EE3008 Test 1

(10:05-11:45am, Oct. 8, 2021)

Question 1 (33 marks)

For each of the following three cases shown in Fig. 1(a) – Fig. 1(c):

1. Plot the Fourier spectrum of $y(t)$; (15 marks)
2. Determine whether the signals $y(t)$ are power-type or energy-type. For energy-type, plot and label the energy spectrum. For power-type, plot and label the power spectrum; (12 marks)
3. Determine the bandwidth of $y(t)$. (6 marks)

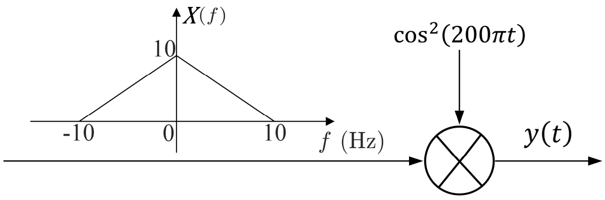

Fig. 1(a)

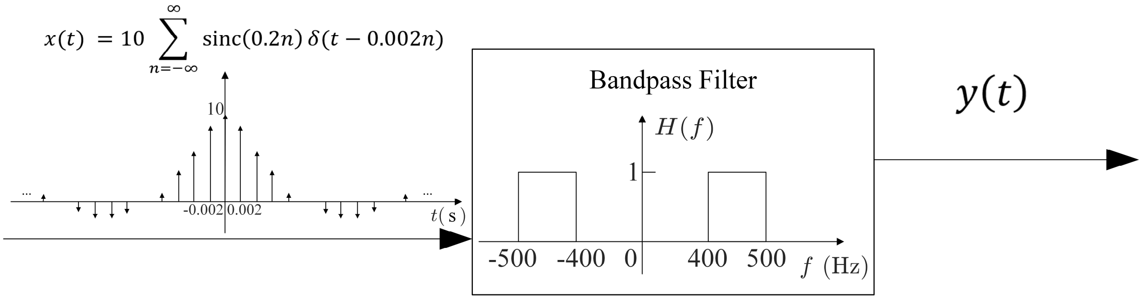

Fig. 1(b)

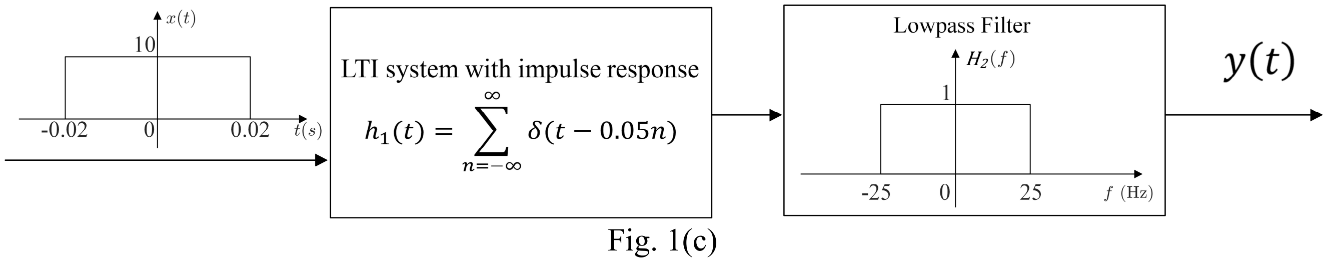

Fig. 1(c)

Question 2 (31 marks)

An input signal $x(t)$ is applied to an AM modulator.

1. Suppose that the input signal $x(t)$ is a sinc-shaped signal, as shown in Fig. 2(a).
   1) If the modulator is an AM-DSB-C modulator, determine the minimum DC offset for the envelope detector to properly work. (3 marks)
   2) If the modulator is an AM-SSB modulator and the carrier frequency is 1 kHz, plot the Fourier spectrum of the output modulated signal $y(t)$. (8 marks)
2. Suppose that the modulator is an AM-DSB-C modulator. The Fourier spectrum $Y(f)$ of the output modulated signal $y(t)$ is shown in Fig. 2(b).
   1) Determine the modulation index; (3 marks)
   2) Determine the expression of $y(t)$; (5 marks)
   3) Determine the output power; (4 marks)

4) Specify whether the modulated signal $y(t)$ can be properly detected by an envelope detector. If yes, sketch and label the output waveform of the envelope detector. If no, determine the minimum DC offset for the envelope detector to properly work. (8 marks)

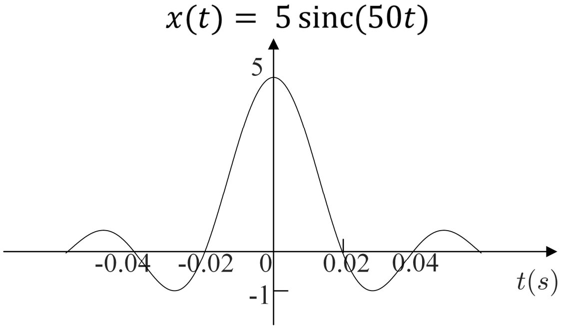

Fig. 2(a)

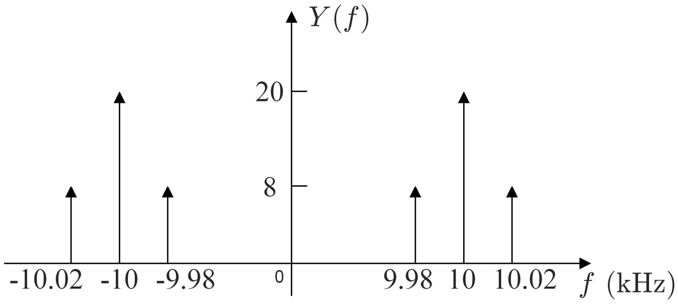

Fig. 2(b)

Question 3 (36 marks)

The output signal of an FM system is given by:

$$s_{FM}(t) = 20\cos\left\{2\pi\left[10^6 t + \int_{-\infty}^{t} A_m \cos(100\pi\tau)\,d\tau\right]\right\}.$$

The modulation index β is equal to 2.

1. Determine the peak amplitude of the message signal, A_m; (4 marks)
2. Determine the effective bandwidth of the modulated signal according to Carson's rule; (3 marks)
3. Determine the output power at 999.95 kHz and 1000.02kHz, respectively; (8 marks)
4. Determine the percentage of the output power at the carrier and the first sidebands, respectively; (6 marks)
5. Determine the minimum required channel bandwidth to include 96% of the output power, and the corresponding channel frequency range; (8 marks)
6. Suppose that the peak amplitude of the input signal is carefully increased until the output signal at 999.9 kHz is zero. Determine the output power at the carrier. (7 marks)

Appendix

Table 6-1 Values of Bessel Function of the First Kind $J_n(\beta)$ for Various Values of n and β

n	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$	$\beta = 9$
0	0.7652	0.2239	−0.2601	−0.3971	−0.1776	0.1506	0.3001	0.1717	−0.0903
1	0.4401	0.5767	0.3391	−0.0660	−0.3276	−0.2767	−0.0047	0.2346	0.2453
2	0.1149	0.3528	0.4861	0.3641	0.0466	−0.2429	−0.3014	−0.1130	0.1448
3	0.0196	0.1289	0.3091	0.4302	0.3648	0.1148	−0.1676	−0.2911	−0.1809
4	0.0025	0.0340	0.1320	0.2811	0.3912	0.3576	0.1578	−0.1054	−0.2655
5	0.0002	0.0070	0.0430	0.1321	0.2611	0.3621	0.3479	0.1858	−0.0550
6	*	0.0012	0.0114	0.0491	0.1310	0.2458	0.3392	0.3376	0.2043
7	*	0.0002	0.0025	0.0152	0.0534	0.1296	0.2336	0.3206	0.3275
8	*	*	0.0005	0.0040	0.0184	0.0565	0.1280	0.2235	0.3051
9	*	*	0.0001	0.0009	0.0055	0.0212	0.0589	0.1263	0.2149
10	*	*	*	0.0002	0.0015	0.0070	0.0235	0.0608	0.1247
11	*	*	*	*	0.0004	0.0020	0.0083	0.0256	0.0622
12	*	*	*	*	0.0001	0.0005	0.0027	0.0096	0.0274
13	*	*	*	*	*	0.0001	0.0008	0.0033	0.0108
14	*	*	*	*	*	*	0.0002	0.0010	0.0039
15	*	*	*	*	*	*	0.0001	0.0003	0.0013
16	*	*	*	*	*	*	*	0.0001	0.0004
17	*	*	*	*	*	*	*	*	0.0001
18	*	*	*	*	*	*	*	*	*
19	*	*	*	*	*	*	*	*	*

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<u>Question 1 (33 marks)</u>

Fig. 1(a):

Let $\cos^2(200\,\pi t)$ be $z(t)$. We have $z(t) = \cos^2(200\,\pi t) = \frac{\cos(400\,\pi t)+1}{2}$ with the Fourier spectrum $Z(f) = \frac{\delta(f-200)}{4} + \frac{\delta(f+200)}{4} + \frac{\delta(f)}{2}$.

Since $y(t) = x(t)cos^2(200\,\pi t)$, by applying convolution property, the Fourier spectrum of $y(t)$ can be given as $Y(f) = X(f) * Z(f) = \frac{X(f-200)}{4} + \frac{X(f+200)}{4} + \frac{X(f)}{2}$, as shown in Fig. 1-1.

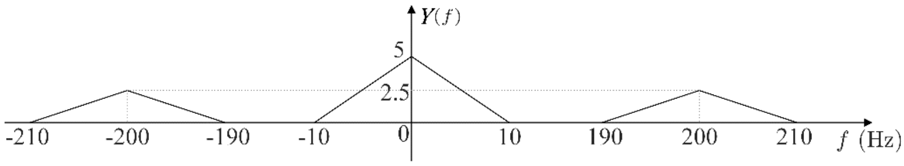

Fig. 1-1

We can see from $Y(f)$ that $y(t)$ is an energy-type signal because its energy is finite. Its energy spectrum is $U(f) = |Y(f)|^2$, which can be plotted as Fig. 1-2 shows.

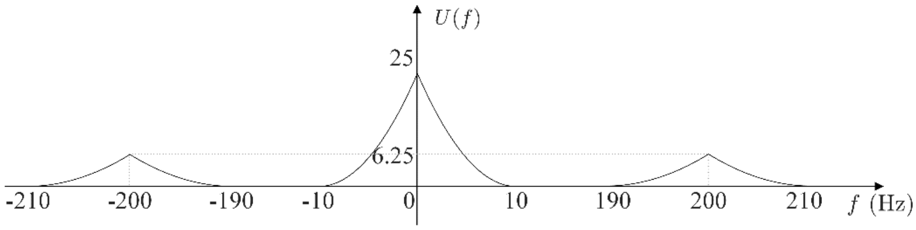

Fig. 1-2

The bandwidth of $y(t)$ is 210 Hz according to $Y(f)$.

Fig. 1(b):

$x(t)$ can be written as $x(t) = 10\text{sinc}(100t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - 0.002n)$. Let $10\text{sinc}(100t)$ be $s_1(t)$ and $\sum_{n=-\infty}^{\infty} \delta(t - 0.002n)$ be $s_2(t)$. According to the convolution property, $X(f) = S_1(f) * S_2(f)$. The Fourier spectrum of $s_1(t)$ is a rectangular pulse (see Tutorial 1, Q1.2 for details) as Fig. 1-3 shows.

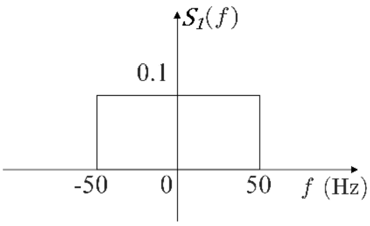

Fig. 1-3

$s_2(t) = \sum_{n=-\infty}^{\infty} \delta(t - 0.002n)$ is an impulse train with the Fourier spectrum $S_2(f) = f_0 \sum_{n=-\infty}^{\infty} \delta(f - nf_0)$, where $f_0 = \frac{1}{T_0} = 500$ Hz (see Lecture 2, Example 6 for details). We then have $X(f) = 500 \sum_{n=-\infty}^{\infty} S_1(f - 500n)$ as plotted in Fig. 1-4.

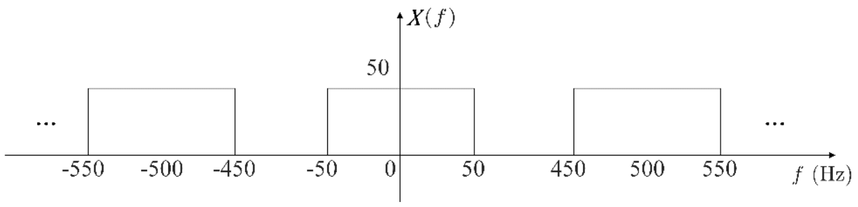

Fig. 1-4

(Note that $X(f)$ can also be obtained by applying the results in Tutorial 1, Q1.3 and then using duality property.)

By passing $x(t)$ through a bandpass filter with frequency range from 400 Hz to 500 Hz, the Fourier spectrum of $y(t)$ can be plotted as Fig. 1-5 shows.

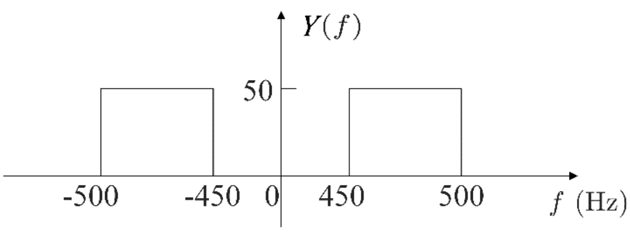

Fig. 1-5

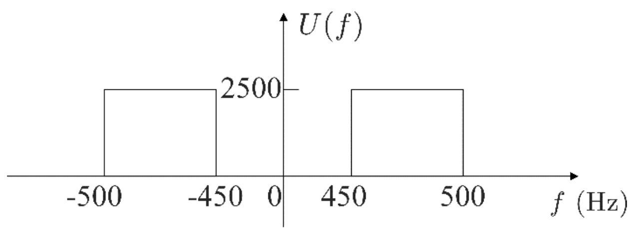

Fig. 1-6

We can see from $Y(f)$ that $y(t)$ is an energy-type signal because its energy is finite.

Its energy spectrum is $U(f) = |Y(f)|^2$, which can be plotted as Fig. 1-6 shows.

The bandwidth of $y(t)$ is 50 Hz according to $Y(f)$.

Fig. 1(c):

The Fourier spectrum of $x(\mathrm{t})$ and $h(t)$ can be given by $X(f) = 0.4\mathrm{sinc}(0.04f)$ (see Lecture 2, Example 5 for details) and $H_1(f) = 20 \sum_{n=-\infty}^{\infty} \delta(f - 20n)$ (see Lecture 2, Example 6 for more details). The output of LTI system is $z(t) = x(t) * h_1(t)$. Using convolution property, we can obtain its Fourier spectrum as $Z(f) = X(f) \cdot H_1(f) = 8 \sum_{n=-\infty}^{\infty} \mathrm{sinc}(0.8n)\, \delta(f - 20n)$, as Fig. 1-7 shows.

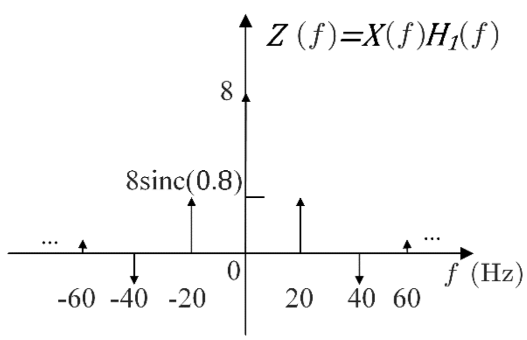

Fig. 1-7

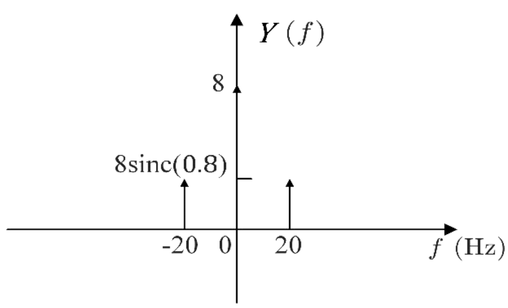

Fig. 1-8

After passing $z(t)$ through the lowpass filter with frequency range from 0 to 25 Hz, the output signal only includes the frequency component at 0, -20, 20 Hz. The Fourier spectrum of $y(t)$ can be given as $Y(f) = 8\delta(f) + 8\mathrm{sinc}(0.8)\delta(f-20) + 8\mathrm{sinc}(0.8)\delta(f+20)$, as shown in Fig. 1-8.

$y(t)$ is a power-type signal because $y(t)$ is a periodic signal in time-domain.

Since it is a periodic signal, its power spectrum is $G(f) = |Y(f)|^2 = 64\delta(f) + 64\mathrm{sinc}^2(0.8)\delta(f-20) + 64\mathrm{sinc}^2(0.8)\delta(f+20)$, which can be plotted as Fig. 1-9 shows.

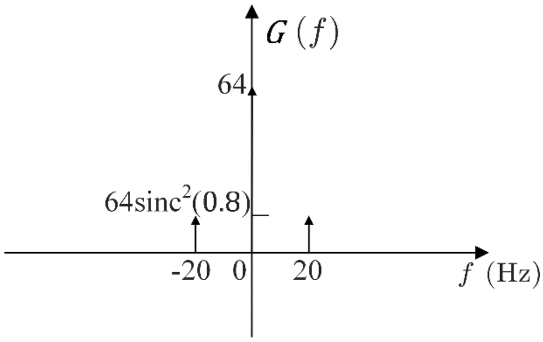

Fig. 1-9

The bandwidth of $y(t)$ is 20 Hz according to $Y(f)$.

Question 2 (31 marks)

Solution:

1. 1) For the envelope detector to properly work, we need $\min_t(x(t)+c) \geq 0$, where c is the DC offset. As shown in Fig. 2(a), $\min_t x(t) = -1$. Thus, the DC offset should be at least 1.

   2) The Fourier spectrum of $x(t)$ is a rectangular pulse (see Tutorial 1, Q1.2 for details) as Fig. 1-10 shows.

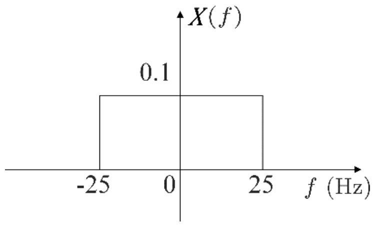

Fig. 1-10

With AM-SSB modulator, either the upper sideband or the lower sideband of $x(t)$ is shifted to the carrier frequency, which is 1 kHz. Then the Fourier spectrum of the output modulated signal $y(t)$ is plotted in Fig. 1-11.

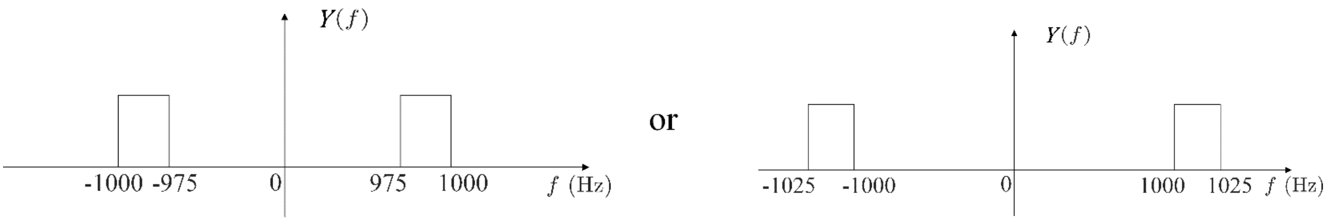

Fig 1-11

2. 1) For $Y(f)$, the magnitude of the carrier components is $Ac/2$ and the magnitude of the sidebands is $Acm/4$ (see Q3, Tutorial 2 for details). According to $\frac{Acm/4}{Ac/2} = \frac{2}{5}$, we have $m = \frac{4}{5}$.

   2) We can see from Fig. 2(b) that the input signal is a sinusoidal signal with frequency 20 Hz, and the carrier frequency is 10 kHz. Moreover, according to $Ac/2 = 20$, we have $Ac = 40$. Therefore, the expression of $y(t)$ can be obtained as $y(t) = 40[\frac{4}{5}\cos(40\pi t) + 1]\cos(20000\pi t)$.

   3) $y(t)$ is a periodic signal, and its power spectrum is $G(f) = |Y(f)|^2$. The output power is $P_y = \int_{-\infty}^{+\infty} G(f)df = 2 \times 20^2 + 4 \times 8^2 = 1056$.

   4) We can use the envelope detector because the modulation index $m \leq 1$. According to $Ac(1+m) = 72$, $Ac(1-m) = 8$ and $f_m = 20$ Hz, the output waveform of the envelope detector is as Fig. 1-12 shows.

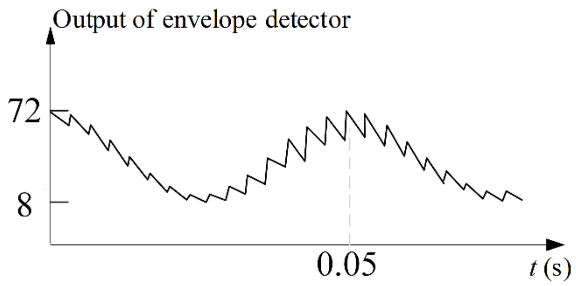

Fig. 1-12

Question 3 (36 marks)

Solution:

1. The instantaneous frequency can be obtained as

$$f(t) = \frac{1}{2\pi} \cdot \frac{d\Psi(t)}{dt} = 10^6 + A_m \cos(100\pi t).$$

   As the carrier frequency is $f_c = 1$ MHz, the peak frequency deviation is $\Delta f = \max_t |f(t) - f_c| = A_m$ Hz.

   The modulation index β can be given by $\Delta f/f_m$. Since $\beta = 2$ and $f_m = 50$ Hz, we have $\Delta f = \beta f_m = 100$ Hz. Then the amplitude of the message signal A_m is 100.

2. According to Carson's rule, the effective bandwidth is $2(\beta + 1)f_m = 300$ Hz.

3. As 999.95 kHz $= f_c - 1f_m$, the output power at 999.95 kHz is then given by $P_t|J_1(2)|^2 = \frac{20^2}{2} \times 0.5767^2 = 66.5166$.

As there is no frequency component at 1000.02 kHz $= f_c + 0.4f_m$, the output power at 1000.02 kHz is 0.

4. The percentage of the output power at the carrier is given by $|J_0(2)|^2 = 0.2239^2 = 0.0501$; the percentage of the output power at the first sidebands is given by $2 \times |J_1(2)|^2 = 2 \times 0.5767^2 = 0.6652$.

5. According to the last question, the total percentage of the output power at the carrier and the first sidebands is $0.0501 + 0.6652 < 0.96$.

   The percentage of the output power at the second sidebands is given by $2 \times |J_2(2)|^2 = 2 \times 0.3528^2 = 0.2489$. Then the total percentage of the output power at the carrier and the first two sidebands is $0.0501 + 0.6652 + 0.2489 = 0.9642 > 0.96$. Therefore, to include at least 96% of the output power, the first two sidebands needs to be transmitted, i.e., from $f_c - 2f_m$ to $f_c + 2f_m$. The minimum required channel bandwidth is then $4f_m = 200\text{ Hz}$. The corresponding channel frequency range is from 999.9 kHz to 1000.1 kHz.

6. 999.9 kHz = $f_c - 2f_m$. According to the table, as β increases from 2, $J_2(\beta)$ first becomes zero when $\beta \approx 5$. Therefore, the output power at the carrier is $P_t|J_0(5)|^2 = \frac{20^2}{2} \times 0.1776^2 = 6.3084$.