

## Solutions to EE3210 Tutorial 9 Problems

**Problem 1:** Using the analysis formula of the discrete-time Fourier transform, we have

$$\begin{aligned} X[\Omega] &= \sum_{n=-\infty}^{+\infty} x[n]e^{-j\Omega n} = \sum_{n=-\infty}^{+\infty} 4^n u[-n]e^{-j\Omega n} = \sum_{l=-\infty}^{+\infty} 4^{-l} u[l]e^{j\Omega l} = \sum_{l=0}^{+\infty} \left(\frac{1}{4}e^{j\Omega}\right)^l \\ &= \frac{1}{1 - \frac{1}{4}e^{j\Omega}}. \end{aligned}$$

**Problem 2:**

(a) Using the time reversal property, we have

$$x(-t) \leftrightarrow X(-\omega).$$

Then, using the time shift property, we have

$$x[-(t-1)] \leftrightarrow e^{-j\omega} X(-\omega)$$

and

$$x[-(t+1)] \leftrightarrow e^{j\omega} X(-\omega).$$

Therefore, using the linearity property, we obtain

$$x(1-t) + x(-1-t) \leftrightarrow e^{-j\omega} X(-\omega) + e^{j\omega} X(-\omega) = 2 \cos \omega X(-\omega).$$

(b) Using the time shift property, we have

$$x(t-6) \leftrightarrow e^{-j6\omega} X(\omega).$$

Then, using the time scaling property, we have

$$x(3t-6) \leftrightarrow \frac{1}{3} e^{-j2\omega} X\left(\frac{\omega}{3}\right).$$

**Problem 3:** Consider the continuous-time Fourier transform pair

$$x(t) = \frac{W}{\pi} \operatorname{sinc}\left(\frac{Wt}{\pi}\right) \leftrightarrow X(\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$$

obtained on Page 14 of Part 8 lecture notes. Letting  $W = \pi$ , we have

$$x(t) = \operatorname{sinc}(t) \leftrightarrow X(\omega) = \begin{cases} 1, & |\omega| < \pi \\ 0, & |\omega| > \pi. \end{cases}$$

Then, using the Parseval's relation, i.e.,

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(\omega)|^2 d\omega$$

we derive the total energy of the continuous-time signal  $x(t) = \operatorname{sinc}(t)$  as

$$\frac{1}{2\pi} \int_{-\pi}^{+\pi} 1 \cdot d\omega = 1.$$