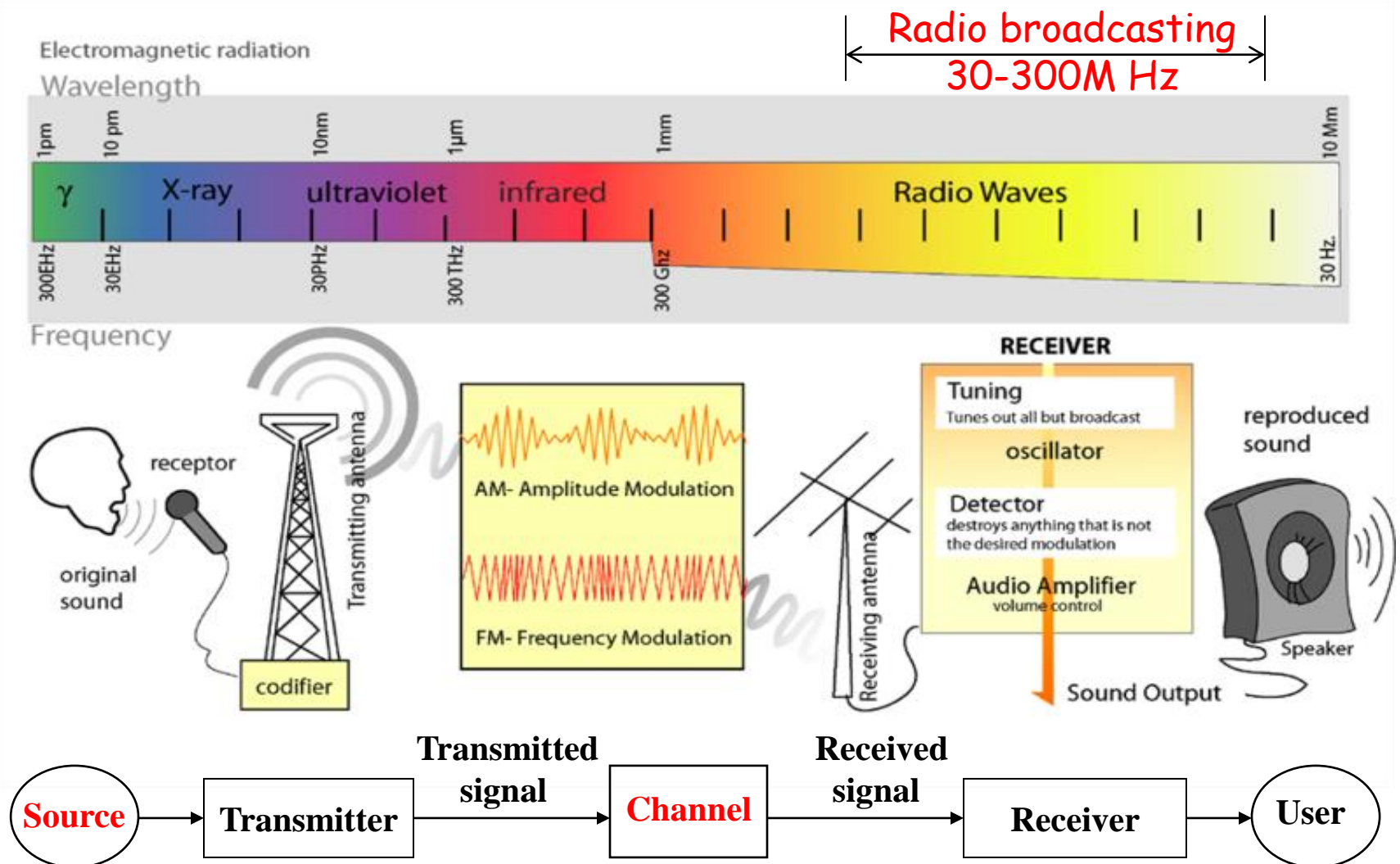


Analog Communications

- Amplitude Modulation (AM)
- Frequency Modulation (FM)



• Analog baseband signal

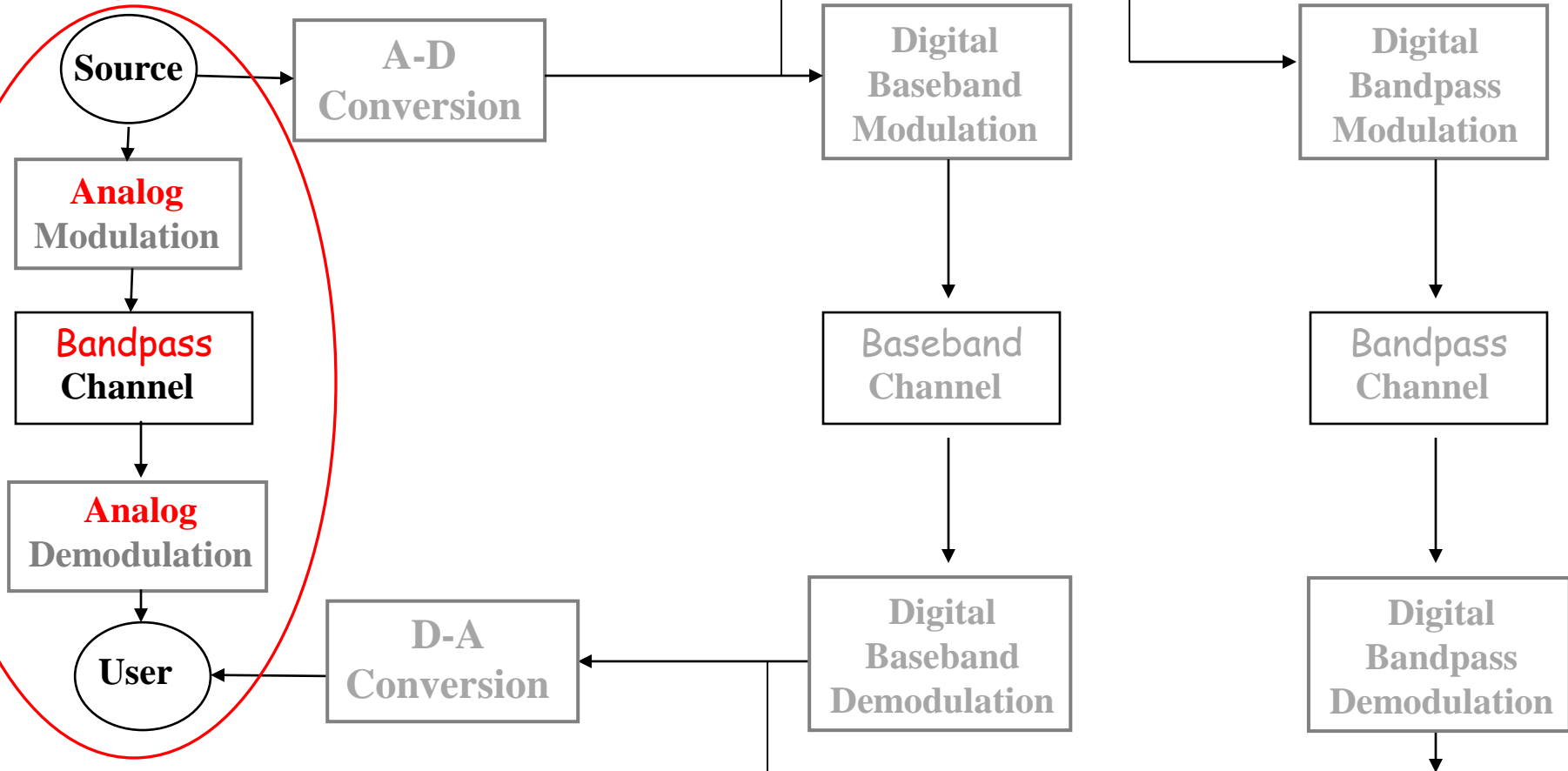
• Bandpass channel

Analog Communications

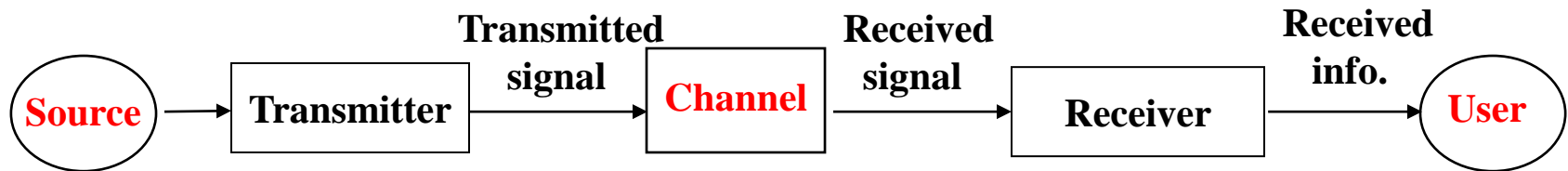
Analog signal



Bit sequence
0001101110.....



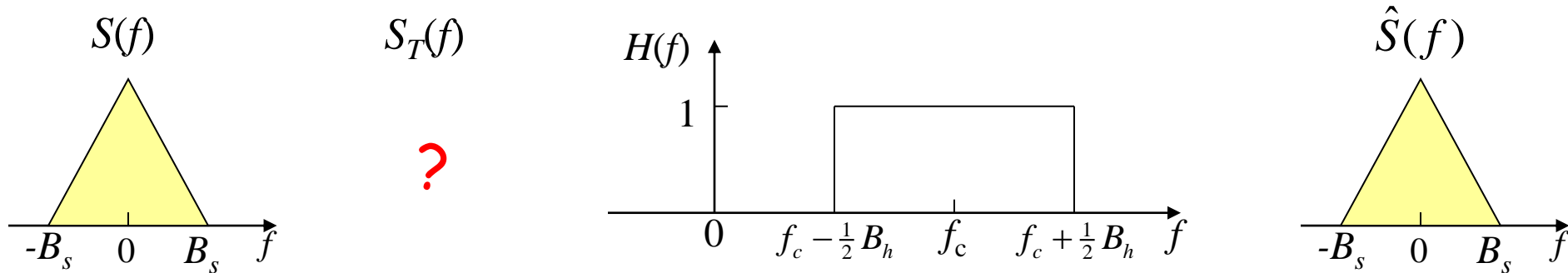
Analog Communications



• Analog baseband signal

• Bandpass channel

• Ignore noise



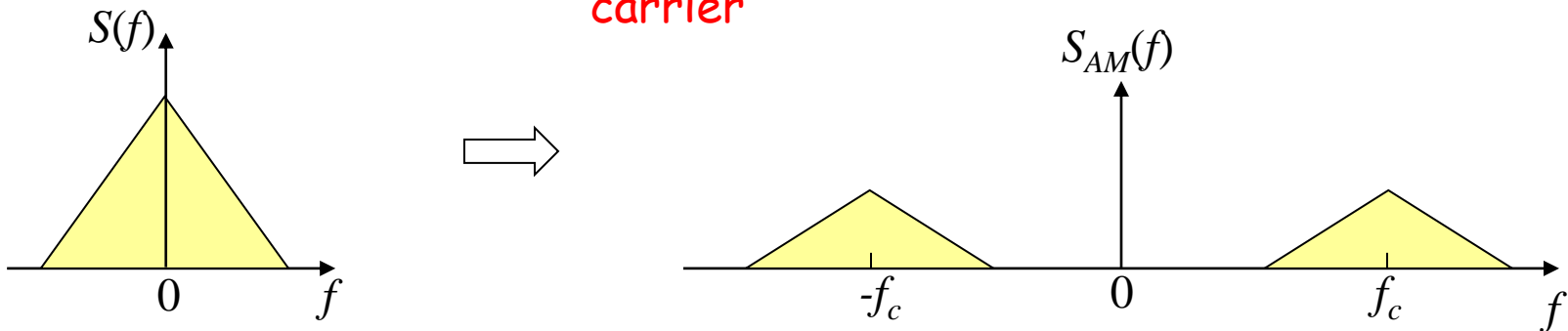
- For reliable communications, i.e., $\hat{S}(f) = S(f)$, all frequency components of the transmitted signal should pass through the channel, which requires:
 - Frequency components of transmitted signal should be centered at f_c ;
 - The channel bandwidth B_h should be no smaller than the bandwidth of transmitted signal B_m .

Analog Modulation

Modulation property of Fourier Transform:

$$s(t) \cos(2\pi f_c t) \Leftrightarrow \frac{1}{2}[S(f - f_c) + S(f + f_c)]$$

carrier



- Amplitude Modulation (AM)** $s_{AM}(t) = A s(t) \cos(2\pi f_c t)$
- Phase Modulation (PM)** $s_{PM}(t) = A \cos(2\pi f_c t + \alpha s(t))$
- Frequency Modulation (FM)** $s_{FM}(t) = A \cos(2\pi(f_c t + k \int_{-\infty}^t s(\tau) d\tau))$

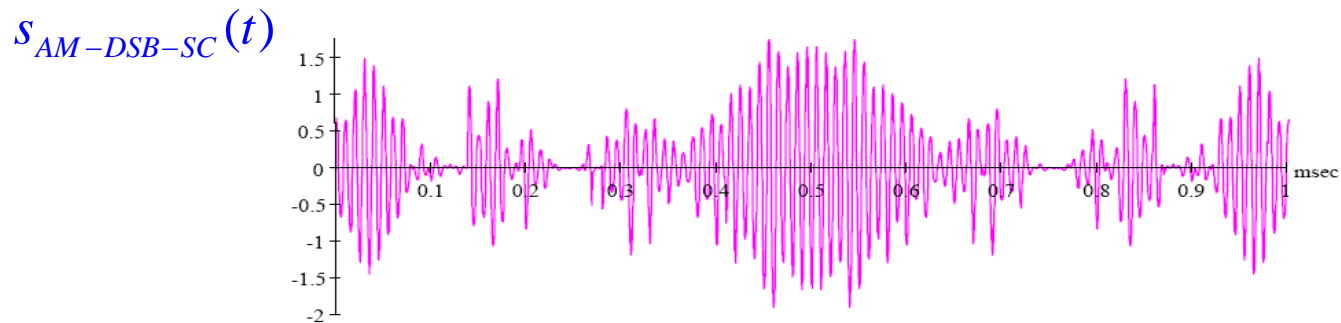
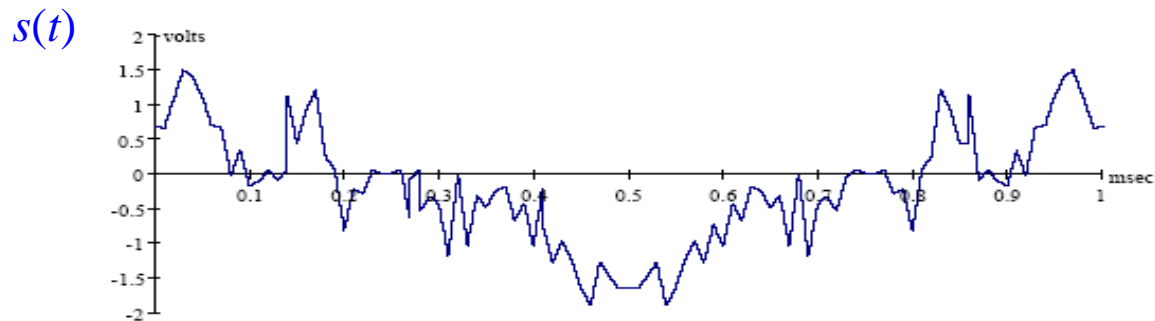
Lecture 3. Analog Communications

Part I. Amplitude Modulation (AM)

- AM-DSB-SC
- AM-DSB-C
- AM-SSB
- AM-VSB

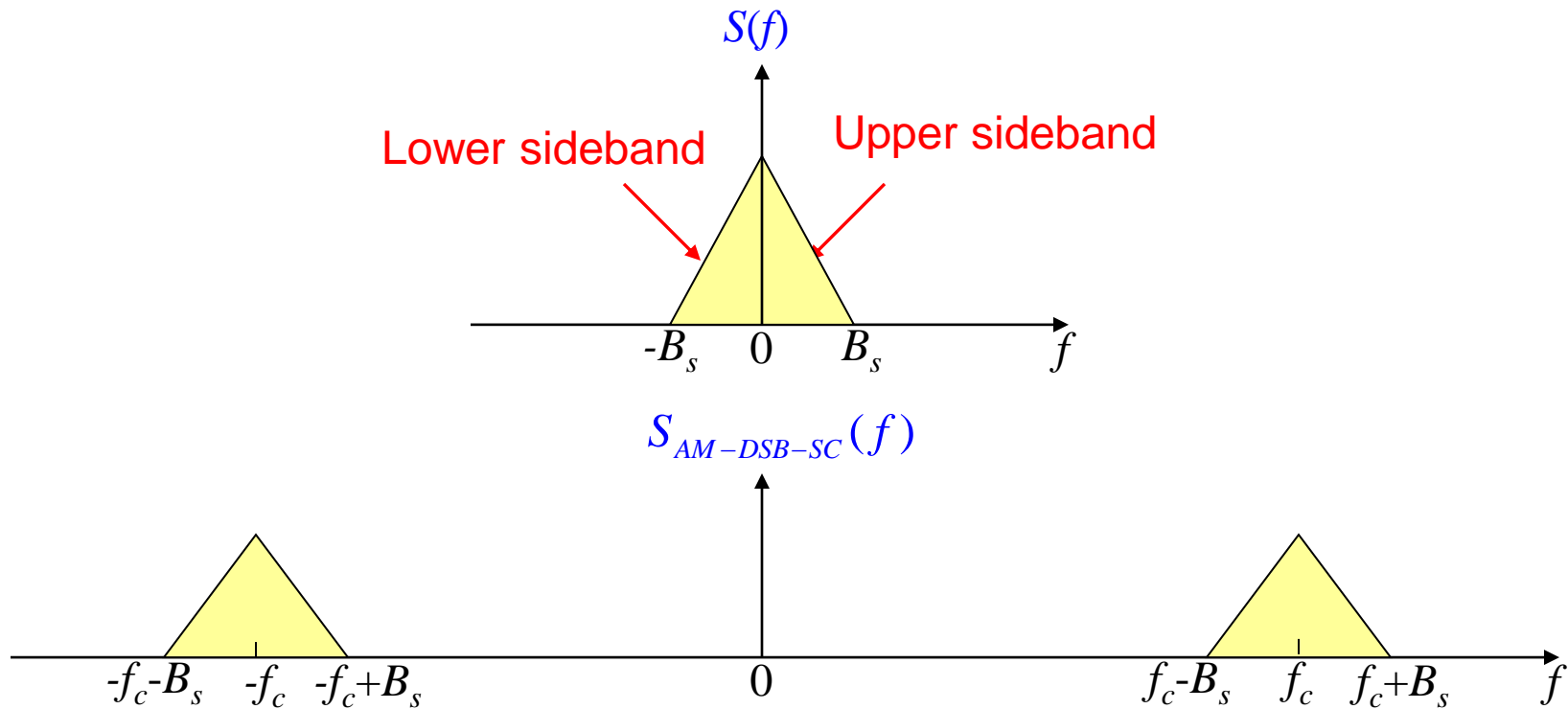
AM-DSB-SC -- Modulation

- Time Domain: $s_{AM-DSB-SC}(t) = As(t) \cos(2\pi f_c t)$



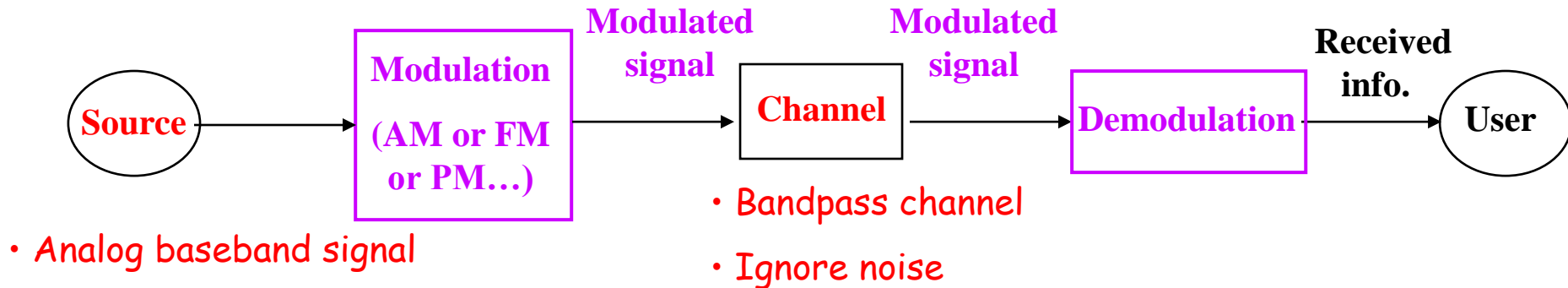
AM-DSB-SC -- Modulation

- Frequency Domain: $S_{AM-DSB-SC}(f) = \frac{A}{2}[S(f - f_c) + S(f + f_c)]$



AM-DSB-SC: Amplitude Modulation-Double SideBand-Suppressed Carrier

Analog Modulation



- Bandwidth efficiency is an important performance metric, which is defined as:

$$\gamma \triangleq \frac{\text{Information Signal Bandwidth } B_s}{\text{Required Channel Bandwidth } B_h} = \frac{\text{Information Signal Bandwidth } B_s}{\text{Modulated Signal Bandwidth } B_m}$$

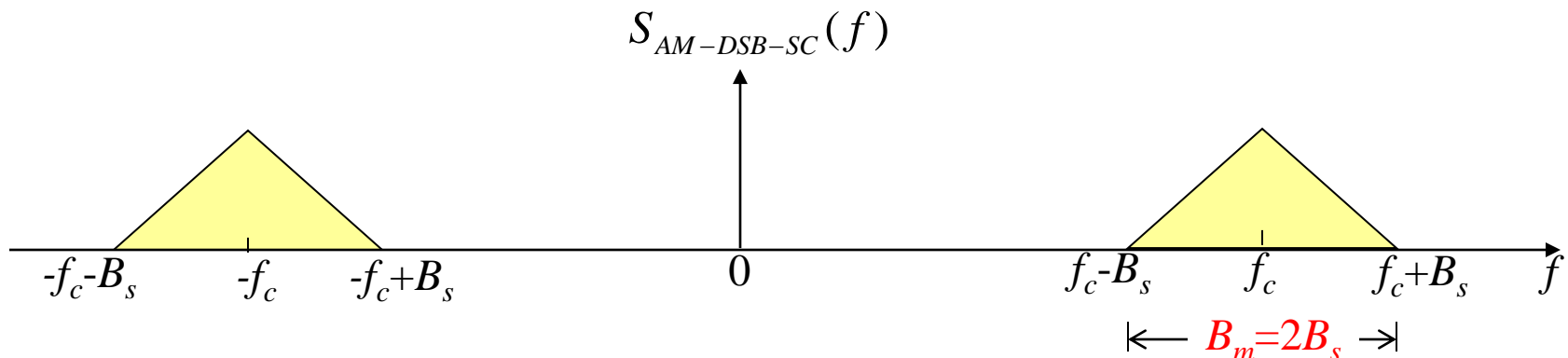
- Required channel bandwidth $B_h = \text{Modulated signal bandwidth } B_m$
- A higher γ indicates a better spectral utilization.

Bandwidth Efficiency of AM-DSB-SC

- Bandwidth Efficiency :

$$\gamma = \frac{\text{Information Signal Bandwidth } B_s}{\text{Required Channel Bandwidth } B_h} = \frac{\text{Information Signal Bandwidth } B_s}{\text{Modulated Signal Bandwidth } B_m}$$

With AM-DSB-SC :



$$\gamma_{AM-DSB-SC} = \frac{B_s}{2B_s} = 50\%$$

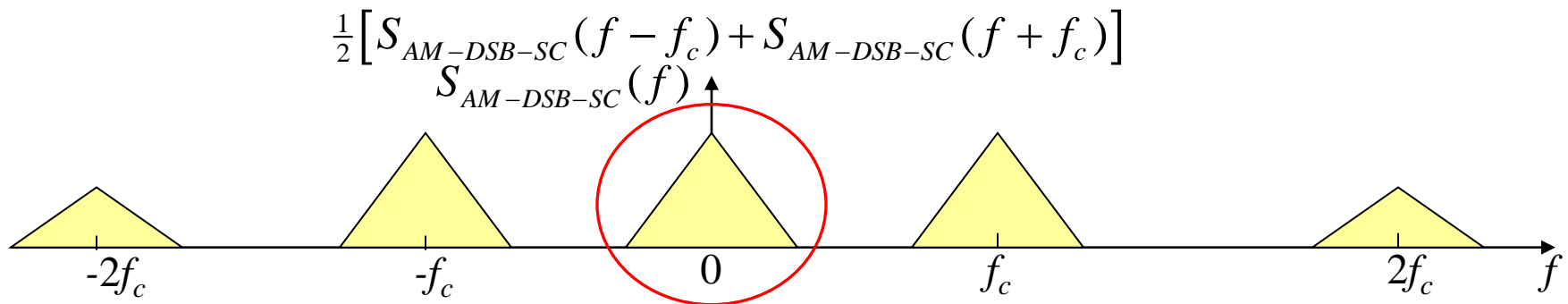
AM-DSB-SC -- Demodulation

• Time Domain: $s_{AM-DSB-SC}(t) \xrightarrow{?} s(t)$

$$s_{AM-DSB-SC}(t) \cos(2\pi f_c t) = As(t) \cos(2\pi f_c t) \cos(2\pi f_c t) = \underbrace{0.5As(t)}_{\text{desired signal}} + 0.5As(t) \cos(2\pi 2f_c t)$$

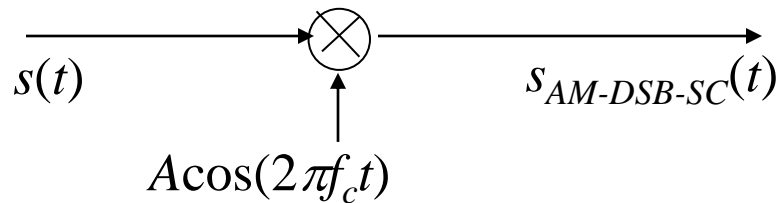
• Frequency Domain: $S_{AM-DSB-SC}(f) \xrightarrow{?} S(f)$

$$\begin{aligned} \frac{1}{2} [S_{AM-DSB-SC}(f - f_c) + S_{AM-DSB-SC}(f + f_c)] &= \frac{A}{4} [S(f - 2f_c) + S(f)] + \frac{A}{4} [S(f) + S(f + 2f_c)] \\ &= \frac{A}{4} [S(f - 2f_c) + S(f + 2f_c)] + \frac{A}{2} \underbrace{S(f)}_{\text{desired signal}} \end{aligned}$$

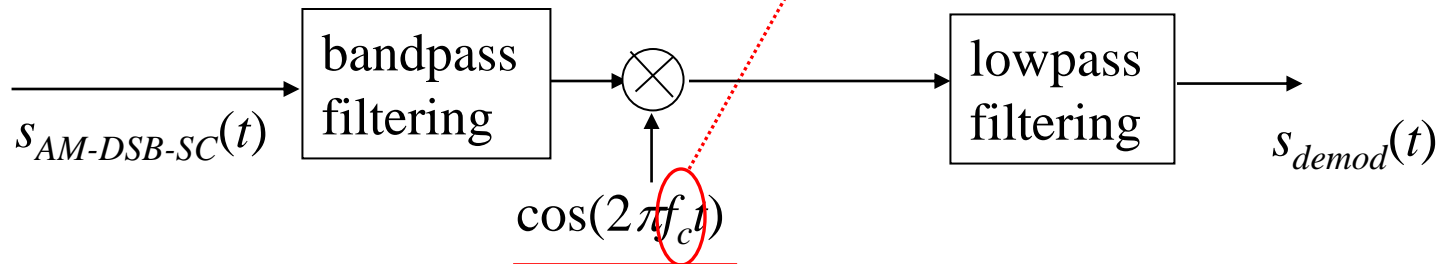


Modulator and Demodulator of AM-DSB-SC

Modulator



Demodulator



What if there is a frequency error?

Coherent Demodulation: the demodulator requires a reference signal which has exactly the same frequency and phase as the carrier signal.

Frequency Error of Coherent Demodulator

Consider that the reference signal has a small frequency error, Δf .

$$\begin{aligned} w(t) &= As(t) \cos(2\pi f_c t) \cos(2\pi (f_c + \Delta f) t) \\ &= 0.5 As(t) (\cos(2\pi \Delta f t) + \cos(2\pi (2f_c + \Delta f) t)) \end{aligned}$$

After lowpass filtering, we have

$$0.5 As(t) \cos(2\pi \Delta f t)$$

$\cos(2\pi \Delta f t) = 1$ when $\Delta f = 0$

$\cos(2\pi \Delta f t)$ changes with t when $\Delta f \neq 0$

The performance of AM-DSB-SC is sensitive to the frequency error of the reference signal.

Pros and Cons of AM-DSB-SC

- Straightforward
-

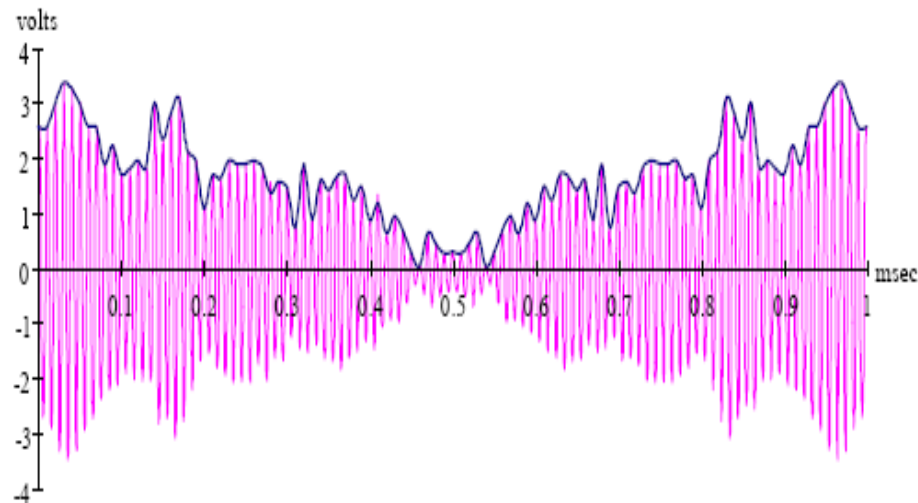
- Sensitive to frequency and phase error of the reference signal (coherent demodulation)
- Bandwidth inefficient ($\eta_{AM-DSB-SC}=50\%$)

AM-DSB-C

Envelope and Envelope Detector

- Envelope

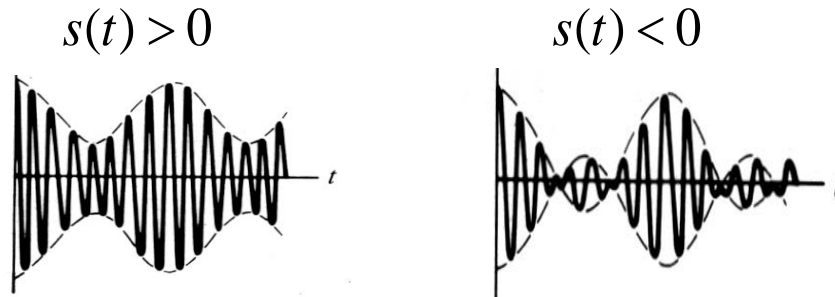
Consider a signal $s(t)\cos 2\pi f_c t$. If $s(t)$ varies slowly in comparison with the carrier $\cos 2\pi f_c t$, the **envelope** of $s(t)\cos 2\pi f_c t$ is $|s(t)|$.



Envelope and Envelope Detector

- Envelope

Consider a signal $s(t)\cos 2\pi f_c t$. If $s(t)$ varies slowly in comparison with the carrier $\cos 2\pi f_c t$, the **envelope** of $s(t)\cos 2\pi f_c t$ is $|s(t)|$.



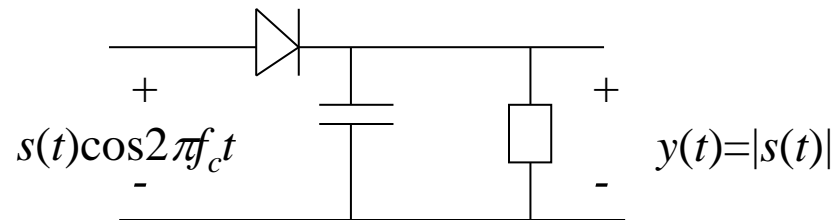
Envelope and Envelope Detector

- Envelope

Consider a signal $s(t)\cos 2\pi f_c t$. If $s(t)$ varies slowly in comparison with the carrier $\cos 2\pi f_c t$, the **envelope** of $s(t)\cos 2\pi f_c t$ is $|s(t)|$.

- The envelope $|s(t)| = s(t)$ if $s(t) \geq 0$.

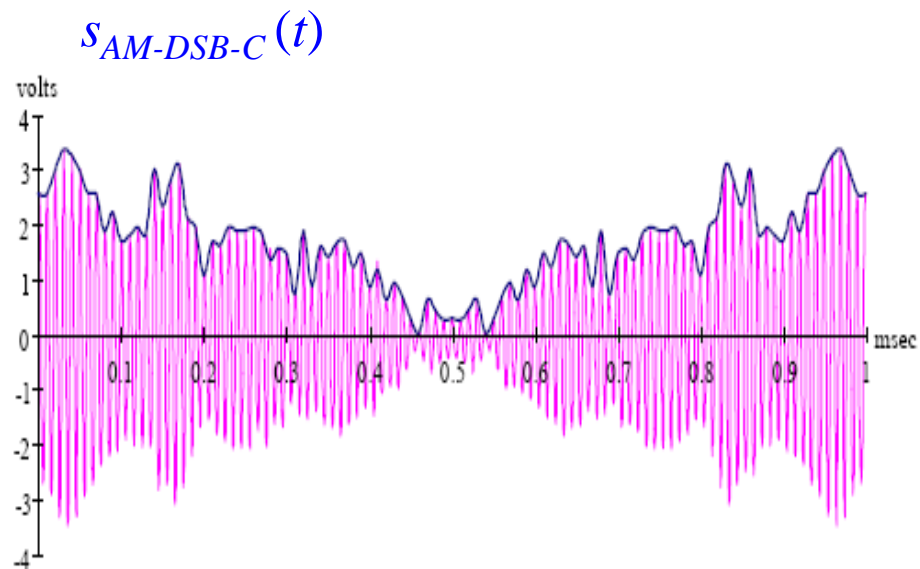
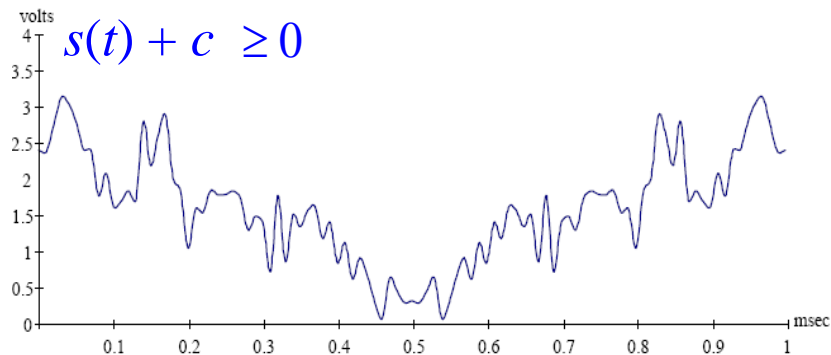
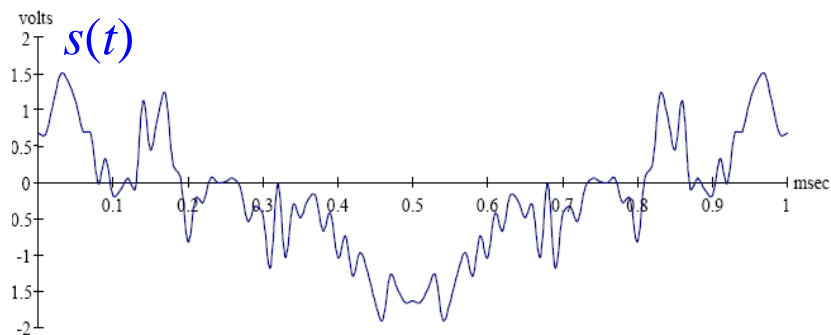
- Envelope Detector:



How to apply the Envelope Detector to AM systems?

AM-DSB-C -- Modulation

- Time Domain: $s_{AM-DSB-C}(t) = A(s(t) + c)\cos(2\pi f_c t)$



c is a dc offset to ensure $s(t) + c \geq 0$ for any time t .

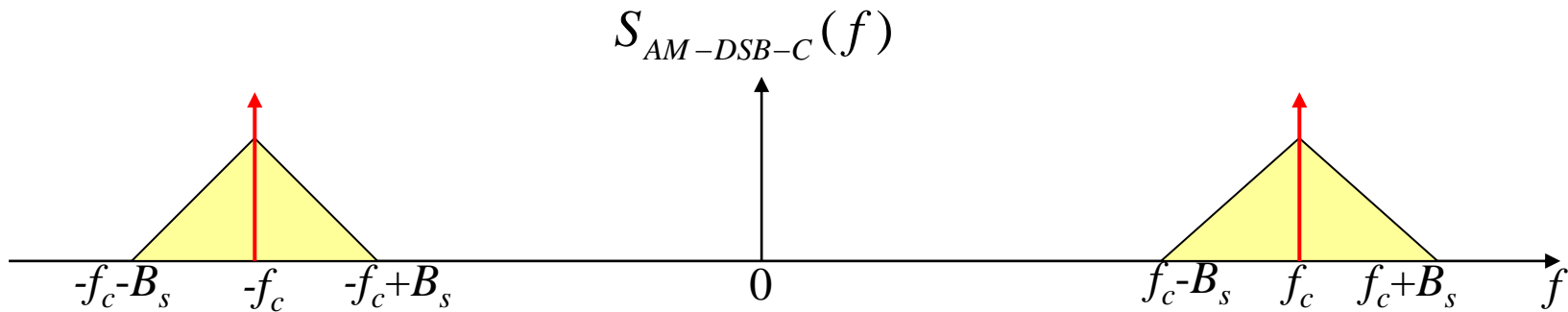
AM-DSB-C -- Modulation

- Frequency Domain:

$$s_{AM-DSB-C}(t) = A(s(t) + c)\cos(2\pi f_c t) = As(t)\cos(2\pi f_c t) + Ac\cos(2\pi f_c t)$$

\Leftrightarrow

$$S_{AM-DSB-C}(f) = \frac{A}{2}[S(f - f_c) + S(f + f_c)] + \frac{Ac}{2}[\delta(f - f_c) + \delta(f + f_c)]$$



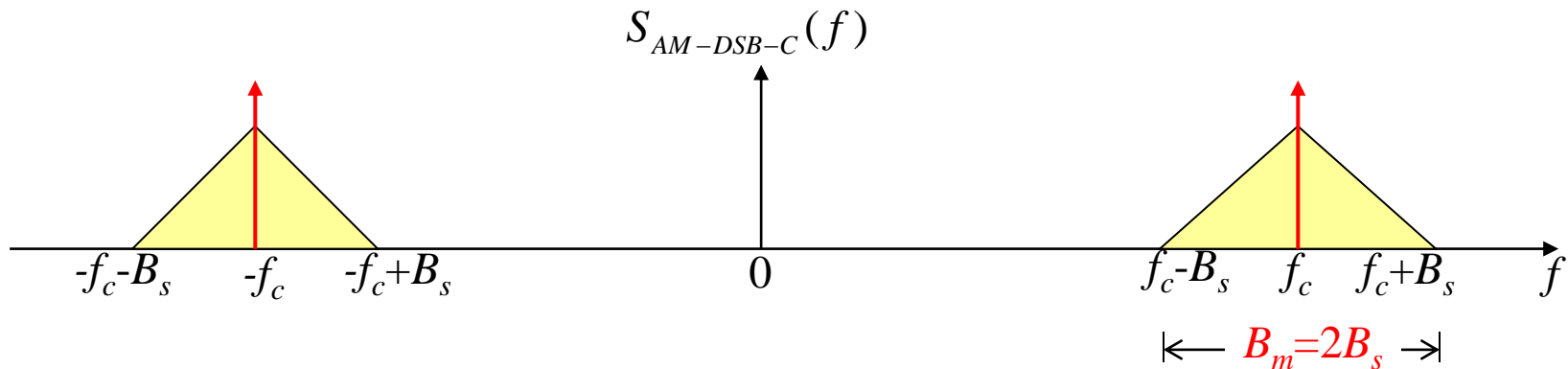
AM-DSB-C: Amplitude Modulation-Double SideBand-Carrier

Bandwidth Efficiency of AM-DSB-C

- Bandwidth Efficiency :

$$\gamma = \frac{\text{Information Signal Bandwidth } B_s}{\text{Required Channel Bandwidth } B_h} = \frac{\text{Information Signal Bandwidth } B_s}{\text{Modulated Signal Bandwidth } B_m}$$

With AM-DSB-C :

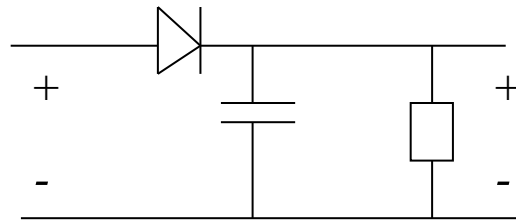


$$\gamma_{AM-DSB-C} = \frac{B_s}{2B_s} = 50\% = \gamma_{AM-DSB-SC}$$

AM-DSB-C -- Demodulation

- Time Domain: $s_{AM-DSB-C}(t) \Rightarrow s(t)$

Envelope Detector:



$$\begin{aligned}
 & s_{AM-DSB-C}(t) \\
 &= A(s(t) + c) \cos(2\pi f_c t)
 \end{aligned}$$

$$\begin{aligned}
 y(t) &= |A(s(t) + c)| = A(s(t) + c) \\
 & \quad (s(t) + c \geq 0)
 \end{aligned}$$

Non-coherent demodulation (no need to generate a reference signal)

- Apply $s_{AM-DSB-C}(t)$ to an envelope detector.
- Remove the dc offset c .

- Simple
- Robust
- Any price to pay?

More about AM-DSB-C

- Define the **power efficiency** of an AM-DSB-C system as:

$$\eta = \frac{\text{power of information signal } s(t)}{\text{power of modulating signal } s(t) + c} = \frac{P_s}{c^2 + P_s} \leq 50\%$$

– η increases as the dc offset c decreases.

– to ensure $s(t) + c \geq 0$, $c^2 \geq P_s$

- Define the **modulation index** of an AM-DSB-C system as:

$$m = \frac{\max[s(t) + c] - \min[s(t) + c]}{\max[s(t) + c] + \min[s(t) + c]} = \frac{\max s(t) - \min s(t)}{\max s(t) + \min s(t) + 2c}$$

– m increases as the dc offset c decreases.

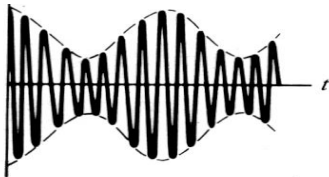
Can m be arbitrarily large?

Modulation Index m of AM-DSB-C

$$m = \frac{\max[s(t) + c] - \min[s(t) + c]}{\max[s(t) + c] + \min[s(t) + c]} = \frac{\max s(t) - \min s(t)}{\max s(t) + \min s(t) + 2c}$$

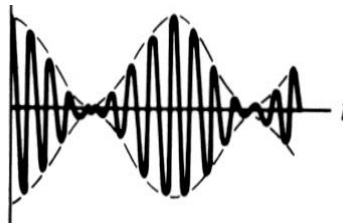
$$m < 1$$

when $\min(s(t) + c) > 0$



$$m = 1$$

when $\min(s(t) + c) = 0$



$$m > 1$$

when $\min(s(t) + c) < 0$



m should not exceed 1 to avoid over-modulation.

Pros and Cons of AM-DSB-C

- Simple and robust receiver design (non-coherent demodulation)



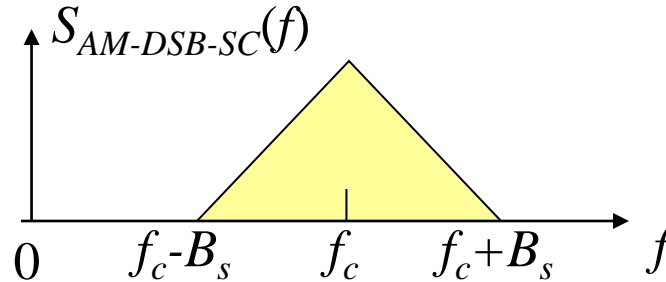
Commercial radio
broadcasting

- Cost of power efficiency ($\eta < 50\%$)
- Bandwidth inefficient ($\gamma_{AM-DSB-C} = \gamma_{AM-DSB-SC} = 50\%$)

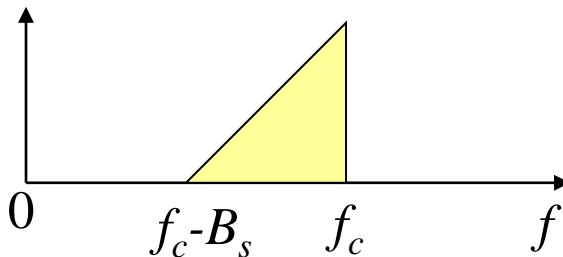
AM-SSB and AM-VSB

How to Improve Bandwidth Efficiency?

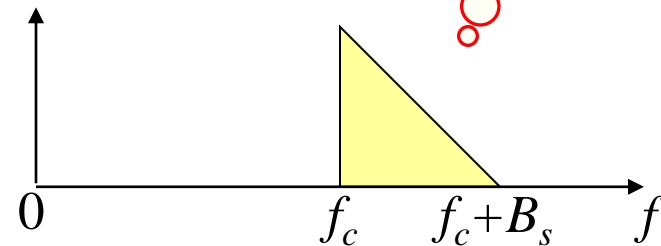
Double sideband
(DSB)



Upper sideband
carries the same
information as the
lower sideband!



Lower sideband



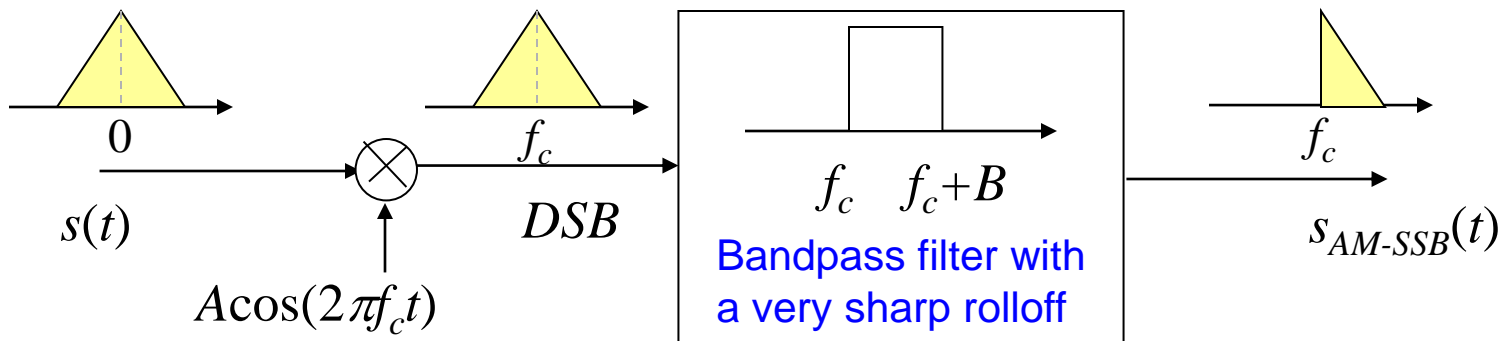
Upper sideband

Use a bandpass filter to select the desired sideband, and only transmit the desired sideband.

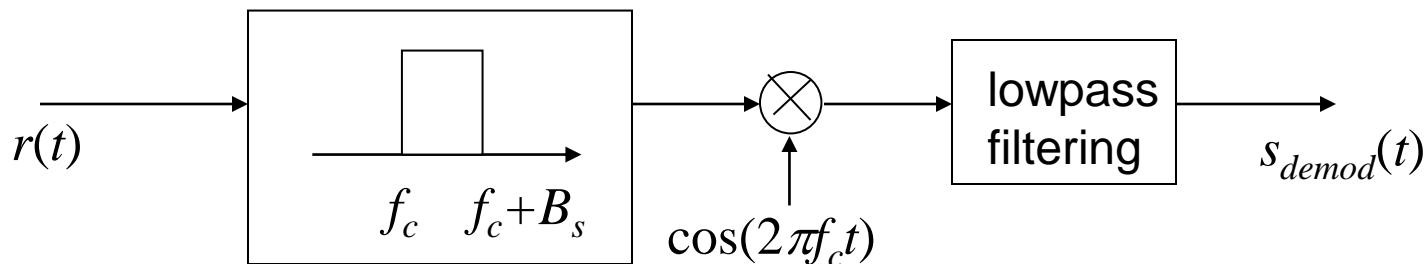
AM-SSB

Amplitude Modulation-Single SideBand (AM-SSB)

Modulation (Frequency Discrimination Method)



Demodulation (coherent)



Pros and Cons of AM-SSB

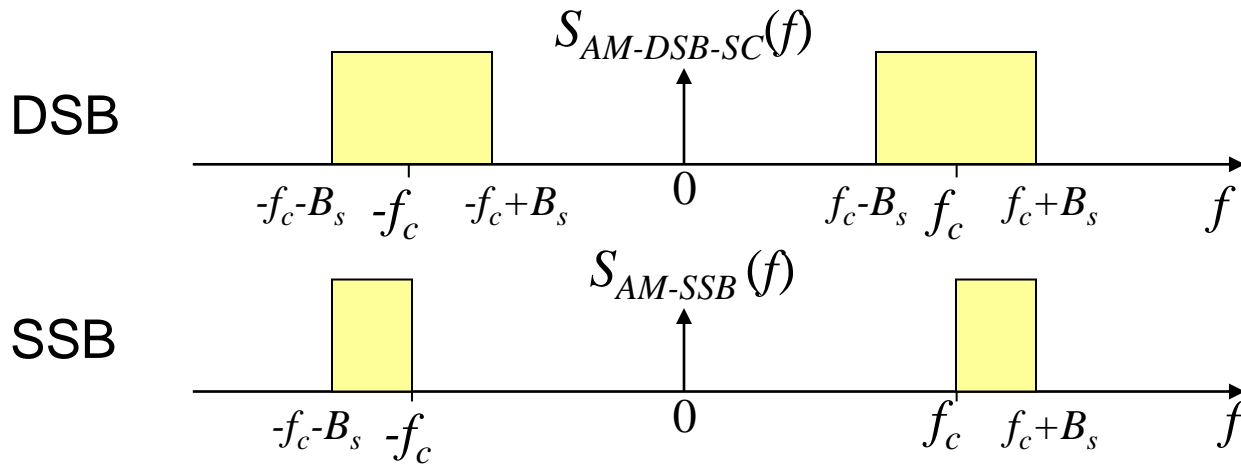
- Bandwidth efficient ($\gamma_{AM-SSB}=100\%$)



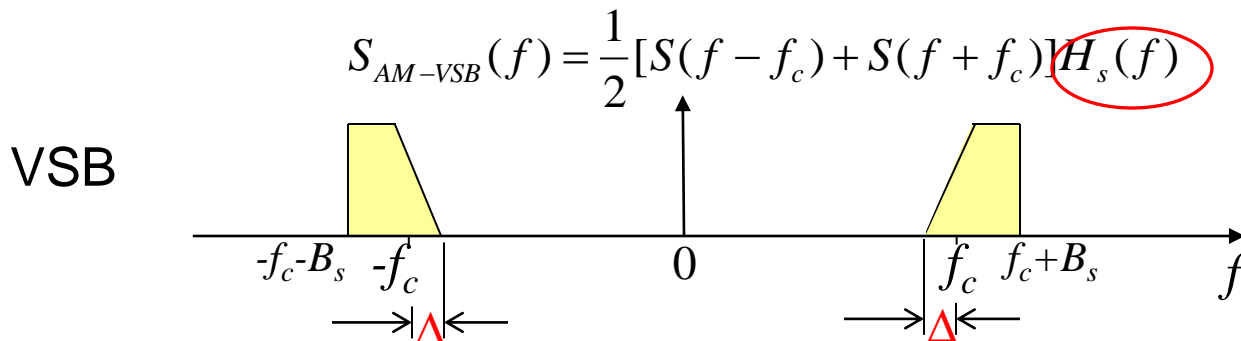
Mobile communications,
military communications, ...

-
- High requirement on filtering (sharp rolloff)
 - Sensitive to frequency and phase error of the reference signal (coherent demodulation)

AM-VSB



Amplitude Modulation-Vestigial SideBand (AM-VSB)



Allow a small portion (or *vestige*) of the lower sideband, Δ , along with the upper sideband.

Spectrum of AM-VSB Signal

modulation:

$$\begin{aligned}
 S_{AM-VSB}(f) &= \frac{1}{2}[S(f - f_c) + S(f + f_c)]H_s(f) \\
 &= \frac{1}{2}S(f - f_c)H_s(f) + \frac{1}{2}S(f + f_c)H_s(f)
 \end{aligned}$$

Demodulation:

$$\begin{aligned}
 S_{\text{demod}}(f) &= \frac{1}{2}[S_{AM-VSB}(f - f_c) + S_{AM-VSB}(f + f_c)] \\
 &= \frac{1}{4}[S(f - 2f_c) + S(f)]H_s(f - f_c) + \frac{1}{4}[S(f) + S(f + 2f_c)]H_s(f + f_c) \\
 &= \frac{1}{4}S(f)[H_s(f - f_c) + H_s(f + f_c)] + \frac{1}{4}S(f - 2f_c)H_s(f - f_c) + \frac{1}{4}S(f + 2f_c)H_s(f + f_c)
 \end{aligned}$$

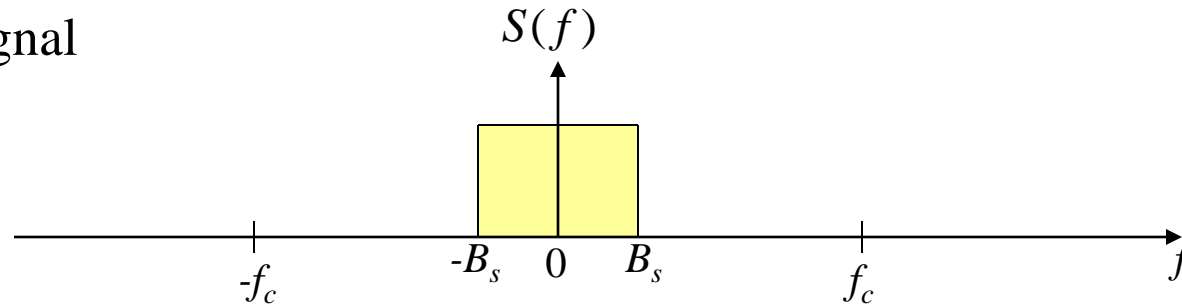
After lowpass filtering:

$$S_{\text{demod}}(f) = S(f)[H_s(f - f_c) + H_s(f + f_c)]$$

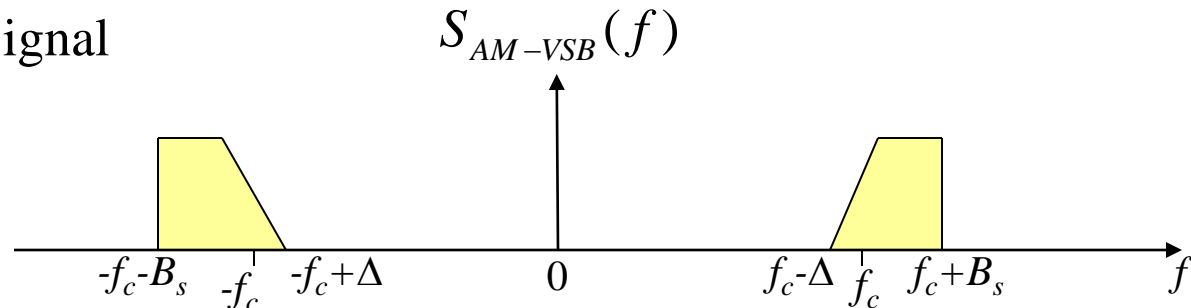
$H_s(f - f_c) + H_s(f + f_c) = k \quad 0 \leq |f| \leq B$

Spectrum of AM-VSB Signal

Baseband signal

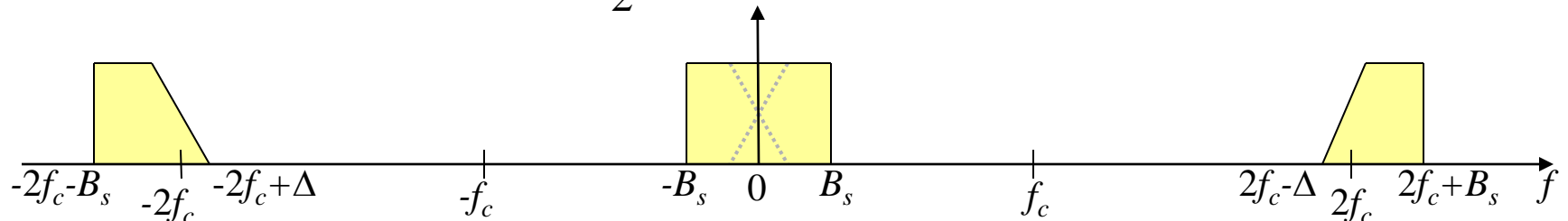


Modulated Signal

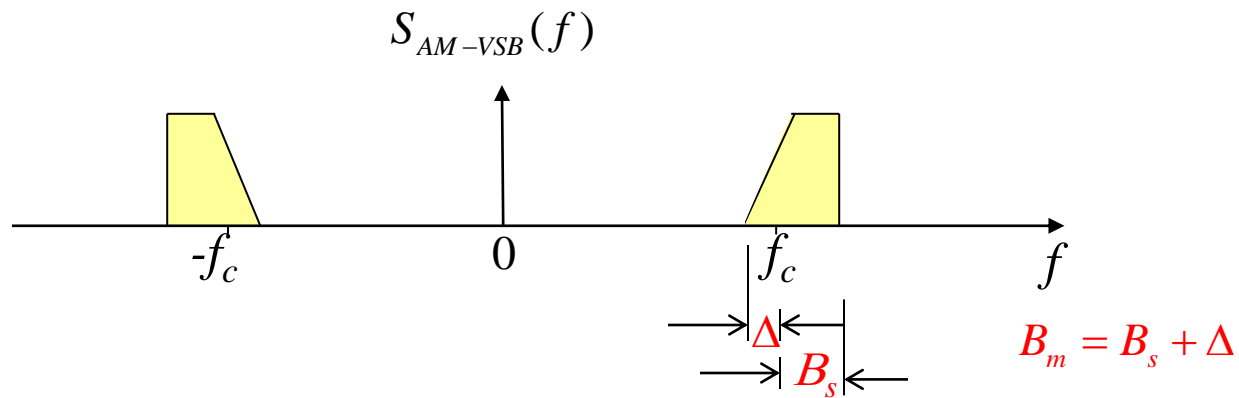


De-modulation

$$S_{\text{demod}}(f) = \frac{1}{2} [S_{AM-VSB}(f - f_c) + S_{AM-VSB}(f + f_c)]$$



Tradeoff between Complexity and Bandwidth Efficiency



$$50\% < \gamma_{AM-VSB} = \frac{B_s}{B_s + \Delta} < 100\%$$

$$0 < \Delta < B_s$$

- The bandwidth efficiency γ decreases as the vestige Δ increases.
- The larger vestige Δ , the lower receiver complexity.

Summary of AM

AM-DSB-SC	Power efficient	Coherent demodulation	Bandwidth inefficient $\gamma=50\%$
AM-DSB-C Commercial radio broadcasting	Power inefficient	Non-coherent demodulation (simple and robust)	Bandwidth inefficient $\gamma=50\%$
AM-SSB Mobile and military communications	Power efficient	Coherent demodulation	Bandwidth efficient $\gamma=100\%$
AM-VSB Public television systems	Power efficient	Coherent demodulation	Tradeoff between bandwidth and complexity $50\% < \gamma = \frac{B_s}{B_s + \Delta} < 100\%$