EE2000 Logic Circuit Design

Mid-Term Test 1 Solution

Question 1 (a)

$$f(a,b,c) = \{(b' + c')'(c + bc')(ab'c' + b' + c') + [(a + c')(b + c)]'\}'$$

$$f(a,b,c) = \{(b' + c')'(c + b)(b' + c') + [(a + c')(b + c)]'\}'$$

0

$$f(a,b,c) = \{[(a + c')(b + c)]'\}'$$

$$f(a,b,c) = (a + c')(b + c)$$

$$f(a,b,c) = ab + ac + bc'$$

$$f(a,b,c) = ac + bc'$$

Question 1 (b)

$$f(a,b,c,d) = \{[(a+b')' + c](c'd')'\}' + [b'(c'd')']'$$

$$f(a,b,c,d) = [(a+b')' + c]' + c'd' + b + c'd'$$

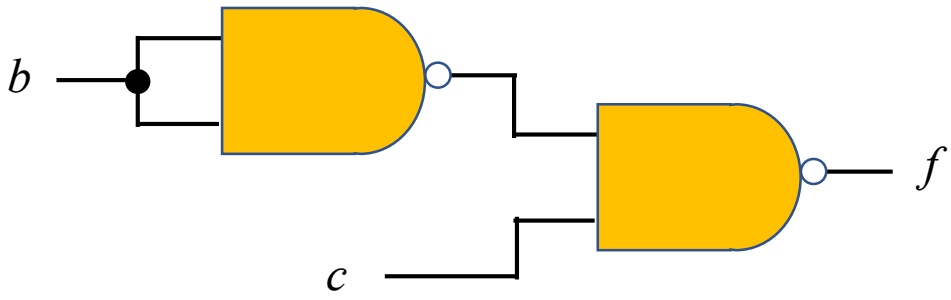
$$f(a,b,c,d) = (a+b')c' + b + c'd'$$

$$f(a,b,c,d) = ac' + b'c' + b + c'd' = ac' + c' + b + c'd'$$

$$f(a,b,c,d) = c' + b$$

Question 1 (c)

$f(a, b, c, d) = c' + b$



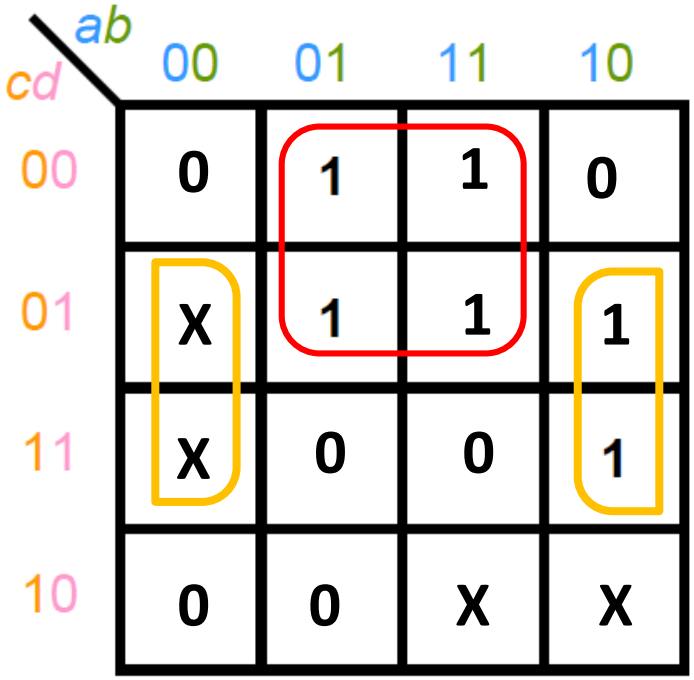
Question 2 (a)

$f(a, b, c, d) =$
$\Sigma m(4, 5, 9, 11, 12, 13) +$
$\Sigma d(1, 3, 10, 14)$

a	b	c	d	f
0	0	0	0	0
0	0	0	1	X
0	0	1	0	0
0	0	1	1	X
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	X
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	X
1	1	1	1	0

Question 2 (b)

$f(a,b,c,d) = \Sigma m(4,5,9,11,12,13) + \Sigma d(1,3,10,14)$



Note: you do not need to draw several K-maps to show PIs and EPIs unless for your own reference.

Or use ink for K-map and pencil for grouping.

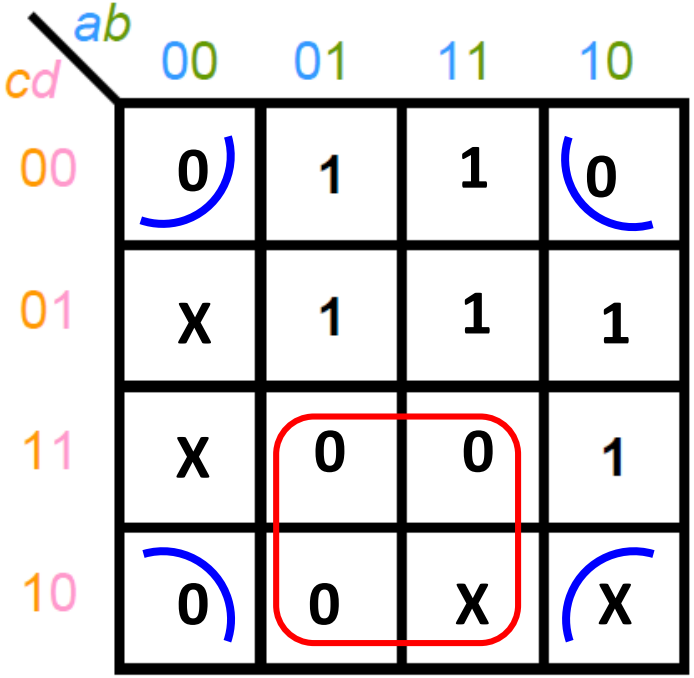
$f(a,b,c,d) = bc' + b'd$

Question 2 (c)

$$f(a,b,c,d) = \Pi M(0,2,6,7,8,15)\Pi d(1,3,10,14)$$

*A common mistake: No ‘+’ sign for Canonical Product.

Question 2 (d)



$$f(a,b,c,d) = (b' + c')(b + d)$$

Question 3 (a)

$$(21\% - 19.5\%) \times \frac{3}{9} + 19.5\%$$
$$= 20\%$$

Question 3 (b)

Minterms	a b c d
m0	0 0 0 0 √
m1	0 0 0 1 √
m5	0 1 0 1 √
m6	0 1 1 0 √
m12	1 1 0 0 √
m7	0 1 1 1 √
m13	1 1 0 1 √
m14	1 1 1 0 √
m15	1 1 1 1 √

Minterms	a b c d
m0 m1	0 0 0 - PI4
m1 m5	0 - 0 1 PI5
m5 m7	0 1 – 1 √
m5 m13	- 1 0 1 √
m6 m7	0 1 1 - √
m6 m14	- 1 1 0 √
m12 m13	1 1 0 - √
m12 m14	1 1 – 0 √
m7 m15	- 1 1 1 √
m13 m15	1 1 – 1 √
m14 m15	1 1 1 - √

Minterms	a b c d
m5 m7 m13 m15	- 1 – 1 PI1
m6 m7 m14 m15	- 1 1 - PI2
m12 m13 m14 m15	1 1 - - PI3

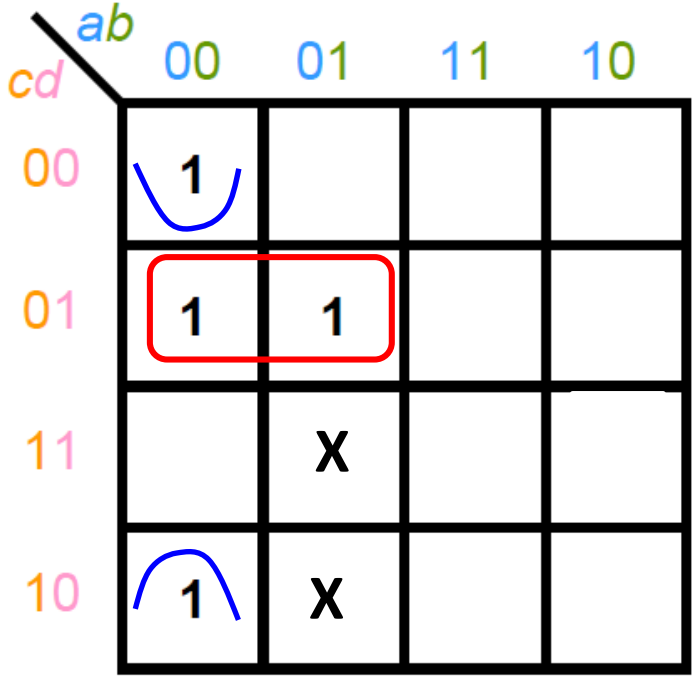
Question 3 (b)

PI	Minterms	a b c d	0	1	5	12	13	14	15
PI1	m5 m7 m13 m15	- 1 – 1			x		x		x
PI2	m6 m7 m14 m15	- 1 1 -						x	x
PI3	m12 m13 m14 m15	1 1 - -				x	x	x	x
PI4	m0 m1	0 0 0 -	x	x					
PI5	m1 m5	0 - 0 1		x	x				

$$\text{LED7} = \text{PI3} + \text{PI4} + \text{PI1} = a'b'c' + ab + bd$$

*A common mistake: PI3 + PI4 + PI5 is not a solution as it has an additional literal.

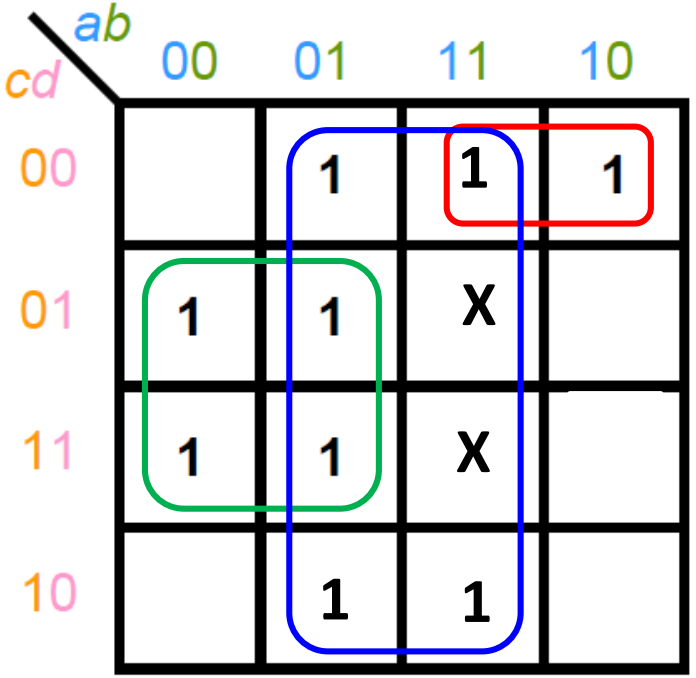
Question 3 (b)



$$\text{LED8} = \Sigma m(0,1,2,5) + \Sigma d(6,7)$$

$$\text{LED8} = a'c'd + a'b'd'$$

Question 3 (b)



$$\text{LED9} = \Sigma m(1,3,4,5,6,7,8,12,14) + \Sigma d(13,15)$$

$$\text{LED9} = b + ac'd' + a'd$$

Question 3 (c)

LED8

Include another product term $a'b'c'$

Question 4 (a)

Consider odd parity, if a Hamming code of 1100011 is received, determine whether single error bit is present, work out the correct Hamming code and the original data code.

$$c_1 = (H_7 \oplus H_5 \oplus H_3 \oplus H_1)' = (1 \oplus 0 \oplus 0 \oplus 1)' = 1$$
$$c_2 = (H_7 \oplus H_6 \oplus H_3 \oplus H_2)' = (1 \oplus 1 \oplus 0 \oplus 1)' = 0$$
$$c_3 = (H_7 \oplus H_6 \oplus H_5 \oplus H_4)' = (1 \oplus 1 \oplus 0 \oplus 0)' = 1$$

$$c_3 c_2 c_1 = (101)_2 = 5$$

Decimal Numbers	8421
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Correct Code: 1110011
Data Code: 1110

This table is given. You can easily work out which bits are required. Don't just memorize formulae. Understand what should be done.
c1: bit with LSB = '1' (7, 5, 3, 1)
For odd parity, if total number of '1' is even c1 = 1; elseif odd c1 = 0.

Question 4 (b)

Consider even parity, if a Hamming code of 1100100 is received, determine whether single error bit is present, work out the correct Hamming code and the original data code.

$$c_1 = H_7 \oplus H_5 \oplus H_3 \oplus H_1 = 1 \oplus 0 \oplus 1 \oplus 0 = 0$$
$$c_2 = H_7 \oplus H_6 \oplus H_3 \oplus H_2 = 1 \oplus 1 \oplus 1 \oplus 0 = 1$$
$$c_3 = H_7 \oplus H_6 \oplus H_5 \oplus H_4 = 1 \oplus 1 \oplus 0 \oplus 0 = 0$$

$$c_3 c_2 c_1 = (010)_2 = 2$$

Decimal Numbers	8421
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Correct Code: 1100110
Data Code: 1101

This table is given. You can easily work out which bits are required.
c1: bit with LSB = '1' (7, 5, 3, 1)
For even parity, if total number of '1' is even c1 = 0; elseif odd c1 = 1.