Solutions to EE3210 Quiz 8 Problems

Problem 1: The signal $z[n] = x[n]y[n]$ is also periodic with period $N = 3$. Applying the multiplication property of the discrete-time Fourier series and choosing the limits of the summation to be $0 \leq l \leq 2$, we obtain the Fourier series coefficients c_k of $z[n]$ as

$$c_k = \sum_{l=0}^{2} a_l b_{k-l} = a_0 b_k + a_1 b_{k-1} = \begin{cases} a_0 b_0 + a_1 b_{-1} = 0, & k = 0 \\ a_0 b_1 + a_1 b_0 = 1, & k = 1 \\ a_0 b_2 + a_1 b_1 = 1, & k = 2. \end{cases}$$

Alternatively, we can obtain c_k as

$$c_k = \sum_{l=0}^{2} b_l a_{k-l} = b_1 a_{k-1} = \begin{cases} b_1 a_{-1} = 0, & k = 0 \\ b_1 a_0 = 1, & k = 1 \\ b_1 a_1 = 1, & k = 2. \end{cases}$$

Problem 2:

(a) Using the analysis formula of the continuous-time Fourier transform, we have

$$X(\omega) = \int_{-\infty}^{+\infty} \delta(t-1) e^{-j\omega t} dt = e^{-j\omega}.$$

(b) Using the synthesis formula of the continuous-time Fourier transform, we have

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} [\delta(\omega - 1) + \delta(\omega + 1)] e^{j\omega t} d\omega = \frac{1}{2\pi} \left(e^{jt} + e^{-jt} \right) = \frac{1}{\pi} \cos t.$$