



Day, Feb 2020

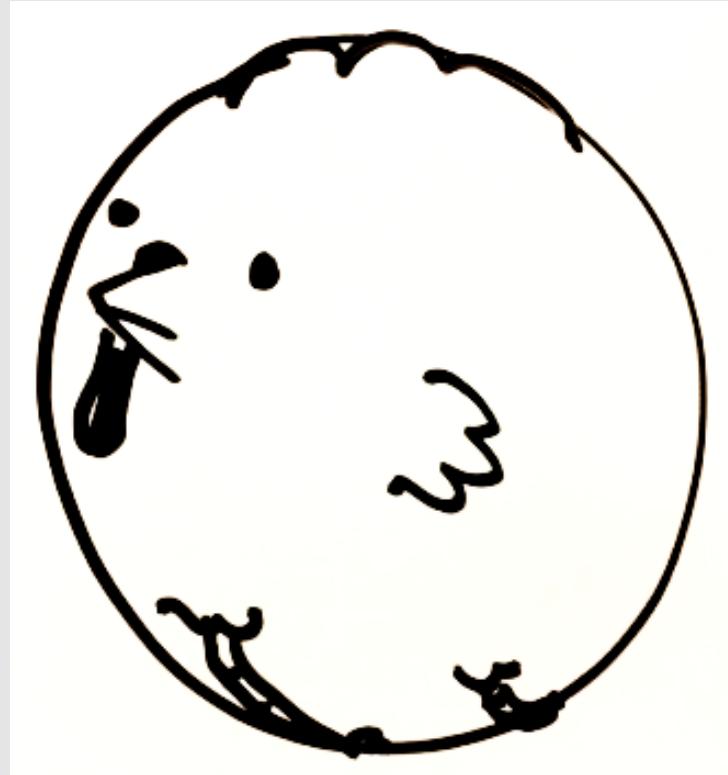
How to cook **delicious** spherical chicken? feat. thermodynamics



Dominik Kufel

What is a spherical chicken?

What is a spherical chicken?



What is the story?

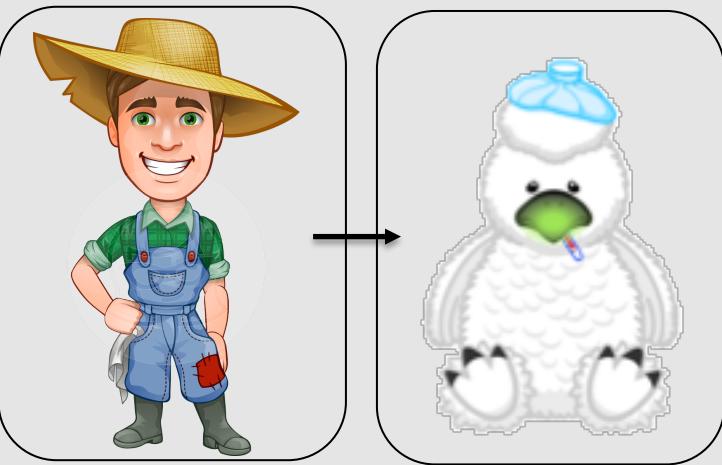
What is the story?



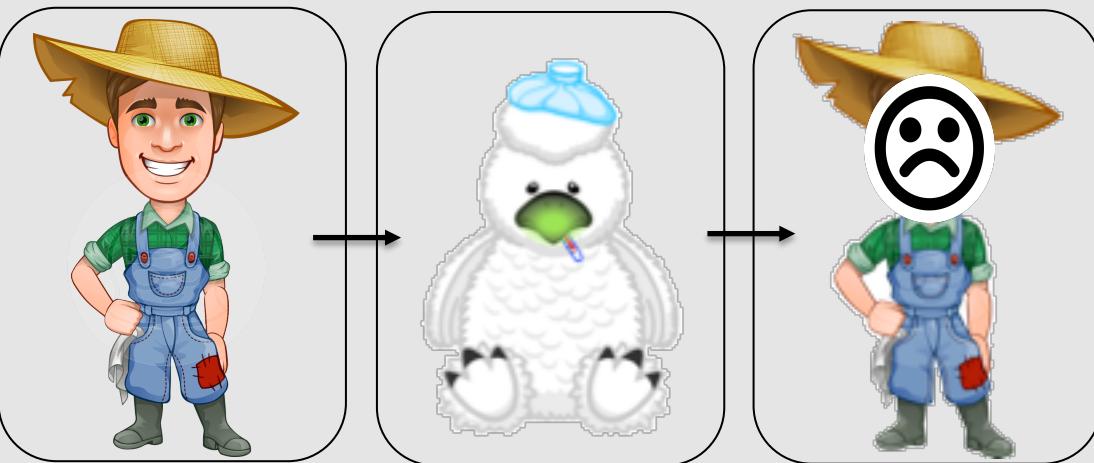
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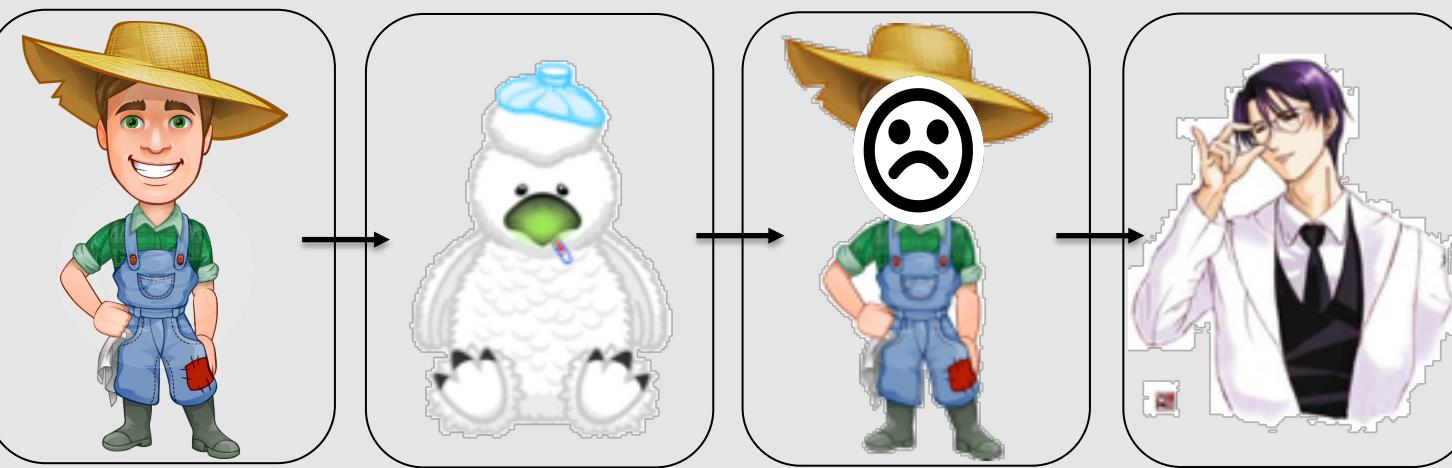
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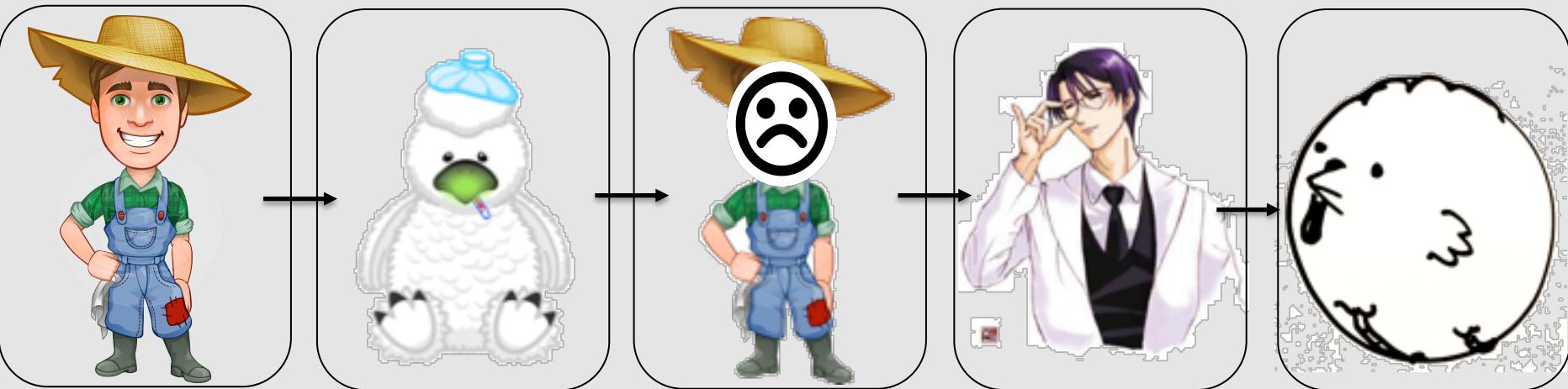
What is the story?



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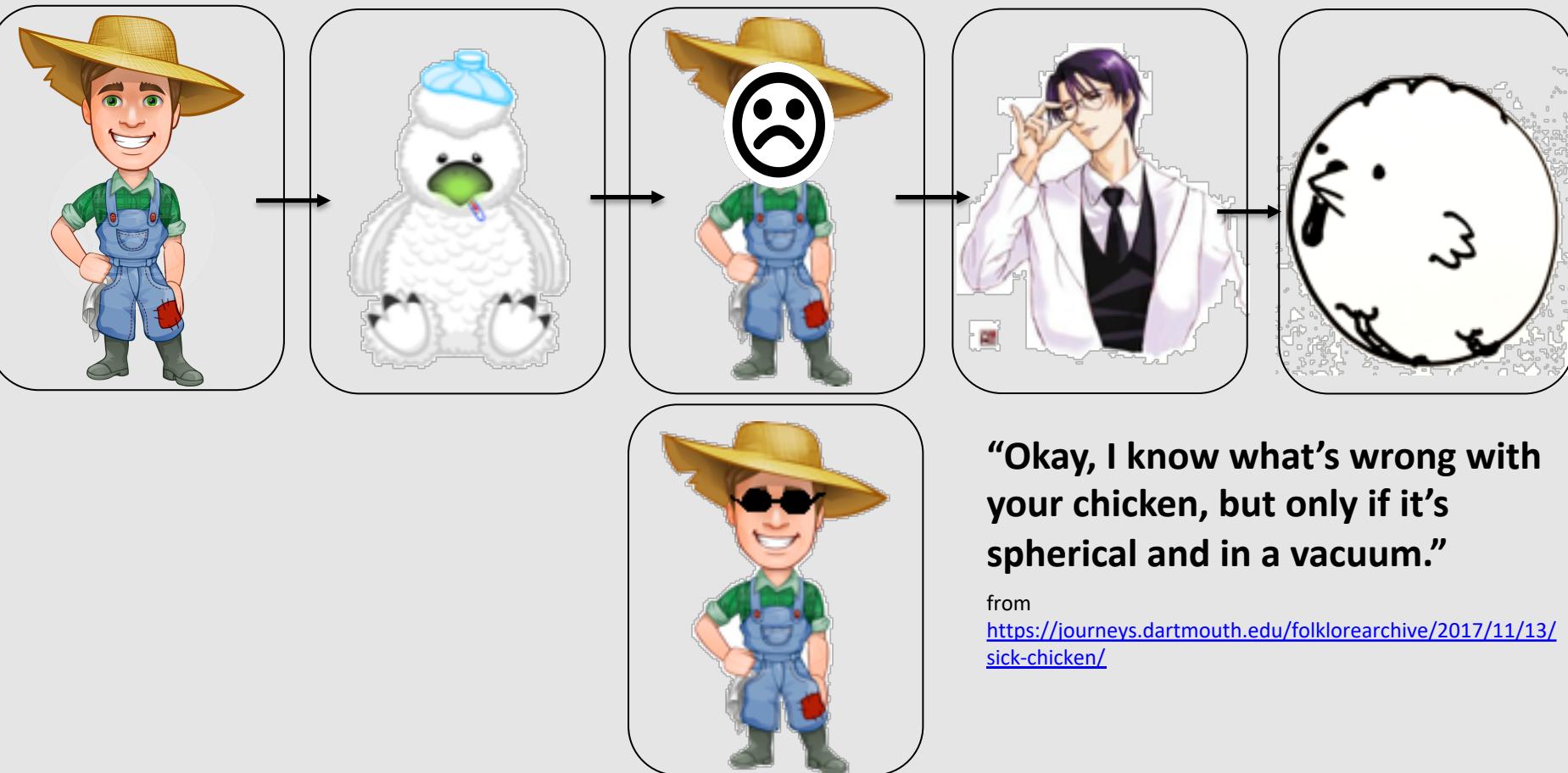


“Okay, I know what’s wrong with your chicken, but only if it’s spherical and in a vacuum.”

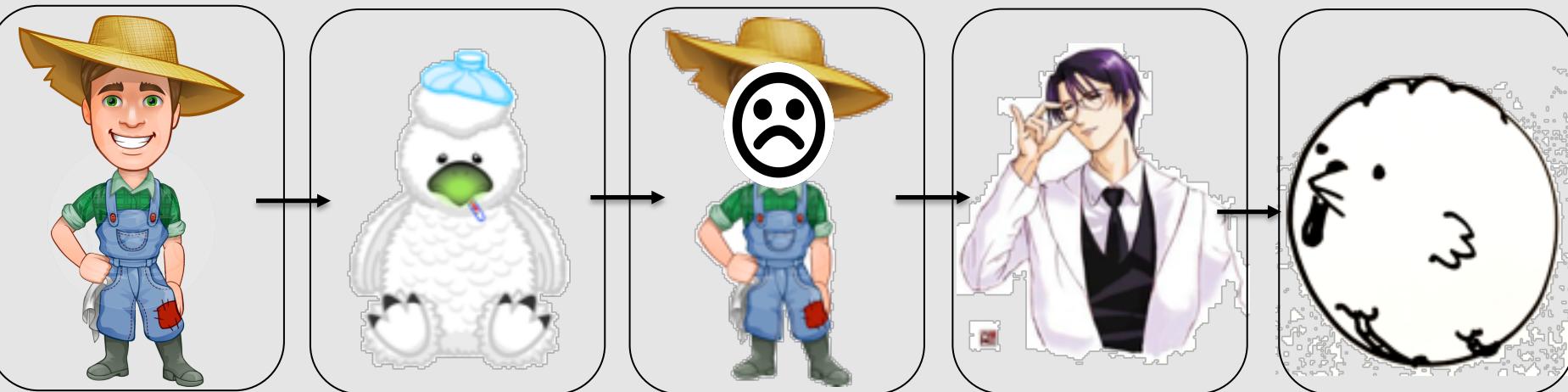
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[https://journeys.dartmouth.edu/folklorearchive/2017/11/13/
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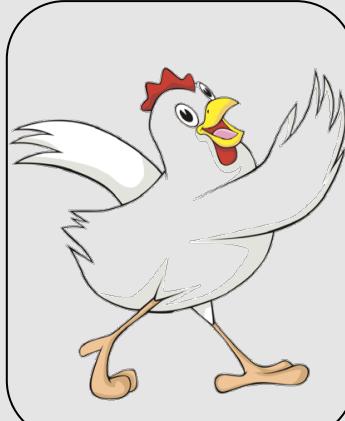
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THE
END



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your chicken, but only if it’s
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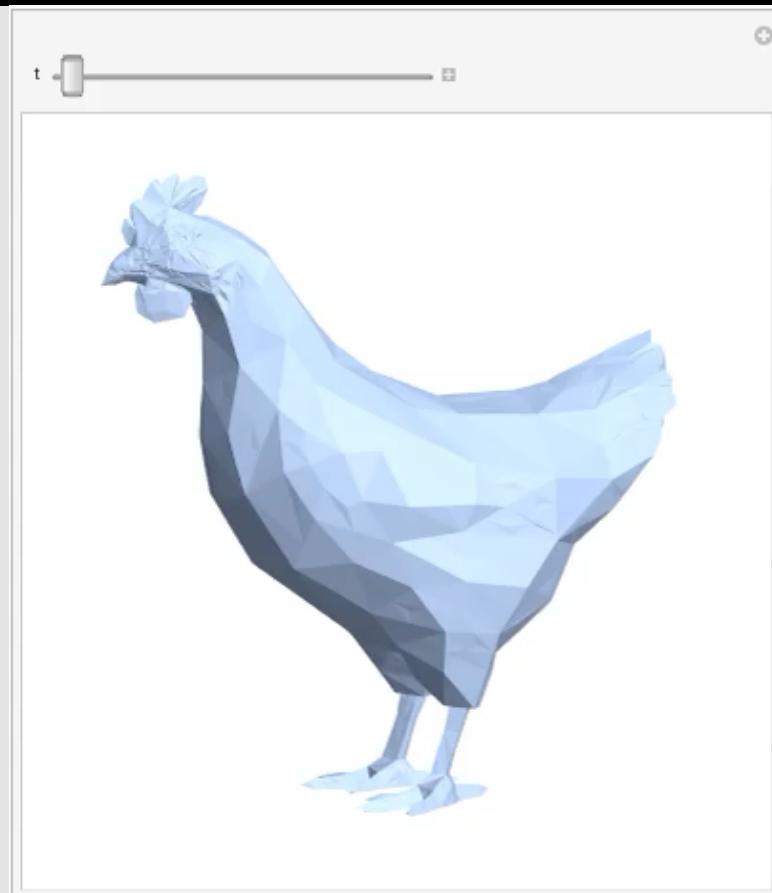
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How to transform normal chicken into spherical chicken? (and inverse transform)

How to transform normal chicken into spherical chicken? (and inverse transform)

**Use Wolfram
Mathematica!!!**



Some spherical chicken related-papers

nature > news & views > article

a nature research journal

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nature

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News & Views | Published: 04 June 1998

Planetary science

Making and braking asteroids

Alan W. Harris

Nature 393, 418–419(1998) | Cite this article

26 Accesses | 8 Citations | 0 Altmetric | Metrics

There is an old joke about the limitations of physics and its need for mathematical abstraction, the punchline of which runs, “We assume a spherical chicken”. In trying to model the collisional evolution of asteroids, however, theorists have been unable to avoid assuming a spherical chicken. To create the present asteroid belt, asteroids must have been undergoing collisions throughout the age of the Solar System, but to make tractable analytical calculations of what happens when one asteroid hits another, one is forced into the unrealistic assumption that the bodies are spherical and rigid.

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Associated Content

Nature | Letter

Disruption of kilometre-sized asteroids by energetic collisions

E. Asphaug, S.J. Ostro [...] W. Benz

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Why you should really bother? Cooking a delicious chicken!!!

Chicken cooking 'law': "Pre-heat your oven to 190°C/375°F/Gas Mark 5. Remove all packaging. Place chicken in a roasting tin in the centre of the oven and cook for 40 minutes per kilo, plus 30 minutes, basting occasionally (refer to guidelines on front of label)."

from <https://groceries.morrisons.com/webshop/product/Morrisons-Whole-Chicken-/232830011>

Result?

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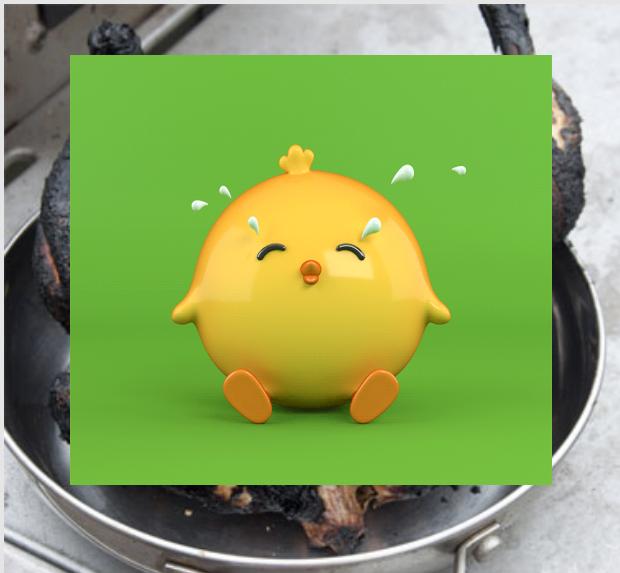
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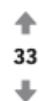
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Goal

Use thermodynamics
to show what's wrong!

What do we care about?



Posted by u/tripostrophe 6 years ago



Every damn time. Pan-fried chicken raw on the inside.



“Cutting into the chicken breast shows that it is definitely not cooked. Don’t eat this!”

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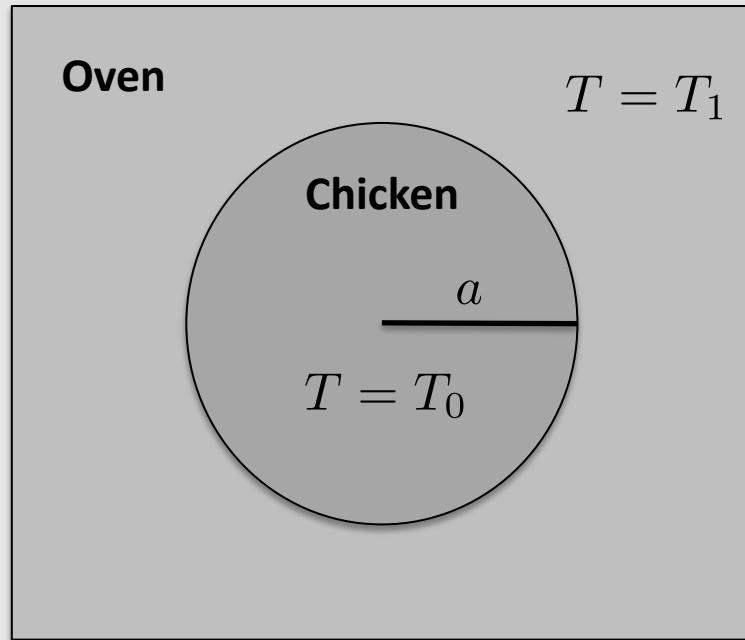


“Cutting into the chicken breast shows that it is definitely not cooked. Don’t eat this!”

Objective: the meat should be well-cooked at the center of the chicken!

Our **delicious** spherical chicken problem

a - chicken radius



Heat equation

$$\frac{\partial T}{\partial t} = D \nabla^2 T$$

If $\frac{\partial T}{\partial t} = 0 \longrightarrow \nabla^2 T = 0$

Laplace's equation!

Crucial in electrostatics,
gravitation, fluid dynamics etc.

In spherical coordinates the solution to the steady-state equation is:

$$T(r) = A + \frac{B}{r}$$

with $A, B = \text{const.}$

Laplace's equation - digression

$$\nabla^2 T = 0$$

Solution in spherical coordinates is:

$$T(r) = A + \frac{B}{r}$$

with $A, B = \text{const.}$

Laplace's equation - digression

$$\nabla^2 V = 0$$

Solution in spherical coordinates is:

$$V(r) = \frac{B}{r}$$

Which is equivalent to:

$$E(r) = \frac{Q}{4\pi\epsilon_0} \frac{1}{r^2}$$

Coulomb's law in electrostatics!

Heat equation with proper symmetry

For a sphere T will have no angular and azimuthal dependence which will come in handy*:

$$\frac{\partial T}{\partial t} = D \nabla^2 T \longrightarrow \boxed{\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = \frac{\partial T}{\partial t}}$$

Heat equation with proper symmetry

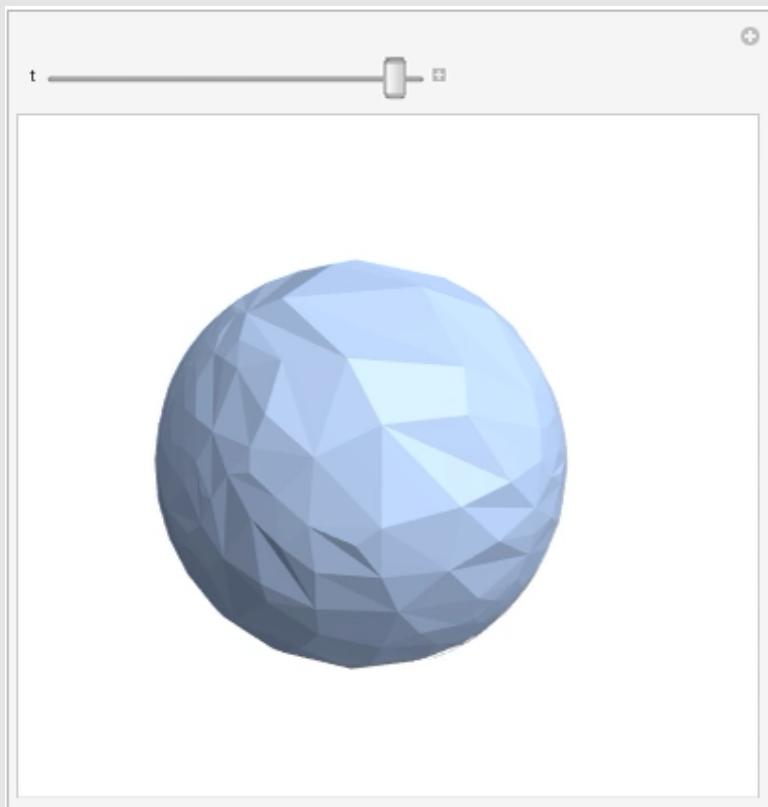
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* however, later we may
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the sphericity
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How to solve the equation?

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) = \frac{\partial T}{\partial t}$$

Subject to boundary conditions:

$$T(r, 0) = T_0 \quad (r < a)$$

$$T(a, t) = T_1$$

A neat trick to simplify the equation

Recall the solution to the steady state:

$$T(r) = A + \frac{B}{r}$$

Generalize it:

$$T(r, t) = T_1 + \frac{B(r, t)}{r}$$

Plug-into the equation:

$$\frac{\partial B}{\partial t} \frac{1}{r} = -\frac{1}{r^2} \cancel{\frac{\partial B}{\partial r}} + \frac{1}{r^2} \cancel{\frac{\partial B}{\partial r}} + r \frac{1}{r^2} \frac{\partial^2 B}{\partial r^2}$$

Hence:

$$\frac{\partial B}{\partial t} = D \frac{\partial^2 B}{\partial r^2}$$

Just a **1D heat equation**

General solution + plug-in of the boundary conditions

Typical technique: separation of variables + superposition principle

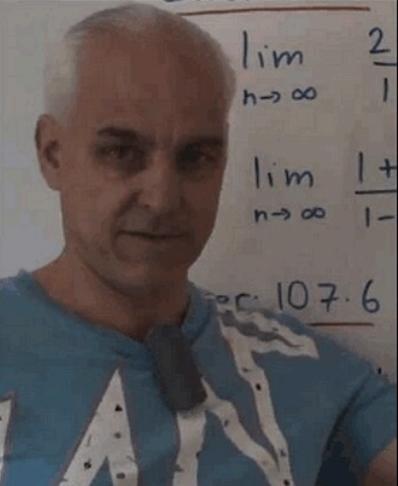
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General solution + plug-in of the boundary conditions

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Mathematicians Hate Him!



He does **MATH**
Without **REALS**

Local professor exposes shocking
math secret. Learn the rational tricks
to his stunning results.

LEARN THE TRUTH NOW

General solution + plug-in of the boundary conditions

Typical technique: separation of variables + superposition principle
(Or just magically guess the solution)

$$B(r, t) = \sum_{n=-\infty}^{+\infty} A_n e^{i(k_n r - \omega_n t)}$$

a.k.a. Fourier series

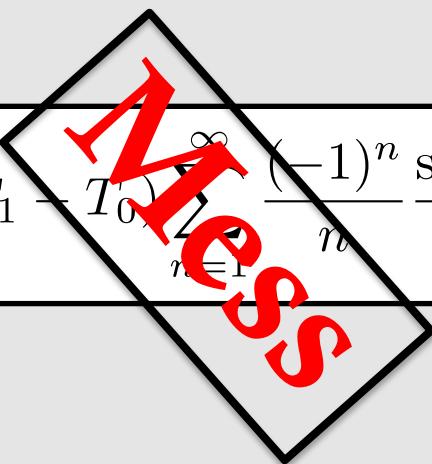
Impose boundary conditions by Fourier analysis methods!

The solution for arbitrary point within the sphere

$$T(r, t) = T_1 + \frac{2a}{\pi} (T_1 - T_0) \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \frac{\sin(n\pi r/a)}{r} e^{-D(n\pi/a)^2 t}$$

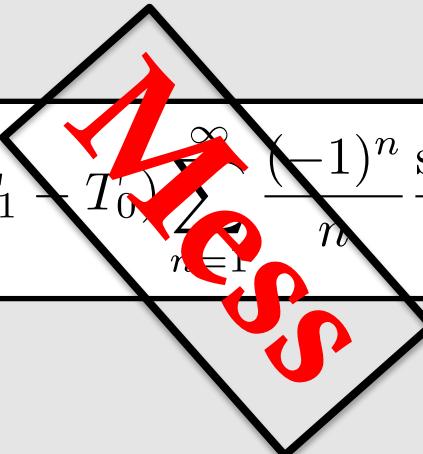
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Can we make it simpler?

The solution at the centre of the sphere

At the center of the chicken previous equation reduces to:

$$T(0, t) = T_1 + 2(T_1 - T_0) \sum_{n=1}^{\infty} \frac{(-1)^n}{n} e^{-D(n\pi/a)^2 t}$$

by using Taylor expansion:

$$\lim_{r \rightarrow 0} \frac{\sin(n\pi r/a)}{r} = \frac{n\pi r}{ar} = \frac{n\pi}{a}$$

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Truncating the sum

$$T(0, t) \approx T_1 - 2(T_1 - T_0)e^{-D(\pi/a)^2 t} \quad \text{for } t \gg \tau$$

For large times the lowest coefficient for the exponent will dominate!

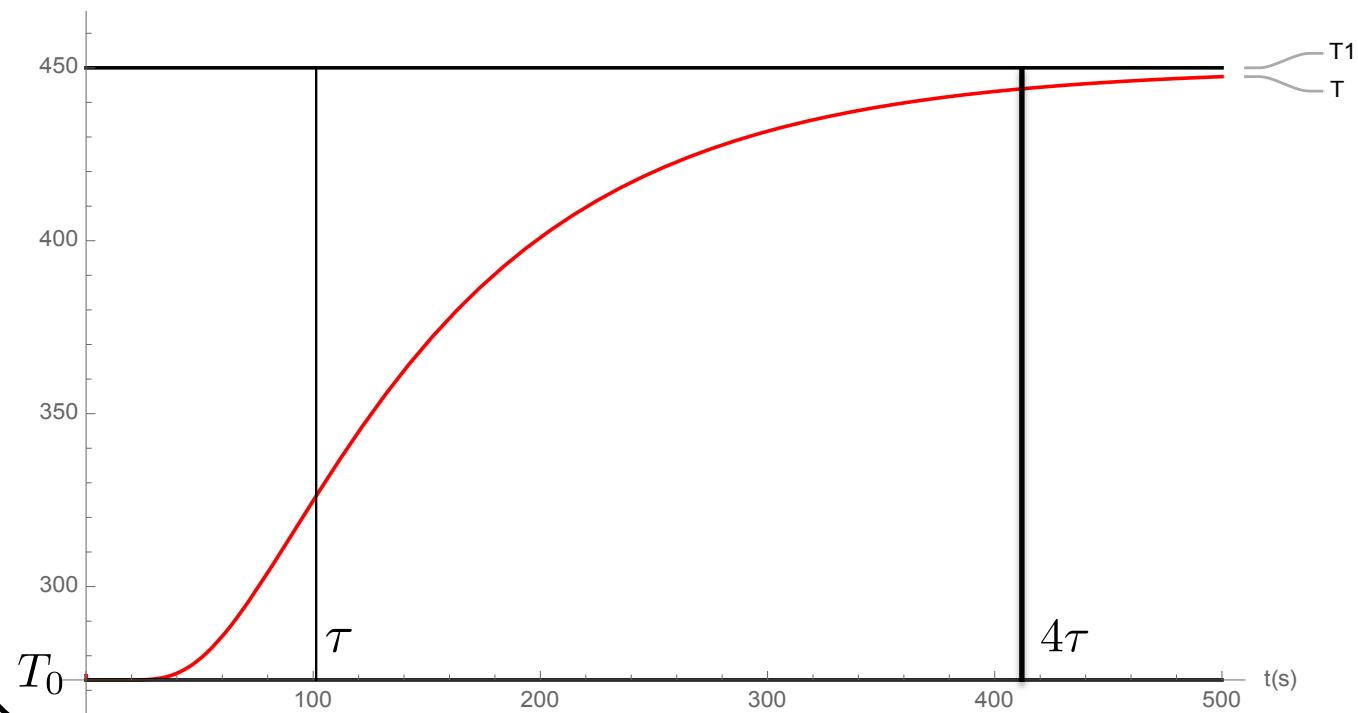
$$e^{-(1)^2} = \frac{1}{e} \approx 0.37$$

$$e^{-(2)^2} = \frac{1}{e^4} \approx 0.018$$

Nice

Characteristic timescale

$$\tau = \frac{a^2}{D\pi^2}$$



Could this be predicted?

Yes! By dimensional analysis – always use dimensional analysis!!!

Relevant parameters: D, a

$$[D] = m^2/s \quad [a] = m$$

What is such combination of parameters that produces unit of time?



$$\left[\frac{a^2}{D} \right] = [s]$$

$$\tau \propto \frac{a^2}{D}$$

Cooking time vs chicken mass law

$$\tau = \frac{a^2}{D\pi^2}$$

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They propose the linear relationship:

$$\tau(m) = Fm + \text{const.}$$

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They propose the linear relationship:

$$\tau(m) = Fm + C,$$

**FAKE
NEWS**

Time vs mass laws comparisons

timescale

Tiny
(cute)
chickens



Most
chickens



Big chickens



m

$m^{(2/3)}$

m

Conclusions

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- Heat equation gives us a nice rigorous way to justify for how long to cook chicken

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- Heat equation gives us a nice rigorous way to justify for how long to cook chicken
- Dimensional analysis would give us the result up to a constant with less hassle
- While solving this problem we have used couple of nice tricks
- You have seen or you will see the above tricks all over the undergraduate course
- **Cookery books discriminate small and very large chickens!**

Why to bother?

Spherical Chicken > “Normal” Chicken



Now
MORE delicious
than ever!!!

Submitted by the Chief
Meme Officer (CMO)
Alex Smola

Questions?

