EE5410 Signal Processing

Solution for Assignment 2

1.(a)

Using the relationship between z transform and DTFT, i.e., $z = e^{j\omega}$, we get:

$$H(e^{j\omega}) = \frac{e^{-2j\omega}}{1+0.7e^{-j\omega}+0.1e^{-2j\omega}} \Rightarrow H(z) = \frac{z^{-2}}{1+0.7z^{-1}+0.1z^{-2}} = \frac{z^{-2}}{(1+0.5z^{-1})(1+0.2z^{-1})}$$

As the DTFT converges, the ROC should include the unit circle. Hence the ROC is $|z| > 0.5$.

1.(b)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-2}}{1+0.7z^{-1}+0.1z^{-2}} \Rightarrow y[n]+0.7y[n-1]+0.1y[n-2] = x[n-2]$$

1.(c)

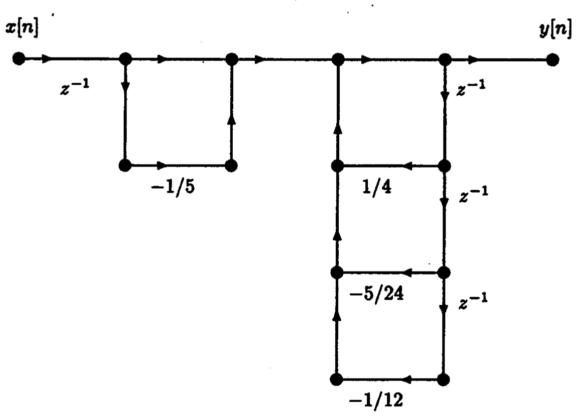
$$H(z) = -\frac{10}{3}\frac{z^{-1}}{1+0.5z^{-1}} + \frac{10}{3}\frac{z^{-1}}{1+0.2z^{-1}}$$

With the use of time-shifting property and ROC of $|z| > 0.5$, we get:

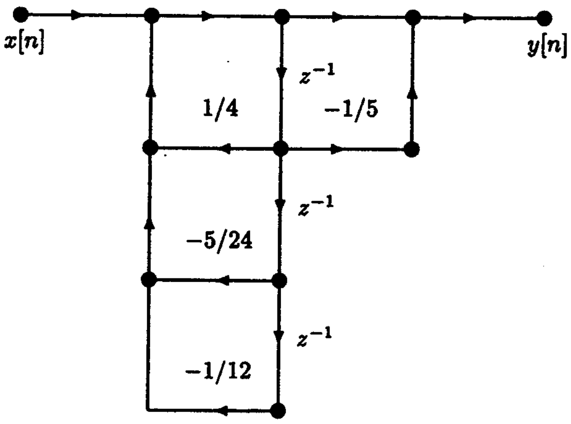
$$h[n] = -\frac{10}{3}\left(-\frac{1}{2}\right)^{n-1}u[n-1] + \frac{10}{3}\left(-\frac{1}{5}\right)^{n-1}u[n-1]$$

2.(a)

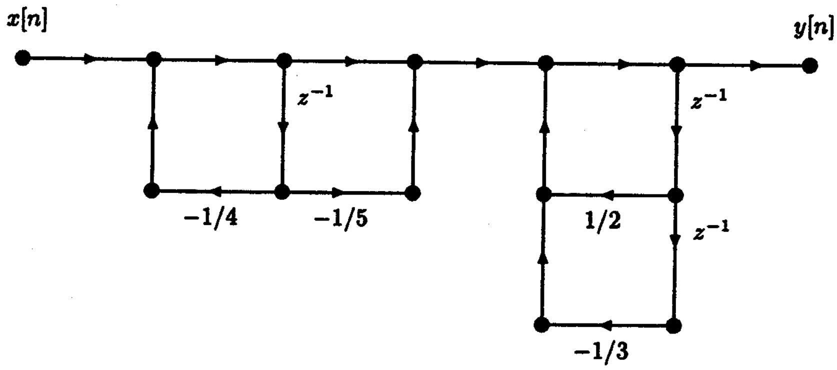
$$H(z) = \frac{1-\frac{1}{5}z^{-1}}{1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3}}$$



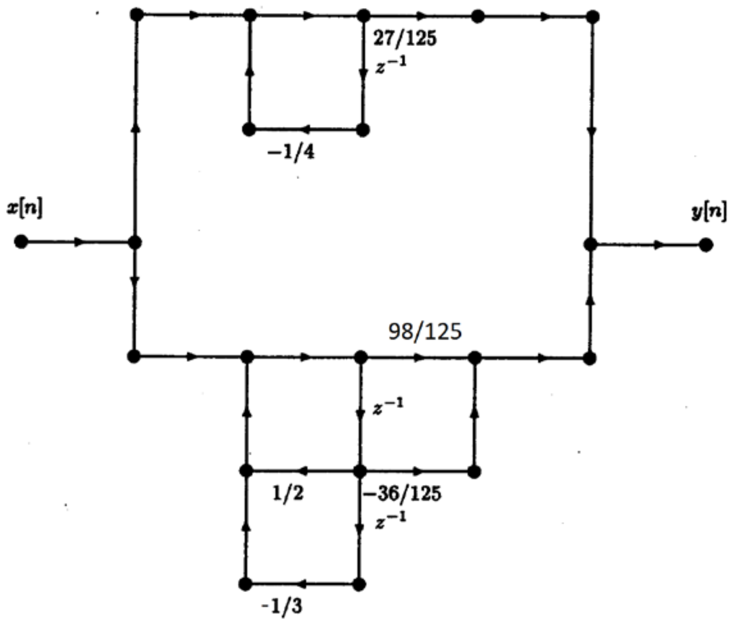
2.(b)



2.(c) $H(z)=\left(\dfrac{1-\frac{1}{5}z^{-1}}{1+\frac{1}{4}z^{-1}}\right)\left(\dfrac{1}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}\right)$

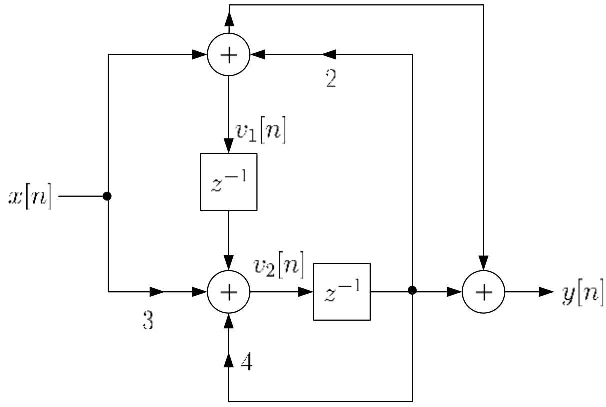


2.(d) $H(z)=\dfrac{\frac{27}{125}}{1+\frac{1}{4}z^{-1}}+\dfrac{\frac{98}{125}-\frac{36}{125}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}$



3.(a)

We introduce two intermediate signals $v_1[n]$ and $v_2[n]$ which are the outputs of unit delays.



We have:

$$v_1[n-1] + 3x[n] + 4v_2[n-1] = v_2[n]$$
$$x[n] + 2v_2[n-1] = v_1[n]$$
$$y[n] = v_1[n] + v_2[n-1]$$

Taking their z transforms yields

$$z^{-1}V_1(z) + 3X(z) + 4z^{-1}V_2(z) = V_2(z)$$
$$X(z) + 2z^{-1}V_2(z) = V_1(z)$$
$$Y(z) = V_1(z) + z^{-1}V_2(z)$$

From the first two equations, we can solve $V_2(z)$ in terms of $X(z)$ as:

$$V_2(z) = \frac{3+z^{-1}}{1-4z^{-1}-2z^{-2}}X(z)$$

Then $V_1(z)$ is computed as:

$$V_2(z) = \frac{1+2z^{-1}}{1-4z^{-1}-2z^{-2}}X(z)$$

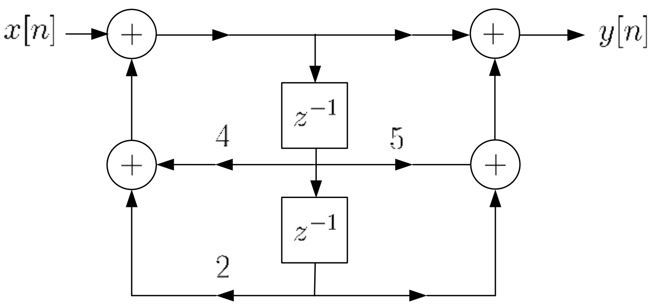
Putting the result into the third equation, we get

$$Y(z) = \left(\frac{1+2z^{-1}}{1-4z^{-1}-2z^{-2}} + \frac{3z^{-1}+z^{-2}}{1-4z^{-1}-2z^{-2}}\right)X(z) \Rightarrow \frac{Y(z)}{X(z)} = \frac{1+5z^{-1}+z^{-2}}{1-4z^{-1}-2z^{-2}}$$

3.(b)

$$Y(z)\left(1-4z^{-1}-2z^{-2}\right) = X(z)\left(1+5z^{-1}+z^{-2}\right)$$
$$\Rightarrow y[n] - 4y[n-1] - 2y[n-2] = x[n] + 5x[n-1] + x[n-2]$$

3.(c)



3.(d)
The system is not stable.
Solving the quadratic equation, the poles of the filter are 4.45 and -0.45. If the system is stable, the ROC should include the unit circle. However, as the system is causal, the ROC is $|z| > 4.45$, which does not include the unit circle.

4.(a)
According to (10.26), the ideal impulse response is:

$$\begin{aligned} h_d[n] &= \frac{\omega_b}{\pi}\text{sinc}\left(\frac{\omega_b n}{\pi}\right) - \frac{\omega_a}{\pi}\text{sinc}\left(\frac{\omega_a n}{\pi}\right) \\ &= 0.75\text{sinc}(0.75n) - 0.25\text{sinc}(0.25n) \end{aligned}$$

For a filter of length 5, we extract $h_d[-2]$, $h_d[-1]$, $h_d[0]$, $h_d[1]$ and $h_d[2]$. The causal filter impulse response is then:

$h[n] = h_d[n-2] = [-0.3183\ 0\ 0.5\ 0\ \ -0.3183]$

With the first element being $h[n]$ at *n* = 0. As a result, $H_1(z)$ is:

$$H_1(z) = -0.3183 + 0.5z^{-2} - 0.3183z^{-4}$$

4.(b)
In the implementation, we make use of the symmetry of the impulse response:

$$y[n] = -0.3183(x[n] + x[n-4]) + 0.5x[n-2]$$

As a result, the minimum numbers of additions and multiplications are both equal to 2.