EE2302 Foundations of Information Engineering Semester A, 2022/23

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1) (9 marks) Let $A = \{1,2,4,5,9\}$, $B = \{2,3,6\}$, $C = \{2,3,5,7,8\}$. $D = \{3,7,\ldots\}$ $|D| = 5$

a) Find the power set of B.

$P(B) = \{1,2,3,4,5,6,7,8,\cancel{9}\} = A \cup C$ −3

b) Find $B \times (A \cap C)$.

$A \cap C = \{2,5\}$

$B \times (A \cap C) = \{(2,2),(2,5),(3,2),(3,5),(6,2),(6,5)\}$

c) Let D be a set with cardinality $|D| = 5$, and $C \cap D = \{3,7\}$. What is the cardinality of $C \cup D$?

2 in common

$|C \cup D| = 5 + 5 - 2 = 8$

+3

2) (10 marks) Let $A = \{n \in \mathbb{Z}: n \equiv 7 \pmod 8\}$ and $B = \{n \in \mathbb{Z}: n \equiv 3 \pmod 4\}$.

a) Is $A \subseteq B$? Prove or disprove it.

For A: $7 \pmod 8 = \cancel{0}$

For B: $3 \pmod 4 = \cancel{0}$

$\because 0 = 0$

$\therefore A \subseteq B$

−2

b) Is $B \subseteq A$? Prove or disprove it.

Yes, $7 \pmod 8 = 3 \pmod 4$, $0 = 0$

$\therefore B \subseteq A$

−5

0.75, $4\overline{)3.0}$, 28, 20, 20, 0

0.875, $8\overline{)7.0}$, 64, 60, 56, 40, 40, 0

[0, 1] (4]

+1

3) (9 marks) Let $A = \{x \in \mathbb{R}: 0 \le x \le 1\}$ and $B = \{x \in \mathbb{R}: 3 < x \le 4\}$.

a) What is $A \cup B$?

$A \cup B = \{0, 1, \cancel{4}\}$

b) Compare $|A|$ with $|A \cup B|$. Which one is larger? Explain your answer.

$|A| = 2$ $|A \cup B| = 3$

$\because 3 > 2$

$\therefore |A \cup B|$ is larger

+6

4) (9 marks) Let $A = \{1,2,3,4\}$ and $B = \{5,6,7\}$. Let f be the relation from A to B defined by

$$f = \{(1,5),(2,6),(3,5),(p,q)\},$$

In each of the following case, find a value for p and a value for q so that the statement is true. If such values cannot be found, explain why.

a) The relation f is not a function.

suppose (p,q) is $(4,7)$, there is no relation between $(1,5), (2,6), (3,5)$, hence f cannot be a function, value of (p,q) must not be a functional relation

7 -3

b) The relation f is an injective function from A to B.

False statement, no value, f is not injective, as 1 and 3 both maps to 5, not satisfy injection. Cannot find (p,q)

c) The relation f is a surjective function from A to B.

If $p \in A$ and $q = 7$, then it forms surjection

example $(p,q) = (4,7)$

+3

5) (12 marks) For each of the following, determine whether the function is injective, surjective, or both. Prove your assertions.

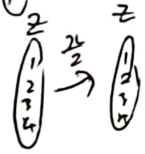
a) $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = 2x$.

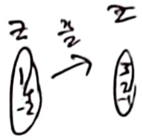
$\forall y$ in $\mathbb{Z}$, there is a $2y$ in $\mathbb{Z}$ that can be mapped distinctivly, hence injection.

$\forall x$ in co-domain, every element has at least one inverse image, hence surjection

$\therefore$ Bijection

b) $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(x) = x/2$ if x is even and $f(x) = (x-1)/2$ if x is odd.





$\frac{x}{2}$ when x is even $=$ $\frac{x-1}{2}$ when x is odd ;

$\frac{x}{2}$ will always be even - $f(x) = 2x$

$\therefore$ It is bijection

$[0,1]$ $[1,\infty)$

+10

6) (15 marks) Let $X = \{x \in \mathbb{R}: 0 < x \leq 1\}$ and $Z = \{z \in \mathbb{R}: z \geq 1\}$. The function $f: X \to Y$ is a bijection defined by $f(x) = x^2 + 4x - 3$. The function $g: Z \to X$ is defined by $g(z) = 1/z$. $y = \frac{1}{z}$

a) Determine the co-domain Y of the function f.

$Y = \{Z\}$

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b) Is g a bijection? If so, find its inverse function. If not, give your reason.

$\frac{1}{z}$ is always in image of $[0,1]$, since $\frac{1}{1} = 1$ and $\frac{1}{\infty} > 0$, hence surjection

distinct $\frac{1}{z}$ always maps to distinct elements in X,

$\therefore$ $g^{-1}(z) = \frac{1}{z}$

c) Let $Y = \mathbb{R}$, determine $f \circ g$. Is $f \circ g$ an injection? State your reason.

$f(g(x)) = f(\frac{1}{z}) = (\frac{1}{z})^2 + 4(\frac{1}{z}) - 3$, $\in \mathbb{R}$

suppose $x^2 + 4x - 3$ ⇒ $2x + 4 = 0$ $x = -2$ $x < 0$, there are 2 distinct outputs

Therefore for each distinct z, there can map to distinct real numbers

$\therefore$ Is injection

+2

7) (8 marks) Prove by contraposition that if a sum of two integers is less than 100, then at least one of the numbers is less than 50.

$\sim p \to \sim q$

If sum > 100 → only 1 number < 50

$x+y > 100$

let $x < 50$, then $y > 50$

If $x < 50$, $y < 50$, cannot satisfy

i.e. can be only one ~~number~~ < 50

∴ Proved by contraposition

+8+1

(+9)

8) (12 marks) Consider these two relations:

- Define a relation R on $\mathbb{R}_+$, the set of all non-negative real numbers, as follows: For all $x, y \in \mathbb{R}_+$, $xRy \leftrightarrow x^2 \le y^2$.
- Let S be the set of all binary strings. Define a relation T on S as follows: For all $s, t \in S$, $sTt \leftrightarrow l(s) \le l(t)$, where $l(x)$ denotes the length of a string x.

a) Which one is not a partial order relation? Justify your answer.

T is not, Not antisymmetric

$sTt, tTz \Rightarrow l(s) \le l(t)$, $l(t) \le l(z)$ −3

$l(s) \le l(t) \not\Rightarrow l(t) \le l(s)$

b) Prove that the other one is a partial order relation.

R: $x^2 \le x^2$, $x^2 = x^2$, $x = x$ ⇒ reflexive

$xRy, yRx \Rightarrow x^2 = y^2 \Rightarrow x = y$, antisymmetric

suppose $xRy, yRz \Rightarrow x^2 \le y^2, y^2 \le z^2, \Rightarrow x^2 \le z^2$, transitive

∴ R is partial order relation

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9) (16 marks) Let R and S be relations on $\mathbb{B}^\infty$, where $\mathbb{B}^\infty$ is the set of all infinite binary sequences. Define $g: \mathbb{B}^\infty \to \mathbb{Z}$ where $g(x)$ is determined by treating the first three bits of x as the binary representation of an integer, as shown in the following table:

First three bits of x	000	001	010	011	100	101	110	111
$g(x)$	0	1	2	3	4	5	6	7

For example, if $x = 010000...$ and $y = 010111...$ then $g(x) = g(y) = 2$. Define R by xRy if $g(x) = g(y)$. Furthermore, define S by xSy if $g(x) \leq g(y)$.

a) Is R an equivalence relation? If so, determine the number of its distinct equivalence classes and list one member of each of them. If not, determine whether it is a partial order. Explain your answers.

If $x = x$, then $g(x) = g(x)$, reflexive

let xRy and yRz, $g(x) = g(y)$, $g(y) = g(z)$, then $g(x) = g(z)$, transitive

Suppose xRy : $g(x) = g(y)$, then yRx : $g(y) = g(x)$, Symmetric

∴ R is equivalence

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$\{x \in \mathbb{Z} \mid x > 0\}$: $[1]$

$\{x \in \mathbb{Z} \mid x = 0\}$: $[0]$ 2 equivilant classes

b) Repeat (a) by considering the relation S.

xSx : $g(x) \leq g(x)$, $x = x$ reflexive

xSy, ySz : $g(x) \leq g(y)$, $g(y) \leq g(z)$ → $g(x) \leq g(z)$ ⇒ transitive

If xSy → ySx ⇒ ~~Symmetric~~

∴ S equivilant

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