Supplementary Explanation of Unbounded Knapsack Problem

Definition

Unbounded Knapsack Problem (UKP) is one of many modeled version of the original Knapsack Problem. The difference from 0/1 Knapsack Problem (0/1 KP) is that there are no limits to the number of items available. In other words, as along the sum is within weight limit, you can take as many copies of each item as possible.

Both UKP and 0/1 KP are fundamental problems in combinatorial optimization and have practical applications in real life. Whilst 0/1 KP handles binary decision scenarios, such as profit maximization within budget constraint, UKP handles scenarios where resources are unlimited, such as production planning.

Formulation

$$
\begin{aligned}
&\text{Let:} \\
&n \text{ be number of items} \\
&w_i \text{ be the weight of the i-th item} \\
&v_i \text{ be value of i-th item} \\
&C \text{ be the knapsack capacity} \\
&dp \text{ be a 2D array where } dp[i][j] \text{ is maximum value}
\end{aligned}
$$

Formula

$$
dp[i][j] = \max(dp[i-1][j], dp[i][j-w_i] + v_i)
$$

Formula Explanation

$$
\begin{aligned}
&dp[i-1][j]: \text{ Maximum value obtained not including i-th item} \\
&dp[i][j-w_i] + v_i: \text{ Maxumum value obtained including i-th item.} \\
&\text{Subtract weight } w_i \text{ from current capacity } j
\end{aligned}
$$

Example

Item	Weight	Value
1	3	4
2	1	2
3	2	3

Item	Weight	Value
4	3	5
5	1	1

	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	4	4	4	8
2	0	2	2	4	6	6	8
3	0	2	3	4	6	7	8
4	0	2	3	5	6	7	9
5	0	2	3	5	6	7	10