Solutions to EE3210 Quiz 1 Problems

Problem 1: Note that

$$\cos[(n-1)\pi] = (-1)^{n-1}.$$

Thus, the input $x[n]$ and output $y[n]$ of this system are also related by

$$y[n] = (-1)^{n-1}x[n].$$

(a) The system is memoryless. Only the current value of the input $x[n]$ influences the current value of the output $y[n]$.

(b) The system is invertible. Its inverse system is $w[n] = y[n]/\cos[(n-1)\pi]$, or $w[n] = y[n]/(-1)^{n-1}$, or $w[n] = (-1)^{n-1}y[n]$.

(c) The system is causal, since it is memoryless.

(d) The system is stable. For $0 < B < \infty$, given $|x[n]| \le B$ for all n, we have $|(-1)^{n-1}x[n]| = |(-1)^{n-1}||x[n]| = |x[n]| \le B$, and therefore $|y[n]| \le B$.

(e) The system is not time invariant. Given $x_1[n]$ and letting $y_1[n] = (-1)^{n-1}x_1[n]$, consider $x_2[n] = x_1[n-n_0]$. Then, we have $y_2[n] = (-1)^{n-1}x_2[n] = (-1)^{n-1}x_1[n-n_0]$, but we have $y_1[n-n_0] = (-1)^{n-n_0-1}x_1[n-n_0]$. Thus, $y_2[n] \ne y_1[n-n_0]$ for any odd number n_0.

(f) The system is linear. Consider $x_1[n] \to y_1[n] = (-1)^{n-1}x_1[n]$ and $x_2[n] \to y_2[n] = (-1)^{n-1}x_2[n]$. Let $x_3[n] = ax_1[n] + bx_2[n]$. Then,

$$y_3[n] = (-1)^{n-1}x_3[n] = a(-1)^{n-1}x_1[n] + b(-1)^{n-1}x_2[n] = ay_1[n] + by_2[n].$$

Problem 2: From the associative property of convolution sum, this series interconnection of two LTI systems is equivalent to a single system where $h[n] = h_1[n] * h_2[n]$. Using the convolution sum formula, the unit impulse response $h[n]$ of the overall system is obtained as

$$\begin{aligned} h[n] &= h_1[n] * h_2[n] \\ &= \sum_{k=-\infty}^{+\infty} h_1[k]h_2[n-k] \\ &= -h_2[n] + h_2[n-1] \\ &= -0.5^n u[n] + 0.5^{n-1} u[n-1]. \end{aligned}$$

Problem 3: Given $x(t) = u(t)$ and $h(t) = u(t)$, we have $x(\tau) = u(\tau)$ and $h(t-\tau) = u(t-\tau)$. Then, using the convolution integral formula, we have

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{+\infty} u(\tau)u(t-\tau)d\tau.$$

We observe that

$$u(\tau) = \begin{cases} 1, & \tau > 0 \\ 0, & \tau < 0. \end{cases}$$

With t fixed,

$$u(t-\tau) = \begin{cases} 1, & \tau < t \\ 0, & \tau > t. \end{cases}$$

Therefore,

$$u(\tau)u(t-\tau) = \begin{cases} 1, & 0 < \tau < t \\ 0, & \text{otherwise.} \end{cases}$$

Then:

- For $t < 0$, since $u(\tau)u(t-\tau) = 0$ for all τ, we have

$$y(t) = 0.$$

- For $t > 0$, since $u(\tau)u(t-\tau) = 1$ for $0 < \tau < t$, we have

$$y(t) = \int_0^t 1 d\tau = t.$$

Thus, for all t, we obtain

$$y(t) = tu(t).$$