Solutions to EE3210 Tutorial 7 Problems

Problem 1: Recall pages 13 and 14 of Part 1 lecture notes. The signal $x_2(t) = x_1(1-t)$ can be obtained from $x_1(t)$ in two alternative ways:

(a) Time shift first followed by time reversal, i.e.:

$$x_1(t) \Rightarrow y(t) = x_1(t+1) \Rightarrow x_2(t) = y(-t) = x_1(-t+1).$$

In this way, the time shift property of the continuous-time Fourier series indicates that, if $x_1(t) \leftrightarrow a_k$, the Fourier coefficients c_k of $x_1(t+1)$ can be expressed as

$$c_k = \left[e^{jk(2\pi/T)}\right] a_k. \tag{1}$$

Then, the time reversal property of the continuous-time Fourier series indicates that, if $x_1(t+1) \leftrightarrow c_k$, the Fourier coefficients b_k of $x_2(t) = x_1(-t+1)$ can be expressed as

$$b_k = c_{-k}. \tag{2}$$

Thus, by (1) and (2), we have

$$b_k = \left[e^{-jk(2\pi/T)}\right] a_{-k}. \tag{3}$$

(b) Time reversal first followed by time shift, i.e.:

$$x_1(t) \Rightarrow y(t) = x_1(-t) \Rightarrow x_2(t) = y(t-1) = x_1(-t+1).$$

In this way, the time reversal property of the continuous-time Fourier series indicates that, if $x_1(t) \leftrightarrow a_k$, the Fourier series coefficients c_k of $x_1(-t)$ can be expressed as

$$c_k = a_{-k}. \tag{4}$$

Then, the time shift property of the continuous-time Fourier series indicates that, if $x_1(-t) \leftrightarrow c_k$, the Fourier series coefficients b_k of $x_2(t) = x_1(-t+1)$ can be expressed as

$$b_k = \left[e^{-jk(2\pi/T)}\right] c_k. \tag{5}$$

Thus, by (4) and (5), we have

$$b_k = \left[e^{-jk(2\pi/T)}\right] a_{-k}$$

which is exactly the same as (3).

Problem 2: Recall pages 9 and 10 of Part 6 lecture notes. Note that the signal $x(t)$ in this problem is in the form of a periodic square wave with $\alpha = 1/6$ and $T = 3$. Therefore, we have

$$x(t) \leftrightarrow a_k = \frac{\sin(k\pi/3)}{k\pi}.$$

Then, using the linearity and time shift properties of the continuous-time Fourier series, the Fourier series coefficients b_k of the signal $2x(t-0.5) + x(t-1.5)$ can be obtained as

$$b_k = \left[2e^{-jk\pi/3} + e^{-jk\pi}\right] a_k = \frac{2 - e^{-jk2\pi/3} - e^{-jk4\pi/3}}{jk2\pi}.$$

Problem 3: Recall Problem 1 in Tutorial 6. We have the Fourier series coefficients a_k of the signal $x(t) = \cos(4\pi t)$ (periodic with period $T = 1/2$) as

$$a_k = \begin{cases} \frac{1}{2}, & k = -1 \\ \frac{1}{2}, & k = 1 \\ 0, & \text{otherwise.} \end{cases}$$

and the Fourier series coefficients b_k of the signal $y(t) = \sin(4\pi t)$ (periodic with period $T = 1/2$) as

$$b_k = \begin{cases} -\frac{1}{2j}, & k = -1 \\ \frac{1}{2j}, & k = 1 \\ 0, & \text{otherwise.} \end{cases}$$

Since the signal $z(t) = x(t)y(t)$ is also periodic with period $T = 1/2$, applying the multiplication property of the continuous-time Fourier series, we obtain the Fourier series coefficients c_k of $z(t)$ as

$$c_k = \sum_{l=-\infty}^{+\infty} a_l b_{k-l} = a_{-1}b_{k+1} + a_1 b_{k-1} = \begin{cases} a_{-1}b_1 + a_1 b_{-1} = 0, & k = 0 \\ a_{-1}b_{-1} = -\frac{1}{4j}, & k = -2 \\ a_1 b_1 = \frac{1}{4j}, & k = 2 \\ 0, & \text{otherwise.} \end{cases}$$

Problem 4: This signal is periodic with a fundamental period $N = 6$. To determine the Fourier series coefficients a_k, we use the analysis formula of the discrete-time Fourier series

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$

and choose the limits of the summation to be $-2 \leq n \leq 3$. Then, we have

$$
\begin{aligned}
a_k &= \frac{1}{6} \sum_{n=-2}^{3} x[n] e^{-jk(\pi/3)n} \\
&= \frac{1}{6} + \frac{1}{3}\left(e^{j\pi k/3} + e^{-j\pi k/3}\right) - \frac{1}{6}\left(e^{j2\pi k/3} + e^{-j2\pi k/3}\right) \\
&= \frac{1}{6} + \frac{2}{3}\cos\left(\frac{\pi}{3}k\right) - \frac{1}{3}\cos\left(\frac{2\pi}{3}k\right)
\end{aligned}
$$

for $-2 \leq k \leq 3$.