Homework #9

This assignment is rather straightforward. As such, it suffices to provide some hints.

Prob. 1

(i) We know the transform pair

$$e^{-\alpha t}\, u(t) \longleftrightarrow \frac{1}{s+\alpha}$$

Use the property

$$\mathcal{L}\{t\, x(t)\} = -\frac{dX(s)}{ds}$$

$$F_1(s) = -\frac{d}{ds}\left(\frac{1}{s+3}\right)$$

ii $f_3(t) = \cos 2\omega t \cos 3\omega t\, u(t)$

Use the trigonometric identity

$$\cos\alpha \cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$\Rightarrow f_3(t) = \frac{1}{2}\left[\cos 5\omega t + \cos\omega t\right] u(t)$$

Use the transform pair

$$\mathcal{L}\{\cos\omega_0 t\, u(t)\} = \frac{s}{s^2+\omega_0^2}$$

2. Do partial fraction

$$F_1(s) = \frac{A}{s} + \frac{B_1}{s+(1+j)} + \frac{B_2}{s+(1-j)}$$

Note That $B_2 = B_1^*$

3.

$$2y''(t) + 2y'(t) + y(t) = u(t)$$

Take Laplace transform:

$$2s^2Y(s) - 2sy(0^+) - 2y'(0^+) + 2sY(s) - 2y(0^+)$$
$$+ Y(s) = U(s)$$

$$(2s^2+2s+1)Y(s) - [2sy(0^+) + 2y'(0^+) + 2y(0^+)]$$
$$= U(s)$$

Transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{2s^2+2s+1}$$

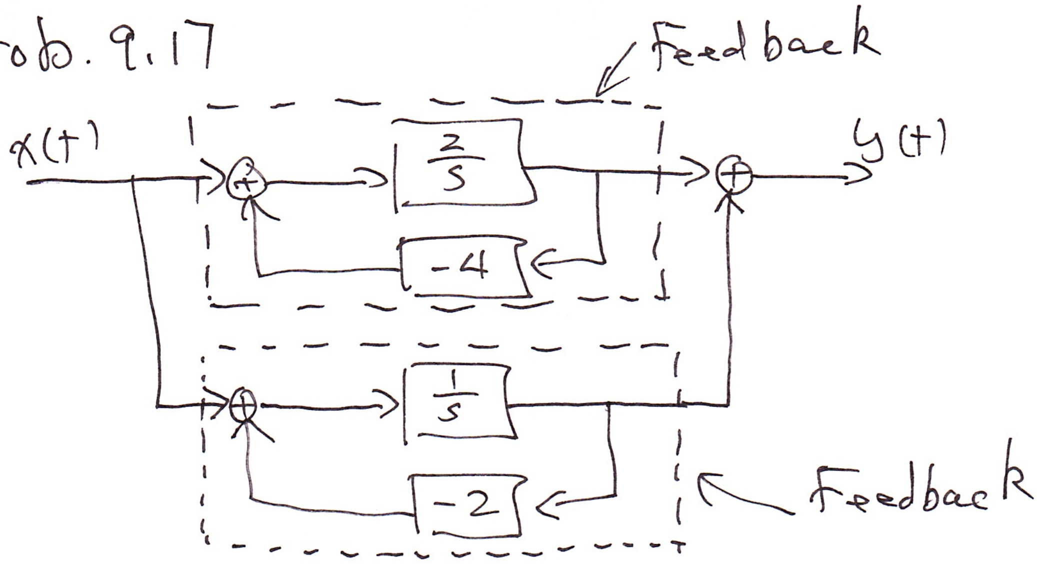
Zero-state Response

$$Y(s) = H(s)U(s) = \frac{1}{s(2s^2+2s+1)}$$
$$= \text{Partial Fraction}$$

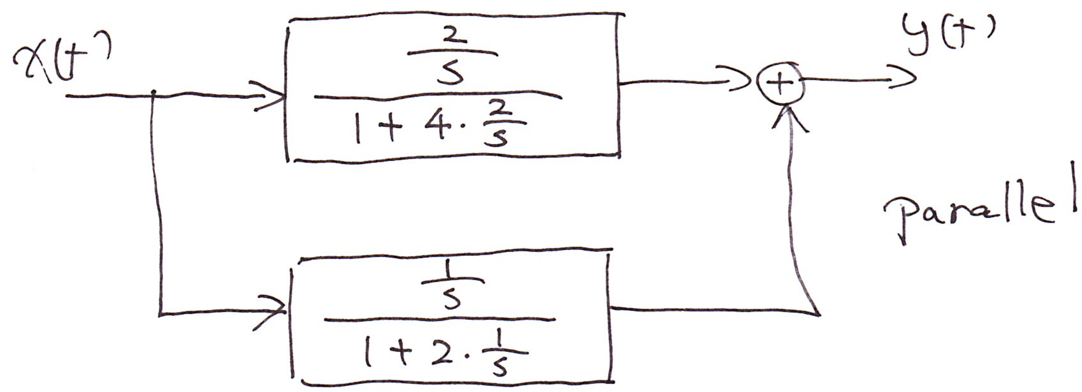
Zero-input Response

$$Y(s) = \frac{2sy(0^+) + 2y'(0^+) + 2y(0^+)}{2s^2+2s+1}, \quad y(0^+)=0, y'(0^+)=2$$
$$= \frac{4}{2s^2+2s+1} = \text{Partial Fraction}$$

• Prob. 9.17



$\Downarrow$



$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{2}{s}}{1+4\cdot\frac{2}{s}} + \frac{\frac{1}{s}}{1+2\cdot\frac{1}{s}}$$

$$= \frac{2}{s+8} + \frac{1}{s+2}$$

$$= \frac{(2s+4)+(s+8)}{(s+2)(s+8)}$$

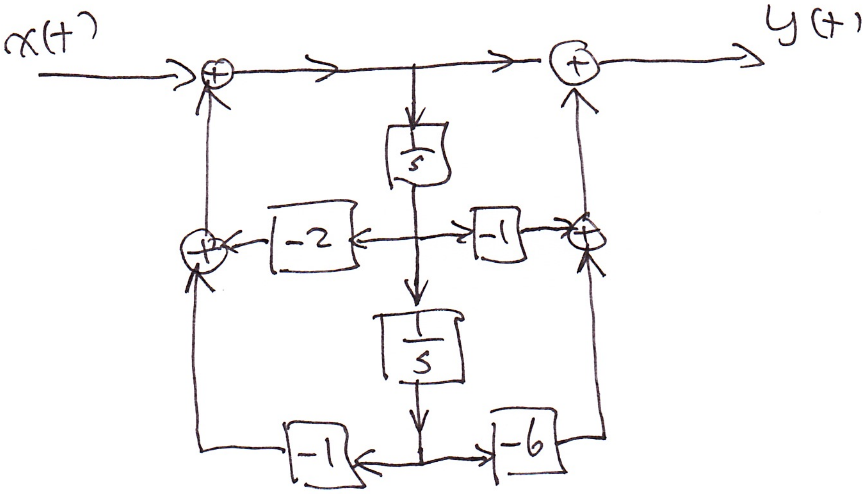
$$= \frac{3s+12}{s^2+10s+16}$$

Do cross-multiplication

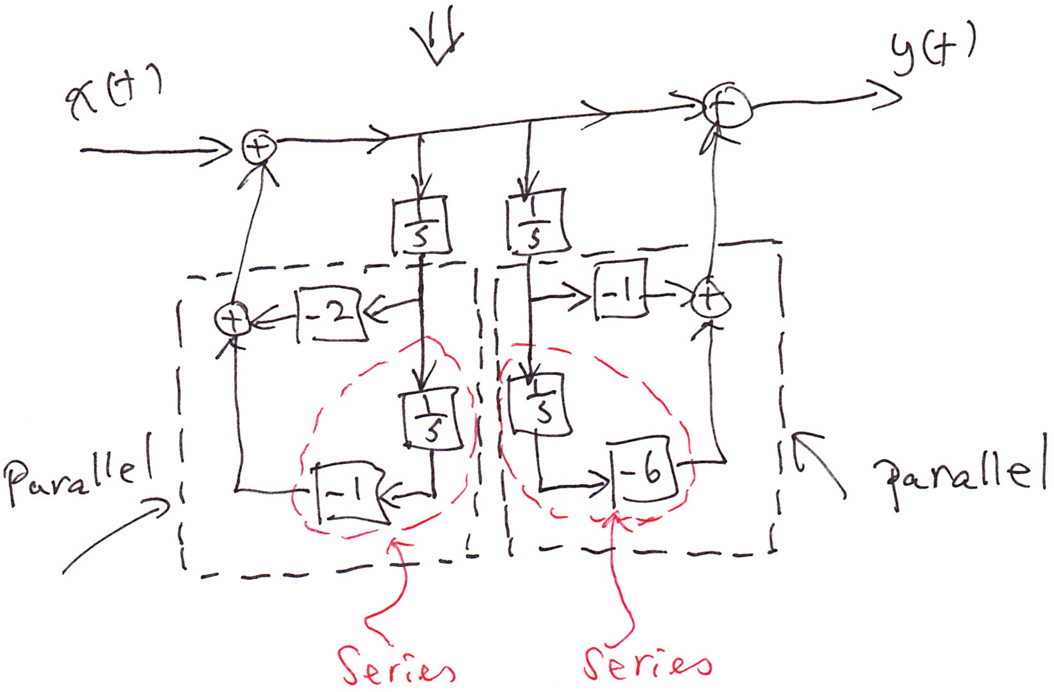
$$(s^2+10s+16)\,Y(s) = (3s+12)\,X(s)$$

$$\Rightarrow \quad \frac{d^2y(t)}{dt^2} + 10\frac{dy(t)}{dt} + 16y(t) = 3\frac{dx(t)}{dt} + 12x(t)$$

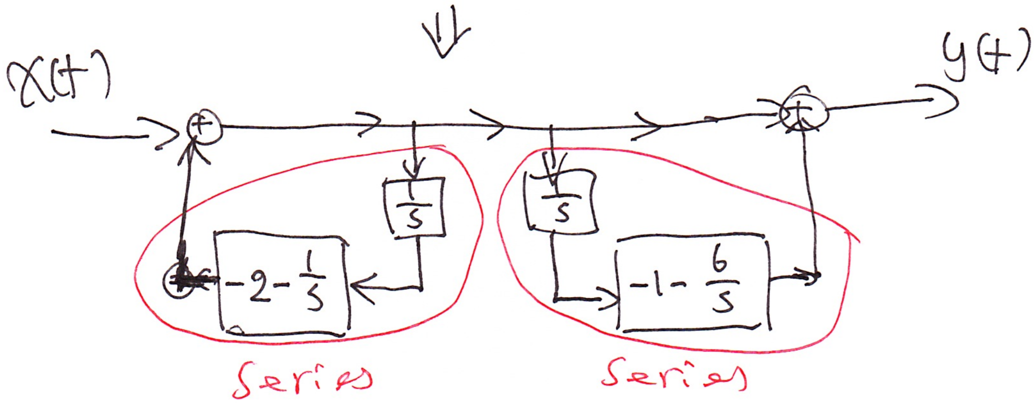
Prob. 9.35



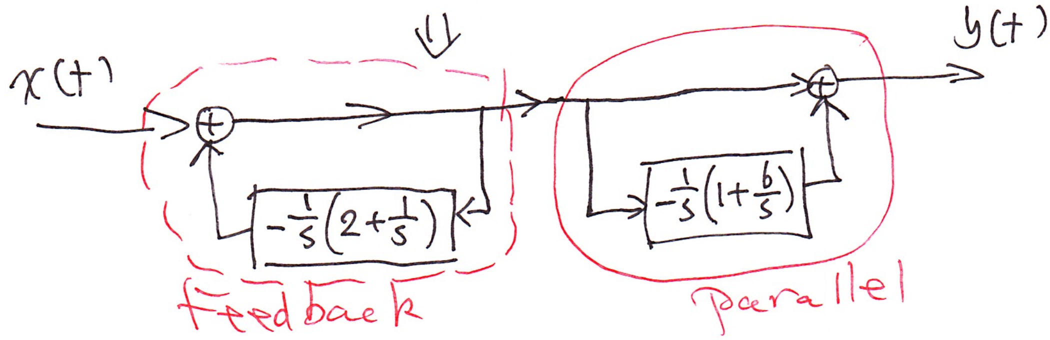
x(t)
y(t)
1/s
-2
-1
1/s
-1
-6



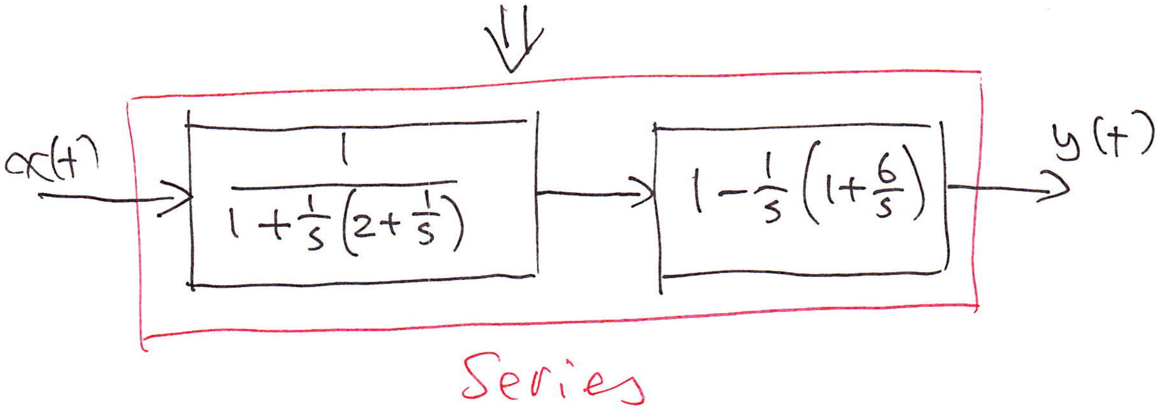
x(t)
y(t)
1/s
1/s
-2
-1
1/s
1/s
-1
-6
Parallel
parallel
Series
Series



x(t)
y(t)
1/s
1/s
-2-1/s
-1-6/s
Series
Series



x(t)
y(t)
-1/s(2+1/s)
-1/s(1+6/s)
feedback
parallel



$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{1+\frac{1}{s}\left(2+\frac{1}{s}\right)}\left[1-\frac{1}{s}\left(1+\frac{6}{s}\right)\right]$$

$$= \frac{\cancel{s^2}}{s^2+2s+1} \cdot \frac{s^2-s-6}{\cancel{s^2}}$$

$$= \frac{s^2-s-6}{s^2+2s+1}$$

Cross-multiplication reveals the differential equation.