

## EE2302 Foundations of Information Engineering

### Assignment 9 (Solution)

1.

a) The codewords are

0 0 0 0 0

1 0 1 1 0

0 1 0 1 1

1 1 1 0 1

b) Yes. It can be checked that the following two conditions hold:

- i. Closed under Addition: It is straightforward to check that the sum of any two codewords is equal to another codeword. For example,  $10110 + 01011 = 11101$ .
- ii. Closed under Scalar Multiplication: Multiplying 0 to any codeword  $c$  gives 00000, which is a codeword. Multiplying 1 to any codeword  $c$  gives  $c$  itself, which is, of course, a codeword.

c) The minimum distance  $d_{min}$  is 3. It can correct one error bit.

d) The generator matrix  $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ .

The parity check matrix  $H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ .

e) The syndrome is  $s = yH^T = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$ .

f) The first two components are non-zero and the last component is zero, which indicate that the first two parity-check equations are in error while the last parity-check equation is without error. Since only  $u_1$  occurs in the first two parity-check equation but not in the last equation, the error bit is  $u_1$  (i.e., the first received bit).

2.

a) Yes. It is systematic because the information bit is embedded in the corresponding codeword.

b)  $G = [1 \ 1 \ 1 \ 1 \ 1]; H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ . (The answer for  $H$  is not unique.)

c) For this code, the code rate is  $1/5$  and the minimum distance is 5 (which can correct two error bits).

For the code in Q.1, the code rate is  $2/5$  and the minimum distance is 3 (which can correct one error bit).

Therefore, this code is less efficient in sending information, but has a better error correction capability.