Tutorial 6

Modulo

Question 1: Divisibility by 9

Let x be an n-digit number. Prove that

$$x \equiv a_{n-1} + a_{n-2} + \cdots + a_1 + a_0 \pmod 9,$$

where a_i is the $(i+1)$-th digit of x.

❑ Example 1:
 ○ Suppose $x = 6213$. $x \bmod 9 = 6 + 2 + 1 + 3 \bmod 9 = 3$.

❑ Example 2:
 ○ Suppose $x = 7218$. Since the digit sum $\pmod 9 = 7 + 2 + 1 + 8 \pmod 9 = 0$, x must be divisible by 9.

Question 2: Diophantine Equation

❑ Solve the equation

$$98x + 35y = 14,$$

where x and y are integers.

Question 3: Repeat-and-Multiply

a) Use the Repeat-and-Multiply method to compute $3^{94} \bmod 17$.

b) User Fermat's Little Theorem to compute $40^{110} \bmod 37$.

Question 4: Fermat's Little Theorem

❑ Solve $x^{103} \equiv 4 \bmod 11$.