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# ***CS3402 – Chapter 5***

## ***Integrity Constraints***

# Integrity Constraints

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- Constraints determine which values are permissible and which are not in the database (table)
  - ◆ Constraints are conditions that must hold on all valid relation states
- A relational database schema  $S$  is a set of relation scheme  $S = \{R_1, R_2, \dots, R_n\}$  and a set of integrity constraints  $IC$
- Valid state Vs. invalid state
  - ◆ Invalid state: A database state that does not obey all the integrity constraints
  - ◆ Valid state: a state that satisfies all the constraints in the defined set of integrity constraints

# ***Relational Integrity Constraints***

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- They are of three main types of constraints:
  - **Inherent or Implicit Constraints:** These are based on the data model itself. (E.g., relational model does not allow a **list** as a value for any attribute)
  - **Schema-based or Explicit Constraints:** They are expressed in the schema by using the facilities provided by the model. (E.g., **max. cardinality ratio constraint** in the ER model)
  - **Application-based or Semantic constraints:** These are beyond the expressive power of the model and must be specified and enforced by the application programs

# Relational Integrity Constraints

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- There are three *main types* of schema-based constraints that can be expressed in the relational model:
  - ◆ Key constraints
  - ◆ Entity integrity constraints
  - ◆ Domain constraint
  - ◆ Referential integrity constraints

# Key Constraints

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- Superkey of R:
  - ◆ A set of attributes that **contains a key** is called a **superkey**
  - ◆ It is a set of attributes SK, e.g., {A1, A2} of R with the following condition:
    - ◆ No two tuples in any valid relation state  $r(R)$  will have the same value for SK
    - ◆ For any distinct tuples  $t1$  and  $t2$  in  $r(R)$ ,  $t1[SK] \neq t2[SK]$
- Key (Primary key, Candidate Key) of R:
  - ◆ A "minimal" superkey
  - ◆ A key is a superkey K such that **removal** of any attribute from K results in a set of attributes that is not a superkey (does not possess the superkey uniqueness property)

# Key Constraints

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- Example: Consider the CAR relation schema:
  - ◆ CAR(State, Reg#, SerialNo, Make, Model, Year)
  - ◆ CAR has two keys:
    - ◆ Key1 = {State, Reg#}
    - ◆ Key2 = {SerialNo}
  - ◆ Both are also superkeys of CAR
  - ◆ {SerialNo, Make} is a superkey but *not* a key
- In general:
  - ◆ Any *key* is a *superkey* (but not vice versa)
  - ◆ Any set of attributes that *includes a key* is a *superkey*
  - ◆ A *minimal superkey* is also a key

# Key Constraints

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- If a relation has several **candidate keys**, one is chosen arbitrarily to be the **primary key**
  - ◆ The primary key attributes are underlined
- Example: Consider the CAR relation schema:
  - ◆ CAR(State, Reg#, SerialNo, Make, Model, Year)
  - ◆ We chose SerialNo as the primary key
- The primary key value is used to *uniquely identify* each tuple in a relation
- General rule: Choose as primary key the smallest of the candidate keys (in terms of size)

# Key Constraints

## ■ Key constraints

CAR

<u>License_number</u>	Engine_serial_number	Make	Model	Year
Texas ABC-739	A69352	Ford	Mustang	02
Florida TVP-347	B43696	Oldsmobile	Cutlass	05
New York MPO-22	X83554	Oldsmobile	Delta	01
California 432-TFY	C43742	Mercedes	190-D	99
California RSK-629	Y82935	Toyota	Camry	04
Texas RSK-629	U028365	Jaguar	XJS	04

**Figure 5.4**

The CAR relation, with two candidate keys: License\_number and Engine\_serial\_number.



# Keys of Relations

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- A set of one or more attributes  $\{A_1, A_2, \dots, A_n\}$  is key for a relation if:
  - ◆ The attributes **functionally determine** all other attributes of the relation
  - ◆ Relations are sets. It is impossible for two distinct tuples of R to agree on all  $A_1, A_2, \dots, A_n$
  - ◆ No **proper subset** of  $\{A_1, A_2, \dots, A_n\}$  functionally determines all other attributes of R, i.e., a key must be **minimal**

# Entity Integrity Constraints

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- Entity Integrity:

- ◆ The *primary key attributes* PK of each relation schema R cannot have null values in any tuple of R
  - ◆ Primary key values are used to *identify* the individual tuples
  - ◆  $t[PK] \neq \text{null}$  for any tuple t in R
  - ◆ If PK has several attributes, null is not allowed in any of these attributes
- ◆ Note: Other attributes of R may be constrained to disallow null values, even though they are not members of the primary key

# Domain Constraints

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- ◆ *Domain constraint*: Every value in a tuple must be from the *domain of its attribute* (or it could be **null**, if allowed for that attribute)

E.g.,

C-Name: string of char (30)

Balance: Number (6,2)

...

# Referential Integrity

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- Key and entity integrity constraints are specified on individual relations
- Referential integrity is a constraint involving two relations
  - ◆ To specify a relationship among tuples in two relations
  - ◆ The referencing relation and the referenced relation ( $R1 \rightarrow R2$ )
- Tuples in the referencing relation  $R1$  have attributes FK (called foreign key attributes) that reference the primary key attributes PK of the referenced relation  $R2$  if it satisfies:
  - ◆ The attributes in FK have the same domain(s) as the primary key attributes PK of  $R2$

# Referential Integrity

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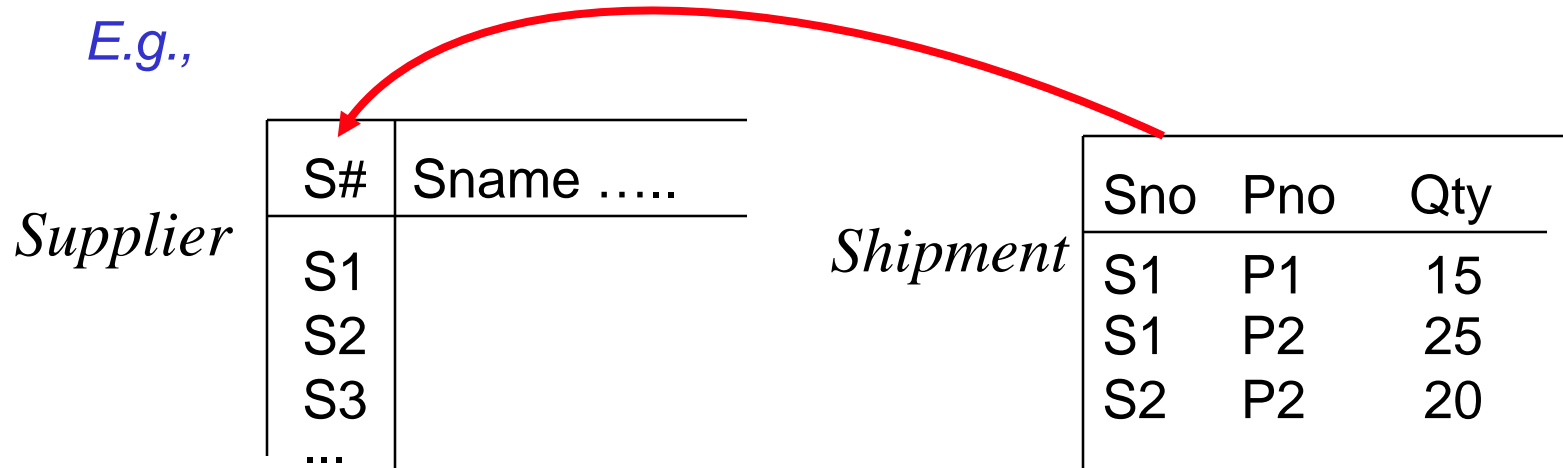
- Referential integrity constraints typically arise from the **relationships** among the entities represented by the relation
- For example, in the EMPLOYEE relation, the attribute Dno refers to DEPARTMENT for which an employee works
- We designate Dno to be a foreign key of EMPLOYEE referencing the DEPARTMENT
- A value of **Dno** in any tuple t1 of the EMPLOYEE relation **must match** a value of the primary key of DEPARTMENT, **Dnumber**, in the same tuple t2 of the DEPARTMENT relation
- Or the value of Dno can be NULL if the employee does not belong to a department or will be assigned to a department later

# Integrity Constraints

## ■ Referential Integrity Constraints

- ◆ typically this implies some “subset dependency” relationships between 2 sets of attributes in 2 tables

*E.g.,*



*ie, Supplier[S#]  $\supseteq$  Shipment[Sno]*

# *Displaying a relational database schema and its constraints*

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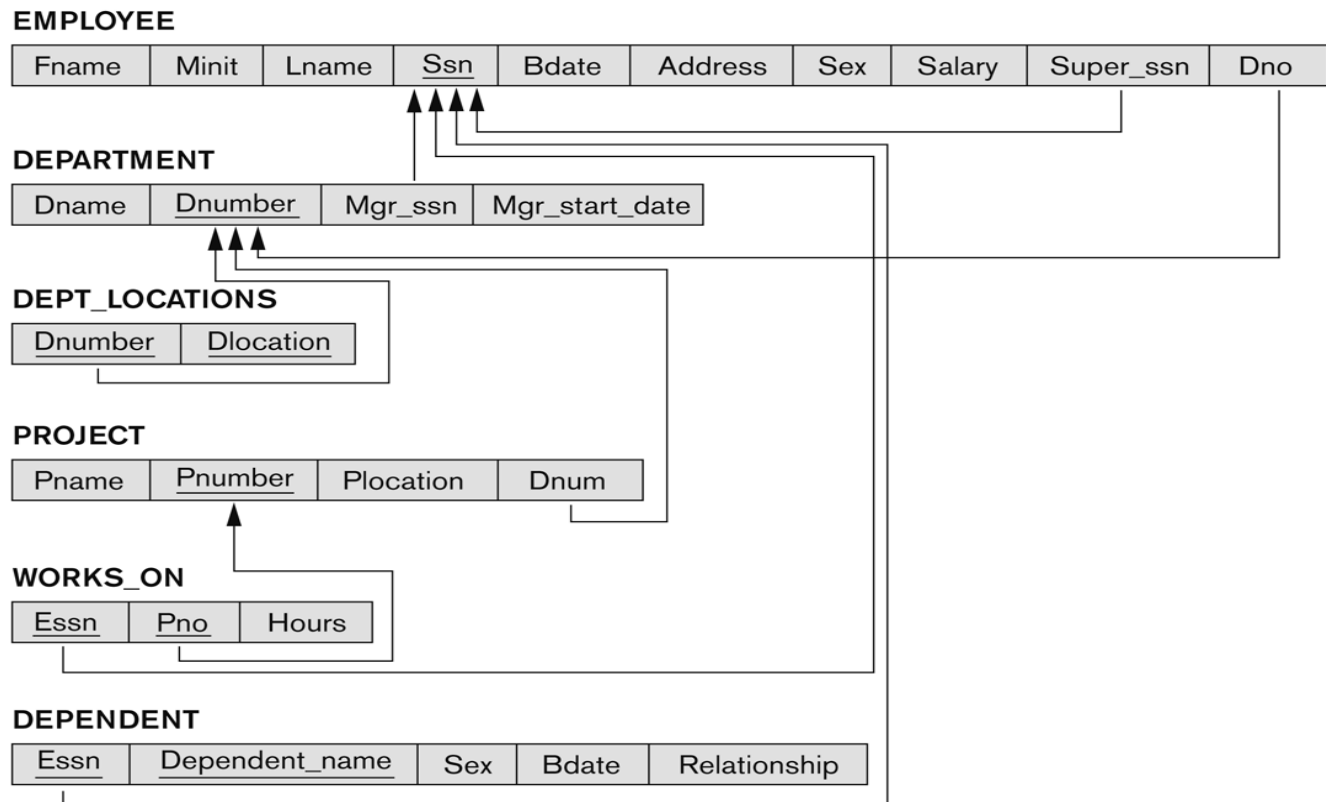
- Each relation schema can be displayed as a row of attribute names
- The name of the relation is written above the attribute names
- The primary key attribute (or attributes) will be underlined
- A foreign key (referential integrity) constraints is displayed as a directed arc (arrow) from the foreign key attributes to the referenced table
- Next slide shows the COMPANY relational schema diagram with referential integrity constraints

# Database State for COMPANY

- All examples discussed below refer to the COMPANY database shown here

**Figure 5.7**

Referential integrity constraints displayed on the COMPANY relational database schema.





# Populated database state for COMPANY

**Figure 5.6**

One possible database state for the COMPANY relational database schema.

## EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

## DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

## DEPT\_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

## WORKS\_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

## PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

## DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

# ***SQL CREATE TABLE data definition statements for defining the COMPANY schema***

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## **CREATE TABLE EMPLOYEE**

( Fname	VARCHAR(15)	NOT NULL,
Minit	CHAR,	
Lname	VARCHAR(15)	NOT NULL,
Ssn	CHAR(9)	NOT NULL,
Bdate	DATE,	
Address	VARCHAR(30),	
Sex	CHAR,	
Salary	DECIMAL(10,2),	
Super_ssn	CHAR(9),	
Dno	INT	NOT NULL,

**PRIMARY KEY (Ssn),**

## **CREATE TABLE DEPARTMENT**

( Dname	VARCHAR(15)	NOT NULL,
Dnumber	INT	NOT NULL,
Mgr_ssn	CHAR(9)	NOT NULL,
Mgr_start_date	DATE,	

**PRIMARY KEY (Dnumber),**

**UNIQUE (Dname),**

**FOREIGN KEY (Mgr\_ssn) REFERENCES EMPLOYEE(Ssn) );**

## **CREATE TABLE DEPT\_LOCATIONS**

( Dnumber	INT	NOT NULL,
Dlocation	VARCHAR(15)	NOT NULL,

**PRIMARY KEY (Dnumber, Dlocation),**

**FOREIGN KEY (Dnumber) REFERENCES DEPARTMENT(Dnumber) );**

# ***SQL CREATE TABLE data definition statements for defining the COMPANY schema***

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## **CREATE TABLE PROJECT**

( Pname	VARCHAR(15)	NOT NULL,
Pnumber	INT	NOT NULL,
Plocation	VARCHAR(15),	
Dnum	INT	NOT NULL,

**PRIMARY KEY** (Pnumber),

**UNIQUE** (Pname),

**FOREIGN KEY** (Dnum) **REFERENCES** DEPARTMENT(Dnumber) );

## **CREATE TABLE WORKS\_ON**

( Essn	CHAR(9)	NOT NULL,
Pno	INT	NOT NULL,
Hours	DECIMAL(3,1)	NOT NULL,

**PRIMARY KEY** (Essn, Pno),

**FOREIGN KEY** (Essn) **REFERENCES** EMPLOYEE(Ssn),

**FOREIGN KEY** (Pno) **REFERENCES** PROJECT(Pnumber) );

## **CREATE TABLE DEPENDENT**

( Essn	CHAR(9)	NOT NULL,
Dependent_name	VARCHAR(15)	NOT NULL,
Sex	CHAR,	
Bdate	DATE,	
Relationship	VARCHAR(8),	

**PRIMARY KEY** (Essn, Dependent\_name),

**FOREIGN KEY** (Essn) **REFERENCES** EMPLOYEE(Ssn) );

# *Update Operations on Relations*

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- INSERT a tuple
- DELETE a tuple
- MODIFY a tuple
  
- Integrity constraints should **not** be violated by the update operations
- Several update operations may have to be grouped together
- Updates may **propagate** to cause other updates automatically. This may be necessary to maintain integrity constraints

# Possible violations for each operation

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- DELETE may violate only referential integrity:
  - ◆ If the primary key value of the tuple being deleted is referenced from other tuples in the database
- INSERT may violate any of the constraints:
  - ◆ Domain constraint: if one of the attribute values provided for the new tuple is not of the specified attribute domain
  - ◆ Key constraint: if the value of a key attribute in the new tuple already exists in another tuple in the relation
  - ◆ Referential integrity: if a foreign key value in the new tuple references a primary key value that does not exist in the referenced relation
  - ◆ Entity integrity: if the primary key value is null in the new tuple

# *Integrity Constraints*

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- In case of integrity violation, several actions can be taken:
  - ◆ cancel the operation that causes the violation
  - ◆ perform the operation but inform the user of the violation
  - ◆ trigger additional updates so the violation is corrected
  - ◆ execute a user-specified error-correction routine

# *Adding Constraints in SQL*

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- CREATE TABLE TOY

```
(toy_id NUMBER(10),  
description VARCHAR(15) NOT NULL,  
last_purchase_date DATE,  
remaining_qnt NUMBER(6));
```

- CREATE TABLE TAB1

```
(col1 NUMBER(10) PRIMARY KEY,  
col2 NUMBER(4) NOT NULL,  
col3 VARCHAR(5) REFERENCES zipcode(zip)  
ON DELETE CASCADE,  
  
col4 DATE,  
col5 VARCHAR(20) UNIQUE,  
col6 NUMBER(5) CHECK (col6 < 100));
```

# Naming Constraints in SQL

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```
■ CREATE TABLE TAB1
    (col1 NUMBER(10)          PRIMARY KEY,
     col2 NUMBER(4)          NOT NULL,
     col3 VARCHAR(5)         REFERENCES zipcode(zip)
                               ON DELETE CASCADE,

     col4 DATE,
     col5 VARCHAR(20)        UNIQUE,
     col6 NUMBER(5)          CHECK (col6 < 100)),
    CONSTRAINT TAB1_PK       PRIMARY KEY(col1)
    CONSTRAINT TAB1_ZIPCODE_FK FOREIGN KEY(col3)
                               REFERENCES ZIPCODE(ZIP)

    CONSTRAINT TAB1_COL5_UK UNQIUE(col5),
    CONSTRAINT TAB1_COL6_CK CHECK (col6 < 100);
```



# Reference Constraints in SQL

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## ■ CREATE TABLE COUNTRY

cntry_cd	CHAR(3)	NOT NULL,
cname	VARCHAR2(32)	NOT NULL,
ename	VARCHAR2(32)	NOT NULL,
curr_cd	CHAR(3)	NOT NULL,
upd_dt	DATE DEFAULT SYSDATE	NOT NULL ,
upd_uid	VARCHAR2(16)	NOT NULL);

## ■ CREATE TABLE EXCHANGE

exchg_cd	VARCHAR2(8)	NOT NULL,
cname	VARCHAR2(32)	NOT NULL,
ename	VARCHAR2(32)	NOT NULL,
cntry_cd	CHAR(3)	NOT NULL);

# *Reference Constraints in SQL*

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- CREATE TABLE COUNTRY

ALTER TABLE COUNTRY ADD CONSTRAINT PK\_country PRIMARY KEY (cntry\_cd);

ALTER TABLE EXCHANGE ADD CONSTRAINT PK\_exchange PRIMARY KEY (exchg\_cd);

ALTER TABLE EXCHANGE ADD CONSTRAINT FK\_exchg\_cntry FOREIGN KEY (cntry\_cd) REFERENCES COUNTRY (cntry\_cd) ;

# Functional Dependency

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- Functional dependency is a constraint between two sets of attributes from the database
- E.g., In DEPARTMENT, *deptno* and *dname*
  - If you know the department number, you know the department name
- A functional dependency denotes by  $X \rightarrow Y$ , between two sets of attributes X and Y that are subsets of a relation R specifies a constraint on the possible tuples that can form a relation state r of R
- The constraint is that, for any two tuples t1 and t2 in r that have  $t1[X] = t2[X]$ , they must also have  $t1[Y] = t2[Y]$
- The values of the Y component of a tuple in r depend on, or are determined by the values of the X component
- If you know his student ID, then I know his name ( $X \rightarrow Y$ )

# Functional Dependency

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- Formal definition:

- ◆ Let  $R$  be a relation **schema**, and  $\alpha \subseteq R$ ,  $\beta \subseteq R$  (i.e.,  $\alpha$  and  $\beta$  are sets of  **$R$ 's attributes**). We say:

$$\alpha \rightarrow \beta$$

- ◆ If in any relation instance  $r(R)$ , for all pairs of tuples  $t1$  and  $t2$  in  $r$ , we have:

$$(t1[\alpha] = t2[\alpha]) \Rightarrow (t1[\beta] = t2[\beta])$$

- E.g., for the table Borrow (B-name, Loan#, C-name, Amount),  
we have:  $\text{Loan\#} \rightarrow \text{Amount}$   
(read as “uniquely determines”)

# Functional Dependency

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- Some usages of FDs:

- (1) to set constraints on legal relations. (e.g. Key constrains)

- Eg,  $\text{Loan\#} \rightarrow \text{Amount}$  (as in the Borrow table example)

- (2) to test relations to see if they are “legal” under a given set of FDs.

- (3) to be used in designing the database schema.

# *Functional Dependency: keys*

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- Candidate key
  - ◆ If a constraint on R states X is a **candidate key** of R, then  $X \rightarrow Y$  for any **subset of attributes** Y of R
  - ◆ A candidate key uniquely identifies a tuple
  - ◆ **The values of all remaining attributes are determined**
  
- A functional dependency is property of the **semantics** or meaning of the attributes

# ***Functional Dependency: Keys***

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Movies(title, year, length, type, studioName, starName)

Title, year, starName  $\rightarrow$  length, type, studioName

- Attributes {title, year, starName} form a key for the relation Movie
- Suppose two tuples agree on these three attributes: title, year, starName
- They must agree on the other attributes, length, type and studioName
- No proper subset of {title, year, starName} functionally determines all other attributes
- {title, year} does not determine starName since many movies have more than one star
- {year, starName} is not a key because we could have a star in two movies in the same year

# Functional Dependency: properties

- E.g., for the table Borrow (B-name, Loan#, C-name, Amount)
- If  $X \rightarrow Y$  in R, this does not say whether or not  $Y \rightarrow X$  in R
  - ◆ Amount  $\rightarrow$  Loan#? No
- If  $X = \text{Loan\#}$ ;  $Y = \text{Amount}$  and  $\text{Loan\#} \rightarrow \text{Amount}$ 
  - ◆ Can it be  $\{\text{Loan\#}, \text{B-name}\} \rightarrow \text{Amount}$ ? Yes
- Some FDs are “trivial”, since they are always satisfied by all relations:
  - ◆ E.g.,  $A \rightarrow A$ ,  $AB \rightarrow A$ ,
  - ◆ E.g.,  $\{\text{C-name}, \text{Amount}\} \rightarrow \text{C-name}$
- An FD is trivial if and only if the right-hand side (the dependent) is a subset of the left-hand side (the determinant), e.g.,  $AB \rightarrow A$



# Inference Rules for FDs

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- Given a set of FDs  $F$ , we can **infer additional FDs** that hold whenever the FDs in  $F$  hold
- **Armstrong's inference** rules:
  - ◆ IR1. (**Reflexive**) If  $Y$  is a *subset* of  $X$ , then  $X \rightarrow Y$
  - ◆ IR2. (**Augmentation**) If  $X \rightarrow Y$ , then  **$XZ \rightarrow YZ$**   
✓ (Notation:  $XZ$  stands for  $X \cup Z$ )
  - ◆ IR3. (**Transitive**) If  $X \rightarrow Y$  and  $Y \rightarrow Z$ , then  $X \rightarrow Z$
- IR1, IR2, IR3 form a **sound** and **complete** set of inference rules
  - ◆ Sound: These rules are true
  - ◆ Complete: All the other rules that are true can be deduced from these rules

# *Inference Rules for FDs*

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- Some additional inference rules that are useful:
  - ◆ **Decomposition**: If  $X \rightarrow YZ$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$
  - ◆ **Union**: If  $X \rightarrow Y$  and  $X \rightarrow Z$ , then  $X \rightarrow YZ$
  - ◆ **Pseudotransitivity**: If  $X \rightarrow Y$  and  $WY \rightarrow Z$ , then  $WX \rightarrow Z$

# *Inference Rules for FDs*

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IR1 (reflective rule)	If $X$ is a subset of $Y$ , then $X \rightarrow Y$
IR2 (augmentation rule)	If $X \rightarrow Y$ , then $XZ \rightarrow YZ$
IR3 (transitive rule)	If $X \rightarrow Y$ and $Y \rightarrow Z$ , then $X \rightarrow Z$
IR4 (decomposition rule)	If $X \rightarrow YZ$ , then $X \rightarrow Y$ and $X \rightarrow Z$
IR5 (union rule)	If $X \rightarrow Y$ and $X \rightarrow Z$ , then $X \rightarrow YZ$
IR6 (pseudotransitive rule)	If $X \rightarrow Y$ and $WY \rightarrow Z$ , then $WX \rightarrow Z$

# Example

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- Suppose we are given a schema R with attributes A, B, C, D, E, F and the FDs are:
  - ◆  $A \rightarrow BC$
  - ◆  $B \rightarrow E$
  - ◆  $CD \rightarrow EF$
  - ◆ Show that the FD  $AD \rightarrow F$  holds
- 1.  $A \rightarrow BC$  (given)
- 2.  $A \rightarrow C$  (1, decomposition)
- 3.  $AD \rightarrow CD$  (2, augmentation)
- 4.  $CD \rightarrow EF$  (given)
- 5.  $AD \rightarrow EF$  (3 and 4, transitivity)
- 6.  $AD \rightarrow F$  (5, decomposition)

# Inference Rules for FDs

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- **Closure** of a set  $F$  of **FDs** is the set  $F^+$  of **all FDs** that can be inferred from  $F$
- E.g., suppose we specify the following set  $F$  of obvious functional dependencies
  - ◆  $F = \{Ssn \rightarrow \{Ename, Bdate, Address, Dnumber\}, Dnumber \rightarrow \{Dname, Dmgr\_ssn\}\}$
  - ◆ Then,
    - ◆  $Ssn \rightarrow \{Dname, Dmgr\_ssn\}$
    - ◆  $Ssn \rightarrow Ssn$
    - ◆  $Dnumber \rightarrow Dname$

# *Inference Rules for FDs*

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- **Closure** of a set of **attributes**  $X$  with respect to  $F$  is the set  $X^+$  of all attributes that are functionally determined by  $X$ 
  - ◆ Note both  $X$  and  $X^+$  are a set of attributes
- If  $X^+$  consists of all attributes of  $R$ ,  $X$  is a **superkey** for  $R$ 
  - ◆ From the value of  $X$ , we can determine the values the whole tuple
- $X^+$  can be calculated by repeatedly applying IR1, IR2, IR3 using the FDs in  $F$
- From  $X$  to find out  $X^+$

# Example

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- Suppose we are given a schema R with attributes A, B, C, D, E, F, and FDs
- $A \rightarrow BC$
- $E \rightarrow CF$
- $B \rightarrow E$
- $CD \rightarrow EF$
- $\{A\}^+ \Rightarrow$

$\{A\}^+ = \{A, B, C, E, F\}$   
Not a superkey or a  
key

# Example

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- $F = \{ \text{Ssn} \rightarrow \text{Ename}$   
 $\text{Pnumber} \rightarrow \{ \text{Pname}, \text{Plocation} \}$   
 $\{ \text{Snn}, \text{Pnumber} \} \rightarrow \text{Hours} \}$
  
- The following closure sets with respect to  $F$ 
  - ◆  $\{ \text{Ssn} \}^+ = \{ \text{Ssn}, \text{Ename} \}$
  - ◆  $\{ \text{Pnumber} \}^+ = \{ \text{Pnumber}, \text{Pname}, \text{Plocation} \}$
  - ◆  $\{ \text{Ssn}, \text{Pnumber} \}^+ = \{ \text{Ssn}, \text{Pnumber}, \text{Ename}, \text{Pname}, \text{Plocation}, \text{Hours} \}$



# Equivalence of Sets of FDs

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- A set of functional dependencies  $F$  is said to **cover** another set of functional dependencies  $E$  if every FD in  $E$  is also in  $F^+$  ( $E$  is a subset of  $F^+$ )
- Two sets of FDs  $F$  and  $G$  are **equivalent** if:
  - ◆ Every FD in  $F$  can be inferred from  $G$ , and
  - ◆ Every FD in  $G$  can be inferred from  $F$
  - ◆ Hence,  $F$  and  $G$  are equivalent if  $F^+ = G^+$
- Example:
  - ◆  $F: A \rightarrow BC; \{A \rightarrow B, A \rightarrow C \text{ (decomposition rule)}\}$
  - ◆  $G: A \rightarrow B, A \rightarrow C$
  - ◆  $F^+ = G^+$

# Summary

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- Closure of a set  $F$  of FDs
  - ◆ The set  $F^+$  of all FDs that can be inferred from  $F$
- Closure of a set of attributes  $X$  with respect to  $F$ 
  - ◆ The set  $X^+$  of all attributes that are functionally determined by  $X$
- A set of functional dependencies  $F$  is said to cover another set of functional dependency  $E$ 
  - ◆ If every FD in  $E$  is also in  $F^+$
- Two sets of FDs  $F$  and  $G$  are equivalent
  - ◆ If  $F$  and  $G$  are equivalent if  $F^+ = G^+$

# *References*

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- 6e
  - ◆ Ch. 3, p. 63 – 70