EE3331 Probability Models in Information Engineering

Semester B 2022-2023

Assignment 3

Due Date: 20 March 2023
Important Note: Only writing the answers without steps will get zero mark.

1. The joint probability distribution function (PDF) of two random variables X and Y has the form of:

$$p(x,y) = \begin{cases} \alpha e^{-(4x+5y)}, & x>0,\ y>0, \\ 0, & \text{otherwise} \end{cases}$$

   (a) Find the value of α.
   (b) Determine the joint cumulative distribution function (CDF) of X and Y.
   (c) Find $P(X<1,\ Y<2)$.

2. Consider two independent standard Gaussian random variables $X \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(0,1)$. With the use of $\int_0^\infty u e^{-u^2/4} du = 2$, compute the variance of $|X-Y|$. Note that a linear combination of Gaussian random variables is also a Gaussian random variable.

3. Consider tossing a fair coin twice. Denote X as the total number of tail(s) in two tosses and Y as the number of head at the second toss. Find the joint probability mass function (PMF) $P_{XY}(x,y)$.

4. The joint probability mass function (PMF) of random variables N and K is given as:

$$P_{NK}(n,k) = \begin{cases} \dfrac{100^n e^{-100}}{n!} \dbinom{100}{k} p^k (1-p)^{100-k}, & n=0,1,\cdots,\ k=0,1,\cdots,100 \\ 0, & \text{otherwise} \end{cases}$$

   Find the marginal PMFs of N and K.

5. The joint probability distribution function (PDF) of two random variables X and Y has the form of:

$$p(x,y) = \begin{cases} cxy^2, & 1 \ge x \ge 0,\ 1 \ge y \ge 0, \\ 0, & \text{otherwise} \end{cases}$$

   (a) Find the value of c.
   (b) Find $P(X>Y)$.
   (c) Find $P(X^2>Y)$.
   (d) Find $P(\min(X,Y) \le 0.5)$.
   (e) Find $P(\max(X,Y) \le 0.75)$.

6. Suppose random variables X and Y are independent of each other where $X \sim \mathcal{U}(0, 0.2)$, and the probability distribution function (PDF) of Y is:

$$p_Y(y) = \begin{cases} 5e^{-5y}, & y > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find $P(Y \leq X)$.

7. Consider the problem of estimating a constant A from N observations:

$$r_n = A + w_n, \quad n = 1, 2, \cdots, N$$

where w_n is a white noise with mean 0 and variance σ_w^2. It is suggested to estimate A using $\hat{A}$ which is given by:

$$\hat{A} = \frac{1}{N-1} \sum_{n=1}^{N} r_n$$

Compute the mean, variance and mean square error of $\hat{A}$.

8. The joint probability mass function (PMF) of two random variables X and Y is given as:

$$P_{XY}(x, y) = \begin{cases} 0.01, & x = 1, 2, \cdots, 10, \ y = 1, 2, \cdots, 10 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the PMF of $W = \min(X, Y)$.
(b) Let A be the event of $\min(X, Y) > 5$. Find the conditional PMF $P_{XY|A}(x, y)$.

9. The joint probability distribution function (PDF) of two random variables X and Y is given as:

$$P_{XY}(x, y) = \begin{cases} 6e^{-(2x+3y)}, & x \geq 0, \ y \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Let A be the event that $X + Y \leq 1$. Find the conditional PDF $P_{XY|A}(x, y)$.

10. The joint probability distribution function (PDF) of two random variables X and Y is given as:

$$P_{XY}(x, y) = \begin{cases} x + y, & 0 \leq x \leq 1, \ 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional PDFs $P_{X|Y}(x|y)$ and $P_{Y|X}(y|x)$.