In-Class Exercise 9

1. Determine the Laplace transform of

$$x(t) = e^{-5t}u(t-1)$$

   Specify its region of convergence (ROC). Determine all pole and zero locations.

2. Determine the Laplace transform of

$$x(t) = -ae^{at}u(-t)$$

   where a is complex number. Specify its ROC. Determine all pole and zero locations.

3. Consider the continuous-time signal $x(t)$:

$$x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$$

Determine the constraints on the complex number β given that the ROC of $X(s)$ is $\Re\{s\} > -3$.

4. Determine the Fourier transforms of $x(t) = e^{-0.5t}u(t)$ and $y(t) = e^{2t}u(t)$.

5. Given the Laplace transform of a continuous-time signal $h(t)$:

$$H(s) = \frac{s+10}{(s+2)^2(s-1)(s-10)(s-20)}$$

Determine all the possible ROCs for $H(s)$.

Solution

1.

$$
\begin{aligned}
X(s) &= \int_{-\infty}^{\infty} e^{-5t}u(t-1)e^{-st}dt = \int_{1}^{\infty} e^{-(s+5)t}dt \\
&= -\frac{1}{s+5}e^{-(s+5)t}\Big|_{1}^{\infty} \\
&= \frac{e^{-(s+5)}}{s+5}, \quad \Re\{s\} > -5
\end{aligned}
$$

There is no zero and there is one pole at $s=-5$.

2.

$$X(s) = \int_{-\infty}^{\infty} -ae^{at}u(-t)e^{-st}dt = -a\int_{-\infty}^{0} e^{-(s-a)t}dt$$
$$= \frac{a}{s-a}e^{-(s-a)t}\bigg|_{-\infty}^{0} = \frac{a}{s-a}, \quad \Re\{s\} < \Re\{a\}$$

There is no zero and there is one pole at $s = a$.

3.
From the ROC properties, we can see that $\Re\{s\} > -3$ is due to $e^{-\beta t}u(t)$.

As a result, we can deduce that $\Re\{\beta\} = 3$ while there is no restriction for the value of $\Im\{\beta\}$.

4.

As discrete-time Fourier transform (DTFT) can be computed using (6.4) and z transform, Fourier transform can be computed using (5.1):

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$

or Laplace transform. Here we only apply the latter.

$$x(t) = e^{-0.5t}u(t) \leftrightarrow X(s) = \frac{1}{s+0.5}, \quad \Re\{s\} > -0.5$$

As the ROC includes the $j\Omega$-axis, the Fourier transform exists.

Substituting $s = j\Omega$, we obtain:

$$X(j\Omega) = \frac{1}{j\Omega + 0.5}$$

On the other hand:

$$y(t) = e^{2t}u(t) \leftrightarrow Y(s) = \frac{1}{s-2}, \quad \Re\{s\} > 2$$

As the ROC does not include the $j\Omega$-axis, $Y(j\Omega)$ does not exist.

Note that using (5.1) will give the same conclusion.

5.

Among the 5 poles, only 4 of them have distinct real parts. As a result, there are 5 ROC possibilities:

$$\Re\{s\} < -2$$
$$1 > \Re\{s\} > -2$$
$$10 > \Re\{s\} > 1$$
$$20 > \Re\{s\} > 10$$
$$\Re\{s\} > 20$$