4.10. **(a)** Use Tables 4.1 and 4.2 to help determine the Fourier transform of the following signal:

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$$

(b) Use Parseval's relation and the result of the previous part to determine the numerical value of

$$A = \int_{-\infty}^{+\infty} t^2 \left(\frac{\sin t}{\pi t}\right)^4 dt$$

4.11. Given the relationships

$$y(t) = x(t) * h(t)$$

and

$$g(t) = x(3t) * h(3t),$$

and given that $x(t)$ has Fourier transform $X(j\omega)$ and $h(t)$ has Fourier transform $H(j\omega)$, use Fourier transform properties to show that $g(t)$ has the form

$$g(t) = Ay(Bt).$$

Determine the values of A and B.

4.12. Consider the Fourier transform pair

$$e^{-|t|} \stackrel{\mathcal{F}}{\longleftrightarrow} \frac{2}{1+\omega^2}.$$

(a) Use the appropriate Fourier transform properties to find the Fourier transform of $te^{-|t|}$.

(b) Use the result from part (a), along with the duality property, to determine the Fourier transform of

$$\frac{4t}{(1+t^2)^2}.$$

Hint: See Example 4.13.

4.13. Let $x(t)$ be a signal whose Fourier transform is

$$X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5),$$

and let

$$h(t) = u(t) - u(t-2).$$

(a) Is $x(t)$ periodic?

(b) Is $x(t) * h(t)$ periodic?

(c) Can the convolution of two aperiodic signals be periodic?

BASIC PROBLEMS

4.21. Compute the Fourier transform of each of the following signals:

(a) $[e^{-\alpha t}\cos\omega_0 t]u(t),\ \alpha > 0$

(b) $e^{-3|t|}\sin 2t$

(c) $x(t) = \begin{cases} 1 + \cos\pi t, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}$

(d) $\sum_{k=0}^{\infty} \alpha^k\,\delta(t - kT),\ |\alpha| < 1$

(e) $[te^{-2t}\sin 4t]u(t)$

(f) $\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$

(g) $x(t)$ as shown in Figure P4.21(a)

(h) $x(t)$ as shown in Figure P4.21(b)

(i) $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

(j) $\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$

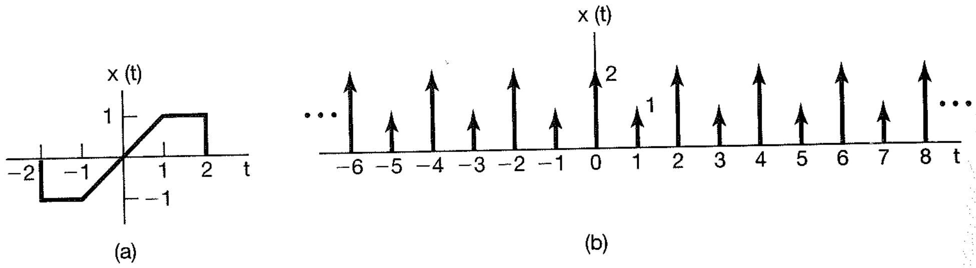

Figure P4.21

4.22. Determine the continuous-time signal corresponding to each of the following transforms.

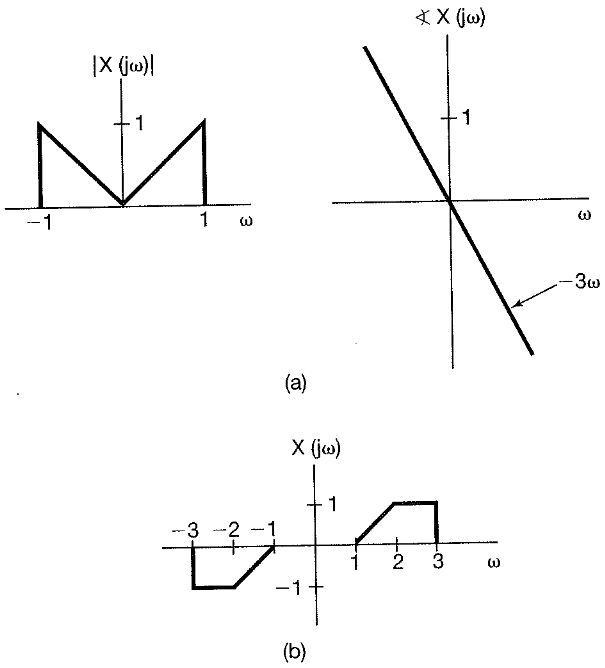

Figure P4.22

(a) $X(j\omega) = \frac{2\sin[3(\omega-2\pi)]}{(\omega-2\pi)}$
(b) $X(j\omega) = \cos(4\omega + \pi/3)$
(c) $X(j\omega)$ as given by the magnitude and phase plots of Figure P4.22(a)
(d) $X(j\omega) = 2[\delta(\omega-1) - \delta(\omega+1)] + 3[\delta(\omega-2\pi) + \delta(\omega+2\pi)]$
(e) $X(j\omega)$ as in Figure P4.22(b)

4.23. Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}.$$

Determine the Fourier transform of each of the signals shown in Figure P4.23. You should be able to do this by explicitly evaluating *only* the transform of $x_0(t)$ and then using properties of the Fourier transform.

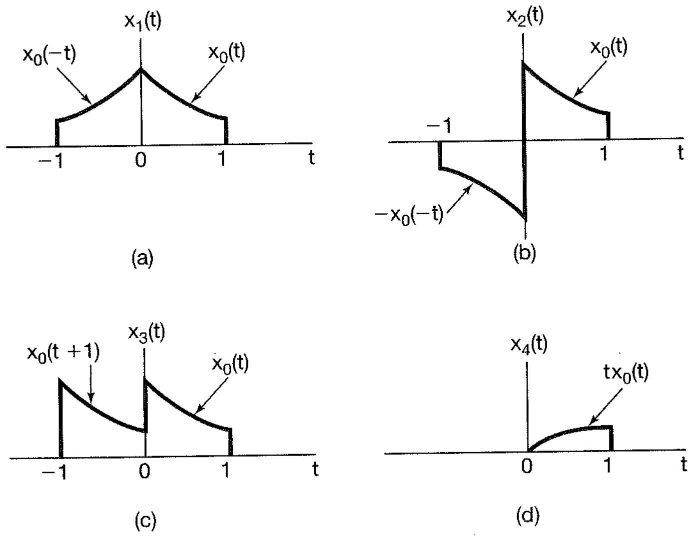

Figure P4.23

4.24. **(a)** Determine which, if any, of the real signals depicted in Figure P4.24 have Fourier transforms that satisfy each of the following conditions:
(1) $\mathcal{R}e\{X(j\omega)\} = 0$
(2) $\mathcal{I}m\{X(j\omega)\} = 0$
(3) There exists a real α such that $e^{j\alpha\omega}X(j\omega)$ is real
(4) $\int_{-\infty}^{\infty} X(j\omega)\, d\omega = 0$
(5) $\int_{-\infty}^{\infty} \omega X(j\omega)\, d\omega = 0$
(6) $X(j\omega)$ is periodic

(b) Construct a signal that has properties (1), (4), and (5) and does *not* have the others.

4.25. Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ depicted in Figure P4.25.

(a) $X(j\omega)$ can be expressed as $A(j\omega)e^{j\Theta(j\omega)}$, where $A(j\omega)$ and $\Theta(j\omega)$ are both real-values. Find $\Theta(j\omega)$.

(b) Find $X(j0)$.

(c) Find $\int_{-\infty}^{\infty} X(j\omega)\,d\omega$.

(d) Evaluate $\int_{-\infty}^{\infty} X(j\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}\,d\omega$.

(e) Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2\,d\omega$.

(f) Sketch the inverse Fourier transform of $\mathcal{R}e\{X(j\omega)\}$.

Note: You should perform all these calculations without explicitly evaluating $X(j\omega)$.

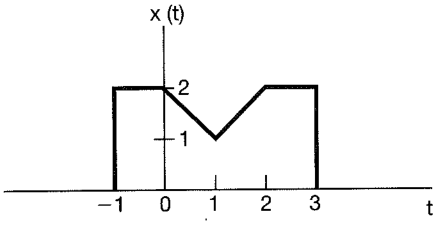

Figure P4.25

4.26. **(a)** Compute the convolution of each of the following pairs of signals $x(t)$ and $h(t)$ by calculating $X(j\omega)$ and $H(j\omega)$, using the convolution property, and inverse transforming.

(i) $x(t) = te^{-2t}u(t)$, $h(t) = e^{-4t}u(t)$

(ii) $x(t) = te^{-2t}u(t)$, $h(t) = te^{-4t}u(t)$

(iii) $x(t) = e^{-t}u(t)$, $h(t) = e^{t}u(-t)$

(b) Suppose that $x(t) = e^{-(t-2)}u(t-2)$ and $h(t)$ is as depicted in Figure P4.26. Verify the convolution property for this pair of signals by showing that the Fourier transform of $y(t) = x(t) * h(t)$ equals $H(j\omega)X(j\omega)$.

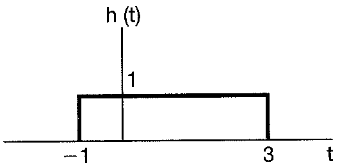

Figure P4.26

4.27. Consider the signals

$$x(t) = u(t-1) - 2u(t-2) + u(t-3)$$

and

$$\tilde{x}(t) = \sum_{k=-\infty}^{\infty} x(t-kT),$$

where $T > 0$. Let a_k denote the Fourier series coefficients of $\tilde{x}(t)$, and let $X(j\omega)$ denote the Fourier transform of $x(t)$.

(a) Determine a closed-form expression for $X(j\omega)$.

(b) Determine an expression for the Fourier coefficients a_k and verify that $a_k = \frac{1}{T}X\left(j\frac{2\pi k}{T}\right)$.

4.28. **(a)** Let $x(t)$ have the Fourier transform $X(j\omega)$, and let $p(t)$ be periodic with fundamental frequency ω_0 and Fourier series representation

$$p(t) = \sum_{n=-\infty}^{+\infty} a_n e^{jn\omega_0 t}.$$

Determine an expression for the Fourier transform of

$$y(t) = x(t)p(t). \qquad \text{(P4.28–1)}$$

(b) Suppose that $X(j\omega)$ is as depicted in Figure P4.28(a). Sketch the spectrum of $y(t)$ in eq. (P4.28–1) for each of the following choices of $p(t)$:

(i) $p(t) = \cos(t/2)$

(ii) $p(t) = \cos t$

(iii) $p(t) = \cos 2t$

(iv) $p(t) = (\sin t)(\sin 2t)$

(v) $p(t) = \cos 2t - \cos t$

(vi) $p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - \pi n)$

(vii) $p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - 2\pi n)$

(viii) $p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - 4\pi n)$

(ix) $p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - 2\pi n) - \frac{1}{2}\sum_{n=-\infty}^{+\infty} \delta(t - \pi n)$

(x) $p(t) =$ the periodic square wave shown in Figure P4.28(b).

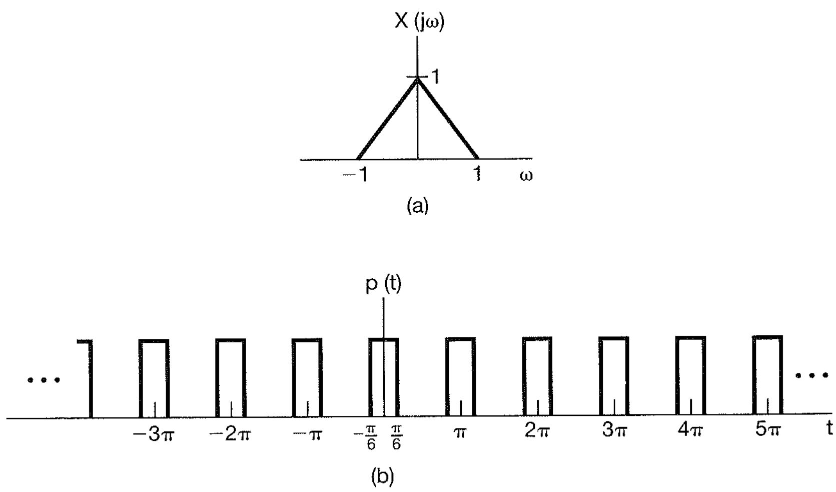

Figure P4.28