EE3331 Probability Models in Information Engineering

Semester B 2022-2023

Assignment 2

Due Date: 20 February 2023
Important Note: Only writing the answers without steps will get zero mark.

1. Assuming that one shot has a probability of 0.2 for a hit and successive shots are independent, determine the minimum number of shots such that the probability of at least one hit is not less than 0.9.

2. The results of a sample survey in a certain place show that the candidates' foreign language scores follow a Gaussian distribution, with a mean score of 72 and 2.5% of the total number of candidates scoring 96 or above. Determine the probability that the candidates' foreign language score is between 60 points and 84 points.

3. What is the probability that the equation $y^2 + Xy + 1 = 0$ has real roots if $X \sim \mathcal{U}(1,6)$ is a uniform random variable?

4. Consider the experiment of rolling 2 fair dice and the outcome is the absolute difference of the two faces. Assign a random variable (RV) for this experiment and then compute the probability mass function (PMF) for all admissible values of the RV. Then determine the probability that the absolute difference is an odd number.

5. With the use of the binomial distribution, compute the probability of obtaining 10 heads and 90 tails when flipping 100 biased coins with probability of head equals 0.1. Then use Poisson distribution to obtain the approximate probability. Briefly explain why Poisson distribution can provide a valid approximation in this case.

6. Let X be a discrete random variable with possible values of 0, 1, 2, 3, …. Suppose the probability mass function (PMF) of X has the form of:

$$p(x) = \begin{cases} \frac{1}{2}, & x = 0 \\ \\ \frac{1}{\alpha^x}, & x = 1, 2, \cdots \end{cases}$$

   (a) Find the value of α.
   (b) Determine the cumulative distribution function (CDF) of X.

7. A discrete random variable X obeys the following probability function:

$$P(X = x) = \begin{cases} \frac{1}{4}, & x = -2, -1, 0, 1 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Compute $\mathbb{E}\{X\}$ and $\mathrm{var}(X)$.
   (b) Define $Y = |2X - 4|$. Compute $\mathbb{E}\{Y\}$ and $\mathrm{var}(Y)$.

8. Consider the experiment of rolling 2 fair dice and the outcome is the sum of the two faces. We define an event of sum equaling 10. Compute the probability when this event occurs within 5 trials.

9. The cumulative distribution function (CDF) of a continuous random variable X is expressed as:

$$F(x) = \begin{cases} 0, & x < -2 \\ \dfrac{x+2}{4}, & -2 \le x \le 2 \\ 1, & x > 2 \end{cases}$$

   (a) Determine the probability density function (PDF) of X.
   (b) Then compute $\mathbb{E}\{X^4\}$.

10. Given that the probability of Peter winning blackjack is 0.6. Let X be his number of wins in m games.
   (a) Determine the probability mass function (PMF) of X.
   (b) Determine the expected number of wins if Peter plays m games.

11. A gambler tosses a coin 3 times and the tosses are independent. Each time a Head comes, he gains \$1,000 while he loses \$2,000 if the result is Tail. Suppose that the coin is biased and the probability of Head is p. Let K be the amount of money that the gambler gains/loses.
   (a) Determine the probability mass function (PMF) of K.
   (b) Determine the expected amount of the money the gambler gains/loses. Then find the range of p such that the gambler will not lose any money.

12. Two baseball teams Eagles and Gladiators play a best out of five playoff series. The series ends as soon as one of the teams has won three games. It is assumed that either team is equally likely to win any game independently of any other game played.
   (a) Determine $P_N(n)$, that is, the probability mass function (PMF) of the total number N of games played in the series.
   (b) Let W be the number of Eagles' wins in the series. Determine the PMF of W, that is, $P_W(w)$.
   (c) Let L be the number of Eagles' losses in the series. Determine the PMF of L, that is, $P_L(l)$.

13. X is a uniform random variable with $\mathbb{E}\{X\} = 7$ and $\mathrm{var}(X) = 3$. Determine the probability density function (PDF) of X.