Solutions to EE3210 Quiz 2 Problems

Problem 1:

(a) Given that $h[n] = \delta[n+1] - \delta[n]$, we have

$$h[n] = \begin{cases} 1, & n = -1 \\ -1, & n = 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Thus:

- System A is not memoryless, because $h[n] = 1$ for $n = -1 \neq 0$.
- System A is not causal, because $h[n] = 1$ for $n = -1 < 0$.
- System A is stable, because

$$\sum_{n=-\infty}^{+\infty} |h[n]| = |h[-1]| + |h[0]| = 2 < \infty.$$

(b) There are two ways in showing this result. Either way is an acceptable solution.

- Given that the input-output relationship of system B is

$$y[n] = \sum_{k=-\infty}^{n-1} x[k], \tag{1}$$

comparing (1) with the convolution sum formula, we observe that for (1) to hold it requires that the unit impulse response $h_1[n]$ of system B satisfies

$$h_1[n-k] = \begin{cases} 1, & k \leq n-1 \\ 0, & k > n-1 \end{cases}$$

which implies that

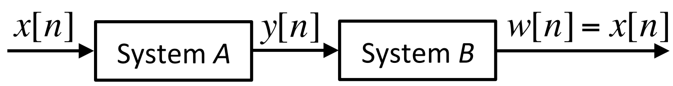
$$\begin{aligned} h_1[n] &= \begin{cases} 1, & n \geq 1 \\ 0, & n < 1 \end{cases} \\ &= u[n-1]. \end{aligned}$$

Then, we have

$$h[n]*h_1[n] = \sum_{k=-\infty}^{+\infty} h[k]h_1[n-k] = h[-1]h_1[n+1]+h[0]h_1[n] = u[n]-u[n-1] = \delta[n],$$

which verifies that system B is an inverse system of system A.

– By definition, system B is an inverse system of system A if



In this case, we have

$$y[n] = h[n]*x[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k] = h[-1]x[n+1]+h[0]x[n] = x[n+1]-x[n].$$

From (1), we have

$$w[n] = \sum_{k=-\infty}^{n-1} y[k].$$

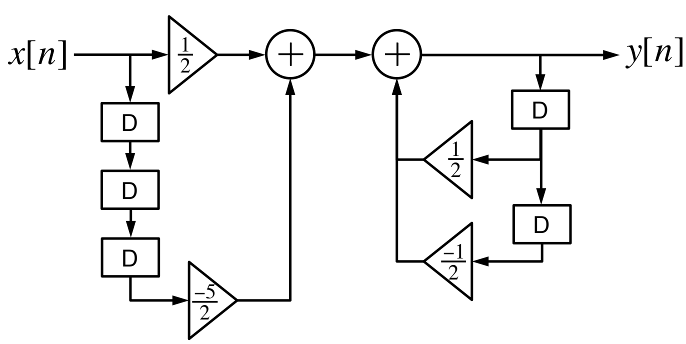
Thus, we obtain

$$w[n] = \sum_{k=-\infty}^{n-1} (x[k+1] - x[k]) = x[n]$$

which verifies that system B is an inverse system of system A.

Problem 2:

(a) The direct form I block diagram representation of the system is



(b) The linear constant-coefficient difference equation that describes the relationship between the input $x[n]$ and the output $y[n]$ of the system is

$$y[n] = \frac{1}{2}y[n-1] - \frac{1}{2}y[n-2] + \frac{1}{2}x[n] - \frac{5}{2}x[n-3].$$

Problem 3: This signal is periodic with a fundamental period $T = 2$. To determine the Fourier series coefficients a_k, we use the analysis formula of the continuous-time Fourier series, and choose $-1 < t < 1$ as the interval over which the integration is performed. Then, it follows that:

- For $k = 0$,
$$a_0 = \frac{1}{2}\int_{-1}^{1} e^{-t}dt = -\frac{1}{2}\left[e^{-t}\right]_{-1}^{1} = \frac{e - e^{-1}}{2}.$$

- For $k \neq 0$,
$$\begin{aligned} a_k &= \frac{1}{2}\int_{-1}^{1} e^{-t}e^{-jk\pi t}dt = \frac{1}{2}\int_{-1}^{1} e^{-(1+jk\pi)t}dt \\ &= -\frac{1}{2(1+jk\pi)}\left[e^{-(1+jk\pi)t}\right]_{-1}^{1} = -\frac{1}{2(1+jk\pi)}\left[e^{-(1+jk\pi)} - e^{(1+jk\pi)}\right] \\ &= \frac{(-1)^k\left(e - e^{-1}\right)}{2(1+jk\pi)}. \end{aligned}$$

Problem 4:

(a) 1. The time reversal property of the discrete-time Fourier series indicates that, if $x[n] \leftrightarrow a_k$, the Fourier series coefficients b_k of $x[-n]$ can be expressed as

$$b_k = a_{-k}.$$

Then, the time shift property of the discrete-time Fourier series indicates that, if $x[-n] \leftrightarrow b_k$, the Fourier series coefficients c_k of $x[-(n-1)] = x[1-n]$ can be expressed as
$$c_k = \left[e^{-jk(2\pi/N)}\right] b_k = \left[e^{-jk(2\pi/N)}\right] a_{-k}.$$
Similarly, the Fourier series coefficients d_k of $x[-(n+1)] = x[-1-n]$ can be expressed as
$$d_k = \left[e^{jk(2\pi/N)}\right] b_k = \left[e^{jk(2\pi/N)}\right] a_{-k}.$$
Finally, using the linearity property of the discrete-time Fourier series, the Fourier series coefficients e_k of $x[1-n] + x[-1-n]$ can be obtained as

$$e_k = c_k + d_k = \left[e^{-jk(2\pi/N)} + e^{jk(2\pi/N)}\right] a_{-k} = 2\cos\left(\frac{2\pi k}{N}\right) a_{-k}.$$

2. Note that

$$\mathcal{E}\{x[n]\} = \frac{1}{2}\left(x[n] + x[-n]\right).$$

The time reversal property of the discrete-time Fourier series indicates that, if $x[n] \leftrightarrow a_k$, the Fourier series coefficients b_k of $x[-n]$ can be expressed as

$$b_k = a_{-k}.$$

Then, using the linearity property of the discrete-time Fourier series, the Fourier series coefficients c_k of $\mathcal{E}\{x[n]\}$ can be obtained as

$$c_k = \frac{a_k + b_k}{2} = \frac{a_k + a_{-k}}{2}.$$

(b) Given that $x[n]$ is a discrete-time periodic signal with fundamental period N, by definition, we have

$$x[n+N] = x[n]$$

for all values of n.

Let $y[n] = x[n] + x[n+N/2]$. Then, we have

$$\begin{aligned} y[n+N/2] &= x[n+N/2] + x[n+N/2+N/2] \\ &= x[n+N/2] + x[n+N] \\ &= x[n+N/2] + x[n] \\ &= y[n] \end{aligned}$$

for all values of n.

Therefore, by definition, $y[n]$ is periodic with period $N/2$, assuming that N is even.