**CITY UNIVERSITY OF HONG KONG**

Course code & title : EE 3301 Optimization Methods for Engineering

Session : Semester A 2022/23  
Time allowed : 2 hours (plus 30 minutes for uploading)

* This paper has **EIGHT** pages (including these 2 cover pages).
* Online questions are not to be distributed or retained in any form after the exam.

**Instructions**

Please make sure you follow all instructions from the University, ARRO, and EE. Please note the following:

1. This paper consists of **SIX** questions that may have multiple sub-questions. ALL the questions and sub-questions are compulsory. Make sure that you attempt all of them. The total score is 100.
2. This is an **open-book exam.** Students can read the lecture notes and/or other materials available online.
3. You will receive the questions on your seat and also on Canvas. Hand-write\* all answers in the answer-book or on blank answer sheets, compile the answers into, preferably, a single PDF file and **submit the PDF file and three Excel files to the Canvas Assignment called “Final Exam” before the exam deadline** **separately**, See additional instructions on Page 2.
4. **Stay in your seat after the deadline** until the examiner allows you to leave.
5. You were given a 10-digit code to be used in questions as instructed. You must use only the new code. See additional instructions on Page 2.

\* You are allowed to copy and paste text or figures and to fill in/replace numbers by typing. You must cite the source if the material is not part of the exam questions. **Soft copy submission in one pdf + Excel files to Canvas + email of all the material to** [**moshezu@gmail.com**](mailto:moshezu@gmail.com) **(e.g. if you write using iPad) is allowed without using hard-copy answer book.**

**On the first page of your answer book,** copy the following sentence and sign it: *I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties.*

Answering this exam paper implies your acknowledgment of the Pledge for following the Rules on Academic Honesty:

“I pledge that the answers in this examination are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

1. I will not plagiarize (copy without citation) from any source;
2. I will not communicate or attempt to communicate with any other person during the examination; neither will I give or attempt to give assistance to another student taking the examination; and
3. I will use only approved devices (e.g., calculators) and/or approved device models.
4. I understand that any act of academic dishonesty can lead to disciplinary action.”

(Signature) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_­ (Date) \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**Additional Instructions**

You **MUST** use the 10-digit code that you received **in the Module called Final Exam.** Do **NOT** use any other sequence of numbers as your 10-digit number. Using the wrong number will incur **PENALTY** in grading.

You must submit **Excel files** as instructed. Do not use software by other vendors to solve optimization problems. Note that in Group Learning, you are encouraged to use other software tools, but in the exam, you must follow the instructions.

Should you have any technical problems during the exam, contact your course leader or invigilator.

**Final Exam Submission**

Before the exam end-time, submit the following:

1. All the pages with your answers to questions 1-6 in the answer book and possibly additional answer sheet only if the answer book is full.
2. Upload your answers also to Canvas Assignments (preferably in one pdf file, but this is not a requirement). Include also the declaration "I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties…." with signature and date.
3. Upload the three Excel files: "Q3\_Regression", “utility\_Q4”, and “knapsack\_Q5”.
4. Submit all these files (including your written answers and the three Excel files) to the Canvas Assignment called “Final Exam” preferable separately, andnotin a Zip file.

Students who do not use iPad, and write their answers in the hard-copy answer book will also be able to copy and paste material from online sources or lecture notes and include it in the material submitted to Canvas. If you are one of those students and a part of your answer to question number m-n about XYZ can be answered by copying and pasting material from online sources or lecture notes, in your answer book, you write: "for question m-n regarding XYZ, please see soft copy uploaded to Canvas Assignments". If you do this, you are requested to email me to the email address: [**moshezu@gmail.com**](mailto:moshezu@gmail.com) another copy of the softcopy submitted to Canvas Assignments so that I will have a backup of your complete answer to question m-n.

**Question 1 (25 marks)**

* 1. **(1 mark)** Write down your 10-digit number in the following format.If your number is **9753689124,** write**:**

**N1 = 9, N2 = 7, N3 = 5, N4 = 3, N5 = 6, N6 = 8, N7 = 9, N8 = 1, N9 = 2, N10= 4.**

Consider four cities (nodes). The cost of a cable (link) (in units of 100,000 dollars) that connects any pair of these four cities is give in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 |  | N1 | N2 | N3 |
| 2 |  |  | N4 | N5 |
| 3 |  |  |  | N6 |
| 4 |  |  |  |  |

For example, we can see from the table that the cost of the cable to connect City 1 and City 2 is N1.

**Remark:** The value Ni is the ith digit of your 10-digit number.

Two cities are said to be connected if there is a path between them.

* 1. **(24 marks)** Answer the following questions

1. You are required to connect the four cities by cables (links) at minimum cost such that all the four cities are connected to each other. What algorithm will you use to find the optimal solution? Provide the relevant optimal graph and the total cost. Show all steps. (5 marks)
2. You are required to connect the four cities by cables (links) at minimum cost such that all the four cities remain connected to each other even if any one of the links break. Provide the relevant optimal graph and the total cost. Justify your answer. (5 marks)
3. Consider the fact that while a repair team is sent to repair a broken cable, another cable may break. Now you are required to connect the four cities by cables (links) at minimum cost such that all the four cities remain connected to each other even if any two of the links break. Provide the relevant optimal graph and the total cost. Justify your answer. (5 marks)
4. Is the graph obtained in (a) above bipartite or non-bipartite? Justify your answer. (3 marks)
5. Is the graph obtained in (b) above bipartite or non-bipartite? Justify your answer. (3 marks)
6. Does the graph obtained in (c) above have a Euler cycle? If the answer is yes, find it, and if the answer is no, justify your answer. (3 marks)

**Question 2 (15 marks)**

Consider again the four cities (nodes) from Problem 1. Problem 1 was about connectivity. This problem (Problem 2) is about capacity assignments. The aim here is to find the minimal capacity required on each link considering the resilience requirements discussed in Problem 1 and the end-to-end demands between the various pairs of nodes. The demands as well as the capacity assignments are bidirectional. The traffic demands {*Dij*} between the various pairs of nodes are given in the following table (in units of Tb/s).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 |
| 1 |  | N9 | N1 | N6 |
| 2 |  |  | N3 | N7 |
| 3 |  |  |  | N8 |
| 4 |  |  |  |  |

**Remark:** The value Ni is the ith digit of your 10-digit number.

For example, we can see from the table that the total traffic demand (in both directions) between City 1 and City 2 is N9. The units of the traffic demands could be Tb/s, but as the units are well understood, so you do not need to include them in your solutions.

1. **(6 Marks)** Consider a network based on the graph obtained in Problem 1 (a). Find the minimal value of the capacity assigned to each link in that network such that the traffic demands between all pair of nodes are satisfied. Show clearly how you obtain the optimal link capacities.
2. **(9 Marks)** Consider a network based on the graph obtained in Problem 1 (b). Explain in no more than 100 words how you will find the minimal value of the capacity assigned to each link in that network such that the traffic demands between all pair of nodes are satisfied subject to the resilient requirement that these traffic demands are satisfied even if any single link in the network fails. Choose one of the links in the graph obtained in Problem 1 (b) and find the optimal capacity on this link. Show all steps.

**Question 3 (15 marks)**

A company aims to assess the benefit they obtain in terms of its revenue from sales as a function of its investment in advertising (advertising budget). The following table provides data for revenues in thousands of dollars from sales and advertising budget for six different markets.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Market | 1 | 2 | 3 | 4 | 5 | 6 |
| Revenue | 32 | 50 | 498 | 249 | 297 | 95 |
| Advertising  budget | 6 + N1 | 10 + N2 | 99 +N3 | 47 +N4 | 59 +N5 | 19 +N6 |

**Remark:** The value Ni is the ith digit of your 10-digit number.

For example, if your 10-digit number is 5697483715, the table should be updated to:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Market | 1 | 2 | 3 | 4 | 5 | 6 |
| Revenue | 32 | 50 | 498 | 249 | 297 | 95 |
| Advertising  budget | 11 | 16 | 108 | 54 | 63 | 27 |

**3.1. (1 mark)** Write down your 10-digit number.

**3.2. (1 mark)** Update the table using your specific Ni values.

**3.3 (5 marks)** The aim is to use linear regression and to find a linear function in the form of to fit these data. In particular, provide a convex optimization formulation for this regression with an objective to find the optimal values for *a* and *b* using the least squares method. The objective function must include the specific numbers in the above table - not just a formula.

**3.4. (8 marks)** Solve the convex optimization problem formulated in sub-question 3.3 in two ways: (1) using Excel Solver and (2) obtain the optimal values of *a* and *b* using the analytical solution by Excel and observe the consistency between (1) and (2).

**Remark**: *Name the Excel file "*Q3\_Regression*" and upload it with your answers on Canvas.*

**Question 4 (15 marks)**

Consider a communications system that serves three users.

The first of the three users has the utility function *U1(x1) =* 1 *– a -x1* .

The second has the utility function of *U2(x2) =* 1 *– b –x2* .

The third has the utility function of *U3(x3) =* 1 *– c –x3* .

Where the values of the parameters *a*, *b* and *c* are based on your first three digits N1, N2and N3in your 10-digit code. The value of parameter *a* is according to the following table.

|  |  |
| --- | --- |
| N1 | *a* |
| 1 | 1.61 |
| 2 | 1.65 |
| 3 | 1.71 |
| 4 | 1.73 |
| 5 | 1.76 |
| 6 | 1.81 |
| 7 | 1.82 |
| 8 | 1.85 |
| 9 | 1.89 |

The values of parameters *b* and *c* are given by *b* = 3 + N2/9 (rounded to 2 decimal places) and c = 50 + N3.

**4.1 (1 mark)** Write down the values of N1, N2 and N3 according to your 10-digit number and provide the values of *a, b,* and *c* based on the values of N1, N2 and N3.

**Remark:**As in previous questions, the value Ni is the ith digit of your 10-digit number.

**4.2 (5 marks)** These utility functions represent a measure of satisfaction of the users as a function of the service rate measured in units of [10 Mb/s]. The total service rate provided by the system is 30 Mb/s. This means that the sum *x1* + *x2*+ *x3* cannot be more than 3. The service provider aims to find the optimal values for *x1*, *x2*and *x3* to maximize the total utility U = *U1(x1)* + *U2(x2)* + *U3(x3)* of the three users.

Formulate this problem as a convex optimization problem and solve it using Excel Solver. Use the File Name: “utility\_Q4”. In your formulation, substitute the relevant values of *a,* *b* and *c* for of the symbols *a,* *b* and *c.* Write down the optimal values of *x1*, *x2*and *x3* and of *U1(x1), U2(x2)* and *U3(x3)* as well as the optimal sum of the utilities *U1(x1)* + *U2(x2)* + *U3(x3).*

**4.3 (9 marks)** Now reduce the total capacity limitation from 3 to 2 **and solve the new problem in the same Excel file** “utility\_Q4”. Again, write down the new optimal values of *x1*, *x2*and *x3* and of *U1(x1), U2(x2)* and *U3(x3)* as well as the optimal sum of the utilities *U1(x1)* + *U2(x2)* + *U3(x3).*

Provide interpretation to the differences in the optimal results – in particular, observe how the different optimal values of the variables vary as the capacity limitation change and provide explanation. Consider TCP (non-real-time) versus UDP (real-time) connections.  And explain their relationships to utility functions of different services. When you upload the Excel file“utility\_Q4”, keep the last value of 2 for the total capacity limitation.

**Question 5 (20 marks)**

Consider a telecommunications provider that offers a connection service between two points through one link.

In this question, we will use units of 10 Gb/s, so the capacity of the link available to the provider is equal to , where

7 if N1 4,

if 7 > N1 4,

and if N1 .

Various enterprises will bid for capacity on this link and the provider will choose the set of enterprises that will receive the service so that its revenue is maximized. That is, the objective of the provider is to maximize the total monthly income it receives from the enterprises. The following table provides a list of bids made by four enterprises that includes capacity requirements in units of 10 Gb/s and monthly payment offers in units of 1000 dollars. Note that the bids are confidential during the bidding process, so that one bidder does not know the bids made by other bidders. After the provide makes the final decision on the choice of the set of enterprises that will receive the service, all the chosen enterprises receive their bitrate capacity in full and those who are not chosen received nothing.

|  |  |  |
| --- | --- | --- |
| Enterprise | Bitrate capacity | Monthly payment |
| 1 | 2 | 17 |
| 2 | 6 | 13 |
| 3 | 3 | 16 |
| 4 | 1 | 18 |

**5.1** **(4 Marks)** Formulate this problem as a 0,1 knapsack problem.

**5.2.** **(4 Marks)** Solve this problem using Excel solver. Call the Excel file “knapsack\_Q5”.

**5.3.** **(8 Marks)** Provide a greedy algorithm for this problem, prove that it gives an optimal solution, use it to solve the problem, and compare your result with the result of 5.2.

**5.4** **(4 Marks)** Provide a dynamic programming formulation **for this problem**. That is, provide: optimal value function, recurrence relation and boundary conditions. Notice the bolded words “for this problem” so do not use general terms like “weight” or the general parameter *N*.

**Question 6 (10 marks)**

In no more than 100 words, discuss limitations of Excel Solver in solving practical problems. In your discussion, consider the fact that Excel Solver is based on mathematical optimization models and algorithms and the limitations of these models and algorithms apply also to Excel Solver.

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