EE3008 Test 2

(1:00-2:30pm, Mar. 30, 2020)

Question 1 (30 marks)

Solution:

1. (a) With $\mu_s(t) = 1\ \ V$ and $\mu_n(t) = 0\ \ V$, we have $\mu_X(t) = \mu_s(t) + \mu_n(t) = 1\ \ V$.

(b) With $\mu_Z = 0\ \ V$, we have $\mu_X(t) = \mu_Z \cdot \sum_{k=-\infty}^{\infty} \omega(t - kT_0) = 0\ \ V$.

(c) With $\mu_m(t) = 0.5\ \ V$, we have $\mu_X(t) = \mu_m(t) \cdot \cos(2\pi f_c t) = 0.5 \cos(2\pi f_c t)\ \ V$.

2. (a) $R_X(t+\tau, t) = E[X(t+\tau)X(t)] = E\big[\big(s(t+\tau) + n(t+\tau)\big) \cdot \big(s(t) + n(t)\big)\big] = E[s(t+\tau)s(t)] + E[n(t+\tau)n(t)] + E[s(t+\tau)n(t)] + E[n(t+\tau)s(t)] = R_s(\tau) + R_n(\tau) + 2\mu_s \cdot \mu_n = R_s(\tau) + R_n(\tau)$.

(b) $R_X(t+\tau, t) = E[X(t+\tau)X(t)] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} E[Z_k Z_l] \cdot \omega(t - kT_0)\omega(t + \tau - lT_0)$. Note that when k=l, $E[Z_k Z_l] = E[Z^2] = 1$. When $k \neq l, E[Z_k Z_l] = E[Z_k]E[Z_l] = 0$. As a result, $R_X(t+\tau, t) = \sum_{k=-\infty}^{\infty} \omega(t - kT_0)\omega(t + \tau - kT_0)$.

(c) $R_X(t+\tau, t) = E[X(t+\tau)X(t)] = E[m(t+\tau)m(t)] \cdot \cos\big(2\pi f_c(t+\tau)\big) \cdot \cos(2\pi f_c t) = R_m(\tau) \cdot \frac{1}{2}\big(\cos(2\pi f_c \tau) + \cos(4\pi f_c t + 2\pi f_c \tau)\big)$.

3. (a) As $X(t)$ is a WSS process, $P_X = R_X(0) = R_s(0) + R_n(0) = P_s + P_n = 2.1\ \ W$.

(b) As $X(t)$ is a cyclostationary process with period $T_0 = 100\ \ ms$, $P_X = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} R_X(t,t)dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \omega^2(t)dt = 100 \cdot \frac{0.1}{100} = 0.1\ \ W$.

(c) As $X(t)$ is a cyclostationary process with period $T_0 = 1/f_c$, $P_X = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} R_X(t,t)dt = R_m(0) \cdot \frac{1}{2}\cos(2\pi f_c \cdot 0) + \frac{1}{2T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \cos(4\pi f_c t)\, dt = \frac{1}{2}P_m = 0.5\ \ W$.

Question 2 (40 marks)

Solution:

1. The step size is $\Delta = \frac{4-(-4)}{2^4} = 0.5$ V. The maximum quantization error is $\frac{\Delta}{2} = 0.25$ V. The quantization error power is $\sigma_e^2 = \frac{\Delta^2}{12} = \frac{1}{48}\ V^2$.

2. The quantized values are (0.25 V, -0.25 V, -1.75 V, -2.25 V) and the corresponding natural codes are (1000 0111 0100 0011). Therefore, the output sequence is (1100 0100 0110 0010).

3. When the number of quantization levels is doubled, the step size is reduced 50%. The quantization error power is then ¼ of the original one, and the SQNR is thus 4 times larger. Denote the new SQNR as SQNR'. From $\frac{SQNR'}{SQNR} = 4$, we have $10\log_{10} SQNR' - 10\log_{10} SQNR = 10\log_{10}\frac{SQNR'}{SQNR} = 6.02\ dB$.

4. 1) Suppose the dynamic range of $x(t)$ is $[-A, A]$. The power of $x(t)$ is then $P_x = E[X^2] = \frac{A^2}{3}$. (See Tutorial 4, Q2 and Tutorial 5, Q4 for details). According to $SQNR = \frac{P_x}{\sigma_e^2} = 20\ dB = 100$, we can obtain that $A=\sqrt{3 \cdot \sigma_e^2 \cdot SQNR} = 2.5$ V. The dynamic range is then [-2.5 V, 2.5 V].

   2) The sampling rate is $f_s=2B=4$ ksample/s and so the bit rate is $R_b=bf_s=16$ kb/s. With 4-ary PAM, the symbol rate is $R_s=R_b/2=8$ ksymbols/s. The required channel bandwidth for 95% in-band power is $2R_s= 16$ kHz.

   3) With the bit rate 16 kb/s and the channel bandwidth 5 kHz, the bandwidth efficiency needs to be at least 3.2. With M-ary PAM and 90% in-band power, M should be at least $2^4=16$.

Question 3 (30 marks)

Solution:

1. Suppose that the peak amplitude of the audio signal is A and the power is P. According to "the ratio of its peak amplitude to the square root of its power is 10", we have $A/\sqrt{P} = 10$, i.e., $P = 0.01A^2$.

   With L=65536 levels, we have the number of quantization bits $b=\log_2 L=16$. The step size is then given by $\Delta = \frac{A-(-A)}{2^{16}} = \frac{A}{2^{15}}$, and the quantization error power is $\sigma_e^2 = \frac{\Delta^2}{12} = \frac{A^2}{3\times 2^{32}}$.

The output SQNR is then

$$SQNR = \frac{P}{\sigma_e^2} = \frac{0.01A^2}{\frac{A^2}{3\times 2^{32}}} = 0.03\times 2^{32} = 1.2885\times 10^8.$$

In the form of dB: $10\log_{10}(1.2885\times 10^8) = 81.1$dB.

2. The Nyquist sampling rate is 2 x 15 kHz = 30 ksamples/s. The sampling rate is then f_s = 1.3 * 30 = 39 ksamples/s. Each sample is represented using $b=\log_2 L$= 16 bits. Therefore, the bit rate is $R_b=bf_s$= 624 kb/s. The bit period is then $\tau=1/R_b$= 1.6 µs.

3. For a digital amplitude modulated signal, the power can be obtained as $P_s = E[Z^2]\cdot\frac{1}{\tau}\int_0^\tau v^2(t)dt$ (see Tutorial 4, Q2 and Tutorial 6, Q1-2 for details). According to Fig. 2, Binary OOK is adopted. For Binary OOK, Z has equal probability to be 0 and 1, and $v(t)$ is a rectangular pulse with width τ = 1.6 µs and amplitude 5V according to Fig. 2. We then have $E[Z^2]=1/2$, and $\frac{1}{\tau}\int_0^\tau v^2(t)dt = 25$. The power is then $P_s = E[Z^2]\cdot\frac{1}{\tau}\int_0^\tau v^2(t)dt = 12.5$ W.

4. For Binary OOK, the required channel bandwidth for 90% in-band power is $B_{h\text{-}90\%}=R_b$= 624 kHz. The required channel frequency range is then [0, 624 kHz].

5. To reduce the required channel bandwidth without changing the bit rate, a higher-level modulation should be adopted. To reduce the channel bandwidth in half, 4-ary PAM can be adopted.

6. The signal cannot pass through a channel with frequency range of [99.4 MHz, 100.4 MHz] without significant distortion because it is a bandpass channel. To pass through such a bandpass channel, we should choose a bandpass modulation scheme. Moreover, as the channel bandwidth is 1MHz and the bit rate is 624 kbit/s, the bandwidth efficiency needs to be at least 0.624. To ensure that 90% of the signal power can pass through the channel, QPSK should be adopted.