EE3008 Quiz 1

(12:05-1:40pm, Feb. 23, 2018)

<u>Question 1 (36 marks)</u>

For **each** of the following three cases shown in Fig. 1:

1. Plot the Fourier spectrum of $y(t)$; (9 marks)
2. Determine whether the signal $y(t)$ is a power-type or energy-type signal. State your reason; (6 marks)
3. If $y(t)$ is an energy-type signal, determine its signal energy and plot the energy spectrum. If $x(t)$ is a power-type signal, determine its signal power and plot the power spectrum; (15 marks)
4. Determine the bandwidth of $x(t)$. (6 marks)

Fig. 1a

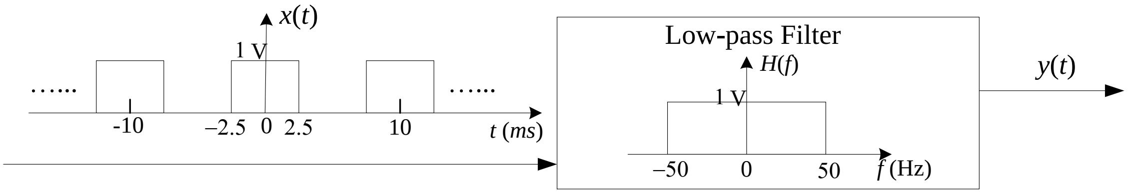


Fig. 1b

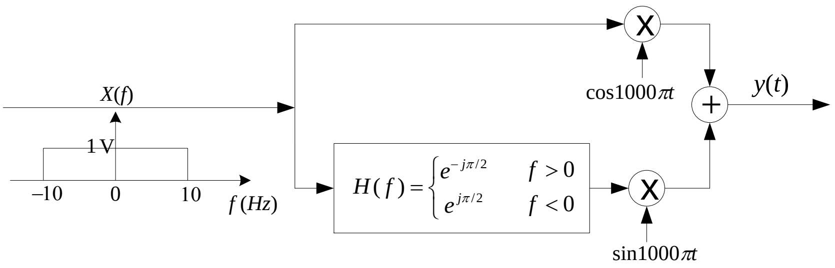
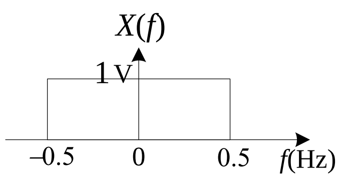


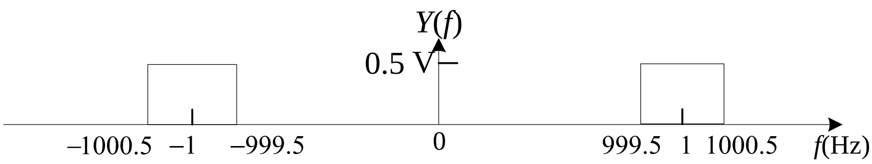
Fig. 1c

Solution:

a) Because the spectrum of $x(t)$ is (see Tutorial 1, Q1.2 for details)

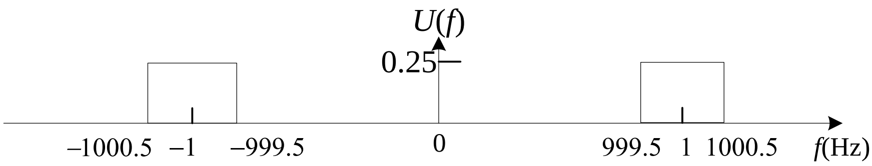


according to $Y(f) = \frac{1}{2}(X(f - f_c) + X(f + f_c))$, f_c=1000Hz, we can plot $Y(f)$ as



We can see from $Y(f)$ that $y(t)$ is an energy-type signal because its energy is finite.

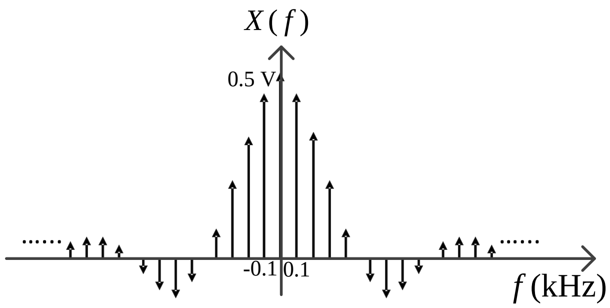
Its energy spectrum is $U(f) = |Y(f)|^2$, which can be plotted as



and its energy is $E_y = \int_{-\infty}^{\infty} U(f)df = 0.5\,\mathrm{J}$.

The bandwidth of $x(t)$ is 0.5Hz according to $X(f)$.

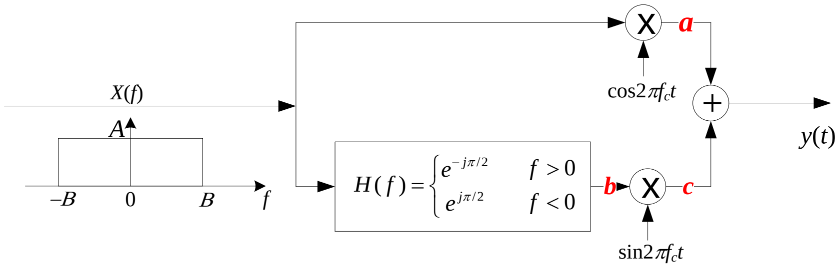
b) The spectrum of $X(t)$ is given by $X(f) = 0.5\sum_{n=-\infty}^{\infty} \mathrm{sinc}(n/2)\delta(f-100n)$ (See Tutorial 1, Q1.3 for details).



After $X(t)$ passes through the low-pass filter shown in Fig. 1, $Y(f) = X(f)H(f) = 0.5\delta(f)$, i.e., only the DC frequency component is left. We can see from $Y(f)$ that $y(t)$ is a power-type signal because it is a dc (constant) in the time domain.

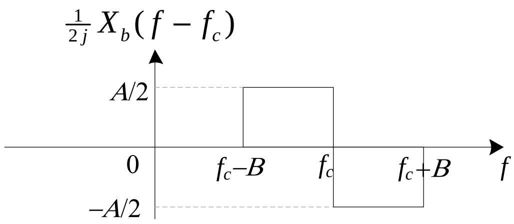
The power spectrum $G(f) = |Y(f)|^2 = 0.25\delta(f)$ and the power is $P_y = \int_{-\infty}^{\infty} G(f)df = 0.25\,\mathrm{W}$.
The bandwidth of $x(t)$ is infinite.

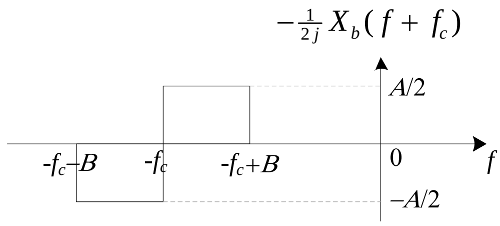
c) Let us re-plot Fig. 1c as



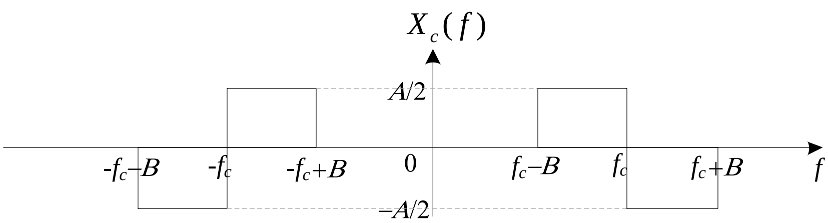
where A=1 V, B=10Hz, and f_c=500Hz.

We can easily obtain the Fourier spectrum of the signal at point b as $X_b(f)=X(f)H(f)=\begin{cases} -jX(f) & f>0 \\ jX(f) & f<0 \end{cases}$. Because $\sin(2\pi f_c t) \Leftrightarrow \frac{1}{2j}(\delta(f-f_c)-\delta(f+f_c))$, the Fourier spectrum of signal at point c is $X_c(f)=\frac{1}{2j}(X_b(f-f_c)-X_b(f+f_c))$. Note that $\frac{1}{2j}X_b(f-f_c)$ and $-\frac{1}{2j}X_b(f+f_c)$ can be plotted as

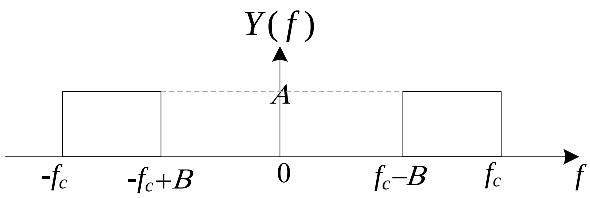




We then have



Finally, according to $Y(f)=X_a(f)+X_c(f)$ and $X_a(f)=\frac{1}{2}(X(f-f_c)+X(f+f_c))$, we have

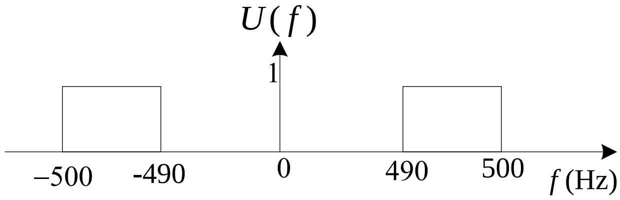


where A=1 V, B=10Hz, and f_c=500Hz.

(We can see from $Y(f)$ that this is indeed a SSB modulator. In this case, we obtain the lower sideband. How to obtain the upper sideband?)

We can see from $Y(f)$ that $y(t)$ is an energy-type signal because its energy is finite.

Its energy spectrum is $U(f) = |Y(f)|^2$, which can be plotted as



and the energy is $E_y = \int_{-\infty}^{\infty} U(f)df = 20\,\text{J}$.

The bandwidth of $x(t)$ is 10 Hz according to $X(f)$.

Question 2 (32 marks)

The output signal $y(t)$ of an AM-DSB-C modulator is shown in Fig. 2.

1. Determine the carrier frequency f_c, and the minimum required channel bandwidth such that all the frequency components of $y(t)$ can pass through; (4 marks)
2. Determine the modulation index; (4 marks)
3. Plot the Fourier spectrum of $y(t)$; (4 marks)
4. Determine the output power; (4 marks)
5. Specify whether the modulated signal $y(t)$ can be properly detected by an envelope detector. If yes, sketch and label the output waveform of the envelope detector. If no, determine the minimum dc offset for the envelope detector to properly work. (10 marks)
6. Now adjust the dc offset until at the output side, the magnitude of the carrier line is found to be equal to the magnitude of the sidebands. Determine the new modulation index, and specify whether the modulated signal $y(t)$ can be properly detected by an envelope detector. State the reason. (6 marks)

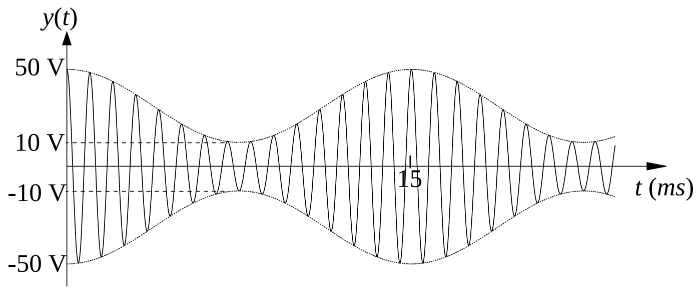


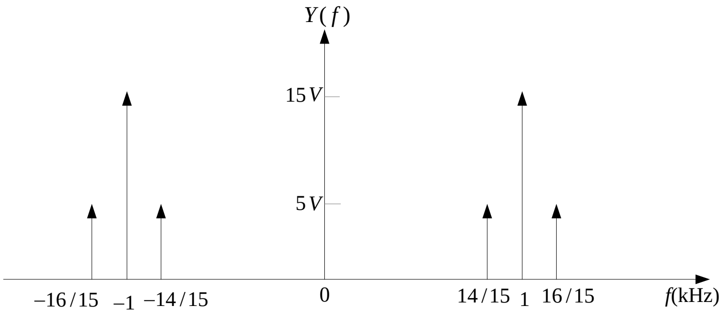
Fig. 2.

Solution:

1. We can see from Fig. 2 that the information signal $x(t)$ is a sinusoidal signal, and its period is 15ms, which is 15 times larger than that of the carrier. Therefore, the carrier frequency is 1kHz; and the frequency of the information signal is 1/15 kHz. The minimum required channel bandwidth is 2/15 kHz.

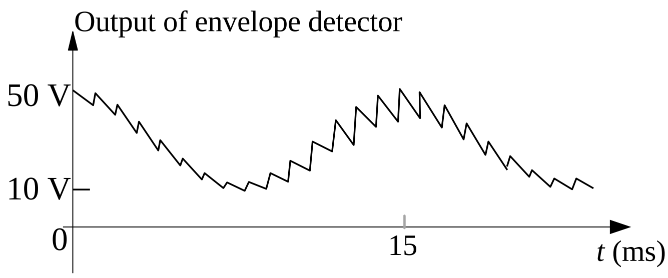
2. We can see from the figure that the maximum and minimum of the envelop are 50V and 10V, respectively. The modulation index is then $m = \frac{50-10}{50+10} = 2/3$.

3. It can be obtained from Fig. 2 that $Ac = 30\,\text{V}$ (see Tutorial 2, Q3.2 for details). The spectrum of $Y(f)$ can then be plotted as (see Tutorial 2, Q3.3 for details)



4. The output power is $P = 2\times15^2 + 4\times5^2 = 550\ \text{W}$.

5. We can use the envelope detector because the modulation index $m \le 1$. The output waveform of the envelope detector is



6. The magnitudes of the carrier line and the sidebands are $Ac/2$ and $Acm/4$, respectively. Therefore, according to "the magnitude of the carrier line is found to be equal to the magnitude of the sidebands", we know that the modulation index m is 2. As $m>1$, over-modulation will occur, and the modulated signal $y(t)$ cannot be properly detected by an envelope detector.

Question 3 (32 marks)

The output signal of an FM system is given by:

$$s_{FM}(t) = 20\cos[10^8 \pi t + 1000\pi \int_{-\infty}^{t} \cos(10^3 \pi \tau) d\tau].$$

1. Determine the peak frequency deviation; (4 marks)
2. Determine the modulation index; (4 marks)
3. Determine the output power at the second sidebands; (4 marks)
4. Determine the output power at 49.9999 MHz and 49.9995 MHz, respectively; (6 marks)
5. Determine the minimum channel bandwidth required for transmitting those sidebands whose magnitudes are larger than 5% of the magnitude of the carrier component; (6 marks)
6. Suppose that the amplitude of the input signal is carefully increased until the output signal at 50.0005MHz is zero. Determine the effective bandwidth of the output signal according to Carson's rule. (8 marks)

Solution:

1. The instantaneous frequency can be obtained as

$$f(t) = \frac{1}{2\pi} \cdot \frac{d\Psi(t)}{dt} = 5 \times 10^7 + 500\cos(10^3 \pi t).$$

As the carrier frequency is f_c=50 MHz, the peak frequency deviation is $\Delta f = \max_t |f(t) - f_c| = 500\text{Hz}$.

2. The input message signal is a sinusoidal signal with frequency f_m=500Hz. The modulation index is then $\beta=\Delta f/f_m=1$.

3. The total power is $P_t = 20^2/2 = 200$ W. The output power at the second sidebands can be therefore obtained as $2 \times P_t |J_2(1)|^2 = 400 \times 0.1149^2 = 5.28$ W.

4. As there is no frequency component at 49.9999M=f_c-0.2f_m, the output power at 49.9999MHz is 0. On the other hand, 49.9995M=f_c-1f_m. The output power at 49.9995MHz is then given by $P_t |J_1(1)|^2 = 200 \times 0.44^2 = 38.72$ W.

5. From the table we can see that when $n > 2$, $|J_n(1)| < |J_0(1)| \cdot 5\% = 0.03826$. Therefore, the sidebands from f_c-2f_m to f_c+2f_m will be transmitted, and the required channel bandwidth is 4f_m=2kHz.

6. 50.0005M=f_c+1f_m. According to the table, as β increases from 1, $J_1(\beta)$ first becomes zero when $\beta \approx 4$. Therefore, the effective bandwidth is $2(\beta+1)f_m = 5$ kHz.