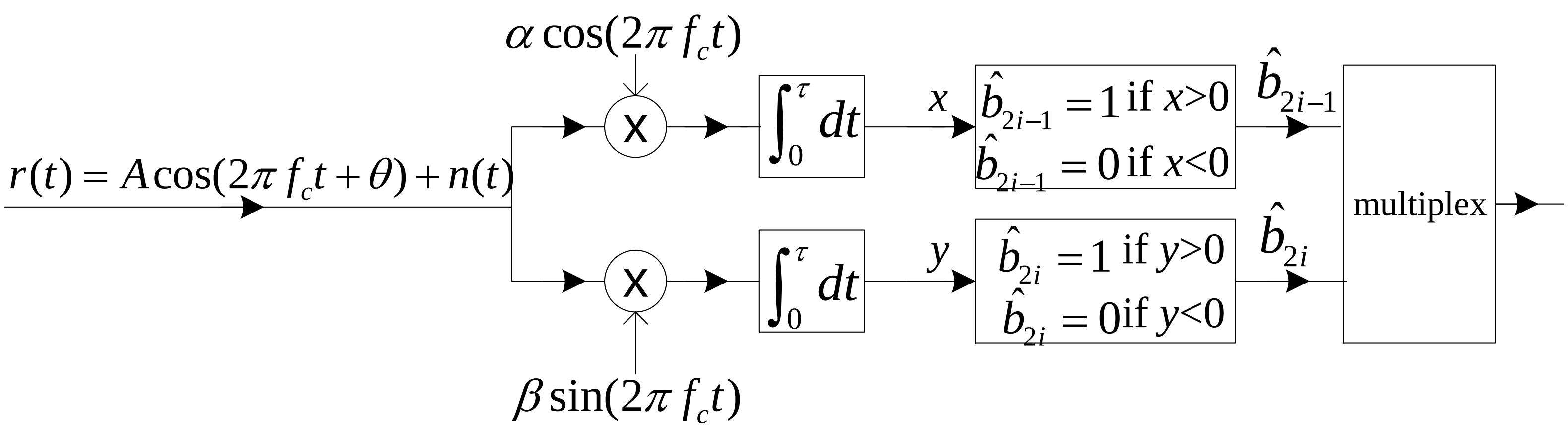
Problem 11.2 (BER of QPSK)

4) What is the probability of bit error if the bit stream is transmitted using BPSK with the same amplitude and the same bit rate? What is the required channel bandwidth?

5) What is the probability of bit error if the bit stream is transmitted using BPSK with the same amplitude and the same bandwidth?

Problem 12 (Demodulator of QPSK)

A coherent receiver for the detection of QPSK signal with AWGN is sketched in the following figure, where decisions on information bits are made at the upper and lower branches, separately, using the decision rules shown in the diagram.



Problem 12 (Demodulator of QPSK)

Suppose that information symbols are mapped into the carrier phase θ according to the following Table I:

Information Symbol	(1 1)	(0 1)	(0 0)	(1 0)
Phase θ	$\pi/4$	$3\pi/4$	$5\pi/4$	$7\pi/4$

Determine the sign of α and β.