EE 3210 Signals and Systems

Semester A 2023 – 2024

4:00 p.m. – 5:30 p.m.

Answer **ALL SIX** questions:

Question 1

Consider a discrete-time signal $x[n]$ which is expressed as:

$$x[n] = \begin{cases} n+1, & -2 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch $x[n]$. **(2 marks)**

(b) Determine the even and odd components of $x[n]$, $x_e[n]$ and $x_o[n]$, respectively. **(10 marks)**

Question 2

Given a continuous-time signal $x(t)$ and for all values of t, it has the form of:

$$x(t) = A_1 \sin(Bt + C_1) + A_2 \cos(Bt + C_2)$$

where A_1, A_2, B, C_1 and C_2 are real-valued constants.

(a) Is $x(t)$ a periodic signal? **(2 marks)**

(b) Is $x(t)$ an energy signal? **(2 marks)**

(c) Compute the power and energy of $x(t)$. Useful trigonometric identities are provided below. **(10 marks)**

$$2\sin(\alpha)\sin(\beta) = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$
$$2\cos(\alpha)\cos(\beta) = \cos(\alpha + \beta) + \cos(\alpha - \beta)$$
$$2\sin(\alpha)\cos(\beta) = \sin(\alpha + \beta) + \sin(\alpha - \beta)$$
$$\cos^2(\alpha) = [1 + \cos(2\alpha)]/2$$
$$\sin^2(\alpha) = [1 - \cos(2\alpha)]/2$$

Question 3

Given a continuous-time periodic signal $x(t)$ with fundamental period $T = 4$. Within the interval $[0, 4]$, $x(t)$ is described as:

$$x(t) = \begin{cases} -4, & 0 < t < 1 \\ 0, & 1 < t < 2 \\ -2, & 2 < t < 4 \end{cases}$$

(a) Compute the power of $x(t)$. **(4 marks)**

(b) Find all the Fourier series coefficients for $x(t)$. **(10 marks)**

Question 4

Determine $y[n] = x[n] \otimes h[n]$ where $x[n]$ and $h[n]$ are

$$x[n] = \begin{cases} 1, & n = 0 \\ -1, & n = 1 \\ 3, & n = 2 \\ -3, & n = 3 \\ 0, & \text{otherwise} \end{cases}$$

and

$$h[n] = \begin{cases} 1, & n = -2 \\ 1, & n = -1 \\ -2, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

(10 marks)

Question 5

A discrete-time system with input $x[n]$ and output $y[n]$ is described by the following relationship:

$$y[n] = \sum_{k=-\infty}^{n+1} \alpha^{n+1-k} x[k]$$

where α is a real-valued constant.

(a) Is the system memoryless? Explain your answer. **(5 marks)**

(b) Is the system invertible? Explain your answer. **(5 marks)**

(c) Is the system linear? Explain your answer. **(5 marks)**

(d) Is the system time-invariant? Explain your answer. **(5 marks)**

(e) Is the system causal? Explain your answer. **(5 marks)**

(f) If the system input is $x[n] = \delta[n]$, compute the output $y[n]$. **(5 marks)**

(g) If the system input is $x[n] = u[n]$, compute the output $y[n]$. **(5 marks)**

Question 6

A continuous-time system with input $x(t)$ and output $y(t)$ is described by the following relationship:

$$y(t) - a\frac{dy(t)}{dt} = x(t) - \frac{1}{a}\frac{dx(t)}{dt}$$

where a is a real-valued constant.

(a) Determine the system frequency response $H(j\Omega) = Y(j\Omega)/X(j\Omega)$ where $X(j\Omega)$ and $Y(j\Omega)$ are the Fourier transforms of $x(t)$ and $y(t)$, respectively. **(5 marks)**

(b) Determine the magnitude and phase of the system frequency response. **(10 marks)**