EE2302 Foundations of Information Engineering

Assignment 5 (Solution)

1. Any integer n can be written as $n = 3q + r$ using the quotient-remainder theorem, where $0 \le r < 3$. There are three possible cases:
   - For $r = 0$, $n^2 = (3q)^2 = 3(3q^2) = 3k$ for $k = 3q^2$.
   - For $r = 1, n^2 = (3q+1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3k + 1$ for $k = (3q^2 + 2q)$.
   - For $r = 2, n^2 = (3q+2)^2 = 9q^2 + 12q + 4 = 3(3q^2 + 4q + 1) + 1 = 3k + 1$ for $k = (3q^2 + 4q + 1)$.

   Therefore, the square of any integer has the form $3k$ or $3k$+1 for some integer k.

2. $\emptyset(9100) = \emptyset(2^2 * 5^2 * 7 * 13)$

   $= \emptyset(2^2)\emptyset(5^2)\emptyset(7)\emptyset(13)$

   $= (2^2 - 2^1)(5^2 - 5)(7^1 - 7^0)(13^1 - 13^0)$
   $= 2880.$

3. $\gcd(67890{,}12000) = 30.$

5	67890 60000	12000 7890	1
1	7890 4110	4110 3780	1
11	3780 3630	330 300	2
5	150 150	30	
	0		

4.

54321	6789		
1	0	54321	a
0	1	6789	b
1	-8	9	$\mathrm{c = a - 8b}$
-754	6033	3	$\mathrm{d = b - 754c}$ $\quad (= \mathrm{b} - 754 * (\mathrm{a} - 8\mathrm{b}) = -754\mathrm{a} + 6033\mathrm{b})$

$\gcd(54321{,}6789) = 3$, $x = -754$ and $y = 6033$.

Remark: In the last column of the above table, the expressions within the parenthesis are not needed, but can show that for each row, the entries in the first two columns represent the coefficients of a and b, respectively.