Department of Physics and Astronomy

Day, Feb 2020

How to cook delicious spherical chicken? feat. thermodynamics

Dominik Kufel

What is a spherical chicken?

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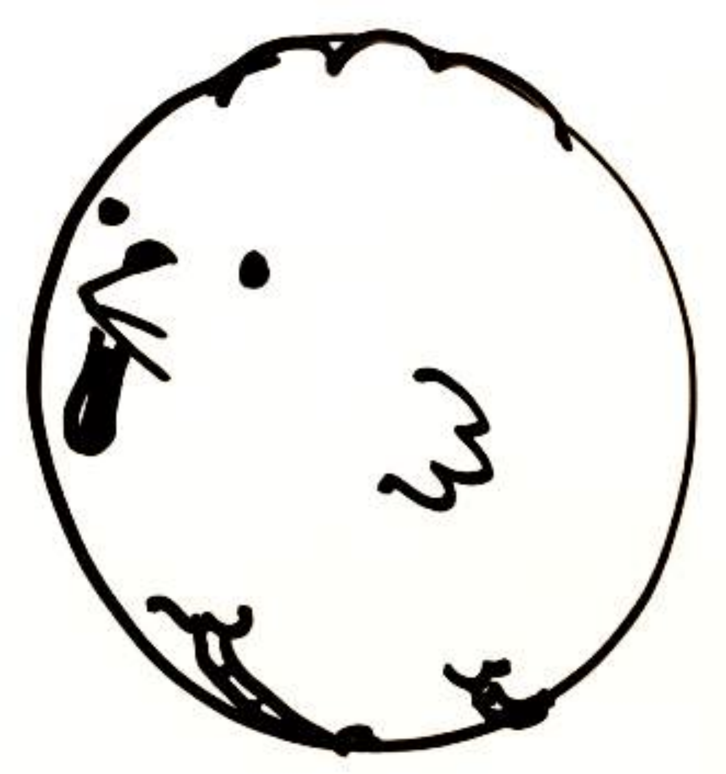

What is the story?

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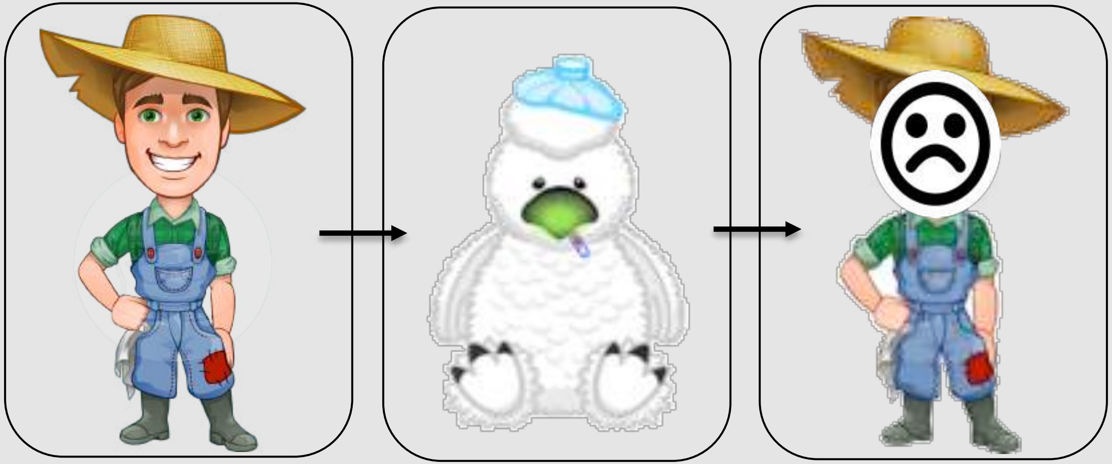

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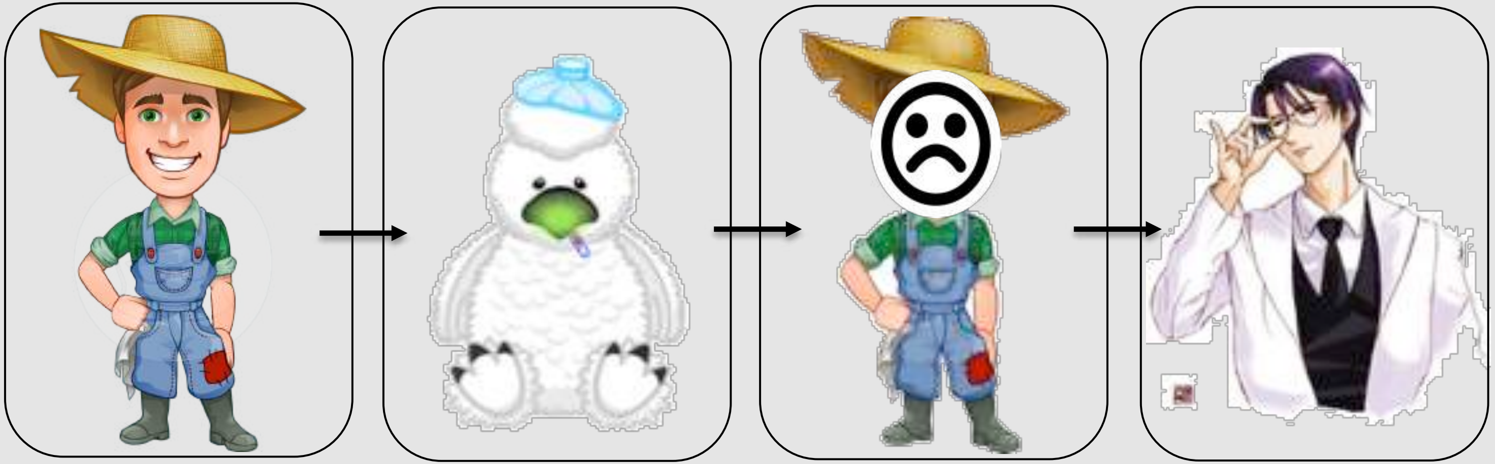

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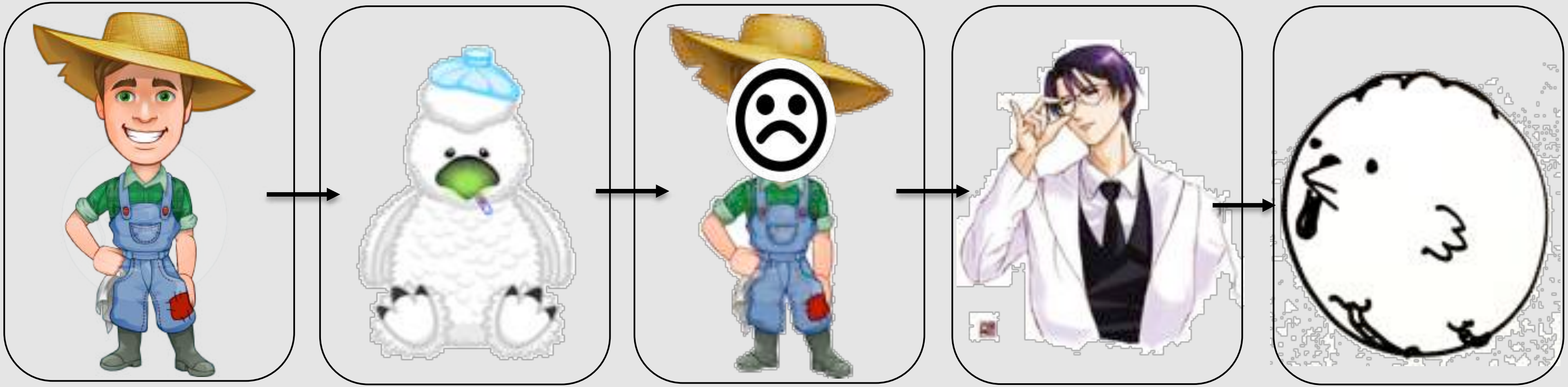

“Okay, I know what’s wrong with your chicken, but only if it’s spherical and in a vacuum.”

from
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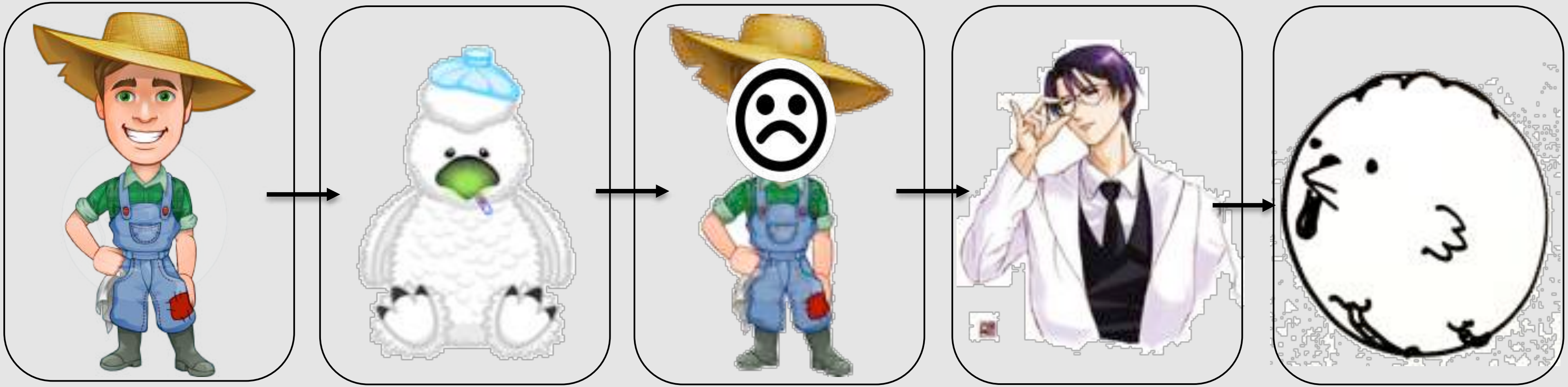

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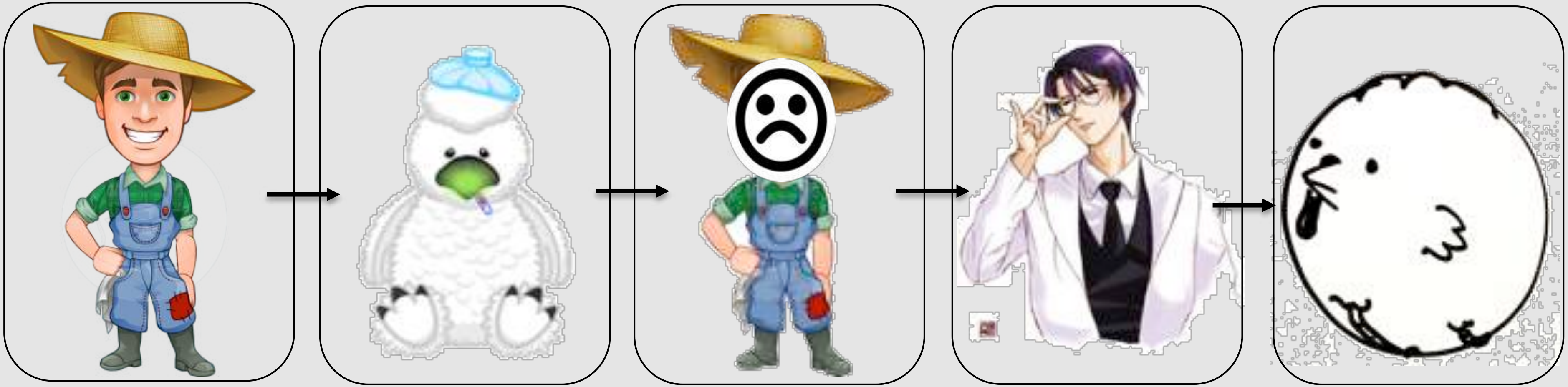

THE END

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How to transform normal chicken into spherical chicken?
(and inverse transform)

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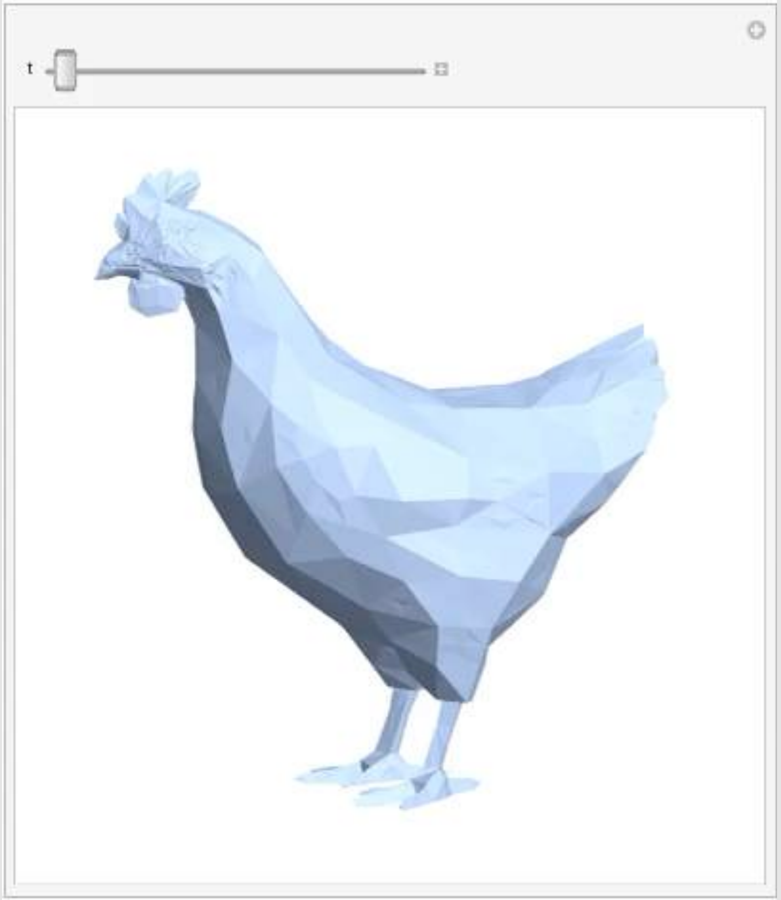
Use Wolfram Mathematica!!!



Some spherical chicken related-papers

News & Views | Published: 04 June 1998

Planetary science

Making and braking asteroids

Alan W. Harris

Nature **393**, 418–419(1998) | Cite this article

There is an old joke about the limitations of physics and its need for mathematical abstraction, the punchline of which runs, "We assume a spherical chicken". In trying to model the collisional evolution of asteroids, however, theorists have been unable to avoid assuming a spherical chicken. To create the present asteroid belt, asteroids must have been undergoing collisions throughout the age of the Solar System, but to make tractable analytical calculations of what happens when one asteroid hits another, one is forced into the unrealistic assumption that the bodies are spherical and rigid.

Why you should really bother?
Cooking a delicious chicken!!!

Chicken cooking 'law': *"Pre-heat your oven to 190°C/375°F/Gas Mark 5. Remove all packaging. Place chicken in a roasting tin in the centre of the oven and cook for 40 minutes per kilo, plus 30 minutes, basting occasionally (refer to guidelines on front of label)."*

from https://groceries.morrisons.com/webshop/product/Morrisons-Whole-Chicken-/232830011

Result?

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Goal

Use thermodynamics

to show what's wrong!

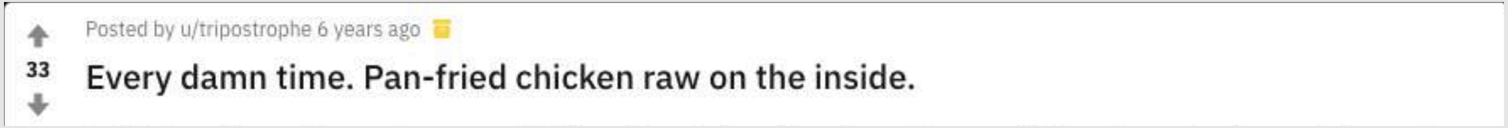
What do we care about?

33 Posted by u/tripostrophe 6 years ago

Every damn time. Pan-fried chicken raw on the inside.

"Cutting into the chicken breast shows that it is definitely not cooked. ***Don't eat this!"***

What do we care about?

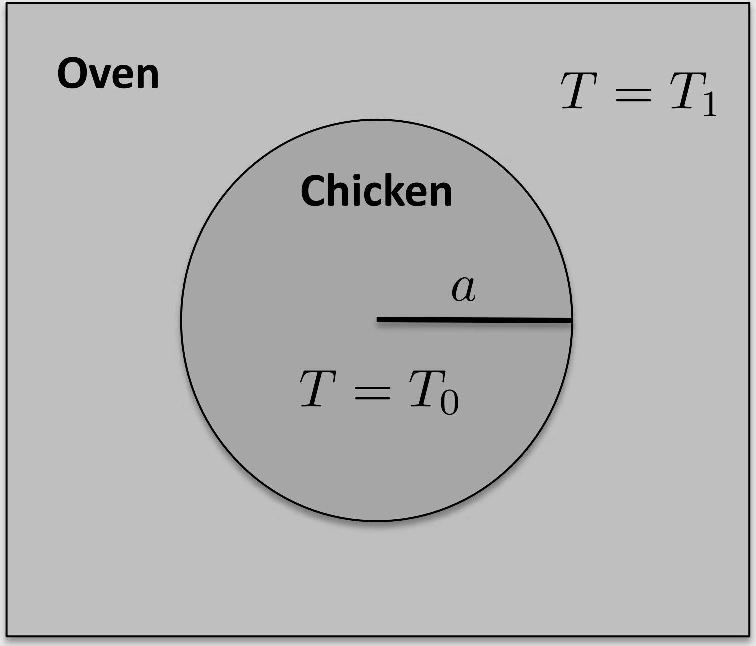


“Cutting into the chicken breast shows that it is definitely not cooked. ***Don’t eat this!”***

Objective: the meat should be well-cooked at the center of the chicken!

Our delicious spherical chicken problem

a - chicken radius



Heat equation

$$\boxed{\frac{\partial T}{\partial t} = D\nabla^2 T}$$

If $\frac{\partial T}{\partial t} = 0 \longrightarrow \nabla^2 T = 0$

Laplace's equation!
Crucial in electrostatics, gravitation, fluid dynamics etc.

In spherical coordinates the solution to the steady-state equation is:

$$T(r) = A + \frac{B}{r}$$

with $A, B = const.$

Laplace’s equation - digression

$$\nabla^2 T = 0$$

Solution in spherical coordinates is:

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Laplace's equation - digression

$$\nabla^2 V = 0$$

Solution in spherical coordinates is:

$$V(r) = \frac{B}{r}$$

Which is equivalent to:

$$E(r) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}$$

Coulomb's law in electrostatics!

Heat equation with proper symmetry

For a sphere T will have no angular and azimuthal dependence which will come in handy*:

$$\frac{\partial T}{\partial t} = D\nabla^2 T \longrightarrow \boxed{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) = \frac{\partial T}{\partial t}}$$

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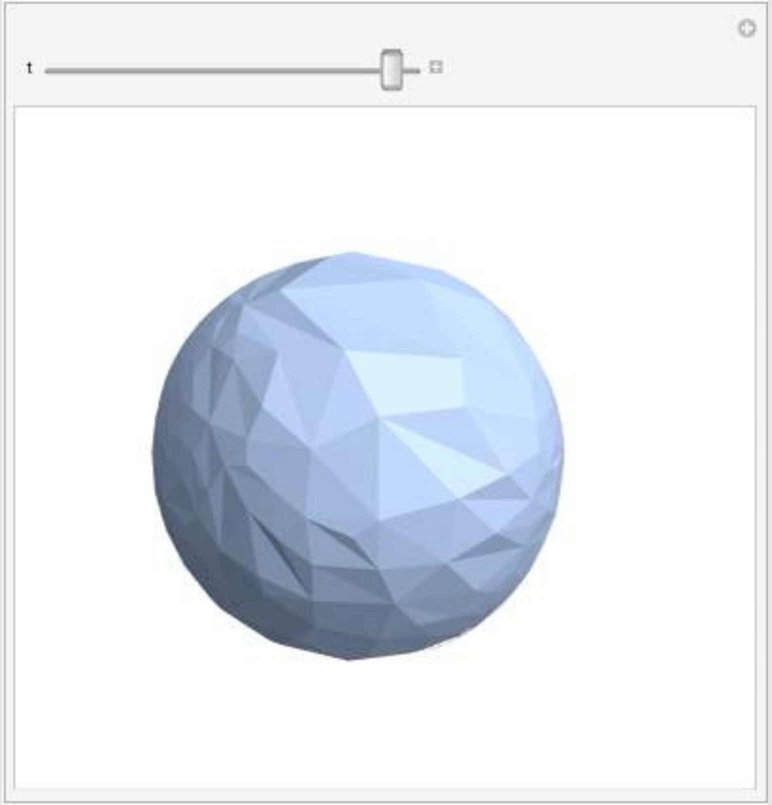

How to solve the equation?

$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) = \frac{\partial T}{\partial t}}$$

Subject to boundary conditions:

$$T(r,0) = T_0 \qquad (r < a)$$

$$T(a,t) = T_1$$

A neat trick to simplify the equation

Recall the solution to the steady state:

$$T(r) = A + \frac{B}{r}$$

Generalize it:

$$T(r,t) = T_1 + \frac{B(r,t)}{r}$$

Plug-into the equation:

$$\frac{\partial B}{\partial t}\frac{1}{r} = -\frac{1}{r^2}\cancel{\frac{\partial B}{\partial r}} + \frac{1}{r^2}\cancel{\frac{\partial B}{\partial r}} + r\frac{1}{r^2}\frac{\partial^2 B}{\partial r^2}$$

Hence:

$$\boxed{\frac{\partial B}{\partial t} = D\frac{\partial^2 B}{\partial r^2}}$$

Just a 1D heat equation

General solution + plug-in of the boundary conditions

Typical technique: separation of variables + superposition principle

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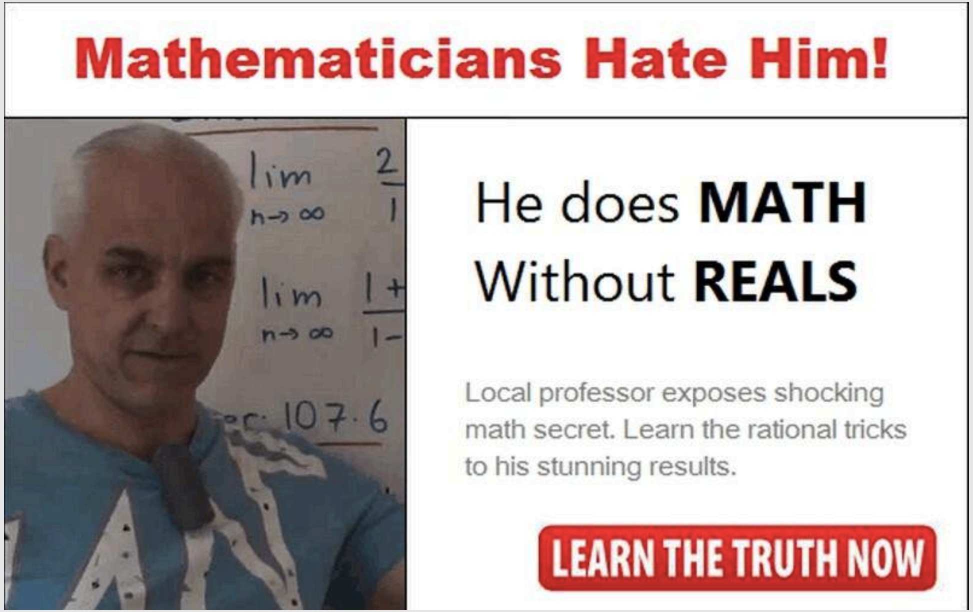

General solution + plug-in of the boundary conditions

Typical technique: separation of variables + superposition principle

(Or just magically guess the solution)

$$B(r,t) = \sum_{n=-\infty}^{+\infty} A_n e^{i(k_n r - \omega_n t)}$$

a.k.a. Fourier series

Impose boundary conditions by Fourier analysis methods!

The solution for arbitrary point within the sphere

$$T(r,t) = T_1 + \frac{2a}{\pi}(T_1 - T_0)\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\frac{\sin(n\pi r/a)}{r}e^{-D(n\pi/a)^2 t}$$

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Mess

Can we make it simpler?

The solution at the centre of the sphere

At the center of the chicken previous equation reduces to:

$$\boxed{T(0,t) = T_1 + 2(T_1 - T_0)\sum_{n=1}^{\infty}\frac{(-1)^n}{n}e^{-D(n\pi/a)^2 t}}$$

by using Taylor expansion: $$\lim_{r\to 0}\frac{\sin(n\pi r/a)}{r} = \frac{n\pi r}{ar} = \frac{n\pi}{a}$$

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Truncating the sum

$$\boxed{T(0,t) \approx T_1 - 2(T_1 - T_0)e^{-D(\pi/a)^2 t}} \qquad \text{for} \quad t \gg \tau$$

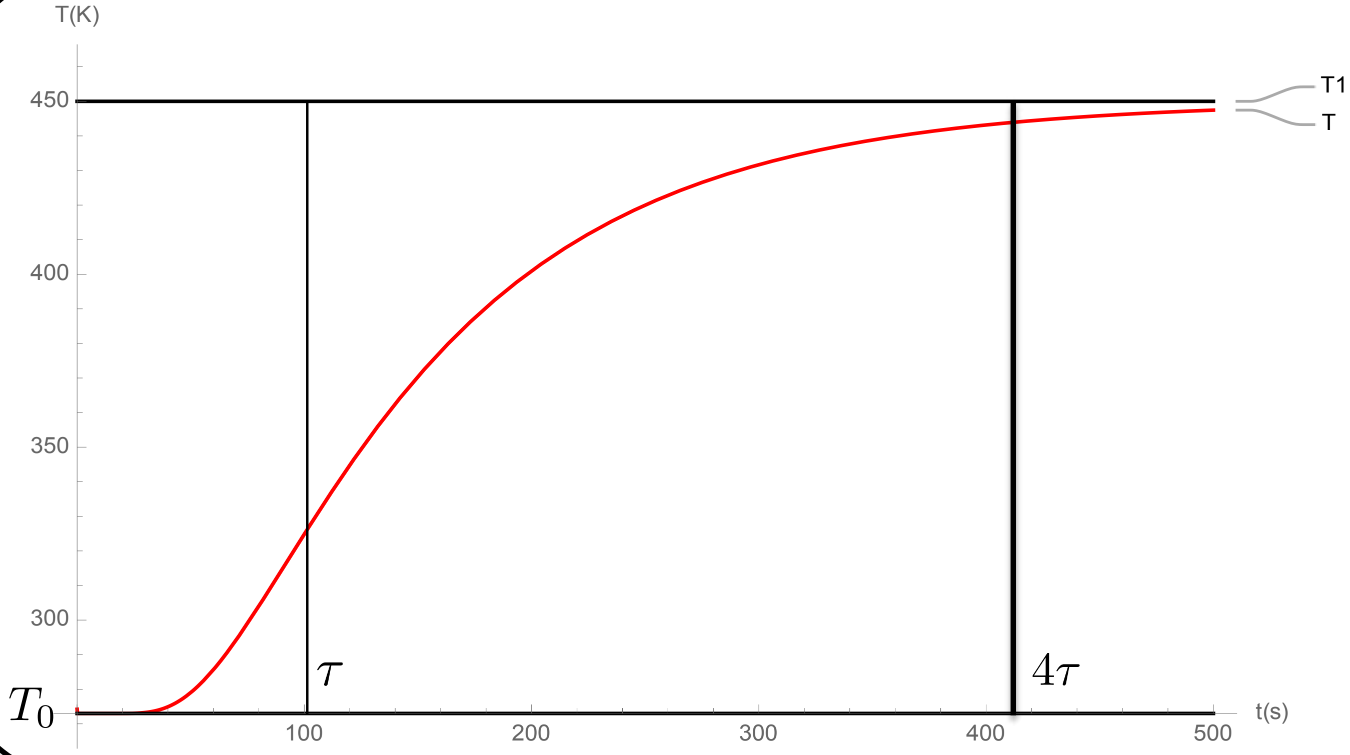
For large times the lowest coefficient for the exponent will dominate!

$$e^{-(1)^2} = \frac{1}{e} \approx 0.37 \qquad\qquad e^{-(2)^2} = \frac{1}{e^4} \approx 0.018$$

Nice

Characteristic timescale

$$\tau = \frac{a^2}{D\pi^2}$$



Could this be predicted?

Yes! By dimensional analysis – always use dimensional analysis!!!

Relevant parameters: D, a $[D] = m^2/s$ $[a] = m$

What is such combination of parameters that produces unit of time?

$$[\frac{a^2}{D}] = [s] \longrightarrow \boxed{\tau \propto \frac{a^2}{D}}$$

Cooking time vs chicken mass law

$$\tau = \frac{a^2}{D\pi^2}$$

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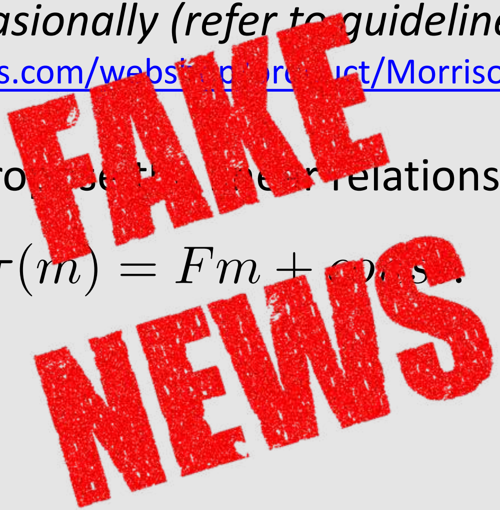
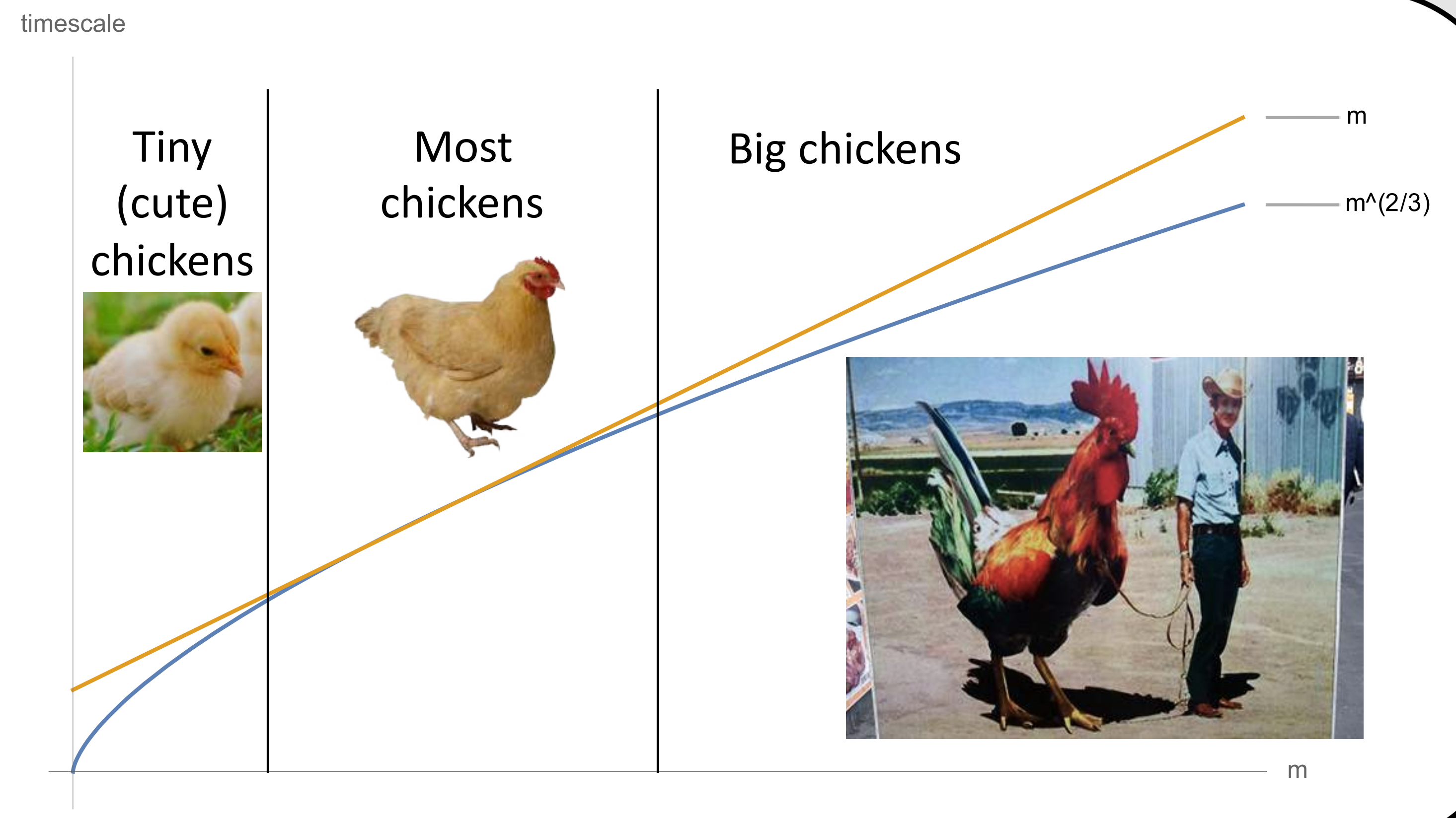


Time vs mass laws comparisons



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- Dimensional analysis would give us the result up to a constant with less hassle
- While solving this problem we have used couple of nice tricks
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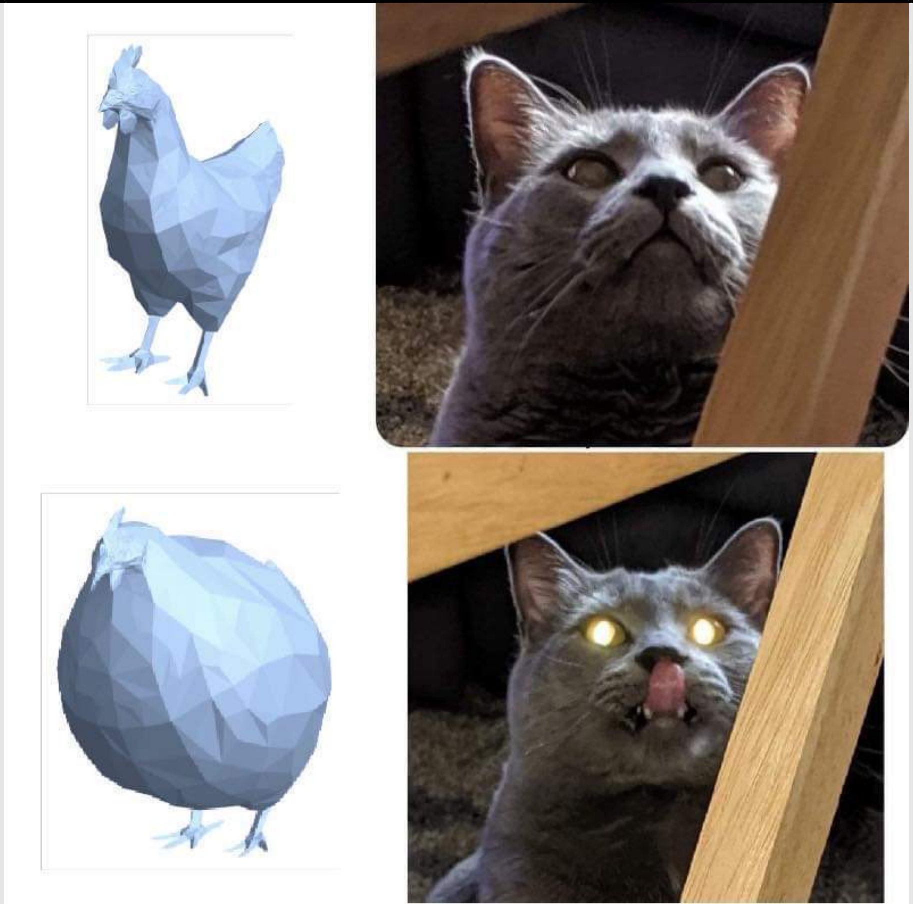
- **Cookery books discriminate small and very large chickens!**

Why to bother?

Spherical Chicken > “Normal” Chicken



Now
MORE delicious
than ever!!!

Submitted by the Chief Meme Officer (CMO)
Alex Smola

Questions?

https://i.ytimg.com/

*Rare photo of two chickens with PTSD after being transformed into a sphere and back.