Solutions to EE3210 Tutorial 8 Problems

Problem 1: Recall pages 13 and 14 of Part 2 lecture notes. The signal $x_2(t) = x_1(1-t)$ can be obtained from $x_1(t)$ in two alternative ways:

(a) Time shift first followed by time reversal, i.e., $x_1(t) \Rightarrow x_1(t+1) \Rightarrow x_1(-t+1)$. In this way, the time shift property of the continuous-time Fourier series indicates that, if $x_1(t) \leftrightarrow a_k$, the Fourier coefficients c_k of $x_1(t+1)$ can be expressed as

$$c_k = \left[e^{jk(2\pi/T)}\right] a_k. \tag{1}$$

Then, the time reversal property of the continuous-time Fourier series indicates that, if $x_1(t+1) \leftrightarrow c_k$, the Fourier coefficients b_k of $x_2(t) = x_1(-t+1)$ can be expressed as

$$b_k = c_{-k}. \tag{2}$$

Thus, by (1) and (2), we have

$$b_k = \left[e^{-jk(2\pi/T)}\right] a_{-k}. \tag{3}$$

(b) Time reversal first followed by time shift, i.e., $x_1(t) \Rightarrow x_1(-t) \Rightarrow x_1[-(t-1)]$. In this way, the time reversal property of the continuous-time Fourier series indicates that, if $x_1(t) \leftrightarrow a_k$, the Fourier coefficients c_k of $x_1(-t)$ can be expressed as

$$c_k = a_{-k}. \tag{4}$$

Then, the time shift property of the continuous-time Fourier series indicates that, if $x_1(-t) \leftrightarrow c_k$, the Fourier coefficients b_k of $x_2(t) = x_1[-(t-1)]$ can be expressed as

$$b_k = \left[e^{-jk(2\pi/T)}\right] c_k. \tag{5}$$

Thus, by (4) and (5), we have

$$b_k = \left[e^{-jk(2\pi/T)}\right] a_{-k}$$

which is exactly the same as (3).

Problem 2: This signal is periodic with a fundamental period $T = 2$. To determine the Fourier series coefficients a_k, we use the analysis formula of the continuous-time Fourier series, and select the interval of integration to be $-1/2 < t < 3/2$, avoiding the placement of impulses at the integration limits. Within this interval, $x(t)$ is the same as $\delta(t) - 2\delta(t-1)$. Thus, it follows that

$$\begin{aligned} a_k &= \frac{1}{2}\int_{-1/2}^{3/2} [\delta(t) - 2\delta(t-1)]\, e^{-jk\pi t} dt = \frac{1}{2}\int_{-1/2}^{3/2} \delta(t) e^{-jk\pi t} dt - \int_{-1/2}^{3/2} \delta(t-1) e^{-jk\pi t} dt \\ &= \frac{1}{2} - e^{-jk\pi} = \frac{1}{2} - \left(e^{-j\pi}\right)^k = \frac{1}{2} - (-1)^k. \end{aligned}$$

Problem 3:

(a) $x(t) = \cos(4\pi t)$ is a periodic signal with fundamental period $T = 1/2$. Using Euler's formula, we can rewrite $x(t)$ as

$$x(t) = \frac{1}{2} e^{j4\pi t} + \frac{1}{2} e^{-j4\pi t}. \tag{6}$$

Comparing the right-hand sides of (6) and the synthesis formula of the continuous-time Fourier series, we obtain the Fourier series coefficients a_k of $x(t)$ as

$$a_k = \begin{cases} \frac{1}{2}, & k = -1 \\ \frac{1}{2}, & k = 1 \\ 0, & \text{otherwise.} \end{cases}$$

(b) $y(t) = \sin(4\pi t)$ is a periodic signal with fundamental period $T = 1/2$. Using Euler's formula, we can rewrite $y(t)$ as

$$y(t) = \frac{1}{2j} e^{j4\pi t} - \frac{1}{2j} e^{-j4\pi t}. \tag{7}$$

Comparing the right-hand sides of (7) and the synthesis formula of the continuous-time Fourier series, we obtain the Fourier series coefficients b_k of $y(t)$ as

$$b_k = \begin{cases} -\frac{1}{2j}, & k = -1 \\ \frac{1}{2j}, & k = 1 \\ 0, & \text{otherwise.} \end{cases}$$

(c) The signal $z(t) = x(t)y(t)$ is also periodic with period $T = 1/2$. Applying the multiplication property of the continuous-time Fourier series, we obtain the Fourier series coefficients c_k of $z(t)$ as

$$c_k = \sum_{l=-\infty}^{+\infty} a_l b_{k-l} = a_{-1} b_{k+1} + a_1 b_{k-1} = \begin{cases} a_{-1}b_1 + a_1 b_{-1} = 0, & k = 0 \\ a_{-1}b_{-1} = -\frac{1}{4j}, & k = -2 \\ a_1 b_1 = \frac{1}{4j}, & k = 2 \\ 0, & \text{otherwise.} \end{cases}$$

(d) Using the trigonometric identity

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

and Euler's formula, we obtain

$$z(t) = \cos(4\pi t)\sin(4\pi t) = \frac{1}{2}\sin(8\pi t) = \frac{1}{4j}e^{j8\pi t} - \frac{1}{4j}e^{-j8\pi t}. \tag{8}$$

Then, comparing the right-hand sides of (8) and the synthesis formula of the continuous-time Fourier series, we obtain the Fourier series coefficients c_k of $z(t)$ as

$$c_k = \begin{cases} -\frac{1}{4j}, & k = -2 \\ \frac{1}{4j}, & k = 2 \\ 0, & \text{otherwise.} \end{cases}$$