Tutorial 3
Frequency Modulation (FM)

Problem 1 (Frequency Deviation)

Consider the following FM signal:

$$s_{\mathrm{FM}}(t) = 100\cos(2\pi(f_c t + \sin f_m t + 2\sin 2 f_m t))$$

where f_c = 100 kHz and f_m=1 kHz. Determine:

(i) Instantaneous phase;

(ii) Instantaneous frequency;

(iii) Peak frequency deviation.

Solution

(i) Instantaneous phase $\Psi(t) = 2\pi(f_c t + \sin f_m t + 2\sin 2 f_m t)$

(ii) Instantaneous frequency

$$f(t) = \frac{1}{2\pi} \cdot \frac{d\Psi(t)}{dt} = f_c + f_m \cos f_m t + 4 f_m \cos 2 f_m t$$

(iii) Peak frequency deviation

$$\Delta f = \max_t | f(t) - f_c | = \max_t \left| f_m \cos f_m t + 4 f_m \cos 2 f_m t \right|$$

$$= f_m + 4 f_m = 5 f_m = 5k\text{Hz}$$

Problem 2 (Modulation Index)

A 1-GHz carrier is frequency-modulated by a 10-kHz sinusoid so that the peak frequency deviation is 100 Hz. Determine

(i) the modulation index β;

(ii) the modulation index if the modulating signal amplitude was doubled;

(iii) the modulation index if the modulating signal frequency was doubled;

(iv) the modulation index if both the amplitude and the frequency of the modulating signal were doubled.

Solution

(i) Modulation Index $\beta = \dfrac{\Delta f}{f_m} = 100/(10\times10^3) = 0.01$

(ii) $\Delta f = 2\times100 \Rightarrow \beta = 2\times100/(10\times10^3) = 0.02$

(iii) $f_m = 2\times10^4 \Rightarrow \beta = 100/(2\times10^4) = 0.005$

(iv) $\Delta f = 2\times100,\ f_m = 2\times10^4 \Rightarrow \beta = 0.01$

Problem 3 (Power Distribution)

Consider an FM transmitter with a sinusoidal input. The total transmission power is 100W. The peak frequency deviation is carefully increased from zero until the first sideband amplitude at the output is zero. Under these conditions, determine

(i) the transmission power at the carrier frequency;

(ii) the transmission power at the sidebands;

(iii) the transmission power at the second sidebands.

Table 6-1 Values of Bessel Function of the First Kind $J_n(\beta)$ for Various Values of n and β

n	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 4$	$\beta = 5$	$\beta = 6$	$\beta = 7$	$\beta = 8$	$\beta = 9$
0	0.7652	0.2239	−0.2601	−0.3971	−0.1776	0.1506	0.3001	0.1717	−0.0903
1	0.4401	0.5767	0.3391	−0.0660	−0.3276	−0.2767	−0.0047	0.2346	0.2453
2	0.1149	0.3528	0.4861	0.3641	0.0466	−0.2429	−0.3014	−0.1130	0.1448
3	0.0196	0.1289	0.3091	0.4302	0.3648	0.1148	−0.1676	−0.2911	−0.1809
4	0.0025	0.0340	0.1320	0.2811	0.3912	0.3576	0.1578	−0.1054	−0.2655
5	0.0002	0.0070	0.0430	0.1321	0.2611	0.3621	0.3479	0.1858	−0.0550
6	*	0.0012	0.0114	0.0491	0.1310	0.2458	0.3392	0.3376	0.2043
7	*	0.0002	0.0025	0.0152	0.0534	0.1296	0.2336	0.3206	0.3275
8	*	*	0.0005	0.0040	0.0184	0.0565	0.1280	0.2235	0.3051
9	*	*	0.0001	0.0009	0.0055	0.0212	0.0589	0.1263	0.2149
10	*	*	*	0.0002	0.0015	0.0070	0.0235	0.0608	0.1247
11	*	*	*	*	0.0004	0.0020	0.0083	0.0256	0.0622
12	*	*	*	*	0.0001	0.0005	0.0027	0.0096	0.0274
13	*	*	*	*	*	0.0001	0.0008	0.0033	0.0108
14	*	*	*	*	*	*	0.0002	0.0010	0.0039
15	*	*	*	*	*	*	0.0001	0.0003	0.0013
16	*	*	*	*	*	*	*	0.0001	0.0004
17	*	*	*	*	*	*	*	*	0.0001
18	*	*	*	*	*	*	*	*	*
19	*	*	*	*	*	*	*	*	*

Solution

"The peak frequency deviation is carefully increased from zero until the first sideband amplitude in the output is zero" can be interpreted as:

We increase the value of modulation index β from zero until $J_1(\beta)=0$.

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15	*	*	*	*	*	*	0.0001	0.0003	0.0013
16	*	*	*	*	*	*	*	0.0001	0.0004
17	*	*	*	*	*	*	*	*	0.0001
18	*	*	*	*	*	*	*	*	*
19	*	*	*	*	*	*	*	*	*

Solution

“The peak frequency deviation is carefully increased from zero until the first sideband amplitude in the output is zero” can be interpreted as:

We increase the value of modulation index β from zero until $J_1(\beta)=0$.

From the table, $J_1(\beta)=0$ first occurs at $\beta \approx 4$. Therefore,

(i) The transmission power at f_c is $P_0 = 100 \times J_0^2(4) \approx 16W$

(ii) The transmission power at the sidebands is

$$P_s = P_t - P_0 = 100 - 16 = 84W$$

(iii) The transmission power at the second sidebands is

$$P_{s2} = 2 \times \left[100 \times J_2^2(4) \right] = 26.5W$$

Problem 4

The sinusoidal signal $s(t)=x\cos(2\pi f_m t)$ is applied to the input of an FM system. The corresponding modulated signal output (in volts) with x=1V, f_m=1kHz, is

$$s_{FM}(t)=100\cos(2\pi\times10^7 t+4\sin 2000\pi t).$$

(i) Determine the peak frequency deviation, the modulation index, the carrier frequency, and the total power of $s_{FM}(t)$;

(ii) What is the percentage of the power at 10MHz?

(iii) What is the effective bandwidth, according to Carson's rule?

Solution

f_m=1kHz

(i) $s_{FM}(t)=A\cos[2\pi f_c t+\frac{\Delta f}{f_m}\sin(2\pi f_m t)]=A\cos[2\pi f_c t+\beta\sin(2\pi f_m t)]$

$=100\cos(2\pi\times10^7 t+4\sin 2000\pi t)$

$\Rightarrow f_c=10^7=10MHz,\ \beta=4\ \Rightarrow\ \Delta f=\beta f_m=4kHz$

$\Rightarrow A=100V\ \Rightarrow P_t=100^2/2=5\times10^3W$

(ii) $J_0^{\ 2}(\beta)=0.16$

(iii) According to Carson's rule, the effective bandwidth of the modulated signal is given by

$2(\beta+1)f_m$ =10kHz

Problem 5

A certain sinusoidal signal with frequency f_m Hz is used as the modulating signal in both an AM-DSB-C and an FM system. When modulated, the peak frequency deviation of the FM system is set to three times the bandwidth of the AM system. The sum of magnitudes of those sidebands spaced $\pm f_m$ Hz from carrier in both systems are equal, and the total transmission powers are equal in both systems.

(i) Determine the modulation index of the FM system;

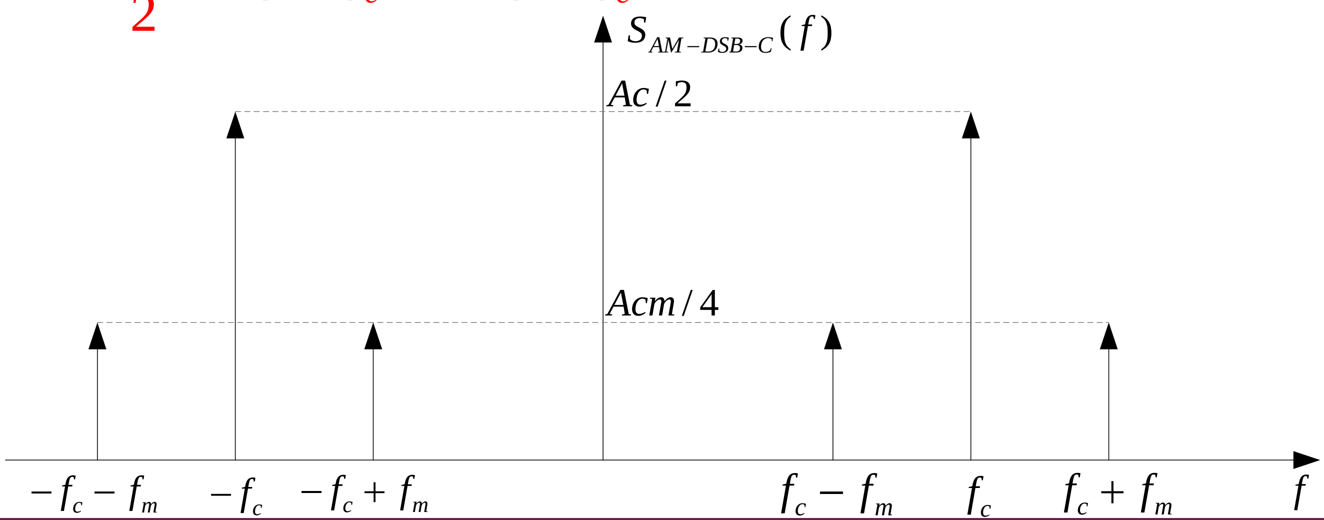
(ii) Determine the modulation index of the AM-DSB-C system.

Solution

In Problem 3, Tutorial 2, we have known that a sinusoidally modulated AM-DSB-C signal can be written as

$$s_{AM-DSB-C}(t) = Ac(m\cos(2\pi f_m t)+1)\cos 2\pi f_c t$$

$$S_{AM-DSB-C}(f) = \frac{Acm}{4}[\delta(f-f_m-f_c)+\delta(f-f_m+f_c)+\delta(f+f_m-f_c)+\delta(f+f_m+f_c)] + \frac{Ac}{2}[\delta(f-f_c)+\delta(f+f_c)]$$



Solution

1. Bandwidth of the AM-DSB-C signal is: $2f_m$.

2. The total power of the AM-DSB-C signal is given by:

$$P_{AM} = 2\times(Ac/2)^2 + 4\times(Acm/4)^2 = \frac{(Ac)^2}{2}(1+\frac{1}{2}m^2)$$

Solution

(i) According to " the peak frequency deviation of the FM system is set to three times the bandwidth of the AM system", we have

$$\Delta f = 3\times(2f_m) \Rightarrow \beta = \Delta f / f_m = 6$$

(ii) According to "the total transmission powers are equal in both systems", we have

$$\underbrace{\frac{1}{2}(Ac)^2(1+\frac{1}{2}m^2)}_{P_{AM}} = \underbrace{\frac{1}{2}A_{FM}^2}_{P_{FM}} \Rightarrow A_{FM}^2 = (Ac)^2(1+\frac{1}{2}m^2)$$

Besides, "the sum of magnitudes of those sidebands spaced $\pm f_m$ Hz from carrier in both systems are equal" means

$$4\times\frac{Acm}{4} = 4\times\frac{A_{FM}}{2}|J_1(\beta)| \Rightarrow A_{FM} = Acm/(2\times|J_1(\beta)|)$$

Solution

$$\left.\begin{aligned} A_{FM}^2 &= (Ac)^2\left(1+\frac{1}{2}m^2\right) \\ A_{FM} &= Acm/(2\times|J_1(\beta)|) \end{aligned}\right\} \quad \left.\begin{aligned} m &= \frac{2|J_1(\beta)|}{\sqrt{1-2J_1^2(\beta)}} \\ \beta &= 6 \end{aligned}\right\} \quad m = 0.6$$