EE2302 Foundations of Information Engineering

Assignment 8 (Solution)

1. Consider two arbitrary matrices in the subset, $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$, where $a_{12} = -a_{21}$ and $b_{12} = -b_{21}$.

   First, addition is closed, since $A + B = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{pmatrix}$ and $a_{12} + b_{12} = -(a_{21} + b_{21})$.
   Second, scalar multiplication is closed, since $cA = \begin{pmatrix} ca_{11} & ca_{12} \\ ca_{21} & ca_{22} \end{pmatrix}$ and $ca_{12} = -ca_{21}$.
   Hence, the subset is a subspace of all 2×2 real matrices.

2. Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix}$. Then $A^T A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ and $A^T b = \begin{bmatrix} 18 \\ 5 \end{bmatrix}$. The normal equation is

$$\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} c \\ m \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \end{bmatrix}.$$

   Solving the equations, we obtain $c = 6$ and $m = \frac{5}{2}$.

3.
   a) The columns are linearly dependent because $2\begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} -4 \\ 6 \end{bmatrix} = 0$.
   b) $\mathcal{C}(A) = \{v \in \mathbb{R}^2 : v = \alpha \begin{bmatrix} 2 \\ -3 \end{bmatrix}$, where $\alpha \in \mathbb{R}\}$ and $\left\{\begin{bmatrix} 2 \\ -3 \end{bmatrix}\right\}$ is a basis.
   c) $\text{rank}(A) = 1$.
   d) $Ax = 0 \Rightarrow \begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow 2x_1 - 4x_2 = 0$.

      Hence, $\mathcal{N}(A) = \left\{v \in \mathbb{R}^2 : v = \alpha \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \text{where } \alpha \in \mathbb{R}\right\}$.

   e) $Ax = b \Rightarrow \begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 8 \\ -12 \end{bmatrix} \Rightarrow 2x_1 - 4x_2 = 8$.
      Direct observation gives a particular solution $x_p = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$.
      General solution: $x = \begin{bmatrix} 4 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, where $\alpha \in \mathbb{R}$.