EE3331 Probability Models in Information Engineering

Semester B 2022 – 2023

Test 2

4:00 p.m. – 5:30 p.m.

Answer **ALL FIVE** questions:

Question 1

The joint probability density function (PDF) of two random variables X and Y is given as:

$$p(x,y) = \begin{cases} C + 3y^2 & -0.5 < x < 0.5,\ -0.5 < y < 0.5 \\ 0, & \text{otherwise} \end{cases}$$

where C is an unknown constant.

(a) Find the value of C. **(8 marks)**

(b) Find the marginal PDF of Y. **(7 marks)**

Question 2

A discrete random variable K has the following cumulative density function (CDF):

$$F(k) = \begin{cases} 0, & k < -2 \\ 0.1, & -2 \le k < 4 \\ 0.3, & 4 \le k < 6 \\ 0.6, & 6 \le k < 7 \\ 1, & k \ge 7 \end{cases}$$

(a) Find the conditional probability mass function (PMF) of K given $K > 0$, namely, $P_{K|K>0}(k)$. **(5 marks)**

(b) Find the conditional CDF of K given $K > 0$, namely, $F_{K|K>0}(k)$. **(5 marks)**

(c) Compute the conditional expected value $\mathbb{E}\{K|K>0\}$ and conditional variance $\mathrm{var}\{K|K>0\}$. **(5 marks)**

Question 3

Two four-sided fair dice with face numbers "1", "2", "3" and "4" are rolled. Let O_1 and O_2 be the random variables representing the two face numbers. Two random variables are then generated from O_1 and O_2, namely, $X = \min(O_1, O_2)$ and $Y = \max(O_1, O_2)$.

(a) Compute probability mass functions (PMFs) of X and Y. **(10 marks)**

(b) Compute the joint PMF of X and Y, and write down the results in a table. **(10 marks)**

(c) Compute the probability $P(Y \ge X + 1)$. **(4 marks)**

(d) Compute the conditional PMFs $P_{X|Y}(x|y)$ and $P_{Y|X}(y|x)$ at $X = Y = 1$. **(6 marks)**

Question 4

Consider tossing a coin and the probability of getting a head is p. The coin is repeatedly tossed until a tail, followed by a head, in two successive trials occur. Let X be the total number of coin tosses.

(a) Determine $\mathbb{E}\{X\}$. **(12 marks)**

(b) What is the expected value of X for a fair coin? **(3 marks)**

Question 5

Consider two independent current sources X and Y passing through a resistor with resistance R. The power dissipated at the resistor is given by $P = (X+Y)^2 R$. The current sources are modelled as uniform random variables, namely, $X \sim \mathcal{U}(X_L, X_U)$ and $Y \sim \mathcal{U}(Y_L, Y_U)$.

(a) Suppose R is a constant. Compute the value of the mean power, namely, $\mathbb{E}\{P\}$, in terms of X_L, X_U, Y_L, Y_U and R. **(15 marks)**

(b) Determine $\mathbb{E}\{P\}$ when $X \sim \mathcal{U}(9, 11)$, $Y \sim \mathcal{U}(0.5, 1.5)$ and $R = 1$. **(5 marks)**

(c) Suppose R is another uniform random variable, i.e., $R \sim \mathcal{U}(R_L, R_U)$, which is independent of X and Y. Write down the mean power $\mathbb{E}\{P\}$ in terms of X_L, X_U, Y_L, Y_U, R_L and R_U. **(5 marks)**