CITY UNIVERSITY OF HONG KONG
Department of Electronic Engineering

**EE 3210 Signals and Systems**

Homework #3

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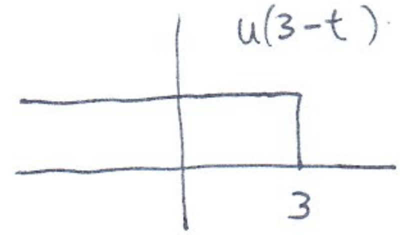
# Homework #3

## Problem 2.29, PP. 144




(b) $h(t) = e^{-6t} u(3-t)$

Solution: The LTI system is non-causal, since $h(t) \not\equiv 0$ for $t < 0$.

For stability, check

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{3} e^{-6t}\, dt = \infty$$

The system is unstable.

(g) $h(t) = \left(2e^{-t} - e^{(t-100)/100}\right) u(t)$

It is causal, since $h(t) \equiv 0$ for $t < 0$.

It is unstable since

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \infty$$

## Problem 2.40, PP. 148

(a) Solution; To find the impulse response, let $x(t) = \delta(t)$. Then,

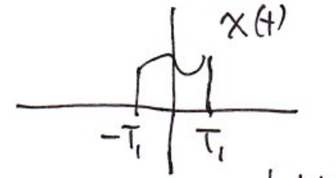
$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau - 2)\, d\tau$$

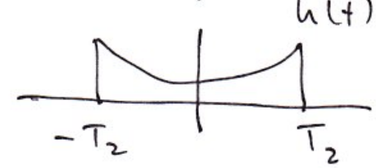
$$= \begin{cases} e^{-(t-\tau)}\Big|_{\tau=2} & t > 2 \\ 0 & t < 2 \end{cases} = e^{-(t-2)} u(t-2)$$

# Problem 2.44

(a) Given $x(t) = 0, \; |t| > T_1$

$h(t) = 0, \; |t| > T_2$

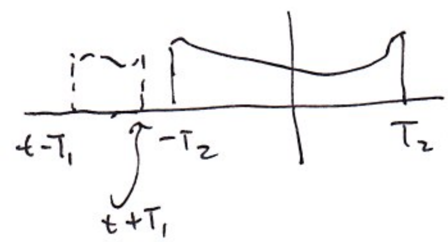


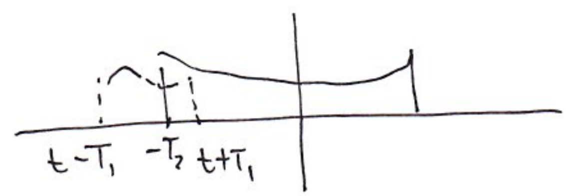

Case 1: $t + T_1 \leq -T_2$

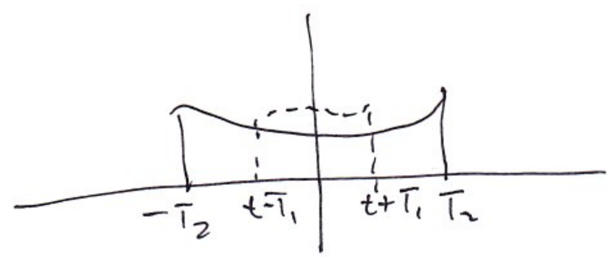
$\iff t < -(T_1 + T_2)$

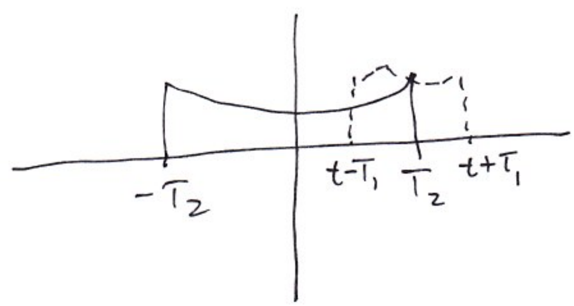
$x(t) * h(t) = 0$




Case 2:



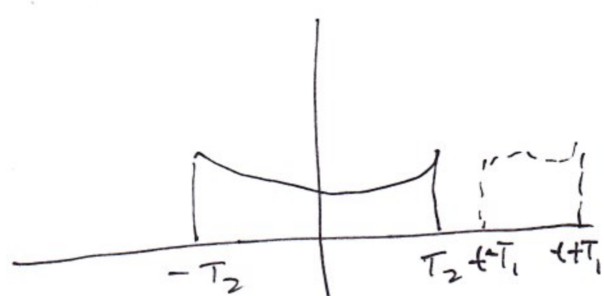




Case 5: $t - T_1 > T_2$

$\iff t > T_1 + T_2$

$x(t) * h(t) = 0$



(d) $$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$y(0) = \int_{-\infty}^{\infty} h(\tau)\, x(-\tau)\, d\tau$$

$$= \int_{-2}^{-1} x(-\tau)\, d\tau + \int_{-\infty}^{\infty} \delta(\tau-6)\, x(-\tau)\, d\tau$$

$$= \int_{-2}^{-1} x(-\tau)\, d\tau + x(-\tau)\Big|_{\tau=6}$$

Let $\underline{-\tau = \lambda}$

$$= \int_{2}^{1} x(\lambda)(-d\lambda) + x(-6)$$

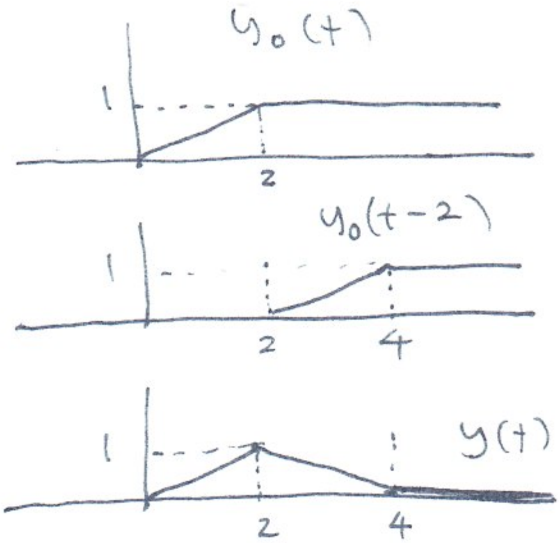
$$= \int_{1}^{2} x(\lambda)\, d\lambda + x(-6)$$

We must know $x(t)$ at $t=-6$ and over the interval $[1, 2]$.

## Problem 2.47, pp. 152

(b) Solution: Since the system is LTI, and since $x(t) = x_0(t) - x_0(t-2)$, we know that the output is

$$y(t) = y_0(t) - y_0(t-2)$$



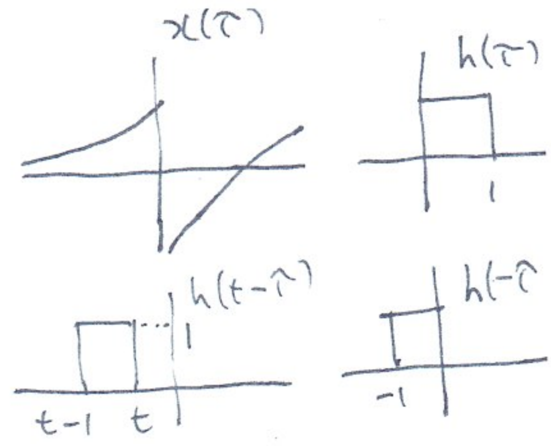

~~$y(t)$~~

$$y(t) = \begin{cases} \frac{1}{2}t & 0<t\le 2 \\ 2-\frac{t}{2} & 2<t\le 4 \\ 0 & \text{otherwise} \end{cases}$$

## Convolution: Problem 2.22

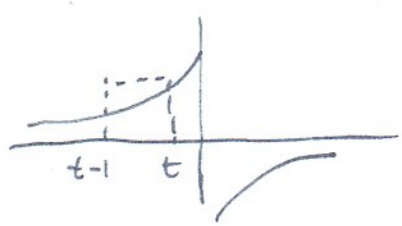
$$h(t) = u(t) - u(t-1)$$

$$x(t) = \begin{cases} e^{t} & t<0 \\ e^{3t}-2e^{-t} & t>0 \end{cases}$$




Solution:

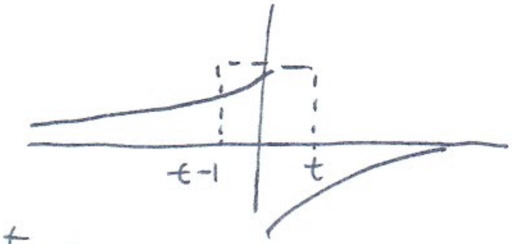
Case 1, $t<0$




$$x(t)*h(t) = \int_{t-1}^{t} 1\cdot e^{\tau}\,d\tau$$

$$= e^{t} - e^{t-1}$$

Case 2: $\begin{cases} t>0 \\ t-1<0 \end{cases}$, i.e., $0<t<1$

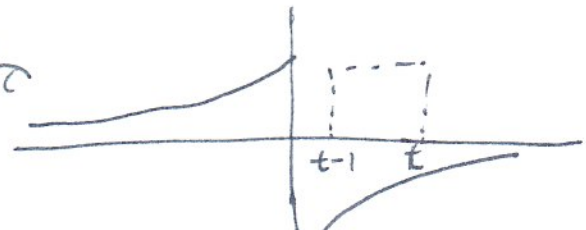



$$x(t) * h(t) = \int_{t-1}^{0} e^{\tau}\, d\tau + \int_{0}^{t} \left(e^{5\tau} - 2e^{-\tau}\right) d\tau$$

$$= 1 - e^{t-1} + \frac{1}{5}\left(e^{5t} - 1\right) - 2\left(1 - e^{-t}\right)$$

Case 3: $t-1>0$, i.e., $t>1$

$$x(t) * h(t) = \int_{t-1}^{t} \left(e^{5\tau} - 2e^{-\tau}\right) d\tau$$




$$= \frac{1}{5}\left[e^{5t} - e^{5(t-1)}\right] - 2\left[e^{-(t-1)} - e^{-t}\right]$$

As a result

$$x(t) * h(t) = \begin{cases} e^{t} - e^{t-1} & t<0 \\ 1 - e^{t-1} + \frac{1}{5}\left(e^{5t} - 1\right) - 2\left(1 - e^{-t}\right) & 0<t<1 \\ \frac{1}{5}\left[e^{5t} - e^{5(t-1)}\right] - 2\left[e^{-(t-1)} - e^{-t}\right] & t>1 \end{cases}$$