EE2302 Foundations of Information Engineering

Assignment 1 (Solution)

1. The statement is true.
   Proof: Suppose m is any even integer and n is any odd integer.
   [We must show that $m+n$ is odd.]
   By the definition of even numbers, there exists an integer k such that $m = 2k$.
   By the definition of odd numbers, there exists an integer h such that $n = 2h + 1$.
   By substitution,
   $$m + n = (2k) + (2h + 1) = 2(h + k) + 1.$$
   Since $h + k$ is an integer, $m + n$ is odd. *Q.E.D.*

2. Let p = "the number is prime", q = "the number is either odd or 2". We should prove by contraposition: $p \to q \equiv {\sim}q \to {\sim}p$. Suppose the number is neither odd nor 2. Then it is even but not 2, i.e., 4, 6, 8, or 10, …. Therefore, it can be divided by 2 and is not prime. By contraposition, the statement is true. *Q.E.D.*

3. Proof:
   a) Assume $x \in C$, then $x = 9^r = 3^{2r}$ for some integer r. Rewrite it as $x = 3^s$ where $s = 2r$ is an integer, so $x \in D$. Since $x \in C$ implies $x \in D$, it follows that $C \subseteq D$.

   Disproof:
   b) To disprove this statement, we need to show $\exists x \in D$ and this $x \notin C$. It is easy to see that $3 \in D$ but $3 \notin C$, which is a counterexample.

4. Proof: We need to prove both $B \subseteq C$ and $C \subseteq B$.
   i. Let m be an element of B, so there is an integer b such that $m = 10b - 4 = 10(b-1) + 6$. Since $(b-1)$ is an integer, by the definition of C, m is an element of C. Therefore, $B \subseteq C$.
   ii. Let n be an element of C, so there is an integer c such that $n = 10c + 6 = 10(c+1) - 4$. Since $(c+1)$ is an integer, be the definition of B, n is an element of B. Therefore, $C \subseteq B$.

5. The description "the smallest integer not describable in fewer than twelve English words" contains only eleven English words. If n is well defined, then it is describable in only eleven English words, which is a contradiction.

6. No. Suppose there were a computer program P that had as output a list of all computer programs that do not list themselves in their output. Consider the following two cases:
   a) If P lists itself as output, then it would be on the output list of P, which means that P would not list itself in its output. A contradiction.
   b) If P does not list itself as output, then it would be a member of the list of all programs that do not list themselves in their output, and this list is exactly the output of P. Hence, P would list itself as output. Again, a contradiction.

   In both cases, the assumption of the existence of such a program P is contradictory, and so no such program exists.