CITY UNIVERSITY OF HONG KONG
Department of Electronic Engineering

**EE 3118 Linear Systems and Signal Analysis**

Homework #3

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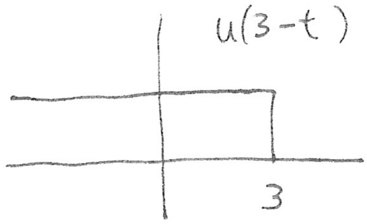
# Homework #3

## Problem 2.29, PP. 144




(b) $h(t) = e^{-6t} u(3-t)$

Solution: The LTI system is non-causal, since $h(t) \not\equiv 0$ for $t<0$.

For stability, check

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{3} e^{-6t}\, dt = \infty$$

The system is unstable.

(g) $h(t) = \left(2e^{-t} - e^{(t-100)/100}\right) u(t)$

It is causal, since $h(t) \equiv 0$ for $t<0$.

It is unstable since

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \infty$$

## Problem 2.40, PP. 148

(a) Solution: To find the impulse response, let $x(t) = \delta(t)$. Then,

$$h(t) = \int_{-\infty}^{t} e^{-(t-\tau)} \delta(\tau-2)\, d\tau$$

$$= \begin{cases} e^{-(t-\tau)}\Big|_{\tau=2} & t>2 \\ 0 & t<2 \end{cases} = e^{-(t-2)} u(t-2)$$

Prob. 3.3

Using Euler identity

$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j},$$

We ~~write~~ write

$$x(t) = 2+\cos\frac{2\pi}{3}t + 4\sin\frac{5\pi}{3}t$$

$$= 2 + \frac{e^{j2\frac{\pi}{3}t}+e^{-j2\frac{\pi}{3}t}}{2} + 4\cdot\frac{e^{j5\frac{\pi}{3}t}-e^{-j5\frac{\pi}{3}t}}{2j}$$

$$= 2j\, e^{-j5\frac{\pi}{3}t} + \frac{1}{2}e^{-j2\frac{\pi}{3}t} + 2 + \frac{1}{2}e^{j2\frac{\pi}{3}t} - 2je^{j5\frac{\pi}{3}t}$$

$\Rightarrow$

$$\omega_0 = \frac{\pi}{3}$$

$$a_{-5} = 2j$$

$$a_{-2} = \frac{1}{2}$$

$$a_0 = 2$$

$$a_2 = \frac{1}{2}$$

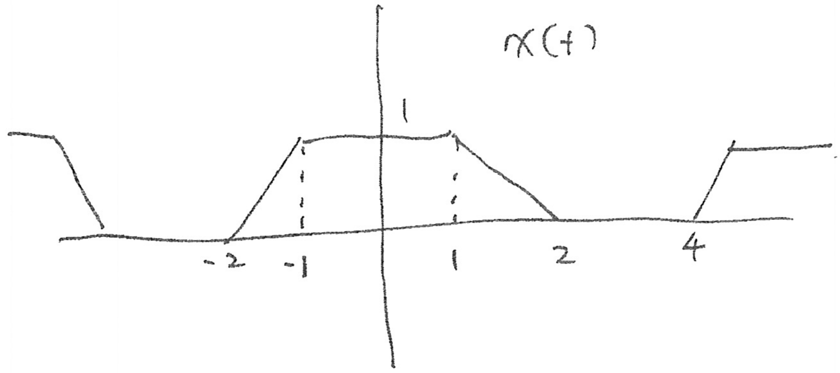
$$a_5 = -2j$$

The key to this problem is twofold:

(i) All sinusoids can be expressed using Euler identity. As a result it is unnecessary to go through integration to find $a_k$.

(ii) Any Fourier series is unique! Thus, if we can expand $x(t)$ in the standard form (as above), we need to identify only the coefficients.

Prob. 3.22 (a), Fig. (b)




Within one period, $T=6$, (say, $[-2, 4]$), $\omega_0 = \frac{2\pi}{T} = \frac{\pi}{3}$

$$x(t) = \begin{cases} 0 & 2 \le t \le 4 \\ 2-t & 1 \le t \le 2 \\ 1 & -1 \le t \le 1 \\ t+2 & -2 \le t \le -1 \end{cases}$$

$$x_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\left[\int_{-2}^{-1}(t+2)\, e^{-jk\omega_0 t}\, dt + \int_{-1}^{1} e^{-jk\omega_0 t}\, dt + \int_{1}^{2}(2-t)\, e^{-jk\omega_0 t}\, dt\right]$$

Use integration by parts

$$\int_a^b u\, dv = uv\Big|_a^b - \int_a^b v\, du$$

$$\int_{-2}^{-1} t\, e^{-jk\omega_0 t}\, dt = -\frac{1}{jk\omega_0}\, t\, e^{-jk\omega_0 t}\Big|_{-2}^{-1} + \int_{-2}^{-1} \frac{1}{jk\omega_0} e^{-jk\omega_0 t}$$

$\left( u = t,\ e^{-jk\omega_0 t} dt = dv \Rightarrow v = -\frac{1}{jk\omega_0} e^{-jk\omega_0 t},\ du = dt \right)$

Prob. 3.26

$$a_k = \begin{cases} 2 & k=0 \\ j\left(\frac{1}{2}\right)^{|k|} & \text{otherwise} \end{cases}$$

$$\Rightarrow \quad x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$= \sum_{k=-\infty}^{-1} j\left(\tfrac{1}{2}\right)^{|k|} e^{jk\omega_0 t} + 2 + \sum_{k=1}^{\infty} j\left(\tfrac{1}{2}\right)^{|k|} e^{jk\omega_0 t}$$

$$= \sum_{k=-\infty}^{-1} j\left(\tfrac{1}{2}\right)^{-k} e^{jk\omega_0 t} + 2 + \sum_{k=1}^{\infty} j\left(\tfrac{1}{2}\right)^{k} e^{jk\omega_0 t}$$

$(k<0,\ |k|=-k)$ $\qquad$ $(k>0,\ |k|=k)$

$$= \sum_{k=1}^{\infty} j\left(\tfrac{1}{2}\right)^{k} e^{-jk\omega_0 t} + 2 + \sum_{k=1}^{\infty} j\left(\tfrac{1}{2}\right)^{k} e^{jk\omega_0 t}$$

( Variable change )

$$= 2 + \sum_{k=1}^{\infty} j\left(\tfrac{1}{2}\right)^{k} \left[e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right]$$

$$= 2 + \sum_{k=1}^{\infty} j\left(\tfrac{1}{2}\right)^{k} \cdot 2\cos k\omega_0 t$$

( Euler Identity )

(a) No, $x(t)$ is not real

(b) Yes, $x(t)$ is even, since

$$x(-t) = 2 + 2\sum_{k=1}^{\infty} j\left(\tfrac{1}{2}\right)^{k} \cos k\omega_0(-t)$$

$$= 2 + 2\sum_{k=1}^{\infty} j\left(\tfrac{1}{2}\right)^{k} \cos k\omega_0 t$$

$$= x(t)$$

(c) $\frac{dx(t)}{dt} = \sum_{k=1}^{\infty} j\left(\frac{1}{2}\right)^k \cdot 2\left(-k\omega_0 \sin k\omega_0 t\right)$

$= -2j\omega_0 \sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k \sin k\omega_0 t$

Let $y(t) = \frac{dx(t)}{dt}$. Then,

$y(-t) = -2j\omega_0 \sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k \sin k\omega_0(-t)$

$= 2j\omega_0 \sum_{k=1}^{\infty} k\left(\frac{1}{2}\right)^k \sin k\omega_0 t$

$= -y(t)$.

No, $\frac{dx(t)}{dt}$ is not even; in fact, it is odd.

1. Prob. 3.40

(a) $x(t-t_0)+x(t+t_0)$

$$x(t-t_0) \longleftrightarrow X_k e^{-jk\omega_0 t_0}$$

$$x(t+t_0) \longleftrightarrow X_k e^{-jk\omega_0(-t_0)} = X_k e^{jk\omega_0 t_0}$$

$$x(t-t_0)+x(t+t_0) \longleftrightarrow X_k e^{-jk\omega_0 t_0} + X_k e^{jk\omega_0 t_0}$$

$$= X_k \left(e^{jk\omega_0 t_0} + e^{-jk\omega_0 t_0}\right)$$

$$= 2X_k \cos k\omega_0 t_0$$

(e) $x(3t-1)$

Let $y(t)=x(3t)$, $z(t)=x(3t-1)$. Then,

$$z(t) = x(3t-1)$$

$$= x\left[3\left(t-\tfrac{1}{3}\right)\right]$$

$$= y\left(t-\tfrac{1}{3}\right)$$

Time-shifting: $z_k = y_k e^{-jk\omega_y \cdot \frac{1}{3}}$

Time-Scaling: $y_k = x_k$

$3T_y = T_x$

$\omega_x = \dfrac{2\pi}{T_x} = \dfrac{2\pi}{3T_y} = \dfrac{1}{3}\omega$

$\Rightarrow z_k = X_k e^{-jk\omega_x}$

Prob. 3.43

(i) For an odd harmonic signal, $a_k = 0$ for any even $k$.

Consider

$$x\left(t+\frac{T}{2}\right) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}\left(t+\frac{T}{2}\right)}$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t} \cdot e^{jk\pi}$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t} \cos k\pi$$

$$= \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t} (-1)^k$$

$$= -\sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

(since $a_k = 0$ for even and $(-1)^k = -1$ for odd $k$)

$$= -x(t)$$

(ii) If $x(t) = -x\left(t+\frac{T}{2}\right)$, in addition to $x(t) = x(t+T)$

Then

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

where

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t}\, dt, \quad \omega_0 = \frac{2\pi}{T}$$

$$= -\frac{1}{T}\int_T x\left(t+\frac{T}{2}\right) e^{-jk\omega_0 t}\, dt$$

Let $\tau = t + \frac{T}{2}$

$$= -\frac{1}{T}\int_T x(\tau) e^{-jk\omega_0\left(\tau-\frac{T}{2}\right)}\, d\tau$$

$$= -\frac{1}{T}\int_T x(\tau)\, e^{-jk\omega_0\tau} \cdot e^{jk\frac{\omega_0 T}{2}}\, d\tau$$

$$= -\left(\frac{1}{T}\int_T x(\tau)\, e^{-jk\omega_0\tau}\, d\tau\right) \cos k\pi$$

$$\cancel{= -(-1)^k \tfrac{1}{T}}$$

$$= -(-1)^k a_k$$

$$\Rightarrow \left[1 + (-1)^k\right] a_k = 0$$

For any even $k$, This means that

$$2a_k = 0,$$

that is, $a_k = 0$. As such, $x(t)$ is odd harmonic.