Find the Fourier series representation of the output $y[n]$ for each of the following inputs:

(a) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$

(b) $x[n]$ is periodic with period 6 and

$$x[n] = \begin{cases} 1, & n = 0, \pm 1 \\ 0, & n = \pm 2, \pm 3 \end{cases}$$

3.38. Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1, & 0 \leq n \leq 2 \\ -1, & -2 \leq n \leq -1 \\ 0, & \text{otherwise} \end{cases}.$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k],$$

determine the Fourier series coefficients of the output $y[n]$.

3.39. Consider a discrete-time LTI system S whose frequency response is

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |\omega| < \pi \end{cases}.$$

Show that if the input $x[n]$ to this system has a period $N = 3$, the output $y[n]$ has only one nonzero Fourier series coefficient per period.

ADVANCED PROBLEMS

3.40. Let $x(t)$ be a periodic signal with fundamental period T and Fourier series coefficients a_k. Derive the Fourier series coefficients of each of the following signals in terms of a_k:

(a) $x(t - t_0) + x(t + t_0)$

(b) $\mathcal{E}v\{x(t)\}$

(c) $\mathcal{R}e\{x(t)\}$

(d) $\frac{d^2x(t)}{dt^2}$

(e) $x(3t - 1)$ [for this part, first determine the period of $x(3t - 1)$]

3.41. Suppose we are given the following information about a continuous-time periodic signal with period 3 and Fourier coefficients a_k:

1. $a_k = a_{k+2}$.
2. $a_k = a_{-k}$.
3. $\int_{-0.5}^{0.5} x(t)dt = 1$.
4. $\int_{1}^{2} x(t)dt = 2$.

Determine $x(t)$.

3.42. Let $x(t)$ be a real-valued signal with fundamental period T and Fourier series coefficients a_k.

(a) Show that $a_k = a^*_{-k}$ and a_0 must be real.

(b) Show that if $x(t)$ is even, then its Fourier series coefficients must be real and even.

(c) Show that if $x(t)$ is odd, then its Fourier series coefficients are imaginary and odd and $a_0 = 0$.

(d) Show that the Fourier coefficients of the even part of $x(t)$ are equal to $\mathcal{R}e\{a_k\}$.

(e) Show that the Fourier coefficients of the odd part of $x(t)$ are equal to $j\mathcal{I}m\{a_k\}$.

3.43. **(a)** A continuous-time periodic signal $x(t)$ with period T is said to be *odd harmonic* if, in its Fourier series representation

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}, \tag{P3.43–1}$$

$a_k = 0$ for every non-zero even integer k.

(i) Show that if $x(t)$ is odd harmonic, then

$$x(t) = -x\left(t + \frac{T}{2}\right). \tag{P3.43–2}$$

(ii) Show that if $x(t)$ satisfies eq. (P3.43–2), then it is odd harmonic.

(b) Suppose that $x(t)$ is an odd-harmonic periodic signal with period 2 such that

$$x(t) = t \quad \text{for } 0 < t < 1.$$

Sketch $x(t)$ and find its Fourier series coefficients.

(c) Analogously, to an odd-harmonic signal, we could define an even-harmonic signal as a signal for which $a_k = 0$ for k odd in the representation in eq. (P3.43–1). Could T be the fundamental period for such a signal? Explain your answer.

(d) More generally, show that T is the fundamental period of $x(t)$ in eq. (P3.43–1) if one of two things happens:

(1) Either a_1 or a_{-1} is nonzero;

or

(2) There are two integers k and l that have no common factors and are such that both a_k and a_l are nonzero.

3.44. Suppose we are given the following information about a signal $x(t)$:

1. $x(t)$ is a real signal.
2. $x(t)$ is periodic with period $T = 6$ and has Fourier coefficients a_k.
3. $a_k = 0$ for $k = 0$ and $k > 2$.
4. $x(t) = -x(t-3)$.
5. $\frac{1}{6}\int_{-3}^{3}|x(t)|^2\,dt = \frac{1}{2}$.
6. a_1 is a positive real number.

Show that $x(t) = A\cos(Bt + C)$, and determine the values of the constants A, B, and C.

BASIC PROBLEMS

4.21. Compute the Fourier transform of each of the following signals:

(a) $[e^{-\alpha t}\cos\omega_0 t]u(t)$, $\alpha > 0$

(b) $e^{-3|t|}\sin 2t$

(c) $x(t) = \begin{cases} 1 + \cos\pi t, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}$

(d) $\sum_{k=0}^{\infty} \alpha^k \delta(t - kT)$, $|\alpha| < 1$

(e) $[te^{-2t}\sin 4t]u(t)$

(f) $\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$

(g) $x(t)$ as shown in Figure P4.21(a)

(h) $x(t)$ as shown in Figure P4.21(b)

(i) $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$

(j) $\sum_{n=-\infty}^{+\infty} e^{-|t-2n|}$

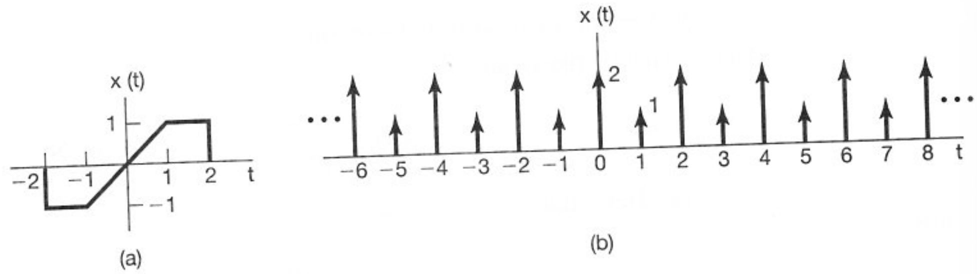

Figure P4.21

4.22. Determine the continuous-time signal corresponding to each of the following transforms.

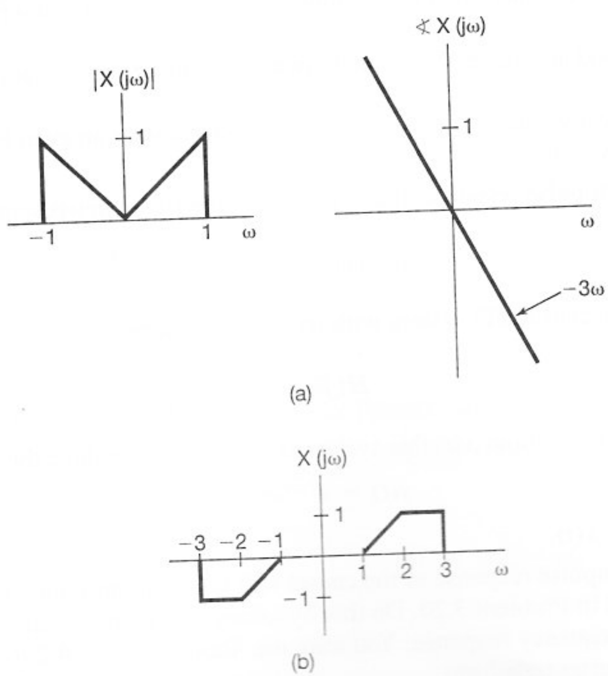

Figure P4.22

(a) $X(j\omega) = \frac{2\sin[3(\omega - 2\pi)]}{(\omega - 2\pi)}$
(b) $X(j\omega) = \cos(4\omega + \pi/3)$
(c) $X(j\omega)$ as given by the magnitude and phase plots of Figure P4.22(a)
(d) $X(j\omega) = 2[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$
(e) $X(j\omega)$ as in Figure P4.22(b)

4.23. Consider the signal

$$x_0(t) = \begin{cases} e^{-t}, & 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}.$$

Determine the Fourier transform of each of the signals shown in Figure P4.23. You should be able to do this by explicitly evaluating *only* the transform of $x_0(t)$ and then using properties of the Fourier transform.

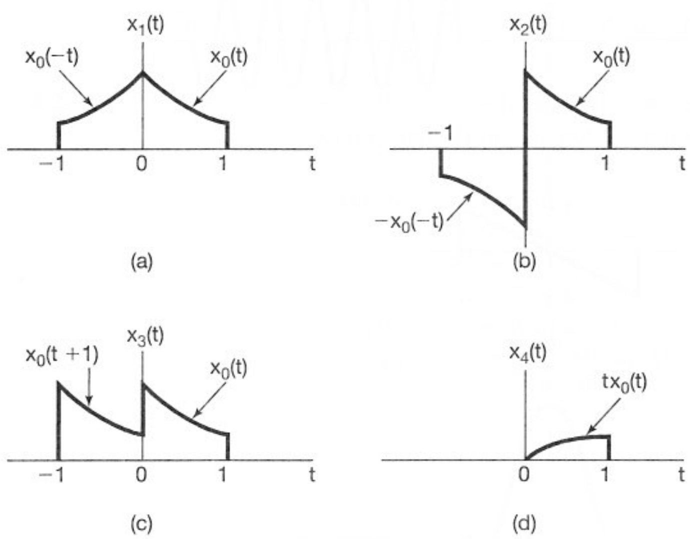

Figure P4.23

4.24. **(a)** Determine which, if any, of the real signals depicted in Figure P4.24 have Fourier transforms that satisfy each of the following conditions:
(1) $\mathcal{Re}\{X(j\omega)\} = 0$
(2) $\mathcal{Im}\{X(j\omega)\} = 0$
(3) There exists a real α such that $e^{j\alpha\omega}X(j\omega)$ is real
(4) $\int_{-\infty}^{\infty} X(j\omega)\,d\omega = 0$
(5) $\int_{-\infty}^{\infty} \omega X(j\omega)\,d\omega = 0$
(6) $X(j\omega)$ is periodic

(b) Construct a signal that has properties (1), (4), and (5) and does *not* have the others.