Optz Cheat Sheet

Graph Theory

Eulerian Trail / Path

A trail which passes through every edge exactly once.

If such trail starts and end on same node = Eulerian Circuit / Cycle

Walk

Sequence of edges connecting vertices without gaps

Trail

A walk where all edges are different

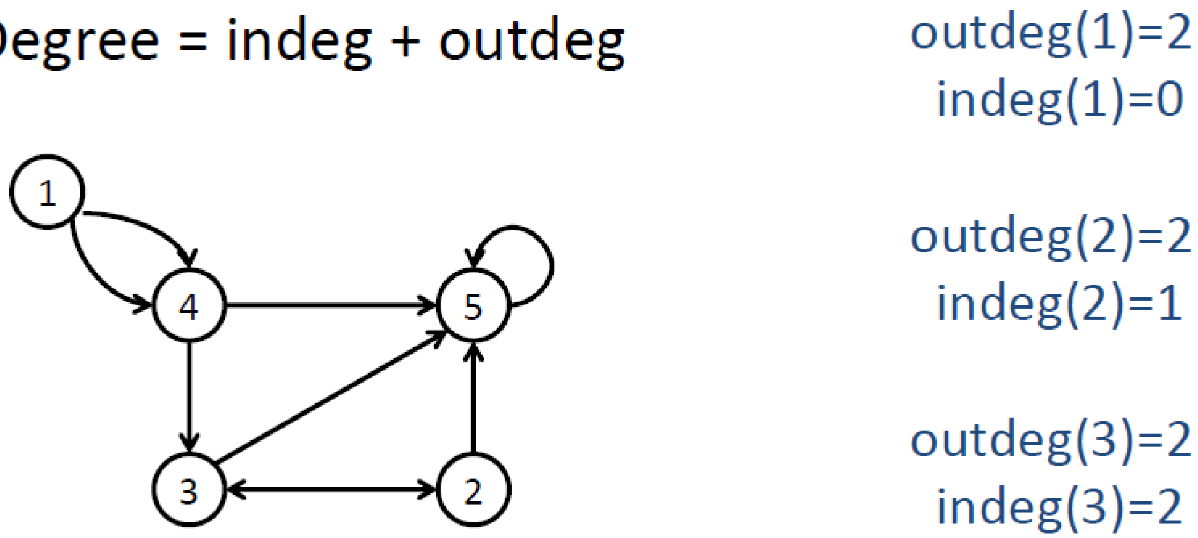
Path

Trail where all vertices are different

Degree

Number of edges to adjacent nodes

Degree = indeg + outdeg



Extra Conditions (Euler)

Node with odd degree → first / last

If fist = last → even degree

If odd degree nodes > 2 → No Euler path

Hamiltonian Cycle

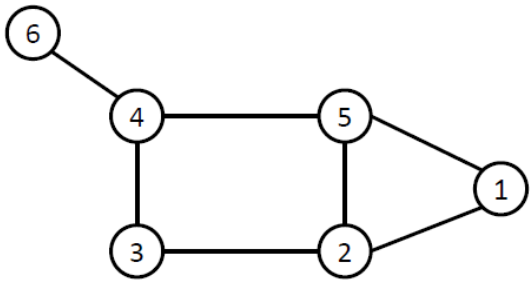
一筆畫

Condition of Existence

1) Connected graph, num of E = num of V

2) No Isolated V

3) No V with deg = 1



- n = 6, m = 7
- Set of Vertices (V) = {1,2,3,4,5,6}
- Set of Edges (E) = {{1,2},{1,5},{2,3},{2,5},{3,4},{4,5},{4,6}}
- *N(4) = Neighborhood (4) = {6,5,3}*

Simple Graph

No loop or multiple edges

Total Degree: $\sum deg(v) = 2\,|E|$, where $|E| = \frac{n(n-1)}{2}$

Complete Graph

Simple undirected graph K_n which every pair of distinct vertices connected by unique edge

Degree per Node = $n-1$ = Num of edges connected to each node

Total Degree = $n(n-1)$

Num of E = $|E| = \frac{n(n-1)}{2}$

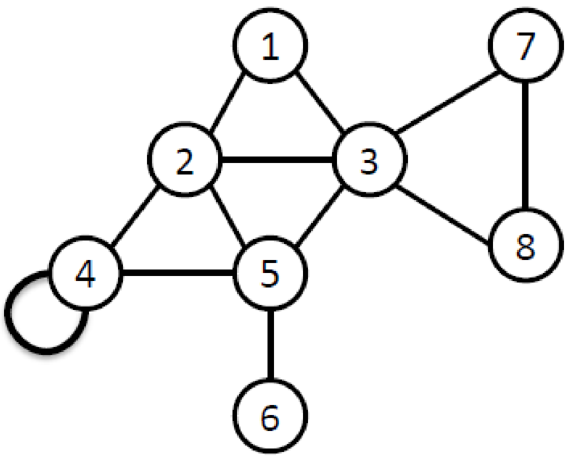
Bipartite

V can be partitioned into 2 sets $V_1 V_2$

Tree

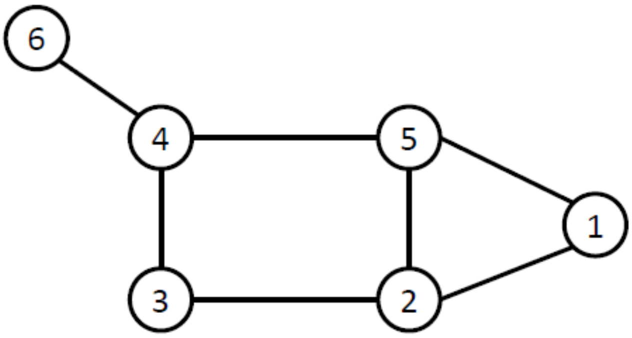
Path between 2 nodes $\equiv$ 1

Adjacency Matrix

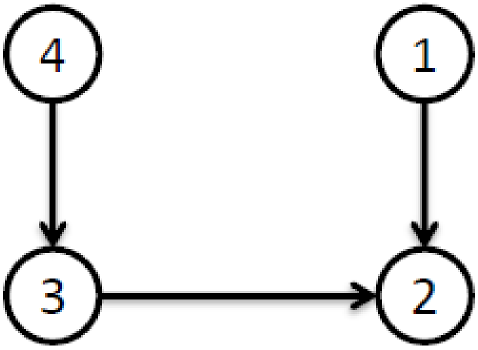


	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	0	0
2	1	0	1	1	1	0	0	0
3	1	1	0	0	1	0	1	1
4	0	1	0	1	1	0	0	0
5	0	1	1	1	0	1	0	0
6	0	0	0	0	1	0	0	0
7	0	0	1	0	0	0	0	1
8	0	0	1	0	0	0	1	0

Incidence Matrix



	1,2	1,5	2,3	2,5	3,4	4,5	4,6
1	1	1	0	0	0	0	0
2	1	0	1	1	0	0	0
3	0	0	1	0	1	0	0
4	0	0	0	0	1	1	1
5	0	1	0	1	0	1	0
6	0	0	0	0	0	0	1



	1,2	2,3	3,4
1	−1	0	0
2	1	1	0
3	0	−1	1
4	0	0	−1

Isomorphic

Same num of nodes, vertices and edge connectivity

Diameter

Longest shortest path between any pairs

NP-Complete

No known polynomial time solution

Spanning Tree

Connected subgraph including all nodes without cycle

Prim's Algorithm

Random choose vertex → add minimum weight adjacent edge