Solutions to EE3210 Quiz 7 Problems

Problem 1: Given $x_1(t) \leftrightarrow a_k$, using the time reversal property, we have

$$x_1(-t) \leftrightarrow a_{-k}.$$

Then, using the time shift property, we have

$$x_1(1 - t) = x_1[-(t - 1)] \leftrightarrow \left[e^{-jk(2\pi/T)} \right] a_{-k}$$

and

$$x_1(-1 - t) = x_1[-(t + 1)] \leftrightarrow \left[e^{jk(2\pi/T)} \right] a_{-k}.$$

Therefore, using the linearity property, we obtain

$$x_2(t) = x_1(1 - t) + x_1(-1 - t) \leftrightarrow b_k = \left[e^{-jk(2\pi/T)} \right] a_{-k} + \left[e^{jk(2\pi/T)} \right] a_{-k} = 2 \cos \left(\frac{2\pi}{T} k \right) a_{-k}.$$

Problem 2: Using the analysis formula of the discrete-time Fourier series

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$

and choosing the limits of the summation to be $0 \leq n \leq 9$, we have

$$a_k = \frac{1}{10} \sum_{n=0}^{7} e^{-jk(\pi/5)n} = \begin{cases} \dfrac{4}{5}, & k = 0 \\[2mm] \dfrac{1 - e^{-jk(8\pi/5)}}{10 \left[1 - e^{-jk(\pi/5)} \right]}, & 1 \leq k \leq 9. \end{cases}$$