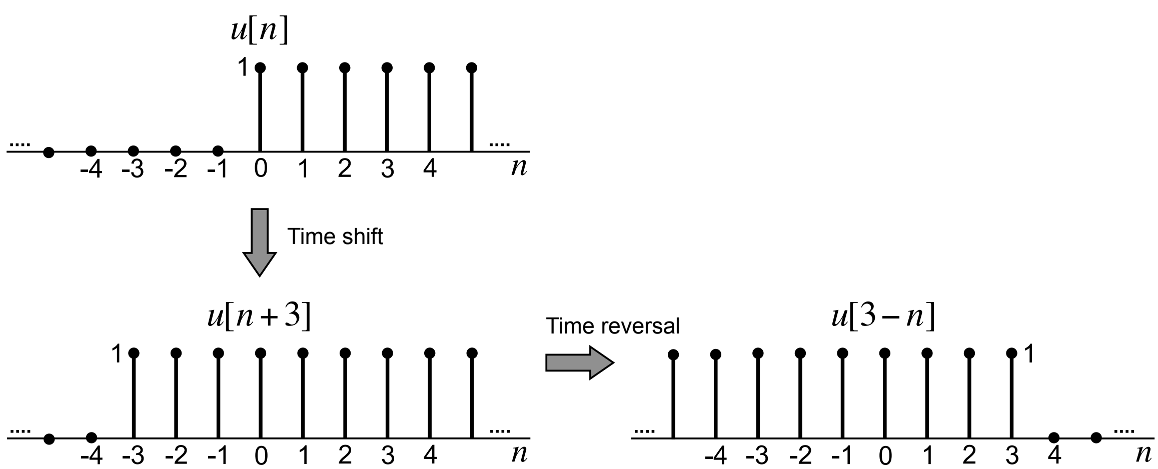
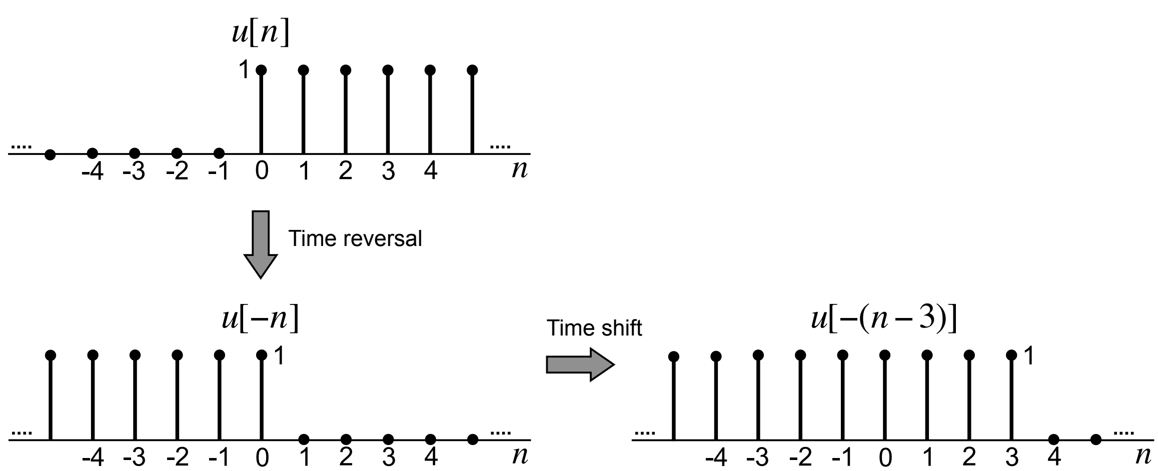
Solutions to EE3210 Assignment 1

Problem 1:

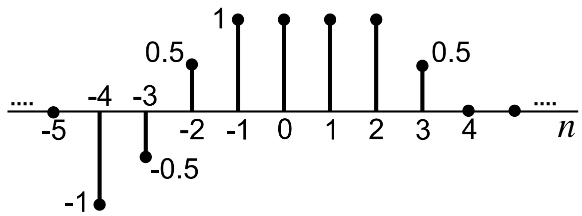
(a) The signal $u[3-n]$ is obtained from $u[n]$ as below:



Alternatively, $u[3-n]$ can be obtained from $u[n]$ by doing:

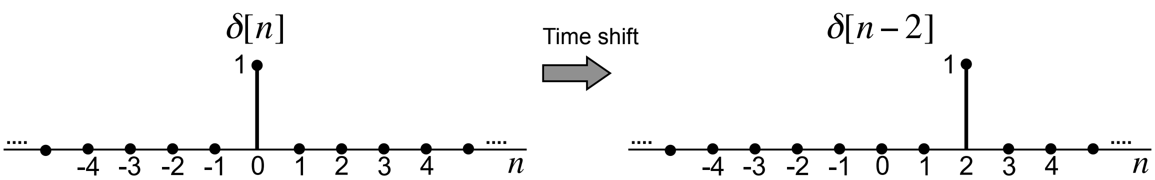


Thus, we obtain $x[n]u[3-n]$ as



which is equivalent to $x[n]$.

(b) The signal $\delta[n-2]$ is obtained from $\delta[n]$ as below:



Using the sampling property of the discrete-time unit impulse signal $\delta[n]$

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$$

we have

$$x[n-2]\delta[n-2] = x[2-2]\delta[n-2] = x[0]\delta[n-2].$$

Thus, we obtain $x[n-2]\delta[n-2]$ simply as $\delta[n-2]$ since in this case $x[0]=1$.

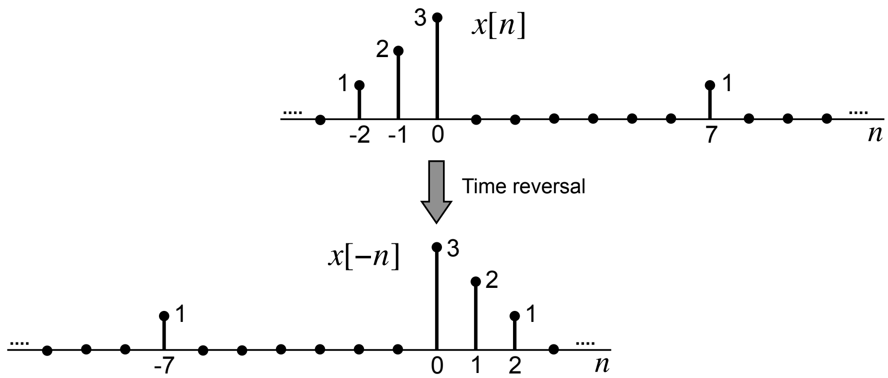
Problem 2: Let the even and odd parts of $x[n]$ be denoted by

$$x_e[n] = \mathcal{E}\{x[n]\} = \frac{1}{2}(x[n]+x[-n])$$

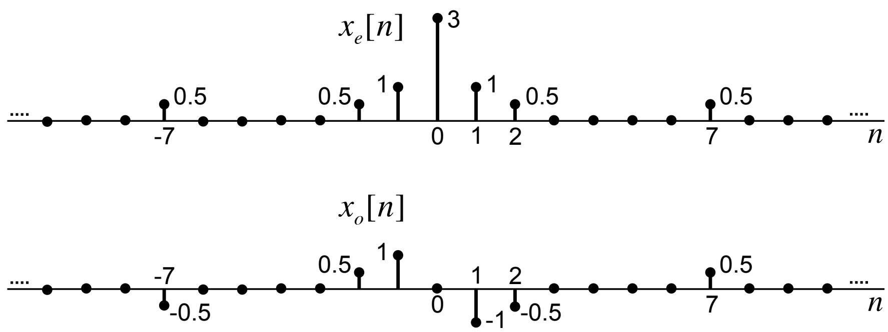
and

$$x_o[n] = \mathcal{O}\{x[n]\} = \frac{1}{2}(x[n]-x[-n]).$$

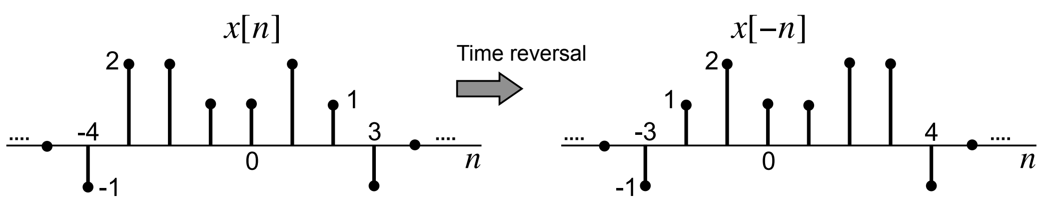
(a) The signal $x[-n]$ is obtained from $x[n]$ as below:



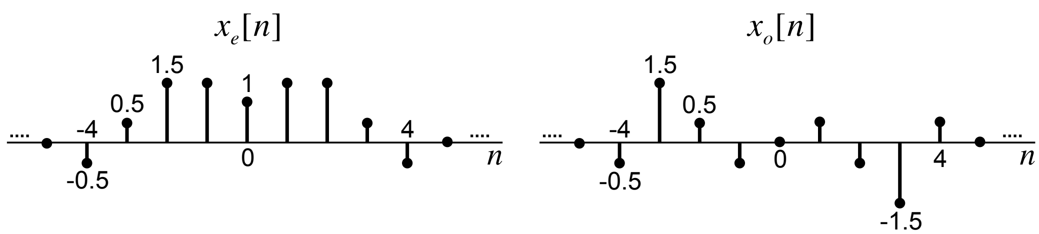
Then, we have



(b) The signal $x[-n]$ is obtained from $x[n]$ as below:



Then, we have



Problem 3:

(a) Consider

$$\sum_{n=-\infty}^{+\infty} x[n] = \sum_{n=-\infty}^{-1} x[n] + x[0] + \sum_{n=1}^{+\infty} x[n] \\ = x[0] + \sum_{n=1}^{+\infty} (x[n] + x[-n]). \tag{1}$$

If $x[n]$ is odd, $x[n] + x[-n] = 0$ for all n, and hence $x[0] = 0$. Therefore, (1) evaluates to zero.

(b) Let $y[n] = x_1[n]x_2[n]$. Then

$$y[-n] = x_1[-n]x_2[-n] = -x_1[n]x_2[n] = -y[n].$$

This implies that $y[n]$ is odd.

(c) Consider

$$\sum_{n=-\infty}^{+\infty} x^2[n] = \sum_{n=-\infty}^{+\infty} (x_e[n] + x_o[n])^2 \\ = \sum_{n=-\infty}^{+\infty} x_e^2[n] + \sum_{n=-\infty}^{+\infty} x_o^2[n] + 2\sum_{n=-\infty}^{+\infty} x_e[n]x_o[n].$$

Using the result of part (b), we know that $x_e[n]x_o[n]$ is an odd signal. Then, using the result of part (a), we have

$$2\sum_{n=-\infty}^{+\infty} x_e[n]x_o[n] = 0.$$

Therefore,

$$\sum_{n=-\infty}^{+\infty} x^2[n] = \sum_{n=-\infty}^{+\infty} x_e^2[n] + \sum_{n=-\infty}^{+\infty} x_o^2[n].$$

Alternatively, we can show the result by considering

$$x_e[n] = \mathcal{E}\{x[n]\} = \frac{1}{2}(x[n] + x[-n])$$

and

$$x_o[n] = \mathcal{O}\{x[n]\} = \frac{1}{2}(x[n] - x[-n]).$$

Then,

$$\sum_{n=-\infty}^{+\infty} x_e^2[n] + \sum_{n=-\infty}^{+\infty} x_o^2[n] = \sum_{n=-\infty}^{+\infty} (x_e^2[n] + x_o^2[n]) \\ = \sum_{n=-\infty}^{+\infty} \frac{1}{2}(x^2[n] + x^2[-n]).$$

Since

$$\sum_{n=-\infty}^{+\infty} \frac{1}{2} x^2[-n] = \sum_{m=+\infty}^{-\infty} \frac{1}{2} x^2[m] = \sum_{n=-\infty}^{+\infty} \frac{1}{2} x^2[n]$$

we have

$$\sum_{n=-\infty}^{+\infty} x_e^2[n] + \sum_{n=-\infty}^{+\infty} x_o^2[n] = \sum_{n=-\infty}^{+\infty} x^2[n].$$