Solutions to EE3210 Tutorial 4 Problems

Problem 1: Given $x[n] = (-\frac{1}{2})^n u[n-4]$ and $h[n] = 4^n u[2-n]$, we have $x[k] = (-\frac{1}{2})^k u[k-4]$ and $h[n-k] = 4^{n-k} u[k-(n-2)]$. So we have

$$\begin{aligned} y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] &= \sum_{k=-\infty}^{+\infty} \left(-\frac{1}{2}\right)^k 4^{n-k} u[k-4]u[k-(n-2)] \\ &= 4^n \sum_{k=-\infty}^{+\infty} \left(-\frac{1}{8}\right)^k u[k-4]u[k-(n-2)]. \end{aligned}$$

Note that

$$u[k-4] = \begin{cases} 1, & k \geq 4 \\ 0, & k < 4 \end{cases}$$

and, with n fixed,

$$u[k-(n-2)] = \begin{cases} 1, & k \geq n-2 \\ 0, & k < n-2. \end{cases}$$

Then:

- For $n-2 \leq 4$, i.e., $n \leq 6$, we have

  $$u[k-4]u[k-(n-2)] = \begin{cases} 1, & k \geq 4 \\ 0, & k < 4 \end{cases}$$

  and hence

  $$y[n] = 4^n \sum_{k=4}^{+\infty} \left(-\frac{1}{8}\right)^k = 4^n \sum_{k=0}^{+\infty} \left(-\frac{1}{8}\right)^{k+4} = \left(\frac{8}{9}\right)\left(-\frac{1}{8}\right)^4 4^n.$$

- For $n-2 > 4$, i.e., $n > 6$, we have

  $$u[k-4]u[k-(n-2)] = \begin{cases} 1, & k \geq n-2 \\ 0, & k < n-2 \end{cases}$$

  and hence

  $$y[n] = 4^n \sum_{k=n-2}^{+\infty} \left(-\frac{1}{8}\right)^k = 4^n \sum_{k=0}^{+\infty} \left(-\frac{1}{8}\right)^{k+n-2} = \left(\frac{8^3}{9}\right)\left(-\frac{1}{2}\right)^n.$$

Problem 2: Given $x(t) = u(t) - 2u(t-2)$, as a consequence of both the commutative and distributive properties, we have in this case

$$y(t) = x(t) * h(t) = u(t) * h(t) - 2u(t-2) * h(t).$$

First, we have

$$u(t) * h(t) = \int_{-\infty}^{+\infty} u(\tau)h(t-\tau)d\tau = \int_{-\infty}^{+\infty} e^{2(t-\tau)}u(\tau)u(\tau - [t-1])d\tau.$$

Note that

$$u(\tau) = \begin{cases} 1, & \tau > 0 \\ 0, & \tau < 0 \end{cases}$$

and, with t fixed,

$$u(\tau - [t-1]) = \begin{cases} 1, & \tau > t-1 \\ 0, & \tau < t-1. \end{cases}$$

Then:

- For $t - 1 < 0$, i.e., $t < 1$, we have

$$u(\tau)u(\tau - [t-1]) = \begin{cases} 1, & \tau > 0 \\ 0, & \tau < 0 \end{cases}$$

  and hence

$$u(t) * h(t) = \int_{0}^{+\infty} e^{2(t-\tau)}d\tau = e^{2t}\int_{0}^{+\infty} e^{-2\tau}d\tau = \frac{1}{2}e^{2t}.$$

- For $t - 1 > 0$, i.e., $t > 1$, we have

$$u(\tau)u(\tau - [t-1]) = \begin{cases} 1, & \tau > t-1 \\ 0, & \tau < t-1 \end{cases}$$

  and hence

$$u(t) * h(t) = \int_{t-1}^{+\infty} e^{2(t-\tau)}d\tau = e^{2t}\int_{t-1}^{+\infty} e^{-2\tau}d\tau = \frac{1}{2}e^{2}.$$

Second, we have

$$-2u(t-2) * h(t) = -2\int_{-\infty}^{+\infty} u(\tau-2)h(t-\tau)d\tau = -2\int_{-\infty}^{+\infty} e^{2(t-\tau)}u(\tau-2)u(\tau - [t-1])d\tau.$$

Note that

$$u(\tau-2) = \begin{cases} 1, & \tau > 2 \\ 0, & \tau < 2 \end{cases}$$

and, with t fixed,

$$u(\tau - [t-1]) = \begin{cases} 1, & \tau > t-1 \\ 0, & \tau < t-1. \end{cases}$$

Then:

- For $t-1<2$, i.e., $t<3$, we have

$$u(\tau-2)u(\tau-[t-1]) = \begin{cases} 1, & \tau > 2 \\ 0, & \tau < 2 \end{cases}$$

and hence

$$-2u(t-2)*h(t) = -2\int_{2}^{+\infty} e^{2(t-\tau)}d\tau = -2e^{2t}\int_{2}^{+\infty} e^{-2\tau}d\tau = -e^{2t-4}.$$

- For $t-1>2$, i.e., $t>3$, we have

$$u(\tau-2)u(\tau-[t-1]) = \begin{cases} 1, & \tau > t-1 \\ 0, & \tau < t-1 \end{cases}$$

and hence

$$-2u(t-2)*h(t) = -2\int_{t-1}^{+\infty} e^{2(t-\tau)}d\tau = -2e^{2t}\int_{t-1}^{+\infty} e^{-2\tau}d\tau = -e^{2}.$$

Therefore, we obtain $y(t)$ as

$$y(t) = \begin{cases} \frac{1}{2}e^{2t} - e^{2t-4}, & t < 1 \\ \frac{1}{2}e^{2} - e^{2t-4}, & 1 < t < 3 \\ -\frac{1}{2}e^{2}, & t > 3. \end{cases}$$