EE2302 Foundations of Information and Data Engineering

Assignment 1 (Solution)

1.

a. $C \subseteq D$ Proof: [We will show that every element of C is in D.] Suppose n is any element of C. Then n = 6r - 5 for some integer r. Let s = 2r - 2. Then s is an integer (because products and differences of integers are integers), and

$$3s + 1 = 3(2r - 2) + 1 = 6r - 6 + 1 = 6r - 5,$$

which equals n. Thus n satisfies the condition for being in D. Hence, every element in C is in D.

- **b.** $D \nsubseteq C$ because there are elements of D that are not in C. For example, 4 is in D because $4 = 3 \cdot 1 + 1$. But 4 is not in C because if it were, then 4 = 6r 5 for some integer r, which would imply that 9 = 6r, or, equivalently, that r = 3/2, and this contradicts the fact that r is an integer.
- 2. Proof: We need to prove both $B \subseteq C$ and $C \subseteq B$.
 - i. Let m be an element of B, so there is an integer b such that m = 10b 3 = 10(b-1) + 7. Since (b-1) is an integer, by the definition of C, m is an element of C. Therefore, $B \subseteq C$.
 - ii. Let n be an element of C, so there is an integer c such that n = 10c + 7 = 10(c + 1) 3. Since (c + 1) is an integer, be the definition of B, n is an element of B. Therefore, $C \subseteq B$.
- 3. domain = $\mathbb{N} \cup \{0\}$, range = $\{0, 1, 2, ..., 9\}$.
- 4. The description "the smallest integer not describable in fewer than twelve English words" contains only eleven English words. If n is well defined, then it is describable in only eleven English words, which is a contradiction.
- 5. No. Suppose there were a computer program P that had as output a list of all computer programs that do not list themselves in their output. Consider the following two cases:
 - a) If P lists itself as output, then it would be on the output list of P, which means that P would not list itself in its output. A contradiction.
 - b) If P does not list itself as output, then it would be a member of the list of all programs that do not list themselves in their output, and this list is exactly the output of P. Hence, P would list itself as output. Again, a contradiction.

In both cases, the assumption of the existence of such a program P is contradictory, and so no such program exists.