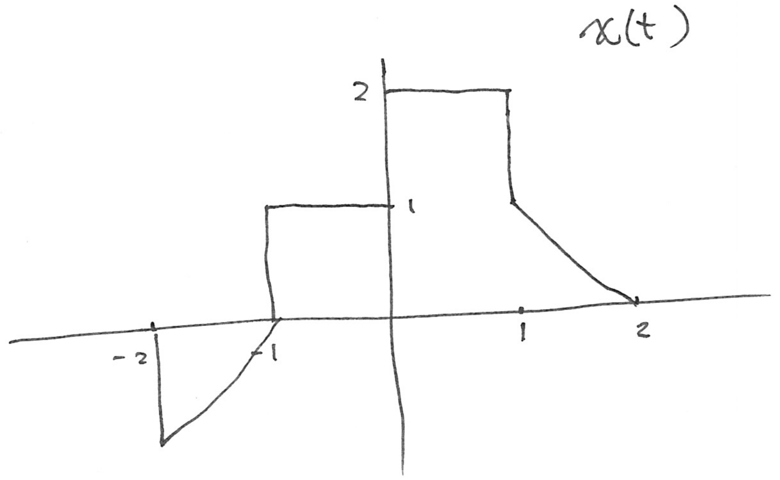
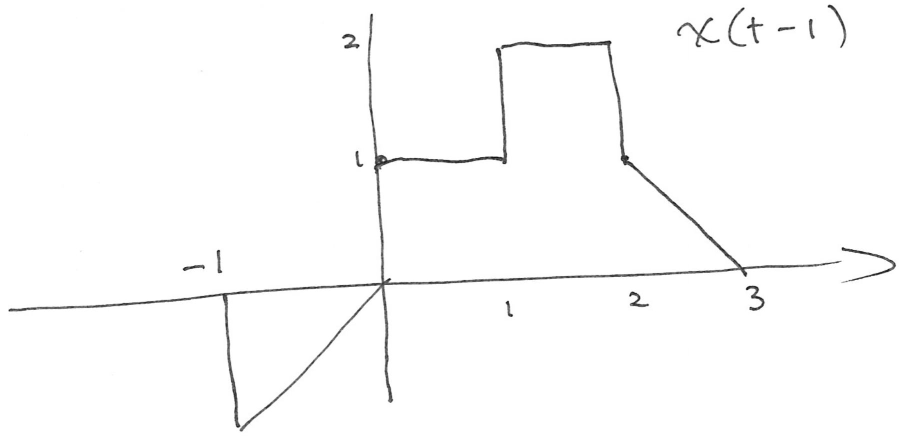
Solutions to Homework #1

Prob. 1



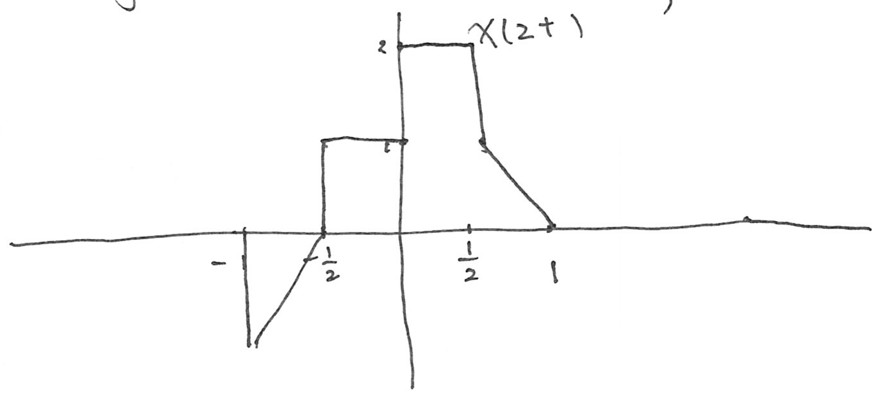
This problem involves time shift, time scaling, and time reversal of signals.

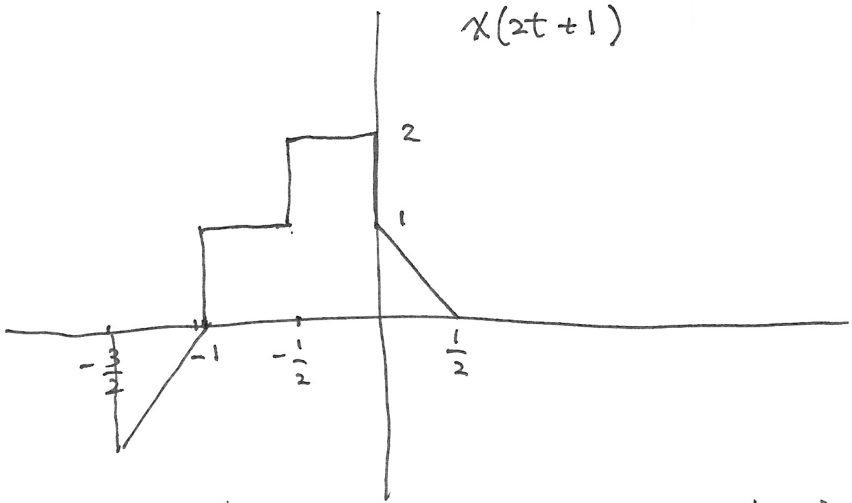
(a)



(c) Let $y(t) = x(2t)$. Then $x(2t+1) = x\left[2\left(t+\frac{1}{2}\right)\right]$
$= y\left(t+\frac{1}{2}\right)$

Thus, $x(2t+1)$ can be determined by first finding $y(t) = x(2t)$ — time scaling, and next by shifting $y(t)$ to $y\left(t+\frac{1}{2}\right)$ — time advance, backward shifting.





To facilitate solving this problem, it is useful to pick on particular (e.g., corner) points and draw the graph, since the signal is piecewise constant or a linear slope.

Prob. 3

(a) $x(t) = 3\cos\left(4t + \frac{\pi}{3}\right)$

This signal fits to the general real sinusoidal signal

$$x(t) = A\cos(\omega t + \phi)$$

and hence is periodic. The fundamental period, consequently, is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$$

(c) $x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2$

$$= \cos^2 2\left(t - \frac{\pi}{6}\right)$$

$$= \frac{1 + \cos 4\left(t - \frac{\pi}{6}\right)}{2} \quad \text{(Trigonometric Identity)}$$

$$= \underbrace{\frac{1}{2}}_{\text{constant}} + \underbrace{\frac{1}{2}\cos\left(4t - \frac{2}{3}\pi\right)}_{\text{Periodic}}$$

$\Rightarrow x(t)$ is periodic. $T = \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$

(d) $x(t) = \mathcal{E}v\{\cos(4\pi t)\,u(t)\}$

$$\cos(4\pi t)\,u(t) = \begin{cases} \cos 4\pi t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

By definition,

$$\mathcal{E}v\{z(t)\} = \frac{z(t) + z(-t)}{2}$$

Set $z(t) = \cos(4\pi t)\,u(t)$.

Then,

$$z(-t) = \begin{cases} 0 & t > 0 \\ \cos 4\pi t & t < 0 \end{cases}$$

(since $\cos(-4\pi t) = \cos 4\pi t$)

$$\Rightarrow \quad \mathcal{E}v\{\cos(4\pi t)\,u(t)\} = \frac{1}{2}\cos 4\pi t$$

Periodic!

Prob. 4

$$y_1(t) = x(2t), \quad y_2(t) = x\left(\frac{t}{2}\right)$$

(1) Let $x(t)$ be periodic with period T_x. Consider

$$\begin{aligned} y_1(t+T_y) &= x[2(t+T_y)] \\ &= x(2t+2T_y) \\ &= x(2t+T_x) \quad \text{if } 2T_y = T_x \\ &= x(2t) \\ &= y_1(t) \end{aligned}$$

Yes, $y_1(t)$ is periodic with period $T_y = \frac{T_x}{2}$.

(4) Set t by $2t$, then $x(t) = y_2(2t)$. Thus if $y_2(t)$ is periodic, then $x(t)$ is periodic.

Prob. 5

(a) Let $z(t) = x(t) + y(t)$. Consider for some T_z to be determined,

$$z(t+T_z) = x(t+T_z) + y(t+T_z).$$

Now, if there are integers m, n such that

$$T_z = mT_x = nT_y$$

where T_x and T_y are the fundamental periods of $x(t)$ and $y(t)$, respectively, i.e.,

$$\cancel{x(t+T_x) =}\ x(t) = x(t+T_x) = x(t+mT_x)$$
$$y(t) = y(t+T_y) = y(t+nT_y),$$

then

$$z(t+T_z) = x(t+mT_x) + y(t+nT_y)$$
$$= x(t) + y(t)$$
$$= z(t);$$

In other words, $z(t)$ is periodic. Thus, a sufficient condition is that

$$mT_x = nT_y \quad \text{for some integers } m, n,$$

or equivalently

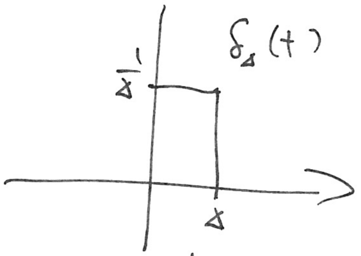
$$\frac{T_x}{T_y} = \frac{n}{m} = \text{a rational number.}$$

Prob 6

This problem examines your understanding about the definition of the unit impulse signal.

Note by definition

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t).$$



$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & 0 \le t \le \Delta \\ 0 & \text{otherwise} \end{cases}$$

Hence,

$$\delta(2t) = \lim_{\Delta \to 0} \delta_\Delta(2t)$$

$$\cancel{\lim_{\Delta \to 0} \delta}$$

Note the change of t to 2t

$$\delta_\Delta(2t) = \begin{cases} \frac{1}{\Delta} & 0 \le 2t \le \Delta \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{1}{\Delta} & \boxed{0 \le t \le \frac{\Delta}{2}} \\ 0 & \text{otherwise} \end{cases}$$

← put This into the form in the definition

$$= \begin{cases} \frac{1}{2} \cdot \boxed{\frac{1}{\left(\frac{\Delta}{2}\right)}} & 0 \le t \le \boxed{\frac{\Delta}{2}} \\ 0 & \text{otherwise} \end{cases}$$

These two ought to be the same in the definition

$$= \frac{1}{2} \begin{cases} \frac{1}{\left(\frac{\Delta}{2}\right)} & 0 \le t \le \frac{\Delta}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$= \frac{1}{2} \delta_{\frac{\Delta}{2}}(t)$$ —— by definition

Thus, $\delta(2t) = \lim_{\Delta \to 0} \delta_\Delta(2t) = \frac{1}{2} \lim_{\Delta \to 0} \delta_{\frac{\Delta}{2}}(t) = \frac{1}{2}\delta(t)$