EE2331 Data Structures and Algorithms

Trees

Trees

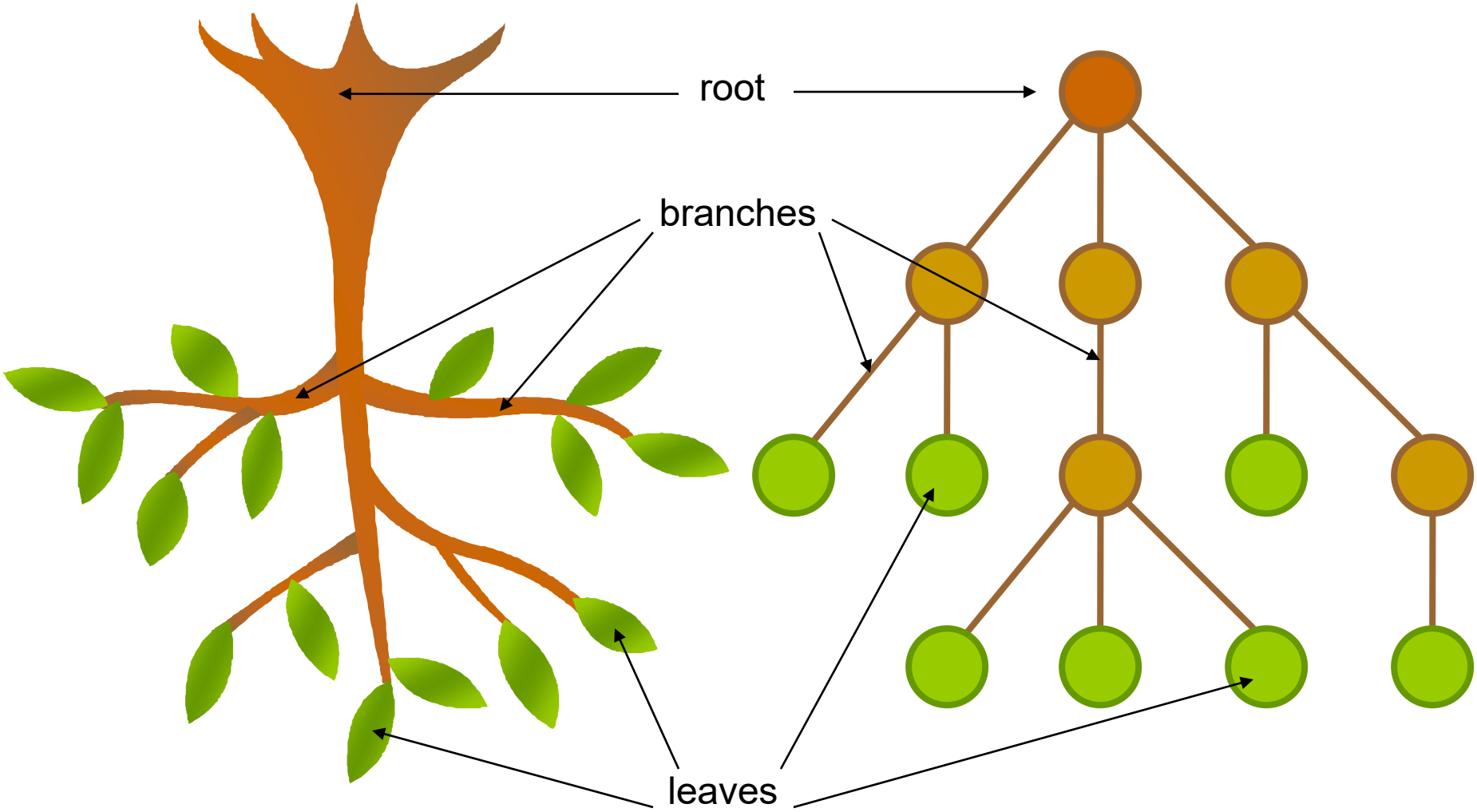
- Tree is a non-linear linked data structure
  - Multiple succeeding elements
  - Tree structure is recursively defined, so tree operations often involve **recursion** and **linked list**

Outline

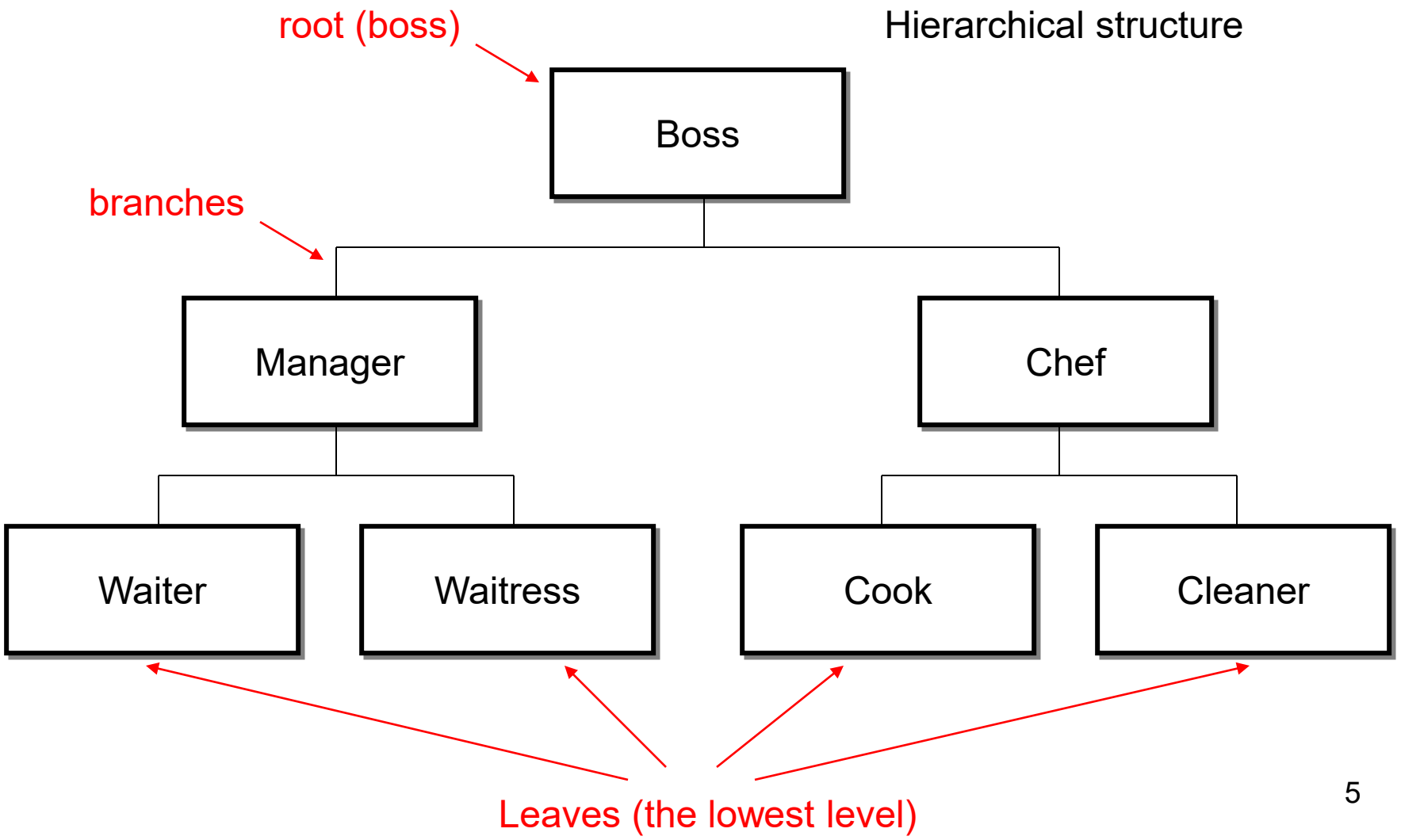
- Terminology
- Representation
- Binary Trees
- Implementations with Array and Linked List
- Common Operations of Binary Tree
- Trees Traversal
  - Preorder, inorder, postorder, level order
- Reconstruction of Binary Trees
- Special Binary Trees
  - Binary Search Trees

- Applications
- General Trees and Other tree structures

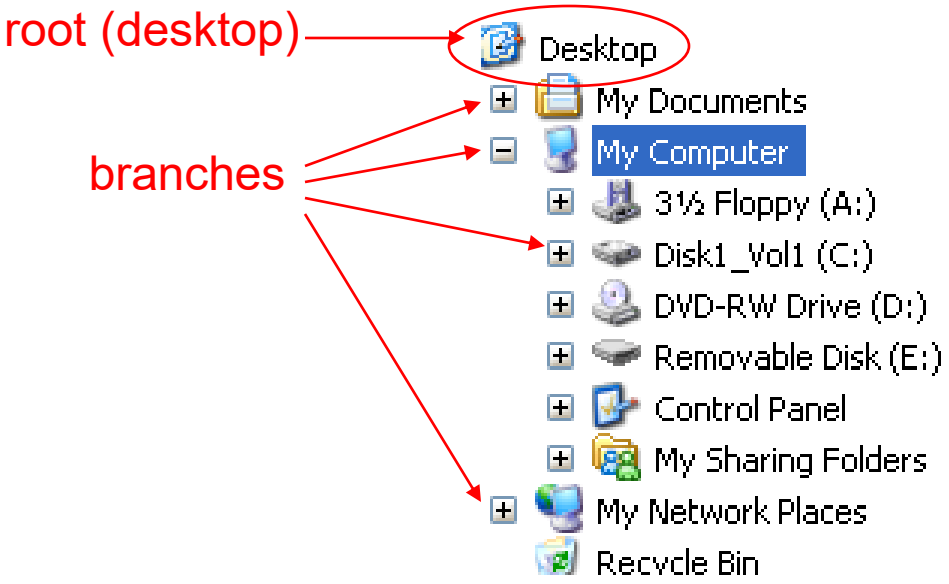
An Inverted Tree

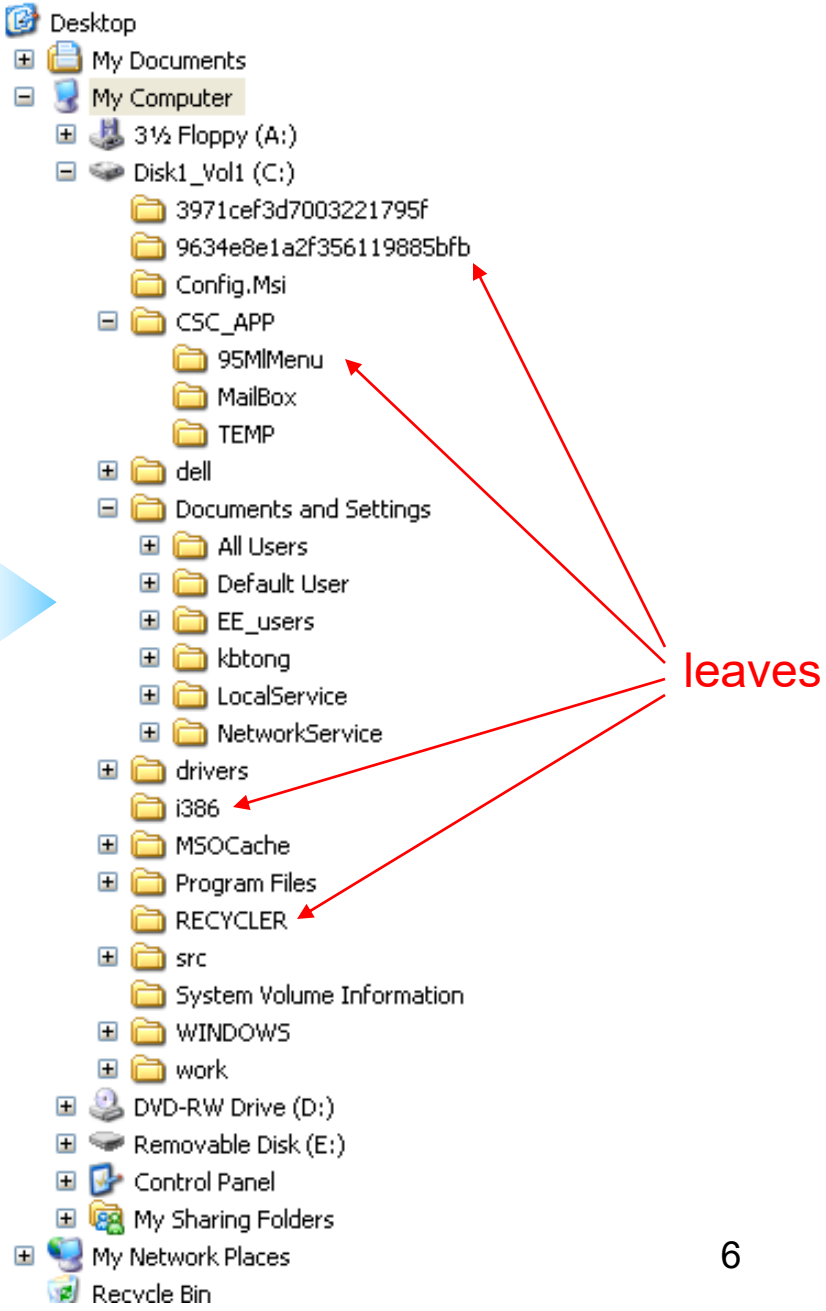


Tree Example: Restaurant



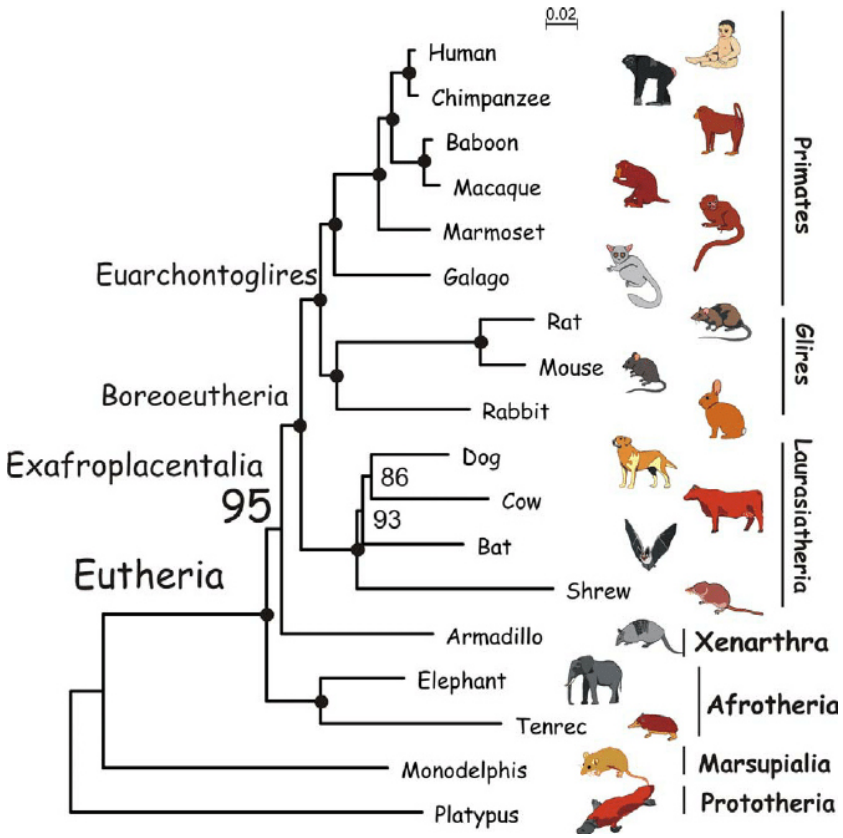
File System

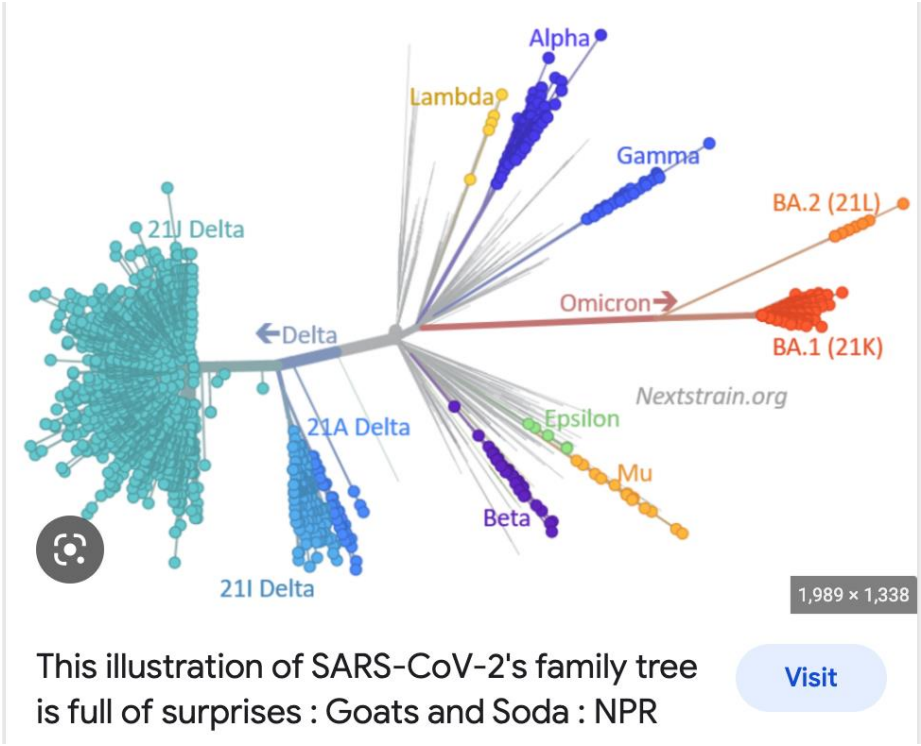




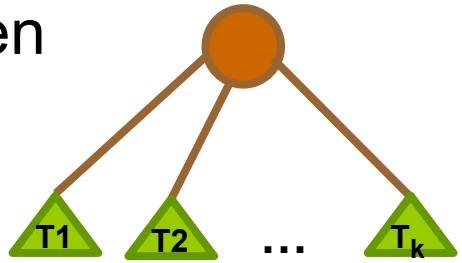
Can you think of other examples of “tree” structures?

Trees in biology

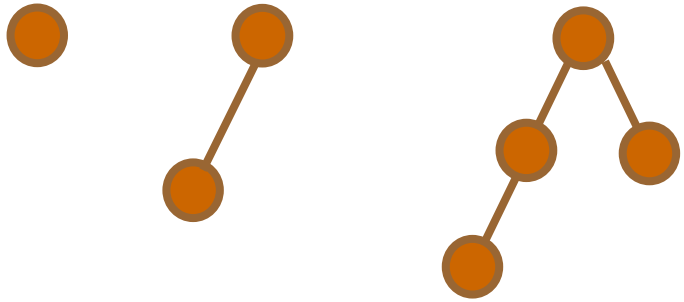




Definition of trees

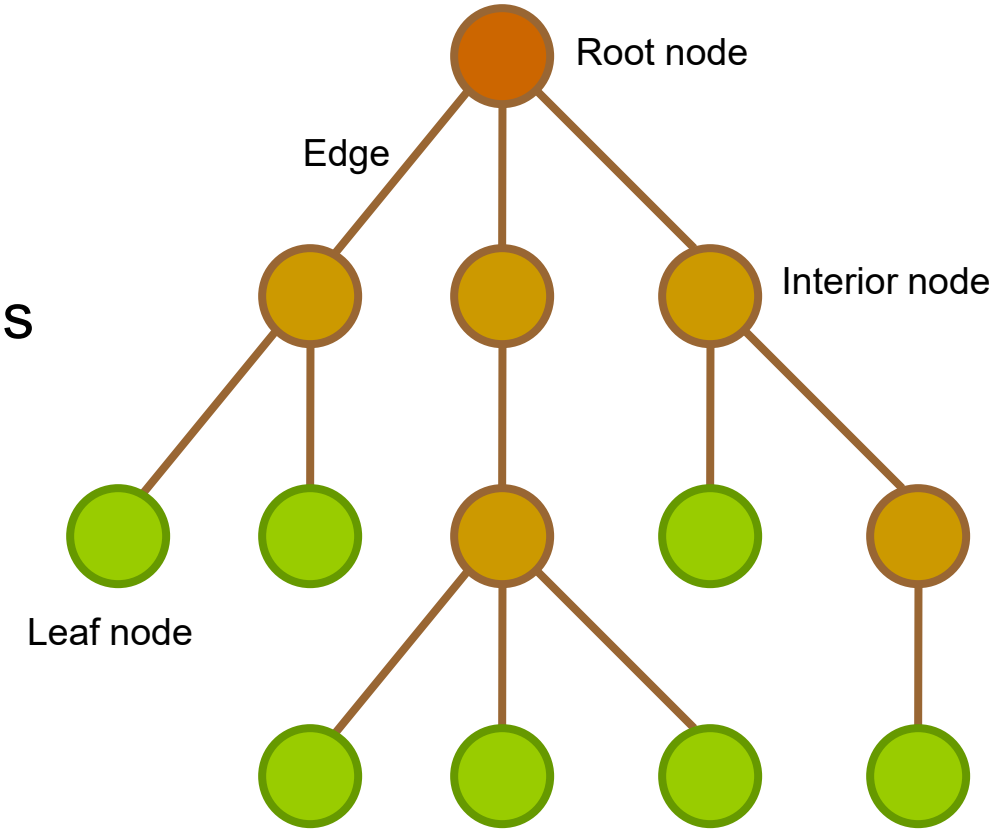
-  is a tree. It is called an "empty tree"
- If  is a node, and  are non-empty trees ($k \geq 0$), then  is a tree.

  When k=0, there is no subtree → a single node is a tree.
- Tree examples:



Composition of a Tree

- Types of tree node
  - Root node (the top node in a tree)
  - Interior nodes (nodes with at least one child)
  - Leaf nodes (nodes with no children)

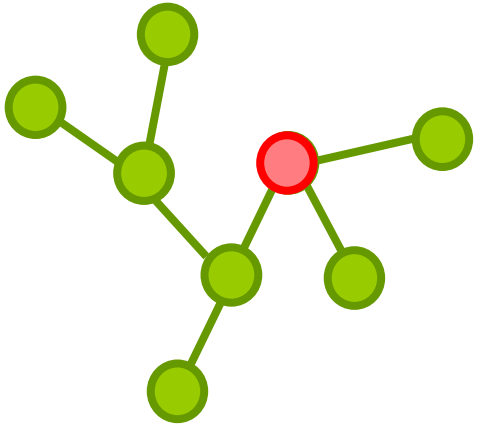


Tree Criteria (vs. graph)

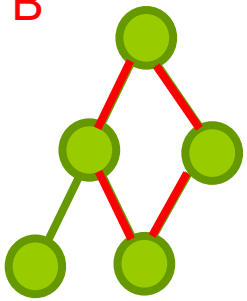
- Always a single root node
- A single path from the root to node

- Exercise: which of the following is not a tree?

A

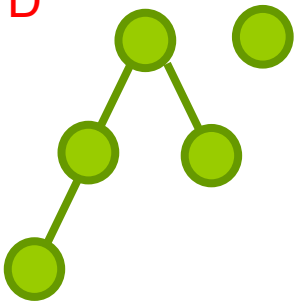


B



C

D



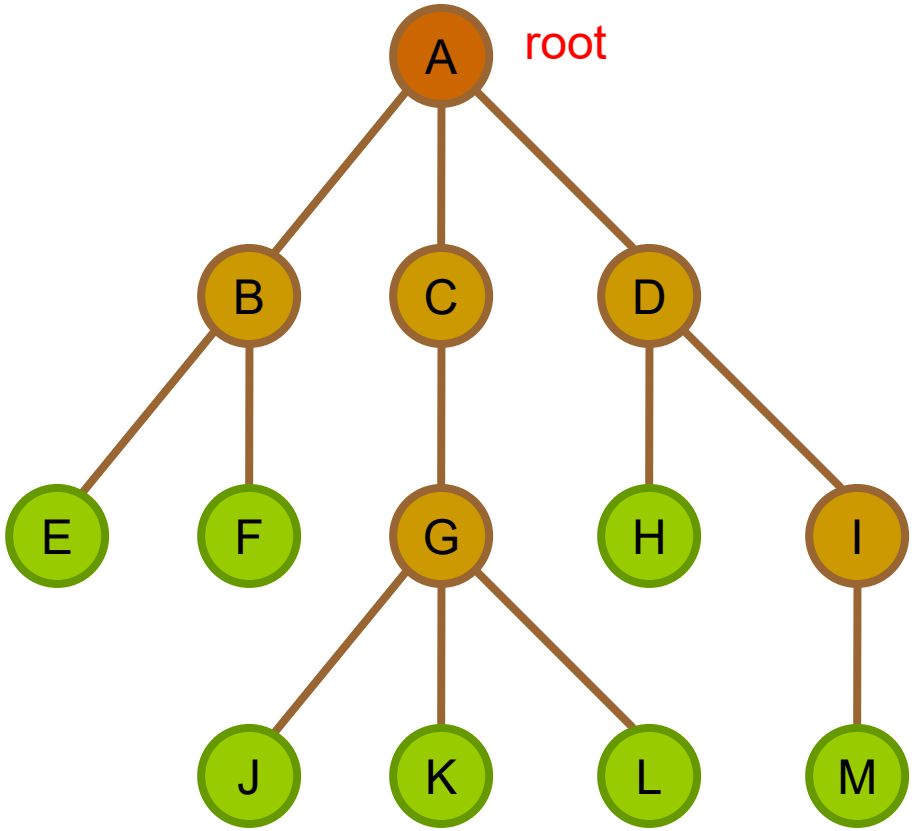
Answer: B and D

Property of Trees

- Nodes represent information (data)
- Branches represent links between the nodes
- If the total number of nodes (i.e. **root node**, **interior nodes** and **leaf nodes**) is *n*, how many branches in the tree?
  - Number of branches is ?

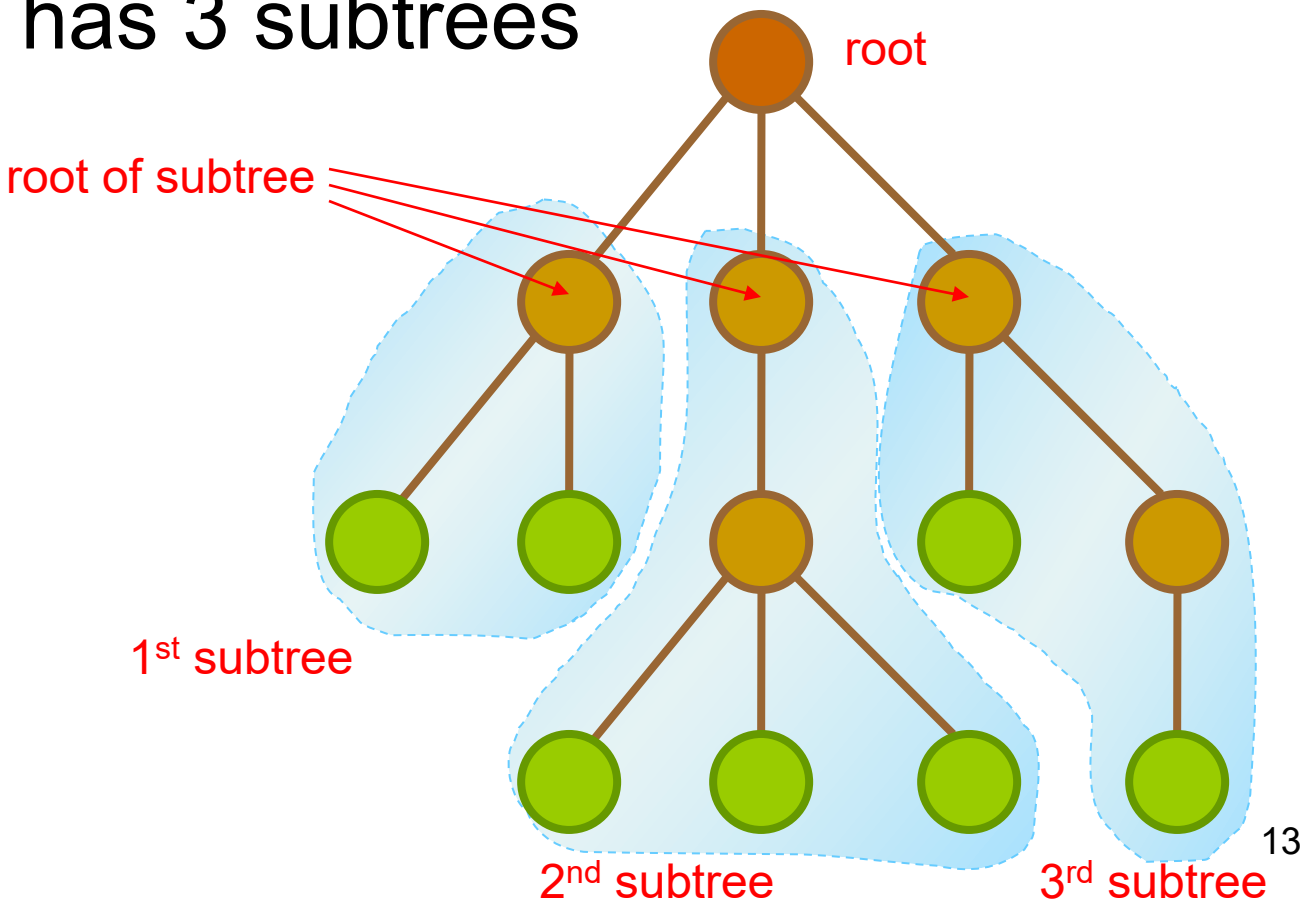
Parent, Children & Sibling

- This tree has 13 nodes
- Node A has 3 children
  - Nodes B, C and D
- Node A is the parent of
  - B, C and D
- Node G is the parent of
  - Nodes J, K and L
- Node G is the child of C
- J, K and L are sibling nodes (share the same parent)



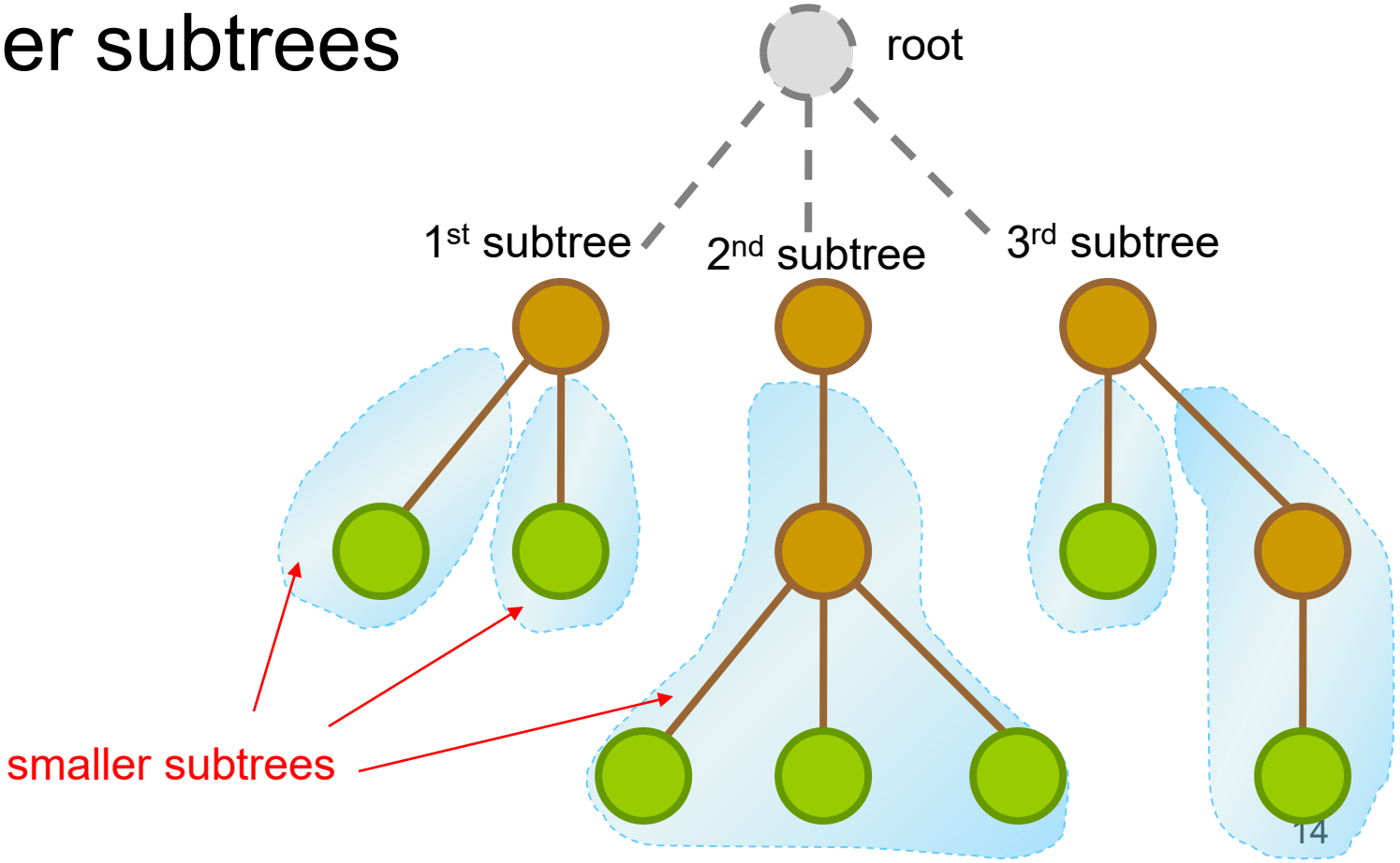
Subtrees

- A tree is composed of several subtrees
- e.g. root has 3 subtrees



Smaller subtrees

- A subtree can be further broken down into smaller subtrees

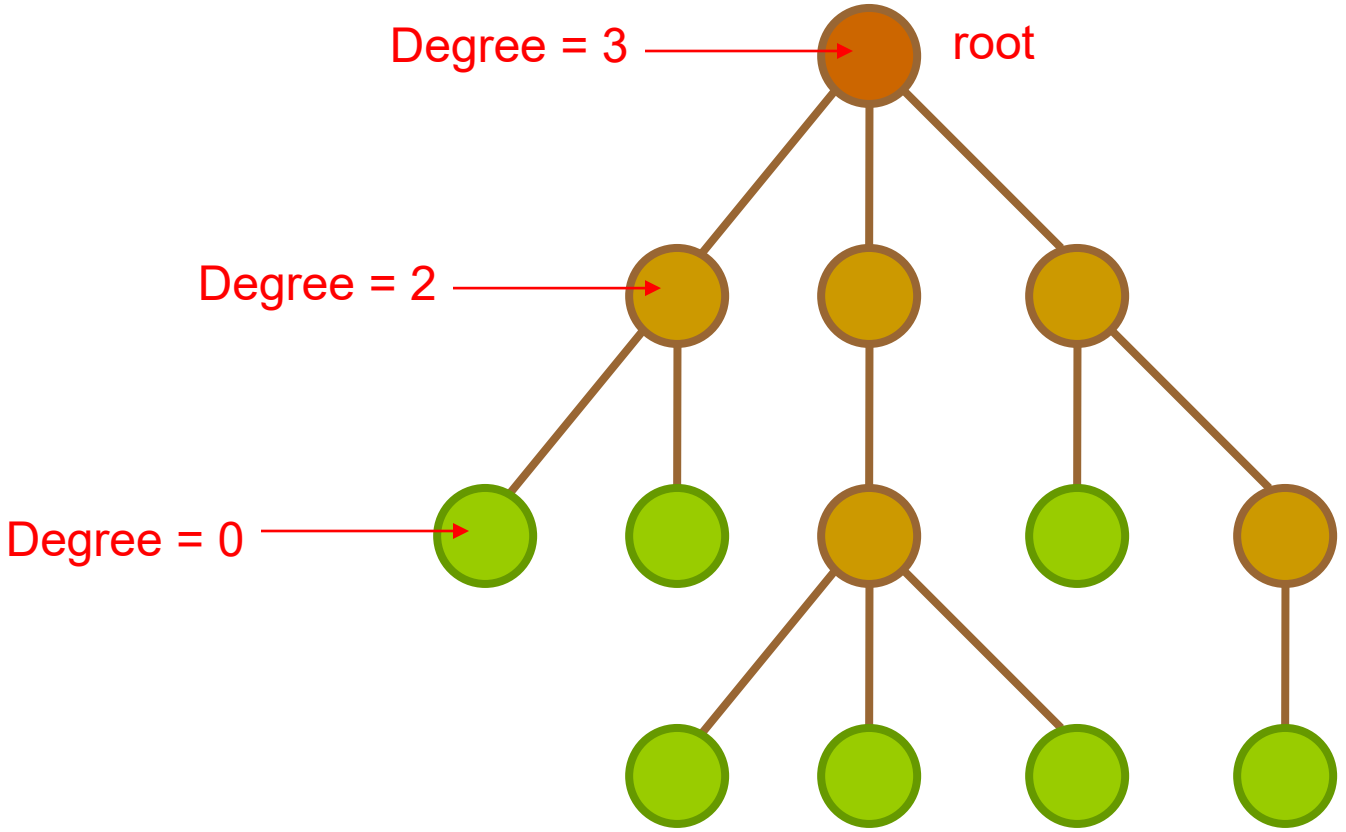


Ancestor and Descendant

- A **simple path** is a sequence of nodes $n_1, n_2, \ldots, n_k$ such that the nodes are all distinct and there is an edge between each pair of nodes (n_1, n_2), (n_2, n_3), …, (n_{k-1}, n_k)
- The nodes along the simple path from the root to node x are the **ancestors** of x
- The **descendants** of a node x are the nodes in the subtrees of x
- **Length of a path** = no. of edges on the path

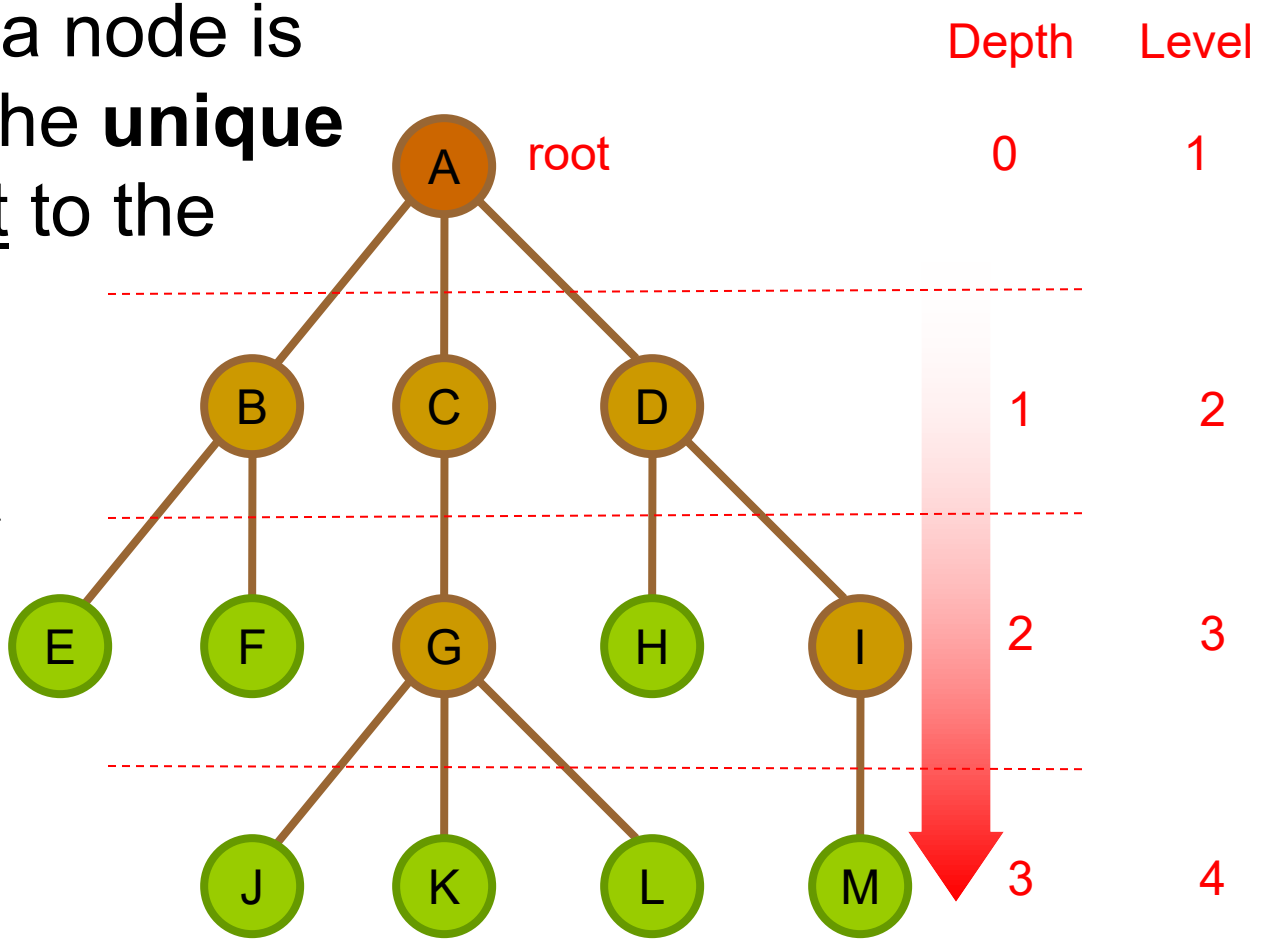
Degree

- The number of subtrees of a node



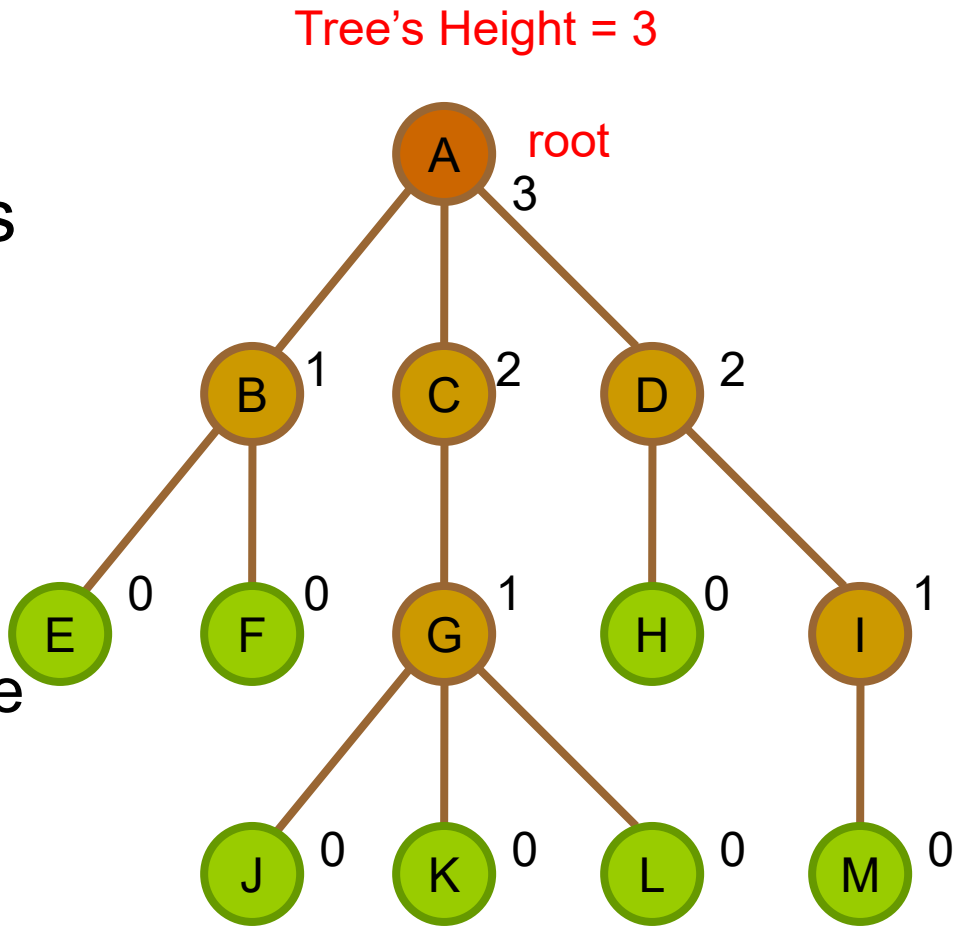
Depth and Level

- The **depth** of a node is the length of the **unique** path from root to the node.
- The **level** of a node is defined by its depth plus 1.



Height

- The **height** of a node is the number of edges from the node to the deepest leaf.
- The height of a tree is a height of the root.
  - = the longest path in the tree



In-class Exercise (no submission needed)

- A node having no parent is called Root
- A node having no children is called Leaf
- A node having both parent and children is called Branch

- If a tree has 5 branches, how many nodes does this tree contain? 5+1

- What is the degree of a leaf node? _____
- Can a node have more than one parent in a tree? _____
- Is there a unique path from root to every node? _____

Tree Representation

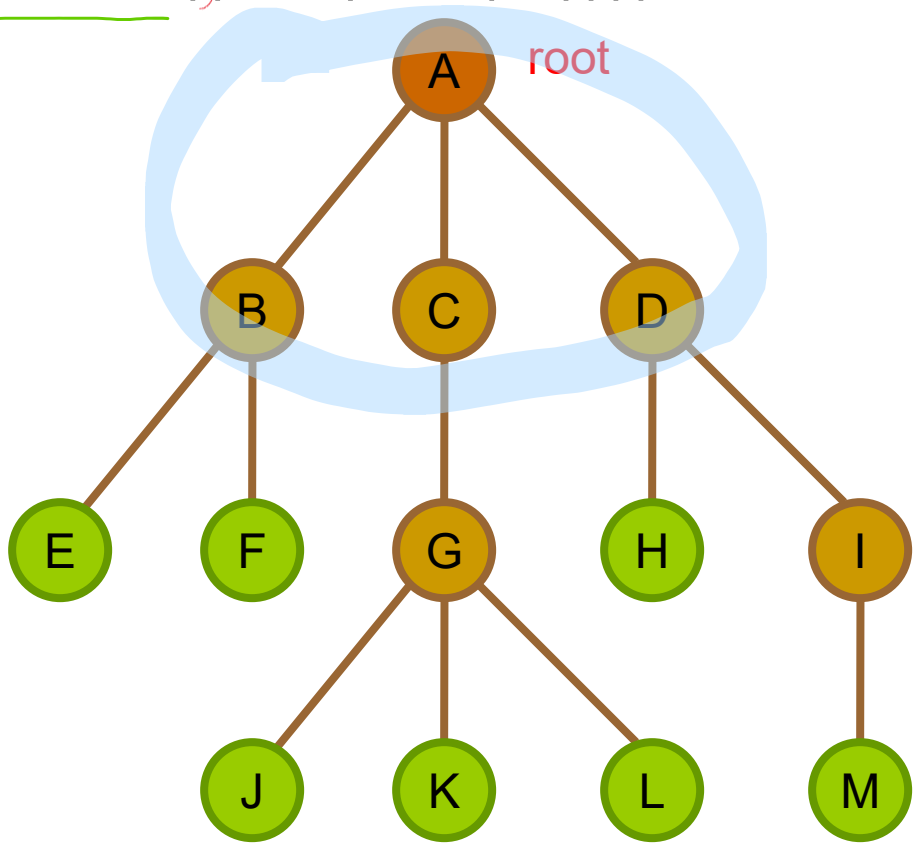
Representation of Trees

- List representation
- Set representation
- Indentation

List Representation (recursive)

- The tree can be represented by this list
  - (A(B(E, F), C(G(J, K, L)), D(H, I(M))))

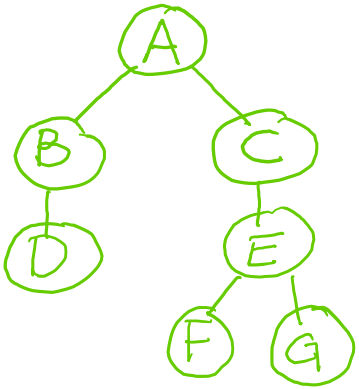
Starting from the root A. A=A(B,C,D). B, C, and D are not leaf nodes. They are the roots of subtrees. We need to recursively represent them using root(child1, child2, …, the last child). Thus,
B=B(E,F)
C=C(G), G=G(J,K,L)
D=D(H,I), I=I(M)

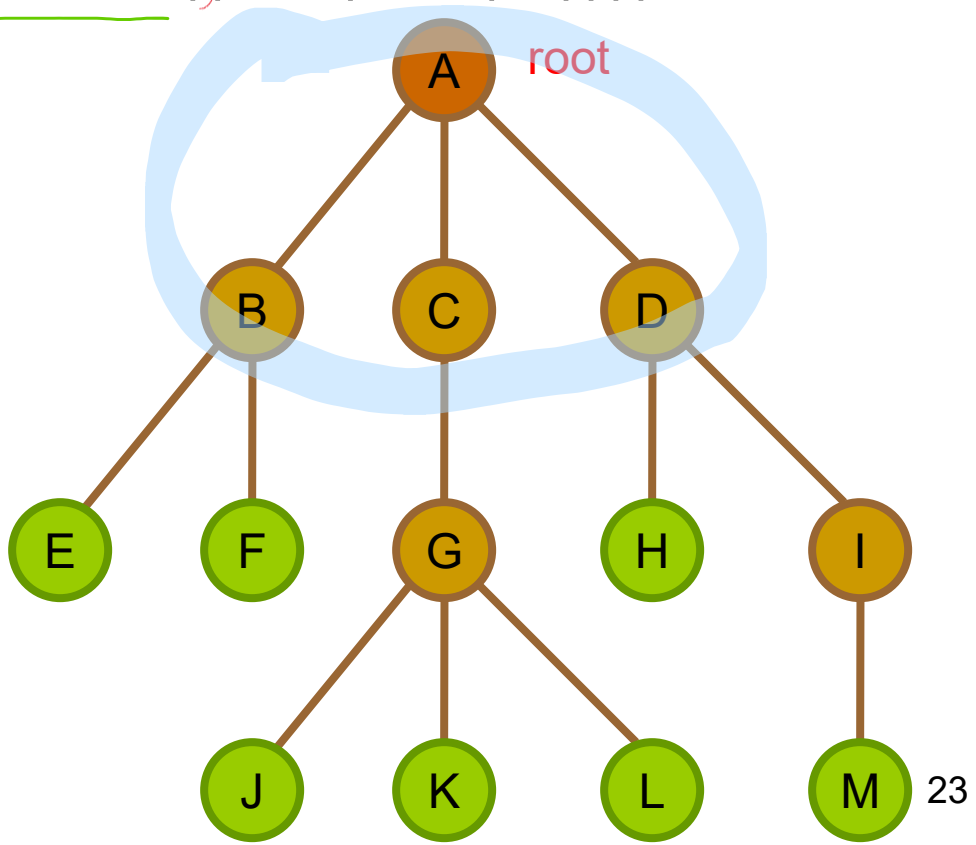


List Representation (recursive)

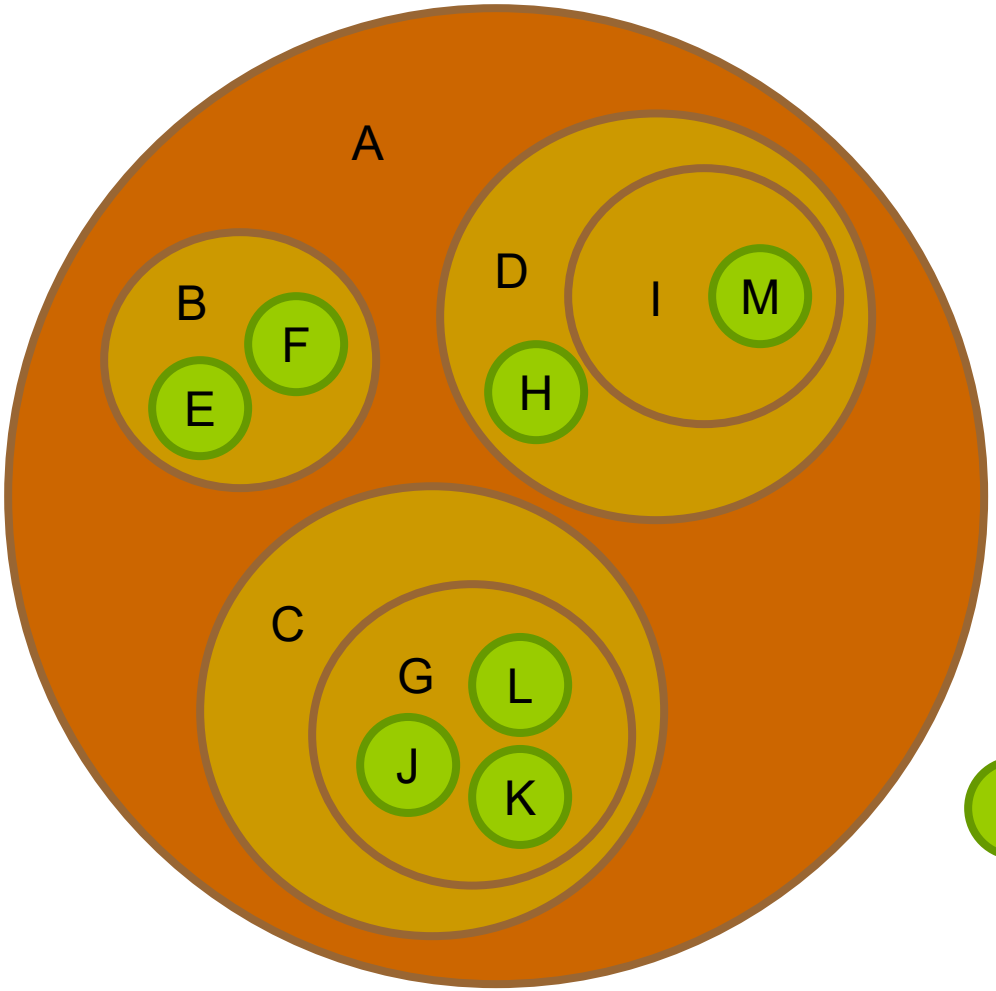
- The tree can be represented by this list
  - (A(B(E, F), C(G(J, K, L)), D(H, I(M))))

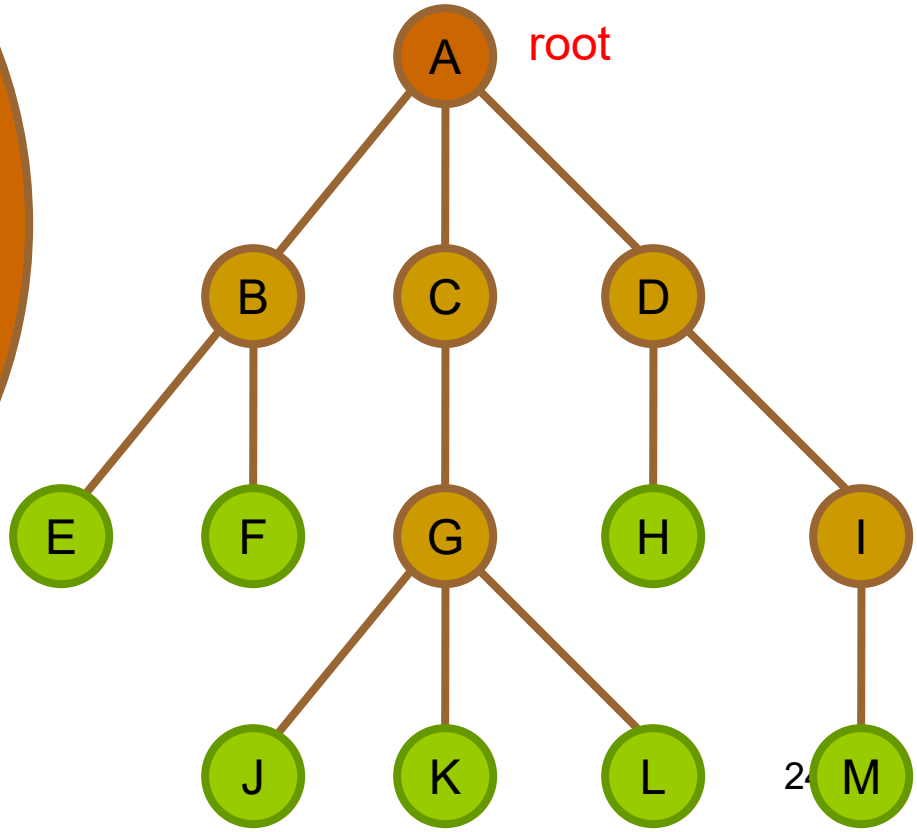
What is the list representation of the following tree?



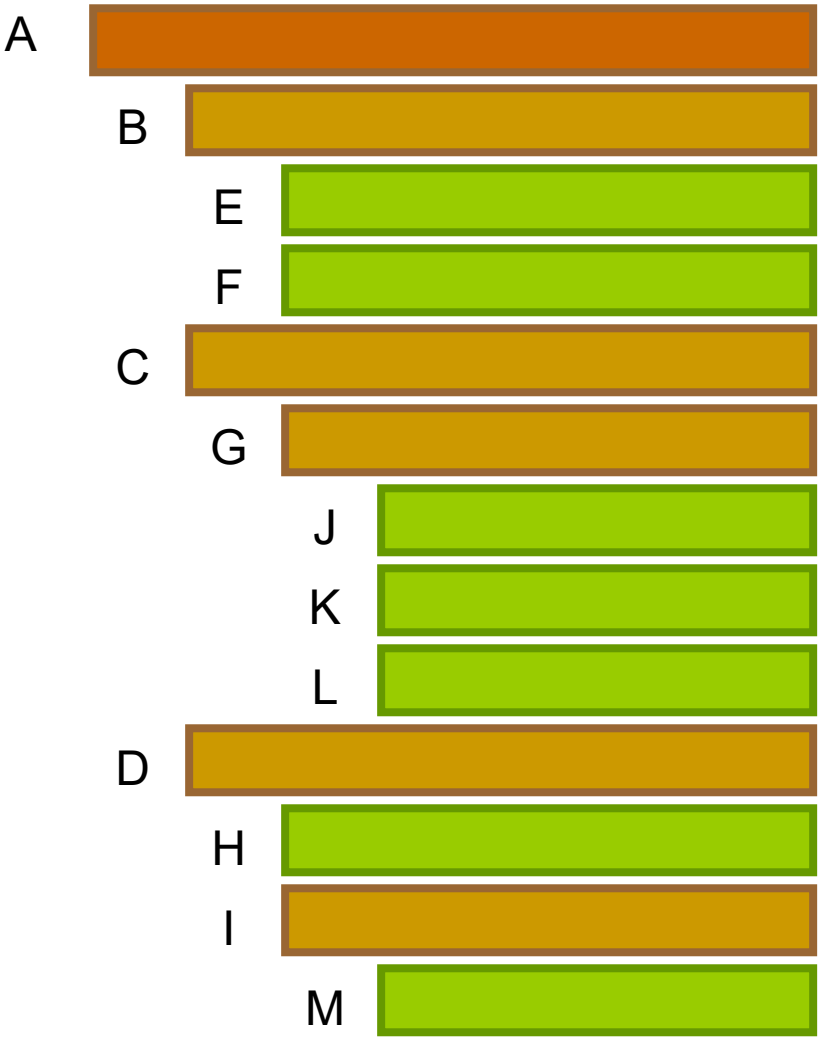


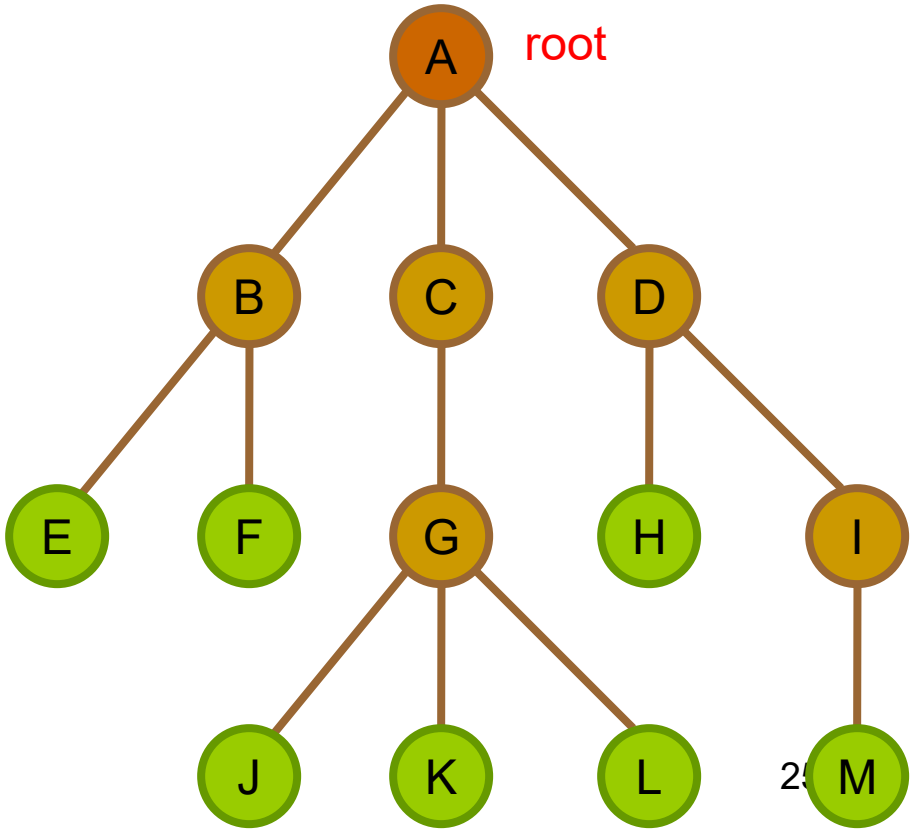
Set Representation





Indentation Representation





They Are Also Indentation

- A
  - B
    - E
    - F
  - C
    - G
      - J
      - K
      - L
  - D
    - H
    - I
      - M