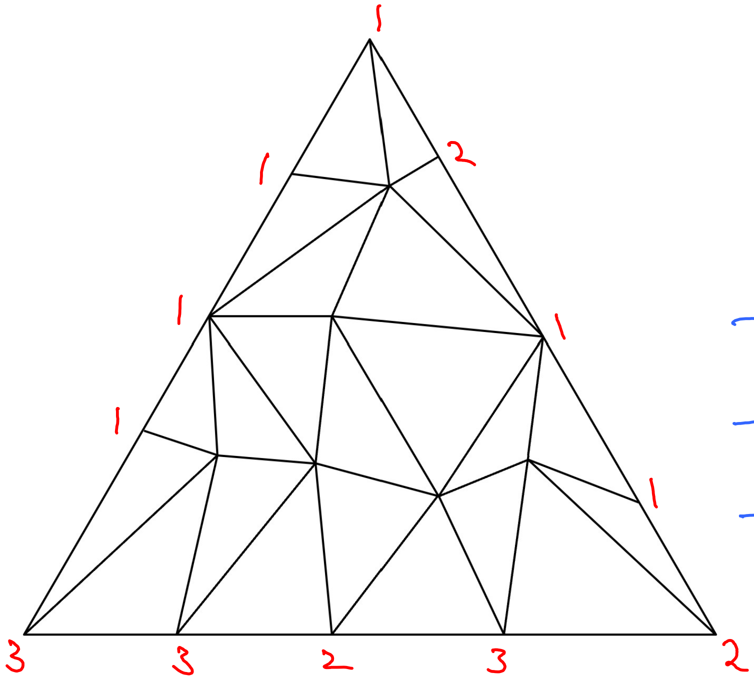
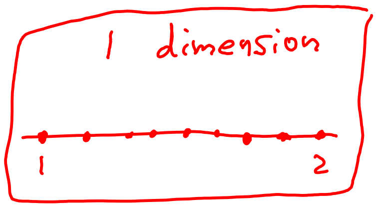
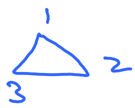
Sperner's Lemma:



Sperner labelling of a triangulation of a triangle T.

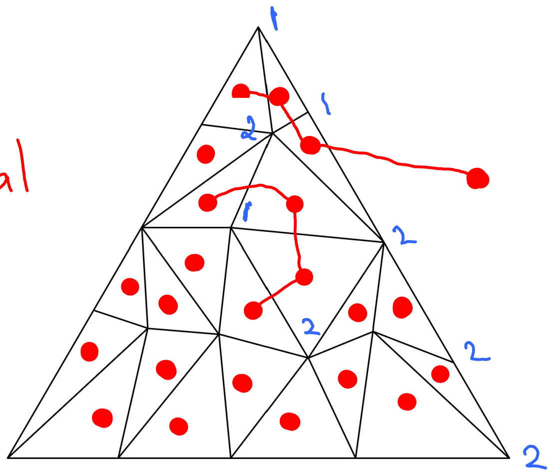
- Labelling of all vertices with 1, 2, 3.
- Vertices of T labelled 1, 2, 3.
- Vertices on side ij labelled i or j

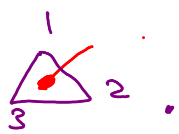


Sperner's Lemma: Cannot avoid 

Proof: We prove that the number of triangles with 3 distinct labels is odd.

H is subgraph of dual graph with edges only for edges labelled 1, 2



For internal vertex x, $\deg(x) = 1$ iff x is 

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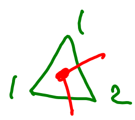
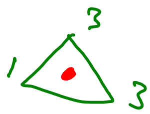
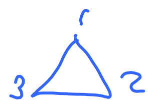
Other vertices have degree 0 or 2. 



External vertex has odd degree.

$\Rightarrow$ number of internal vertices with odd degree is odd.

$\Rightarrow$ number of internal vertices with degree 1 is odd $\Longrightarrow$ 

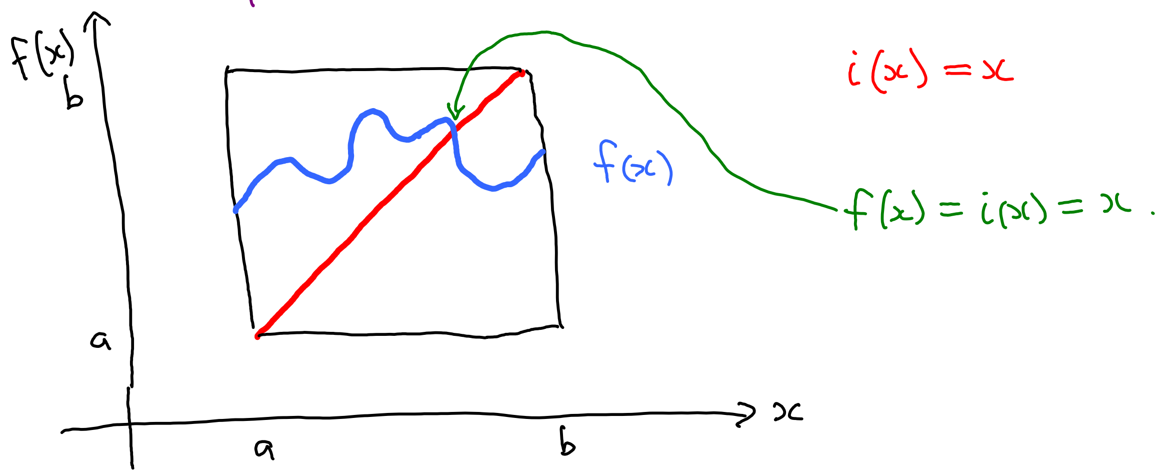
$\square$

Sperner's Lemma generalises to n-dimensions.

Can be used to prove Brouwer's Fixed Point Theorem (see notes).

BFPT: Every continuous function $f: B_n \to B_n$ has a fixed point ($x \in B_n$ such that $f(x) = x$), where B_n is any closed ball in $\mathbb{R}^n$.

1 dimension: Every continuous function $f: [a,b] \to [a,b]$ has a fixed point.



2 dimensions: Every continuous function f:  $\to$  has a fixed point.