EE2302 Foundations of Information Engineering

Assignment 4 (Solution)

1. The statement is true.
   Proof: Suppose m is any even integer and n is any integer. [We must show that mn is even.]
   By the definition of even numbers, there exists an integer k such that $m = 2k$.
   By substitution,
   $$mn = (2k)n = 2(kn).$$
   Since kn is an integer, by the definition of even numbers, mn is even. *Q.E.D.*

2. Let p = "the number is prime", q = "the number is either odd or 2". We should prove by contraposition: $p \rightarrow q \equiv \sim q \rightarrow \sim p$. Suppose the number is neither odd nor 2. Then it is even but not 2, i.e., 4, 6, 8, or 10, …. Therefore, it can be divided by 2 and is not prime. By contraposition, the statement is true. *Q.E.D.*

3. Yes. Define a function $f: (0,1) \rightarrow (1,100)$ such that $f(x) = 99x + 1$. Suppose $f(x_1) = f(x_2)$. Then $99x_1 + 1 = 99x_2 + 1$, which implies that $x_1 = x_2$. Hence, f is one-to-one. Given any $y \in (1, 100)$, there exists $x = \frac{y-1}{100} \in (0,1)$ such that $f(x) = y$. Hence, f is onto. Therefore, f is a one-to-one correspondence. Hence, the two sets have the same cardinality.