Name:___________________

Student Number #: ___________________

City University of Hong Kong
Department of Electronic Engineering

EE 3210 Systems and Signals

Midterm Quiz 1

NOTE:

1. This is an open book, open notes midterm quiz.
2. There are 4 problems altogether, which amount to 20 points.
3. In addition to final answers, you need to include necessary steps to show your derivations. An answer without any supporting development will unlikely be awarded with a point. You do receive partial credits if you write down the steps despite that you may not complete your solution.
4. Write your work in the blank space. If necessary, write on the back sheet.
5. Enjoy the test, and good luck!

1. (5 pts) (i) Let $x(t)$ and $y(t)$ be periodic signals with fundamental periods T_1 and T_2. Under what conditions is the following signal periodic? If it is periodic, find the fundamental period.

$$w(t) = \sqrt{x(t)} + y^2(t), \qquad x(t) \geq 0 \text{ for all } t.$$

(ii) Show that

$$\int_{-\infty}^{\infty} f(t)\delta(at - bt_0)dt = \frac{1}{|a|} f\left(\frac{b}{a}t_0\right).$$

Solution:

(i) Consider some $T > 0$,

$$w(t+T) = \sqrt{x(t+T)} + y^2(t+T).$$

If there exist integers m and n such that

1 pt $\quad mT_1 = nT_2,$

then set $T = mT_1 = nT_2$. It follows that

$$w(t+T) = \sqrt{x(t+mT_1)} + y^2(t+nT_2)$$
$$= \sqrt{x(t)} + y^2(t)$$
$$= w(t).$$

Consequently, $w(t)$ can be made a periodic signal provided that $mT_1 = nT_2$ for some integers m and n, or alternatively

1 pt $\quad \frac{T_1}{T_2} = \frac{n}{m}$ = rational number

The fundamental period: $T = mT_1 = nT_2$, where m and n should be selected as the smallest integers.

(ii) $$\int_{-\infty}^{\infty} f(t)\delta(at - bt_0)\,dt = \begin{cases} \int_{-\infty}^{\infty} f\left(\frac{\tau + bt_0}{a}\right)\delta(\tau)\frac{1}{a}d\tau, & a > 0 \\ \int_{\infty}^{-\infty} f\left(\frac{\tau + bt_0}{a}\right)\delta(\tau)\frac{1}{a}d\tau, & a < 0 \end{cases}$$

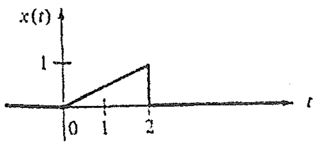
1 pt variable substitution

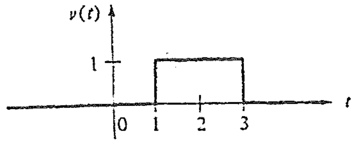
1 pt

1 pt.

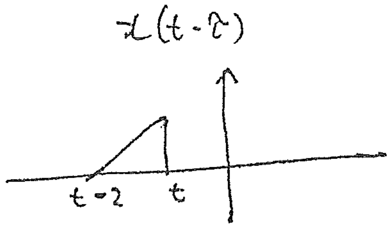
$$= \frac{1}{|a|} f\left(\frac{b}{a}t_0\right)$$

2. (5 pts) The impulse response of a LTI system and an input signal applied to the system are shown in the following figure. Determine the output response.



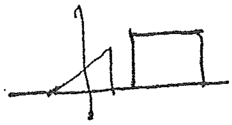


Solution:

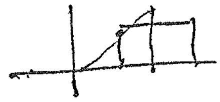


1 pt $\quad x(t) = \begin{cases} \frac{1}{2}t & 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$

1 pt $\quad t \le 1$: $\quad v(\tau)\, x(t-\tau) = 0, \quad y(t) = v(t) * x(t) = 0$



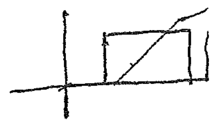
1 pt $\quad 1 < t \le 3$: $\quad y(t) = \int_1^t \frac{1}{2}(t-\tau)\, d\tau$



$$= \frac{1}{2}t(t-1) - \frac{1}{4}(t^2-1) = \frac{1}{4}t^2 - \frac{1}{2}t + \frac{1}{4}$$

$$= \frac{1}{4}(t-1)^2$$

1 pt $\quad 3 < t \le 5$: $\quad y(t) = \int_{t-2}^{3} \frac{1}{2}(t-\tau)\, d\tau$



$$= \frac{1}{2}t(5-t) - \frac{1}{4}\left[3^2 - (t-2)^2\right]$$

$$= -\frac{1}{2}t^2 + \frac{3}{2}t - \frac{7}{2}$$

1 pt $\quad t > 5$: $\quad y(t) = 0$

$$y(t) = \begin{cases} 0 & t \le 1 \\ \frac{1}{4}(t-1)^2 & 1 < t \le 3 \\ -\frac{1}{2}t^2 + \frac{3}{2}t - \frac{7}{2} & 3 < t \le 5 \\ 0 & t > 5 \end{cases}$$

3. (6 pts) Consider the system described by the equation

$$y(t) = \int_{-\infty}^{t} x(\tau - 5)d\tau.$$

Determine if this system is (i) memoryless, (ii) linear, (iii) time invariant, (iv) causal, and (v) stable. Justify your answers. If the system is linear time-invariant, find also the impulse response $h(t)$, and use $h(t)$ to determine the causality and stability of the system.

Solution:

1 pt { (i) Not memoryless
(ii) Linear

1 pt (iii) Time-invariant

1 pt (iv) Causal

1 pt (v) unstable.

Impulse Response

1 pt $$h(t) = \int_{-\infty}^{t} \delta(\tau-5)\, d\tau = \begin{cases} 1 & t>5 \\ 0 & t<5 \end{cases} = u(t-5)$$

The system is causal since $h(t) \equiv 0$ for $t<5$.

1 pt It is not stable since

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{5}^{\infty} 1\, dt = \infty$$

4. (4 pts) (i) Let $x(n)$ be a periodic signal with fundamental period N. For some given positive integer m, define the new signal

$$y(n) = e^{j(2\pi/N)mn} x(mn).$$

Is $y(n)$ a periodic signal? If it is, what is its fundamental period? Furthermore, is it possible that the fundamental period of $y(n)$ less than N? Under what conditions can this be true and what is the fundamental period?

Solution:

$$y(n+N) = e^{j\frac{2\pi}{N}m(n+N)} x[m(n+N)]$$

$$= e^{j\frac{2\pi}{N}mn} \cdot e^{j\frac{2\pi}{N}mN} x(mn+mN)$$

2 pts

$$= e^{j\frac{2\pi}{N}mn} \cdot e^{j2\pi m} x(mn)$$

$$= e^{j\frac{2\pi}{N}mn} x(mn)$$

$$= y(n)$$

Hence, $y(n)$ is periodic.

Fundamental period: If $\frac{N}{m}$ is an integer, Then

$$y\left(n+\frac{N}{m}\right) = e^{j\frac{2\pi}{N}m\left(n+\frac{N}{m}\right)} x\left[m\left(n+\frac{N}{m}\right)\right]$$

$$= e^{j\frac{2\pi}{N}mn} \cdot e^{j2\pi} x(mn+N)$$

2 pts

$$= e^{j\frac{2\pi}{N}mn} x(mn)$$

$$= y(n).$$

In this case, $\frac{N}{m}$ is the fundamental period.

Otherwise N is the fundamental period.