Tutorial 9

Codes (with solution)

Question 1

You want to encode the information bits 1010101010101011 by a two-dimensional parity scheme. You put the information bits into a 4×4 array, and even parity is assumed.

a) Determine the parity bits.

b) When determining the parity bit at the bottom right corner, do you use column parity or row parity? Is there any inconsistency?

Q. 1 (solution)

a) The rightmost column and bottom row are the parity bits.

1	0	1	0	0
1	0	1	0	0
1	0	1	0	0
1	0	1	1	1
0	0	0	1	1

b) There won't be any inconsistency. (Why?)

Question 2

- Suppose a transmission channel operates at 3 Mbps and has a bit error rate of 10^{-3}. Bit errors occur at random and are independent of each other. Suppose the (3, 1) repetition code is used. The receiver takes the three received bits and decides which bit was sent by taking the majority vote of the three bits.
  a) What is the generator matrix of this code?
  b) What is the parity-check matrix of this code?
  c) What is the effective data rate if this repetition code is used?
  d) Find the probability that the receiver makes a decoding error.

Q. 2 (solution)

a) $G = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$

b) $H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

c) Code rate = 1/3, so Effective data rate = 1 Mbps

d) The receiver will make a decoding error if two or three bits are in error. Hence,

$$\Pr\{\text{decoding error}\} = 3(1-p)p^2 + p^3,$$

where $p = 10^{-3}$.

$$\Pr\{\text{decoding error}\} = 2.997 \times 10^{-6} + 10^{-9}$$
$$= 2.998 \times 10^{-6}$$

Question 3: Hamming Code

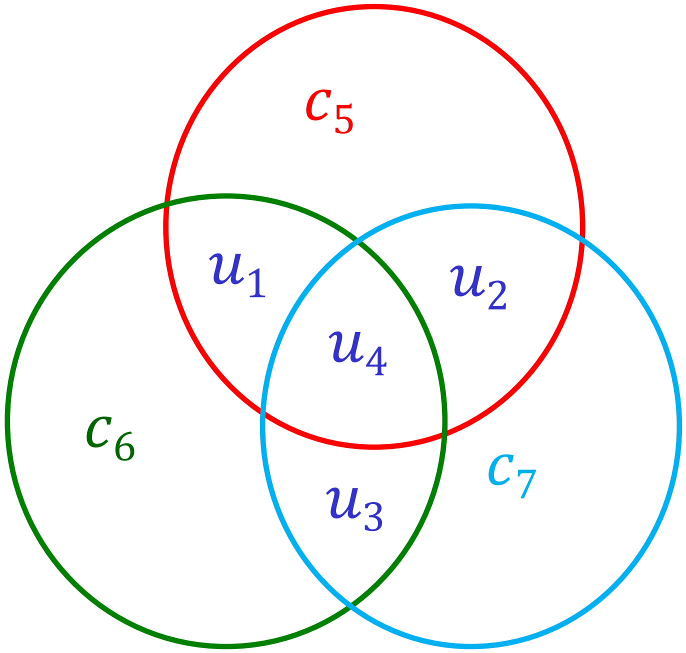
Consider the (7, 4) Hamming code.
The encoding equations are

$$c_1 = u_1, c_2 = u_2, c_3 = u_3, c_4 = u_4$$
$$c_5 = u_1 + u_2 + u_4$$
$$c_6 = u_1 + u_3 + u_4$$
$$c_7 = u_2 + u_3 + u_4$$

a) Write down its generator matrix and parity-check matrix.

b) List all its codewords and find its minimum distance. Explain its error detection/correction capability.

c) If the received vector is 1001010, what are the decoded data bits?



Q.3 (solution)

a) The generator matrix is

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}_{k \times n}$$

The parity-check matrix is

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}_{r \times n}$$

Q.3 (solution)

b) The codewords are shown in the table. $d_{\min} = 3$.

Either detect all single-bit errors and double-bit errors, **or** correct all single-bit errors, **but not** both.

c_1	c_2	c_3	c_4	c_5	c_6	c_7
0	0	0	0	0	0	0
0	0	0	1	1	1	1
0	0	1	0	0	1	1
0	0	1	1	1	0	0
0	1	0	0	1	0	1
0	1	0	1	0	0	1
0	1	1	0	1	1	0
0	1	1	1	0	0	1
1	0	0	0	1	1	0
1	0	0	1	0	0	1
1	0	1	0	1	0	1
1	0	1	1	0	1	0
1	1	0	0	0	1	1
1	1	0	1	1	0	1
1	1	1	0	0	0	0
1	1	1	1	1	1	1

Q.3 (solution)

c) The decoded data bits should be 1011.