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EE2302 Foundations of Information Engineering

Semester A, 2022/23

Test 3 (75 min.) Name: 18 (hung Well Student ID: 5747463

Answer ALL questions.

1. (10 marks) Define a function $f: \mathbb{R}^2 \to \mathbb{R}$ by $f(x) = x_1y_2 - x_2y_1$, where y_1, y_2 are constants and $x = (x_1, x_2)$. Is f a linear function? Prove or disprove it.

Addition:

mti:

2. (10 marks) Using Cauchy-Schwarz inequality to prove that

$$\sum_{m=1}^{M} (b_m - 1)^2 \ge \frac{1}{M} \left(\sum_{m=1}^{M} (b_m - 1) \right)^2$$

(Hint: Cauchy-Schwarz inequality: $|a^T b| \le ||a|| ||b||$.)

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$$\frac{2(b_{m}-1)^{2}}{m[2(b_{m}-1)]^{2}} = \frac{1}{m[(b_{m}-1)^{2}-...+(b_{n}-1)]^{2}}$$

3. (10 marks) Let x and y be vectors in \mathbb{R}^3 , where x = (1,2,3) and y = (3,2,1). Is the span of x and y a subspace of \mathbb{R}^3 ? Prove or disprove it.

4. Consider the equation
$$Ax = b$$
, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$.

$$C(A) = \{n \in \mathbb{R}^2 : [\frac{1}{3}]\} \quad \text{basis is } [\frac{1}{3}] \text{ since } 2[\frac{1}{3}] = [\frac{7}{6}]$$

$$AR = 0 \quad [\frac{1}{3} \frac{2}{6}][\frac{x_1}{n_2}] = [\frac{9}{9}]$$

$$N(A) = [\frac{7}{3}]$$

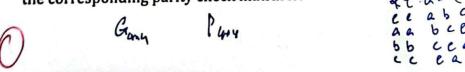
$$N(A) = [\frac{7}{3}]$$

b) (8 marks) Determine the general solution for
$$x$$
.

5. Consider the binary linear code C which appends four parity bits c_5 , c_6 , c_7 , c_8 to four information bits u_1 , u_2 , u_3 , u_4 in the following way. Put the four information bits in a 2×2 array and the four parity bits are obtained by two-dimensional even parity as shown below (i.e., each row and each column has an even number of 1's):

u_1	u_2	C ₅
u_3	u_4	C ₆
C7	C ₈	

a) (6 marks) The encoding function $f: \mathbb{B}^4 \to \mathbb{B}^8$ can be expressed in the form of f(u) = uG, where u is the row vector (u_1, u_2, u_3, u_4) . Determine the generator matrix G and the corresponding parity check matrix H.



b) (6 marks) Is the function f surjective? Prove or disprove it.

Surjective

c) (6 marks) Determine the minimum distance of the code and state its error correction capability. Explain how you obtain the minimum distance.

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d) (6 marks) Suppose the vector (1,0,0,0,1,1,1,1) is received. Determine its syndrome.

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e) (6 marks) Suppose nearest-neighbor decoding is used. Determine the decoder output for the received vector given in (d). Explain how you obtain the answer.

- 6. Consider the function $f_{\theta} : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $f_{\theta}(x) = A_{\theta}x$, where x is a 2-dim column vector, $A_{\theta} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ is the rotation matrix with $0 \le \theta < 2\pi$.
- a) (5 marks) Given any value of θ , is f_{θ} a linear function of x? Prove or disprove it.

 Solve f_{θ} is f_{θ} a linear function of f_{θ} ? Prove or disprove it. f_{θ} is f_{θ} a linear function of f_{θ} ? Prove or disprove it. f_{θ} is f_{θ} a linear function of f_{θ} ? Prove or disprove it. f_{θ} is f_{θ} a linear function of f_{θ} ? Prove or disprove it. f_{θ} is f_{θ} a linear function of f_{θ} ? Prove or disprove it. f_{θ} is f_{θ} a linear function of f_{θ} ? Prove or disprove it. f_{θ} is f_{θ} a linear function of f_{θ} ? Prove or disprove it. f_{θ} is f_{θ} a linear function of f_{θ} ? Prove or disprove it. f_{θ} is f_{θ} is f_{θ} a linear function of f_{θ} ? Prove or disprove it.
 - b) (5 marks) Assume $\theta = \frac{\pi}{4}$. Write down the inverse function of $f_{\pi/4}$. Steps are not required.

- c) (5 marks) Determine $f_{\pi/4} \circ f_{\pi/2}$, where the operator \circ denotes function composition. Steps are not required.
- d) (5 marks) Let $F = \{f_0, f_{\pi/4}, f_{\pi/2}, f_{3\pi/4}, f_{\pi}, f_{5\pi/4}, f_{3\pi/2}, f_{7\pi/4}\}$. Explain why $\langle F, \circ \rangle$ is a group. Your explanation can be based on the properties of the rotation matrix. Rigorous mathematical derivation is not required. (AIN properties of the rotation matrix.) User Addition

e) (5 marks) List all subgroups of $\langle F, \circ \rangle$.