Solutions to EE3210 Tutorial 9 Problems

Problem 1: This signal is periodic with a fundamental period $N = 6$. To determine the Fourier series coefficients a_k, we use the analysis formula of the discrete-time Fourier series

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$

and choose the limits of the summation to be $-2 \leq n \leq 3$. Then, we have

$$\begin{aligned}
a_k &= \frac{1}{6} \sum_{n=-2}^{3} x[n] e^{-jk(\pi/3)n} \\
&= \frac{1}{6} \left[-e^{jk(2\pi/3)} + 2e^{jk(\pi/3)} + 1 + 2e^{-jk(\pi/3)} - e^{-jk(2\pi/3)} \right] \\
&= \frac{1}{6} \left[1 + 4\cos\left(\frac{\pi}{3}k\right) - 2\cos\left(\frac{2\pi}{3}k\right) \right]
\end{aligned}$$

for $-2 \leq k \leq 3$.

Problem 2: To determine the signal $x[n]$, we use the synthesis formula of the discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$$

and choose the limits of the summation to be $-3 \leq k \leq 4$. Then, we have

$$\begin{aligned}
x[n] &= \sum_{n=-3}^{4} a_k e^{jk(\pi/4)n} \\
&= \frac{1}{4}\left(e^{j3\pi n/4} + e^{-j3\pi n/4}\right) + \frac{1}{2}\left(e^{j2\pi n/4} + e^{-j2\pi n/4}\right) + \left(e^{j\pi n/4} + e^{-j\pi n/4}\right) + 2 \\
&= 2 + 2\cos\left(\frac{\pi}{4}n\right) + \cos\left(\frac{\pi}{2}n\right) + \frac{1}{2}\cos\left(\frac{3\pi}{4}n\right)
\end{aligned}$$

for $-3 \leq n \leq 4$.

Problem 3:

(a) $x[n] = 1 + \cos\left(\frac{2\pi}{6}n\right)$ is a periodic signal with fundamental period $N = 6$. Using Euler's formula, we can rewrite $x[n]$ as

$$x[n] = 1 + \frac{1}{2}e^{j(2\pi n/6)} + \frac{1}{2}e^{-j(2\pi n/6)}. \tag{1}$$

Comparing the right-hand sides of (1) and the synthesis formula of the discrete-time Fourier series with the limits of the summation chosen to be $-2 \leq k \leq 3$, i.e.,

$$x[n] = \sum_{k=-2}^{3} a_k e^{jk(2\pi n/6)} \tag{2}$$

we obtain the Fourier series coefficients a_k of $x[n]$ as $a_0 = 1$, $a_{-1} = a_1 = 1/2$, and $a_k = 0$ for $k = -2, 2, 3$.

(b) $y[n] = \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right)$ is a periodic signal with fundamental period $N = 6$. Using Euler's formula, we can rewrite $y[n]$ as

$$y[n] = \frac{1}{2j}e^{j\pi/4}e^{j(2\pi n/6)} - \frac{1}{2j}e^{-j\pi/4}e^{-j(2\pi n/6)}. \tag{3}$$

Comparing the right-hand sides of (3) and (2), we obtain the Fourier series coefficients b_k of $y[n]$ as $b_{-1} = -\frac{1}{2j}e^{-j\pi/4}$, $b_1 = \frac{1}{2j}e^{j\pi/4}$, and $b_k = 0$ for $k = -2, 0, 2, 3$.

(c) The signal $z[n] = x[n]y[n]$ is also periodic with period $N = 6$. Applying the multiplication property of the discrete-time Fourier series, we obtain the Fourier series coefficients c_k of $z[n]$ as

$$c_k = \sum_{l=-2}^{3} a_l b_{k-l} = a_{-1}b_{k+1} + a_0 b_k + a_1 b_{k-1}$$
$$= \begin{cases} a_{-1}b_{-1} = -\frac{1}{4j}e^{-j\pi/4}, & k = -2 \\ a_0 b_{-1} = -\frac{1}{2j}e^{-j\pi/4}, & k = -1 \\ a_{-1}b_1 + a_1 b_{-1} = \frac{1}{4j}e^{j\pi/4} - \frac{1}{4j}e^{-j\pi/4} = \frac{1}{2}\sin\left(\frac{\pi}{4}\right), & k = 0 \\ a_0 b_1 = \frac{1}{2j}e^{j\pi/4}, & k = 1 \\ a_1 b_1 = \frac{1}{4j}e^{j\pi/4}, & k = 2 \\ 0, & k = 3. \end{cases}$$

(d) Through direct evaluation of $z[n]$, we have

$$
\begin{aligned}
z[n] = x[n]y[n] &= \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right)\cos\left(\frac{2\pi}{6}n\right) \\
&= \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right) + \frac{1}{2}\sin\left(\frac{4\pi}{6}n + \frac{\pi}{4}\right) + \frac{1}{2}\sin\left(\frac{\pi}{4}\right).
\end{aligned}
$$

This implies that the Fourier series coefficients c_k of $z[n]$ are

$$
c_k = \begin{cases}
-\dfrac{1}{4j}e^{-j\pi/4}, & k = -2 \\
-\dfrac{1}{2j}e^{-j\pi/4}, & k = -1 \\
\dfrac{1}{2}\sin\left(\dfrac{\pi}{4}\right), & k = 0 \\
\dfrac{1}{2j}e^{j\pi/4}, & k = 1 \\
\dfrac{1}{4j}e^{j\pi/4}, & k = 2 \\
0, & k = 3.
\end{cases}
$$