

- 4.31. (a) Show that the three LTI systems with impulse responses

$$h_1(t) = u(t),$$

$$h_2(t) = -2\delta(t) + 5e^{-2t}u(t),$$

and

$$h_3(t) = 2te^{-t}u(t)$$

all have the same response to $x(t) = \cos t$.

- (b) Find the impulse response of another LTI system with the same response to $\cos t$.

This problem illustrates the fact that the response to $\cos t$ cannot be used to specify an LTI system uniquely.

- 4.32. Consider an LTI system S with impulse response

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}.$$

Determine the output of S for each of the following inputs:

- (a) $x_1(t) = \cos(6t + \frac{\pi}{2})$
 (b) $x_2(t) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \sin(3kt)$
 (c) $x_3(t) = \frac{\sin(4(t+1))}{\pi(t+1)}$
 (d) $x_4(t) = (\frac{\sin 2t}{\pi t})^2$
- 4.33. The input and the output of a stable and causal LTI system are related by the differential equation

$$\frac{d^2 y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

- (a) Find the impulse response of this system.
 (b) What is the response of this system if $x(t) = te^{-2t}u(t)$?
 (c) Repeat part (a) for the stable and causal LTI system described by the equation

$$\frac{d^2 y(t)}{dt^2} + \sqrt{2}\frac{dy(t)}{dt} + y(t) = 2\frac{d^2 x(t)}{dt^2} - 2x(t)$$

- 4.34. A causal and stable LTI system S has the frequency response

$$H(j\omega) = \frac{j\omega + 4}{6 - \omega^2 + 5j\omega}.$$

- (a) Determine a differential equation relating the input $x(t)$ and output $y(t)$ of S .
- (b) Determine the impulse response $h(t)$ of S .
- (c) What is the output of S when the input is

$$x(t) = e^{-4t}u(t) - te^{-4t}u(t)?$$

4.35. In this problem, we provide examples of the effects of nonlinear changes in phase.

- (a) Consider the continuous-time LTI system with frequency response

$$H(j\omega) = \frac{a - j\omega}{a + j\omega},$$

where $a > 0$. What is the magnitude of $H(j\omega)$? What is $\angle H(j\omega)$? What is the impulse response of this system?

- (b) Determine the output of the system of part (a) with $a = 1$ when the input is

$$\cos(t/\sqrt{3}) + \cos t + \cos \sqrt{3}t.$$

Roughly sketch both the input and the output.

4.36. Consider an LTI system whose response to the input

$$x(t) = [e^{-t} + e^{-3t}]u(t)$$

is

$$y(t) = [2e^{-t} - 2e^{-4t}]u(t).$$

- (a) Find the frequency response of this system.
- (b) Determine the system's impulse response.
- (c) Find the differential equation relating the input and the output of this system.

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4.37. Consider the signal $x(t)$ in Figure P4.37.

- (a) Find the Fourier transform $X(j\omega)$ of $x(t)$.
- (b) Sketch the signal

$$\tilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k).$$

- (c) Find another signal $g(t)$ such that $g(t)$ is not the same as $x(t)$ and

$$\tilde{x}(t) = g(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k).$$