Tutorial 1

Sets

Question 1: Inclusion & Exclusion

- What is the formula for $|A \cup B \cup C|$?

a) $|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

b) $|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + 3|A \cap B \cap C|$

c) $|A| + |B| + |C| - 2|A \cap B| - 2|A \cap C| - 2|B \cap C| + 3|A \cap B \cap C|$

d) $|A| + |B| + |C| - 3|A \cap B| - 3|A \cap C| - 3|B \cap C| + 3|A \cap B \cap C|$

Question 2: Subset Relationship

Let $A = \{n \in \mathbf{Z} \mid n = 5r \text{ for some integer } r\}$

and $B = \{m \in \mathbf{Z} \mid m = 20s \text{ for some integer } s\}$.

i. Is $A \subseteq B$?

ii. Is $B \subseteq A$?

a) Both are true.

b) Both are false.

c) (i) is true while (ii) is false

d) (i) is false while (ii) is true

Question 3: Cartesian Product

- Consider two nonempty sets A and B.

- Is it true that $A \times B \neq B \times A$?

  a) Yes
  b) No
  c) Cannot be determined

  Justify your answer.

Question 4: Union and Intersection

Let $R_j = \left\{x \in \mathbb{R} \mid 1 \leq x \leq 1 + \frac{1}{j}\right\} = \left[1, 1 + \frac{1}{j}\right]$.

i. What is $\bigcup_{j=1}^{4} R_j$?

ii. What is $\bigcap_{j=1}^{4} R_j$?

iii. Are R_1, R_2, R_3, and R_4 mutually disjoint? Why?

iv. What is $\bigcup_{j=1}^{\infty} R_j$?

v. What is $\bigcap_{j=1}^{\infty} R_j$?