Tutorial 3
Frequency Modulation (FM)

Problem 1 (Frequency Deviation)

Consider the following FM signal:

$$s_{FM}(t) = 100\cos(2\pi(f_c t + \sin f_m t + 2\sin 2f_m t))$$

where f_c = 100 kHz and f_m=1 kHz. Determine:

(i) Instantaneous phase;

(ii) Instantaneous frequency;

(iii) Peak frequency deviation.

Problem 2 (Modulation Index)

A 1-GHz carrier is frequency-modulated by a 10-kHz sinusoid so that the peak frequency deviation is 100 Hz. Determine

(i) the modulation index β;

(ii) the modulation index if the modulating signal amplitude was doubled;

(iii) the modulation index if the modulating signal frequency was doubled;

(iv) the modulation index if both the amplitude and the frequency of the modulating signal were doubled.

Problem 3 (Power Distribution)

Consider an FM transmitter with a sinusoidal input. The total transmission power is 100W. The peak frequency deviation is carefully increased from zero until the first sideband amplitude at the output is zero. Under these conditions, determine

(i) the transmission power at the carrier frequency;

(ii) the transmission power at the sidebands;

(iii) the transmission power at the second sidebands.

Table 6-1 Values of Bessel Function of the First Kind $J_n(\beta)$ for Various Values of n and β

n	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$	$\beta = 9$
0	0.7652	0.2239	−0.2601	−0.3971	−0.1776	0.1506	0.3001	0.1717	−0.0903
1	0.4401	0.5767	0.3391	−0.0660	−0.3276	−0.2767	−0.0047	0.2346	0.2453
2	0.1149	0.3528	0.4861	0.3641	0.0466	−0.2429	−0.3014	−0.1130	0.1448
3	0.0196	0.1289	0.3091	0.4302	0.3648	0.1148	−0.1676	−0.2911	−0.1809
4	0.0025	0.0340	0.1320	0.2811	0.3912	0.3576	0.1578	−0.1054	−0.2655
5	0.0002	0.0070	0.0430	0.1321	0.2611	0.3621	0.3479	0.1858	−0.0550
6	*	0.0012	0.0114	0.0491	0.1310	0.2458	0.3392	0.3376	0.2043
7	*	0.0002	0.0025	0.0152	0.0534	0.1296	0.2336	0.3206	0.3275
8	*	*	0.0005	0.0040	0.0184	0.0565	0.1280	0.2235	0.3051
9	*	*	0.0001	0.0009	0.0055	0.0212	0.0589	0.1263	0.2149
10	*	*	*	0.0002	0.0015	0.0070	0.0235	0.0608	0.1247
11	*	*	*	*	0.0004	0.0020	0.0083	0.0256	0.0622
12	*	*	*	*	0.0001	0.0005	0.0027	0.0096	0.0274
13	*	*	*	*	*	0.0001	0.0008	0.0033	0.0108
14	*	*	*	*	*	*	0.0002	0.0010	0.0039
15	*	*	*	*	*	*	0.0001	0.0003	0.0013
16	*	*	*	*	*	*	*	0.0001	0.0004
17	*	*	*	*	*	*	*	*	0.0001
18	*	*	*	*	*	*	*	*	*
19	*	*	*	*	*	*	*	*	*

Problem 4

The sinusoidal signal $s(t)=x\cos(2\pi f_m t)$ is applied to the input of an FM system. The corresponding modulated signal output (in volts) with x=1V, f_m=1kHz, is

$$s_{FM}(t) = 100\cos(2\pi \times 10^7 t + 4\sin 2000\pi t).$$

(i) Determine the peak frequency deviation, the modulation index, the carrier frequency, and the total power of $s_{\mathrm{FM}}(t)$;

(ii) What is the percentage of the power at 10MHz?

(iii) What is the effective bandwidth, according to Carson's rule?

Problem 5

A certain sinusoidal signal with frequency f_m Hz is used as the modulating signal in both an AM-DSB-C and an FM system. When modulated, the peak frequency deviation of the FM system is set to three times the bandwidth of the AM system. The sum of magnitudes of those sidebands spaced $\pm f_m$ Hz from carrier in both systems are equal, and the total transmission powers are equal in both systems.

(i) Determine the modulation index of the FM system;

(ii) Determine the modulation index of the AM-DSB-C system.