<u>EE 5410 Signal Processing</u>

Semester A 2017-2018

Assignment 1

Due Date: <u>10 October 2017</u>

1. Find the Fourier series coefficients for the following continuous-time signal:

$$x(t) = \begin{cases} 2, & 2 > t > 0 \\ 1, & 4 > t > 2 \end{cases}$$

   with fundamental period of $T = 4$.

2. The impulse response of a RL circuit which corresponds to a continuous-time linear time-invariant (LTI) system, is given as:

$$h(t) = e^{\frac{-t}{L/R}} u(t)$$

   where R and L represent the values of resistor and inductor, respectively. Find the Fourier transform of $h(t)$. Then determine its magnitude, phase, real part and imaginary part of $H(j\Omega)$.

3. Consider a continuous-time LTI system with continuous-time input $x(t)$ and impulse response $h(t) = -2\delta(t-2) + \delta(t-10)$. Determine the system continuous-time output $y(t)$ in terms of $x(t)$. Is the system stable? Is the system causal? Is the system memoryless?

4. Given a discrete-time system with input $x[n]$ and output $y[n]$:

$$y[n] = T(x[n]) = x[n] + \frac{1}{x[n]}$$

   Determine whether the system is memoryless, stable, causal, linear, and/or time-invariant.

5. Consider two discrete-time signals $x[n] = u[-1-n]$ and $h[n] = (0.5)^n u[n]$.

   (a) Compute $y[n] = x[n] \otimes h[n]$ using the convolution formula.
   (b) Compute $y[n] = x[n] \otimes h[n]$ using z transform.

6. Given a continuous-time signal:

$$x(t) = 2\sin\left(\frac{\pi}{2}t + \frac{\pi}{5}\right)$$

   We sample it with a sampling period $T = 1$ sec. to produce the discrete-time signal $x[n]$. Find $x[1]$, $x[2]$, $x[3]$, $x[4]$ and $x[5]$. Is $x[n]$ a periodic signal?

7. Figure 1 shows a discrete-time system which consists of an interconnection of four LTI systems with impulse responses $h_1[n]$, $h_2[n]$, $h_3[n]$ and $h_4[n]$.

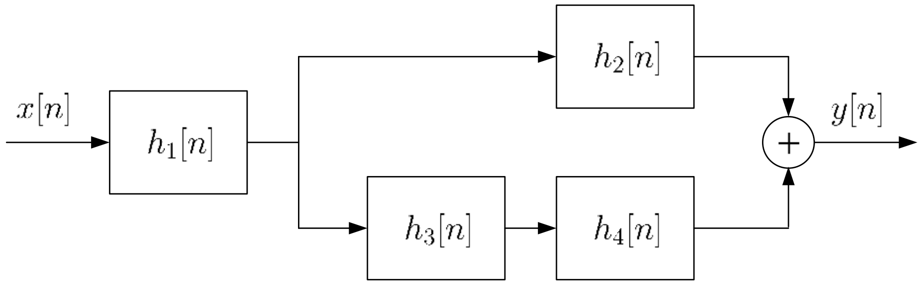

Figure 1

(a) Determine the overall impulse response of the system, $h[n]$, in terms of $h_1[n]$, $h_2[n]$, $h_3[n]$ and $h_4[n]$.

(b) Determine $h[n]$ when

$$h_1[n] = \delta[n] + \delta[n-1]$$

$$h_2[n] = h_3[n] = u[n]$$

and

$$h_4[n] = \delta[n-2]$$

(c) Determine $y[n]$ in (b) if the input has the form of

$$x[n] = \delta[n+2] + 3\delta[n-1]$$

8. Determine the convolution of the following two discrete-time signals:

$$x[n] = \begin{cases} n^2 - 1, & -2 \le n \le 2 \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad h[n] = \begin{cases} n - 4, & 0 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

9. When the input to a discrete-time LTI system is

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1]$$

The corresponding output is

$$y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n]$$

(a) Find the system function $H(z)$ of the system and specify its region of convergence (ROC).

(b) Determine the pole(s) and zero(s) of $H(z)$.

(c) Find the impulse response $h[n]$ of the system.

(d) Determine the discrete-time Fourier transform (DTFT) of $h[n]$.

(e) Write a difference equation which relates $x[n]$ and $y[n]$.

(f) Is the system stable? Why?

(g) Is the system causal? Why?