Some Basic Concepts in Computational Complexity Theory

Moshe Zukerman
Electronic Engineering Dept.,
City University of Hong Kong
Email: moshezu@cityu.edu.hk

“Computational Complexity Theory

focuses on classifying computational problems according to their resource usage, and relating these classes to each other.”

Resource usage indicates difficulty of a problem.

Source: https://en.wikipedia.org/wiki/Computational_complexity_theory

Big O notation (in CS)

Let f and g be functions from the positive integers to the nonnegative real numbers.

f(x) = O(g(x)) if there are positive integers M and k, such that $f(n) \leq Mg(n)$ for all $n \geq k$.

Credits: https://en.wikipedia.org/wiki/Big_O_notation

Michael Sipser (1997). Introduction to the Theory of Computation. Boston/MA: PWS Publishing Co. Here: Def.7.2, p.227

Notation	Name
$O(n)$	Linear
$O(n^2)$ $O(n^c)$ $O(c^n)$, $c>1$	Quadratic Polynomial Exponential
$O(n!)$	Factorial

Examples:

The running time is $O(c^n)$, $c>1$ <=> The running time is exponential

The running time is $O(n^c)$, $c>1$ <=> The running time is polynomial

P or Class P

The class of problems for which some algorithm can provide a solution in polynomial time (“quickly”).

This does not mean that if such an algorithm is not known the problem does not belong in P.

Therefore, the Class P is no yet fully known.

NP or Class NP

The class of problems for which a solution can be verified in polynomial time (“quickly”).

NP (Nondeterministic Polynomial Time)

Credit: https://en.wikipedia.org/wiki/P_versus_NP_problem

P versus NP Problem

A major unsolved problem in computer science and mathematics.

“It asks whether every problem whose solution can be quickly verified can also be solved quickly.”

“Quickly” = in polynomial time

Credit: https://en.wikipedia.org/wiki/P_versus_NP_problem

NP Complete

A class of problems that are informally "the hardest problems in NP”.

If we could find a quick solution to one NP-Complete problem, then

$$P = NP$$

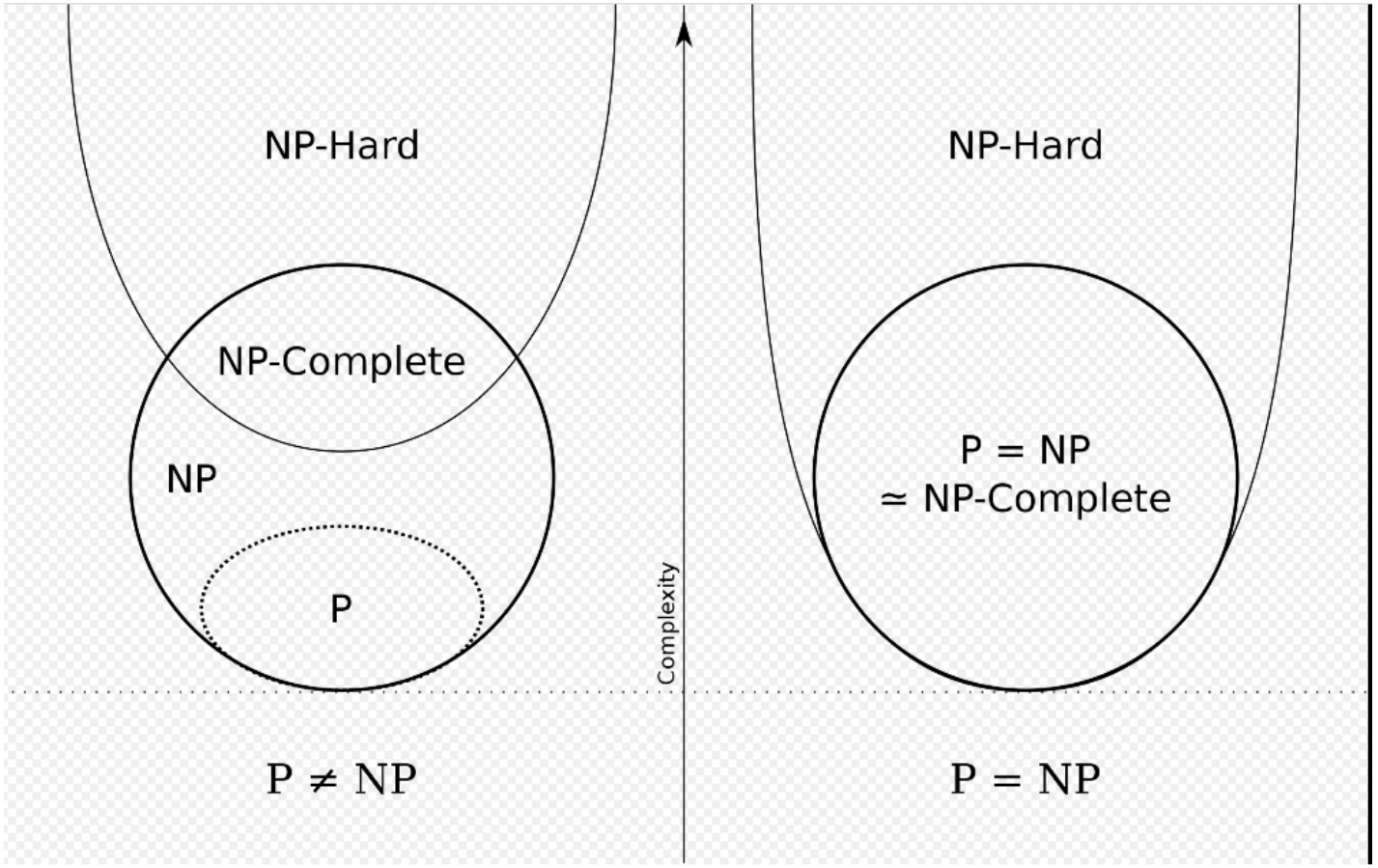
https://en.wikipedia.org/wiki/NP-completeness

NP Hard

A class of problems that are informally "at least as hard as the hardest problems in NP”.

Credit: https://en.wikipedia.org/wiki/NP-hardness

P versus NP Problem (cont'ed.)



Credit: https://en.wikipedia.org/wiki/P_versus_NP_problem