Name: ____________________

Student ID: ____________________

Signature: ____________________

CITY UNIVERSITY OF HONG KONG

Semester B 2013/2014

EE3210: Signals and Systems

Quiz 2

1. Time allowed: One hour
2. Total number of problems: 4
3. Total marks available: 35
4. This paper may not be retained by candidates

Special Instructions

5. This is a closed book exam
   - A list of possibly relevant equations is attached at the end of this paper
6. Attempt all questions from each problem
7. Show all equations, calculations involved in your solutions
   - If you just provide the final answer, you won't receive full marks even if it is correct
   - If you provide intermediate steps and they are correct, you will receive partial marks even if the final answer is incorrect

Problem 1: [9 marks = (a) 4.5 marks + (b) 4.5 marks]

Consider a discrete-time LTI system, called system A, whose unit impulse response $h[n]$ is given by

$$h[n] = \delta[n+1] - \delta[n].$$

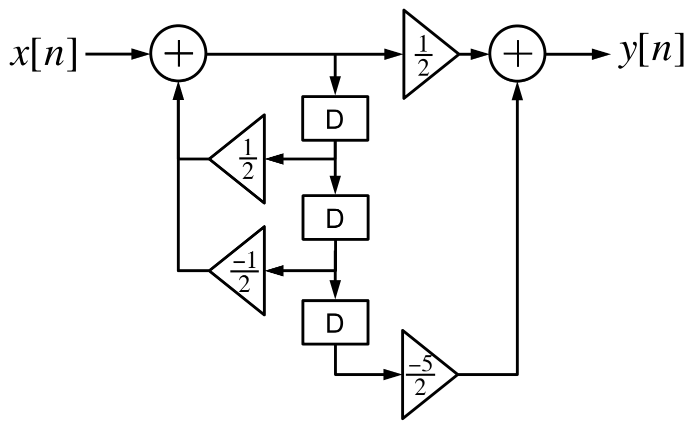
(a) Determine whether system A is memoryless, causal and/or stable. Justify your answers.

(b) Show that system A has an inverse system, called system B, whose input-output relationship is defined by

$$y[n] = \sum_{k=-\infty}^{n-1} x[k].$$

Problem 2: [9 marks = (a) 5 marks + (b) 4 marks]

Consider a discrete-time LTI system whose direct form II realization is shown in the figure below:



Determine:

(a) The direct form I block diagram representation of the system.

(b) The linear constant-coefficient difference equation that describes the relationship between the input $x[n]$ and the output $y[n]$ of the system.

Problem 3: [8 marks]

Consider a continuous-time periodic signal $x(t)$ with period 2 and

$$x(t) = e^{-t} \text{ for } -1 < t < 1.$$

Determine the Fourier series coefficients of $x(t)$.

Problem 4: [9 marks = (a) 5 marks + (b) 4 marks]

Let $x[n]$ be a discrete-time periodic signal with fundamental period N and Fourier series coefficients a_k.

(a) Derive the Fourier series coefficients of each of the following two signals in terms of a_k:

1. $x[1-n] + x[-1-n]$
2. $\mathcal{E}\{x[n]\}$

(b) Show that the signal $x[n] + x[n+N/2]$ is periodic with period $N/2$, assuming that N is even.

Appendix – A list of possibly relevant equations

- Complex number:
  - Euler's formula: $e^{j\theta} = \cos\theta + j\sin\theta$
- Fundamental period of a periodic signal:
  - Continuous-time sinusoidal of the form $x(t) = A\cos(\omega t + \phi)$: $T_0 = 2\pi/\omega$
  - Discrete-time sinusoidal of the form $x[n] = A\cos(\Omega n + \phi)$: $N_0 = 2\pi m/\Omega$ if N_0 and m have no factors in common.
  - Continuous-time complex exponential of the form $x(t) = e^{j\omega t}$: $T_0 = 2\pi/|\omega|$
  - Discrete-time complex exponential of the form $x[n] = e^{j\Omega n}$: $N_0 = 2\pi m/|\Omega|$ if N_0 and m have no factors in common.
- Even/odd part of a signal:
  - Continuous-time signal $x(t)$:
    - Even part: $\mathcal{E}\{x(t)\} = \frac{1}{2}[x(t) + x(-t)]$
    - Odd part: $\mathcal{O}\{x(t)\} = \frac{1}{2}[x(t) - x(-t)]$
  - Discrete-time signal $x[n]$:
    - Even part: $\mathcal{E}\{x[n]\} = \frac{1}{2}(x[n] + x[-n])$
    - Odd part: $\mathcal{O}\{x[n]\} = \frac{1}{2}(x[n] - x[-n])$
- Convolution sum: $x[n] * h[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$
  - Commutative property: $x[n] * h[n] = h[n] * x[n]$
  - Distributive property: $x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$
  - Associative property: $x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$
- Convolution integral: $x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$
  - Commutative property: $x(t) * h(t) = h(t) * x(t)$
  - Distributive property: $x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$
  - Associative property: $x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$

- Properties of continuous-time LTI systems:
  - Memoryless: $h(t) = 0$ for $t \neq 0$.
  - Invertibility: $h(t) * h_1(t) = \delta(t)$ where $h_1(t)$ is the unit impulse response of the inverse system.
  - Causality: $h(t) = 0$ for $t < 0$.
  - Stability: $\int_{-\infty}^{+\infty} |h(t)|\, dt < \infty$

- Properties of discrete-time LTI systems:
  - Memoryless: $h[n] = 0$ for $n \neq 0$.
  - Invertibility: $h[n] * h_1[n] = \delta[n]$ where $h_1[n]$ is the unit impulse response of the inverse system.
  - Causality: $h[n] = 0$ for $n < 0$.
  - Stability: $\sum_{n=-\infty}^{+\infty} |h[n]| < \infty$

- Continuous-time Fourier series:
  - Formulas: Consider $x(t)$ periodic with fundamental period $T_0 = T$.
    * Synthesis: $x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$
    * Analysis: $a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt = \frac{1}{T}\int_T x(t) e^{-jk(2\pi/T)t} dt$
  - Properties: Consider $x(t)$ and $y(t)$ periodic with period T, $x(t) \leftrightarrow a_k$, $y(t) \leftrightarrow b_k$.
    * Linearity: $Ax(t) + By(t) \leftrightarrow Aa_k + Bb_k$
    * Time shift: $x(t - t_0) \leftrightarrow \left[e^{-jk(2\pi/T)t_0}\right] a_k$
    * Time reversal: $x(-t) \leftrightarrow a_{-k}$
    * Time scaling: $x(\alpha t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk(\alpha\omega_0)t}$
    * Multiplication: $x(t)y(t) \leftrightarrow \sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
    * Differentiation: $\frac{dx(t)}{dt} \leftrightarrow (jk\omega_0) a_k$
    * Parseval's relation: $\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$

- Discrete-time Fourier series:
  - Formulas: Consider $x[n]$ periodic with fundamental period $N_0 = N$.
    - Synthesis: $x[n] = \sum_{k=\langle N\rangle} a_k e^{jk\Omega_0 n} = \sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$
    - Analysis: $a_k = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk\Omega_0 n} = \frac{1}{N}\sum_{n=\langle N\rangle} x[n]e^{-jk(2\pi/N)n}$
  - Properties: Consider $x[n]$ and $y[n]$ periodic with period N, $x[n] \leftrightarrow a_k$, $y[n] \leftrightarrow b_k$.
    - Linearity: $Ax[n] + By[n] \leftrightarrow Aa_k + Bb_k$
    - Time shift: $x[n-n_0] \leftrightarrow \left[e^{-jk(2\pi/N)n_0}\right] a_k$
    - Time reversal: $x[-n] \leftrightarrow a_{-k}$
    - Multiplication: $x[n]y[n] \leftrightarrow \sum_{l=\langle N\rangle} a_l b_{k-l}$
    - First difference: $x[n] - x[n-1] \leftrightarrow \left[1 - e^{-jk(2\pi/N)}\right] a_k$
    - Parseval's relation: $\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$

— End of Paper —