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EE2302 Foundations of Information Engineering Semester A, 2022/23

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Answer ALL questions.

1. (10 marks) Define a function $f: \mathbb{R}^2 \to \mathbb{R}$ by $f(x) = x_1y_2 - x_2y_1$, where y_1, y_2 are constants and $x = (x_1, x_2)$. Is f a linear function? Prove or disprove it.

$f(\alpha x + by) = \alpha f(x) + b f(y)$ where $\alpha = y_1,\ b = y_2$

$\alpha f(x) + b f(y) = \alpha(x_1 y_2 - x_2 y_1) + b(x_1 y_2 - x_2 y_1) = (\alpha+\beta)(x_1 y_1 - x_2 y_2)$

2

Addition:

Multi:

2. (10 marks) Using Cauchy-Schwarz inequality to prove that

$$\sum_{m=1}^{M}(b_m - 1)^2 \geq \frac{1}{M}\left(\sum_{m=1}^{M}(b_m - 1)\right)^2$$

(Hint: Cauchy-Schwarz inequality: $|a^T b| \leq \|a\|\,\|b\|$.)

0

$\Sigma(b_m - 1)^2 = \Sigma[(b_M - 1)^2 + \dots + (b_1 - 1)^2]$

$\frac{1}{M}[\Sigma(b_m - 1)]^2 = \frac{1}{M}[(b_M - 1) + \dots + (b_1 - 1)]^2$

$\frac{1}{M}[\Sigma(b_m - 1)]^2 \leq \Sigma(b_m - 1)^2$

$[\Sigma(b_m - 1)]^2 \leq M\Sigma(b_m - 1)^2$

$\Sigma(b_m - 1)^T \Sigma(b_m - 1) \leq M(b_m - 1)^2$

3. (10 marks) Let x and y be vectors in $\mathbf{R}^3$, where $x = (1,2,3)$ and $y = (3,2,1)$. Is the span of x and y a subspace of $\mathbf{R}^3$? Prove or disprove it.

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Yes, $x \in \mathbb{R}^3$, $y \in \mathbb{R}^3$, span of x and $y \in \mathbb{R}$

Addition: $(1,2,3) + (3,2,1) = (1,2,3)$

Multiplication: $c(1,2,3) = c(3,2,1)$

4. Consider the equation $Ax = b$, where $A = \begin{bmatrix}1 & 2\\3 & 6\end{bmatrix}$ and $b = \begin{bmatrix}4\\12\end{bmatrix}$.

a) (7 marks) Determine the column space and the null space of A.

6

$C(A) = \{x \in \mathbb{R}^2 : \begin{bmatrix}1\\3\end{bmatrix}\}$ basis is $\begin{bmatrix}1\\3\end{bmatrix}$ since $2\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}2\\6\end{bmatrix}$

$Ax = 0$ $\quad \begin{bmatrix}1 & 2\\3 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$

$x_1 + 2x_2 = 0$

$N(A) = \begin{bmatrix}2\\1\end{bmatrix}$

b) (8 marks) Determine the general solution for x.

3

$Ax = b$ $\quad \begin{bmatrix}1 & 2\\3 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}4\\12\end{bmatrix}$

$x_1 + 2x_2 = 4$

GS: $x = 4 + \alpha\begin{bmatrix}2\\1\end{bmatrix}$, where $\alpha \in \mathbb{R}$

5. Consider the binary linear code C which appends four parity bits c_5, c_6, c_7, c_8 to four information bits u_1, u_2, u_3, u_4 in the following way. Put the four information bits in a 2×2 array and the four parity bits are obtained by two-dimensional even parity as shown below (i.e., each row and each column has an even number of 1's):

u_1	u_2	c_5
u_3	u_4	c_6
c_7	c_8	

a) (6 marks) The encoding function $f: \mathbb{B}^4 \rightarrow \mathbb{B}^8$ can be expressed in the form of $f(u) = uG$, where u is the row vector (u_1, u_2, u_3, u_4). Determine the generator matrix G and the corresponding parity check matrix H.

0

$G_{8\times4}$ $\quad P_{4\times4}$

ee ab c
ee abc
aa bce
bb cca
cc eab

b) (6 marks) Is the function f surjective? Prove or disprove it.

Surjective

0

c) **(6 marks) Determine the minimum distance of the code and state its error correction capability. Explain how you obtain the minimum distance.**

$\|x-y\|$

d) **(6 marks) Suppose the vector $(1,0,0,0,1,1,1,1)$ is received. Determine its syndrome.**

Stockholmes syndrome
Alzymers Syndrome

e) **(6 marks) Suppose nearest-neighbor decoding is used. Determine the decoder output for the received vector given in (d). Explain how you obtain the answer.**

6. Consider the function $f_\theta: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f_\theta(x) = A_\theta x$, where x is a 2-dim column vector, $A_\theta = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$ is the rotation matrix with $0 \le \theta < 2\pi$.

0

a) (5 marks) Given any value of θ, is f_θ a linear function of x? Prove or disprove it.

Suppose $B_\theta = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$

$B_\theta + A_\theta = \begin{pmatrix}2\cos\theta & -2\sin\theta \\ 2\sin\theta & 2\cos\theta\end{pmatrix} = 2\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$

b) (5 marks) Assume $\theta = \frac{\pi}{4}$. Write down the inverse function of $f_{\pi/4}$. Steps are not required.

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

0

$f_{\frac{\pi}{4}} = A_{\frac{\pi}{4}} x$

$= \begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix} x$

$= \frac{1}{\sqrt{2}}\begin{bmatrix}1 & -1 \\ 1 & 1\end{bmatrix} x$

$x = \sqrt{2}\begin{bmatrix}1 & -1 \\ 1 & 1\end{bmatrix} = f^{-1}_{\frac{\pi}{4}}$

c) (5 marks) Determine $f_{\pi/4} \circ f_{\pi/2}$, where the operator $\circ$ denotes function composition. Steps are not required.

0

d) (5 marks) Let $F = \{f_0, f_{\pi/4}, f_{\pi/2}, f_{3\pi/4}, f_\pi, f_{5\pi/4}, f_{3\pi/2}, f_{7\pi/4}\}$. Explain why $\langle F, \circ\rangle$ is a group. Your explanation can be based on the properties of the rotation matrix. Rigorous mathematical derivation is not required. CAIN properties

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Closure
Addition
Identity matrix

e) (5 marks) List all subgroups of $\langle F, \circ\rangle$.

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