Stat3004 Assignment 3

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Q1

First we note that $N((a,b]) \sim Poi(2 \cdot (b-a))$.

a)

$$\begin{aligned}
\mathbb{P}(N_{t_1} = n_1, N_{t_2} = n_2) &= \mathbb{P}(N_2 = 7, N_{12} = 10) \\
&= \mathbb{P}(N_2 = 7, N_{12-2} = 10 - 7) \\
&= \mathbb{P}(N_2 = 7)\mathbb{P}(N_{10} = 3) \\
&= e^{-2\cdot 2}\frac{(2\cdot 2)^7}{7!} e^{-2\cdot 10}\frac{(2\cdot 10)^3}{3!} \\
&\approx 1.636291121 \times 10^{-7}
\end{aligned}$$

b)

$$\begin{aligned}
\mathbb{P}(N(1,t_1] = n_1, N(t_1 - 1, t_2] = n_2) &= \mathbb{P}(N(1,2] = 7, N(1,12] = 10) \\
&= \mathbb{P}(N(1,2] = 7, N(2,12] = 3) \\
&= \mathbb{P}(N(1,2] = 7)\mathbb{P}(N(2,12] = 3) \\
&= \frac{e^{-2}(2)^7}{7!}\frac{e^{-20}(20)^3}{3!} \\
&\approx 9.4458178806 \times 10^{-9}
\end{aligned}$$

c)

$$\begin{aligned}
\mathbb{E}[N(1,t_1] \mid N(t_1-1,t_2]=n_2] &= \mathbb{E}[N(1,2] \mid N(1,12]=10] \\
&= \sum_{t\geq 0} t\cdot \mathbb{P}(N(1,2]=t|N(1,12]=10) \\
&= \sum_{t=0}^{10} t\cdot \frac{\mathbb{P}(N(1,2]=t, N(1,12]=10)}{\mathbb{P}(N(1,12]=10)} \\
&= \frac{1}{\mathbb{P}(N(1,12]=10)} \sum_{t=0}^{10} t\cdot \mathbb{P}(N(1,2]=t, N(2,12]=10-t) \\
&= \frac{1}{\mathbb{P}(N(1,12]=10)} \sum_{t=0}^{10} t\cdot \mathbb{P}(N(1,2]=t)\mathbb{P}(N(2,12]=10-t) \\
&= \frac{1}{\frac{e^{-22}(22)^{10}}{10!}} \sum_{t=0}^{10} t\cdot \frac{e^{-2}(2)^t}{t!}\frac{e^{-20}(20)^{10-t}}{(10-t)!} \\
&= \frac{10}{11}
\end{aligned}$$

d)

$$\begin{aligned}
\mathbb{E}[N(t_1-1,t_2] \mid N(1,t_1]=n_1] &= \mathbb{E}[N(1,12] \mid N(1,2]=7] \\
&= \sum_{t\geq 0} t\cdot \mathbb{P}(N(1,12]=t|N(1,2]=7) \\
&= \sum_{t\geq 0} t\cdot \mathbb{P}(N(1,12]=t|N(1,2]=7) \\
&= \sum_{t\geq 0} t\cdot \frac{\mathbb{P}(N(1,12]=t, N(1,2]=7)}{\mathbb{P}(N(1,2]=7)} \\
&= \sum_{t\geq 0} t\cdot \frac{\mathbb{P}(N(2,12]=t+7, N(1,2]=t)}{\mathbb{P}(N(1,2]=7)}
\end{aligned}$$

Q2

Q3

a)

b)

c)

Q4

a)

b)

c)

d)