EE2302 Foundations of Information Engineering

Assignment 2 (Solution)

1.

a) No, because the elements *c* and *d* map to the same element in the co-domain of *F*.
b) Yes, because all the elements in *Y* are images of some element in *X*.
c) {*e*, *f*, *g*}.
d) No, because the elements *a* and *b* map to the same element in the co-domain of *G*.
e) No, because the element *g* is not an image of any element in the domain of *G*.
f) {*e*, *f*}.

2.

a) No. We can give a counterexample $g(1) = g(-1) = 1$ and $1 \neq -1$.
b) Yes. For $\forall y \in \mathbb{R}_+$, there exists $x = \pm\sqrt{y} \in \mathbb{R}$ such that $g(x) = (\pm\sqrt{y})^2 = y$.

3. Let x_1 and x_2 be elements in X such that $g \circ f(x_1) = g \circ f(x_2)$. That is equivalent to $g(f(x_1)) = g(f(x_2))$. Since g is injective, $f(x_1) = f(x_2)$. Since f is also injective, $x_1 = x_2$. Hence, $g \circ f$ is injective.

4. Yes. Define a function $f: (0,1) \to (10, 100)$ such that $f(x) = 90x + 10$. Suppose $f(x_1) = f(x_2)$. Then $90x_1 + 10 = 90x_2 + 10$, which implies that $x_1 = x_2$. Hence, f is one-to-one. Given any $y \in (10, 100)$, there exists $x = \frac{y-10}{90} \in (0,1)$ such that $f(x) = y$. Hence, f is onto. Therefore, f is a one-to-one correspondence. Hence, the two sets have the same cardinality.