

## EE2302 Foundations of Information Engineering

### Assignment 4 (Solution)

- Any integer  $n$  can be written as  $n = 3q + r$  using the quotient-remainder theorem, where  $0 \leq r < 3$ . There are three possible cases:
  - For  $r = 0$ ,  $n^2 = (3q)^2 = 3(3q^2) = 3k$  for  $k = 3q^2$ .
  - For  $r = 1$ ,  $n^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3k + 1$  for  $k = (3q^2 + 2q)$ .
  - For  $r = 2$ ,  $n^2 = (3q + 2)^2 = 9q^2 + 12q + 4 = 3(3q^2 + 4q + 1) + 1 = 3k + 1$  for  $k = (3q^2 + 4q + 1)$ .
 Therefore, the square of any integer has the form  $3k$  or  $3k+1$  for some integer  $k$ .

$$\begin{aligned}
 2. \quad \phi(560) &= \phi(2^4 * 5 * 7) \\
 &= \phi(2^4)\phi(5)\phi(7) \\
 &= 2^3(2 - 1)(5 - 1)(7 - 1) \\
 &= 192.
 \end{aligned}$$

$$3. \quad \gcd(46288, 2046) = 22$$

22	46288 45012	2046 1276	1
1	1276 770	770 506	1
1	506 264	264 242	1
1	242 22	22 0	22

4.

10245	1689		
1	0	10245	$a$
0	1	1689	$b$
1	-6	111	$c = a - 6b$
-15	91	24	$d = b - 15c$
61	-370	15	$e = c - 4d$
-76	461	9	$f = d - e$
137	-831	6	$g = e - f$
-213	1292	3	$h = f - g$

$$\gcd(10245, 1689) = 3, \quad x = -213 \text{ and } y = 1292.$$