S&S Cheat Sheet

DTFT

$$\text{Defining } \omega = \Omega T$$
$$X(j\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Inverse DTFT

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})^{j\omega n}\, dw$$

Magnitude and Phase

$$\left|X(e^{j\omega})\right| = \sqrt{[\mathrm{Im}(X(e^{j\omega}))]^2 + [\mathrm{Re}(X(e^{j\omega})]^2}$$
$$phase X(e^{jw}) = \tan^{-1}\left(\frac{[\mathrm{Im}(X(e^{j\omega}))]}{[\mathrm{Re}(X(e^{j\omega})]}\right)$$

Frequency Response

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$$

CT → DT (Sampling)

$$x[n] = x(t)\Big|_{t=nT} = x(nT)$$

DT → CT

$$h(t) = \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} H(j\Omega)e^{j\Omega t}\, d\Omega = sinc\left(\frac{t}{T}\right), sinc(u) = \frac{\sin(\pi u)}{\pi u}$$

Z-Transform

$$X(z) = \sum_{n-\infty}^{\infty} x[n]z^{-n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{j\omega n}, z = re^{j\omega}$$

$$\text{if } \left|X(e^{j\omega})\right| \to \infty, \text{ DTFT does not exist}$$

$$\text{X(z) Converges When: } |X(z)| < \infty$$

$$\text{Zero : Values of z such that } X(z) = 0$$

$$\text{Poles: Values of z such that } X(z) = \pm\infty$$

DTFT Existence Condition

$$|a| < 1$$

$$if\, |z| > |a| \text{ includes } |z| = 1, |a| < 1 \text{ required}$$

Causality

$$h[n] = 0, n < 0, \text{ ROC include } \infty$$

Stability

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

Inverse z transform

Inspection

$$X(z) = \frac{z}{2z-1}$$

$$\text{rewrite } z \to z^{-1}$$

$$\text{use table}$$

Partial Function:

Case 1:

$$\text{M<N: Chnage } z^{-1} \text{ to } z$$

$$\text{Set denominator} = 0\text{, then partial}$$

Case 2:

$$\text{M } \geq \text{ N : Long Division}$$

$$X(z) = B + \frac{A_1}{1 - 0.5z^{-1}} + \frac{A_2}{1 - z^{-1}}$$

Case 3:

$M < N$ with multiple-order pole

$$X(z) = \frac{4}{(1+z^{-1})(1-z^{-1})^2} = \frac{A_1}{a+z^{-1}} + \frac{C_1}{1-z^{-1}} + \frac{C_2}{(1-z^{-1})^2}$$

Case 4:

$\mathrm{M} \geq \mathrm{N}$:

$$X(z) = \sum_{l=0}^{M-N} B_l z^{-1} + \sum_{k=1,k\neq i}^{N} \frac{A_k}{1-c_k z^{-1}} + \frac{\sum_{m=1}^{s} C_m}{(1-c_i z^{-1})^m}$$

Transfer Function of LTI

$$H(z) = \frac{Y(z)}{X(z)}$$

Geometric Series formulas

Interval	Sum	Condition	Interval	Sum	Condition
Infinite	$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$	$\lvert a\rvert<1$	*Finite on [1,N]*	$\sum_{k=1}^{N} a^k = \frac{a(1-a^{N+1})}{1-a}$	None
Finite on [0,N]	$\sum_{k=0}^{N} a^k = \frac{1-a^{N+1}}{1-a}$	None	*Finite on $[N_1,N_2]$*	$\sum_{k=N_1}^{N_2} a^k = \frac{a^{N_1}-a^{N_2+1}}{1-a}$	None
Infinite	$\sum_{k=1}^{\infty} a^k = \frac{a}{1-a}$	$\lvert a\rvert<1$	*Finite on [1,N]*	$\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$	None

■ Partial fractions.

$\frac{f(x)}{(x-a)(x-b)}$	$\frac{A}{x-a} + \frac{B}{x-b}$
$\frac{f(x)}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
$\frac{f(x)}{(x-a)\left(x^2+bx+c\right)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$
$\frac{f(x)}{(x-a)(x+d)^2}$	$\frac{A}{x-a} + \frac{B}{x+d} + \frac{C}{(x+d)^2}$
$\frac{f(x)}{(x+d)^2}$	$\frac{A}{x+d} + \frac{B}{(x+d)^2}$
$\frac{f(x)}{(x-a)\left(x^2-b^2\right)}$	$\frac{A}{x+d} + \frac{Bx+C}{x^2-b^2}$
$\frac{f(x)}{\left(x^2-a\right)\left(x^2-b\right)}$	$\frac{Ax+B}{x^2-a} + \frac{Cx+D}{x^2-b}$
$\frac{f(x)}{\left(x^2-a\right)^2}$	$\frac{Ax+B}{x^2-a} + \frac{Cx+D}{\left(x^2-a\right)^2}$

Sequence	Transform	ROC
$\delta[n]$	1	All z
$\delta[n-m]$	z^{-m}	$\lvert z\rvert > 0$, $m > 0$; $\lvert z\rvert < \infty$, $m < 0$
$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$a^n \cos(bn) u[n]$	$\dfrac{1-a\cos(b)z^{-1}}{1-2a\cos(b)z^{-1}+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$a^n \sin(bn) u[n]$	$\dfrac{a\sin(b)z^{-1}}{1-2a\cos(b)z^{-1}+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

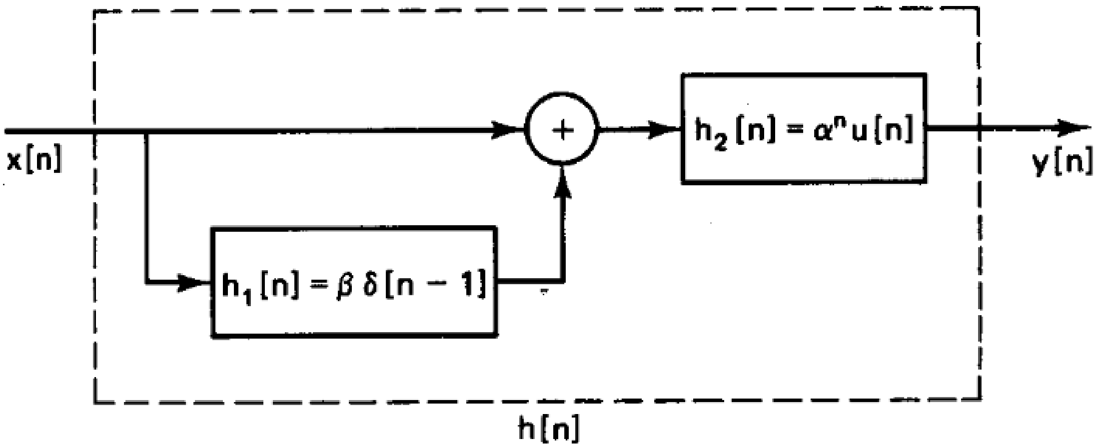

Figure 1

$$
\begin{aligned}
y[n] &= (x[n] + x[n] \otimes h_1[n]) \otimes h_2[n] \\
&= (x[n] \otimes \delta[n] + x[n] \otimes h_1[n]) \otimes h_2[n] \\
&= x[n] \otimes ([\delta[n] + h_1[n]]) \otimes h_2[n]
\end{aligned}
$$