EE2302 Foundations of Information Engineering

Assignment 3 **Due: 6 pm, Sep 21 (Wed)**

Full mark: 18 points

1. Let R be the relation on $\mathbb{Z}$ defined by

$$mRn \text{ if and only if } mn > 0 \text{ or } m = n = 0.$$

   a) (3 points) Prove that R is an equivalence relation.

   b) (3 points) How many distinct equivalence classes are there? What are they?

2. Consider the following two relations defined on $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers:
   - For all $x, y \in \mathbb{R}$, $x\ S\ y$ if and only if $x \geq y$.
   - For all $x, y \in \mathbb{R}$, $x\ T\ y$ if and only if $x - y$ is an integer.

   a) (2 marks) Which one is not an equivalence relation? Justify your answer.

   b) (3 marks) Prove that the other one is an equivalence relation.

   c) (2 marks) Describe its distinct equivalence classes.

3. Let $\mathbb{R}$ and $\mathbb{Z}$ be the sets of real numbers and of integers, respectively. Consider these two relations:
   - For all $(a, b), (c, d) \in \mathbb{R} \times \mathbb{R}$, $(a, b)\ S\ (c, d)$ if and only if either $(a < c)$ or (both $a = c$ and $b \leq d$).
   - For all $m, n \in \mathbb{Z}$, $m\ T\ n$ if and only if $m + n$ is even.

   a) (2 marks) Which one is not a partial order relation? Justify your answer.

   b) (3 marks) Prove that the other one is a partial order relation.