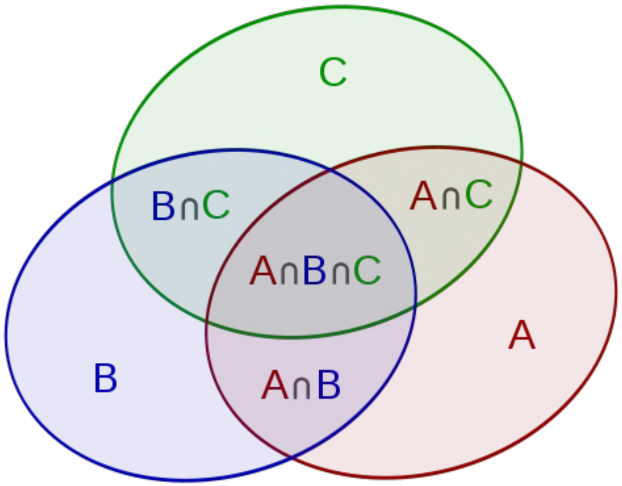
Tutorial 1 (with solution)

Sets

Question 1: Inclusion & Exclusion

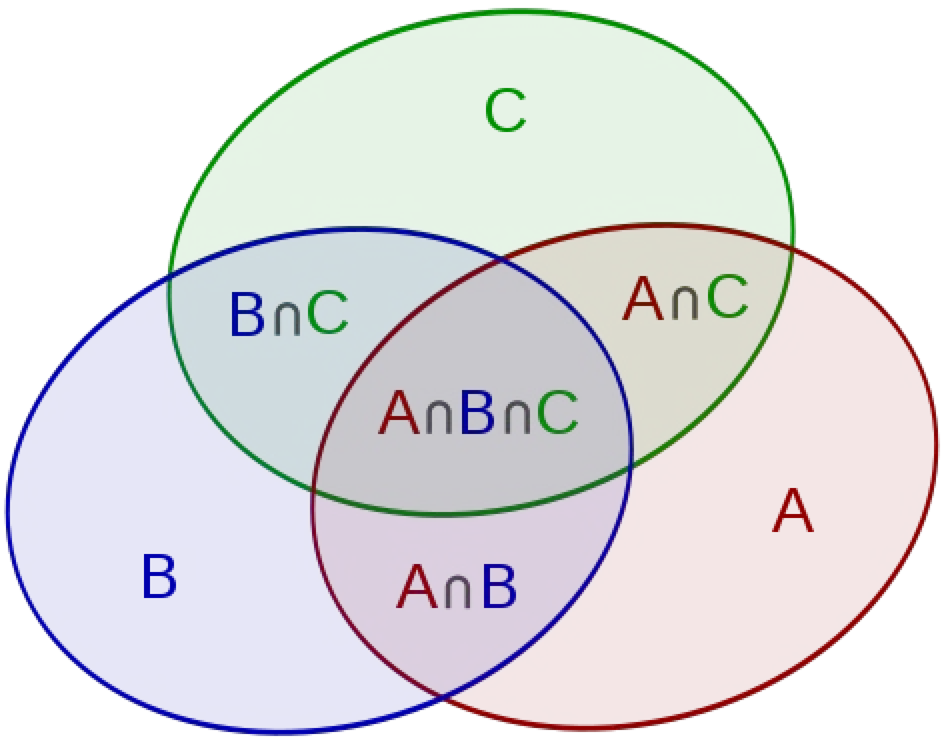
- What is the formula for $|A \cup B \cup C|$? page 1-12

a) $|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
b) $|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + 3|A \cap B \cap C|$
c) $|A| + |B| + |C| - 2|A \cap B| - 2|A \cap C| - 2|B \cap C| + 3|A \cap B \cap C|$
d) $|A| + |B| + |C| - 3|A \cap B| - 3|A \cap C| - 3|B \cap C| + 3|A \cap B \cap C|$



Q.1 Solution

$$|A|+|B|+|C|-|A\cap B|-|A\cap C|-|B\cap C|+|A\cap B\cap C|$$



Q.1 Extension

❑ Inclusion–exclusion principle

For n finite sets $A_1, A_2, \dots, A_n$, the cardinality of the union of n sets:

$$|A_1 \cup \cdots \cup A_n| = \sum_{i=1}^{n} |A_i| - \sum_{1 \le i < j \le n} |A_i \cap A_j| + \sum_{1 \le i < j < k \le n} |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n-1} |A_1 \cap \cdots \cap A_n|$$

i. Include the cardinalities of n sets.
ii. Exclude the cardinalities of the pairwise intersections.
iii. Include the cardinalities of the triple-wise intersections.
iv. Exclude the cardinalities of the quadruple-wise intersections.
v. Continue, until the cardinality of the n-tuple-wise intersection is included (if n is odd) or excluded (n even).

Question 2: Subset Relationship

Let $A = \{n \in \mathbf{Z} \mid n = 5r \text{ for some integer } r\}$

and $B = \{m \in \mathbf{Z} \mid m = 20s \text{ for some integer } s\}$.

i. Is $A \subseteq B$?

ii. Is $B \subseteq A$?

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a) Both are true.
b) Both are false.
c) (i) is true while (ii) is false
d) (i) is false while (ii) is true

Q.2 Solution

i. No.
   - This can be proved by a counter-example.
   - For example, $5 \in A$ (since $5 = 5r$, where $r = 1$).
   - But 5 cannot be written as $20s$, where s is an integer.
   - So, 5 is not an element of B.
   - Therefore, $A \nsubseteq B$.

ii. Yes.
   - Let $n \in B$, so $n = 20s$, where s is an integer.
   - Since $n = 20s = 5(4s)$, where $4s$ is an integer, $n \in A$.
   - Therefore, $B \subseteq A$.

Q.3 Cartesian Product

- Consider two nonempty sets A and B. page 1-14

- Is it true that $A \times B \neq B \times A$?

  a) Yes
  b) No
  c) Cannot be determined

Justify your answer.

Q.3 Solution

- It cannot be determined.

- If $A = B$, then $A \times B = B \times A$.

- If $A \neq B$, then $A \times B \neq B \times A$.
  - For example, $A = \{a\}$ and $B = \{1, 2\}$.
  - $A \times B = \{(a, 1), (a, 2)\}$.
  - $B \times A = \{(1, a), (2, a)\}$.

Question 4: Union and Intersection

Let $R_j = \left\{x \in \mathbb{R} \mid 1 \leq x \leq 1 + \frac{1}{j}\right\} = \left[1, 1 + \frac{1}{j}\right]$.

i. What is $\bigcup_{j=1}^{4} R_j$?

ii. What is $\bigcap_{j=1}^{4} R_j$?

iii. Are they mutually disjoint? page 1-11

iv. What is $\bigcup_{j=1}^{\infty} R_j$?

v. What is $\bigcap_{j=1}^{\infty} R_j$?

Q.4 Solution

i. $[1, 2]$

ii. $\left[1, 1 + \frac{1}{4}\right]$

iii. No, because 1 is in all the four sets.

iv. $[1, 2]$

v. $\{1\}$