Geometric Series formulas

Interval	Sum	Condition	Interval	Sum	Condition
Infinite	$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$	$\lvert a\rvert<1$	***Finite on [1,N]***	$\sum_{k=1}^{N} a^k = \frac{a(1-a^{N+1})}{1-a}$	None
Finite on [0,N]	$\sum_{k=0}^{N} a^k = \frac{1-a^{N+1}}{1-a}$	None	***Finite on $[N_1,N_2]$***	$\sum_{k=N_1}^{N_2} a^k = \frac{a^{N_1}-a^{N_2+1}}{1-a}$	None
Infinite	$\sum_{k=1}^{\infty} a^k = \frac{a}{1-a}$	$\lvert a\rvert<1$	***Finite on [1,N]***	$\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$	None

Elementary signal classification

Name	Continuous	Discrete	Name	Continuous	Discrete
Unit Step function	$u(t) = \begin{cases}1, & t \geq 0\\ 0, & t < 0\end{cases}$	$u[n] = \begin{cases}1, & n \geq 0\\ 0, & n < 0\end{cases}$	**Signum signal**	$Sgn(t) = \begin{cases}1, & t > 0\\ -1, & t < 0\end{cases}$	$Sgn[n] = \begin{cases}1, & n > 0\\ -1, & n < 0\end{cases}$
Ramp signal	$r(t) = \begin{cases}t, & t \geq 0\\ 0, & t < 0\end{cases}$	$r[n]=nu(n) = \begin{cases}n, & n \geq 0\\ 0, & n < 0\end{cases}$	**Sinusoidal signal**	$x(t) = \sin(2\pi f_0 t + \theta)$	$X[n] = \sin(2\pi f_0 n + \theta)$
Impulse function	$\delta(t) = 0, t \neq 0$	$\delta[n] = \begin{cases}1, & n = 0\\ 0, & otherwise\end{cases}$	**Sinc function**	$\sin(\omega_0 t) = \frac{\sin(\pi\omega_0 t)}{\pi\omega_0 t}$	$\sin[\omega_0 n] = \frac{\sin(\pi\omega_0 n)}{\pi\omega_0 n}$
Rectangular pulse function	$\Pi\left(\frac{t}{\tau}\right) = \begin{cases}1, & \lvert t\rvert \leq \tau/2\\ 0, & \lvert t\rvert > \tau/2\end{cases}$	$\Pi[\frac{n}{2N}] = \begin{cases}1, & \lvert n\rvert \leq N\\ 0, & \lvert n\rvert > N\end{cases}$	**Triangular pulse**	$\Lambda\left(\frac{t}{\tau}\right) = \begin{cases}1-\lvert\frac{t}{\tau}\rvert, & t \leq \lvert\tau\rvert\\ 0, & t > \lvert\tau\rvert\end{cases}$	$\Lambda[\frac{n}{N}] = \begin{cases}1-\frac{\lvert n\rvert}{N}, & \lvert n\rvert \leq N\\ 0, & elsewhere\end{cases}$

Important Properties of Signals

Name	Properties	Name	Properties
Impulse properties	$\int_{-\infty}^{\infty}\delta(t)dt = 1$ $\delta(\alpha t) = \frac{1}{\lvert\alpha\rvert}\delta(t)$ $\delta(\alpha t + b) = \frac{1}{\lvert\alpha\rvert}\delta(t + \frac{b}{\alpha})$ $\int_{-\infty}^{\infty}\emptyset(t)\delta(t-\lambda)dt = \emptyset(\lambda)$ $\emptyset(t)\delta(t-\lambda) = \emptyset(\lambda)\delta(t-\lambda)$	**Time period of linear combination of two signals**	Sum of signals is periodic if $\frac{T_1}{T_2} = \frac{m}{n} = rational\ number$ The fundamental period of g(t) is given by nT1 = mT2 provided that the values of m and n are chosen such that the greatest common divisor (gcd) between m and n is 1
Signals in term of unit step and vice versa	$u(t) = \int_{-\infty}^{t}\delta(\tau)d\tau$ $\delta(t) = \frac{d}{dt}u(t)$ $r(t) = tu(t)$ $u(t) = \frac{d}{dt}r(t)$ $u(t) = \frac{d}{dt}r(t)$ $sgn = u(t) - u(-t)$ $sgn = 2u(t) - 1$ $\Pi\left(\frac{t}{\tau}\right) = u\left(t + \frac{t}{\tau}\right) - u(t - \frac{t}{\tau})$	**Odd and even & symmetry**	$x_e(t) = x_e(-t)$ $x_o(t) = -x_o(-t)$ $x(t) = x_e(t) + x_o(t)$ $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$ $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$
Derivative of impulse (doublet)	$\frac{d}{dt}\delta(t) = \delta'(t) = \begin{cases}undefined, & t = 0\\ 0, & otherwise\end{cases}$ $\delta'(\alpha t) = \frac{1}{\alpha\lvert\alpha\rvert}\delta'(t)$ $\int_{-\infty}^{\infty}x(t)\delta'(t-\lambda)dt = -x'(\lambda)$ $x(t)\delta'^{(t)} = x(0)\delta'^{(t)} - x'(0)\delta(t)$	**Combined operation**	*x(t)⇒ Kx(t) +C* Scale by *K* then shift by *C* .... x(t)⇒ x(αt −β) Shift by β : [x(t -β)] Then Compress by a:[x(t-β) ⇒ x (αt-β)] OR Compress by α: [x(t)⇒ x(αt)] then Shift by $\frac{\beta}{\alpha}$:[x(αt) ⇒x{α (t- $\frac{\beta}{\alpha}$)} = x (αt −β)}]
Energy and power	Periodic signals have infinite energy hence power type signals.		

Properties of the Continuous-Time Fourier Series

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$$

$$a_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$$

Property	Time Domain	Fourier series
	$\left.\begin{array}{l} x(t) \\ y(t) \end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time-Shifting	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency-Shifting	$e^{jM\omega_0 t} = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
Conjugation	$x^*(t)$	a^*_{-k}
Time Reversal	$x(-t)$	a_{-k}
Time Scaling	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution	$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation	$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration	$\int_{-\infty}^{t} x(t)dt$ (finite-valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x(t)$ real and even	a_k real and even
Real and Odd Signals	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

Properties of the Discrete-Time Fourier Series

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}$$

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk(2\pi/N)n}$$

Property	Time Domain	Fourier Series
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} a_k \\ b_k \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time shift	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shift	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m] & \text{if } n \text{ is a multiple of } m \\ 0 & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k \left(\begin{matrix}\text{viewed as} \\ \text{periodic with} \\ \text{period } mN\end{matrix}\right)$
Periodic Convolution	$\sum_{r=\langle N \rangle} x[r] y[n-r]$	$N a_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N \rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)}) a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \left(\begin{matrix}\text{finite-valued and} \\ \text{periodic only if } a_0 = 0\end{matrix}\right)$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right) a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{O}d\{x[n]\}$ $[x[n]$ real$]$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{k=\langle N \rangle} |a_k|^2$$

Properties of the Continuous-Time Fourier Transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Property	Aperiodic Signal	Fourier Transform
	$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
Time-shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
Frequency-shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time-Reversal	$x(-t)$	$X(-j\omega)$
Time- and Frequency-Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega) * Y(j\omega)$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re e\{X(j\omega)\} = \Re e\{X(-j\omega)\} \\ \Im m\{X(j\omega)\} = -\Im m\{X(-j\omega)\} \\ \lvert X(j\omega)\rvert = \lvert X(-j\omega)\rvert \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ $[x(t)$ real$]$ $x_o(t) = \mathcal{O}d\{x(t)\}$ $[x(t)$ real$]$	$\Re e\{X(j\omega)\}$ $j\Im m\{X(j\omega)\}$

Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \lvert X(j\omega)\rvert^2 d\omega$$

Basic Continuous-Time Fourier Transform Pairs

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi\delta(\omega)$	$a_0 = 1, \quad a_k = 0, \ k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \leq \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2\sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2\sin\omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t), \Re e\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t), \Re e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$, $\Re e\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

Properties of the Discrete-Time Fourier Transform

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

Property	Aperiodic Signals	Fourier Transform
	$x[n]$ $y[n]$	$\left.\begin{matrix} X(e^{j\omega}) \\ Y(e^{j\omega}) \end{matrix}\right\}$ Periodic with period 2π
Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
Time-Shifting	$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Frequency-Shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Time Reversal	$x[-n]$	$X(e^{-j\omega})$
Time Expansions	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-e^{-j\omega}}X(e^{j\omega})$ $+\pi X(e^{j0})\sum_{k=-\infty}^{+\infty}\delta(\omega - 2\pi k)$
Differentiation in Frequency	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \Re e\{X(e^{j\omega})\} = \Re e\{X(e^{-j\omega})\} \\ \Im m\{X(e^{j\omega})\} = -\Im m\{X(e^{-j\omega})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$
Symmetry for Real, Even Signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{O}d\{x[n]\}$ $[x[n]$ real$]$	$\Re e\{X(e^{j\omega})\}$ $j\Im m\{X(e^{j\omega})\}$

Parseval's Relation for Aperiodic Signals

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

Basic Discrete-Time Fourier Transform Pairs

Signal	Fourier Transform	Fourier Series Co-effecient (if periodic)
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$, $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$, $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$, $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \ldots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & \|n\| \leq N_1 \\ 0, & N_1 < \|n\| \leq N/2 \end{cases}$ and $x[n+N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, \; k \neq 0, \pm N, \pm 2N, \ldots$ $a_k = \frac{2N_1 + 1}{N}, \; k = 0, \pm N, \pm 2N, \ldots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], \quad \|a\| < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] \begin{cases} 1, & \|n\| \leq N_1 \\ 0, & \|n\| > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \leq \|\omega\| \leq W \\ 0, & W < \|\omega\| \leq \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi \delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n+1)a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—

Properties of Laplace Transform

Property	Signal	Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t)+bx_2(t)$	$aX_1(s)+bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0t}x(t)$	$X(s-s_0)$	Shifted version of R [i.e., s is in the ROC if $(s-s_0)$ is in R]
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	"Scaled" ROC (i.e., s is in the ROC if (s/a) is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\Re e\{s\} > 0\}$

Initial- and Final Value Theorems

If $x(t)=0$ for $t<0$ and $x(t)$ contains no impulses or higher-order singularities at $t=0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t)=0$ for $t<0$ and $x(t)$ has a finite limit as $t\to\infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

Laplace Transform of Elementary Functions

Signal	Transform	Roc
1. $\delta(t)$	1	All s
2. $u(t)$	$\frac{1}{s}$	$\Re e\{s\} > 0$
3. $-u(-t)$	$\frac{1}{s}$	$\Re e\{s\} < 0$
4. $\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$
5. $-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re e\{s\} < 0$
6. $e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} > -\alpha$
7. $-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\Re e\{s\} < -\alpha$
8. $\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} > -\alpha$
9. $-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\Re e\{s\} < -\alpha$
10. $\delta(t-T)$	e^{-sT}	All s
11. $[\cos\omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
12. $[\sin\omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
13. $[e^{-\alpha t}\cos\omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
14. $[e^{-\alpha t}\sin\omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$\Re e\{s\} > -\alpha$
15. $u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
16. $u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$

Sequence	Transform	ROC
$\delta[n]$	1	All z
$\delta[n-m]$	z^{-m}	$\|z\|>0$, $m>0$; $\|z\|<\infty$, $m<0$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\|>\|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\|<\|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|>\|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|<\|a\|$
$a^n \cos(bn)u[n]$	$\frac{1-a\cos(b)z^{-1}}{1-2a\cos(b)z^{-1}+a^2z^{-2}}$	$\|z\|>\|a\|$
$a^n \sin(bn)u[n]$	$\frac{a\sin(b)z^{-1}}{1-2a\cos(b)z^{-1}+a^2z^{-2}}$	$\|z\|>\|a\|$

Signal	Transform	ROC
$\delta(t)$	1	All s
$\delta(t-T)$	e^{-sT}	All s
$e^{-at}u(t)$	$\frac{1}{s+a}$	$\Re\{s\}>-a$
$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\Re\{s\}<-a$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(s+a)^n}$	$\Re\{s\}>-a$
$-\frac{t^{n-1}}{(n-1)!}e^{-at}u(-t)$	$\frac{1}{(s+a)^n}$	$\Re\{s\}<-a$
$e^{-at}\cos(bt)u(t)$	$\frac{s+a}{(s+a)^2+b^2}$	$\Re\{s\}>-a$
$e^{-at}\sin(bt)u(t)$	$\frac{b}{(s+a)^2+b^2}$	$\Re\{s\}>-a$

Table 9.1: Laplace transforms for common signals