1 Spiro-atom angle modeling

The cross-angle in a spiro-atom is expliticly modeled if the disjoint cycles are both small, i.e. of size ≤ 5 . Only then is the additional variance when calculating cross-angle bounds particularly harmful for the overall 3D structure. We define:

$$\vec{a} = R_z \left(\frac{\alpha}{2}\right) \begin{pmatrix} 1\\0\\0 \end{pmatrix} = \begin{pmatrix} \cos\frac{\alpha}{2}\\\sin\frac{\alpha}{2}\\0 \end{pmatrix} \tag{1}$$

$$\vec{b} = R_y \left(\frac{\beta}{2}\right) \begin{pmatrix} -1\\0\\0 \end{pmatrix} = \begin{pmatrix} -\cos\frac{\beta}{2}\\0\\\sin\frac{\beta}{2} \end{pmatrix} \tag{2}$$

Where R_y and R_z are the standard three-dimensional rotation matrices, α is the cycle internal angle for the first cycle at the spiro-atom, and β is the cycle internal angle for the second cycle at the spiro-atom.

The angle between these vectors ϕ is then:

$$\vec{a}\vec{b} = ||\vec{a}||||\vec{b}||\cos\phi \tag{3}$$

$$\phi = \arccos\left\{ \left(\cos\frac{\alpha}{2}\right) \left(-\cos\frac{\beta}{2}\right) \right\} \tag{4}$$