Solutions to EE3210 Tutorial 3 Problems

Problem 1:

(a) Periodic, $T_0 = 2\pi/\omega_0 = 2\pi/4 = \pi/2$.

(b) Periodic, $T_0 = 2\pi/\omega_0 = 2\pi/\pi = 2$.

(c) Using the trigonometric identity

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

we obtain

$$\left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2 = \frac{1+\cos(4t - \frac{2\pi}{3})}{2}.$$

Thus, the signal is periodic with $T_0 = 2\pi/\omega_0 = 2\pi/4 = \pi/2$.

(d) We have

$$\begin{aligned} x(t) &= \mathcal{E}\{\cos(4\pi t)u(t)\} \\ &= \frac{1}{2}[\cos(4\pi t)u(t) + \cos(-4\pi t)u(-t)] \\ &= \frac{1}{2}\cos(4\pi t)[u(t) + u(-t)] \\ &= \frac{1}{2}\cos(4\pi t). \end{aligned}$$

Thus, the signal is periodic with $T_0 = 2\pi/\omega_0 = 2\pi/(4\pi) = 1/2$.

(e) We have

$$\begin{aligned} x(t) &= \mathcal{E}\{\sin(4\pi t)u(t)\} \\ &= \frac{1}{2}[\sin(4\pi t)u(t) + \sin(-4\pi t)u(-t)] \\ &= \frac{1}{2}\sin(4\pi t)[u(t) - u(-t)]. \end{aligned}$$

Thus, the signal is not periodic.

Problem 2:

(a) We have $\Omega_0 = 6\pi/7$ so that $\Omega_0/(2\pi) = 3/7$ is rational. Thus, the signal is periodic with $N_0 = 7$.

(b) We have $\Omega_0 = 1/8$ so that $\Omega_0/(2\pi) = 1/(16\pi)$ is irrational. Thus, the signal is not periodic.

(c) To determine whether or not this signal is periodic, we need to find if there exists a positive integer N so that

$$\begin{aligned}\cos(\tfrac{\pi}{8}n^2) &= \cos[\tfrac{\pi}{8}(n+N)^2] \\ &= \cos[\tfrac{\pi}{8}(n^2 + 2nN + N^2)]\end{aligned}$$

for all values of n. This is to find N so that $\frac{\pi}{8}(2nN + N^2)$ is an integer multiple of 2π for all values of n. The smallest integer N that satisfies this condition is 8. Therefore, this signal is periodic and its fundamental period $N_0 = 8$.

(d) Using the trigonometric identity

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

we obtain

$$\cos\left(\frac{\pi}{2}n\right)\cos\left(\frac{\pi}{4}n\right) = \frac{1}{2}\left[\cos\left(\frac{\pi}{4}n\right) + \cos\left(\frac{3\pi}{4}n\right)\right].$$

For the term $\cos(\frac{\pi}{4}n)$, we have

$$\Omega_0 = \pi/4 \Rightarrow \Omega_0/(2\pi) = 1/8 \Rightarrow N_0 = 8.$$

For the term $\cos(\frac{3\pi}{4}n)$, we have

$$\Omega_0 = 3\pi/4 \Rightarrow \Omega_0/(2\pi) = 3/8 \Rightarrow N_0 = 8.$$

Therefore, the overall signal $x[n]$ is periodic with $N_0 = 8$.

(e) For the term $2\cos(\frac{\pi}{4}n)$, we have

$$\Omega_0 = \pi/4 \Rightarrow \Omega_0/(2\pi) = 1/8 \Rightarrow N_0 = 8.$$

For the term $\sin(\frac{\pi}{8}n)$, we have

$$\Omega_0 = \pi/8 \Rightarrow \Omega_0/(2\pi) = 1/16 \Rightarrow N_0 = 16.$$

For the term $2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$, we have

$$\Omega_0 = \pi/2 \Rightarrow \Omega_0/(2\pi) = 1/4 \Rightarrow N_0 = 4.$$

Therefore, the overall signal $x[n]$ is periodic with its fundamental period N_0 being the least common multiple of the periods of the three terms in $x[n]$, which is equal to 16.