EE3210 Signals and Systems

Semester A 2023-2024

Assignment 2

Due Date: 15 November 2023

1. Compute the Fourier transform of $x(t) = e^{-2|t-1|}$.

2. Find the frequency response $H(e^{j\omega})$ of a discrete-time linear time-invariant (LTI) system whose input $x[n]$ and output $y[n]$ satisfy the following difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

3. Determine the difference equation that characterizes a discrete-time LTI system whose frequency response is

$$H(e^{j\omega}) = \frac{1 - \frac{1}{2}e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2}e^{-j\omega} + \frac{3}{4}e^{-j2\omega}}$$

4. Figure 1 shows a system which consists of an interconnection of two discrete-time LTI systems with impulse responses $h_1[n]$ and $h_2[n]$.

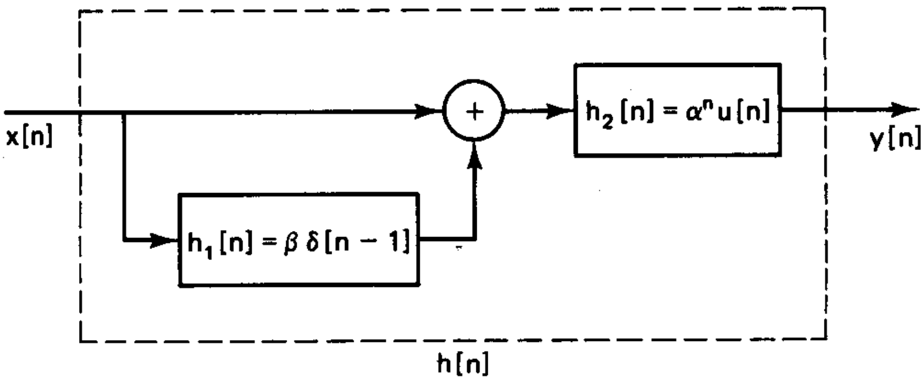

Figure 1

(a) Find the impulse response $h[n]$ of the overall system.
(b) Find the system transfer function $H(z)$ of the overall system, which is equal to $Y(z)/X(z)$ where $X(z)$ and $Y(z)$ are the z transforms of the input $x[n]$ and output $y[n]$, respectively.
(c) Write down the difference equation that relates $x[n]$ and $y[n]$.
(d) Is the system causal?
(e) Under what condition would the system be stable?

5. Given a discrete-time signal $x[n]$ which has the form of:

$$x[n] = \begin{cases} \alpha e^{j(\omega_o n+\phi)}, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

where α, ω_0 and ϕ are real numbers.

(a) Determine $X(e^{j\omega})$ which is the discrete-time Fourier transform of $x[n]$.
(b) Find the maximum value of $|X(e^{j\omega})|$. Determine the value of ω which maximizes $|X(e^{j\omega})|$.

6. Given a continuous-time signal $x(t)$:

$$x(t) = \sin\left(\frac{\pi}{2}t\right)$$

The signal is sampled with a sampling period $T = 1$s to produce the discrete-time signal $x[n]$. Find $x[0]$, $x[1]$, $x[2]$, $x[3]$ and $x[4]$. Is $x[n]$ a periodic signal?

7. Determine the z transform of $x[n]$ which has the form of:

$$x[n] = \begin{cases} na^n, & 1 \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

Specify the region of convergence (ROC).

8. Consider a discrete-time LTI system whose transfer function $H(z)$ is:

$$H(z) = \frac{z^{-2}}{(1-0.5z^{-1})(1-3z^{-1})}$$

(a) If the system is stable, determine the output $y[n]$ when the input is $x[n] = u[n]$.
(b) If the system is causal, determine the output $y[n]$ when $x[n] = \delta[n]$.

9. Use z transform and inverse z transform to compute the convolution of $x[n] = u[-n-1]$ and $h[n] = (0.5)^n u[n]$.

10. Watch the short video of the 2013 Shaw Prize winner for mathematics, Prof. David Donoho (start at 14:50): https://www.youtube.com/watch?v=5wv4grOMgIU

(a) Briefly describe a denoising system, which includes the system input, output and function, as well as the principle to achieve denoising. Use your own words in <u>no more than 100 words</u>.
(b) Suppose you are given the following observed continuous-time signal $x(t)$:

$$x(t) = \cos(100\pi t) + n(t)$$

where $n(t)$ is the unwanted noise. With the use of an appropriate transform you have learned in this course, briefly describe, in theory, how can you extract $\cos(100\pi t)$ from $x(t)$? Also, how can you achieve compression for $\cos(100\pi t)$?