Solutions to EE3210 Tutorial 12 Problems

Problem 1:

(a) No. From Property 3 of ROC, we know that, given that $x(t)$ is absolutely integrable, if it is also of finite duration, then the ROC is the entire s-plane. However, in this case, $X(s)$ has a pole at $s = 2$. Therefore, $x(t)$ cannot be of finite duration.

(b) Yes. Since $x(t)$ is absolutely integrable, the ROC must include the imaginary axis, i.e., $\text{Re}\{s\} = 0$. Since $X(s)$ has a pole at $s = 2$, one valid ROC would be $\text{Re}\{s\} < 2$. From Property 5 of ROC, we know that this could correspond to a left-sided signal.

(c) No. Since $x(t)$ is absolutely integrable, the ROC must include the imaginary axis, i.e., $\text{Re}\{s\} = 0$. From Property 4 of ROC, we know that, if $x(t)$ is right sided and the line $\text{Re}\{s\} = 0$ is in the ROC, then all values of s for which $\text{Re}\{s\} > 0$ will also be in the ROC. However, in this case, $X(s)$ has a pole at $s = 2$. Therefore, $x(t)$ cannot be right sided.

(d) Yes. Since $x(t)$ is absolutely integrable, the ROC must include the imaginary axis, i.e., $\text{Re}\{s\} = 0$. Furthermore, $X(s)$ has a pole at $s = 2$. Therefore, one valid ROC would be $a < \text{Re}\{s\} < 2$ such that $a < 0$. From Property 6 of ROC, we know that this could correspond to a two-sided signal.

Problem 2: The partial fraction expansion of $X(s)$ is

$$X(s) = \frac{4}{s+4} - \frac{2}{s+3}.$$

We know that there are two possible inverse Laplace transforms of the form $1/(s + a)$, depending on whether the ROC is to the left or the right of the pole. That is:

$$e^{-at}u(t) \leftrightarrow \frac{1}{s+a}, \quad \text{Re}\{s\} > -a$$

or

$$-e^{-at}u(-t) \leftrightarrow \frac{1}{s+a}, \quad \text{Re}\{s\} < -a.$$

(a) With $\text{Re}\{s\} > -3$ and hence $\text{Re}\{s\} > -4$, we have

$$x(t) = 4e^{-4t}u(t) - 2e^{-3t}u(t).$$

Therefore, in this case, $x(t)$ is a right-sided signal.

(b) With $\text{Re}\{s\} < -4$ and hence $\text{Re}\{s\} < -3$, we have

$$x(t) = -4e^{-4t}u(-t) + 2e^{-3t}u(-t).$$

Therefore, in this case, $x(t)$ is a left-sided signal.

(c) With $-4 < \text{Re}\{s\} < -3$, we have

$$x(t) = 4e^{-4t}u(t) + 2e^{-3t}u(-t).$$

Therefore, in this case, $x(t)$ is a two-sided signal.

Problem 3: We know that

$$x_1(t) = e^{-2t}u(t) \leftrightarrow X_1(s) = \frac{1}{s+2}, \quad \text{Re}\{s\} > -2$$

and

$$x_2(t) = e^{-3t}u(t) \leftrightarrow X_2(s) = \frac{1}{s+3}, \quad \text{Re}\{s\} > -3.$$

Using the time shift property of the Laplace transform, we obtain

$$x_1(t-2) \leftrightarrow e^{-2s}X_1(s) = \frac{e^{-2s}}{s+2}, \quad \text{Re}\{s\} > -2.$$

Using the time shift property followed by the time reversal property of the Laplace transform, we have

$$x_2(t+3) \leftrightarrow e^{3s}X_2(s), \quad \text{Re}\{s\} > -3$$

and then

$$x_2(-t+3) \leftrightarrow e^{-3s}X_2(-s) = \frac{e^{-3s}}{3-s}, \quad \text{Re}\{s\} < 3.$$

Therefore, using the convolution property of the Laplace transform, we obtain

$$Y(s) = \left(\frac{e^{-2s}}{s+2}\right)\left(\frac{e^{-3s}}{3-s}\right) = \frac{-e^{-5s}}{(s+2)(s-3)}, \quad -2 < \text{Re}\{s\} < 3.$$