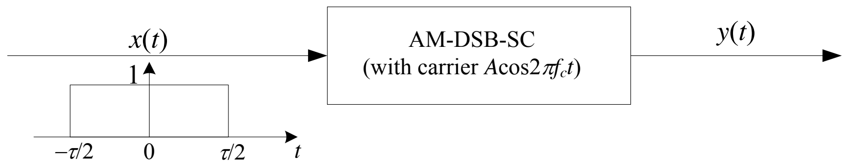
# EE3008 Assignment I

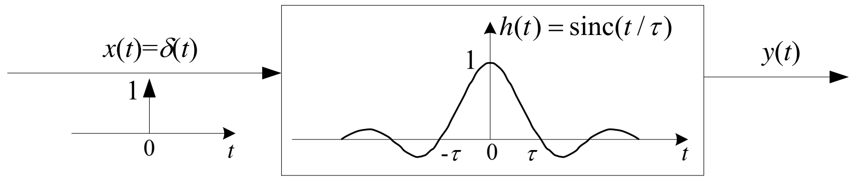
## Q1.

Determine whether the following output signals $y(t)$ are power-type or energy-type signals. For energy-type signals, determine the signal energy. For power-type signals, determine the signal power.

1)




2)

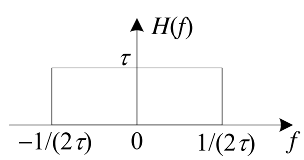



Solution:

1) $y(t) = Ax(t)\cos(2\pi f_c t)$ is an energy signal and its energy is $E_y = \int_{-\infty}^{\infty} |y(t)|^2 = A^2\tau/2 < \infty$ .

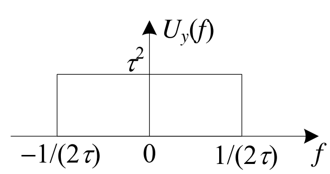
(Note: Here $y(t)$ is a truncated sinusoidal signal which has been discussed in Tutorial 1. Please see Q1, Tutorial 1 for details.)

2) The transfer function of the filter $H(f)$ can be obtained as




(See Q2, Tutorial 1 for details.)

As $X(f)$=1, we have $Y(f)$=$H(f)$. Therefore, $y(t)$ is an energy signal and its energy spectrum is



The energy is $E_y = \int_{-\infty}^{\infty} U_y(f)df = \int_{-\frac{1}{2\tau}}^{\frac{1}{2\tau}} \tau^2 df = \tau < \infty$.

**Q2.**

An AM-DSB-C modulator has a carrier signal with power 512W. When a sinusoidal test signal is applied to this AM-DSB-C modulator, it is found that the magnitude ratio of each sideband and the carrier line of the modulated signal is 25%.

1) Determine the modulation index *m*;
2) Determine the magnitude of each sideband;
3) Derive the mathematical expression of the modulated signal in the time domain;
4) Sketch the output waveform if the modulated signal passes through an envelope detector.

Solution:

The magnitude spectrum of a sinusoidally AM-DSB-C modulated signal can be drawn as:

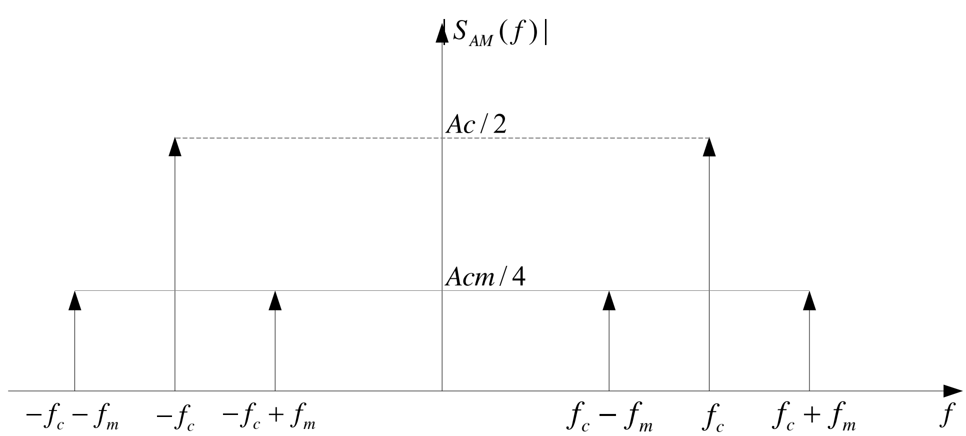



Fig. 1. Magnitude spectrum of sinusoidally AM-DSB-C modulated signal

where $f_c$ is the carrier frequency, $f_m$ is the frequency of the input sinusoidal signal, $m$ is the modulation index, $c$ is the dc offset and $A$ is the scaling factor.

1. According to “it is found that the magnitude ratio of each sideband and the carrier line of the modulated signal is 25%”, we have

$$\frac{Acm/4}{Ac/2} = 0.25 \Rightarrow m = 0.5.$$

2. According to Fig. 1, the carrier power $P_c$ can be written as

$$P_c = 2 \times (Ac/2)^2 = (Ac)^2/2,$$

which is given by 512W. We then have $Ac = \sqrt{2P_c} = 32V$.

The magnitude of each sideband is then $Acm/4 = 4V$.

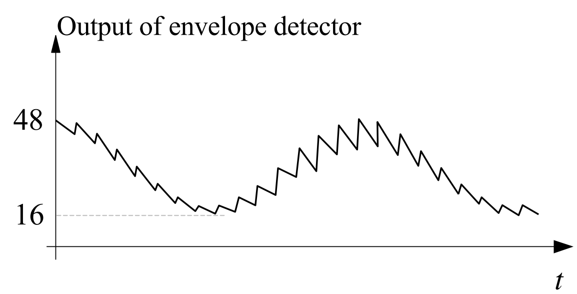
3. The time-domain expression of a sinusoidally AM-DSB-C modulated signal can be written as

$$s_{AM}(t) = Ac(1 + m\cos(2\pi f_m t))\cos(2\pi f_c t)$$

According to $Ac$=32V and $m$=0.5, we have

$$s_{AM}(t) = 32(1 + 0.5\cos(2\pi f_m t))\cos(2\pi f_c t).$$

4. According to the expression of $s_{AM}(t)$, the maximum and minimum values of the envelop are given by $Ac(1+m)$=48V and $Ac(1-m)$=16V, respectively. As a result, output waveform of the envelope detector can be sketched as:




**Q3.**

A frequency-modulated signal with carrier frequency $f_c$ has the following expression:

$$s_{FM}(t) = 100\cos\{10^6 \pi t + 4\sin(1000\pi t) + 3\cos(1000\pi t)\}.$$

1) Determine the modulation index;
2) Determine the effective bandwidth of the modulated signal according to Carson's rule;

3) Determine the output power of the first sideband components;
4) Determine the output power at 500.5kHz and 500.1kHz, respectively.

Solution:

1. The instantaneous phase is $\Psi(t) = 2\pi f_c t + 4\sin(1000\pi t) + 3\cos(1000\pi t)$. Therefore, the instantaneous frequency is given by

$$f(t) = \frac{1}{2\pi}\cdot\frac{d\Psi(t)}{dt} = f_c + 2000\cos(1000\pi t) - 1500\sin(1000\pi t)$$
$$= f_c - 500\cdot 5\sin(1000\pi t - \tan^{-1}(4/3))$$

The peak frequency deviation is

$$\Delta f = \max\left|-500\cdot 5\sin(1000\pi t - \tan^{-1}(4/3))\right| = 2.5\ \ kHz;$$

The frequency of the information signal $f_m$=500Hz and the modulation index

$$\beta = \frac{\Delta f}{f_m} = \frac{2500}{500} = 5.$$

2. According to Carson's rule, the bandwidth of the modulated signal is approximately given by

$$2(\beta + 1) f_m = 6\ \ \text{kHz}.$$

3. According to A=100V, the total power is $P$=$100^2$/2=5000 W. The output power of the first sideband components is

$$P_{s1} = P\cdot 2\,|\,J_1(\beta)\,|^2 = 5000\times 2\times 0.33^2 = 1089\ \ \text{W}.$$

4. 500.5kHz=$f_c$+$f_m$. Therefore, the output power is $P\cdot|\,J_1(\beta)\,|^2 = 5000\times 0.33^2 = 544.5$ W. There is no frequency component at 500.1kHz. The output power is then zero.