9.13. Let

$$g(t) = x(t) + \alpha x(-t),$$

where

$$x(t) = \beta e^{-t} u(t)$$

and the Laplace transform of $g(t)$ is

$$G(s) = \frac{s}{s^2 - 1}, \qquad -1 < \mathcal{R}e\{s\} < 1.$$

Determine the values of the constants α and β.

9.14. Suppose the following facts are given about the signal $x(t)$ with Laplace transform $X(s)$:

1. $x(t)$ is real and even.
2. $X(s)$ has four poles and no zeros in the finite s-plane.
3. $X(s)$ has a pole at $s = (1/2)e^{j\pi/4}$.
4. $\int_{-\infty}^{\infty} x(t)\,dt = 4$.

Determine $X(s)$ and its ROC.

9.15. Consider two right-sided signals $x(t)$ and $y(t)$ related through the differential equations

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t)$$

and

$$\frac{dy(t)}{dt} = 2x(t).$$

Determine $Y(s)$ and $X(s)$, along with their regions of convergence.

9.16. A causal LTI system S with impulse response $h(t)$ has its input $x(t)$ and output $y(t)$ related through a linear constant-coefficient differential equation of the form

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(\alpha+1)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t).$$

(a) If

$$g(t) = \frac{dh(t)}{dt} + h(t),$$

how many poles does $G(s)$ have?

(b) For what real values of the parameter α is S guaranteed to be stable?

9.17. A causal LTI system S has the block diagram representation shown in Figure P9.17. Determine a differential equation relating the input $x(t)$ to the output $y(t)$ of this system.

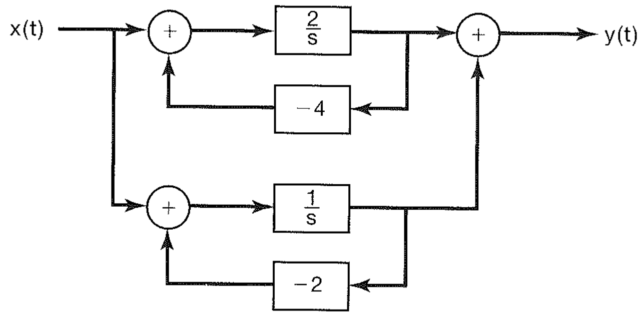

Figure P9.17

9.18. Consider the causal LTI system represented by the *RLC* circuit examined in Problem 3.20.

(a) Determine $H(s)$ and specify its region of convergence. Your answer should be consistent with the fact that the system is causal and stable.

(b) Using the pole-zero plot of $H(s)$ and geometric evaluation of the magnitude of the Fourier transform, determine whether the magnitude of the corresponding Fourier transform has an approximately lowpass, highpass, or bandpass characteristic.

(c) If the value of R is now changed to $10^{-3}\ \Omega$, determine $H(s)$ and specify its region of convergence.

(d) Using the pole-zero plot of $H(s)$ obtained in part (c) and geometric evaluation of the magnitude of the Fourier transform, determine whether the magnitude of the corresponding Fourier transform has an approximately lowpass, highpass, or bandpass characteristic.

9.19. Determine the unilateral Laplace transform of each of the following signals, and specify the corresponding regions of convergence:

(a) $x(t) = e^{-2t}u(t+1)$

(b) $x(t) = \delta(t+1) + \delta(t) + e^{-2(t+3)}u(t+1)$

(c) $x(t) = e^{-2t}u(t) + e^{-4t}u(t)$

9.20. Consider the *RL* circuit of Problem 3.19.

(a) Determine the zero-state response of this circuit when the input current is $x(t) = e^{-2t}u(t)$.

(b) Determine the zero-input response of the circuit for $t > 0^-$, given that

$$y(0^-) = 1.$$

(c) Determine the output of the circuit when the input current is $x(t) = e^{-2t}u(t)$ and the initial condition is the same as the one specified in part (b).

BASIC PROBLEMS

9.21. Determine the Laplace transform and the associated region of convergence and pole-zero plot for each of the following functions of time:

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$ **(b)** $x(t) = e^{-4t}u(t) + e^{-5t}(\sin 5t)u(t)$

(c) $x(t) = e^{2t}u(-t) + e^{3t}u(-t)$ **(d)** $x(t) = te^{-2|t|}$

9.31. Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t).$$

Let $X(s)$ and $Y(s)$ denote Laplace transforms of $x(t)$ and $y(t)$, respectively, and let $H(s)$ denote the Laplace transform of $h(t)$, the system impulse response.

(a) Determine $H(s)$ as a ratio of two polynomials in s. Sketch the pole-zero pattern of $H(s)$.

(b) Determine $h(t)$ for each of the following cases:

1. The system is stable.
2. The system is causal.
3. The system is *neither* stable *nor* causal.

9.32. A causal LTI system with impulse response $h(t)$ has the following properties:

1. When the input to the system is $x(t) = e^{2t}$ for all t, the output is $y(t) = (1/6)e^{2t}$ for all t.
2. The impulse response $h(t)$ satisfies the differential equation

$$\frac{dh(t)}{dt} + 2h(t) = (e^{-4t})u(t) + bu(t),$$

where b is an unknown constant.

Determine the system function $H(s)$ of the system, consistent with the information above. There should be no unknown constants in your answer; that is, the constant b should *not* appear in the answer.

9.33. The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}.$$

Determine and sketch the response $y(t)$ when the input is

$$x(t) = e^{-|t|}, \qquad -\infty < t < \infty.$$

9.34. Suppose we are given the following information about a causal and stable LTI system S with impulse response $h(t)$ and a rational system function $H(s)$:

1. $H(1) = 0.2$.
2. When the input is $u(t)$, the output is absolutely integrable.
3. When the input is $tu(t)$, the output is not absolutely integrable.
4. The signal $d^2h(t)/dt^2 + 2\,dh(t)/dt + 2h(t)$ is of finite duration.
5. $H(s)$ has exactly one zero at infinity.

Determine $H(s)$ and its region of convergence.

9.35. The input $x(t)$ and output $y(t)$ of a causal LTI system are related through the block-diagram representation shown in Figure P9.35.

(a) Determine a differential equation relating $y(t)$ and $x(t)$.

(b) Is this system stable?

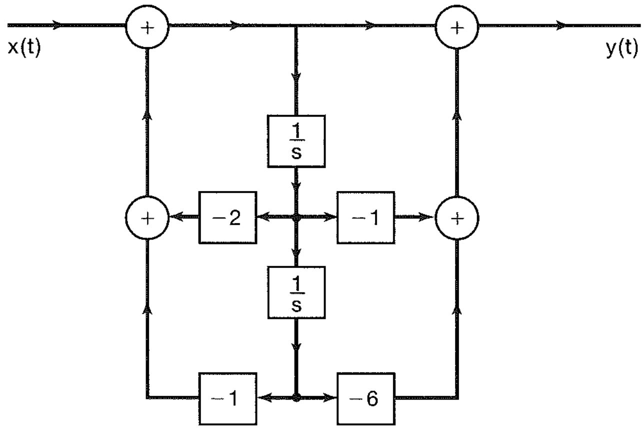

Figure P9.35

9.36. In this problem, we consider the construction of various types of block diagram representations for a causal LTI system S with input $x(t)$, output $y(t)$, and system function

$$H(s) = \frac{2s^2 + 4s - 6}{s^2 + 3s + 2}.$$

To derive the direct-form block diagram representation of S, we first consider a causal LTI system S_1 that has the same input $x(t)$ as S, but whose system function is

$$H_1(s) = \frac{1}{s^2 + 3s + 2}.$$

With the output of S_1 denoted by $y_1(t)$, the direct-form block diagram representation of S_1 is shown in Figure P9.36. The signals $e(t)$ and $f(t)$ indicated in the figure represent respective inputs into the two integrators.

(a) Express $y(t)$ (the output of S) as a linear combination of $y_1(t)$, $dy_1(t)/dt$, and $d^2y_1(t)/dt^2$.

(b) How is $dy_1(t)/dt$ related to $f(t)$?

(c) How is $d^2y_1(t)/dt^2$ related to $e(t)$?

(d) Express $y(t)$ as a linear combination of $e(t)$, $f(t)$, and $y_1(t)$.

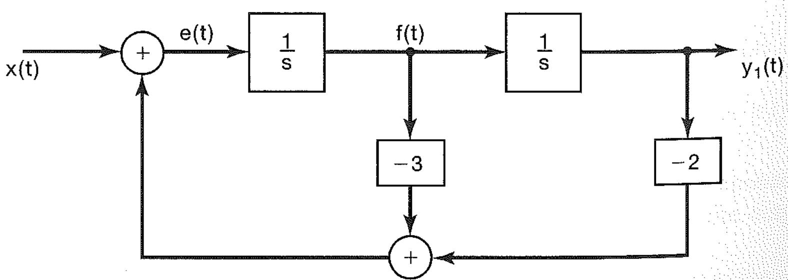

Figure P9.36