EE3210 Signals and Systems

Semester A 2023-2024

Assignment 1

Due Date: 11 October 2023

1. Let $x(t)$, $x_1(t)$ and $x_2(t)$ be three continuous-time signals.

   (a) Show that if $x(t)$ is an odd signal, then

   $$\int_{-\infty}^{\infty} x(t)dt = 0$$

   (b) Show that if $x_1(t)$ is an odd signal and $x_2(t)$ is an even signal, then $x_1(t)x_2(t)$ is an odd signal.

   (c) Decompose $x(t)$ into even and odd parts as $x(t) = x_e(t) + x_o(t)$ where

   $$x_e(t) = \frac{1}{2}[x(t) + x(-t)] \qquad \text{and} \qquad x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

   Show that

   $$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt$$

2. A capacitor is discharged by connecting a resistor across its terminals at $t = 0$. The voltage across the terminals is:

   $$v(t) = e^{-3t}u(t)$$

   (a) Compute the energy of $v(t)$. Is $v(t)$ an energy signal?

   (b) Compute the power of $v(t)$. Is $v(t)$ a power signal?

3. Consider a continuous-time linear time-invariant system with input $x(t)$ and impulse response $h(t) = -2\delta(t-2) + \delta(t-10)$. Determine the system output $y(t)$ in terms of $x(t)$. Is the system stable? Is the system causal? Is the system memoryless? Briefly explain your answers.

4. Given a discrete-time system with input $x[n]$ and output $y[n]$:

   $$y[n] = T(x[n]) = x[n] + \frac{1}{x[n]}$$

   Determine whether the system is memoryless, invertible, stable, causal, linear, and/or time-invariant. Briefly explain your answers.

5. Consider a discrete-time linear time-invariant system with input $x[n] = u[-1-n]$ and impulse response $h[n] = (0.5)^n u[n]$. Compute the system output $y[n]$. Is the system stable? Why? Is the system causal? Why?

6. Determine the convolution of the following two discrete-time signals:

$$x[n] = \begin{cases} n^2 - 1, & -2 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}$$

and

$$h[n] = \begin{cases} n - 4, & 0 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

7. Consider a continuous-time periodic signal $x(t)$ with fundamental period of $T = 2$. Within one period, $x(t)$ is expressed as:

$$x(t) = \begin{cases} 1, & 1 > t > 0 \\ 2, & 2 > t > 1 \end{cases}$$

(a) Compute the power of $x(t)$.

(b) Determine the Fourier series coefficients for $x(t)$.

(c) With the use of appropriate Fourier series properties or otherwise, find the Fourier series coefficients for $x(t)$ if it is modified as:

$$x(t) = \begin{cases} 10, & 1 > t > 0 \\ 20, & 2 > t > 1 \end{cases}$$

(d) With the use of appropriate Fourier series properties or otherwise, find the Fourier series coefficients for $x(t)$ if it is modified as:

$$x(t) = \begin{cases} 2, & 1 > t > 0 \\ 1, & 2 > t > 1 \end{cases}$$

(e) With the use of appropriate Fourier series properties or otherwise, find the Fourier series coefficients for $x(t)$ if it is modified as:

$$x(t) = \begin{cases} 2, & 0.5 > t > 0 \\ 1, & 1.5 > t > 0.5 \\ 2, & 2 > t > 1.5 \end{cases}$$

8. Choose a real-world system in daily life. The system can be a software or hardware. Identify the input, output and function of this system. Briefly explain if your chosen system is linear and time-invariant.

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Solution for Assignment 1

1(a)
If $x(t)$ is an odd, then $x(-t)+x(t)=0$. As a result, we have:

$$\begin{aligned}\int_{-\infty}^{\infty} x(t)dt &= \int_{-\infty}^{0} x(t)dt + \int_{0}^{\infty} x(t)dt \\ &= \int_{0}^{\infty} x(-u)du + \int_{0}^{\infty} x(t)dt, \quad t=-u \\ &= \int_{0}^{\infty} [x(-t)+x(t)]dt = 0\end{aligned}$$

1(b)
Let $y(t)=x_1(t)x_2(t)$. Then

$$y(-t)=x_1(-t)x_2(-t)=-x_1(t)x_2(t)=-y(t)$$

This implies that $y(t)$ is odd.

1(c)

$$\begin{aligned}\int_{-\infty}^{\infty} x^2(t)dt &= \int_{-\infty}^{\infty} [x_e(t)+x_o(t)]^2 dt \\ &= \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt + 2\int_{-\infty}^{\infty} x_e(t)x_o(t)dt\end{aligned}$$

Using the result of part (b), we know that $x_e(t)x_o(t)$ is an odd signal. Then, using the result of part (a), we have

$$\int_{-\infty}^{\infty} x_e(t)x_o(t)dt = 0$$

Therefore,

$$\int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^2(t)dt + \int_{-\infty}^{\infty} x_o^2(t)dt$$

2(a)

$$E_v = \int_{-\infty}^{\infty} |v(t)|^2 dt = \int_{-\infty}^{\infty} (e^{-3t}u(t))^2 dt = \int_{0}^{\infty} e^{-6t} dt = -\frac{1}{6}e^{-6t}\bigg|_0^{\infty} = \frac{1}{6}$$

It has finite energy and hence it is an energy signal.

2(b)
As energy is finite, its power is $P_v = 0$ and thus it is not a power signal.

3.

$$y(t) = x(t) \otimes h(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$$
$$= \int_{-\infty}^{\infty} \left[-2\delta(\lambda-2) + \delta(\lambda-10)\right]x(t-\lambda)d\lambda$$
$$= \int_{-\infty}^{\infty} -2\delta(\lambda-2)x(t-\lambda)d\lambda + \int_{-\infty}^{\infty} \delta(\lambda-10)x(t-\lambda)d\lambda$$
$$= -2x(t-2) + x(t-10)$$

The system is **stable** because if $x(t)$ is bounded, $y(t)$ will also be bounded. (Or, the system is **stable** because $\int_{-\infty}^{\infty} |h(t)|\, dt = 3 < \infty$, that is, the impulse response is absolutely summable.)

The system is **causal** because the output $y(t)$ does not depend on any future input values. (Or, the system is **causal** because $h(t) = 0$ for $t < 0$.)

The system is **not memoryless** because the output at time t does not only depend on the input at time t.

4.
The system is **memoryless** because the output at time n only depends on the input at time n.

The system is **not invertible**. Reorganizing the input-output relationship as: $x^2[n] - y[n]x[n] + 1 = 0$, we can find that $x[n]$ is expressed in terms of $y[n]$ with two possibilities in solving the quadratic equation.

The system is **not stable**. It is because for a bounded input $x[n] = 0$, the output will be unbounded.

The system is **causal** because the output does not depend on the future input value.

The system is **not linear**. The proof is as follows:
Let $y_1[n] = T\{x_1[n]\}$, $y_2[n] = T\{x_2[n]\}$ and $y_3[n] = T\{x_3[n]\}$ with $x_3[n] = a \cdot x_1[n] + b \cdot x_2[n]$.

The system outputs for $x_1[n]$ and $x_2[n]$ are:

$$y_1[n] = x_1[n] + 1/x_1[n] \quad \text{and} \quad y_2[n] = x_2[n] + 1/x_2[n]$$

The system output for $x_3[n]$ is then:

$$y_3[n] = x_3[n] + 1/x_3[n] = ax_1[n] + bx_2[n] + 1/\left(ax_1[n] + bx_2[n]\right)$$
$$\neq ax_1[n] + bx_2[n] + 1/(ax_1[n]) + 1/(bx_2[n]) = ay_1[n] + by_2[n]$$

The system is **time-invariant**. The proof is as follows:

First, we have $y[n-n_0] = x[n-n_0] + 1/x[n-n_0]$
Consider $x_1[n] = x[n-n_0]$, its system output is

$$y_1[n] = x_1[n] + 1/x_1[n] = x[n-n_0] + 1/x[n-n_0] = y[n-n_0]$$

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$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] = \sum_{m=-\infty}^{\infty} u[-m-1]\cdot(0.5)^{n-m}u[n-m]$$
$$= \sum_{m=-\infty}^{-1} (0.5)^{n-m}u[n-m]$$
$$= \sum_{l=1}^{\infty} (0.5)^{n+l}u[n+l]$$

For $n \geq -1$, all $\{u[n+l]\}$ correspond to 1 and we have:

$$y[n] = \sum_{l=1}^{\infty} (0.5)^{n+l} = (0.5)^n \sum_{l=1}^{\infty} (0.5)^l = (0.5)^n \cdot \frac{0.5}{1-0.5} = (0.5)^n$$

For $n < -1$, $u[n+l]=1$ when $n+l \geq 0$ or $l \geq -n$

$$y[n] = \sum_{l=-n}^{\infty} (0.5)^{n+l} = (0.5)^n \sum_{l=-n}^{\infty} (0.5)^l = (0.5)^n \cdot \frac{(0.5)^{-n}}{1-0.5} = 2$$

Combining the results, we have:

$$y[n] = \begin{cases} (0.5)^n, & n \geq -1 \\ 2, & n < -1 \end{cases}$$

The system is stable because $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$.

The system is causal because $h[n] = 0$ for $n < 0$.

6.
Let $y[n]$ be the convolution output. Starting from $n = -2$, $y[n]$ = -12, -9, -2, 0, -10, -8, -6, -3. At other time instants, the output is 0.

7(a)

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{1}{T}\int_0^T x^2(t)dt$$
$$= \frac{1}{2}\left[\int_0^1 1^2 dt + \int_1^2 2^2 dt\right] = 2.5$$

7(b)
The signal has period $T=2$ and fundamental frequency $\omega_0 = \pi$. Consider the period from $t=-1$ to $t=1$ and use (4.4):

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}dt = \frac{1}{2}\left[\int_{-1}^{0} 2e^{-jk\pi t}dt + \int_0^1 e^{-jk\pi t}dt\right]$$

For $k = 0$,

$$a_0 = \frac{1}{2}\left[\int_{-1}^{0} 2dt + \int_{0}^{1} 1dt\right] = \frac{1}{2}[2+1] = 1.5$$

For $k \neq 0$,

$$\begin{aligned} a_k &= \frac{1}{2}\left[\int_{-1}^{0} 2e^{-jk\pi t}dt + \int_{0}^{1} e^{-jk\pi t}dt\right] \\ &= \frac{1}{2}\left[\frac{2}{-jk\pi}e^{-jk\pi t}\Big|_{-1}^{0} + \frac{1}{-jk\pi}e^{-jk\pi t}\Big|_{0}^{1}\right] \\ &= -\frac{1}{2jk\pi}\left[2 - 2e^{jk\pi} + e^{-jk\pi} - 1\right] \\ &= -\frac{1}{2jk\pi}\left[1 - 2e^{jk\pi} + e^{-jk\pi}\right] \end{aligned}$$

Combining the results, we have:

$$a_k = \begin{cases} 1.5, & k = 0 \\ -\dfrac{1}{2jk\pi}\left[1 - 2e^{jk\pi} + e^{-jk\pi}\right] & k \neq 0 \end{cases}$$

7(c)
Using the linearity property, now $x(t)$ is modified as $10x(t)$, so we have:

$$a_k = \begin{cases} 15, & k = 0 \\ -\dfrac{5}{jk\pi}\left[1 - 2e^{jk\pi} + e^{-jk\pi}\right] & k \neq 0 \end{cases}$$

7(d)
Using the time reversal property, we can just change a_k to a_{-k}

$$a_k = \begin{cases} 1.5, & k = 0 \\ \dfrac{1}{2jk\pi}\left[1 - 2e^{-jk\pi} + e^{jk\pi}\right] & k \neq 0 \end{cases}$$

(You can also use time-shift property with a shift of 1)

7(e)

Using the time shifting property, $x(t) \leftrightarrow a_k \Rightarrow x(t-0.5) \leftrightarrow e^{-j0.5k\pi}a_k$:

$$a_k = \begin{cases} 1.5, & k = 0 \\ -\dfrac{e^{-j0.5k\pi}}{2jk\pi}\left[1 - 2e^{jk\pi} + e^{-jk\pi}\right] & k \neq 0 \end{cases}$$

8.
There are many possible answers.

One example is a software that computes the moving average of stock prices. In this software system, the input is the close price of a stock to be investigated. The output is the moving average based on the previous close prices including the current one. It is used for analysis the stock, particularly to see if it is on the rising or falling trend.

Consider N-day moving average where $x[n]$ is the input and $y[n]$ is the output, the input-output relationship is then:

$$y[n] = \frac{1}{N}\left(x[n] + x[n-1] + \cdots + x[n-N+1]\right)$$

It is a difference equation, and hence the system is both linear and time-invariant.

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Assignment 2

Due Date: 15 November 2023

1. Compute the Fourier transform of $x(t) = e^{-2|t-1|}$.

2. Find the frequency response $H(e^{j\omega})$ of a discrete-time linear time-invariant (LTI) system whose input $x[n]$ and output $y[n]$ satisfy the following difference equation:

$$y[n] - \frac{1}{2}y[n-1] = x[n] + 2x[n-1] + x[n-2]$$

3. Determine the difference equation that characterizes a discrete-time LTI system whose frequency response is

$$H(e^{j\omega}) = \frac{1 - \frac{1}{2}e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2}e^{-j\omega} + \frac{3}{4}e^{-j2\omega}}$$

4. Figure 1 shows a system which consists of an interconnection of two discrete-time LTI systems with impulse responses $h_1[n]$ and $h_2[n]$.

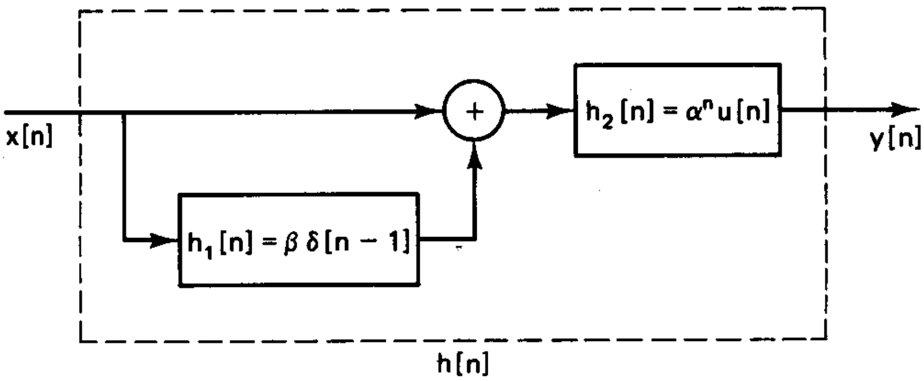

Figure 1

(a) Find the impulse response $h[n]$ of the overall system.
(b) Find the system transfer function $H(z)$ of the overall system, which is equal to $Y(z)/X(z)$ where $X(z)$ and $Y(z)$ are the z transforms of the input $x[n]$ and output $y[n]$, respectively.
(c) Write down the difference equation that relates $x[n]$ and $y[n]$.
(d) Is the system causal?
(e) Under what condition would the system be stable?

5. Given a discrete-time signal $x[n]$ which has the form of:

$$x[n] = \begin{cases} \alpha e^{j(\omega_o n+\phi)}, & 0 \le n \le N-1 \\ 0, & \text{otherwise} \end{cases}$$

where α, ω_0 and ϕ are real numbers.

(a) Determine $X(e^{j\omega})$ which is the discrete-time Fourier transform of $x[n]$.
(b) Find the maximum value of $|X(e^{j\omega})|$. Determine the value of ω which maximizes $|X(e^{j\omega})|$.

6. Given a continuous-time signal $x(t)$:

$$x(t) = \sin\left(\frac{\pi}{2}t\right)$$

The signal is sampled with a sampling period $T = 1$s to produce the discrete-time signal $x[n]$. Find $x[0]$, $x[1]$, $x[2]$, $x[3]$ and $x[4]$. Is $x[n]$ a periodic signal?

7. Determine the z transform of $x[n]$ which has the form of:

$$x[n] = \begin{cases} na^n, & 1 \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

Specify the region of convergence (ROC).

8. Consider a discrete-time LTI system whose transfer function $H(z)$ is:

$$H(z) = \frac{z^{-2}}{(1-0.5z^{-1})(1-3z^{-1})}$$

(a) If the system is stable, determine the output $y[n]$ when the input is $x[n] = u[n]$.
(b) If the system is causal, determine the output $y[n]$ when $x[n] = \delta[n]$.

9. Use z transform and inverse z transform to compute the convolution of $x[n] = u[-n-1]$ and $h[n] = (0.5)^n u[n]$.

10. Watch the short video of the 2013 Shaw Prize winner for mathematics, Prof. David Donoho (start at 14:50): https://www.youtube.com/watch?v=5wv4grOMgIU

(a) Briefly describe a denoising system, which includes the system input, output and function, as well as the principle to achieve denoising. Use your own words in no more than 100 words.
(b) Suppose you are given the following observed continuous-time signal $x(t)$:

$$x(t) = \cos(100\pi t) + n(t)$$

where $n(t)$ is the unwanted noise. With the use of an appropriate transform you have learned in this course, briefly describe, in theory, how can you extract $\cos(100\pi t)$ from $x(t)$? Also, how can you achieve compression for $\cos(100\pi t)$?

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Solution for Assignment 2

1.
Re-expressing $x(t)$ as

$$x(t) = e^{-2|t-1|} = \begin{cases} e^{-2(t-1)}, & t > 1 \\ e^{2(t-1)}, & t < 1 \end{cases}$$

We then apply (5.1) to obtain:

$$\begin{aligned} X(j\Omega) &= \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt \\ &= \int_{-\infty}^{1} e^{2(t-1)} e^{-j\Omega t} dt + \int_{1}^{\infty} e^{-2(t-1)} e^{-j\Omega t} dt \\ &= \frac{e^{-j\Omega}}{2 - j\Omega} + \frac{e^{-j\Omega}}{2 + j\Omega} \\ &= \frac{4e^{-j\Omega}}{4 + \Omega^2} \end{aligned}$$

2.
Taking the DTFT on both sides of

$$y[n] - \frac{1}{2} y[n-1] = x[n] + 2[n-1] + x[n-2]$$

yields:

$$Y(e^{j\omega})\left(1 - 0.5e^{-j\omega}\right) = X(e^{j\omega})\left(1 + 2e^{-j\omega} + e^{-j2\omega}\right) \Rightarrow H(e^{j\omega}) = \frac{1 + 2e^{-j\omega} + e^{-j2\omega}}{1 - 0.5e^{-j\omega}}$$

3.
Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ be the DTFTs of the system input $x[n]$ and output $y[n]$. We then have:

$$\begin{aligned} & H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - \frac{1}{2} e^{-j\omega} + e^{-j3\omega}}{1 + \frac{1}{2} e^{-j\omega} + \frac{3}{4} e^{-j2\omega}} \\ & \Rightarrow Y(e^{j\omega}) \left(1 + \frac{1}{2} e^{-j\omega} + \frac{3}{4} e^{-j2\omega}\right) = X(e^{j\omega}) \left(1 - \frac{1}{2} e^{-j\omega} + e^{-j3\omega}\right) \\ & \Rightarrow y[n] + \frac{1}{2} y[n-1] + \frac{3}{4} y[n-2] = x[n] - \frac{1}{2} x[n-1] + x[n-3] \end{aligned}$$

Alternatively, we can convert $X(e^{j\omega})$ and $Y(e^{j\omega})$ to $X(z)$ and $Y(z)$ and then apply inverse z transform.

4.(a)
From the figure, we have

$$\begin{aligned} y[n] &= (x[n] + x[n] \otimes h_1[n]) \otimes h_2[n] \\ &= (x[n] \otimes \delta[n] + x[n] \otimes h_1[n]) \otimes h_2[n] \\ &= x[n] \otimes ([\delta[n] + h_1[n]]) \otimes h_2[n] \end{aligned}$$

As a result, the overall impulse response $h[n]$ is:

$$h[n] \quad = \quad ([\delta[n] + h_1[n]]) \otimes h_2[n] = [\delta[n] + \beta\delta[n-1]] \otimes \alpha^n u[n] = \alpha^n u[n] + \beta\alpha^{n-1} u[n-1]$$

4.(b)
Taking the z transform of $h[n]$ yields

$$H(z) = \frac{1}{1-\alpha z^{-1}} + \frac{\beta z^{-1}}{1-\alpha z^{-1}}, \quad |z| > |\alpha|$$

4.(c)
Apply cross-multiplying and perform inverse z transform, we get:

$$\begin{aligned} &H(z) = \frac{Y(z)}{X(z)} = \frac{1+\beta z^{-1}}{1-\alpha z^{-1}} \\ &\Rightarrow Y(z)(1-\alpha z^{-1}) = X(z)(1+\beta z^{-1}) \\ &\Rightarrow y[n] - \alpha y[n-1] = x[n] + \beta x[n-1] \end{aligned}$$

4.(d)
As $h[n] = 0$ for $n < 0$, the system is causal.

4.(e)
The system is stable if the ROC of $H(z)$ includes the unit circle, i.e., $|\alpha| < 1$.

5.(a)

$$\begin{aligned} X(e^{j\omega}) &= \alpha e^{j\phi} \sum_{n=0}^{N-1} e^{j(\omega_0-\omega)n} \\ &= \alpha e^{j\phi} \frac{1-e^{j(\omega_0-\omega)N}}{1-e^{j(\omega_0-\omega)}} \\ &= \alpha e^{j(\phi-(\omega_0-\omega)(N-1)/2)} \frac{\sin(\frac{(\omega_0-\omega)N}{2})}{\sin(\frac{\omega_0-\omega}{2})} \end{aligned}$$

5.(b)
Using the result in Example 6.3, we have

$$\frac{\sin((\omega_0-\omega)N/2)}{\sin((\omega_0-\omega)/2)} = N \cdot \frac{\mathrm{sinc}((\omega_0-\omega)N/(2\pi))}{\mathrm{sinc}((\omega_0-\omega)/(2\pi))}$$

where its maximum appears at $\omega = \omega_0$, with a value of N. As a result, the maximum value of $|X(e^{j\omega})|$ is $|\alpha|N$. The value of ω which maximizes $|X(e^{j\omega})|$ is thus $\omega = \omega_0$.

6.
$x[0]=0,\ x[1]=1,\ x[2]=0,\ x[3]=-1$ and $x[4]=0$.

Yes. $x[n]$ is a periodic signal.

7.
The z transform is:

$$X(z)=\sum_{n=1}^{N} na^n z^{-n} = az^{-1}+2a^2z^{-2}+\cdots Na^Nz^{-N}$$

Clearly, the ROC is $|z|>0$.

Considering $X(z)=X_1(z)+X_2(z)+\cdots+X_N(z)$ where

$$X_1(z)=az^{-1}+a^2z^{-2}+\cdots a^Nz^{-N}=\frac{az^{-1}(1-(az^{-1})^N)}{1-az^{-1}}$$
$$X_2(z)=a^2z^{-2}+a^3z^{-3}+\cdots a^Nz^{-N}=\frac{a^2z^{-2}(1-(az^{-1})^{N-1})}{1-az^{-1}}$$
…………………………………..
$$X_N(z)=a^Nz^{-N}=\frac{a^Nz^{-N}(1-az^{-1})}{1-az^{-1}}$$

As a result, we have:

$$\begin{aligned}
X(z) &= \frac{az^{-1}(1-(az^{-1})^N)}{1-az^{-1}}+\frac{a^2z^{-2}(1-(az^{-1})^{N-1})}{1-az^{-1}}+\cdots+\frac{a^Nz^{-N}(1-az^{-1})}{1-az^{-1}}\\
&= \frac{az^{-1}+a^2z^{-2}+\cdots+a^Nz^{-N}-N(az^{-1})^{N+1}}{1-az^{-1}}\\
&= \frac{\frac{az^{-1}(1-(az^{-1})^N)}{1-az^{-1}}-N(az^{-1})^{N+1}}{1-az^{-1}}\\
&= \frac{az^{-1}-(N+1)(az^{-1})^{N+1}+N(az^{-1})^{N+2}}{(1-az^{-1})^2},\quad |z|>0
\end{aligned}$$

Alternatively, you can find $X(z)$ by first expressing $x[n]$ as:

$$x[n]=na^n(u[n-1]-u[n-N+1])$$

and then make use of Table 8.1 and time shifting property.

8.(a)
If the system is stable, then the ROC for $H(z)$ is $0.5<|z|<3$. On the other hand, for the unit step input, we have:

$$X(z)=\frac{1}{1-z^{-1}},\quad |z|>1$$

The z transform for $y[n]$ is $Y(z)=H(z)X(z)$. Using partial fraction expansion, we get:

$$Y(z) = H(z)X(z) = \frac{0.8}{1-0.5z^{-1}} + \frac{0.2}{1-3z^{-1}} - \frac{1}{1-z^{-1}}, \quad 1 < |z| < 3$$

Taking the inverse z transform, we get:

$$y[n] = (0.8)(0.5)^n u[n] - 0.2(3)^n u[-n-1] - u[n]$$

8.(b)
If the system is causal, then the ROC for $H(z)$ is $|z| > 3$. For $x[n] = \delta[n]$, $y[n] = h[n]$. Using partial fraction expansion, we get:

$$Y(z) = H(z) = \frac{-0.4z^{-1}}{1-0.5z^{-1}} + \frac{0.4z^{-1}}{1-3z^{-1}}, \quad |z| > 3$$

Taking the inverse z transform, we get:

$$y[n] = -(0.4)(0.5)^{n-1}u[n-1] + 0.4(3)^{n-1}u[n-1]$$

9.
The z transforms of $x[n] = u[-n-1]$ and $h[n] = (0.5)^n u[n]$ are:

$$X(z) = -\frac{1}{1-z^{-1}}, \quad |z| < 1 \qquad \text{and} \qquad H(z) = \frac{1}{1-0.5z^{-1}}, \quad |z| > 0.5$$

So we have:

$$Y(z) = -\frac{1}{1-z^{-1}} \cdot \frac{1}{1-0.5z^{-1}} = \frac{-2}{1-z^{-1}} + \frac{1}{1-0.5z^{-1}}, \quad 0.5 < |z| < 1$$

Taking the inverse z transform yields:

$$y[n] = 2u[-n-1] + (0.5)^n u[n]$$

10.(a)
In a denoising system, the input contains a signal-of-interest and additive noise, and the system attempts to extract the signal-of-interest while removing/suppressing the noise, thus it is expected that the output is a good approximation of the signal-of-interest. The principle is that, under a certain transform, the signal-of-interest is sparse, meaning that there are only a few non-zero entries and we only need to keep them and ignoring the rest.

10.(b)
In theory, we can transform the continuous-time signal to frequency domain using Fourier transform. The $\cos(100\pi t)$ corresponds to two impulses at -100π and 100π in the frequency domain. We keep only the two components at -100π and 100π, and assign the rest to zero, and convert this resultant frequency-domain signal to time domain using the inverse Fourier transform.

Compression is easily achieved. Instead of storing $\cos(100\pi t)$, we only need to store the amplitudes and locations of the two impulses in the frequency-domain signal.