Solutions to EE3210 Quiz 4 Problems

Problem 1: Given $x[n] = \beta^n u[n]$ and $h[n] = \beta^n u[n]$, we have $x[k] = \beta^k u[k]$ and $h[n-k] = \beta^{n-k} u[n-k]$. So we have

$$\begin{aligned} y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] &= \sum_{k=-\infty}^{+\infty} \beta^k u[k] \beta^{n-k} u[n-k] \\ &= \beta^n \sum_{k=-\infty}^{+\infty} u[k]u[n-k]. \end{aligned}$$

We observe that

$$u[k]u[n-k] = \begin{cases} 1, & 0 \le k \le n \\ 0, & \text{otherwise.} \end{cases}$$

Then:

- For $n < 0$, since $u[k]u[n-k] = 0$ for all k, we have
$$y[n] = 0.$$
- For $n \ge 0$, since $u[k]u[n-k] = 1$ for $0 \le k \le n$, we have
$$y[n] = \beta^n \sum_{k=0}^{n} 1 = (n+1)\beta^n.$$

Thus, for all n, we obtain

$$y[n] = (n+1)\beta^n u[n].$$

Problem 2: Given

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau$$

for any continuous-time signal $x(t)$, we have

$$u(t) = \int_{-\infty}^{+\infty} u(\tau)\delta(t-\tau)d\tau = \int_{0}^{+\infty} \delta(t-\tau)d\tau$$

since

$$u(\tau) = \begin{cases} 1, & \tau > 0 \\ 0, & \tau < 0. \end{cases}$$