EE2302 Foundations of Information Engineering

Assignment 7 **Due: 6 pm, Nov 3**

Full Mark: 22 points

1. (4 points) Determine whether each of the following scalar-valued functions $n$-vectors is linear. If it is linear, give its inner product representation, i.e., an $n$-vector $a$ for which $f(x) = a^T x$ for all $x$. If it is not linear, give specific $x, y, \alpha, \beta$ for which superposition fails, i.e.,
$$f(\alpha x + \beta y) \neq \alpha f(x) + \beta f(y).$$
   a) $f(x) = \max_k x_k$
   b) $f(x) = x_n - x_1$

2. (4 points) What 3 by 3 matrices represent the transformations that
   a) reflect every vector through the $x$-$y$ plane?
   b) rotate the $x$-$y$ plane through $90^o$, leaving the $z$-axis alone?

3. (4 points) Consider the 2-dimensional space and the projection of $b$ on the line passing through the origin and $a$, where $a = (10, 10)$.
   a) Determine the corresponding projection matrix P.
   b) Suppose $b = (3, 5)$. Determine the result after the projection.

4. (6 points) Let $x$ be a real $n$-vector and define $y$ as the real, non-negative vector (i.e., the vector with non-negative real entries) closest to $x$.
   a) Give an expression for each element of $y$.
   b) Show that $z = y - x$ is also a non-negative vector.
   c) Show that $z^T y = 0$.

5. (4 points) By choosing the right vector $b$ in the Cauchy-Schwarz inequality, prove that
$$(a_1 + a_2 + \cdots + a_n)^2 \leq n(a_1^2 + a_2^2 + \cdots a_n^2).$$