Sample Code - Mathematica

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Introduction

This sample code involves two fragments of the separate codes, both of them related to my BSc dissertation about the charge-resonant-enhanced-ionization and quantum bridges in phase space.

Calculations involving specified tridiagonal determinant of an arbitrary size

Defining the tridiagonal determinant

```
In[•]:= γ := -1/2

In[•]:= α := -116563/9531

In[•]:= maxbeta := 4.707

In[•]:= μ[α_, β_] := 1/4 * (α * (α + 2) + 2 α β - β (β + 1))

In[•]:= upperdiagonal[R_, beta_] := Table[(r - 1) * (r - 1 + beta), {r, 2, R}]

In[•]:= diagonal[R_, alpha_, beta_, gamma_, mu_] :=
         Table[mu - (r - 1) * (r + beta + gamma) + (r - 1) * (alpha), {r, 1, (R), 1}]

In[•]:= lowerdiagonal[R_, alpha_] := Reverse[Table[(r), {r, alpha, (R - 1) * alpha, alpha}]]

In[•]:= MM[alpha_, beta_, gamma_, mu_, Rterminate_] :=
         Total[{DiagonalMatrix[lowerdiagonal[Rterminate, alpha], -1],
           DiagonalMatrix[diagonal[Rterminate, alpha, beta, gamma, mu]],
           DiagonalMatrix[upperdiagonal[Rterminate, beta], 1]}]

In[•]:= MM[A, β, Γ, M, 3] // MatrixForm
Out[•]//MatrixForm=
```

$$\begin{pmatrix} M & 1+\beta & 0 \\ 2A & -2+A-\beta-\Gamma+M & 2\,(2+\beta) \\ 0 & A & 2A-2\,(3+\beta+\Gamma)+M \end{pmatrix}$$

```
(* Tri-diagonal matrix reported by Downing (2013)
  to specify one of the conditions for the Heun's power series
  termination to a polynomial of degree Rterminate (see Downing,
   C.A. (2013), Journal of Mathematical Physics,54(7),072101; page 7) *)
```

Finding the determinant fulfilling certain conditions

```
In[•]:= Select[Select[Select[Values@Flatten[Solve[Det[MM[α, β, γ, μ[α, β], 9]] == 0, β]] ,
    Im[#] == 0 &], # > 0 &], # < maxbeta &]

Out[•]= {0.310..., 2.11...}

(* I am searching for such β values that make determinant of a matrix
  0 and in addition which are real and fulfilling 0 < β < maxbeta  *)
```

Wigner quasi-probability distribution in position-momentum phase space by means of numerical integration

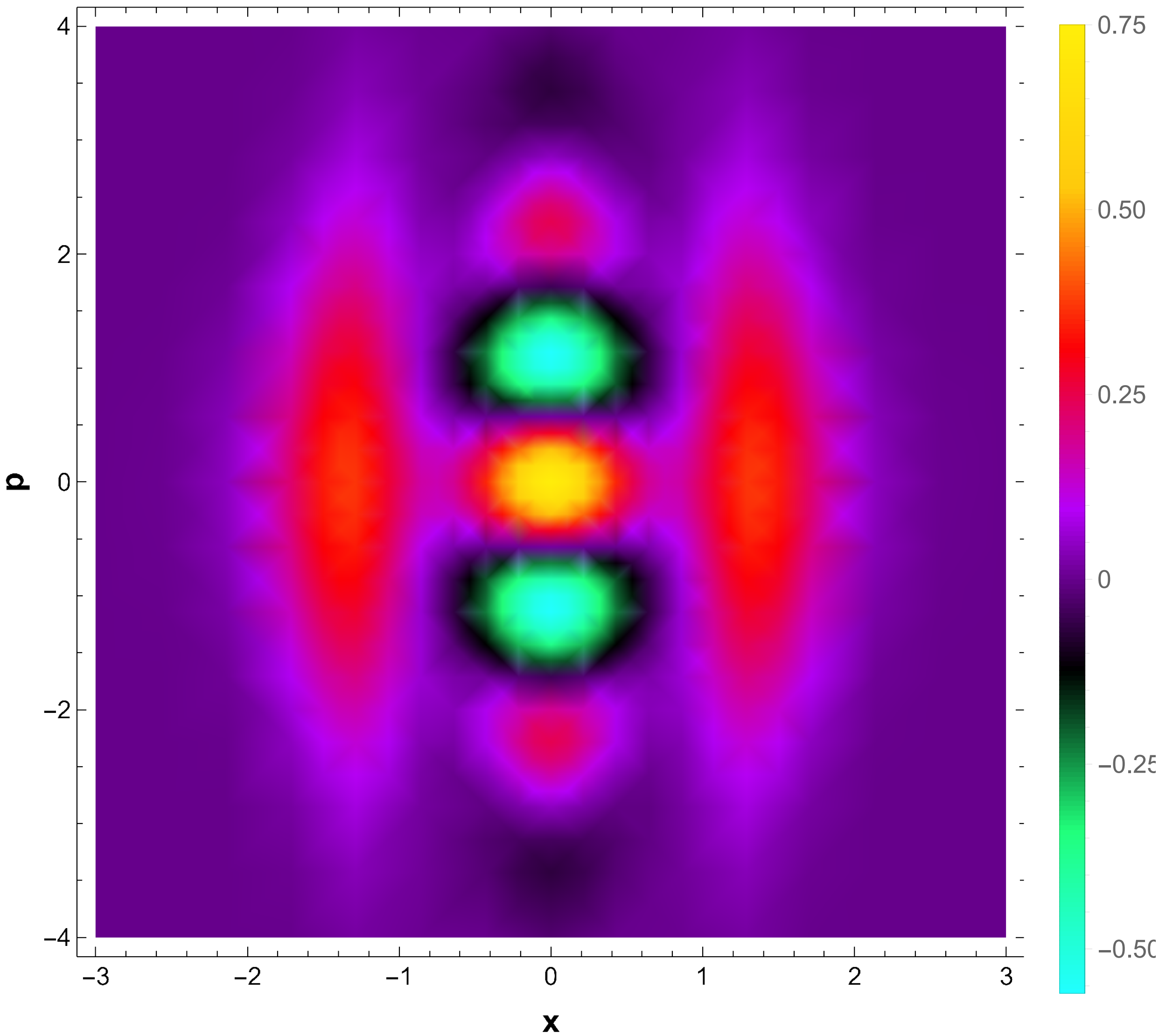
```
In[•]:= ψdelocalized[ξ_, c_, Ω_] :=
```

$$\frac{1}{\sqrt{-\text{ExpIntegralE}[-2\,c\,\Omega,\ 2\,c] + 4^{-c\,\Omega}\,c^{-2\,c\,\Omega}\,\Omega\,\text{Gamma}[2\,c\,\Omega]}} * \xi\,\hat{}\,(c\,\Omega) * \text{Exp}[-c\,\xi]$$

```
(* Wavefunction to be plotted in phase-space *)

In[•]:= cf[u_] =
  Blend[{RGBColor[0.13192950331883727, 0.9998168917372396, 0.9990386816205081],
    RGBColor[0.1300984206912337, 0.9975127794308385, 0.481177996490425],
    RGBColor[0., 0., 0.], RGBColor[0.7097581445029374, 0., 0.970443274586099],
    RGBColor[0.9859616998550393, 0., 0.02696269169146258],
    RGBColor[0.9957122148470283, 0.7894560158693827, 0.04928664072632944],
    RGBColor[0.998992904554818, 0.9320515754940109, 0.03978027008468757]}, u]

••• Blend: u should be a real number or a list of non−negative numbers, which has the same length as {■, ■, ■, ■, ■, ■, ■}.

Out[•]= Blend[{■, ■, ■, ■, ■, ■, ■}, u]

(* Home-made colormap; *)
```

In[•]:=
```
iF4[δ_, x_, p_, c_, Ω_] :=
  NDSolveValue[{F3'[δ] == 1/(2*Pi) * ψdelocalized[1/(Cosh[x + δ])^2, c, Ω] * ψdelocalized[
      1/(Cosh[x - δ])^2, c, Ω] * Exp[2 ⅈ p δ], F3[0] == 1}, F3, {δ, -10 000, +10 000}];
wignerplot = DensityPlot[Re[iF4[δ, x, p, 7, 1/4][10 000] -
     iF4[δ, x, p, 7, 1/4][-10 000]], {x, -3, 3}, {p, -4, 4},
  ImageSize → Large, PlotRange → All, ImageSize → Large, ColorFunction → cf,
  PlotLegends → {LabelStyle → FontSize → 14},
  FrameLabel → {Style["x", 18, Black, Bold], Style["p", 18, Black, Bold]},
  FrameTicksStyle → {14, 14}]
```

Out[•]=




```
(* Wigner quasi-probability plot above *)

(* Note that the above way of calculating the definite integral is several
 orders of magnitude quicker compared to naively using NIntegrate! *)
```