Solutions to EE3210 Tutorial 6 Problems

Problem 1:

(a) $h[n] = \left(\frac{1}{5}\right)^n u[n] = \begin{cases} 0, & n < 0 \\ \left(\frac{1}{5}\right)^n, & n \geq 0 \end{cases} \quad \Rightarrow \quad \text{Causal}$

$$\sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{n=0}^{+\infty} \left(\frac{1}{5}\right)^n = \frac{5}{4} < \infty \quad \Rightarrow \quad \text{Stable}$$

(b) $h[n] = \left(\frac{1}{2}\right)^n u[-n] > 0 \text{ for all } n < 0 \quad \Rightarrow \quad \text{Not causal}$

$$\sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^n = \sum_{n=0}^{+\infty} 2^n = \infty \quad \Rightarrow \quad \text{Not stable}$$

Problem 2:

(a) $h(t) = e^{-4t} u(t-2) = \begin{cases} 0, & t < 2 \\ e^{-4t}, & t > 2 \end{cases} \quad \Rightarrow \quad \text{Causal}$

$$\int_{-\infty}^{+\infty} |h(t)|\, dt = \int_{2}^{+\infty} e^{-4t} dt = \frac{1}{4} e^{-8} < \infty \quad \Rightarrow \quad \text{Stable}$$

(b) $h(t) = e^{-6|t|} = \begin{cases} e^{6t}, & t < 0 \\ e^{-6t}, & t > 0 \end{cases} \quad \Rightarrow \quad \text{Not causal}$

$$\int_{-\infty}^{+\infty} |h(t)|\, dt = \int_{-\infty}^{0} e^{6t} dt + \int_{0}^{+\infty} e^{-6t} dt = \frac{1}{3} < \infty \quad \Rightarrow \quad \text{Stable}$$

Problem 3: Because of the commutative property of convolution sum, and given that $x[n] = u[n]$, we have

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k] = \sum_{k=-\infty}^{+\infty} h[k]u[n-k] = \sum_{k=-\infty}^{n} h[k].$$

Thus, we obtain

$$y[n] - y[n-1] = h[n].$$

(a) Since

$$y[n-1] = \begin{cases} 1, & 1 \le n \le 8 \\ 0, & \text{elsewhere} \end{cases}$$

we obtain

$$h[n] = y[n] - y[n-1] = \begin{cases} 1, & n = 0 \\ -1, & n = 8 \\ 0, & \text{elsewhere.} \end{cases}$$

This system is stable because $\displaystyle\sum_{n=-\infty}^{+\infty} |h[n]| = |h[0]| + |h[8]| = 2 < \infty$.

(b) Using

$$y[n] = \sum_{k=-\infty}^{+\infty} h[k]x[n-k]$$

with

$$h[k] = \begin{cases} 1, & k = 0 \\ -1, & k = 8 \\ 0, & \text{elsewhere.} \end{cases}$$

we obtain the linear constant-coefficient difference equation as

$$y[n] = x[n] - x[n-8].$$

(c) The block diagram representation of this system is: