Proving Feasibility of Optimizable Problem Using Dykstra's Algorithm

Initiative

I was wondering if there is a standard method to determine whether a problem is feasible for optimization, and I came across Dykstra's Algorithm which determines feasibility of a problem. Below is my attempt at understanding it.

Concept

To be optimizable, the problem must first be feasible. If Dykstra's Algorithm proves that there is no feasible solution, then optimization cannot be carried out.

Dykstra's Algorithm

For each r, find the only $\bar{x} \in C \cap D$ such that

$$||\bar{x} - r||^2 \leq ||x - r||^2, \quad \text{for all } x \in C \cap D,$$

where C, D are convex sets.

We can understand this as finding the projection $proj_{C \cap D}$, which is the the projection of r onto set $C \cap D$

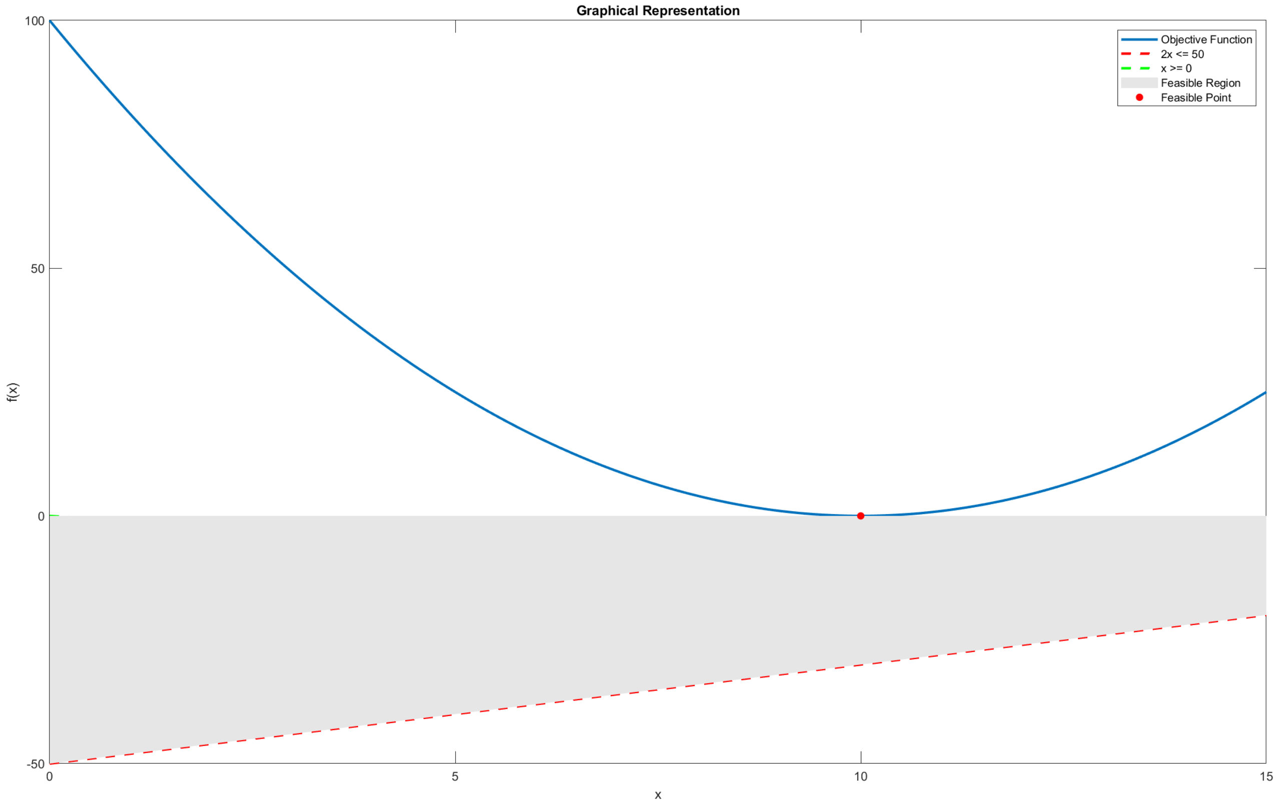
Its concept is to iteratively project onto each of the convex sets and correct error introduced in projection. Until convergence to feasible point is reached.

Example

$$\begin{aligned}
& f(x) = x^2 - 20x + 100 \\
\text{Step 1: } & \text{Set } x = 0 \text{ as a feasible point} \\
\text{Step 2: } & \text{Find projection onto the set of contraints} \\
& \text{Project x onto } 2x \leq 50 \\
& Proj_{2x \leq 50} = \min\left(x, \frac{50}{2}\right) = \min(0, 25) = 0 \\
& \text{Project result onto} x >= 0 \\
& Proj_{x \geq 0}(Proj_{2x \leq 50}(x)) = \max(0, 0) = 0 \\
\text{Step 3: } & \text{There is no deviation in this projection} \\
\text{Step 4: } & \text{Stop iteraiton}
\end{aligned}$$

We can conclude that the algorithm converges in one iteration as the projection onto the set of constraints do not change the point. Hence, the feasible point is x*=0.

Graph



References

Dykstra's projection algorithm. (2023, August 22). Wikipedia, the free encyclopedia. Retrieved November 11, 2023, from https://en.wikipedia.org/wiki/Dykstra%27s_projection_algorithm