EE2302 Foundations of Information Engineering

Assignment 2 (Solution)

1.
   a) No, because the elements c and d map to the same element in the co-domain of F.
   b) Yes, because all the elements in Y are images of some element in X.
   c) $\{e, f, g\}$.
   d) No, because the elements a and b map to the same element in the co-domain of G.
   e) No, because the element g is not an image of any element in the domain of G.
   f) $\{e, f\}$.

2.
   a) Yes. Suppose that $g(n_1) = g(n_2)$ for some elements n_1 and n_2 in $\mathbf{Z}$. By the definition of $g, 5n_1 + 7 = 5n_2 + 7$, which implies that $n_1 = n_2$. Therefore, g is injective.

   b) No. One possible counter example: Choose $0 \in \mathbf{Z}$. There does not exist $n \in \mathbf{Z}$ such that $g(n) = 5n + 7 = 0$, since the only root is -7/5, which is not in $\mathbf{Z}$. Therefore, g is not surjective.

3. Let x_1 and x_2 be elements in X such that $g \circ f(x_1) = g \circ f(x_2)$. That is equivalent to $g(f(x_1)) = g(f(x_2))$. Since g is injective, $f(x_1) = f(x_2)$. Since f is also injective, $x_1 = x_2$. Hence, $g \circ f$ is injective.