Solutions to EE3210 Tutorial 3 Problems

Problem 1:

(a) The system is not causal. For example, when $n = 1$, we have $y[1] = x[2]$.

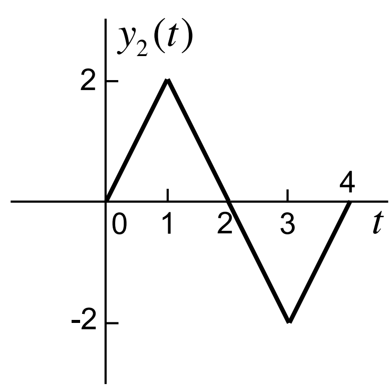
(b) The system is stable. For $0 < B < \infty$, given $|x[n]| \leq B$ for all n, we have $|x[2n]| \leq B$ for all n, and therefore $|y[n]| \leq B$ for all n.

(c) The system is not time invariant. Given $x_1[n]$ and letting $y_1[n] = x_1[2n]$, consider $x_2[n] = x_1[n - n_0]$. Then, we have $y_2[n] = x_2[2n] = x_1[2n - n_0]$, but we have $y_1[n - n_0] = x_1[2n - 2n_0]$. Thus, $y_2[n] \neq y_1[n - n_0]$.

(d) The system is linear. Consider $x_1[n] \to y_1[n] = x_1[2n]$ and $x_2[n] \to y_2[n] = x_2[2n]$. Let $x_3[n] = ax_1[n] + bx_2[n]$. Then,

$$y_3[n] = x_3[2n] = ax_1[2n] + bx_2[2n] = ay_1[n] + by_2[n].$$

Problem 2: Note that $x_2(t) = x_1(t) - x_1(t-2)$. Therefore, using linearity and time invariance, we get $y_2(t) = y_1(t) - y_1(t-2)$, which is shown in the figure below.



Problem 3: In this cascade of two systems, the input to system B is exactly the output of system A.

(a) Since system A is an LTI system, if the output of system A is $ay_1(t) + by_2(t)$, by the property of linearity, this implies that the input to system A is $ax_1(t) + bx_2(t)$. Since system B is the inverse of system A, the response of system B to the input $ay_1(t) + by_2(t)$ must be $ax_1(t) + bx_2(t)$.

(b) Since system A is an LTI system, if the output of system A is $y_1(t - t_0)$, by the property of time invariance, this implies that the input to system A is $x_1(t - t_0)$. Since system B is the inverse of system A, the response of system B to the input $y_1(t - t_0)$ must be $x_1(t - t_0)$.

(c) By definition, an LTI system is one that possesses the properties of linearity and time invariance.

- From (a), we know that system B possesses the property of linearity, since given $y_1(t) \rightarrow x_1(t)$ and $y_2(t) \rightarrow x_2(t)$ it implies that $ay_1(t) + by_2(t) \rightarrow ax_1(t) + bx_2(t)$ for any complex constants a and b.
- From (b), we know that system B possesses the property of time invariance, since given $y_1(t) \rightarrow x_1(t)$ it implies that $y_1(t - t_0) \rightarrow x_1(t - t_0)$ for all t_0.

Therefore, system B is an LTI system.