Tutorial 3 (with solution)

Relations

Q.1 Relation

Let A be the set of all lines in the 2-dimensional plane. Let R be the relation on A defined by

$$l_1 R l_2 \text{ iff } l_1 \text{ is perpendicular to } l_2.$$

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a) Is it reflexive?
b) Is it symmetric?
c) Is it antisymmetric?
d) Is it transitive?

Q.1 (solution)

a) No
b) Yes
c) No
d) No

You are expected to be able to explain the answers.

Q.2 Equivalence Relation

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Let S be the set of all digital logic circuit with two inputs and one output.

Let R be defined on S as follows:

$c_1 R c_2$ iff c_1 has the same input/output table as c_2.

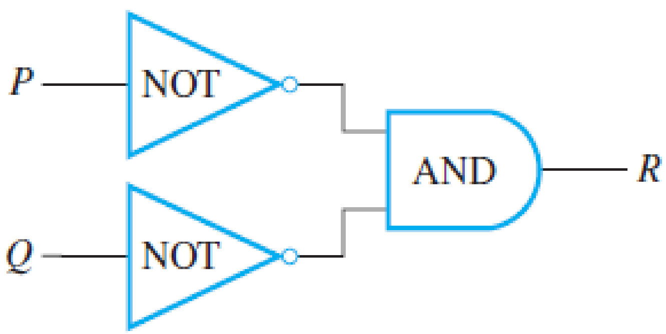
a) Explain why R is an equivalence relation.
b) How many distinct equivalence classes are there?
c) Find two different circuits that are in the same equivalence class.

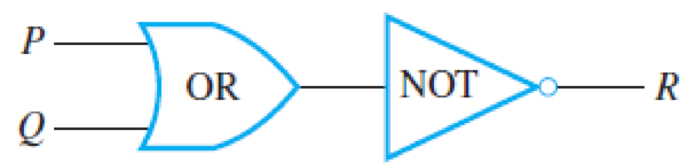
Q.2 (solution)

a) Check the three defining conditions. Details omitted.

b) There are $2^4 = 16$ equivalence classes.

c) An example:





Why are they equivalent? ⟹ De Morgan's laws!

Q.3 Partial Order

Let $A = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$.

Consider the relation R on A defined as

$$xRy \text{ iff } x \text{ is a factor of } y.$$

a) Prove that R is a partial order. page 3-35

b) List all maximal elements. page 3-38

c) List all minimal elements.

Q.3 (solution)

a) Check the three defining conditions. Details omitted.
b) Maximal elements: 8, 9, 10, 11, 12, 13, 14, 15
c) Minimal elements: 2, 3, 5, 7, 11, 13

Q.4 Congruence page 3-24

Let $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$.

Prove that $a + c \equiv b + d \pmod{n}$.

Q.4 (solution)

- By the definition of congruences,

$$a = kn + b \text{ for some integer } k.$$

$$c = hn + d \text{ for some integer } h.$$

- Adding the two equations,

$$\begin{aligned} a + c &= kn + hn + b + d \\ &= (k + h)n + (b + d). \end{aligned}$$

- Since $(k + h)$ is an integer,

$$a + c \equiv b + d \pmod{n}.$$

Q.E.D.