1 Cyclic polygon expansion plane

The plane that the cyclic polygon expands into can be found as follows:
We define the dihedral atom sequence $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ with the angles $\alpha = \angle ABC, \beta = \angle BCD$ and the dihedral $\varphi = \angle ABCD$ with the following conditions:

$$|AB| = a, |BC| = b, |CD| = c$$

$$\alpha, \beta \in [0, \pi], \varphi \in [-\pi, \pi]$$

We arbitrarily choose the following positions for the points:

$$\vec{A} = \mathbf{R}_z(\alpha) a \vec{e}_x \tag{1}$$
$$\vec{B} = \vec{0} \tag{2}$$
$$\vec{C} = b \vec{e}_x \tag{3}$$
$$\vec{D} = \mathbf{R}_x(\varphi) \left[\vec{C} + \mathbf{R}_z(\pi - \beta) c \vec{e}_x \right] \tag{4}$$

where $\vec{e}_x$ is the unit vector along the x axis and $\mathbf{R}_i$ is the rotation matrix along the axis i.
The dihedral distance is formed by the line segment $\overline{AD}$.
In order to find the shortest distance between the line segments $\overline{AD}$ and $\overline{BC}$, we define:

$$\overline{BC} : \vec{r}(\lambda) = \vec{B} + \lambda \left(\vec{C} - \vec{B} \right) = \lambda \vec{C}, \lambda \in [0, 1] \tag{5}$$
$$\overline{AD} : \vec{s}(\mu) = \vec{A} + \mu \left(\vec{D} - \vec{A} \right), \mu \in [0, 1] \tag{6}$$

The shortest distance between the line segments must be orthogonal to both line segments' direction vectors:

$$\left[\lambda \vec{C} - \left(\vec{A} + \mu (\vec{D} - \vec{A}) \right) \right] \cdot \vec{C} = 0 \tag{7}$$
$$\left[\lambda \vec{C} - \left(\vec{A} + \mu (\vec{D} - \vec{A}) \right) \right] \cdot \left(\vec{D} - \vec{A} \right) = 0 \tag{8}$$

Solving this system of equations yields:

$$\mu_0 = -\frac{A_y S_y + A_z S_z}{S_y^2 + S_z^2} \tag{9}$$
$$\lambda_0 = \frac{A_x + \mu_0 S_x}{b} \tag{10}$$
$$\text{with } \vec{S} = \vec{D} - \vec{A} \tag{11}$$

These do not satisfy their conditions yet:

$$\lambda_m = \min\left(\max(\lambda_0, 0), 1\right) \tag{12}$$
$$\mu_m = \left\{ \begin{array}{c|c} -\frac{A_x}{S_x} & \lambda_0 \leq 0 \\ \frac{b - A_x}{S_x} & \lambda_0 \geq 1 \\ \lambda_0 & \text{else} \end{array} \right\} \tag{13}$$

The plane in which the cyclic polygon expands can then be defined via the three points $\vec{A}, \vec{D}$, and $\vec{r}(\lambda_m)$.
For $\varphi = \pm\pi$, these points become collinear and the plane definition is invalid. Under these circumstances, the plane can instead be defined using the point $\vec{A}$ and the two vectors $\left(\vec{D} - \vec{A} \right)$ and $\vec{e}_z$.