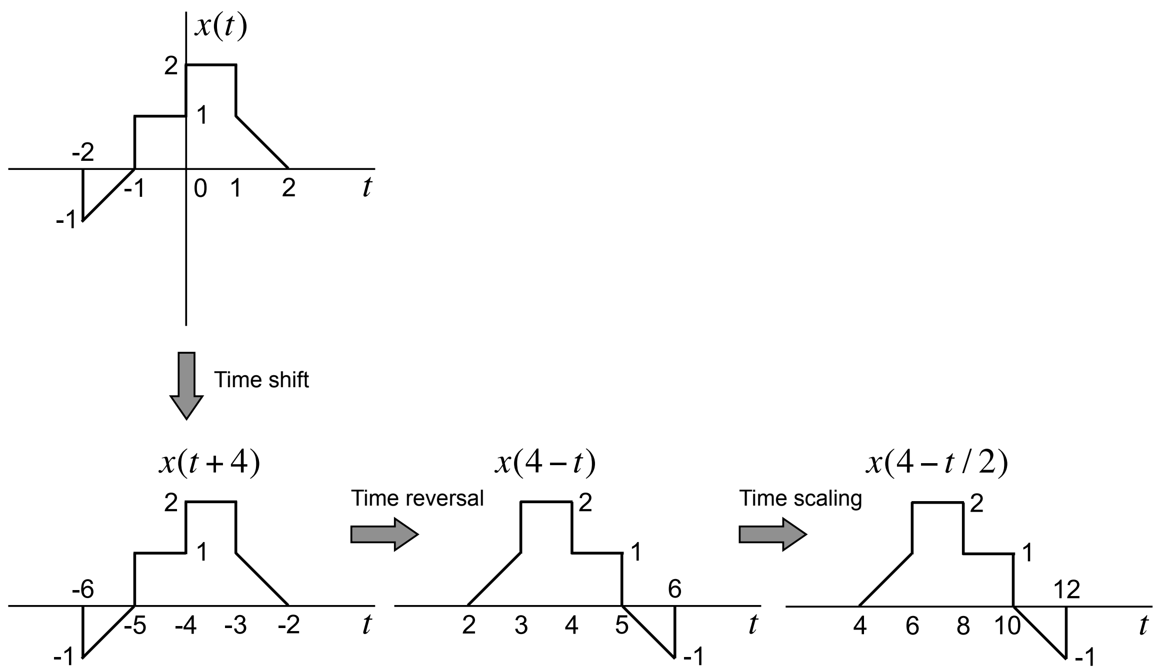
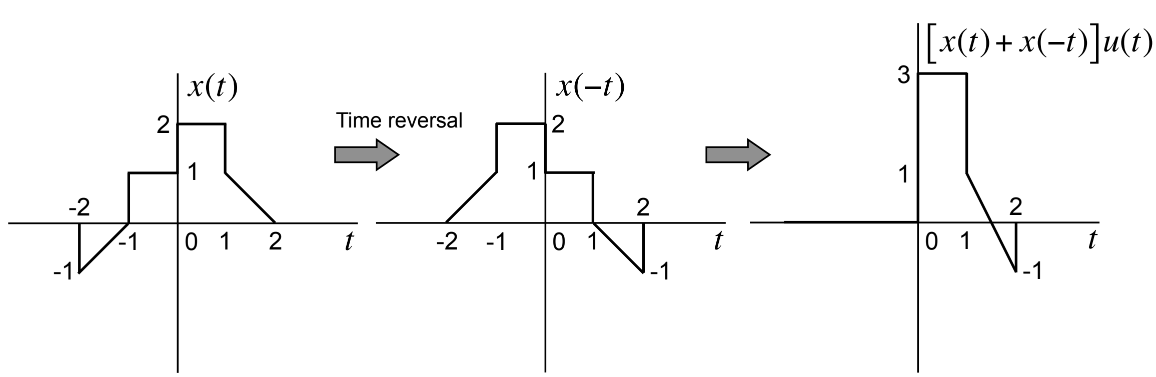
Solutions to EE3210 Tutorial 2 Problems

Problem 1:

(a) The signal $x(4 - \frac{t}{2})$ is obtained from $x(t)$ as below:



(b) The signal $[x(t) + x(-t)]u(t)$ is obtained from $x(t)$ as below:



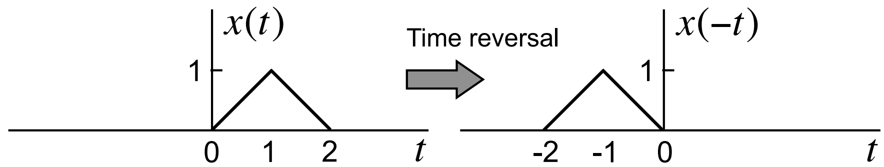
Problem 2: Let the even and odd parts of $x(t)$ be denoted by

$$x_e(t) = \mathcal{E}\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$$

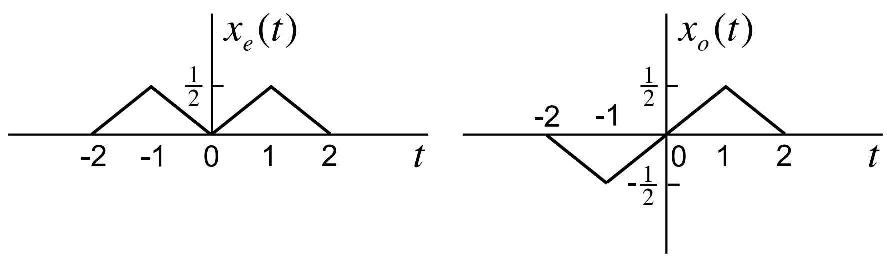
and

$$x_o(t) = \mathcal{O}\{x(t)\} = \frac{1}{2}[x(t) - x(-t)].$$

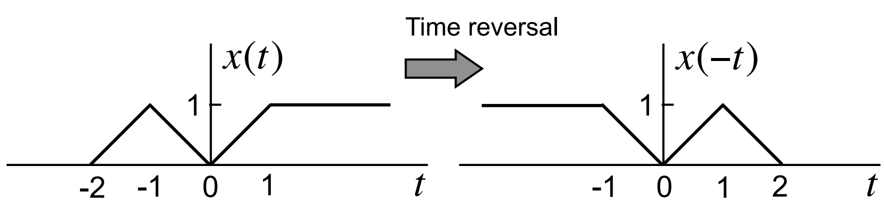
(a) The signal $x(-t)$ is obtained from $x(t)$ as below:



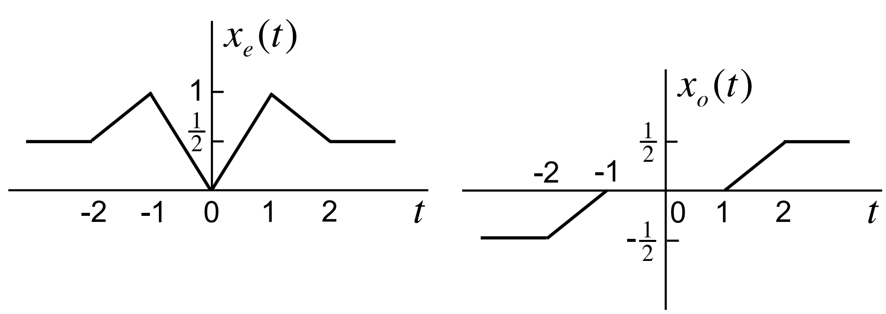
Then, we have



(b) The signal $x(-t)$ is obtained from $x(t)$ as below:



Then, we have



Problem 3:

(a) Consider

$$\begin{aligned}\int_{-\infty}^{+\infty} x(t)dt &= \int_{-\infty}^{0} x(t)dt + \int_{0}^{+\infty} x(t)dt \\ &= \int_{0}^{+\infty} x(-t)dt + \int_{0}^{+\infty} x(t)dt \\ &= \int_{0}^{+\infty} [x(t) + x(-t)]dt.\end{aligned} \tag{1}$$

If $x(t)$ is odd, $x(t) + x(-t) = 0$. Therefore, (1) evaluates to zero.

(b) Let $y(t) = x_1(t)x_2(t)$. Then

$$y(-t) = x_1(-t)x_2(-t) = -x_1(t)x_2(t) = -y(t).$$

This implies that $y(t)$ is odd.

(c) Consider

$$\begin{aligned}\int_{-\infty}^{+\infty} x^2(t)dt &= \int_{-\infty}^{+\infty} [x_e(t) + x_o(t)]^2 dt \\ &= \int_{-\infty}^{+\infty} x_e^2(t)dt + \int_{-\infty}^{+\infty} x_o^2(t)dt + 2\int_{-\infty}^{+\infty} x_e(t)x_o(t)dt.\end{aligned}$$

Using the result of part (b), we know that $x_e(t)x_o(t)$ is an odd signal. Then, using the result of part (a), we have

$$2\int_{-\infty}^{+\infty} x_e(t)x_o(t)dt = 0.$$

Therefore,

$$\int_{-\infty}^{+\infty} x^2(t)dt = \int_{-\infty}^{+\infty} x_e^2(t)dt + \int_{-\infty}^{+\infty} x_o^2(t)dt.$$