Tutorial 10

Groups

Q.1 Group or Not?

Is each of the following cases a group? If so, is it an Abelian group?

a) Even numbers under addition

b) Odd numbers under addition

c) Multiples of 7 under addition

d) 2×2 real matrices under addition

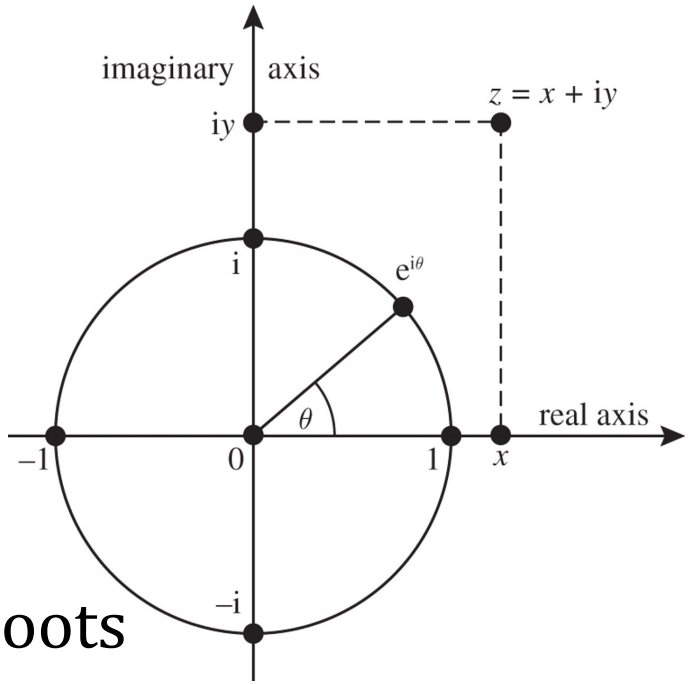
e) 2×2 real matrices under multiplication

Q.2 Unit Circle on Complex Plane

- Consider the set of complex numbers on the unit circle:

$$H = \{z \in \mathbb{C}: |z| = 1\}.$$

- Denote multiplication by $\times$.
  - e.g. $(1 + 2i)(3 - i)$
    $= (3 + 2) + (6 - 1)i$
    $= 5 + 5i.$



a) Show that $\langle H, \times \rangle$ forms a group.

b) Find the cube roots of unity, or roots satisfying the equation: $z^3 = 1$, where $z \in \mathbb{C}$. Do the roots form a subgroup of H?

Q.3 Binary Linear Code

- Recall that a binary linear code C is a subset of $\mathbb{B}^n$.
- It is defined by the encoding function $f: \mathbb{B}^k \to \mathbb{B}^n$, where $f(u) = uG$ and G is the generator matrix.
- It can be checked that $\mathbb{B}^n$ with binary addition is a group.
- Is C a subgroup of $\mathbb{B}^n$?