Name:____________

Last Four Digits of Soc. Sec. #: ____________

UNIVERSITY OF CALIFORNIA, RIVERSIDE
Department of Electrical Engineering

EE 110A Signals and Systems
Fall Quarter 2002

NOTE:

This is a 50-minute, open book, open notes midterm exam. You are allowed to use any appropriate reference which you consider helpful.

2. There are four problems, plus an additional bonus problem. The corresponding points for each problem are indicated. Each point you earned here counts one point in the final score.

3. In addition to final answers, you need to include necessary steps to show your derivations. An answer without any supporting development will unlikely be awarded with a point. You do receive partial credits if you write down the steps despite that you may not complete your solution.

4. Write your work in the blank space. If necessary, write on the back sheet.

5. Do not forget putting down your name! Also include the last four digits of your SSN for purpose of posting grades.

6. Enjoy the exam! Good luck!

1. (5 pts) Let $x(t)$ and $y(t)$ be periodic signals with fundamental periods T_1 and T_2. Define

$$y_1(t) = x(at), \qquad y_2(t) = y(bt+c), \qquad a,\ b > 0.$$

(a) Show that $y_1(t)$ and $y_2(t)$ are periodic signals. Find the fundamental periods for $y_1(t)$ and $y_2(t)$, respectively.

(b) Let $z(t) = y_1(t)y_2(t)$. Under what conditions is $z(t)$ a periodic signal? What is the fundamental period of $z(t)$ if it is periodic?

Proof. (a) Consider for some $T_\alpha > 0$,

$$y_1(t+T_\alpha) = x(at + aT_\alpha)$$

Let $aT_\alpha = T_1$ then

$$y_1(t+T_\alpha) = x(at+T_1) = x(at) = y_1(t)$$

3 pts Similarly, let $bT_\beta = T_2$, then

$$y_2(t+T_\beta) = y(bt+bT_\beta+c)$$
$$= y(bt+c+T_2)$$
$$y(bt+c)$$
$$= y_2(t)$$

Hence, $y_1(t)$ & $y_2(t)$ are both periodic signals, whose periods are respectively,

$$T_\alpha = \frac{T_1}{a}, \quad T_\beta = \frac{T_2}{b}$$

2 pts (b) Consider for some $T > 0$

$$z(t+T) = y_1(t+T)\, y_2(t+T)$$

If $T = mT_\alpha = nT_\beta$ for some integers m and n.

i.e., $\frac{T_\alpha}{T_\beta} = \frac{n}{m} \Rightarrow \frac{T_\alpha}{T_\beta}$ is a rational number,

then $z(t+T) = y_1(t+mT_\alpha)\, y_2(t+nT_\beta) = y_1(t)\, y_2(t) = z(t)$

That is $z(t)$ is periodic. The period $= mT_\alpha$, with m the smallest integer such that $mT_\alpha = nT_\beta$

2. (5 pts) Consider the system described by the following integral relation

$$y(t) \quad \int_{-\infty}^{t} x(2\tau)d\tau,$$

(i) Determine if this system is (i) memoryless, (ii) causal, (iii) linear, (iv) time invariant, and (v) stable.
(ii) What is the impulse response of this system?

Solution

$$y(t) = \frac{1}{2}\int_{\infty}^{2t} x(\tau)\, d\tau$$

pt { (i) Not memoryless
(ii) Non-causal

1 pt (iii) Linear

pt (iv) Time-varying

1 pt v) Unstable

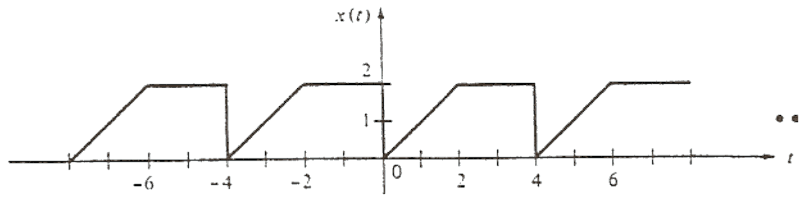
1 pt Impulse Response

$$h(t) = \int_{-\infty}^{t} \delta(2\tau)\, d\tau$$

$$- \int_{-\infty}^{t} \frac{1}{2}\delta(\tau)\, d\tau$$

$$= \frac{1}{2} u(t)$$

3. (5 pts) **Find the exponential Fourier series of the signal depicted in the following figure.**



Solution: Fundamental period = 4, $\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$C_k = \frac{1}{4}\int_0^4 x(t)\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{4}\int_0^2 t\, e^{-jk\frac{\pi}{2}t}\, dt + \frac{1}{4}\int_2^4 2\, e^{-jk\frac{\pi}{2}t}\, dt$$

2 pts

$$= \frac{1}{4}\,\frac{1}{(-jk\frac{\pi}{2})}\, t\, e^{-jk\frac{\pi}{2}t}\Big|_0^2 + \frac{1}{4}\cdot\frac{1}{jk\frac{\pi}{2}}\int_0^2 e^{-jk\frac{\pi}{2}t}\, dt + \frac{1}{4}\int_2^4 2\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$= -\frac{1}{jk\pi}\, e^{-jk\pi} + \frac{1}{2jk\pi}\,\frac{1}{(-jk\frac{\pi}{2})}\, e^{-jk\frac{\pi}{2}t}\Big|_0^2 + \frac{1}{2}\,\frac{1}{(-jk\frac{\pi}{2})}\, e^{-jk\frac{\pi}{2}t}\Big|_2^4$$

integration by parts done correctly 4 pts

$$jk\pi\; e^{-jk\pi} + \frac{\;}{(jk\pi)^2}\left[1 \quad e^{-jk\pi}\right]$$

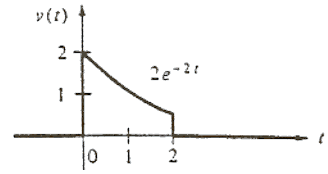
$$+ \; jk\pi \left[e^{-jk\pi} - e^{-jk2\pi}\right]$$

$$(k\pi)^2 + \frac{1}{(k\pi)^2}\cos k\pi - \overline{jk\pi}$$

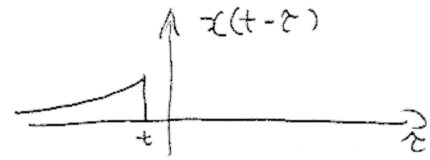
Final answer comparably simplified 5 pts

$$= \begin{cases} j\,\frac{1}{k\pi} & k \text{ s even} \\ -\frac{2}{(k\pi)^2} + j\,\frac{\;}{k\pi} & k \text{ is odd} \end{cases}$$

4. (5 pts) The impulse response of a LTI system and an input signal applied to the system are shown in the following figure. Determine the output response.



Solution



$$y(t) = v(t) * x(t) = \int_{-\infty}^{\infty} v(t)\, x(t-\tau)\, d\tau$$

1pt For $t<0$ $\quad y(t) = 0$

2 pts For $0<t<2$ $\quad y(t) = \int_0^t 2e^{-2\tau}\, e^{-(t-\tau)}\, d\tau$

(1pt for integral limits, 1 pt for integrand)

$$= 2e^{-t}\int_0^t e^{-\tau}\, d\tau$$

$$- 2e^{-t}(1-e^{-t})$$

2pts For $t>2$ $\quad y(t) = \int_0^2 2e^{-2\tau}\, e^{-(t-\tau)}\, d\tau$

$$2e^{-t}\int_0^2 e^{-\tau}\, d\tau$$

$$2e^{-t}(1-e^{-2})$$

$$\Rightarrow \quad y(t) = \begin{cases} 0 & t<0 \\ 2e^{-t}(-e^{-t}) & 0<t<2 \\ 2e^{-t}(1-e^{-2}) & t>2 \end{cases}$$

5. (Bonus Problem, 5 pts) Consider the linear time-invariant system described by the equation

$$y(t) \quad 2x(t-1) + \int_{-\infty}^{t} x(\tau + 3)d\tau.$$

Find the impulse response $h(t)$ for the system. Furthermore, determine and justify (i.e., prove or disapprove) whether or not the system is stable, causal.

Solution

1 pt $$h(t) = 2\delta(t-1) + \int_{\infty}^{t} \delta(\tau+3)\, d\tau$$

p For $t < -3$ $\quad \int_{-\infty}^{t} \delta(\tau+3)\, d\tau \quad 0$

For $t > -3$ $\quad \int_{-\infty}^{t} \delta(\tau+3)\, d\tau =$

$$\Rightarrow \int_{-\infty}^{t} \delta(\tau+3)\, d\tau = \begin{cases} 0 & t < -3 \\ & t > -3 \end{cases}$$

$$= u(t+3)$$

$$\Rightarrow h(t) = 2\delta(t-1) + u(t+3)$$

1 pt Since $h(t) \neq 0$ for $t < 3$, the system is noncausal. Furthermore,

1 pt $$\int_{-\infty}^{\infty} |h(t)|\, dt = 2 + \int_{-\infty}^{\infty} |u(t+3)|\, dt$$

$$\infty$$

The system s unstable.