EE3210 Signal and System

Final Exam

Honor Pledge

Please review the following honor code, then sign your name and write down the date.

1. I pledge that the answers in this exam are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,
   (a) I will not plagiarize (copy without citation) from any source;
   (b) I will not communicate or attempt to communicate with any other person during the exam;
   (c) neither will I give or attempt to give assistance to another student taking the exam; and
   (d) I will use only approved devices (e.g., calculators) and/or approved device models.
2. I understand that any act of academic dishonesty can lead to disciplinary action.

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Signature

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Date

Problem 1.

Questions	Answer
a)	(i)
b)	(ii)
c)	(iv)
d)	(i)

Problem 2.

Question	Answer
a)	(iii)
b)	(iv)
c)	(iv)

Problem 3.

(a)

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t) \quad where\ z(t) = e^{-t}u(t) + \delta(t)$$

$$\frac{dy(t)}{dt} + 10y(t) = x(t)z(t) - x(t)$$

$$(j2\pi f + 10)\mathcal{Y}(f) = \mathcal{X}(f)[z(f) - 1]$$

$$\mathcal{H}(f) = \frac{\mathcal{Y}(f)}{\mathcal{X}(f)} = \frac{z(f) - 1}{j2\pi f + 10} = \frac{1}{(j2\pi f + 10)(j2\pi f + 1)}$$

(b)

$$\mathcal{H}(f) = \frac{1}{(j2\pi f + 10)(j2\pi f + 1)} = \frac{c_1}{j2\pi f + 10} + \frac{c_2}{j2\pi f + 1}$$

$$c_1 = (j2\pi f + 10)\mathcal{H}(f)|_{j2\pi f = -10} = -\frac{1}{9}$$

$$c_2 = (j2\pi f + 1)\mathcal{H}(f)|_{j2\pi f = -1} = \frac{1}{9}$$

$$\mathcal{H}(f) = \frac{1}{9(j2\pi f + 1)} - \frac{1}{9(j2\pi f + 10)} \overset{F^{-1}}{\Longrightarrow} h(t) = \frac{1}{9}(e^{-t} - e^{-10t})u[t]$$

Problem 4. .

(a) The LT of the differential equation is $(s^2+6s+8)\mathcal{Y}(s) = s\mathcal{X}(s)$

$$\mathcal{H}(s) = \frac{\mathcal{Y}(s)}{\mathcal{X}(s)} = \frac{s}{s^2+6s+8} = \frac{c_1}{s+4} + \frac{c_2}{s+2}$$

$$c_1 = (s+4)\mathcal{H}(s)|_{s=-4} = \frac{-4}{-2} = 2$$

$$c_2 = (s+2)\mathcal{H}(s)|_{s=-2} = \frac{-2}{2} = -1$$

By using the LT table,

$$h(t) = 2e^{-4t}u(t) - e^{-2t}u(t)$$

$$\mathcal{Y}(s) = \mathcal{H}(s)\cdot\mathcal{X}(s) \quad x(t) = u(t) \overset{L_I}{\Rightarrow} \mathcal{X}(s) = \frac{1}{s}$$

$$\mathcal{Y}(s) = \frac{1}{(s+4)(s+2)} = \frac{c_1}{s+4} + \frac{c_2}{s+2}$$

$$c_1 = (s+4)\mathcal{H}(z)|_{s=-4} = -1/2$$

$$c_2 = (s+2)\mathcal{H}(z)|_{s=-2} = 1/2$$

The step response of the given equation.

$$y(t) = -\frac{1}{2}e^{-4t}u(t) + \frac{1}{2}e^{-2t}u(t)$$

$$ROC \quad Re(s) > -2$$

(b) $y(t) = e^t\left[4 + 4\int_0^t e^{-\tau}y(\tau)d\tau\right] \quad t \geq 0$

$$\frac{dy(t)}{dt} = e^t\left[4 + 4\int_0^t e^{-\tau}y(\tau)d\tau\right] + 4e^t\left(e^{-t}y(t)\right) = 5y(t)$$

By obtaining the initial condition as $y(0^-) = 4$

$$sY_I(s) - y(0^-) = 4Y_I(s)$$

$$sY_I(s) - 4 = 5Y_I(s)$$

$$Y_I(s) = \frac{4}{s-5}$$

After applying the inverse LT, $y(t) = 4e^{5t}u(t)$.

Problem 5.

(a) Apply the time-shifting property,

$$\left(1-\frac{3}{4}Z^{-1}+\frac{1}{8}Z^{-2}\right)\mathcal{Y}(z)=\mathcal{X}(z)$$

$$\mathcal{H}(z)=\frac{\mathcal{Y}(z)}{\mathcal{X}(z)}=\frac{1}{1-\frac{3}{4}Z^{-1}+\frac{1}{8}Z^{-2}}=\frac{Z^2}{\left(Z-\frac{1}{4}\right)\left(Z-\frac{1}{2}\right)}\quad ROC,|Z|>\frac{1}{2}$$

Let's represent $\mathcal{H}(z)$ in the following form,

$$\mathcal{H}(z)=\frac{1}{1-\frac{3}{4}Z^{-1}+\frac{1}{8}Z^{-2}}=\frac{c_1}{1-\frac{1}{4}Z^{-1}}+\frac{c_2}{1-\frac{1}{2}Z^{-1}}$$

$$c_1=\left(1-\frac{1}{4}Z^{-1}\right)\mathcal{H}(z)\Big|_{\frac{Z^{-1}}{4}=1}=-1$$

$$c_2=\left(1-\frac{1}{2}Z^{-1}\right)\mathcal{H}(z)\Big|_{\frac{Z^{-1}}{2}=1}=2$$

Based on the Z-transform table,

$$h[n]=2\cdot\left(\frac{1}{2}\right)^n u[n]-\left(\frac{1}{4}\right)^n u[n]=\left[2\cdot\left(\frac{1}{2}\right)^n-\left(\frac{1}{4}\right)^n\right]u[n]$$

(b) Since the $|Z|=1/2$, which contain the $|Z|=1$, so, it is a stable system. In addition, we can find that the $ROC>1/2$, which means that it is a casual system.

(c) Since $u[n]\leftrightarrow\frac{1}{1-Z^{-1}},\ |Z|>1$

$$\mathcal{Y}(z)=\mathcal{H}(z)\mathcal{X}(z)\quad where\ \mathcal{X}(z)=Z(u[n])$$

$$\mathcal{Y}(z)=\frac{1}{\left(1-\frac{1}{4}Z^{-1}\right)\left(1-\frac{1}{2}Z^{-1}\right)(1-Z^{-1})}=\frac{c_1}{1-\frac{1}{4}Z^{-1}}+\frac{c_2}{1-\frac{1}{2}Z^{-1}}+\frac{c_3}{1-Z^{-1}}$$

$$c_1=\left(1-\frac{1}{4}Z^{-1}\right)\mathcal{Y}(z)\Big|_{\frac{Z^{-1}}{4}=1}=\frac{1}{3}$$

$$c_2=\left(1-\frac{1}{2}Z^{-1}\right)\mathcal{Y}(z)\Big|_{\frac{Z^{-1}}{2}=1}=-2\qquad \mathcal{Y}(z)=\frac{\frac{1}{3}}{1-\frac{1}{4}Z^{-1}}+\frac{-2}{1-\frac{1}{2}Z^{-1}}+\frac{\frac{8}{3}}{1-Z^{-1}}$$

$$c_3=(1-Z^{-1})\mathcal{Y}(z)|_{Z^{-1}=1}=\frac{8}{3}$$

Using the z-transform table,

$$y[n]=\left[\frac{1}{3}\left(\frac{1}{4}\right)^n-2\left(\frac{1}{2}\right)^n+\frac{8}{3}\right]u[n]$$