Solutions to EE3210 Tutorial 2 Problems

Problem 1:

(a) Using the trigonometric identity

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

we obtain

$$\left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2 = \frac{1+\cos(4t - \frac{2\pi}{3})}{2}.$$

Thus, the signal is periodic with $T_0 = 2\pi/\omega = 2\pi/4 = \pi/2$.

(b) Using the trigonometric identity

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$$

we obtain

$$\cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right) = \frac{1}{2}\left[\cos\left(\frac{\pi}{4}n\right)+\cos\left(\frac{3\pi}{4}n\right)\right].$$

For the term $\cos(\frac{\pi}{4}n)$, we have

$$\Omega = \pi/4 \Rightarrow \Omega/(2\pi) = 1/8 \Rightarrow N_0 = 8.$$

For the term $\cos(\frac{3\pi}{4}n)$, we have

$$\Omega = 3\pi/4 \Rightarrow \Omega/(2\pi) = 3/8 \Rightarrow N_0 = 8.$$

Therefore, the overall signal $x[n]$ is periodic with $N_0 = 8$.

Problem 2:

(a) The system is not memoryless. For example, when $t = 1$, we have $y(1) = x(2)$.

The system is invertible. The inverse system is $w(t) = y(t/2)$.

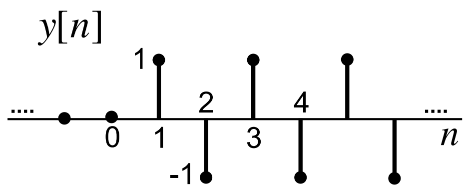
(b) The system is not memoryless. For example, when $n = 1$, we have $y[1] = x[2]$.

The system is not invertible. For example, consider $x_1[n] = 1$, $x_2[n] = (-1)^n$. Then, $y_1[n] = x_1[2n] = 1$, $y_2[n] = x_2[2n] = (-1)^{2n} = 1$, so that $y_1[n] = y_2[n]$ for all n.

Problem 3: Since $e[n] = x[n] - y[n]$, we have

$$y[n] = e[n-1] = x[n-1] - y[n-1].$$

The output $y[n]$ is sketched in the figure below.



It can also be expressed in the following compact form:

$$y[n] = (-1)^{n-1} u[n-1].$$