10.15. Let

$$y[n] = \left(\frac{1}{9}\right)^n u[n].$$

Determine two distinct signals such that each has a z-transform $X(z)$ which satisfies both of the following conditions:

1. $[X(z) + X(-z)]/2 = Y(z^2)$.
2. $X(z)$ has only one pole and only one zero in the z-plane.

10.16. Consider the following system functions for stable LTI systems. Without utilizing the inverse z-transform, determine in each case whether or not the corresponding system is causal.

(a) $\dfrac{1 - \frac{4}{3}z^{-1} + \frac{1}{2}z^{-2}}{z^{-1}(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})}$

(b) $\dfrac{z - \frac{1}{2}}{z^2 + \frac{1}{2}z - \frac{3}{16}}$

(c) $\dfrac{z + 1}{z + \frac{4}{3} - \frac{1}{2}z^{-2} - \frac{2}{3}z^{-3}}$

10.17. Suppose we are given the following five facts about a particular LTI system S with impulse response $h[n]$ and z-transform $H(z)$:

1. $h[n]$ is real.
2. $h[n]$ is right sided.
3. $\lim_{z \to \infty} H(z) = 1$.
4. $H(z)$ has two zeros.
5. $H(z)$ has one of its poles at a nonreal location on the circle defined by $|z| = 3/4$.

Answer the following two questions:

(a) Is S causal? **(b)** Is S stable?

10.18. Consider a causal LTI system whose input $x[n]$ and output $y[n]$ are related through the block diagram representation shown in Figure P10.18.

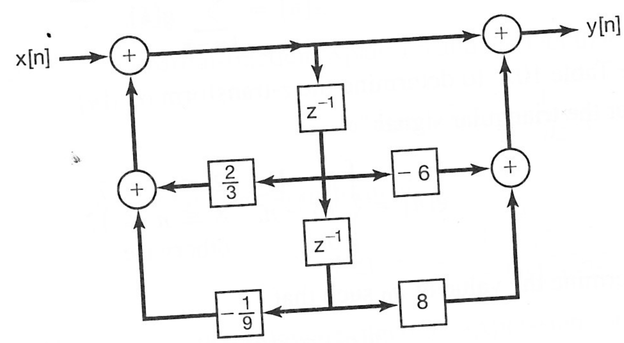

Figure P10.18

(a) Determine a difference equation relating $y[n]$ and $x[n]$.

(b) Is this system stable?

10.19. Determine the unilateral z-transform of each of the following signals, and specify the corresponding regions of convergence:

(a) $x_1[n] = (\frac{1}{4})^n u[n+5]$
(b) $x_2[n] = \delta[n+3] + \delta[n] + 2^n u[-n]$
(c) $x_3[n] = (\frac{1}{2})^{|n|}$

10.20. Consider a system whose input $x[n]$ and output $y[n]$ are related by

$$y[n-1] + 2y[n] = x[n].$$

(a) Determine the zero-input response of this system if $y[-1] = 2$.
(b) Determine the zero-state response of the system to the input $x[n] = (1/4)^n u[n]$.
(c) Determine the output of the system for $n \geq 0$ when $x[n] = (1/4)^n u[n]$ and $y[-1] = 2$.

BASIC PROBLEMS

10.21. Determine the z-transform for each of the following sequences. Sketch the pole-zero plot and indicate the region of convergence. Indicate whether or not the Fourier transform of the sequence exists.

(a) $\delta[n+5]$
(b) $\delta[n-5]$
(c) $(-1)^n u[n]$
(d) $(\frac{1}{2})^{n+1} u[n+3]$
(e) $(-\frac{1}{3})^n u[-n-2]$
(f) $(\frac{1}{4})^n u[3-n]$
(g) $2^n u[-n] + (\frac{1}{4})^n u[n-1]$
(h) $(\frac{1}{3})^{n-2} u[n-2]$

10.22. Determine the z-transform for the following sequences. Express all sums in closed form. Sketch the pole-zero plot and indicate the region of convergence. Indicate whether the Fourier transform of the sequence exists.

(a) $(\frac{1}{2})^n \{u[n+4] - u[n-5]\}$
(b) $n(\frac{1}{2})^{|n|}$
(c) $|n|(\frac{1}{2})^{|n|}$
(d) $4^n \cos[\frac{2\pi}{6}n + \frac{\pi}{4}]u[-n-1]$

10.23. Following are several z-transforms. For each one, determine the inverse z-transform using both the method based on the partial-fraction expansion and the Taylor's series method based on the use of long division.

$$X(z) = \frac{1 - z^{-1}}{1 - \frac{1}{4}z^{-2}}, \quad |z| > \frac{1}{2}.$$

$$X(z) = \frac{1 - z^{-1}}{1 - \frac{1}{4}z^{-2}}, \quad |z| < \frac{1}{2}.$$

$$X(z) = \frac{z^{-1} - \frac{1}{2}}{1 - \frac{1}{2}z^{-1}}, \quad |z| > \frac{1}{2}.$$

$$X(z) = \frac{z^{-1} - \frac{1}{2}}{1 - \frac{1}{2}z^{-1}}, \quad |z| < \frac{1}{2}.$$

$$X(z) = \frac{z^{-1} - \frac{1}{2}}{(1 - \frac{1}{2}z^{-1})^2}, \quad |z| > \frac{1}{2}.$$

$$X(z) = \frac{z^{-1} - \frac{1}{2}}{(1 - \frac{1}{2}z^{-1})^2}, \quad |z| < \frac{1}{2}.$$

10.34. A causal LTI system is described by the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n-1].$$

(a) Find the system function $H(z) = Y(z)/X(z)$ for this system. Plot the poles and zeros of $H(z)$ and indicate the region of convergence.
(b) Find the unit sample response of the system.
(c) You should have found the system to be unstable. Find a stable (noncausal) unit sample response that satisfies the difference equation.

10.35. Consider an LTI system with input $x[n]$ and output $y[n]$ for which

$$y[n-1] - \frac{5}{2}y[n] + y[n+1] = x[n].$$

The system may or may not be stable or causal.

By considering the pole-zero pattern associated with the preceding difference equation, determine three possible choices for the unit sample response of the system. Show that each choice satisfies the difference equation.

10.36. Consider the linear, discrete-time, shift-invariant system with input $x[n]$ and output $y[n]$ for which

$$y[n-1] - \frac{10}{3}y[n] + y[n+1] = x[n].$$

The system is stable. Determine the unit sample response.

10.37. The input $x[n]$ and output $y[n]$ of a causal LTI system are related through the block-diagram representation shown in Figure P10.37.

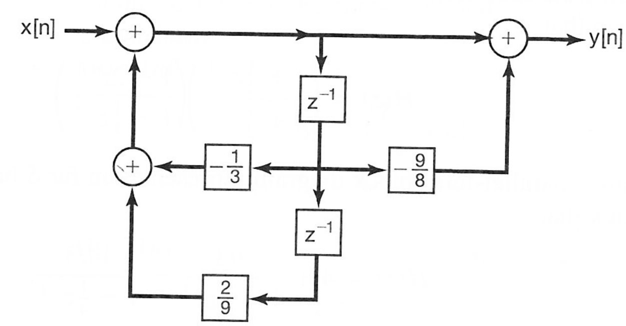

Figure P10.37

(a) Determine a difference equation relating $y[n]$ and $x[n]$.
(b) Is this system stable?

10.38. Consider a causal LTI system S with input $x[n]$ and a system function specified as

$$H(z) = H_1(z)H_2(z),$$

where

$$H_1(z) = \frac{1}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}}$$