DTFT

$$\text{Defining } \omega = \Omega T$$
$$X(j\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Inverse DTFT

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})^{j\omega n}\, dw$$

Magnitude and Phase

$$\left|X(e^{j\omega})\right| = \sqrt{[\mathrm{Im}(X(e^{j\omega}))]^2 + [\mathrm{Re}(X(e^{j\omega})]^2}$$
$$phaseX(e^{jw}) = \tan^{-1}\left(\frac{[\mathrm{Im}(X(e^{j\omega}))]}{[\mathrm{Re}(X(e^{j\omega})]}\right)$$

Frequency Response

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

CT → DT

$$x[n] = x(t)\Big|_{t=nT} = x(nT)$$

■ Partial fractions.

$\dfrac{f(x)}{(x-a)(x-b)}$	$\dfrac{A}{x-a}+\dfrac{B}{x-b}$
$\dfrac{f(x)}{(x-a)^2}$	$\dfrac{A}{x-a}+\dfrac{B}{(x-a)^2}$
$\dfrac{f(x)}{(x-a)\left(x^2+bx+c\right)}$	$\dfrac{A}{x-a}+\dfrac{Bx+C}{x^2+bx+c}$
$\dfrac{f(x)}{(x-a)(x+d)^2}$	$\dfrac{A}{x-a}+\dfrac{B}{x+d}+\dfrac{C}{(x+d)^2}$
$\dfrac{f(x)}{(x+d)^2}$	$\dfrac{A}{x+d}+\dfrac{B}{(x+d)^2}$
$\dfrac{f(x)}{(x-a)\left(x^2-b^2\right)}$	$\dfrac{A}{x+d}+\dfrac{Bx+C}{x^2-b^2}$
$\dfrac{f(x)}{\left(x^2-a\right)\left(x^2-b\right)}$	$\dfrac{Ax+B}{x^2-a}+\dfrac{Cx+D}{x^2-b}$
$\dfrac{f(x)}{\left(x^2-a\right)^2}$	$\dfrac{Ax+B}{x^2-a}+\dfrac{Cx+D}{\left(x^2-a\right)^2}$

Geometric Series formulas

Interval	*Sum*	*Condition*	*Interval*	*Sum*	*Condition*
Infinite	$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$	$\lvert a \rvert<1$	***Finite on [1,N]***	$\sum_{k=1}^{N} a^k = \frac{a(1-a^{N+1})}{1-a}$	None
Finite on [0,N]	$\sum_{k=0}^{N} a^k = \frac{1-a^{N+1}}{1-a}$	None	***Finite on [N_1,N_2]***	$\sum_{k=N_1}^{N_2} a^k = \frac{a^{N_1}-a^{N_2+1}}{1-a}$	None
Infinite	$\sum_{k=1}^{\infty} a^k = \frac{a}{1-a}$	$\lvert a \rvert<1$	***Finite on [1,N]***	$\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$	None

Sequence	Transform	ROC
$\delta[n]$	1	All z
$\delta[n-m]$	z^{-m}	$\lvert z\rvert > 0$, $m > 0$; $\lvert z\rvert < \infty$, $m < 0$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$a^n \cos(bn)u[n]$	$\frac{1-a\cos(b)z^{-1}}{1-2a\cos(b)z^{-1}+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$a^n \sin(bn)u[n]$	$\frac{a\sin(b)z^{-1}}{1-2a\cos(b)z^{-1}+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

Signal	Transform	ROC
$\delta(t)$	1	All s
$\delta(t-T)$	e^{-sT}	All s
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\Re\{s\} > -a$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\Re\{s\} < -a$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(s+a)^n}$	$\Re\{s\} > -a$
$-\frac{t^{n-1}}{(n-1)!}e^{-at}u(-t)$	$\frac{1}{(s+a)^n}$	$\Re\{s\} < -a$
$e^{-at}\cos(bt)u(t)$	$\frac{s+a}{(s+a)^2+b^2}$	$\Re\{s\} > -a$
$e^{-at}\sin(bt)u(t)$	$\frac{b}{(s+a)^2+b^2}$	$\Re\{s\} > -a$

Table 9.1: Laplace transforms for common signals