Tutorial 4 (with solution)

Infinity

Q.1: Proof by Contraposition

❑ Prove that if $7x + 9$ is even, then x is odd.   P4-8

[Q.1: Proof by Contraposition](#)

Proposition: If $7x + 9$ is even, then x is odd.

Proof (by Contraposition):

Suppose x is even.

We can write $x = 2k$, where k is an integer.

Then $7x + 9 = 14k + 9 = 2(7k + 4) + 1$, which is an odd number.

Q.E.D.

Q.2: Comparison of Infinities

❑ Do the intervals $(0,1)$ and $(0,2)$ have the same cardinality? Prove or disprove it. P4-19

    a) Yes

    b) No

Q.2: Comparison of Infinities

Proof:

Define $f: (0,1) \to (0,2)$ such that $f(x) = 2x$.

If $f(x_1) = f(x_2)$, then $2x_1 = 2x_2$, which implies that $x_1 = x_2$. Hence, $f(n)$ is one to one.

Given any $y \in (0,2)$, let $x = y/2$, so $x \in (0,1)$ and $f(x) = y$. Hence, $f(n)$ is onto.

Therefore, f is a one-to-one correspondence.

Hence, the two sets have the same cardinality.

Q.E.D.