Nowcasting Under Structural Breaks

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Overview

- Fore/Now-casting GDP is useful
- GDP is observed at a low frequency
- We can use a factor model for x-casting
- Observed macro, survey, etc. data are often non-stationary
- Rolling windows are a naive way of dealing with this

We will

- Use a piecewise-stationary model
- Identify structural breaks with a new algorithm
- Use estimated break points intelligently for prediction

Model: Factors

For observations $\boldsymbol{X}_t \in \mathbb{R}^d$ we consider the factor model

$$\boldsymbol{X}_t = \chi_t + \varepsilon_t := \boldsymbol{\Lambda} \boldsymbol{F}_t + \varepsilon_t, \quad t = 1, \ldots, n \tag{1}$$

where χ_t and ε_t are common and idiosyncratic components respectively. $\boldsymbol{F}_t \in \mathbb{R}^r$ is an unobservable vector of r factors, $\boldsymbol{\Lambda} \in \mathbb{R}^{d \times r}$ is a fixed matrix of loadings, and ε_t are white noise. $y_{i,t} = \lambda_i^y F_t + \varepsilon_{i,t}^y$ is nowcasted series, observed at e.g. $t = 1, 4, 7, \ldots,$

Model: Piecewise-VAR

Factors follow $VAR(p)$ with structural changes

$$F_t = \begin{cases} \boldsymbol{\Phi}_1 \mathbb{F}_{t-1} + \boldsymbol{\eta}_t, & k_0 = 1 \le t \le k_1 \\ \boldsymbol{\Phi}_2 \mathbb{F}_{t-1} + \boldsymbol{\eta}_t, & k_1 < t \le k_2 \\ \dots & \\ \boldsymbol{\Phi}_{q+1} \mathbb{F}_{t-1} + \boldsymbol{\eta}_t, & k_q < t \le k_{q+1} = n \end{cases}$$

$$\boldsymbol{\Phi}_j = \begin{pmatrix} \boldsymbol{\Phi}_j^T(1) \\ \dots \\ \boldsymbol{\Phi}_j^T(r) \end{pmatrix} \in \mathbb{R}^{r \times rp} \qquad \mathbb{F}_{t-1} = \begin{bmatrix} \mathbb{F}_{t-1}(1) \\ \dots \\ \mathbb{F}_{t-1}(r) \end{bmatrix} \in \mathbb{R}^{rp}$$

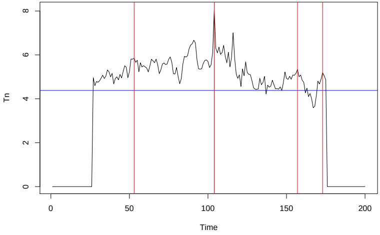
where $\mathbb{F}_{t-1}(i) = [F_{t-1}(i), \dots, F_{t-p}(i)]^T$, and $\boldsymbol{\eta}_t$ are also white noise. Change points are $k_j, j = 0, 1, \dots, q, q+1$ are unknown, to be estimated.

Change Point Analysis

- (1) identify the factor series using Principal Components Analysis (PCA) on $\boldsymbol{X}_t$
- (2) select first $\hat{r}$ components; $\hat{r}$ may be determined through some Information Criterion
- (3) run the `mosumvar` algorithm on the resulting series

Change Point Analysis: mosumvar

Input: series $\{\boldsymbol{F}_t, t = 1, \ldots, n\}$, bandwidth G
At each $k = G, \ldots, n - G$ calculate a test statistic T_k, comparing parameter estimates over window $k - G + 1, \ldots, k$ to window $k + 1, \ldots, k + G$. These will be large at change points, and small elsewhere. Use local maxima of series $\{T_k\}$ as location estimates.
Output: Test outcome, Change point estimates



Change Point Analysis: Forecasting

Having identified change points, how should we make predictions? We consider, for estimation:

- (A) Using the entire sample (problem: multiple regimes)
- (B) Using only the most recent segment (problem: few observations)
- (C) Pooled forecast: Average predictions made by estimating over all intervals between (A) and (B)

Simulations

- `mosumfactorvar` works well in contrived setting
- For certain types of change, performance is less good - have an idea of how to overcome this (future work, more of mathematical interest)

Illustration of forecast comparison

METHOD	(A)	(B) oracle	(B)	(C) oracle	(C)
ERROR	1.153	1.236	1.238	0.739	0.737

Application: Data

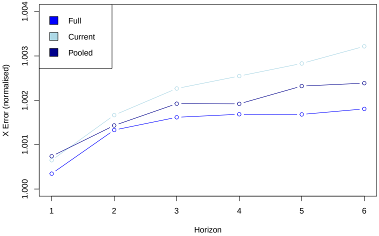
Database combining NYFED and FRED-MD dbs for $d = 147$.
Panel components transformed for stationarity. Use GDP transform

$$y_t = \frac{GDP_t - GDP_{t-12}}{GDP_{t-12}} - \frac{GDP_{t-3} - GDP_{t-15}}{GDP_{t-15}}$$

We want $\hat{y}_{t+h|t}$ for $h = 0, 1, \ldots, 6$, corresponding to nowcasting and one- to six-step ahead forecasting.

Application: x-casts

Headline: pooled forecast makes noticeable improvement for $\hat{y}_{t+1|t}$.



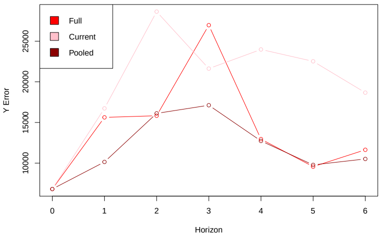


Figure: Forecasts for the panel (left) and GDP (right) up to a 6-step horizon, using different estimation methods

Application: Classifying Recessions

Classification: $-sign(y_t)$

Want to control false positives, i.e flagging a recession when there is none, so use $F_{0.5} = \frac{1.25 \cdot precision \cdot recall}{0.25 \cdot precision + recall}$ measure.

Paucity of data (lack of positive examples, so # TP is low or zero) makes comparison/assessment difficult, still working on this

To Do

- Mathematical proofs (good start, less urgent)
- Simulations: compare to existing methods, challenging change types
- Online change point detection (trivial in practice, needs mathematical justification)
- Classification
- ...