EE3210
Signals and Systems

Part 1: The Math You Need

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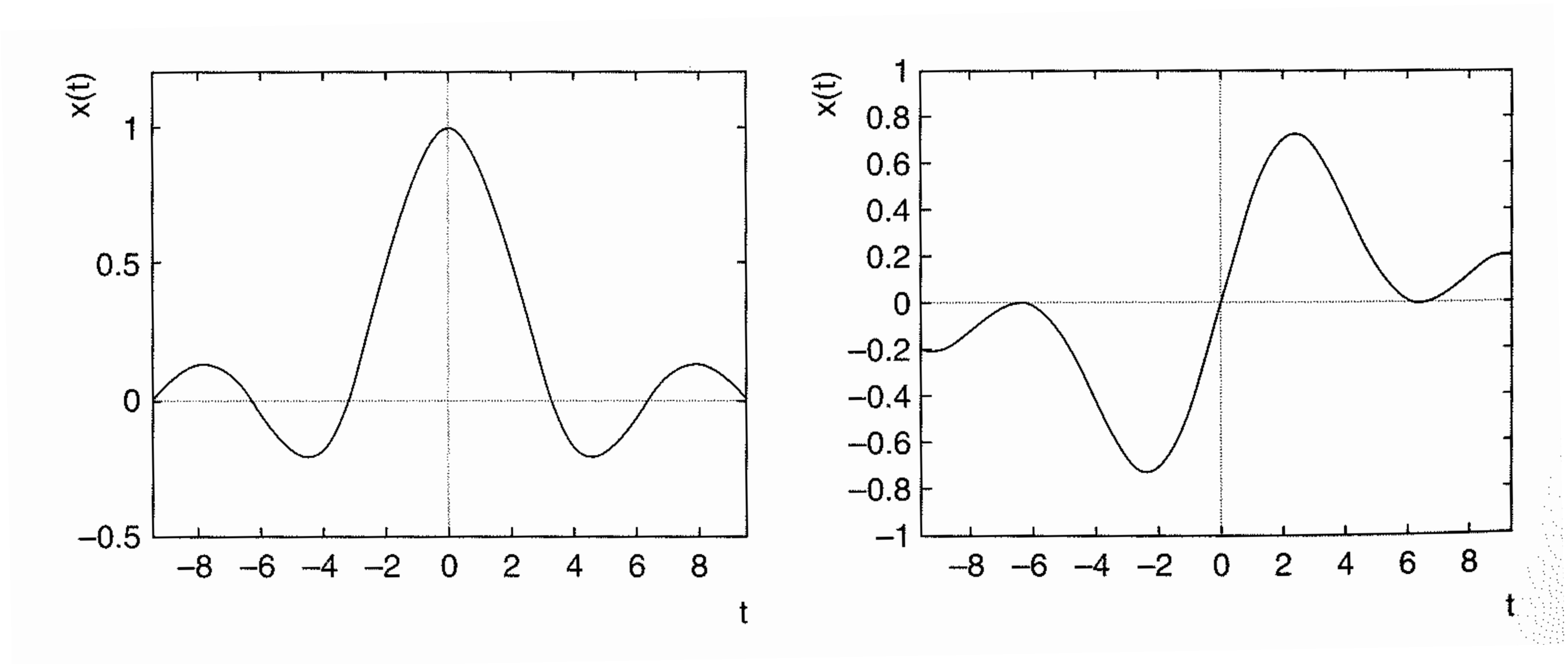
Analog and Digital Functions

- An analog function is designated as $x(t)$, where t stands for time.
  - It provides, for every value of time, the amplitude of a continuous-time signal.
- A digital function is designated as $x[n]$, where n stands for the sample number.
  - It is a means of describing the amplitude of a discrete-time signal, a signal defined only at discrete intervals.

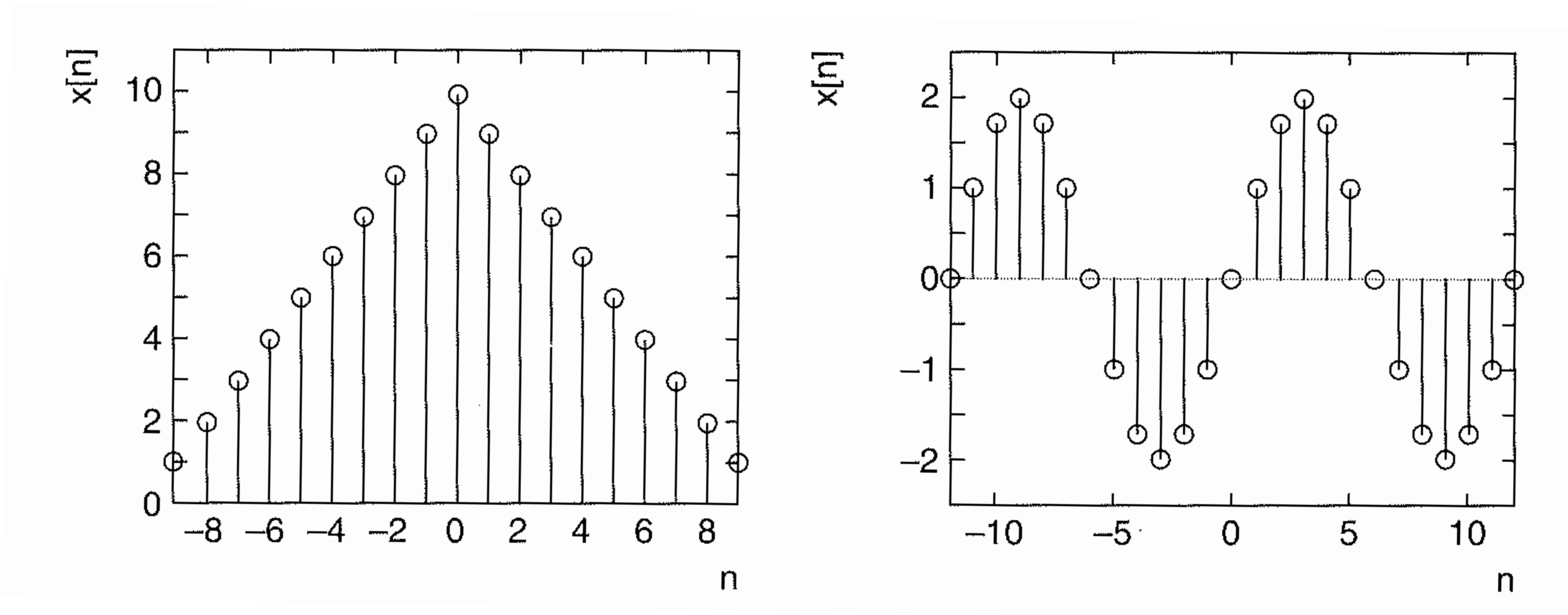
Even and Odd Functions

- An even function satisfies the equation $x(t) = x(-t)$, or $x[n] = x[-n]$.
- An odd function satisfies the equation $x(t) = -x(-t)$, or $x[n] = -x[-n]$.

Examples of Even and Odd Analog Functions



Examples of Even and Odd Digital Functions



Rational Functions

- A function of the form

$$f(x) = \frac{g(x)}{h(x)}$$

is called a rational function if both the numerator $g(x)$ and the denominator $h(x)$ are polynomials in x.

- For example,

$$f(x) = \frac{x^2 - 1}{x + 1}$$

Exponential Functions

- Exponential functions have the form $y = b^x$, where b is the base and x is the exponent.
  - The most important exponential function is e^x, where $e \approx 2.71828$ is known as the Euler's number.
  - Identities:

$$
\begin{aligned}
(b^p)^q &= b^{pq} \\
b^p b^q &= b^{p+q} \\
\frac{b^p}{b^q} &= b^{p-q} \\
b^{-x} &= \frac{1}{b^x}
\end{aligned}
$$

Logarithmic Functions

- Logarithmic functions are by definition the inverse of exponential functions, and have the form $y = \log_b x$, where b is the base.
  - A base e logarithm is called the natural logarithm, or $\ln$, i.e., $\log_e x = \ln x$.
  - Identities:

$$\log_b (c^a) = a \log_b c$$
$$\log_b (gh) = \log_b g + \log_b h$$
$$\log_b \left(\frac{g}{h}\right) = \log_b g - \log_b h$$

Degrees and Radians

- Angles are most commonly expressed in degrees, with 360° in a full circle.
- Angles may also be given in radians, with 2π radians in a full circle.
- Degrees may be converted to radians by multiplying by π radians$/180^\circ$, and radians may be converted to degrees by multiplying by $180^\circ/\pi$ radians.
- Examples:
  - 40° is equivalent to $40(\pi/180) = 2\pi/9$ radians.
  - $\pi/2$ radians is equivalent to $\pi/2(180/\pi) = 90^\circ$.

Complex Numbers

- A complex number z can be defined in Cartesian form as

$$z = x + jy \tag{1}$$

  - j stands for the imaginary unit, satisfying $j^2 = -1$.
  - x is a real number called the real part of z, often noted as $x = \mathrm{Re}\{z\}$.
  - y is a real number called the imaginary part of z, often noted as $y = \mathrm{Im}\{z\}$.

Complex Numbers (cont.)

- A complex number z can also be defined in polar form as

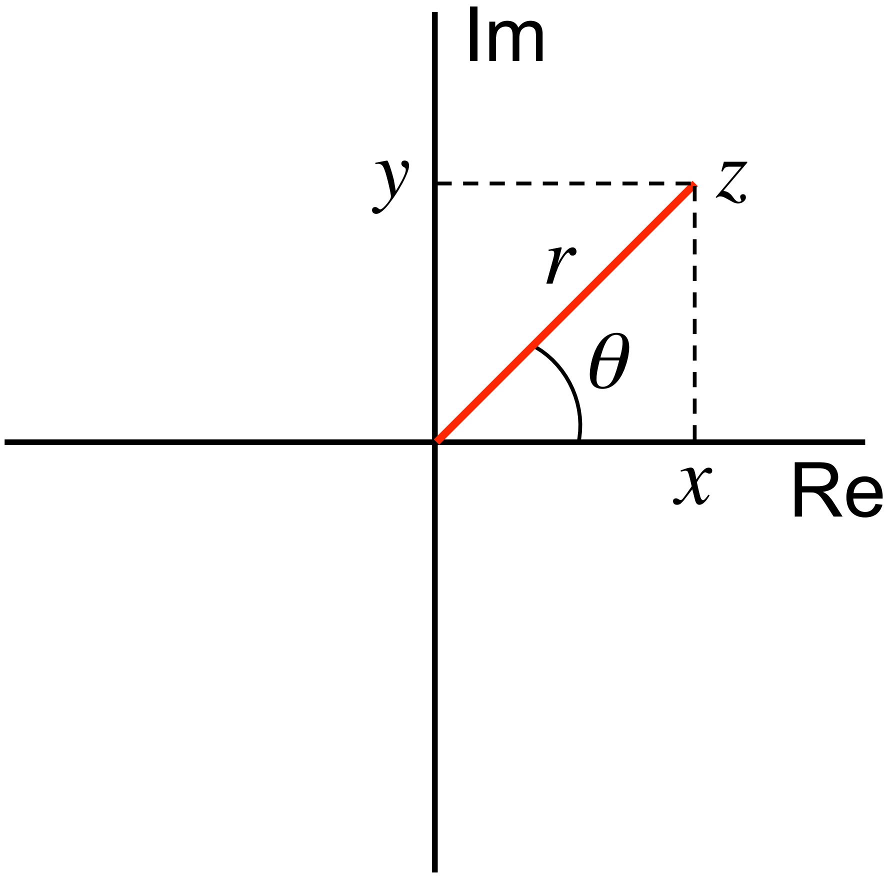
$$z = re^{j\theta} \tag{2}$$

  - $r = |z|$ is the magnitude of z.
  - $\theta = \angle z$ is the phase of z.
  - By Euler's formula, we have $e^{j\theta} = \cos\theta + j\sin\theta$.
  - Thus, the relationship between (1) and (2) is

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \quad \text{and} \quad \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \arctan\dfrac{y}{x} \end{cases}$$

Complex Numbers (cont.)

- Plot of z in the complex plane



Complex Numbers (cont.)

- Two complex numbers $z_1 = x_1 + jy_1$ and $z_2 = x_2 + jy_2$ are equal if and only if $x_1 = x_2$ and $y_1 = y_2$.

- Operations for complex numbers:
  - Addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
  - Subtraction: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
  - Multiplication: $z_1 z_2 = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1)$
  - Division:

$$\frac{z_1}{z_2} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + j\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}$$

Conjugation

- Given $z = x + jy$, the complex conjugate of z is defined as

$$z^* = x - jy = re^{-j\theta}$$

- Properties of conjugation:

$$zz^* = |z|^2$$

$$(z_1 + z_2)^* = z_1^* + z_2^*$$

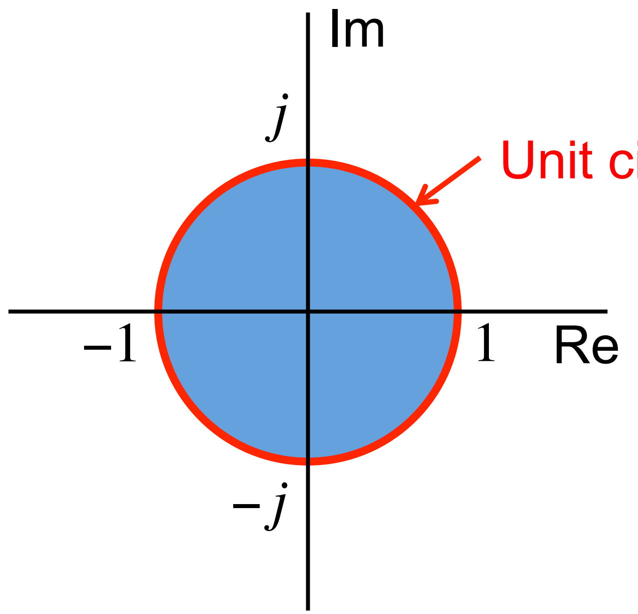
$$(z_1 - z_2)^* = z_1^* - z_2^*$$

$$(z_1 z_2)^* = z_1^* z_2^*$$

$$\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*}$$

Unit Circle

- A unit circle is a circle with radius one on a complex plane, as shown in the figure below.



- Complex numbers with magnitude less than one lie inside the unit circle.
- Thus, the equation

$$|z| < 1$$

describes the shaded area in the figure.