

5. ①  $\text{curry} \triangleq \lambda f. \lambda x. \lambda y. f(x, y)$

(e.g.  $f = \text{fun } (a, b) \rightarrow a + b$ )

$\text{curry } f = \lambda x. \lambda y. f(x, y)$   
 $= \lambda x. \lambda y. (x + y)$

take in  $\rightarrow$  out put 的新 f'

这是帮你理解不用写。

②  $\text{uncurry} \triangleq \lambda f. \lambda p. f(\text{first } p) (\text{second } p)$

所以  $\text{curry}$  也是个  $\lambda$  term.  
 take in  $\rightarrow$  f, out put  $\rightarrow$  新 f

4.

a)

$\text{succ} \triangleq \lambda \hat{n} f x. f \hat{n} (\hat{n} f x)$

$(\text{succ } \hat{n}) \rightarrow (\lambda \hat{n} f x. f \hat{n} (\hat{n} f x)) \hat{n}$

$\rightarrow \lambda f x. f \hat{n} (\hat{n} f x)$

$\rightarrow \lambda f x. f \hat{n} ((\lambda f x. f \hat{n-1} (\hat{n-1} f x)) f x)$

$\rightarrow \lambda f x. f \hat{n} f \hat{n-1} (\hat{n-1} f x)$

$\rightarrow \lambda f x. f \hat{n} f \hat{n-1} f \dots (f \circ x)$

只是帮你理解不用写到 LaTeX.

b)  $\text{Pred} \triangleq \lambda \hat{n}. \hat{n} (\lambda x y. x) \hat{o}$



比如

$$\text{易知 } \hat{n} = \lambda f x. f \hat{n} (\hat{n} + x) \quad \forall n \geq 1.$$

这点你自己可以验证 比如.

$$(\hat{n} = \lambda f x. f \hat{n} (f \hat{o} x) = \lambda f x. f \hat{n} (\hat{n} + x))$$

所以

$$\begin{aligned} \text{Pred } \hat{n} &= (\lambda \hat{n} f x. \hat{n} (\lambda x y. x) x) \hat{n} \\ &= \lambda f x. \hat{n} (\lambda x y. x) x \\ &= \lambda f x. [(\lambda f x. f \hat{n} (\hat{n} + x)) (\lambda x y. x) x] \\ &= \lambda f x. [(\lambda x y. x) \hat{n} (\hat{n} + x)] \\ &= \lambda f x. \hat{n} \end{aligned}$$

帮你理解.

所以

$$\begin{aligned} \text{Pred } \hat{n} &= (\lambda \hat{n}. \hat{n} (\lambda x y. x) \hat{o}) \hat{n} \\ &= \hat{n} (\lambda x y. x) \hat{o} \\ &= (\lambda f x. f \hat{n} (\hat{n} + x)) (\lambda x y. x) \hat{o} \\ &= (\lambda x y. x) \hat{n} (\hat{n} + \hat{o}) \\ &= \hat{n} \quad (\forall n \geq 1) \end{aligned}$$

当  $n = \hat{o}$

$$\begin{aligned} \text{Pred } \hat{o} &= \hat{o} (\lambda x y. x) \hat{o} \\ &= (\lambda f x. x) (\lambda x y. x) \hat{o} = \hat{o} \end{aligned}$$

c)

isZero  $\triangleq \lambda \hat{n}. \hat{n} (\lambda xy. \text{false}) \text{true}$

3、

就是在  $\beta$ -reduction (full) 下,

可以不记顺序的 reduce

不一定挨着在 reduce 到 "V"

正确如下:

[BETA]  $\frac{}{(\lambda x. e_0) e_1 \rightarrow e_0 \{e_1/x\}}$

[BODY]  $\frac{e \rightarrow e'}{\lambda x. e \rightarrow \lambda x. e'}$

[Left]  $\frac{e_1 \rightarrow e_1'}{e_1 e_2 \rightarrow e_1' e_2}$

[Right]  $\frac{e_2 \rightarrow e_2'}{e_1 e_2 \rightarrow e_1 e_2'}$



$$e_0 = x \quad e = y$$

2. a) counter example  $\lambda y. x \{y/x\}$   
 (这题) correct:  $\lambda z. \{x \{z/y\} \{y/x\}\} = \lambda z. y$   
 你写的) wrong:  $\lambda y. x \{y/x\} = \lambda y. (x \{y/x\}) = \lambda y. y$

b) counter example  $(\lambda y. z) \{y/x\}$   
 这题 correct:  $= \lambda t. (z \{t/y\} \{y/x\}) = \lambda t. z$   
 你、correct 的 'm' 笔 wrong:  $= \lambda z. (z \{z/y\} \{y/x\}) = \lambda z. z$   
 误成 "n" 了, 改过来.

c) counter example  $(\lambda y. x) \{zy/x\}$   
 这题(c) correct:  $= \lambda t. (x \{t/y\} \{zy/x\}) = \lambda t. zy$   
 你写的 wrong:  $= \lambda z. \{x \{z/y\} \{zy/x\}\} = \lambda z. zy$

$$1. (\lambda x. \lambda y. xy) / ((\lambda x. \lambda z. xz) (\lambda y. y))$$

$$\rightarrow (\lambda x. \lambda y. xy) (\lambda z. (\lambda y. y) z)$$

CBV

$$\rightarrow \cancel{(\lambda x. \lambda y. xy)}$$

$$\lambda y. (\lambda z. (\lambda y. y) z) \quad y$$

原式

$$CBN \rightarrow \lambda y. ((\lambda x. \lambda z. xz) (\lambda y. y)) \quad y$$

注, 此处不能再 Step!

已经是 lambda Abstraction, 所以  
 你得改!