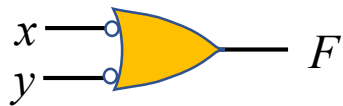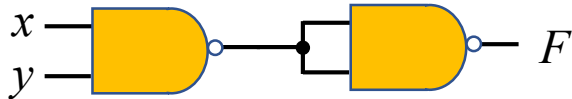# Exercise

Work out the Boolean functions of the following circuits. Which standard logic gate does each of them represent?
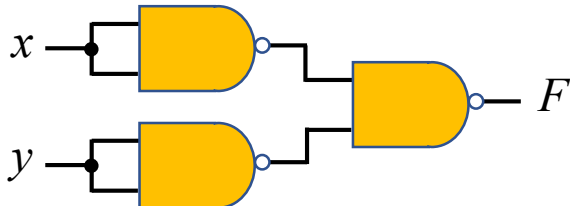

$$F = x' + y' = (xy)'$$
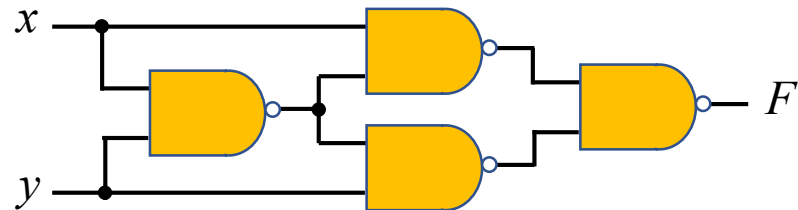
$$F = (xx)' = x'$$

$$F = [(xy)']' = xy$$

$$F = (x'y')' = x'' + y'' = x + y$$

$$F = \{[x(xy)']'[y(xy)']'\}'$$
$$= [x(xy)']'' + [y(xy)']''$$
$$= x(xy)' + y(xy)'$$
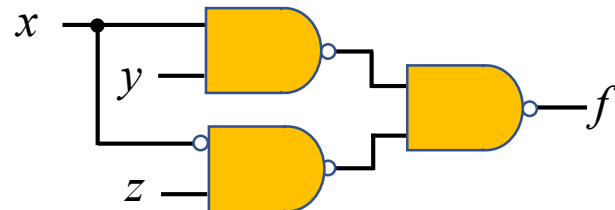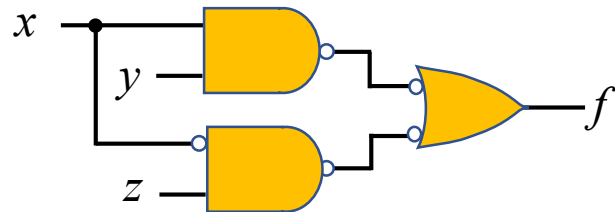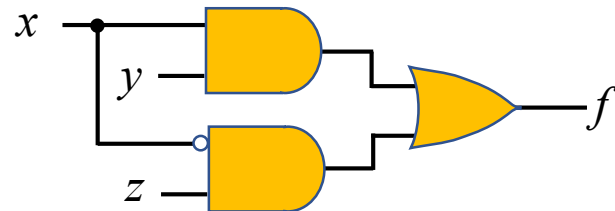$$= x(x' + y') + y(x' + y')$$
$$= xy' + yx' = x \oplus y$$

Exercise

Inputs			Output
x	y	z	$f(x, y, z)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

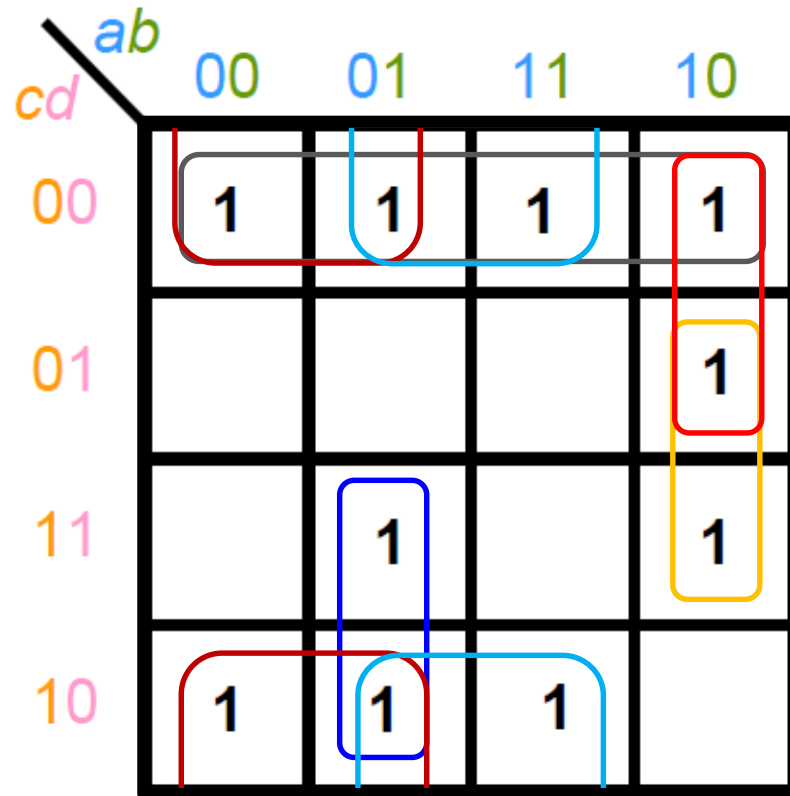$$f(x, y, z) = \sum m(1,3,6,7) = \prod M(0,2,4,5)$$

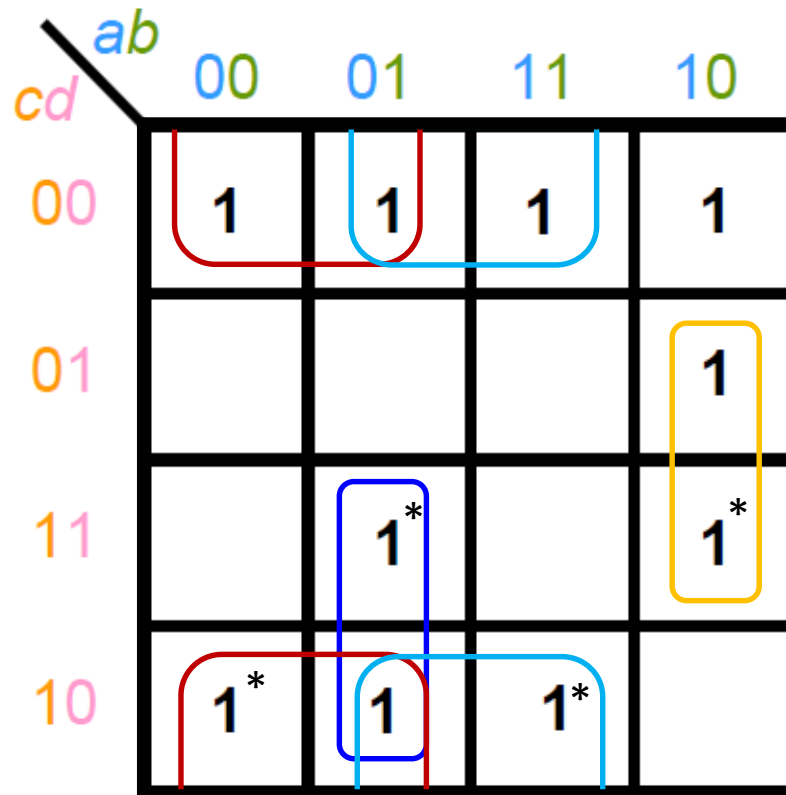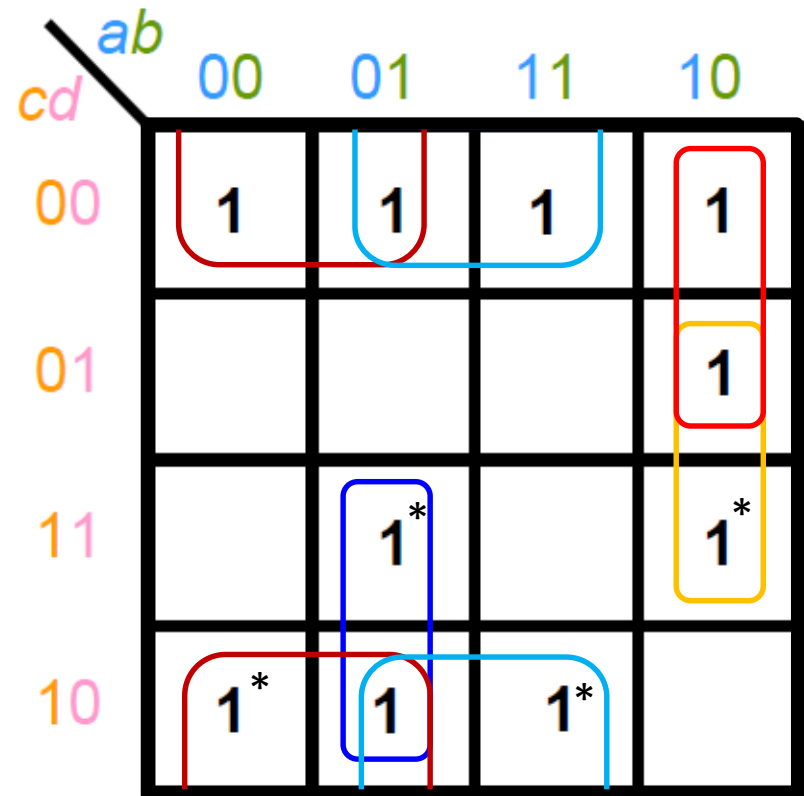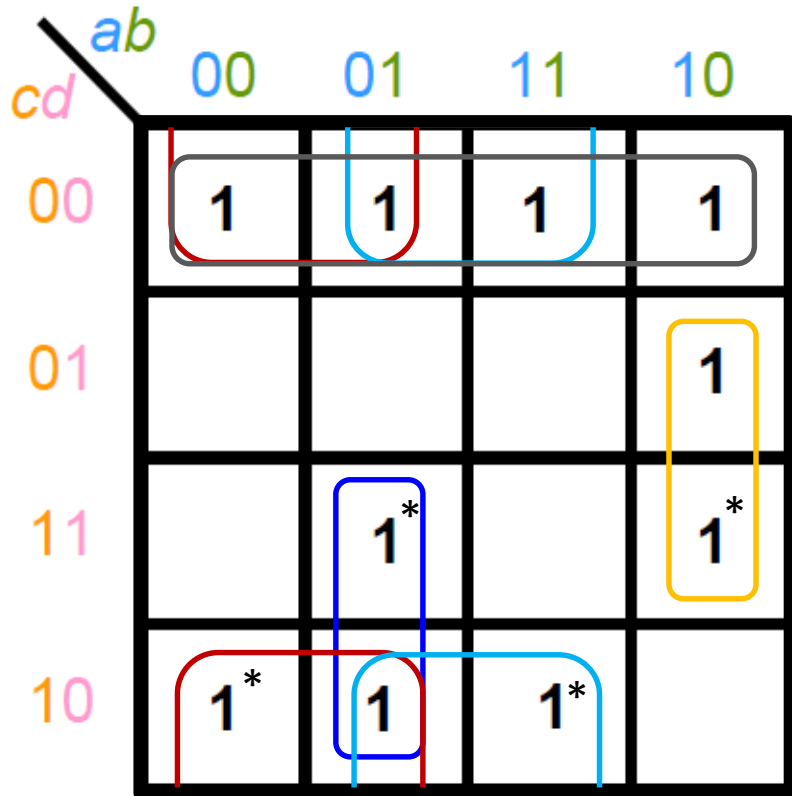$$f(x, y, z) = x'y'z + x'yz + xyz' + xyz$$
$$= x'z + xy$$

Exercise

1. Identify all PIs.
2. Select all EPIs.
3. Add PIs of remaining minterms.

Exercise

1. Identify all PIs.
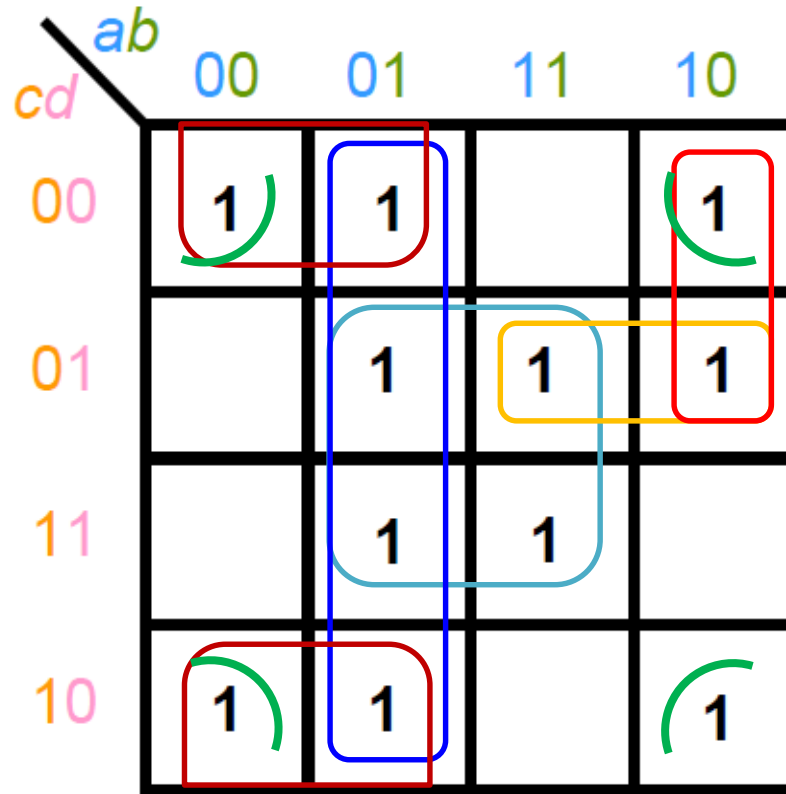2. Select all EPIs.
3. Add PIs of remaining minterms.

Exercise

1. Identify all PIs.
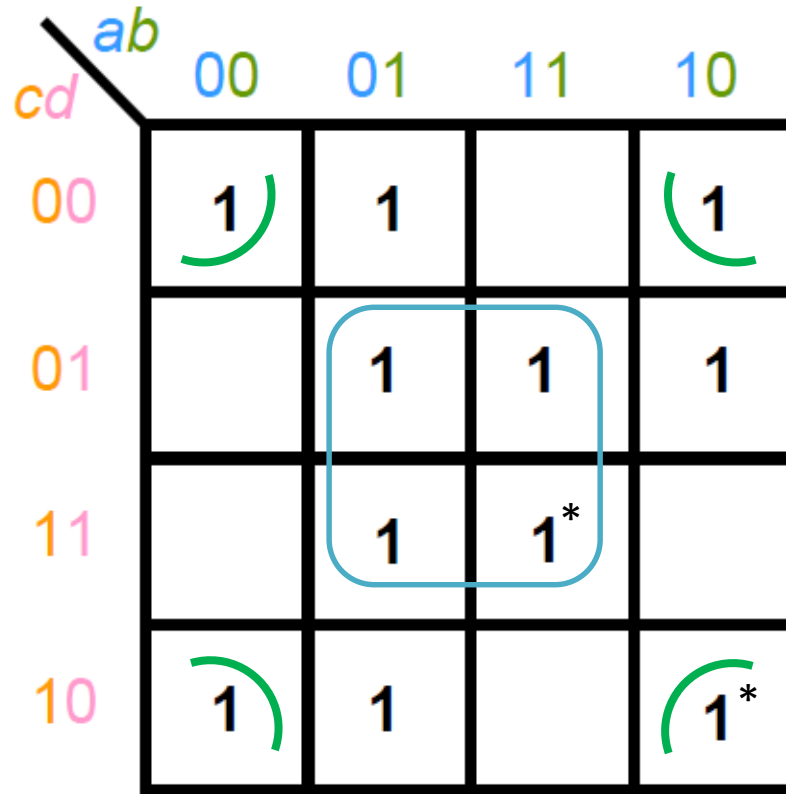2. Select all EPIs.
3. Add PIs of remaining minterms.

Exercise

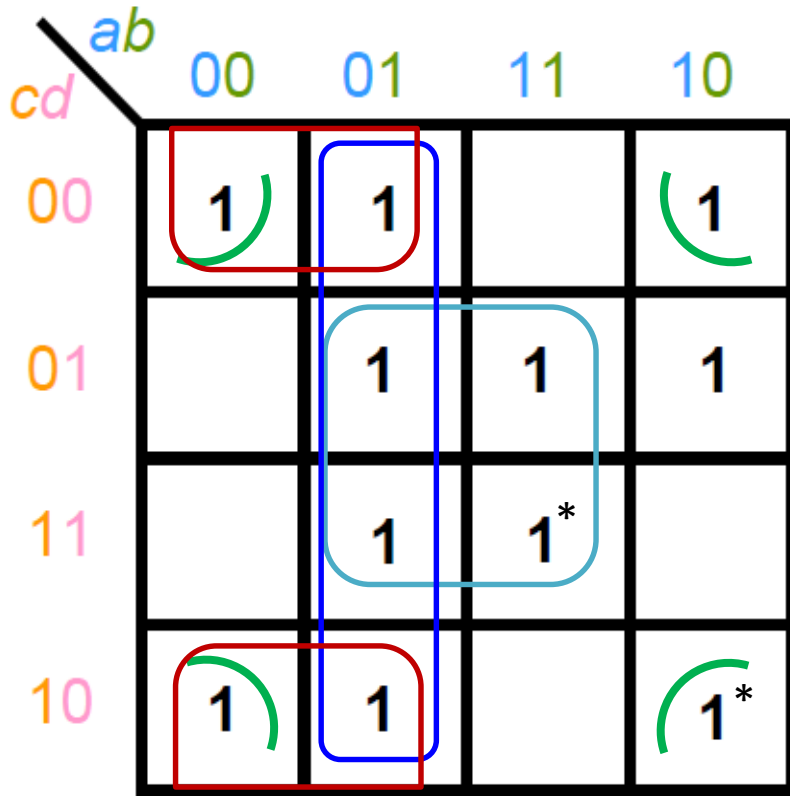Find all minimum sum of products expressions for the following K-map.

Exercise

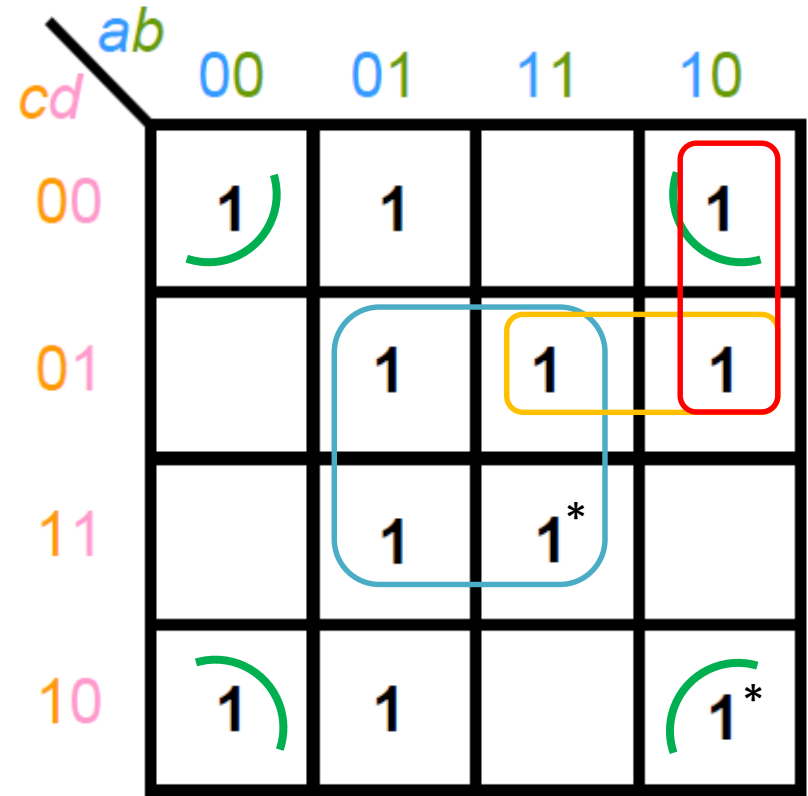Find all minimum sum of products expressions for the following K-map.

Exercise

$$f = bd + b'd' + a'd' + ac'd$$
$$f = bd + b'd' + a'b + ac'd$$

$$f = bd + b'd' + a'd' + ab'c'$$
$$f = bd + b'd' + a'b + ab'c'$$

Exercise

Find all minimum sum of products and all minimum product of sums expressions for the following Boolean Function.

$$f(a, b, c, d) = \sum m(1,3,4,6,11) + \sum d(0,8,10,12,13)$$

ab \ cd	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

ab \ cd	00	01	11	10
00	X	1	X	X
01	1		X	
11	1			1
10		1		X

Exercise

Exercise

$$f = a'bd' + a'b'd + b'cd$$

$$f = a'bd' + a'b'c' + b'cd$$

$$f = a'bd' + a'b'd + ab'c$$

Exercise

Find all minimum sum of products and all minimum product of sums expressions for the following Boolean Function.

Exercise

Find all minimum sum of products and all minimum product of sums expressions for the following Boolean Function.

Exercise

$$f' = bd + b'd' + ac' + ab$$

$$f = (b' + d')(b + d)$$
$$(a' + c)(a' + b')$$

$$f' = bd + b'd' + ac' + ad'$$

$$f = (b' + d')(b + d)$$
$$(a' + c)(a' + d)$$

Exercise (Don't Care Case)

Step 1-3 (Partition, Combine, List PIs): Include Don't Care minterms

Simplify $f(a, b, c, d) = \Sigma m(4, 8, 9, 10, 12, 15) + \Sigma d(2, 6, 13)$

Minterms	$abcd$
m_2	0010 ✔
m_4	0100 ✔
m_8	1000 ✔
m_6	0110 ✔
m_9	1001 ✔
m_{10}	1010 ✔
m_{12}	1100 ✔
m_{13}	1101 ✔
m_{15}	1111 ✔

Minterms	$abcd$
m_2, m_6	0-10 PI_2
m_2, m_{10}	-010 PI_3
m_4, m_6	01-0 PI_4
m_4, m_{12}	-100 PI_5
m_8, m_9	100- ✔
m_8, m_{10}	10-0 PI_6
m_8, m_{12}	1-00 ✔
m_9, m_{13}	1-01 ✔
m_{12}, m_{13}	110- ✔
m_{13}, m_{15}	11-1 PI_7

Minterms	$abcd$
m_8, m_9, m_{12}, m_{13}	1-0- PI_1

Exercise (Don't Care Case)

Step 4: Generate PI chart

- Exclude Don't Care Minterms

Simplify $f(a, b, c, d) = \Sigma m(4, 8, 9, 10, 12, 15) + \Sigma d(2, 6, 13)$

PI	Minterms	$abcd$	4	8	9	10	12	15
PI_1	m_8, m_9, m_{12}, m_{13}	1-0-		x	x		x	
PI_2	m_2, m_6	0-10						
PI_3	m_2, m_{10}	-010				x		
PI_4	m_4, m_6	01-0	x					
PI_5	m_4, m_{12}	-100	x				x	
PI_6	m_8, m_{10}	10-0		x		x		
PI_7	m_{13}, m_{15}	11-1						x

Step 5-6: Reduce PI chart & express the Boolean Function

Exercise (Don't Care Case)

Step 5-6: Reduce PI chart & express the Boolean Function

PI	Minterms	$abcd$	4	8	9	10	12	15
PI$_1$	m_8, m_9, m_{12}, m_{13}	1-0-		x	x		x	
PI$_2$	m_2, m_6	0-10						
PI$_3$	m_2, m_{10}	-010				x		
PI$_4$	m_4, m_6	01-0	x					
PI$_5$	m_4, m_{12}	-100	x				x	
PI$_6$	m_8, m_{10}	10-0		x		x		
PI$_7$	m_{13}, m_{15}	11-1						x

PI	Minterms	$abcd$	4	10
PI$_3$	m_2, m_{10}	-010		x
PI$_4$	m_4, m_6	01-0	x	
PI$_5$	m_4, m_{12}	-100	x	
PI$_6$	m_8, m_{10}	10-0		x

$$f(a, b, c, d) = \text{PI}_1 + \text{PI}_3 + \text{PI}_4 + \text{PI}_7 = ac' + b'cd' + a'bd' + abd$$

$$= \text{PI}_1 + \text{PI}_3 + \text{PI}_5 + \text{PI}_7 = ac' + b'cd' + bc'd' + abd$$

$$= \text{PI}_1 + \text{PI}_4 + \text{PI}_6 + \text{PI}_7 = ac' + a'bd' + ab'd' + abd$$

$$= \text{PI}_1 + \text{PI}_5 + \text{PI}_6 + \text{PI}_7 = ac' + bc'd' + ab'd' + abd$$