
EE3210 Signal and System

Final Exam

Honor Pledge

Please review the following honor code, then sign your name and write down the date.

1. I pledge that the answers in this exam are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,
 - (a) I will not plagiarize (copy without citation) from any source;
 - (b) I will not communicate or attempt to communicate with any other person during the exam;
 - (c) neither will I give or attempt to give assistance to another student taking the exam; and
 - (d) I will use only approved devices (e.g., calculators) and/or approved device models.
2. I understand that any act of academic dishonesty can lead to disciplinary action.

Signature

Date

Problem 1.

Questions Answer

- a) (i)
- b) (ii)
- c) (iv)
- d) (i)

Problem 2.

Question Answer

- a) (iii)
- b) (iv)
- c) (iv)

Problem 3.

(a)

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)d\tau - x(t) \quad \text{where } z(t) = e^{-t}u(t) + \delta(t)$$

$$\frac{dy(t)}{dt} + 10y(t) = x(t)z(t) - x(t)$$

$$(j2\pi f + 10)Y(f) = X(f)[z(f) - 1]$$

$$\mathcal{H}(f) = \frac{Y(f)}{X(f)} = \frac{z(f) - 1}{j2\pi f + 10} = \frac{1}{(j2\pi f + 10)(j2\pi f + 1)}$$

(b)

$$\mathcal{H}(f) = \frac{1}{(j2\pi f + 10)(j2\pi f + 1)} = \frac{c_1}{j2\pi f + 10} + \frac{c_2}{j2\pi f + 1}$$

$$c_1 = (j2\pi f + 10)\mathcal{H}(f)|_{j2\pi f = -10} = -\frac{1}{9}$$

$$c_2 = (j2\pi f + 1)\mathcal{H}(f)|_{j2\pi f = -1} = \frac{1}{9}$$

$$\mathcal{H}(f) = \frac{1}{9(j2\pi f + 1)} - \frac{1}{9(j2\pi f + 10)} \xrightarrow{F^{-1}} h(t) = \frac{1}{9}(e^{-t} - e^{-10t})u[t]$$

Problem 4. .

(a) The LT of the differential equation is $(s^2 + 6s + 8)Y(s) = sX(s)$

$$\mathcal{H}(s) = \frac{Y(s)}{X(s)} = \frac{s}{s^2 + 6s + 8} = \frac{c_1}{s + 4} + \frac{c_2}{s + 2}$$

$$c_1 = (s + 4)\mathcal{H}(s)|_{s=-4} = \frac{-4}{-2} = 2$$

$$c_2 = (s + 2)\mathcal{H}(s)|_{s=-2} = \frac{-2}{2} = -1$$

By using the LT table,

$$h(t) = 2e^{-4t}u(t) - e^{-2t}u(t)$$

$$Y(s) = \mathcal{H}(s) \cdot X(s) \quad x(t) = u(t) \xrightarrow{L_I} X(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{(s + 4)(s + 2)} = \frac{c_1}{s + 4} + \frac{c_2}{s + 2}$$

$$c_1 = (s + 4)\mathcal{H}(z)|_{s=-4} = -1/2$$

$$c_2 = (s + 2)\mathcal{H}(z)|_{s=-2} = 1/2$$

The step response of the given equation.

$$y(t) = -\frac{1}{2}e^{-4t}u(t) + \frac{1}{2}e^{-2t}u(t)$$

$$ROC \quad Re(s) > -2$$

$$(b) \quad y(t) = e^t \left[4 + 4 \int_0^t e^{-\tau} y(\tau) d\tau \right] \quad t \geq 0$$

$$\frac{dy(t)}{dt} = e^t \left[4 + 4 \int_0^t e^{-\tau} y(\tau) d\tau \right] + 4e^t(e^{-t}y(t)) = 5y(t)$$

By obtaining the initial condition as $y(0^-) = 4$

$$sY_I(s) - y(0^-) = 4Y_I(s)$$

$$sY_I(s) - 4 = 5Y_I(s)$$

$$Y_I(s) = \frac{4}{s - 5}$$

After applying the inverse LT, $y(t) = 4e^{5t}u(t)$.

Problem 5.

(a) Apply the time-shifting property,

$$\left(1 - \frac{3}{4}Z^{-1} + \frac{1}{8}Z^{-2}\right)Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4}Z^{-1} + \frac{1}{8}Z^{-2}} = \frac{Z^2}{\left(Z - \frac{1}{4}\right)\left(Z - \frac{1}{2}\right)} \quad \text{ROC, } |Z| > \frac{1}{2}$$

Let's represent $H(z)$ in the following form,

$$H(z) = \frac{1}{1 - \frac{3}{4}Z^{-1} + \frac{1}{8}Z^{-2}} = \frac{c_1}{1 - \frac{1}{4}Z^{-1}} + \frac{c_2}{1 - \frac{1}{2}Z^{-1}}$$

$$c_1 = \left(1 - \frac{1}{4}Z^{-1}\right)H(z)\Big|_{\frac{Z^{-1}}{4}=1} = -1$$

$$c_2 = \left(1 - \frac{1}{2}Z^{-1}\right)H(z)\Big|_{\frac{Z^{-1}}{2}=1} = 2$$

Based on the Z-transform table,

$$h[n] = 2 \cdot \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n] = \left[2 \cdot \left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u[n]$$

(b) Since the $|Z| = 1/2$, which contain the $|Z| = 1$, so, it is a stable system.

In addition, we can find that the $\text{ROC} > 1/2$, which means that it is a casual system.

(c) Since $u[n] \leftrightarrow \frac{1}{1-Z^{-1}}, |Z| > 1$

$$Y(z) = H(z)X(z) \quad \text{where } X(z) = Z(u[n])$$

$$Y(z) = \frac{1}{\left(1 - \frac{1}{4}Z^{-1}\right)\left(1 - \frac{1}{2}Z^{-1}\right)(1 - Z^{-1})} = \frac{c_1}{1 - \frac{1}{4}Z^{-1}} + \frac{c_2}{1 - \frac{1}{2}Z^{-1}} + \frac{c_3}{1 - Z^{-1}}$$

$$c_1 = \left(1 - \frac{1}{4}Z^{-1}\right)Y(z)\Big|_{\frac{Z^{-1}}{4}=1} = \frac{1}{3}$$

$$c_2 = \left(1 - \frac{1}{2}Z^{-1}\right)Y(z)\Big|_{\frac{Z^{-1}}{2}=1} = -2 \quad Y(z) = \frac{\frac{1}{3}}{1 - \frac{1}{4}Z^{-1}} + \frac{-2}{1 - \frac{1}{2}Z^{-1}} + \frac{\frac{8}{3}}{1 - Z^{-1}}$$

$$c_3 = (1 - Z^{-1})Y(z)\Big|_{Z^{-1}=1} = \frac{8}{3}$$

Using the z-transform table,

$$y[n] = \left[\frac{1}{3}\left(\frac{1}{4}\right)^n - 2\left(\frac{1}{2}\right)^n + \frac{8}{3}\right] u[n]$$