CITY UNIVERSITY OF HONG KONG
Department of Electronic Engineering

EE 3118 Linear Systems and Signal Analysis

Homework #4

1. Problem 4.21, (c), (g), pp. 338.
2. Problem 4.22, (c), (d), pp. 338.
3. Problem 4.11, pp. 336. (Hint: use properties 4.3.5 and 4.4.)
4. Problem 4.23, pp. 339.
5. Problem 4.26, (a)-i, (b), pp. 341.
6. Problem 4.28, (a), (b)-ii/vi, pp. 342.

Homework #7

Prob. 4.11

$$y(t) = x(t) * h(t) \implies Y(\omega) = X(\omega)H(\omega)$$

$$g(t) = x(3t) * h(3t)$$

$$\implies G(\omega) = \mathcal{F}\{x(3t)\} \cdot \mathcal{F}\{h(3t)\}$$

$$= \frac{1}{3} X\left(\frac{\omega}{3}\right) \cdot \frac{1}{3} H\left(\frac{\omega}{3}\right)$$

$$= \frac{1}{3} \cdot \frac{1}{3} Y\left(\frac{\omega}{3}\right)$$

$$\implies g(t) = \frac{1}{3} y(3t)$$

$A = \frac{1}{3}$, $B = 3$.

Prob. 4.23

$$x_0(t) = \begin{cases} e^{-t} & 0 \le t \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$X_0(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_0^1 e^{-t} \cdot e^{-j\omega t}\, dt$$

$$= \int_0^1 e^{-(1+j\omega)t}\, dt$$

$$= \frac{1}{-(1+j\omega)} e^{-(1+j\omega)t} \Big|_0^1$$

$$= \frac{1}{1+j\omega}\left[1 - e^{-(1+j\omega)}\right]$$

(a) $x_1(t) = x_0(t) + x_0(-t)$

$$X_1(\omega) = X_0(\omega) + X_0(-\omega) \qquad \left(x_0(-t) \longleftrightarrow X_0(-\omega) \text{ Time-reversal} \right)$$

$$= \frac{1}{1+j\omega}\left[1 - e^{-(1+j\omega)}\right] + \frac{1}{1-j\omega}\left[1 - e^{-(1-j\omega)}\right]$$

$$= \left(\frac{1}{1+j\omega} + \frac{1}{1-j\omega}\right) - e^{-1}\left(\frac{e^{-j\omega}}{1+j\omega} + \frac{e^{j\omega}}{1-j\omega}\right)$$

$$= \frac{2}{1+\omega^2} - e^{-1}\,\frac{2\cos\omega - 2\omega\sin\omega}{1+\omega^2}$$

(c) $x_3(t) = x_0(t) + x_0(t+1)$

$$X_3(\omega) = X_0(\omega) + e^{j\omega} X_0(\omega) \quad \text{— Time-shifting}$$

$$= \cdots$$

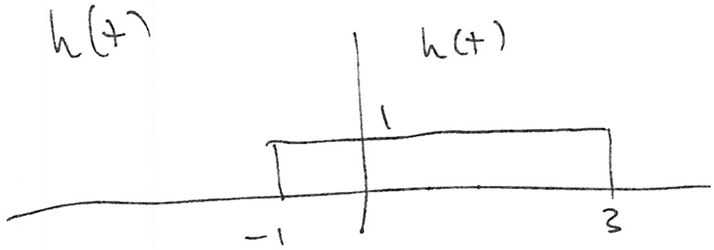
Prob. 4.26

(b) $x(t) = e^{-(t-2)} u(t-2)$

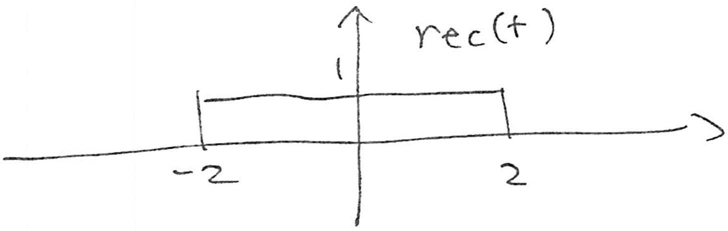
$$X(\omega) = e^{-j2\omega}\, \mathcal{F}\{e^{-t} u(t)\} \quad \text{— Time-shifting}$$

$$= \frac{e^{-j2\omega}}{1+j\omega}$$

h(t)



Let $rec(t) = \begin{cases} 1 & |t| < 2 \\ 0 & \text{elsewhere} \end{cases}$



Then $h(t) = rec(t-1)$

$$H(\omega) = e^{-j\omega} \mathcal{F}\{rec(t)\}$$

$$= e^{-j\omega} \frac{2 \sin 2\omega}{\omega}$$

$$Y(\omega) = H(\omega) X(\omega)$$

$$= e^{-j3\omega} \frac{1}{1+j\omega} \frac{2 \sin 2\omega}{\omega}$$

Prob. 4.28

(a) $y(t) = x(t)\,p(t)$

$$= x(t) \sum_{n=-\infty}^{\infty} a_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} a_n x(t) e^{jn\omega_0 t}$$

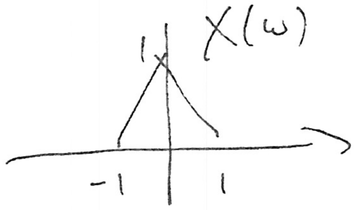
$$x(t) \longleftrightarrow X(\omega)$$

$$x(t) e^{jn\omega_0 t} \longleftrightarrow X(\omega - n\omega_0)$$

$$\Rightarrow \quad Y(\omega) = \sum_{n=-\infty}^{\infty} a_n X(\omega - n\omega_0)$$

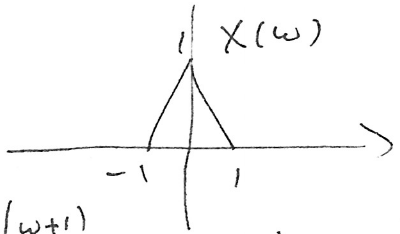
(b) (ii) $p(t) = \cos t = \dfrac{e^{jt} + e^{-jt}}{2}$

$a_{-1} = a_1 = \frac{1}{2}$, $\quad a_n = 0$ for $n \neq \pm 1$, $\omega_0 = 1$.

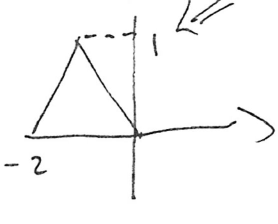


$$Y(\omega) = \frac{1}{2} X(\omega - 1) + \frac{1}{2} X(\omega + 1)$$

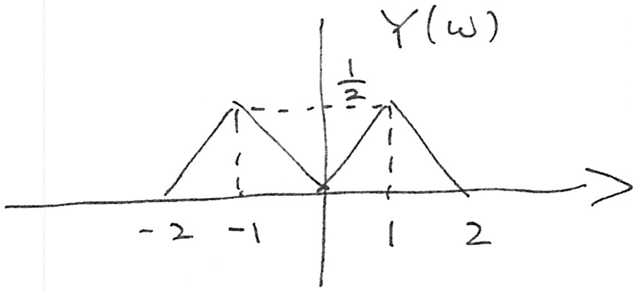
$$= \frac{1}{2}\left[X(\omega - 1) + X(\omega + 1)\right]$$



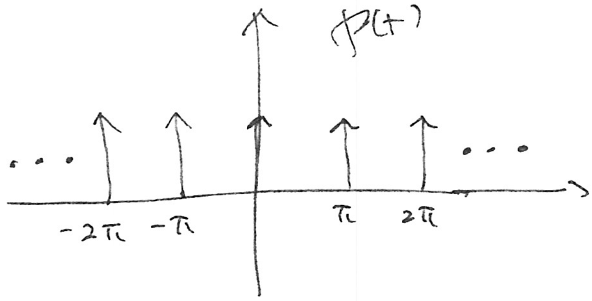
$X(\omega+1)$



$X(\omega-1)$



(vi) $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\pi)$



$p(t)$ is periodic with period equal to π.

$$p_k = \frac{1}{T}\int_T p(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \delta(t)\, e^{-jk\omega_0 t}\, dt$$

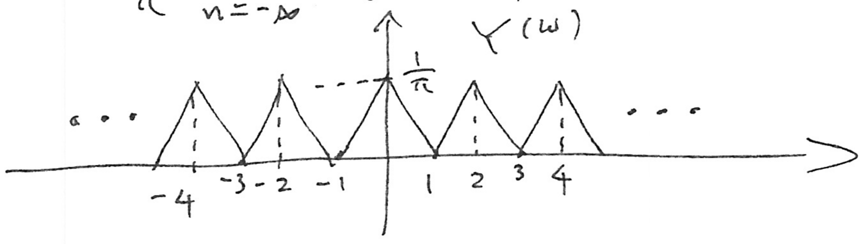
$$= \frac{1}{\pi} e^{-jk\omega_0 t}\Big|_{t=0} = \frac{1}{\pi}$$

$$p(t) = \sum_{k=-\infty}^{\infty} p_k\, e^{jk\omega_0 t} = \frac{1}{\pi}\sum_{k=-\infty}^{\infty} e^{jk\omega_0 t}$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} p_n\, X(\omega - n\omega_0)$$

$$= \frac{1}{\pi}\sum_{n=-\infty}^{\infty} X(\omega - 2n)$$



Problem 4.21 (c)

$$x(t) = \begin{cases} 1+\cos\pi t & |t| \le 1 \\ 0 & |t| > 1 \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{1} (1+\cos\pi t)\, e^{-j\omega t}\, dt$$

$$= \int_{-1}^{1} e^{-j\omega t}\, dt + \int_{-1}^{1} \frac{e^{j\pi t}+e^{-j\pi t}}{2}\, e^{-j\omega t}$$

$$= \frac{1}{(-j\omega)}\left[e^{-j\omega} - e^{j\omega}\right] + \int_{-1}^{1} \frac{1}{2} e^{j(\pi-\cancel{j}\omega)t}\, dt + \int_{-1}^{1} \frac{1}{2} e^{-j(\pi+\cancel{j}\omega)t}\, dt$$

$$= \frac{1}{(-j\omega)}\left[-2j\sin\omega\right] + \frac{1}{j(\pi-\omega)}\,\frac{1}{2}\, e^{j(\pi-\omega)t}\Big|_{-1}^{1}$$

$$+ \frac{1}{[-j(\pi+\omega)]}\,\frac{1}{2}\, e^{-j(\pi+\omega)t}\Big|_{-1}^{1}$$

Prob. 4.22

(d) $X(\omega) = 2\left[\delta(\omega-1) - \delta(\omega+1)\right] + 3\left[\delta(\omega-2\pi) + \delta(\omega+2\pi)\right]$

$$= \frac{2j}{\pi} \cdot \frac{\pi}{j}\left[\delta(\omega-1) - \delta(\omega+1)\right] + \frac{3}{\pi} \cdot \pi\left[\delta(\omega-2\pi) + \delta(\omega+2\pi)\right]$$

$$\sin \omega_0 t \longleftrightarrow \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

$$\cos \omega_0 t \longleftrightarrow \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

$$\Rightarrow \quad x(t) = \frac{2j}{\pi} \sin t + \frac{3}{\pi} \cos 2\pi t$$