CITY UNIVERSITY OF HONG KONG

Course code & title : EE3301 Optimization Methods for Engineering

Session : Semester B 2019/20

Time allowed : Two hours

This paper has **4** pages (including this cover page but excluding the Honor Pledge that was sent separately).

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1. This paper consists of 3 questions each of which has sub-questions
2. Answer **ALL** questions.

This is an **open-book** examination.

This is an open book exam, and you are allowed to search the internet for sources of information, but remember that you must quote your sources if you use them in any part of any solution. Otherwise, you commit plagiarism.

If you have not submitted the honesty pledge you must sign and submit it with before you submit this exam.

In Canvas / Modules / Final Exam you will find the Excel file "10_digit_numbers_for_final_exam". It contains your unique 10-digit number that you will need to use in Question 1.

Before the exam end-time, submit the following:

(1) All the pages with your answers to questions 1-3 in a pdf file.
(2) The Excel files: "knapsack_LP_Q_1" and "utility_Q_2.
(3) Include all these files (the pdf and the two Excel files) in a Zip file and submit to Canvas Assignments.

Question 1 (40 marks)

1.1. (8 Marks) Consider the following Linear Programming (LP) problem.

$$\max_{\{x_i\}} \sum_{i=1}^{N} v_i\, x_i$$

Subject to:

$$\sum_{i=1}^{N} w_i x_i \leq W$$

$$1 \geq x_i \geq 0,\ i = 1,\ 2,\ \ldots, N.$$

Explain how this problem formulation can be applied to the problem a student is faced when starting working on an exam paper and is deciding which questions or portion of questions to attempt, or equivalently, how much time to spend on each question, aiming to maximize the grade achieved on the exam. In particular, write down what each of the parameters (N, W, $v_1, v_2, \ldots, v_N, w_1, w_2, \ldots ,\ w_N$), each of the decision variables ($x_i,\ i = 1,\ 2,\ \ldots, N$) and the objective function represent in this problem that the student faces.

1.2 (2 Mark) Write down your unique 10-digit number.
If your first digit is more than 5 then $v_1 = 2$. Otherwise, it is equal to 3.
If your second digit is more than 5 then $v_2 = 2$. Otherwise, it is equal to 1.
If your third digit is more than 5 then $v_3 = 2$. Otherwise, it is equal to 1.
$N = 3, W = 5,\ \ w_1 = 1, w_2 = 2$, and $w_3 = 3$.
Substitute the values of $N, W, v_1, v_2, v_3, w_1, w_2$ and w_3, in the above LP problem, and write down the LP problem that is resulted from the above substitutions.

1.3. (10 Marks) Solve the LP problem obtained in Item **1.2** above in two ways: (1) using a greedy algorithm, and (2) using Excel solver. Call the Excel file "knapsack_LP_Q_1". Check that the two solutions you obtain are the same so they confirm each other.

1.4 (10 Marks) Now consider the following knapsack problem

$$\max_{\{x_1, x_2, x_3\}} v_1x_1 + v_2x_2 + v_3x_3$$

Subject to:

$$w_1x_1 + w_2x_2 + w_3x_3 \leq 5$$

and x_1, x_2, x_3 are all binary numbers (0, 1 integers).

For this 0,1 knapsack problem, provide a dynamic programming formulation. As mentioned in class and in tests 1 and 2, when you are asked to provide a dynamic programming formulation of a given problem, you are required to provide the following.

1. Definition of the appropriate optimal value function including definition of the function and its arguments;
2. an appropriate recurrence relation;
3. appropriate boundary conditions.

The formulation must be specific to the above specific problem.

1.5. (10 Marks) Now consider the knapsack problem of Item 1.4 with the values of the parameters v_1, v_2, v_3, w_1, w_2 and w_3 as provided and/or obtained in Item 1.2. Solve the problem using dynamic programming. In particular, find a complete solution of the recurrence relation equations; and the optimal policy function. Provide all the steps of your solution.

Question 2 (30 marks)

2.1 (15 marks) Consider a communications system that serves two users. One of the users (User 1) has a utility function of $U_1(x_1) = 1 - (1.5)^{-x_1}\ x_1 \geq 0$, and the other user (User 2) has the utility function of $U_2(x_2) = 1 - (50)^{-(x_2-0.5)}, x_2 \geq 0.5$.

These utility functions represent the level of satisfaction of the users as a function of the service rate measured in units of [50 Mb/s]. The total service rate provided by the system is 100 Mb/s. This means that the sum $x_1 + x_2$ cannot be more than C=2. Also to guarantee minimum service quality for User 2, x_2 cannot be less than 0.5. The service provider aims to find the optimal values of x_1 and x_2 to maximize the total utility ($U_1(x_1) + U_2(x_2)$) of the two users.

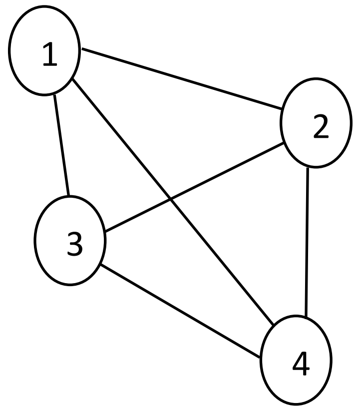
Formulate this problem as a convex optimization problem and solve it using Excel Solver. Use the File Name: "utility_Q_2" and name the sheet: "capacity2". Write down the optimal values of x_1 and x_2 and of $U_1(x_1)$ and $U_2(x_2)$ as well as the optimal sum of the utilities $U_1(x_1) + U_2(x_2)$.

2.2 (15 marks) Now consider two new cases where the capacity limitation changes from C=2 to C=1 and C=3 **and solve the new problems in the Excel file** "utility_Q_2". Use a new sheets called "capacity1" and "capacity3". Again, for each case, write down the new optimal values of x_1 and x_2 and of $U_1(x_1)$ and $U_2(x_2)$ as well as the optimal sum of the utilities $U_1(x_1) + U_2(x_2)$. Provide interpretation to the differences in the optimal results – in particular, observe more significant changes in one of the variables than in the other and provide explanation. Plot the two functions to gain further insight into their behaviors.

In addition, provide for the two utility functions, their modelling applications and implications to TCP (non-real-time) versus UDP (real-time) connections. If one of the two utility functions models TCP connection and the other UDP connection. Which of the two utility functions is more suitable for TCP and which is more suitable for UDP. Justify your answer.

Question 3 (30 marks)

Consider the following fully connected undirected network.



3.1 (5 marks) For each origin-destination (OD) pair of nodes (*i,j*) where the index of *i* is less than the index of *j* (*i*<*j*) write all the loop-free routes that start in *i* and end in *j*. Notice that since the links (and so are the routes) are all bidirectional, so the route 3,2,4 is equivalent (and is the same for all practical purposes) to the route 4,2,3, so we write the route 3,2,4 and we do not write 4,2,3 for the OD pair (3,4). In this way, all the routes for the OD pair (3,4), will start at 3 and with end at 4. This is the reason that we consider only OD pairs (*i,j*) with $i < j$. You are required to write a list of all the routes for each OD pair, but do not include routes where nodes or links are repeated (they all should be **loop free**), and do not include equivalent routes in reverse as mentioned in the example above regarding routes 3,2,4 and 4,2,3. Write every route on a separate line.

3.2 (25 marks) Let D_{ij} be the traffic demand (total in both directions) between nodes *i* and *j*. Let $\mathrm{B_{ij}}$ be the bandwidth capacity of link {ij}. We use Roman notations for links and italic notation for OD pairs, but i and j or *i* and *j* always denote nodes. Traffic between any OD pair (*i,j*) may use any available route, but the cost of transporting a unit of traffic on a route is proportional to the number of hops on that route. In particular, for a one-hop route (using only one link) this cost is equal to 1. For a 2-hop route (using two links) this cost is equal to 2. For a 3-hop route (using three links) this cost is equal to 3. For example, if X_{12} is the amount of traffic transported in a one-hop route (using only one link) between node 1 and node 2, then its cost is assumed to be $1 \times X_{12} = X_{12}$. If X_{132} is the amount of traffic transported in a 2-hop route 1,3,2 between node 1 and node 2, then its cost is assumed to be $2X_{132}$. If X_{1342} is the amount of traffic transported in the 3-hop route 1,3,4,2 between node 1 and node 2, then its cost is assumed to be $3X_{1342}$. Provide a linear programming formulation for the problem of minimizing the total cost of transporting the traffic meeting the demand requirements $D_{ij} > 0$ for all six OD pairs, subject to capacity limitations $\mathrm{B_{ij}} > 0$ of all the six links {ij} in the network. **To save you time**, please provide capacity constraint only for links (1,2), (2,3) and (1,3), and to avoid writing all the non-negativity constraints, you may define R to be the set of routes. Then, for any route $r \in R$, you may write the non-negativity constraint for X_r. You must however provide all the six demand satisfaction constraints associated with the demand requirements $D_{ij} > 0$ for all the six OD pairs.