Solutions to EE3210 Assignment 2

Problem 1: We know that both $\cos(2t)$ and $\sin(2\pi t)$ are periodic. The fundamental period of $\cos(2t)$ is $T_0 = 2\pi/\omega_0 = 2\pi/2 = \pi$. The fundamental period of $\sin(2\pi t)$ is $T_0 = 2\pi/\omega_0 = 2\pi/(2\pi) = 1$. To determine whether or not the signal $x(t) = \cos(2t)+\sin(2\pi t)$ is periodic, it reduces to find if there exist two integers m and k such that

$$m \cdot \pi = k \cdot 1. \tag{1}$$

Since π is not a rational number, (1) does not hold. Therefore, $x(t)$ is not a periodic signal.

Problem 2:

(a) The system is not memoryless. For example, when $n = 1$, we have $y[1] = x[2]$.

(b) The system is not invertible. For example, consider $x_1[n] = 1$, $x_2[n] = (-1)^n$. Then, $y_1[n] = x_1[2n] = 1$, $y_2[n] = x_2[2n] = (-1)^{2n} = 1$, so that $y_1[n] = y_2[n]$ for all n.

Note: Some students tried to show the result by suggesting that $w[n] = y[n/2]$ is not a valid system. However, this is not rigorous enough, since the fact that $w[n] = y[n/2]$ is not a valid system does not necessarily mean that there does not exist an inverse system in this context, unless we rigorously prove that there are no other systems such that $w[n] = x[n]$. As I discussed in the lectures, to show that a system is not invertible, it is much easier if we can make a counterexample and show that distinct inputs lead to same outputs. On the other hand, to show that a system is invertible, which implies that we cannot find any counterexample, we have to form an inverse system such that $w[n] = x[n]$.

(c) The system is not causal. This can be justified using the same example in (a).

(d) The system is stable. For $0 < B < \infty$, given $|x[n]| \leq B$ for all n, we have $|x[2n]| \leq B$ for all n, and therefore $|y[n]| \leq B$ for all n.

(e) The system is not time invariant. Given $x_1[n]$ and letting $y_1[n] = x_1[2n]$, consider $x_2[n] = x_1[n - n_0]$. Then, we have $y_2[n] = x_2[2n] = x_1[2n - n_0]$, but we have $y_1[n - n_0] = x_1[2n - 2n_0]$. Thus, $y_2[n] \neq y_1[n - n_0]$.

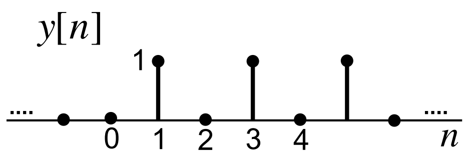
(f) The system is linear. Consider $x_1[n] \to y_1[n] = x_1[2n]$ and $x_2[n] \to y_2[n] = x_2[2n]$. Let $x_3[n] = ax_1[n] + bx_2[n]$. Then,

$$y_3[n] = x_3[2n] = ax_1[2n] + bx_2[2n] = ay_1[n] + by_2[n].$$

Problem 3: Since $e[n] = x[n] - y[n]$, we have

$$y[n] = e[n-1] = x[n-1] - y[n-1].$$

The output $y[n]$ is sketched in the figure below.



The results of $y[n]$ can be expressed in a compact form as

$$y[n] = \frac{1-(-1)^n}{2} u[n].$$

Problem 4: Note that $x_2(t) = x_1(t) + x_1(t+1)$. Therefore, using linearity and time invariance, we get $y_2(t) = y_1(t) + y_1(t+1)$. The two signals $y_1(t)$ and $y_1(t+1)$ overlap in the interval $0 < t < 1$. In particular, when $0 < t < 1$, $y_1(t) = -2t + 2$, $y_1(t+1) = 2t$. Thus, $y_2(t) = 2$ when $0 < t < 1$. The signal $y_2(t)$ is shown in the figure below.