CITY UNIVERSITY OF HONG KONG

Course code & title	:	EE3210 Signals and Systems
Session	:	Semester B 2019/20
Time allowed	:	Two hours

This paper has twelve pages (including this cover page).

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1. This paper consists of 5 questions.
2. Answer ALL questions.

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*This is an **open-book** examination.*

Students are allowed to use the following materials/aids:

Portable battery operated calculator, Course Handout

Materials/aids other than those stated above are not permitted. Students will be subject to disciplinary action if any unauthorized materials or aids are found on them.

** If you face any technical issue during the exam, please contact the departmental hotline (3442-7740).*

Honor Pledge

Please review the following honor code, then sign your name and write down the date.

1. I pledge that the answers in this exam are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,
   (a) I will not plagiarize (copy without citation) from any source;
   (b) I will not communicate or attempt to communicate with any other person during the exam;
   (c) neither will I give or attempt to give assistance to another student taking the exam; and
   (d) I will use only approved devices (e.g., calculators) and/or approved device models.
2. I understand that any act of academic dishonesty can lead to disciplinary action.

_____
Signature

_____
Date

Question 1 (20 points)

Find the correct Fourier Transform (or Inverse FT) of the given signals.

(a) (5 points)

$$\mathcal{F}\left\{e^{-\alpha|t|}\right\}, \quad \alpha > 0$$

(i) $\dfrac{2\alpha}{\alpha^2 + (2\pi f)^2}$

(ii) $\dfrac{1}{\alpha^2 + (2\pi f)^2}$

(iii) $\dfrac{2\alpha}{(\alpha + j2\pi f)^2}$

(iv) $\dfrac{2\alpha}{(\alpha - j2\pi f)^2}$

(b) (5 points)

$$\mathcal{F}\left\{\text{sinc}^2(t) \cdot \cos(4\pi t)\right\}, \quad \alpha > 0$$

(i) $\dfrac{1}{2j}\left[\text{tri}(f-2) - \text{tri}(f+2)\right]$

(ii) $\dfrac{1}{2}\left[\text{tri}(f-2) + \text{tri}(f+2)\right]$

(iii) $\dfrac{4\pi}{(1 + j2\pi f)^2 + 16\pi^2}$

(iv) $\dfrac{1 + j2\pi f}{(1 + j2\pi f)^2 + 16\pi^2}$

(c) (5 points)

$$\mathcal{F}\left\{\left(\tau\ \text{tri}\left(\frac{t}{\tau}\right)\right) * \left(\tau\ \text{tri}\left(\frac{t}{\tau}\right)\right)\right\}, \quad \text{where tri}(t) = \begin{cases} 1 - |t|, & |t| < 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) $\dfrac{1}{2}\left[\text{sinc}(f-1) + \text{sinc}(f+1)\right]$

(ii) $\dfrac{1}{2j}\left[\text{sinc}(f-1) - \text{sinc}(f+1)\right]$

(iii) $\tau^2 \text{sinc}^2(f \cdot \tau)$

(iv) $\tau^4 \text{sinc}^4(f \cdot \tau)$

(d) (5 points)

$$\mathcal{F}^{-1}\left\{\frac{1}{4 + j8\pi f + 4\pi^2(1 - f^2)}\right\}$$

(i) $\dfrac{1}{2\pi} e^{-2t} \sin(2\pi t)\, u(t)$

(ii) $\dfrac{1}{2\pi} e^{-2t} \cos(2\pi t)\, u(t)$

(iii) $\dfrac{1}{2\pi} e^{-2t} \sin(2\pi t)$

(iv) $\dfrac{1}{2\pi} e^{-2t} \cos(2\pi t)$

(Answer Page for Question 1)

Question 2 (20 points)

Consider the following system function $H(s)$ of an LTI system.

(a) (6 points) Choose the ROC for a **stable** system.

$$H(s) = \frac{s}{(s+2)(s+1)(s-1)}$$

(i) $\{\text{Re}(s) < -2\}$ (ii) $\{-2 < \text{Re}(s) < -1\}$

(iii) $\{-1 < \text{Re}(s) < 1\}$ (iv) $\{\text{Re}(s) > 1\}$

(b) (8 points) Choose the ROC for a **causal** and **stable** system.

$$H(s) = \frac{1}{(s+4)(s+3)(s+1)^2}$$

(i) $\{\text{Re}(s) < -4\}$ (ii) $\{-4 < \text{Re}(s) < -3\}$

(iii) $\{-3 < \text{Re}(s) < -1\}$ (iv) $\{\text{Re}(s) > -1\}$

(c) (6 points) Choose the ROC for a **causal** system.

$$H(s) = \frac{s+1}{(s^2+4s+5)(s+3)(s-1)}$$

(i) $\{\text{Re}(s) < -3\}$ (ii) $\{-3 < \text{Re}(s) < -2\}$

(iii) $\{-2 < \text{Re}(s) < 1\}$ (iv) $\{\text{Re}(s) > 1\}$

(Answer Page for Question 2)

Question 3 (20 points)

Consider a continuous LTI system described by the following input-output relationship.

$$\frac{dy(t)}{dt} + 10y(t) = \int_{-\infty}^{\infty} x(\tau)\, z(t-\tau)\, d\tau - x(t), \quad \text{where } z(t) = e^{-t}u(t) + \delta(t).$$

(a) (10 points) Find the frequency response $H(f) = \frac{Y(f)}{X(f)}$ of this system.

(b) (10 points) Determine the impulse response $h(t)$ of the system.

(Answer Page for Question 3)

Question 4 (20 points)

Consider LTI systems whose input-output relationship is described by the given equations.

a) (10 points) Find the step response $y(t)$ using the bilateral Laplace Transform.

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = \frac{dx(t)}{dt}$$

b) (10 points) Solve the following integral equation using the unilateral Laplace Transform.

$$y(t) = e^t \left[4 + 4\int_0^t e^{-\tau} y(\tau)\, d\tau\right], \quad t \geq 0$$

(Answer Page for Question 4)

Question 5 (20 points)

Consider a discrete system described by the following difference equation

$$y[n] = \frac{3}{4}y\left[n-1\right] - \frac{1}{8}y\left[n-2\right] + x[n], \quad |z| > \frac{1}{2}$$

(a) (8 points) Derive the system function $H(z)$ and the impulse response $h[n]$ using Z-transform.

(b) (6 points) Answer whether this system is causal (or not).

(c) (6 points) Determine the step response $y[n]$ given that $|z| > 1$.

(Answer Page for Question 5)