EE3331 Probability Models in Information Engineering

Semester B 2022-2023

Assignment 1

Due Date: 6 February 2023
Important Note: Only writing the answers without steps will get zero mark.

1. An integrated circuit factory has 3 machines, namely, A, B and C. One integrated circuit produced by each machine is tested, which is either "pass" or "fail", denoted by P or F, respectively. An observation is a sequence of 3 test results corresponding to the circuits from A, B and C. For example, the observed sequence PPF means that the circuits from A and B pass the test while the circuit from C fails the test.
(a) Determine the sample space.
(b) Determine the elements of the following sets:
$$U = \{\text{circuit from } C \text{ fails}\}$$
$$V = \{\text{circuit from } A \text{ passes}\}$$
$$X = \{\text{more than one circuit pass}\}$$
$$Y = \{\text{at least two circuits fail}\}$$
(c) Are U and V mutually exclusive?
(d) Are X and Y mutually exclusive?

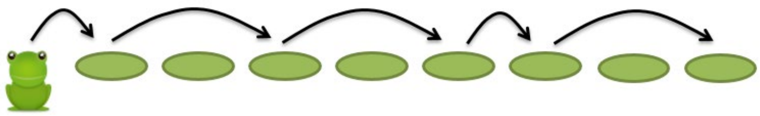
2. Members of a book club only read the following: mysteries, graphic novels and/or epic fiction. Denote the events of reading mysteries, graphic novels and epic fiction as M, G and E, respectively. Suppose $P(M) = 0.6$, $P(G) = 0.4$, $P(E) = 0.3$, $P(M \cap G) = 0.2$, and no one reads both graphic novels and epic fiction.
(a) Find $P(M \cup G)$
(b) Find $P((M \cap G) \cup (M \cap E))$
(c) Are the events $\overline{G}$ and M independent? Explain your answer.

3. Suppose there are four traffic lights on the way from your home to campus. It is assumed that when you encounter a red signal, you must stop 50s, 50s, 30s and 20s for the first, second, third, and fourth light, respectively. In addition, the probabilities of encountering a red signal and green signal (no waiting time) are both 0.5.
(a) Determine the sample space of the waiting time due to the four traffic lights.
(b) Compute the probabilities of all possible outcomes in (a).

4. Cindy has two coins. One is fair, with a head on one side and a tail on the other. The second is a trick coin and has a tail on both sides. She picks up one of the coins at random and flips it.
(a) Find the probability that the result is a head.
(b) Given that the result is a tail, find the probability that she picked up the fair coin.

5. Box A contains 3 red balls and 6 green balls, while box B contains 3 red balls and 1 green ball. You firstly pick one of the boxes at random, then draw a ball multiple times, with replacement of balls after each draw. If red ball is continuously drawn k times, what is the probability that you picked box B?

6. A frog has 8 lily pads in front of her and she can either hop to the next lily pad, or skip one and go to the next. Denote H and S as "hop" and "skip one", respectively. For example, a possible sequence is HSSHS:



Determine the number of possible sequences for the frog to get to the final lily pad.

7. There are 6 people in a room. How many ways are there of forming two teams of people, where each team must have at least one person?

8. Three dices are rolled. What is the probability that the sum of three dices is 8?

9. Alan has 5 different comics books, 4 different novels and 2 different textbooks on a shelf. How many ways are there to arrange the books if he wants to keep the comics books together, the novels together, and the textbooks together?

10. For random events A, B, C, prove $P(AB) + P(AC) - P(BC) \leq P(A)$. Hint: Start with $P(AB) = P(AB\overline{C}) + P(ABC)$.

11. According to the previous analysis of the examination results in a class, students who study hard pass the course with probability of 95%, while students who do not study hard fail the course with probability of 90%. Moreover, 90% of students study hard.
    (a) If a student who passes the course is chosen from the class, what is the probability that he/she does not study hard?
    (b) If a student who fails the course is chosen from the class, what is the probability that he/she is hard-working?

12. Consider travelling from bottom-left hand corner to the top-right corner of a $N \times N$ grid, each time only making up or right movements. Determine the total number of possible paths in terms of N. For example, a possible path for a 4×4 grid is:

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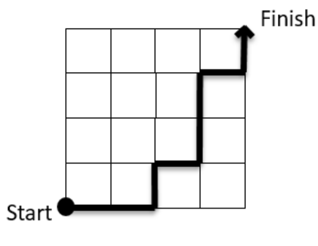

13. A game-show host offers Peter the choices of 4 doors. Behind one of these doors is a luxury Rolex watch, and behind the other three are balloons. The host, who knows what is behind each of the doors, announces that after Peter selects a door without opening it, he will open one of the other three doors corresponding to the balloons. Suppose Peter selected a door, and the host then opened one of the other doors and showed the balloons. Now the host offers Peter the chance to switch his choice to the remaining two doors.
    (a) If Peter does not switch his choice, what is the probability of getting the watch? Explain your answer.
    (b) How about the probability of the getting the watch if Peter switches his choice? Explain your answer.