CITY UNIVERSITY OF HONG KONG
Department of Electronic Engineering

EE 3210 Signals and Systems

Homework #3

1. Problem 1.27, (a), (b), (c), (d), (f), pp. 62.

2. Problem 1.31, pp. 63. (Hint: Express $x_2(t)$ and $x_3(t)$ in terms of the combinations of its shifted and scaled versions)

3. A discrete-time system may or may not be (1) memoryless, (2) time-invariant, (3) linear, (4) causal, and (5) stable. Determine if each of the following systems satisfy these properties. Justify your answers.
(a) $y(n) = x(n)x(n-1)$.
(b) $y(n) = x(n-2) - 2x(n-17)$.
(c) $y(n) = nx(n)$.
(d) $y(n) = x(2n)$.
(e)
$$y(n) = \begin{cases} x(n), & n \geq 1 \\ 0, & n = 0 \\ x(n+1), & n \leq -1 \end{cases}$$

4. Problem 2.22, (a), pp. 141. Additionally, do the same for the following pairs: (b) $x(t) = e^{-3t}u(t)$, $h(t) = u(t-1)$; (c) $h(t) = u(t) - u(t-1)$,
$$x(t) = \begin{cases} e^{t}, & t < 0 \\ e^{5t} - 2e^{-t}, & t > 0 \end{cases}$$

Solution for Selected Homework Problems

Homework 2

Problem 1.27, pp. 61

(c) $$y(t) = \int_{-\infty}^{2t} x(\tau)\, d\tau$$

Solution:

(i) Not memoryless, e.g., $y(0) = \int_{-\infty}^{0} x(t)\, dt$

(ii) Not causal, e.g., $y(1) = \int_{-\infty}^{2} x(t)\, dt$

(iii) Not stable. Consider the bounded input $x(t) = u(t)$. We have, for $t > 0$

$$y(t) = \int_{0}^{2t} d\tau = 2t$$

Thus,

$$\lim_{t \to \infty} |y(t)| = \infty$$

(iv) Linear, since for any $x(t) = \alpha x_1(t) + \beta x_2(t)$,

$$y(t) = \int_{-\infty}^{2t} \left[\alpha x_1(\tau) + \beta x_2(\tau)\right] d\tau$$

$$= \alpha \int_{-\infty}^{2t} x_1(\tau)\, d\tau + \beta \int_{-\infty}^{2t} x_2(\tau)\, d\tau$$

$$= \alpha\, y_1(t) + \beta\, y_2(t)$$

(v) Time-varying. Consider $x(t) = x_1(t - t_0)$

$$y(t) = \int_{-\infty}^{2t} x_1(\tau - t_0)\, d\tau$$

$$(\lambda = \tau - t_0)$$

$$= \int_{-\infty}^{2t - t_0} x_1(\lambda)\, d\lambda$$

$$\neq \int_{-\infty}^{2(t-t_0)} x_1(\lambda)\, d\lambda$$

$$= y_1(t - t_0)$$

(f) $y(t) = x\left(\frac{t}{3}\right)$

Solution: (i) Not memoryless, e.g., $y(1) = x\left(\frac{1}{3}\right)$

(ii) Not causal, e.g., $y(-1) = x\left(-\frac{1}{3}\right)$

(iii) Stable, since for any $x(t)$ such that $|x(t)| \leq M_x$, we have

$$|y(t)| = \left|x\left(\frac{t}{3}\right)\right| \leq M_x.$$

(iv) linear, since for any $x(t) = \alpha x_1(t) + \beta x_2(t)$,

$$y(t) = x\left(\frac{t}{3}\right) = \alpha x_1\left(\frac{t}{3}\right) + \beta x_2\left(\frac{t}{3}\right)$$

$$= \alpha y_1(t) + \beta y_2(t)$$

(v) Time-varying. Consider $x(t) = x_1(t - t_0)$.

$$\Rightarrow y(t) = x\left(\frac{t}{3}\right) = x_1\left(\frac{t}{3} - t_0\right) \neq x_1\left[\frac{t-t_0}{3}\right] = y_1(t-t_0).$$

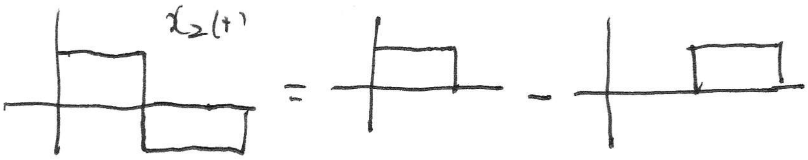
Problem 1.31, PP. 62

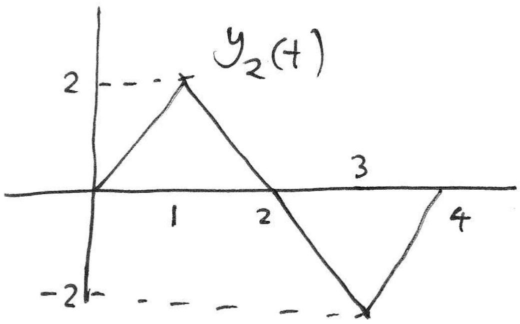
(a) Solution:

$$x_2(t) = x_1(t) - x_1(t-2)$$

Since the system is LTI, it follows that

$$y_2(t) = y_1(t) - y_1(t-2)$$





Problem 1.32, PP. 63

Solution: Let T_x be the period for $x(t)$. Then,

(1) $y_1\left(t + \frac{T_x}{2}\right) = x\left[2\left(t + \frac{T_x}{2}\right)\right] = x(2t + T_x) = x(2t) = y_1(t)$

$\Rightarrow y_1(t)$ is periodic, with period $\frac{T_x}{2}$.

(2) $y_2(t) = x\left(\frac{t}{2}\right) \Longrightarrow x(t) = y_2(2t)$

Let $y_2(t + T_2) = y_2(t)$. Then,

$$x\left(t + \frac{T_2}{2}\right) = y_2\left[2\left(t + \frac{T_2}{2}\right)\right] = y_2(2t + T_2)$$

$$= y_2(2t) = x(t)$$

$\Longrightarrow x(t)$ is periodic, with period $\frac{T_2}{2}$.

Prob. 3

(a) $y(n) = x(n)\,x(n-1)$

(i) Not memoryless, since, e.g., $y(0) = x(0)\,x(-1)$;

(ii) Causal, since $y(n)$ depends on only $x(n)$ and $x(n-1)$;

(iii) Stable, since for any $x(n)$ such that

$$|x(n)| \le M_x,$$

$$\Rightarrow \quad |y(n)| = |x(n)| \cdot |x(n-1)| \le M_x \cdot M_x = M_x^2$$

(iv) Nonlinear; Consider $x(n) = c\,x_1(n)$ where $y_1(n) = x_1(n)\,x_1(n-1)$.

$$\Rightarrow \quad y(n) = x(n)\,x(n-1) = [c\,x_1(n)][c\,x_1(n-1)]$$

$$= c^2\,x_1(n)\,x_1(n-1)$$

$$= c^2\,y_1(n)$$

$$\ne c\,y_1(n) \qquad (\text{for any } c \ne 1)$$

(v) Time-invariant. Let $x(n) = x_1(n-n_0)$. Then

shifted by n_0

$$y(n) = x(n)\,x(n-1)$$

$$= x_1(n-n_0)\,x_1[(n-1)-n_0]$$

$$= x_1(n-n_0)\,x_1[(n-n_0)-1]$$

$$= y_1(n-n_0)$$

shifted by n_0