Question 1: Bandpass Modulation & Demodulation

a.

(i) Important, the bit/symbol energy is calculated directly before the integrator/correlator and not after it. With this in mind,

$$E_b = \int_0^T p^2(t)\cos^2 t\,dt = \frac{1}{2}\left[\left(\frac{A}{4}\right)^2\frac{T}{4} + A^2\frac{T}{2} + \left(\frac{A}{4}\right)^2\frac{T}{4}\right] = \frac{17}{64}A^2T$$

(ii) This follows directly, as

$$BER = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{17A^2T}{32N_0}}\right)$$

b. BPSK:

(i) For $b = +1$, the correlator output (O/P) is

$$B = \int_0^{\tau} A\cos^2(2\pi\times10^9 t)\,dt = \frac{A}{2}\int_0^{\tau}(1+\sin(4\pi\times10^9 t))dt \approx \frac{1}{2}A\tau$$

$$= 2\times10^{-2}\times\frac{10^{-5}}{2} = 1\times10^{-7}\text{ V},\qquad \tau = 10^{-5}$$

For $b = -1$ corresponding to 180-degree phase shift, the correlator O/P is $-B = \frac{1}{2}A\tau = -1\times10^{-7}$V. We are now in a position to calculate the "distance" D_{BPSK} between these two output values, depicted here graphically:

Correlator O/P:

$$D_{\text{BPSK}} \stackrel{\text{def}}{=} B - (-B) = \tfrac{1}{2}A\tau - \tfrac{1}{2}A\tau = A\tau = 2\times10^{-7}\text{V}$$

(ii) The energy per bit is given by:

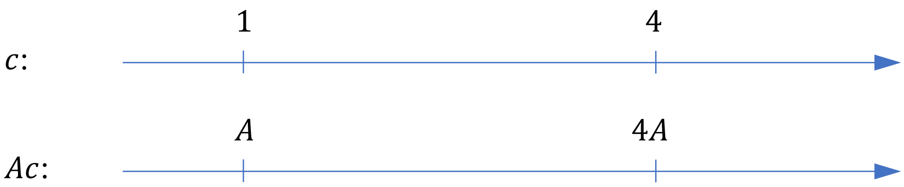
$$E_b = \frac{A^2\tau}{2} = \frac{(2\times10^{-2})^2\,10^{-5}}{2} = 2\times10^{-9}\,J,\qquad \tau = 10^{-5}$$

(iii) The BER is given by:

$$BER_{\text{BPSK}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{\frac{A^2\tau}{N_0}}\right) = Q\left(\sqrt{\frac{2\times2\times10^{-9}}{1\times10^{-9}}}\right) = Q(2).$$

c. 2ASK:

(i) Here, values of c, Ac and correlator O/P are all shown for clarity.



For the correlator O/P,

$$\int_0^{\tau} A\cos^2(2\pi f_c t)\,dt \approx \frac{A}{2}\tau = \frac{1}{2}A\tau \quad \text{and} \quad \int_0^{\tau} 4A\cos^2(2\pi f_c t)\,dt \approx \frac{4A}{2}\tau = 2A\tau$$

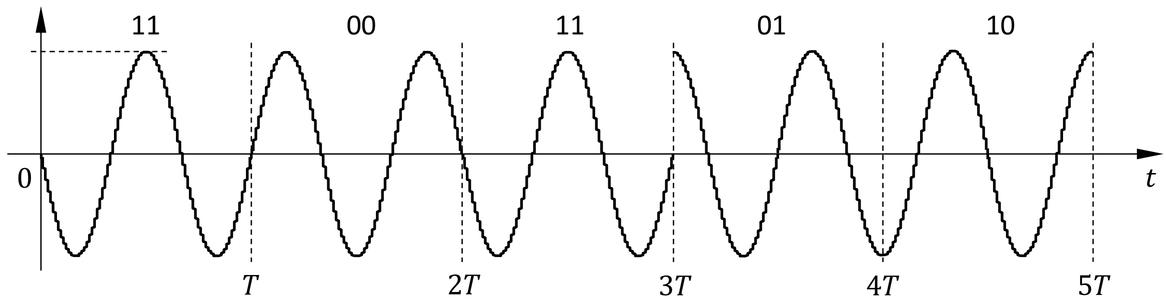
Now we can calculate the distance $D_{2\text{ASK}}$ between these two output values:

Correlator O/P:

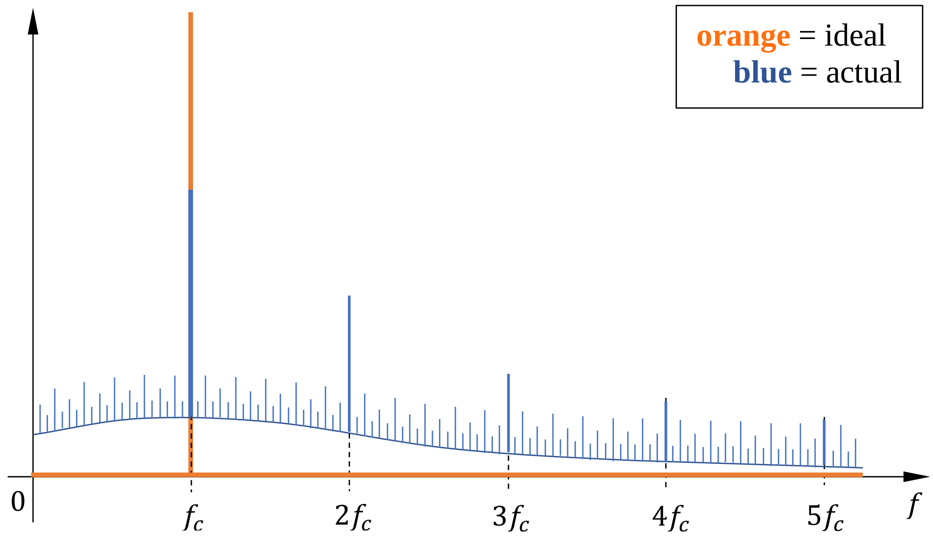
$$D_{2\text{ASK}} = 2A\tau - \tfrac{1}{2}A\tau = \tfrac{3}{2}A\tau = 3\times10^{-7}\text{V}$$

Since $D_{\text{BPSK}} < D_{2\text{ASK}}$, the two Gaussian PDFs (identical for BPSK and 2ASK) are separated farther for 2ASK, the error probabilities must be smaller and therefore we conclude that $BER_{\text{BPSK}} = Q(2) > BER_{2\text{ASK}}$.

d.



e. Deterministic errors are necessarily period (i.e., has discrete line spectral components), containing subharmonic frequencies (i.e., $\frac{m}{n}f_c$, for integers $0 < m < n$), their multiples, and harmonic frequencies (i.e., nf_c, for integers $n > 1$). Non-deterministic errors are necessarily non-periodic and result in continuous spectral components. Qualitatively, this can be graphically depicted as shown below. Here, f_c is the fundamental frequency of the DDS-generated sinewave.



Question 2: Analogue-to-Digital & Digital-to-Analogue Conversion

a.

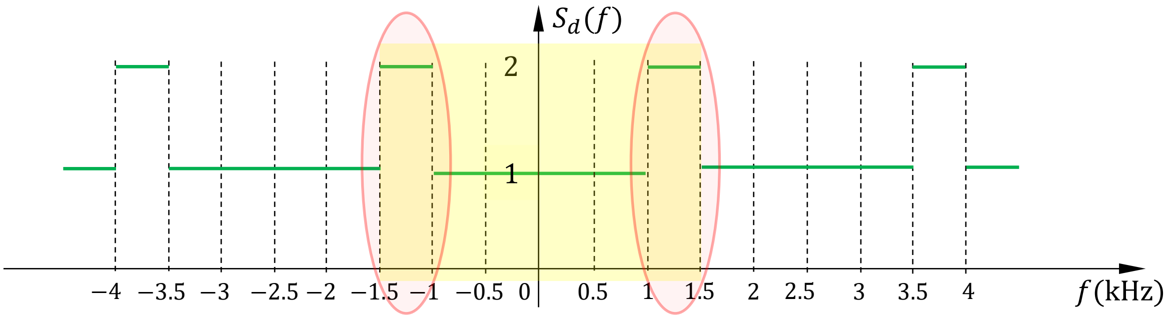
(i) The minimum sampling rate is $R_s = 2f_m$ = 10 kilo-samples/second.

(ii) Number of bits required to quantize this signal is $2^k = 2048$, i.e., $k = 11$ bits.
Thus, the data rate is given by $R = kR_s$ = 110 kilo-bits/second.
For BPSK modulation, the symbol rate is $R_{\text{BPSK}} = R$ = 110 kilo-bits/second.
Thus, the maximum value of T is $T_{\text{BPSK}} = 1/R_{\text{BPSK}} = 9.0909 \times 10^{-6}$ seconds.
The bandwidth of the transmitted signal is $W_{\text{BPSK}} = 2/T_{\text{BPSK}}$ = 220 kHz.

(iii) For 16PSK modulation, the symbol rate is $R_{\text{16PSK}} = R/4$ = 110/4 kilo-bits/second = 27.5 kilo-bits/second.
Thus, maximum value of T is $T_{\text{16PSK}} = 1/R_{\text{16PSK}} \approx 36.3636 \times 10^{-6}$ seconds.
The bandwidth of the transmitted signal is $W_{\text{16PSK}} = 2/T_{\text{16PSK}}$ = 55 kHz.

b.

(i) Due to aliasing (1 kHz to 1.5 kHz) the baseband signal (passband highlighted in yellow) cannot be recovered as the aliased components (circled in red) will be passed by the lowpass filter with passband 1.5 kHz.



(ii) Due to sampling at Nyquist rate (3 kHz), the baseband signal (passband highlighted in yellow) can be recovered.