Solutions to EE3210 Tutorial 6 Problems

Problem 1:

(a) $x(t) = \cos(4\pi t)$ is a periodic signal with fundamental period $T = 1/2$. Using Euler's formula, we can rewrite $x(t)$ as

$$x(t) = \frac{1}{2}e^{j4\pi t} + \frac{1}{2}e^{-j4\pi t}. \tag{1}$$

Comparing the right-hand sides of (1) and the synthesis formula of the continuous-time Fourier series, we obtain the Fourier series coefficients a_k of $x(t)$ as

$$a_k = \begin{cases} \frac{1}{2}, & k = -1 \\ \frac{1}{2}, & k = 1 \\ 0, & \text{otherwise.} \end{cases}$$

(b) $x(t) = \sin(4\pi t)$ is a periodic signal with fundamental period $T = 1/2$. Using Euler's formula, we can rewrite $x(t)$ as

$$x(t) = \frac{1}{2j}e^{j4\pi t} - \frac{1}{2j}e^{-j4\pi t}. \tag{2}$$

Comparing the right-hand sides of (2) and the synthesis formula of the continuous-time Fourier series, we obtain the Fourier series coefficients a_k of $x(t)$ as

$$a_k = \begin{cases} -\frac{1}{2j}, & k = -1 \\ \frac{1}{2j}, & k = 1 \\ 0, & \text{otherwise.} \end{cases}$$

(c) $x(t) = \cos(4\pi t)\sin(4\pi t)$ is also periodic with period $T = 1/2$. Using the trigonometric identity

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

and Euler's formula, we obtain

$$x(t) = \cos(4\pi t)\sin(4\pi t) = \frac{1}{2}\sin(8\pi t) = \frac{1}{4j}e^{j8\pi t} - \frac{1}{4j}e^{-j8\pi t}. \tag{3}$$

Then, comparing the right-hand sides of (3) and the synthesis formula of the continuous-time Fourier series, we obtain the Fourier series coefficients a_k of $x(t)$ as

$$a_k = \begin{cases} -\frac{1}{4j}, & k = -2 \\ \frac{1}{4j}, & k = 2 \\ 0, & \text{otherwise.} \end{cases}$$

Problem 2: This signal is periodic with a fundamental period $T = 2$. To determine the Fourier series coefficients a_k, we use the analysis formula of the continuous-time Fourier series, and select the interval of integration to be $-1/2 < t < 3/2$, avoiding the placement of impulses at the integration limits. Within this interval, $x(t)$ is the same as $\delta(t) - 2\delta(t-1)$. Thus, using the sampling property of $\delta(t)$, it follows that

$$\begin{aligned} a_k &= \frac{1}{2}\int_{-1/2}^{3/2} \left[\delta(t) - 2\delta(t-1)\right] e^{-jk\pi t} dt = \frac{1}{2}\int_{-1/2}^{3/2} \delta(t) e^{-jk\pi t} dt - \int_{-1/2}^{3/2} \delta(t-1) e^{-jk\pi t} dt \\ &= \frac{1}{2} - e^{-jk\pi} = \frac{1}{2} - \left(e^{-j\pi}\right)^k = \frac{1}{2} - (-1)^k. \end{aligned}$$

Problem 3: This signal is periodic with a fundamental period $T = 3$. To determine the Fourier series coefficients a_k, we use the analysis formula of the continuous-time Fourier series, and choose the limits of the integration to include the interval $0 < t < 2$. Within this interval,

$$x(t) = \begin{cases} 2, & 0 < t < 1 \\ 1, & 1 < t < 2. \end{cases}$$

Thus, it follows that:

- For $k = 0$,

$$a_0 = \frac{1}{3}\int_0^1 2dt + \frac{1}{3}\int_1^2 dt = 1.$$

- For $k \neq 0$,

$$\begin{aligned} a_k &= \frac{2}{3}\int_0^1 e^{-jk(2\pi/3)t} dt + \frac{1}{3}\int_1^2 e^{-jk(2\pi/3)t} dt \\ &= \frac{2 - e^{-jk2\pi/3} - e^{-jk4\pi/3}}{jk2\pi}. \end{aligned}$$

  – Note: In this case, we have

$$\lim_{k \to 0} \frac{2 - e^{-jk2\pi/3} - e^{-jk4\pi/3}}{jk2\pi} = 1$$

  following from the l'Hôpital's rule.