EE2302 Foundations of Information Engineering

Assignment 3 (Solution)

1.

a) We check the three conditions:
   i. If $m = 0$, then $m \times m = 0$. If $m \neq 0$, then $m \times m > 0$. Therefore, R is **reflexive**.
   ii. Suppose mRn. We only need to consider the case $m \neq n$. (The case where $m = n$ is the same as reflexivity.) Then, $mn > 0$, which implies that $nm > 0$. Therefore, R is **symmetric**.
   iii. Suppose mRn and nRp. We only need to consider the case where m, n, and p are distinct. (The other cases are the same as reflexivity or symmetry.) Then, $mn > 0$ and $np > 0$. Multiplying these two inequalities gives $mn^2p > 0$. Since $n \neq 0$ (for otherwise we cannot have $mn > 0$), we have $mp > 0$. Therefore, R is **transitive**.

b) There are three equivalence classes. They are
   i. $[1] = \{x \in Z \mid x > 0\}$,
   ii. $[-1] = \{x \in Z \mid x < 0\}$, and
   iii. $[0] = \{0\}$.

2.

a) S is not an equivalence relation. It is not symmetric, since $x \geq y$ does not imply $y \geq x$.

b) T is an equivalence relation.

   (reflexive): $x - x = 0$ is an integer

   (symmetric): if $x - y$ is an integer, then $y - x = -(x - y)$ is also an integer.

   (transitive): if $x - y$ and $y - z$ are integers, then $x - z = (x - y) - (y - z)$, which is a difference of two integers, is also an integer.

c) There is an equivalence class for each real number x, where $0 \leq x < 1$.

   (Note: The answer is not unique. For example, $-0.5 \leq x < 0.5$ is also correct.)

3.

a) T is not a partial order relation because it is not anti-symmetric.
Counter-example: 1 T 3 (because 1 + 3 is even) and 3 T 1 (because 3 + 1 is even), but $1 \neq 3$.

b) S is reflexive: $(a,b)\ S\ (a,b)$ because $a = a$ and $b \leq b$.

S is anti-symmetric: Suppose $(a,b)\ S\ (c,d)$ and $(c,d)\ S\ (a,b)$.
Then

either $(a < c)$ or (both $a = c$ and $b \leq d$). (Condition 1)

and

either $(c < a)$ or (both $c = a$ and $d \leq b$). (Condition 2)

Condition 1 implies $a \leq c$, while Condition 2 implies $c \leq a$. Combining them, we must have $a = c$. As a consequence, Condition 1 implies $b \leq d$ while Condition 2 implies $b \leq d$, so we much have $b = d$. Hence, $(a,b) = (c,d)$.

S is transitive: Suppose $(a,b)\ S\ (c,d)$ and $(c,d)\ S\ (e,f)$.

Then

either $(a < c)$ or (both $a = c$ and $b \leq d$). (Condition 3)

and

either $(c < e)$ or (both $c = e$ and $d \leq f$). (Condition 4)

Conditions 3 and 4 imply that $a \leq e$. There are two cases to consider:
Case 1: If $a < e$, then $(a,b)\ S\ (e,f)$.
Case 2: If $a = e$, then Conditions 3 and 4 imply $b \leq f$. Therefore, $(a,b)\ S\ (e,f)$.
Hence, we must have $(a,b)\ S\ (e,f)$.