Solutions to EE3210 Tutorial 1 Problems

Problem 1: The two equations can be derived by adding or subtracting Euler's formula. Given

$$e^{j\theta} = \cos\theta + j\sin\theta$$

and

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

we obtain

$$e^{j\theta} + e^{-j\theta} = 2\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

and

$$e^{j\theta} - e^{-j\theta} = 2j\sin\theta \quad \Rightarrow \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}.$$

Problem 2: Using Euler's formula, we have

$$z = r(\cos\theta + j\sin\theta) = re^{j\theta}.$$

Then,

$$z^n = \left(re^{j\theta}\right)^n = r^n e^{jn\theta} = r^n[\cos(n\theta) + j\sin(n\theta)].$$

Problem 3: Let $x = \rho e^{j\alpha}$. Then, by definition,

$$re^{j\theta} = \rho^n e^{jn\alpha}.$$

This implies $\rho^n = r$, $n\alpha = \theta + 2\pi k$ where k can be any integer. Thus, we have $\rho = r^{1/n}$, $\alpha = (\theta + 2\pi k)/n$, and therefore

$$x = r^{\frac{1}{n}} e^{j\frac{\theta + 2\pi k}{n}}.$$

Clearly, we have n distinct values of x for $k = 0, 1, \ldots, n-1$.