EE5410 Signal Processing

Semester A 2020-2021

Assignment 2

Due Date: **10 November 2020**

1. Consider a linear time-invariant (LTI) system with impulse response $h[n]$. The discrete-time Fourier transform (DTFT) of $h[n]$ is:

$$H(e^{j\omega}) = \frac{1}{1 + 0.9e^{-j\omega} + 0.2e^{-2j\omega}}$$

(a) Determine the transfer function $H(z)$ and its region of convergence (ROC).
(b) Find $h[n]$.

2. Find the frequency response $H(e^{j\omega})$ of a discrete-time stable system whose input $x[n]$ and output $y[n]$ satisfy the following difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

Then determine the system impulse response $h[n]$.

3. Figure 1 shows the block diagram representation of a causal LTI discrete-time system with input $x[n]$ and output $y[n]$.

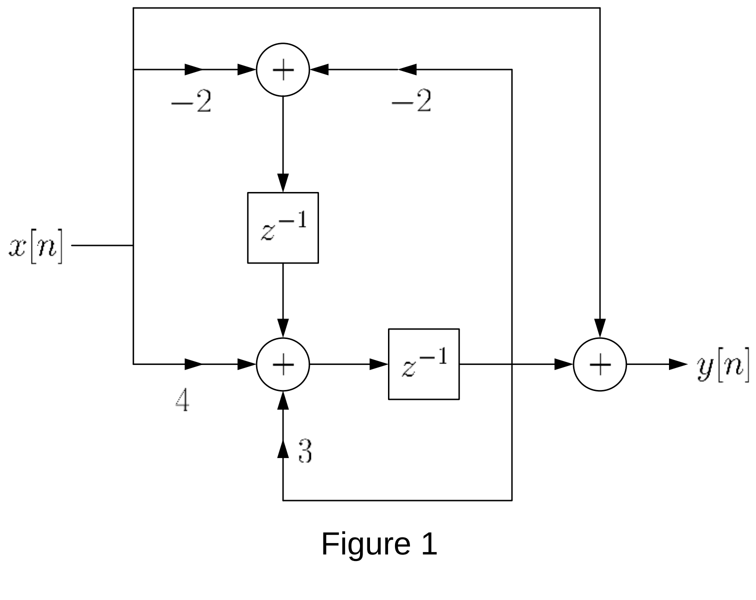

Figure 1

(a) Determine the system transfer function $H(z) = Y(z)/X(z)$ where $X(z)$ and $Y(z)$ are the z transforms of the input $x[n]$ and output $y[n]$, respectively.
(b) Draw the block diagram representation of the system using canonic form.
(c) Is the system stable? Explain your answer.

4. Consider a causal LTI system whose system function is

$$H(z) = \frac{1 + \frac{1}{5}z^{-1}}{\left(1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}\right)\left(1 + \frac{1}{4}z^{-1}\right)}$$

Draw one signal flow graph for the system in each of the following forms:

(a) Direct form
(b) Cascade form using canonic form sections
(c) Parallel form using canonic form sections

5. Consider an ideal bandpass filter whose frequency response in $(-\pi, \pi)$ is:

$$H_d(e^{j\omega}) = \begin{cases} 1, & \omega_a \leq \omega \leq \omega_b, -\omega_b \leq \omega \leq -\omega_a \\ 0, & \text{otherwise} \end{cases}$$

where $\omega_a = 0.3\pi$ and $\omega_b = 0.8\pi$.
(a) Use the window method with rectangular window to design a causal and linear-phase finite impulse response (FIR) filter of length 7 that approximates $H_d(e^{j\omega})$. Write down the filter transfer function $H(z)$ with numerical values.
(b) When implementing the FIR filter with transfer function $H(z)$, determine the minimum numbers of multiplications and additions for computing each output sample.

6. Consider a causal and linear-phase FIR filter of length 3 such that $h[0] = h[2] = \alpha_0$ and $h[1] = \alpha_1$. It is known that the magnitude of the filter frequency response $H(e^{j\omega})$ is $|H(e^{j\omega})| = 0$ at $\omega = 0.1$, while $|H(e^{j\omega})| = 1$ at $\omega = 0.4$. Determine the values of α_0 and α_1.