Solutions to EE3210 Tutorial 13 Problems

Problem 1:

(a) No. From Property 3 of ROC, we know that, if $x[n]$ is of finite duration, then the ROC is the entire z-plane, except possibly $z = 0$ and/or $z = \infty$. However, in this case, $X[z]$ has a pole at $z = 1/2$. Therefore, $x[n]$ cannot be a finite-duration signal.

(b) No. Since $x[n]$ is absolutely summable, the ROC must include the unit circle, i.e., $|z| = 1$. From Property 5 of ROC, we know that, if $x[n]$ is left sided and the circle $|z| = 1$ is in the ROC, then all values of z for which $0 < |z| < 1$ will also be in the ROC. However, in this case, $X[z]$ has a pole at $z = 1/2$. Therefore, $x[n]$ cannot be a left-sided signal.

(c) Yes. Since $x[n]$ is absolutely summable, the ROC must include the unit circle, i.e., $|z| = 1$. Since $X[z]$ has a pole at $z = 1/2$, one valid ROC would be $|z| > 1/2$. From Property 4 of ROC, we know that this could correspond to a right-sided signal.

(d) Yes. Since $x[n]$ is absolutely summable, the ROC must include the unit circle, i.e., $|z| = 1$. Furthermore, $X[z]$ has a pole at $z = 1/2$. Therefore, one valid ROC would be $1/2 < |z| < a$ such that $a > 1$. From Property 6 of ROC, we know that this could correspond to a two-sided signal.

Problem 2: The partial fraction expansion of $X[z]$ is

$$X[z] = \frac{2/9}{1 - z^{-1}} + \frac{7/9}{1 + 2z^{-1}}.$$

We know that there are two possible inverse z-transforms of the form $1/(1 - az^{-1})$, depending on whether the ROC lies inside or outside the pole. That is:

$$a^n u[n] \leftrightarrow \frac{1}{1 - az^{-1}}, \quad |z| > |a|$$

or

$$-a^n u[-n-1] \leftrightarrow \frac{1}{1 - az^{-1}}, \quad |z| < |a|.$$

(a) With $|z| > 2$ and hence $|z| > 1$, we have

$$x[n] = \frac{2}{9}u[n] + \frac{7}{9}(-2)^n u[n].$$

Therefore, in this case, $x[n]$ is a right-sided signal.

(b) With $|z| < 1$ and hence $|z| < 2$, we have

$$x[n] = -\frac{2}{9}u[-n-1] - \frac{7}{9}(-2)^n u[-n-1].$$

Therefore, in this case, $x[n]$ is a left-sided signal.

(c) With $1 < |z| < 2$, we have

$$x[n] = \frac{2}{9}u[n] - \frac{7}{9}(-2)^n u[-n-1].$$

Therefore, in this case, $x[n]$ is a two-sided signal.

Problem 3: We know that

$$x_1[n] = \left(\frac{1}{2}\right)^n u[n] \leftrightarrow X_1[z] = \frac{1}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}$$

and

$$x_2[n] = \left(\frac{1}{3}\right)^n u[n] \leftrightarrow X_2[z] = \frac{1}{1-\frac{1}{3}z^{-1}}, \quad |z| > \frac{1}{3}.$$

Using the time shift property of the z-transform, we obtain

$$x_1[n+3] \leftrightarrow z^3 X_1[z] = \frac{z^3}{1-\frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}.$$

Using the time shift property followed by the time reversal property of the z-transform, we have

$$x_2[n+1] \leftrightarrow zX_2[z], \quad |z| > \frac{1}{3}$$

and then

$$x_2[-n+1] \leftrightarrow z^{-1}X_2[z^{-1}] = \frac{z^{-1}}{1-\frac{1}{3}z}, \quad |z| < 3.$$

Therefore, using the convolution property of the z-transform, we obtain

$$Y[z] = \left(\frac{z^3}{1-\frac{1}{2}z^{-1}}\right)\left(\frac{z^{-1}}{1-\frac{1}{3}z}\right) = \frac{z^2}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z)}, \quad \frac{1}{2} < |z| < 3.$$