CS3402 : Chapter 7
Functional Dependency & Normalization

Functional Dependency

- Functional dependency is a constraint between two sets of attributes from the database

- Formal definition:
  - Let R be a relation schema, and $\alpha \subseteq R$, $\beta \subseteq R$ (i.e., α and β are sets of R's attributes). We say:

$$\alpha \rightarrow \beta$$

  - If in any relation instance r(R), for all pairs of tuples t1 and t2 in r, we have:

$$(t1[\alpha] = t2[\alpha]) \Rightarrow (t1[\beta] = t2[\beta])$$

Inference Rules for FDs

IR1 (reflective rule)	If X is a subset of Y, then X → Y
IR2 (augmentation rule)	If X → Y, then XZ → YZ
IR3 (transitive rule)	If X → Y and Y → Z, then X → Z
IR4 (decomposition rule)	If X →YZ, then X → Y and X → Z
IR5 (union rule)	If X → Y and X → Z, then X → YZ
IR6 (pseudotransitive rule)	If X → Y and WY → Z, then WX → Z

Example

- Suppose we are given a schema R with attributes A, B, C, D, E, F and the FDs are:
  - A → BC
  - B → E
  - CD → EF
  - Show that the FD AD → F holds

1. A → BC (given)
2. A → C (1, decomposition)
3. AD → CD (2, augmentation)
4. CD → EF (given)
5. AD → EF (3 and 4, transitivity)
6. AD → F (5, decomposition)

Inference Rules for FDs

- Closure of a set F of FDs is the set F^+ of all FDs that can be inferred from F

- E.g., suppose we specify the following set F of obvious functional dependencies
  - F = {Ssn →{Ename, Bdate, Address,Dnumber}, Dnumber→{Dname, Dmgr_ssn}}
  - Then,
    - Ssn → {Dname, Dmgr_ssn}
    - Ssn → Ssn
    - Dnumber → Dname

Inference Rules for FDs

- Closure of a set of attributes X with respect to F is the set X^+ of all attributes that are functionally determined by X
  - Note both X and X^+ are a set of attributes
- If X^+ consists of all attributes of R, X is a superkey for R
  - From the value of X, we can determine the values the whole tuple
- X^+ can be calculated by repeatedly applying IR1, IR2, IR3 using the FDs in F
- From X to find out X^+

Example

- Suppose we are given a schema R with attributes A, B, C, D, E, F, and FDs
- A → BC
- E → CF
- B → E
- CD → EF
- $\{A\}^+$ =>

$\{A\}^+ = \{A,B,C,E,F\}$
Not a superkey or a key

Example

- F = {Ssn → Ename
  Pnumber → {Pname, Plocation}
  {Snn, Pnumber} → Hours}

- The following closure sets with respect to F
  - $\{Ssn\}^+ = \{Ssn, Ename\}$
  - $\{Pnumber\}^+ = \{Pnumber, Pname, Plocation\}$
  - $\{Ssn, Pnumber\}^+ = \{Ssn, Pnumber, Ename, Pname, Plocation, Hours\}$

Equivalence of Sets of FDs

- A set of functional dependencies F is said to cover another set of functional dependency E if every FD in E is also in F^+ (E is a subset of F^+)

- Two sets of FDs F and G are equivalent if:
  - Every FD in F can be inferred from G, and
  - Every FD in G can be inferred from F
  - Hence, F and G are equivalent if $F^+ = G^+$

- Example:
  - F: A→BC; {A→B, A→C (decomposition rule)}
  - G: A→B, A→C
  - $F^+ = G^+$

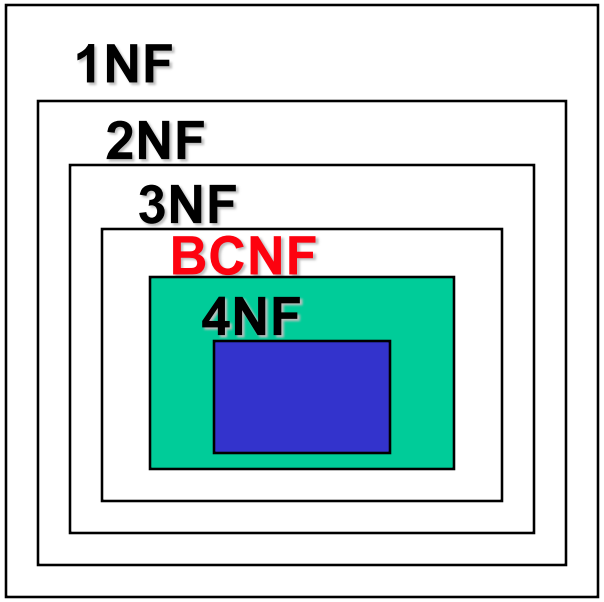
Relational Database Design

- Logical / Conceptual DB Design
  - Schema
    - what relations (tables) are needed?
    - what their attributes should be?

- What is a “bad” DB Design?
  - Repetition of data/information
  - Potential inconsistency
  - Inability to represent certain information
  - Loss of data/information

Relational Database Design

- **Introduction (con'd)**
  - *Normalization theory*
    - based on functional dependencies
      * universe of relations
      * 1st Normal Form (1NF)
      * 2NF
      * 3NF
      * BCNF
      * 4NF
      * ...



Normalization

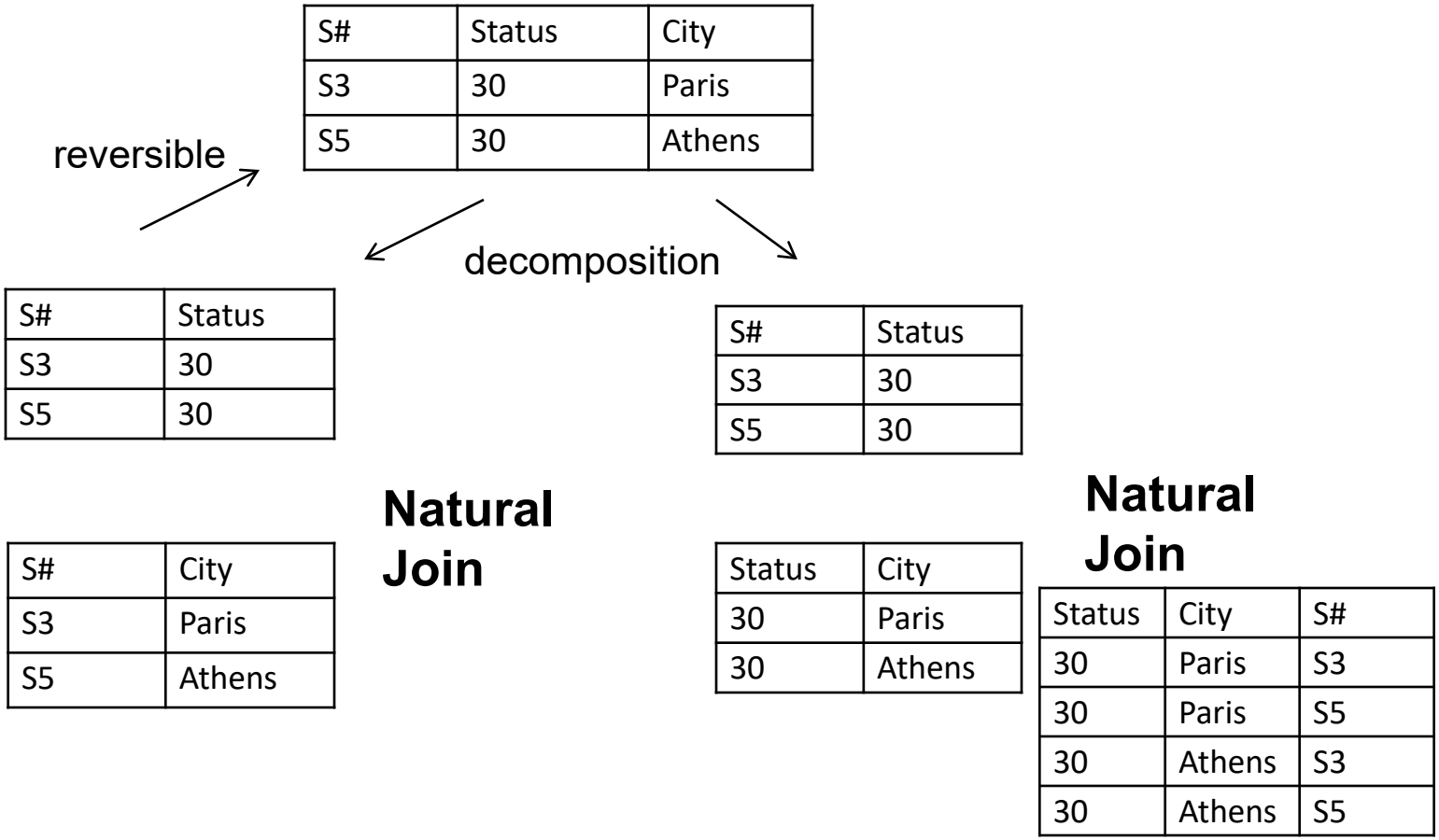
- Normalization
  - Proposed by Codd (1972): take a relation schema through a series of tests to certify whether it satisfies a certain normal form

- Analyzing the relation schema based on FD and primary keys to achieve:
  - Minimizing redundancy
  - Minimizing the insertion, deletion and update anomalies

Normalization

- Normalization requires two properties
  - Non-additive or lossless join
    - Decomposition is reversible and no information is loss
    - No spurious tuples (tuples that should not exist) should be generated by doing a natural-join of any relations (Extremely important)
  - Preservation of the functional dependencies
    - Ensure each functional dependency is represented in some individual relation (Sometimes can be sacrificed)

Lossless Decomposition



S#	Status	City
S3	30	Paris
S5	30	Athens

S#	Status
S3	30
S5	30

S#	Status
S3	30
S5	30

S#	City
S3	Paris
S5	Athens

Status	City
30	Paris
30	Athens

Status	City	S#
30	Paris	S3
30	Paris	S5
30	Athens	S3
30	Athens	S5

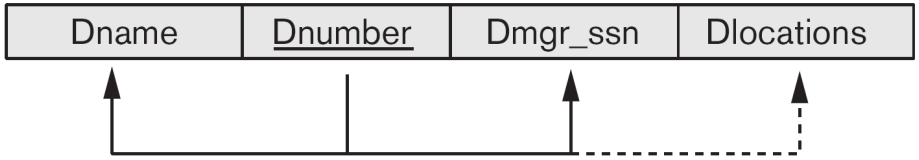
First Normal Form with Primary Key

- First normal form (1NF)
  - Disallow multi-values attributes, composite attributes and their combination
  - The domain of an attribute must be atomic (simple and indivisible) values

- DEPARTMENT: each department can have a number of locations
- 1NF: DEPT_LOCATIONS(Dnumber, Dlocation)
- 1NF also disallows multivalued attributes that are themselves composite

(a)

DEPARTMENT



(b)

DEPARTMENT

Dname	Dnumber	Dmgr_ssn	Dlocations
Research	5	333445555	{Bellaire, Sugarland, Houston}
Administration	4	987654321	{Stafford}
Headquarters	1	888665555	{Houston}

(c)

DEPARTMENT

Dname	Dnumber	Dmgr_ssn	Dlocation
Research	5	333445555	Bellaire
Research	5	333445555	Sugarland
Research	5	333445555	Houston
Administration	4	987654321	Stafford
Headquarters	1	888665555	Houston

Figure 14.9
Normalization into 1NF. (a) A relation schema that is not in 1NF. (b) Sample state of relation DEPARTMENT. (c) 1NF version of the same relation with redundancy.

(a)

EMP_PROJ

		Projs	
Ssn	Ename	Pnumber	Hours

(b)

EMP_PROJ

Ssn	Ename	Pnumber	Hours
123456789	Smith, John B.	1	32.5
		2	7.5
666884444	Narayan, Ramesh K.	3	40.0
453453453	English, Joyce A.	1	20.0
		2	20.0
333445555	Wong, Franklin T.	2	10.0
		3	10.0
		10	10.0
		20	10.0
999887777	Zelaya, Alicia J.	30	30.0
		10	10.0
987987987	Jabbar, Ahmad V.	10	35.0
		30	5.0
987654321	Wallace, Jennifer S.	30	20.0
		20	15.0
888665555	Borg, James E.	20	NULL

(c)

EMP_PROJ1

Ssn	Ename

EMP_PROJ2

Ssn	Pnumber	Hours

Figure 14.10 Normalizing nested relations into 1NF. (a) Schema of the EMP_PROJ relation with a nested relation attribute PROJS. (b) Sample extension of the EMP_PROJ relation showing nested relations within each tuple. (c) Decomposition of EMP_PROJ into relations EMP_PROJ1 and EMP_PROJ2 by propagating the primary key.

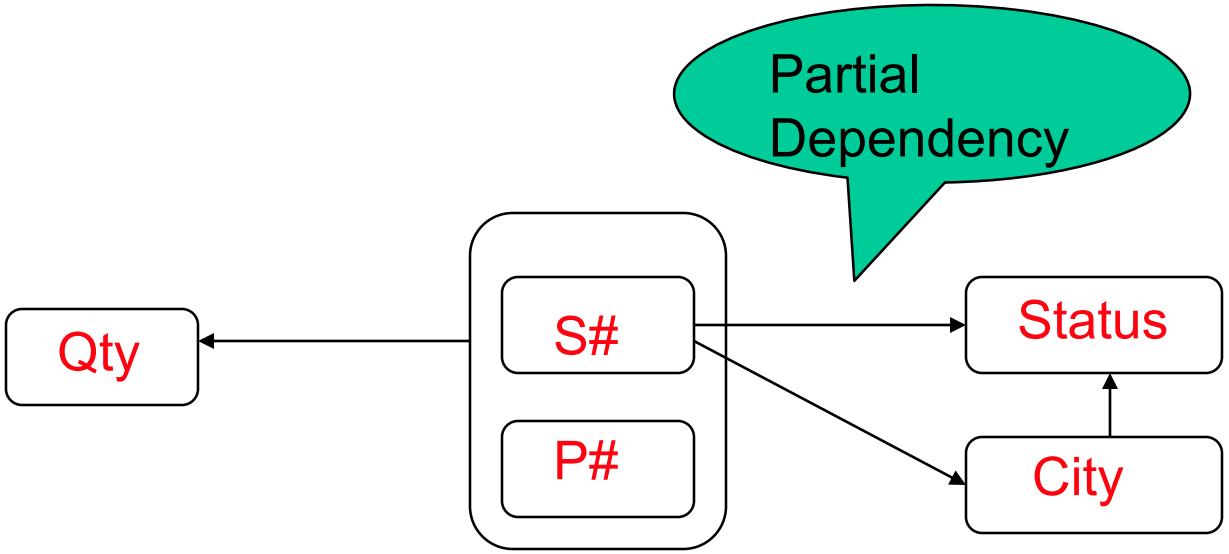
Problems with 1NF

- Example:

  FIRST(S#, Status, City, P#, Qty)

  and its Functional Dependency Diagram:

  What's the primary key? (S#,P#)



Problems with 1NF

- Problems with 1NF
  - Insert Anomalies
    - Inability to represent certain information:

      Eg, cannot enter “Supplier and City” information until Supplier supplies at least one part
  - Delete Anomalies
    - Deleting the “only tuple” for a supplier will destroy all the information about that supplier
  - Update Anomalies
    - “S# and City” could be redundantly represented for each P#, which may cause potential inconsistency when updating a tuple

Solution: 1NF->2NF

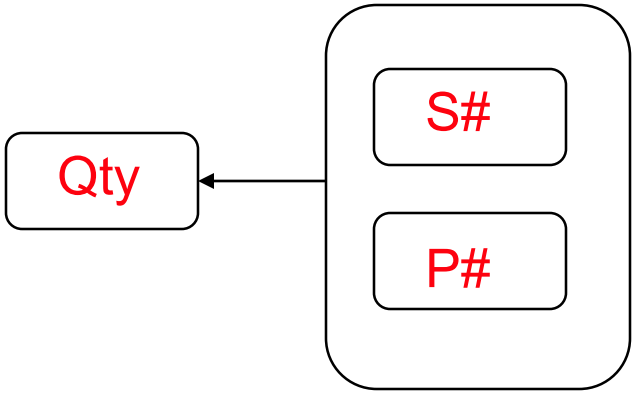
- Possible Solution:

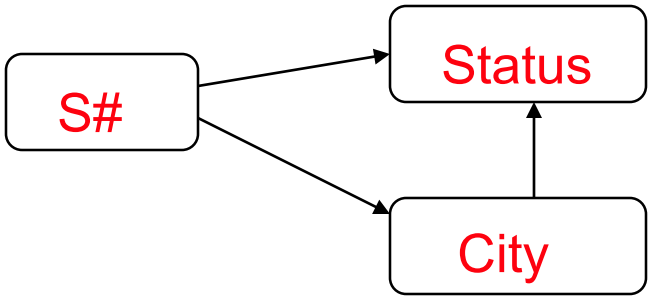
  Replace the original table by two sub-tables

  SECOND(S#, Status, City)

  SP (S#, P#, Qty)

  and now, their FD diagrams are:





Second Normal Form with Primary Key

- X->Y

- Full functional dependency
  - If removal of any attribute A from X means that the dependency does not hold any more
  - E.g., {S#, P#} → Qty

- Partial functional dependency
  - If some attributes A belonging to X can be removed from X and the dependency still holds
  - E.g., {S#, P#} → Status as S# → Status

Second Normal Form with Primary Key

- A relation schema R is in 2NF if every nonprime attributes A in R is fully functional dependent on the primary key of R
- An attribute of R is called prime (key) attribute of R if it is a member of some candidate key of R. Otherwise it is nonprime
- Non-prime (non-key) attributes: Status, City, Qty
- Primary key: {S#, P#}

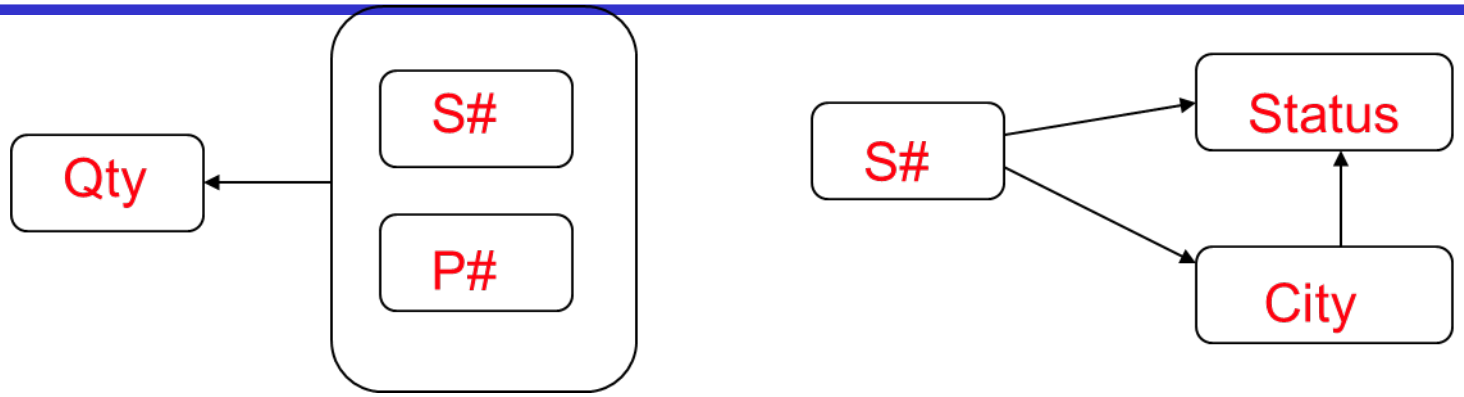
Second Normal Form with Primary Key

- The nonprime attributes Status violates 2NF because of S#->Status
- Similarly, S#->City

- If a relation schema is not in 2NF, it can be 2NF normalized into a number of 2NF relations in which nonprime attributes are associated only with the part of the primary key on which they are fully functional dependent

- Solutions: SP (P#,S#,Qty); Second(S#, Statuts, City);

Problem with 2NF



- Still, there are problems with 2NF:
  - Insert Anomalies
    - Cannot insert "City and Status" information until some Supplier in that city exist
  - Delete Anomalies
    - Deleting an "only tuple" for a particular city will destroy all the information about that city
  - Update Anomalies
    - The supplier move to another city， inconsistency between city and status will happen

Solution: 2NF->3NF

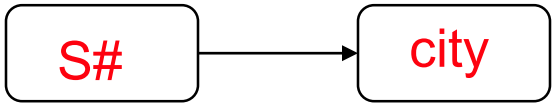
- Possible Solution:

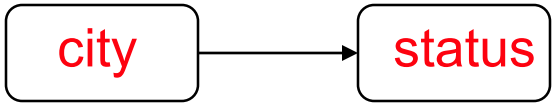
  Replace the SECOND table by two sub-tables

  SC (S#, City)

  CS (City, Status)

  and still keep the table SP (S#, P#, Qty). Now the FD diagrams for the new tables:





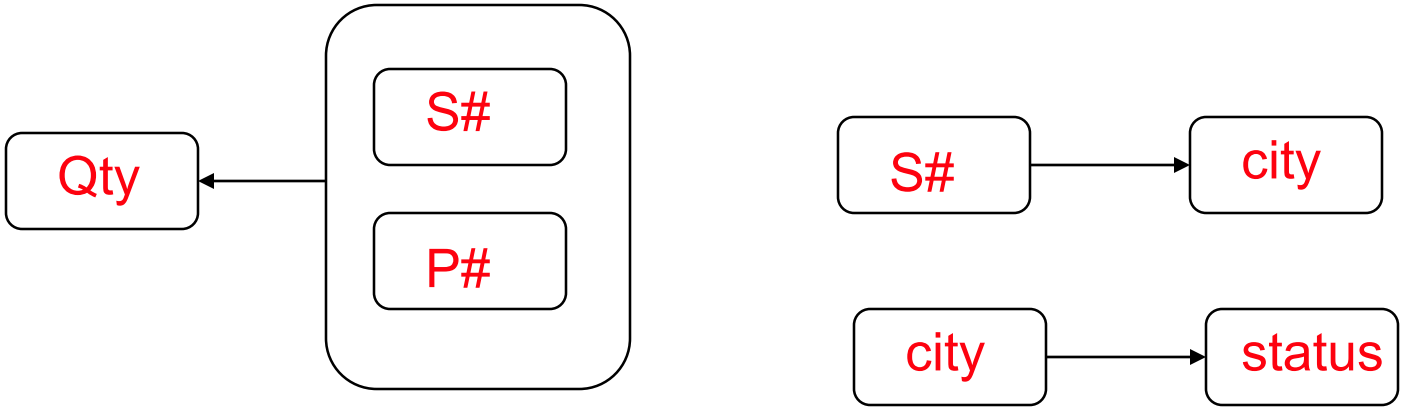
Third Normal Form with Primary Key

- 3NF is based on the concept of transitive dependency
- A functional dependency X->Y in a relation schema R is transitive dependency if there exists a set of attributes Z in R that is neither a candidate key nor a subset of any key of R, and both X->Z and Z->Y hold
- X-> Z ->Y (Z: a non-prime attribute or a set of non-prime attributes)
- E.g., dependency S#->Status is transitive through City
  - S#->City and City->Status, and City is neither a key itself nor a subset of the key
  - S#-> City-> Status

Third Normal Form with Primary Key

- According to Codd's original definition, a relation scheme R is in 3NF if it satisfies 2NF and no non-prime attribute of R is transitively dependent on the primary key

- 3NF: SC (S#, City)， CS (City, Status)， SP (S#, P#, Qty).



Normal Forms Defined Informally

- 1st normal form
  - All attributes depend on the key (no nested relation or multivalued attributes)
- 2nd normal form
  - All non-prime attributes depend on the whole key (no partial dependency)
- 3rd normal form
  - All non-prime attributes only depend on the key (no transitive dependency)

- In general, 3NF is desirable and powerful enough
- But, still there are cases where a stronger normal form is needed

General Definitions of Normal Forms

Table 14.1 Summary of Normal Forms Based on Primary Keys and Corresponding Normalization

Normal Form	Test	Remedy (Normalization)
First (1NF)	Relation should have no multivalued attributes or nested relations.	Form new relations for each multivalued attribute or nested relation.
Second (2NF)	For relations where primary key contains multiple attributes, no nonkey attribute should be functionally dependent on a part of the primary key.	Decompose and set up a new relation for each partial key with its dependent attribute(s). Make sure to keep a relation with the original primary key and any attributes that are fully functionally dependent on it.
Third (3NF)	Relation should not have a nonkey attribute functionally determined by another nonkey attribute (or by a set of nonkey attributes). That is, there should be no transitive dependency of a nonkey attribute on the primary key.	Decompose and set up a relation that includes the nonkey attribute(s) that functionally determine(s) other nonkey attribute(s).

General Definitions of 2NF and 3NF

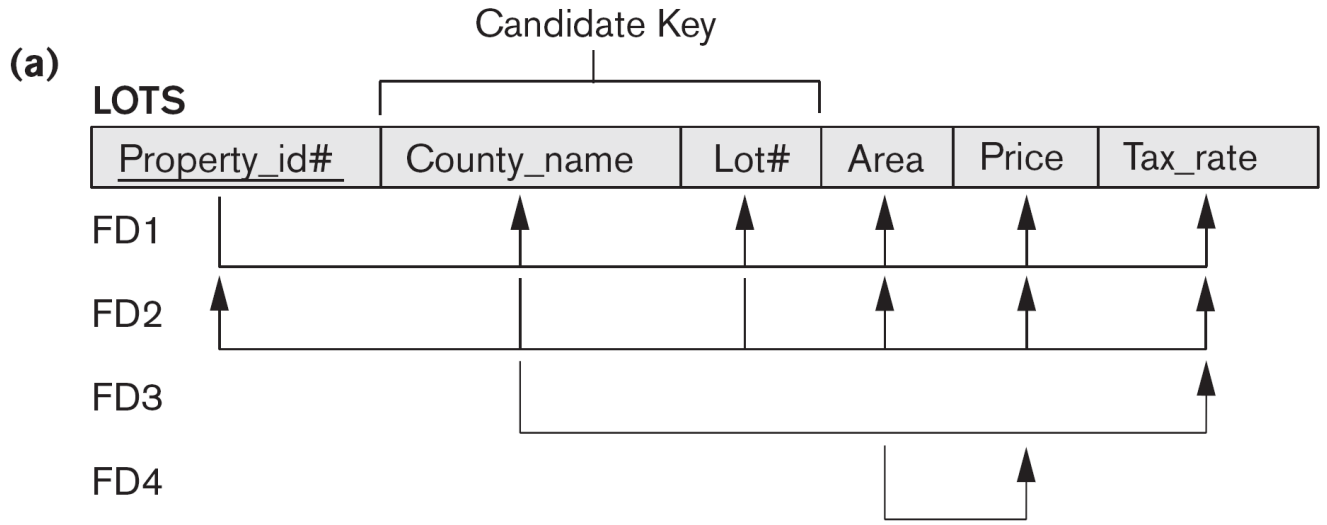
Definition. A relation schema R is in **second normal form** (**2NF**) if every non-prime attribute A in R is not partially dependent on *any* key of R.[11]

- LOTS describes parcels of land for sale in various counties of a state
- Two candidate keys: Property_id# and {County_name, Lot#}
- Property_id# is the primary key
- LOTS violates 2NF due to FD3 as Tax_rate is partially dependent on the candidate key {County_name, Lot#} => {County_name, Tax-rate}
- County_name => Tax-rate (partial dependent)

Consider all candidate keys of a relation
(not just a defined primary key)

General Definition of Second Normal Form

Figure 14.12
Normalization into 2NF and 3NF. (a) The LOTS relation with its functional dependencies FD1 through FD4. (b) Decomposing into the 2NF relations LOTS1 and LOTS2. (c) Decomposing LOTS1 into the 3NF relations LOTS1A and LOTS1B. (d) Summary of the progressive normalization of LOTS.



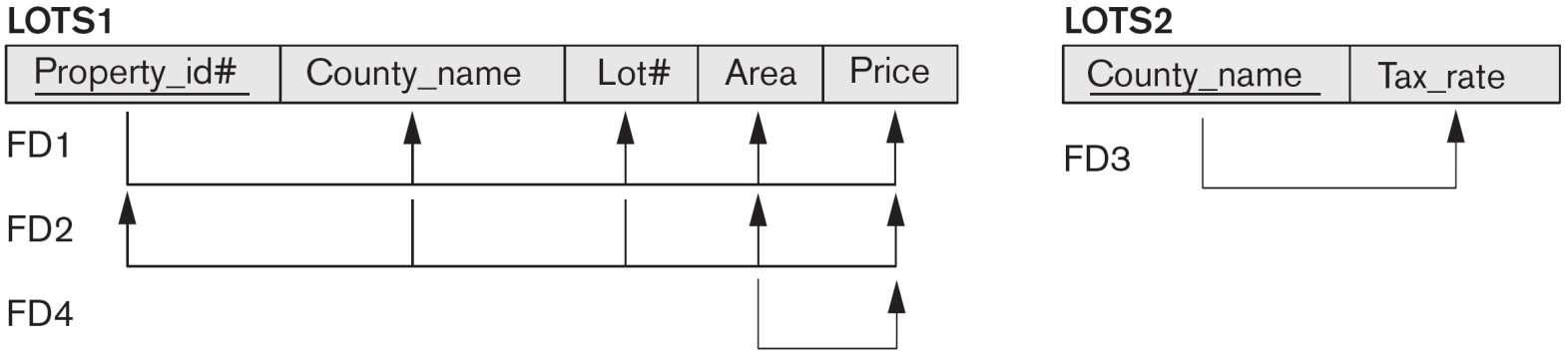
General Definition of Third Normal Form

Definition. A relation schema R is in **third normal form** (3NF) if, whenever a *nontrivial* functional dependency $X \rightarrow A$ holds in R, either (a) X is a superkey of R, or (b) A is a prime attribute of R.

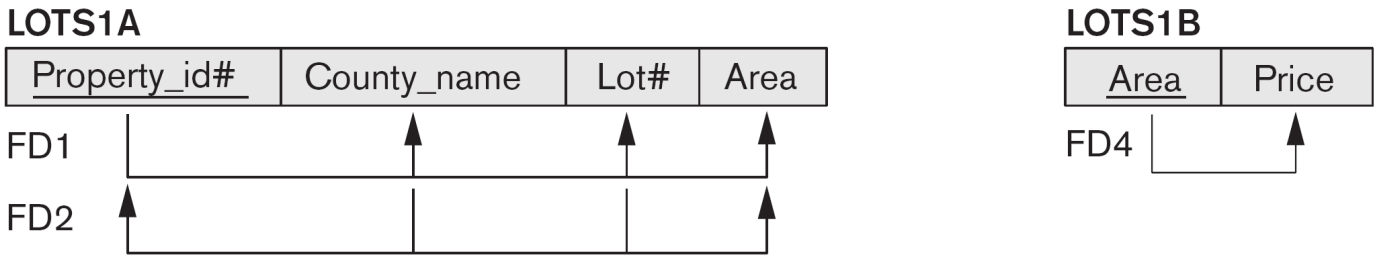
- Superkey => include a key
- Prime attributes => attributes in a key

- LOTS2 is in 3NF
- FD4 in LOTS1 violates 3NF because Area is not a superkey and Price is not a prime attribute in LOTS1
- LOTS1 violates 3NF because Price is transitively dependent on each of the candidate keys of LOTS via the nonprime attribute Area
- To normalize LOTS1 into 3NF, LOTS1B{Area, Price}

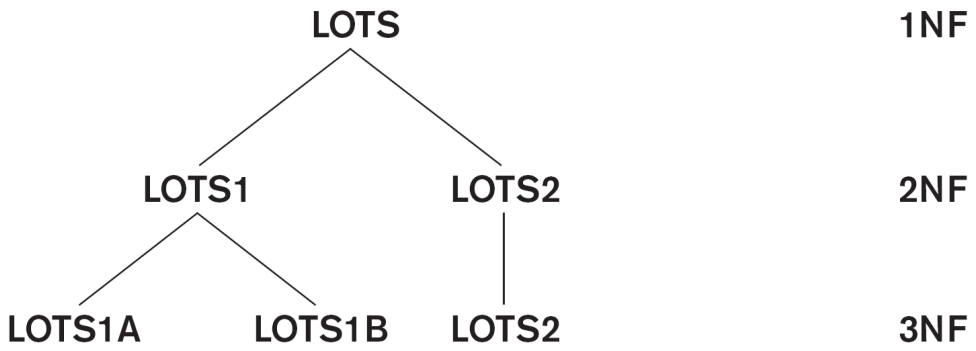
(b)


LOTS1
Property_id#
County_name
Lot#
Area
Price
FD1
FD2
FD4
LOTS2
County_name
Tax_rate
FD3

(c)


LOTS1A
Property_id#
County_name
Lot#
Area
FD1
FD2
LOTS1B
Area
Price
FD4

(d)


LOTS
LOTS1
LOTS2
LOTS1A
LOTS1B
LOTS2
1NF
2NF
3NF

Boyce-Codd Normal Form

- BCNF was proposed as a simpler form of 3NF, but it was found to be stricter than 3NF
- Every relation in BCNF is also in 3NF
  - BUT Relation in 3NF is not necessarily in BCNF

Definition. A relation schema R is in **BCNF** if whenever a *nontrivial* functional dependency $X \rightarrow A$ holds in R, then X is a superkey of R.

- Difference:
  - Condition which allows A to be prime is absent from BCNF

Definition. A relation schema R is in **third normal form** (3NF) if, whenever a *nontrivial* functional dependency $X \rightarrow A$ holds in R, either (a) X is a superkey of R, or (b) A is a prime attribute of R.

Boyce-Codd Normal Form

- Suppose we have thousands of lots in the relation but the lots are from only two counties: China and US
- Lot sizes in China are only 0.5, 0.6, 0.7, 0.8, 0.9 and 2.0 acres whereas lots in US are 1.1, 1.2, …, 1.9
- Then, Area -> County_name
- FD5 satisfies 3NF in LOTS1A because County_name is a prime attribute
- FD5 violates BCNF in LOTS1A because Area is not a superkey of LOTS1A
- Create new relation R(Area, County_name). The area of lots can be represented by 16 tuples in R(Area, County_name) to reduce redundancy

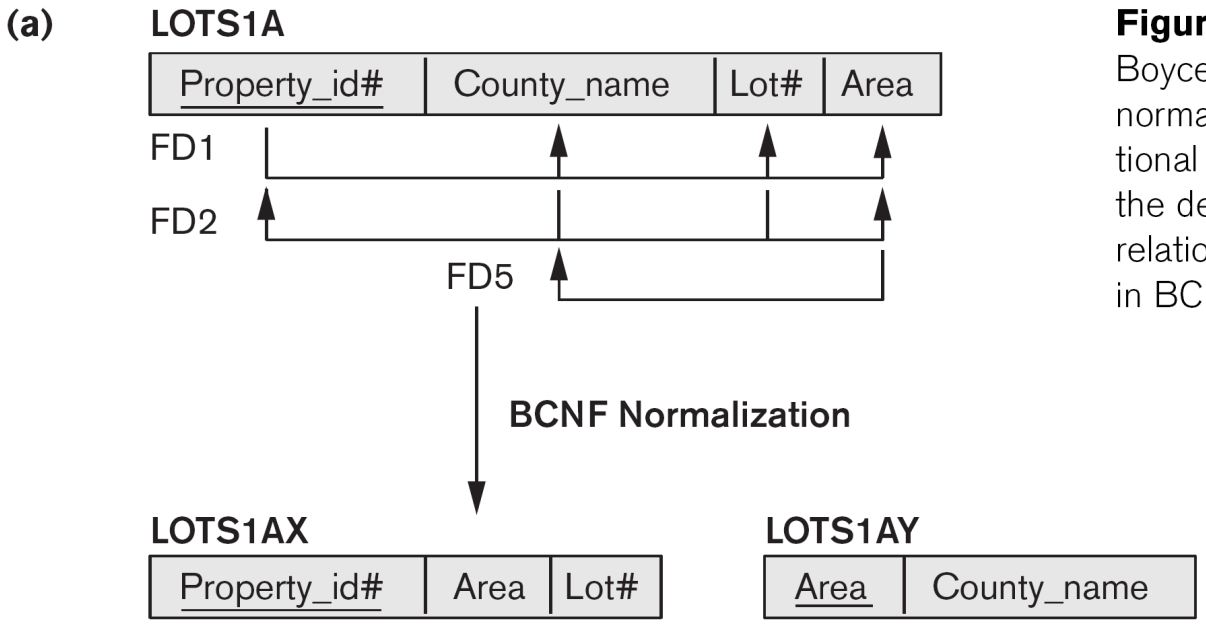
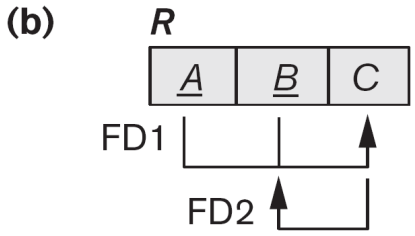


Figure 14.13
Boyce-Codd normal form. (a) BCNF normalization of LOTS1A with the functional dependency FD2 being lost in the decomposition. (b) A schematic relation with FDs; it is in 3NF, but not in BCNF.



County_name is a prime attribute

Boyce-Codd Normal Form

- Some notes on BCNF:
  - BCNF is stronger than 3NF
  - BCNF is useful when a relation has:
    - multiple candidate keys, and
    - these candidate keys are composite ones, and
    - they overlap on some common attributes
  - BCNF reduces to 3NF if the above conditions do not apply

References

- 6e
  - *Ch. 14, p. 487-515*
  - *Ch. 15, p. 529-533*