Tutorial 2
Amplitude Modulation (AM)

Problem 1 (AM-DSB-SC)

Signal $s(t)$ (with Fourier transform $S(f)$) is applied to a double-sideband suppressed-carrier (DSB-SC) modulator operating at a carrier frequency of 200 Hz with a scaling factor of 1. Sketch the spectrum of the resulting AM-DSB-SC waveform and identify the upper and lower sidebands for each of the following cases.

(i) $s(t) = \cos 100\pi t$

(ii) $S(f) = \begin{cases} [1+\cos(\pi f/100)]/2 & |f| < 100 \\ 0 & \text{elsewhere} \end{cases}$

Solution

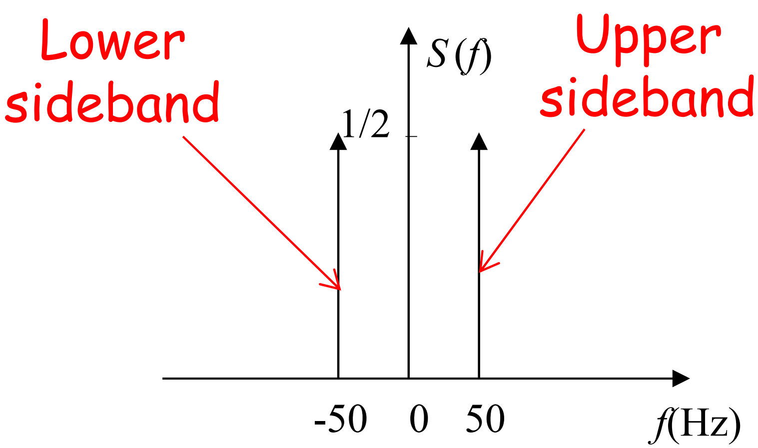
(i) The spectrum of $s(t) = \cos 100\pi t$ is given by

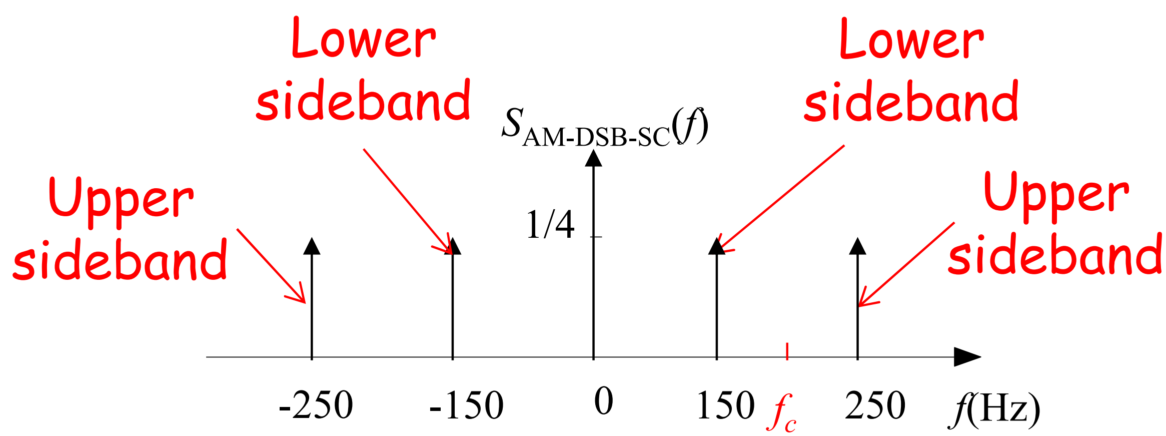
$$S(f) = (1/2)[\delta(f-50)+\delta(f+50)]$$

Therefore, according to $s_{AM\text{-}DSB\text{-}SC}(t)=s(t)\cos(2\pi f_c t)$ and f_c=200Hz,

$$S_{AM\text{-}DSB\text{-}SC}(f)=(1/2)[S(f-f_c)+S(f+f_c)]$$

$$=(1/4)[\delta(f-250)+\delta(f-150)+\delta(f+150)+\delta(f+250)]$$

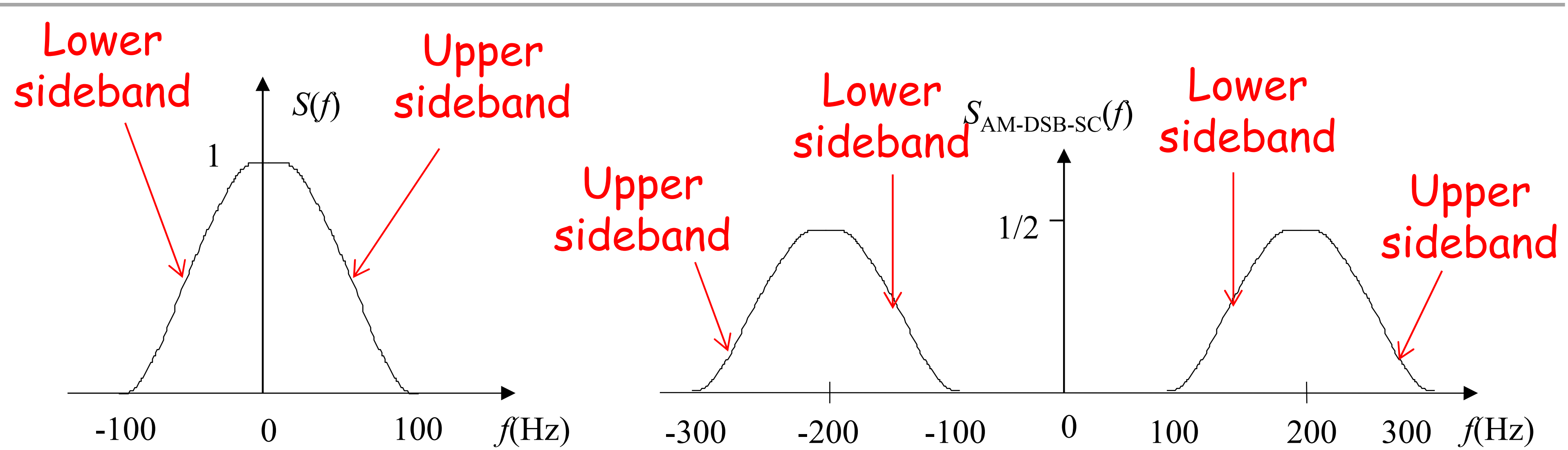




Solution

(ii) $$S(f)=\begin{cases}[1+\cos(\pi f/100)]/2 & |f|<100 \\ 0 & \text{elsewhere}\end{cases}$$

$$S_{AM\text{-}DSB\text{-}SC}(f)=(1/2)S(f+f_c)+(1/2)S(f-f_c)$$

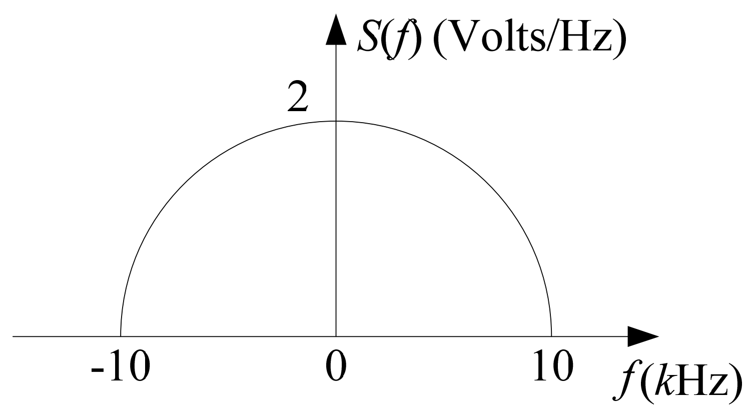


Problem 2 (AM-DSB-C)

Consider an information signal with the spectrum shown below. Suppose we have a channel capable of passing frequencies in the range $300k\text{Hz} \le f \le 320k\text{Hz}$ and we want to transmit the signal across the channel using AM-DSB-C with a scaling factor of 1 and a modulation index of 0.667. Suppose that the maximum amplitude of the information signal is +2 volts and the minimum amplitude is -2 volts.

1) Determine the carrier frequency.

2) Draw the spectrum of the transmitted signal.



Solution

1) Carrier frequency is 310 kHz.

2) $s_{AM-DSB-C}(t) = A(s(t)+c)\cos(2\pi f_c t)$

$\Leftrightarrow$

$$S_{AM-DSB-C}(f) = \tfrac{A}{2}[S(f-f_c)+S(f+f_c)] + \tfrac{Ac}{2}[\delta(f-f_c)+\delta(f+f_c)]$$

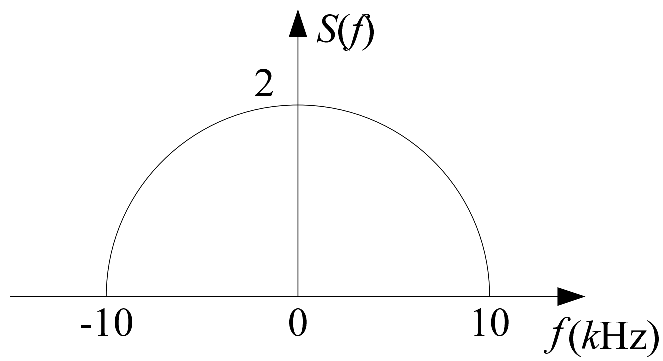
$$A = 1$$

$$m = \frac{\max[s(t)+c]-\min[s(t)+c]}{\max[s(t)+c]+\min[s(t)+c]} = \frac{(2+c)-(-2+c)}{(2+c)+(-2+c)} = \frac{4}{2c} = 0.667 \Rightarrow c = \frac{2}{0.667} = 3 \text{ volts}$$

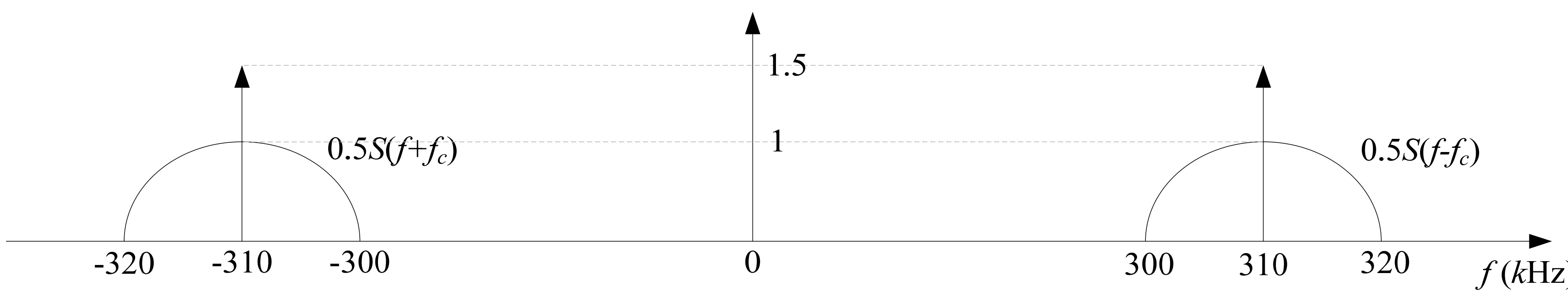
$$S_{AM-DSB-C}(f) = \tfrac{1}{2}[S(f-f_c)+S(f+f_c)] + \tfrac{3}{2}[\delta(f-f_c)+\delta(f+f_c)]$$

Solution

2)

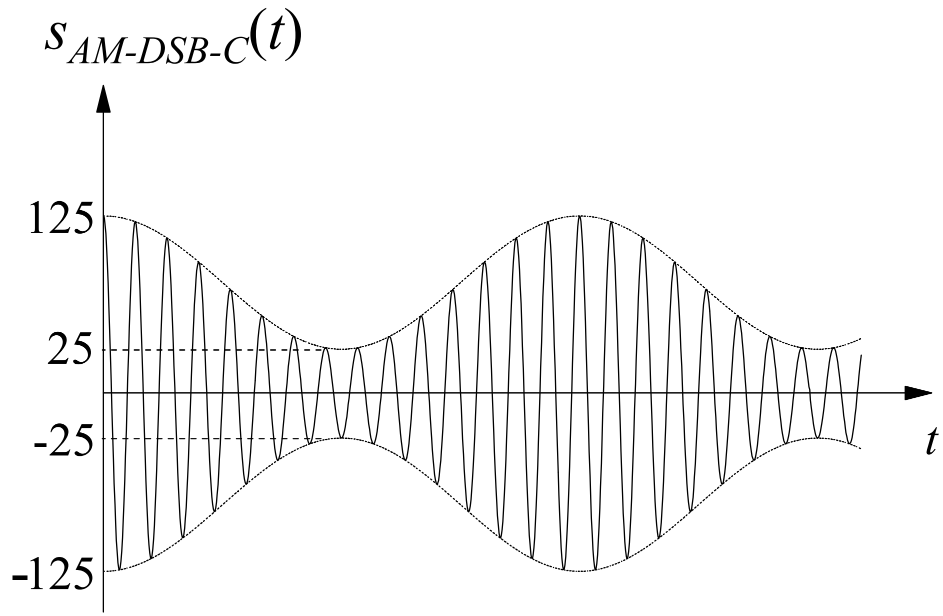


$$S_{AM-DSB-C}(f) = \tfrac{1}{2}[S(f-f_c)+S(f+f_c)] + \tfrac{3}{2}[\delta(f-f_c)+\delta(f+f_c)]$$



Problem 3.1 (AM-DSB-C)

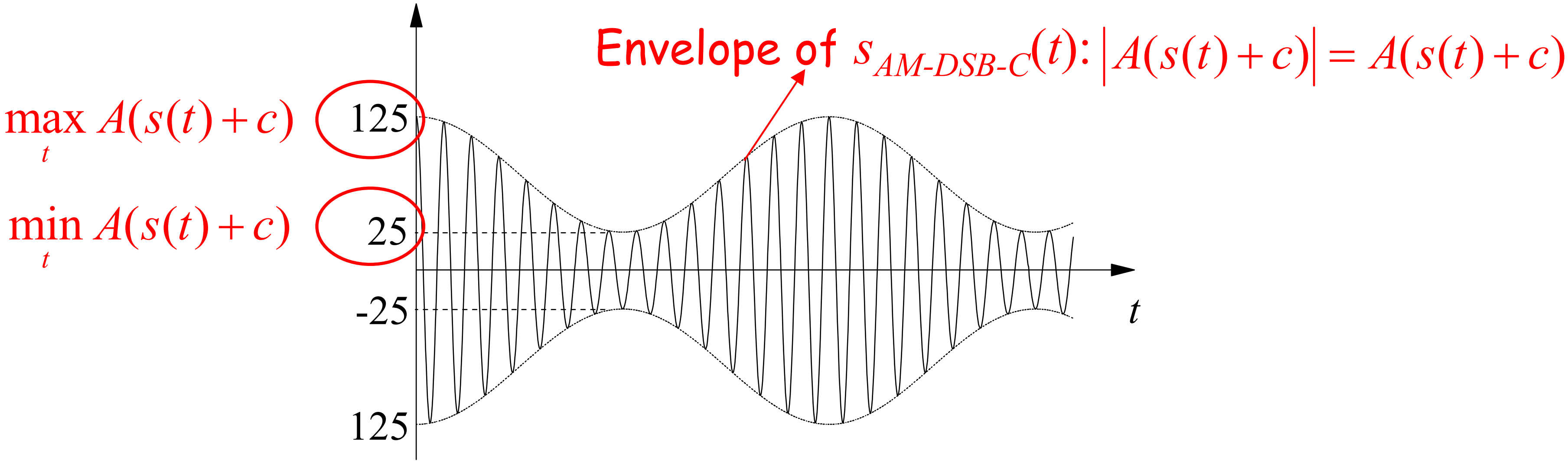
For the sinusoidally modulated AM-DSB-C waveform shown below:



1) Find the modulation index.
2) Find the time-domain expression of the waveform.
3) Sketch the spectrum of the waveform.
4) Show that the sum of the two sideband parts in part (3), divided by the carrier part, yields the modulation index. Explain why.

Solution

$$s_{AM\text{-}DSB\text{-}C}(t) = A(s(t)+c)\cos(2\pi f_c t)$$



1) The modulation index is given by

$$m = \frac{\max[s(t)+c]-\min[s(t)+c]}{\max[s(t)+c]+\min[s(t)+c]} = \frac{\max[A(s(t)+c)]-\min[A(s(t)+c)]}{\max[A(s(t)+c)]+\min[A(s(t)+c)]}$$

$$= \frac{125-25}{125+25} = \frac{2}{3}$$

Solution

2) Suppose that $s(t)=x\cos(2\pi f_m t)$. The modulated signal can be then written as

$$s_{AM-DSB-C}(t) = A(s(t)+c)\cos 2\pi f_c t = A(x\cos(2\pi f_m t)+c)\cos 2\pi f_c t$$

$$m = \frac{\max[s(t)+c]-\min[s(t)+c]}{\max[s(t)+c]+\min[s(t)+c]}$$

$$\max(s(t))=-\min(s(t))=x$$

$$m = \frac{(x+c)-(-x+c)}{(x+c)+(-x+c)} = \frac{x}{c} \Rightarrow x = c\cdot m$$

$$\boxed{s_{AM-DSB-C}(t) = Ac(m\cos(2\pi f_m t)+1)\cos 2\pi f_c t}$$

Solution

2) $s_{AM-DSB-C}(t) = Ac(m\cos(2\pi f_m t)+1)\cos 2\pi f_c t$

$$\left.\begin{array}{r}\max_t A(s(t)+c) = 125 \Rightarrow A(x+c) = 125 \\ x = c\cdot m\end{array}\right\} \left.\begin{array}{r}Ac(m+1) = 125 \\ m = 2/3\end{array}\right\} Ac = 75$$

Finally, the time-domain expression of the modulated waveform is

$$s_{AM-DSB-C}(t) = 75(\tfrac{2}{3}\cos(2\pi f_m t)+1)\cos(2\pi f_c t)$$

Solution

3) For AM-DSB-C modulated signal $s_{AM-DSB-C}(t) = A(s(t)+c)\cos(2\pi f_c t)$:

$$S_{AM-DSB-C}(f) = \tfrac{A}{2}[S(f-f_c)+S(f+f_c)] + \tfrac{Ac}{2}[\delta(f-f_c)+\delta(f+f_c)]$$

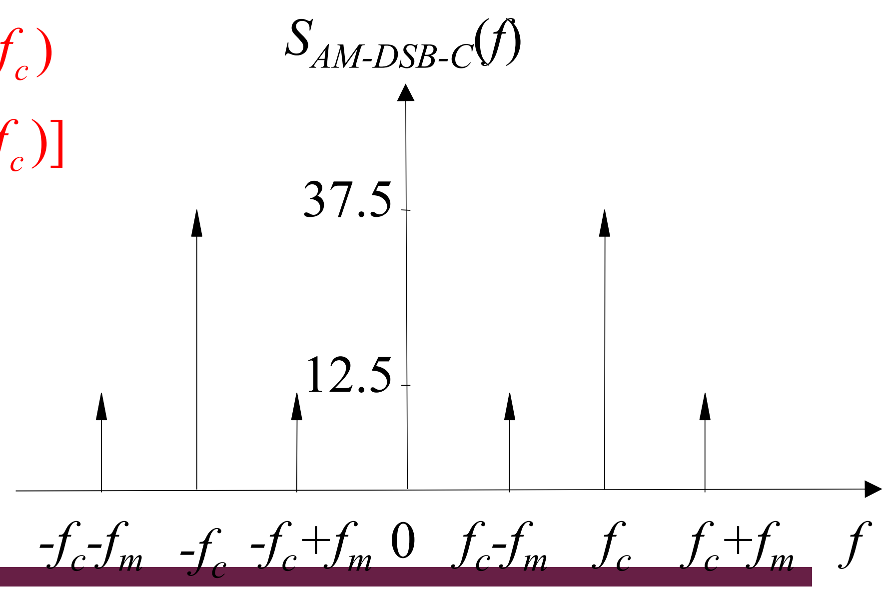
$$s(t) = x\cos(2\pi f_m t) \Leftrightarrow S(f) = \frac{x}{2}[\delta(f-f_m)+\delta(f+f_m)]$$

$$x = c \cdot m$$

$$S_{AM-DSB-C}(f) = \frac{Acm}{4}[\delta(f-f_m-f_c)+\delta(f-f_m+f_c) + \delta(f+f_m-f_c)+\delta(f+f_m+f_c)] + \frac{Ac}{2}[\delta(f-f_c)+\delta(f+f_c)]$$

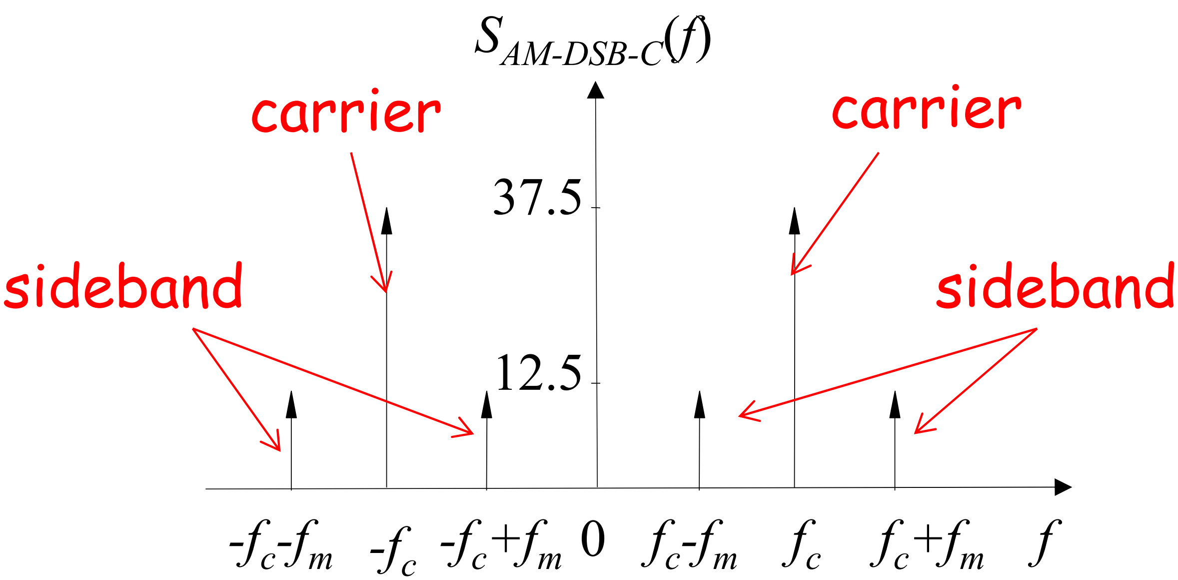
12.5

37.5



Solution

4) "Show that the sum of the two sideband parts, divided by the carrier part, yields the modulation index."



Modulation index:

$$m = \frac{12.5 + 12.5}{37.5} = 2/3$$

Why?

$$\left.\begin{aligned} A_{carrier} &= \frac{Ac}{2} \\ A_{sideband} &= \frac{Acm}{4} \end{aligned}\right\} m = \frac{2A_{sideband}}{A_{carrier}}$$

Problem 3.2 (AM-DSB-C)

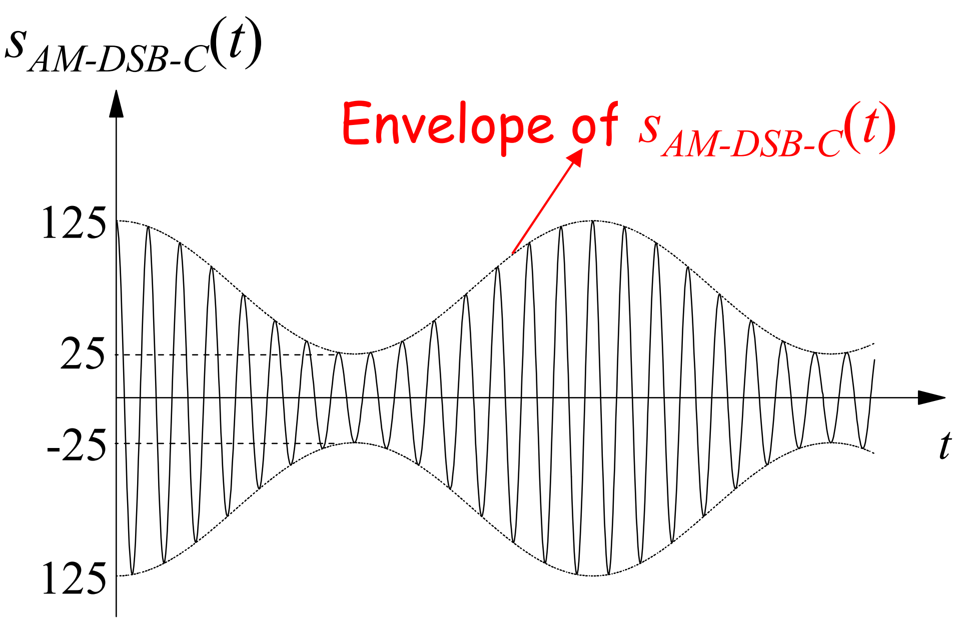
5) Sketch the output of the envelope detector .

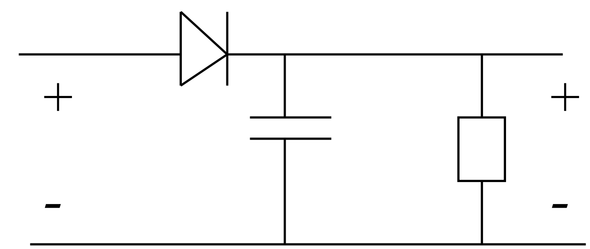
6) If an additional carrier is added to the waveform $s_{AM\text{-}DSB\text{-}C}(t)$ to attain a modulation index of 20%, determine the peak amplitude of this additional carrier.

7) Sketch the output of the envelope detector that takes the waveform in (6) as the input.

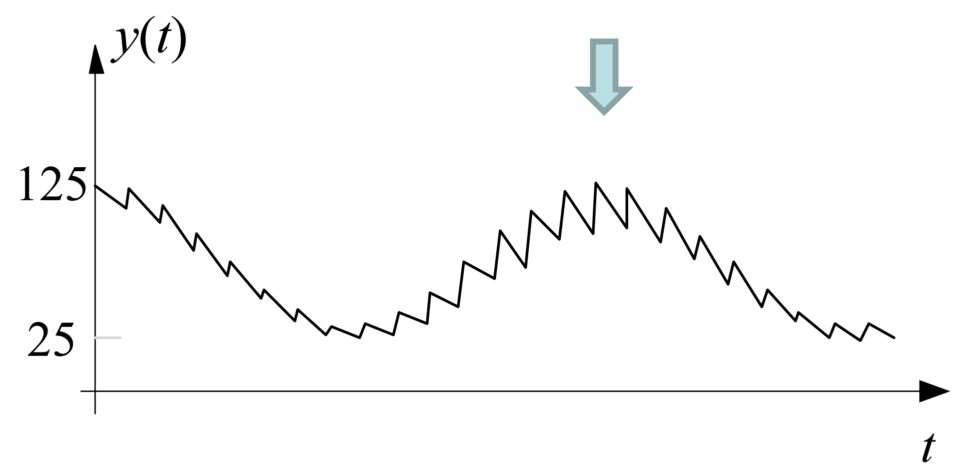
Solution

5)





Envelope Detector



Solution

6) Suppose the peak amplitude of the additional carrier is *B*.

According to

$$\tilde{m} = \frac{(125+B)-(25+B)}{(125+B)+(25+B)} = 0.2$$

we have $B = 175.$

7) Output of the envelope detector: