# regularized-regression

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## 1 Fundamentals of Machine Learning - Exercise 6

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```
In [1]: import numpy as np
        from scipy.sparse import linalg
        from scipy.sparse import coo_matrix
        from scipy.sparse import csc_matrix
        import matplotlib.pyplot as plt
        import scipy.sparse
        from scipy.ndimage.filters import gaussian_filter
        import random
        from sklearn.datasets import load_digits
        from sklearn import model_selection
        from matplotlib import colors
        from sklearn.metrics import confusion_matrix
```

### 1.1 Bias and variance of ridge regression

Only included in the PDF file

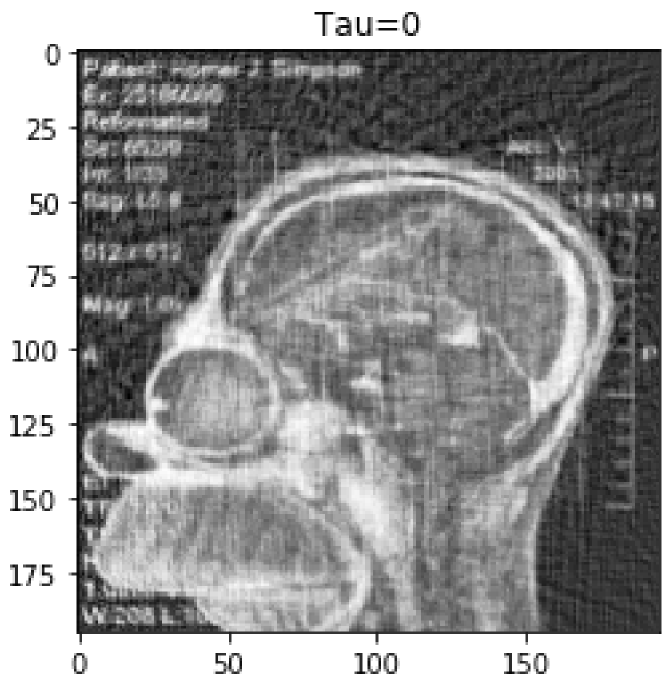
### 1.2 Denoising of a CT image

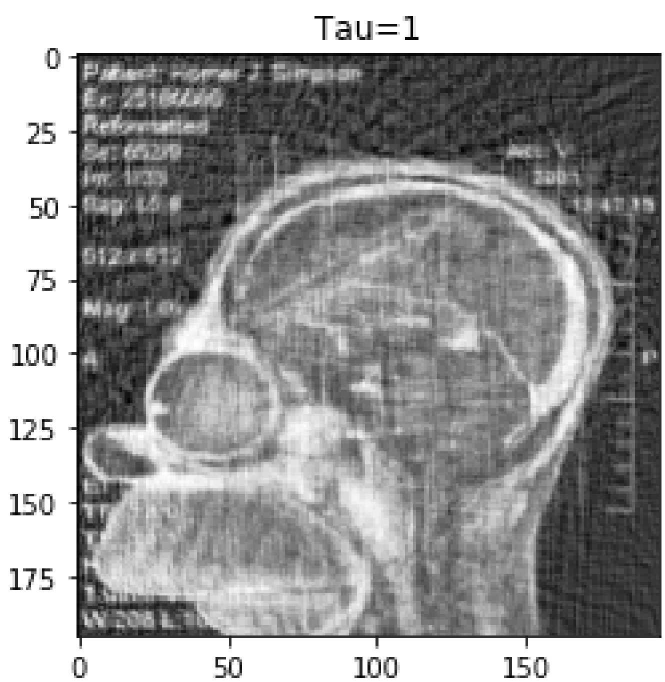
```
In [2]: def construct_X(M, alphas, Np=None, tau=0):
            if not Np:
                Np = np.ceil(np.sqrt(2) * M).astype(int)
            if Np % 2 == 0:
                Np += 1
            D = M * M
            No = len(alphas)
            N = Np * No
            C1 = (np.mgrid[0:M, 0:M][0]).flatten()
            C2 = (np.mgrid[0:M, 0:M][1]).flatten()
            C = np.vstack((C1, C2))
            # centralise the coordinates in C
            C = C - (M - 1) / 2
            # convert alphas to radian
```

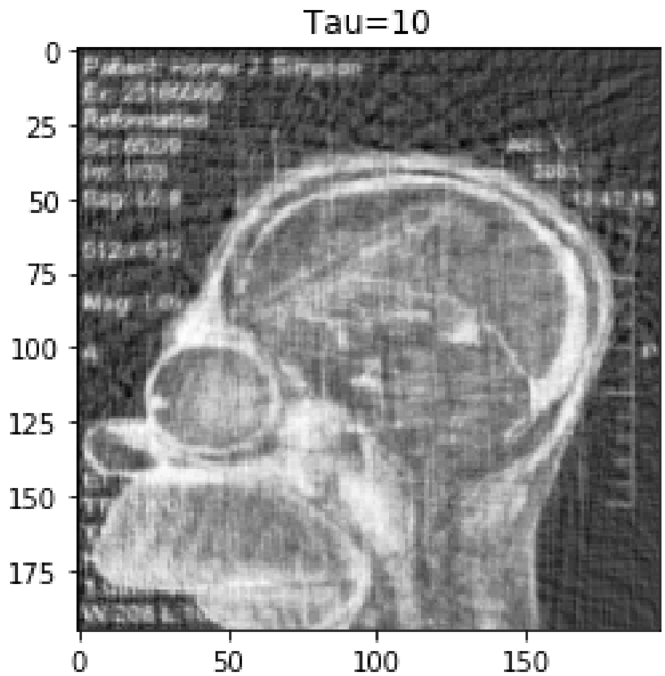
```
    alphas_rad = np.radians(alphas)
    # now make vectors out of the angles
    n = np.zeros((2, No))
    n[0] = -np.sin(alphas_rad)
    n[1] = np.cos(alphas_rad)
    p = n.T.dot(C) + (Np - 1) / 2
    lower_element = np.floor(p).astype(int)
    upper_element = np.ceil(p).astype(int)
    lower_value = upper_element - p
    upper_value = p - lower_element
    weights = []
    i_indices = []
    j_indices = []
    for i in range(No):
        for j in range(len(lower_element[i])):
            if (lower_element[i][j] == upper_element[i][j]):
                weights.append(1)
                i_indices.append(i*Np+lower_element[i][j])
                j_indices.append(j)
            else:
                weights.append(lower_value[i][j])
                i_indices.append((i*Np+lower_element[i][j]))
                j_indices.append(j)
                weights.append(upper_value[i][j])
                i_indices.append(((i*Np+upper_element[i][j])))
                j_indices.append(j)
    X = coo_matrix((weights, (i_indices, j_indices)), shape=(N, D), dtype=np.float32)
    # if tau is given construct the diagonal matrix and append it to X
    if tau != 0:
        x_indices = np.arange(D)
        y_indices = np.arange(D)
        taus = np.full((D), np.sqrt(tau))
        tauMatrix = coo_matrix((taus, (x_indices, y_indices)), shape=(D, D), dtype=np.
        X = scipy.sparse.vstack((X, tauMatrix))
    return X
```

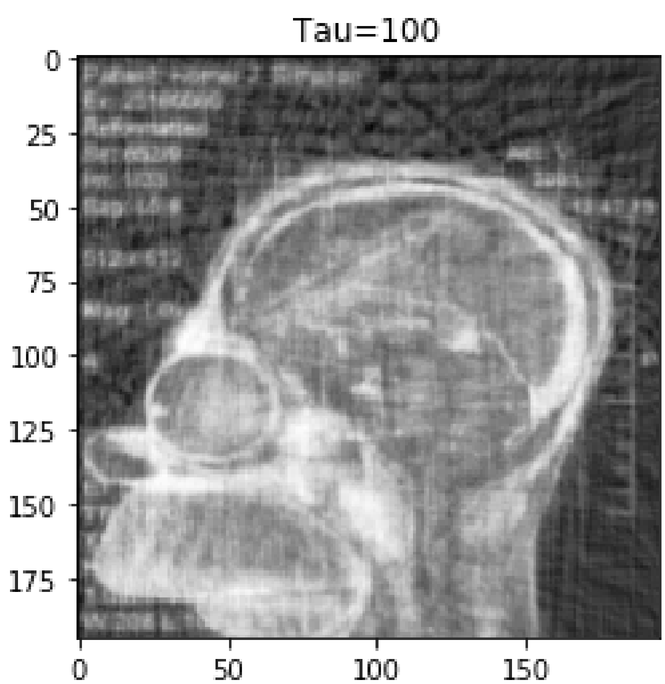
In [3]:
```
alphas = np.load('hs_tomography/alphas_195.npy')
y = np.load('hs_tomography/y_195.npy')
Np = 275
M = 195
newy = np.array([])
No = len(alphas)
# out of all alphas draw 64 at random
randalphas = random.sample(range(0, No), 64)
newalphas = []
for i in randalphas:
    newalphas.append(alphas[i])
    data = y[np.arange(i*Np, i*Np+Np)]
```

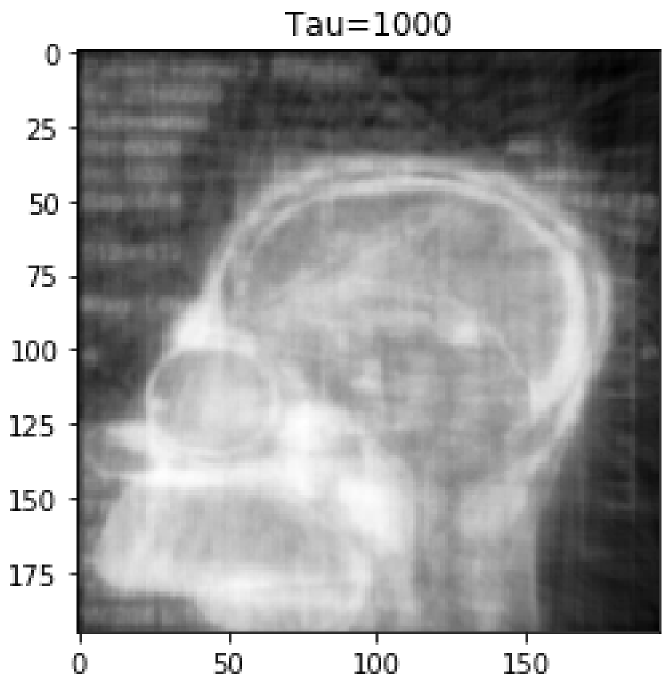
```
    newy = np.concatenate((newy, data))

# don't include tau=0 it will be handede separately because
# the image is needed later for the gaussian filtering
taus = [1, 10, 100, 1000, 10000]
# now start with tau = 0
X = construct_X(M, newalphas, Np, 0)
X_csc = X.tocsc()
img_tau_zero = linalg.lsqr(X_csc, newy, atol=1e-05, btol=1e-05)
img_tau_zero = img_tau_zero[0]
img_tau_zero = img_tau_zero.reshape((M, M))
plt.imshow(img_tau_zero, cmap=plt.cm.Greys_r)
plt.title('Tau=0')
plt.show()

# now construct a new y vector with zeros appendend
# for the cases where tau is not zero
zeros = np.zeros(M*M)
newy = np.concatenate((newy, zeros))
for tau in taus:
    X = construct_X(M, newalphas, Np, tau)
    X_csc = X.tocsc()
    img = linalg.lsqr(X_csc, newy, atol=1e-05, btol=1e-05)
    img = img[0]
    img = img.reshape((M, M))
    plt.imshow(img, cmap=plt.cm.Greys_r)
    plt.title('Tau=%i' %(tau))
    plt.show()
```

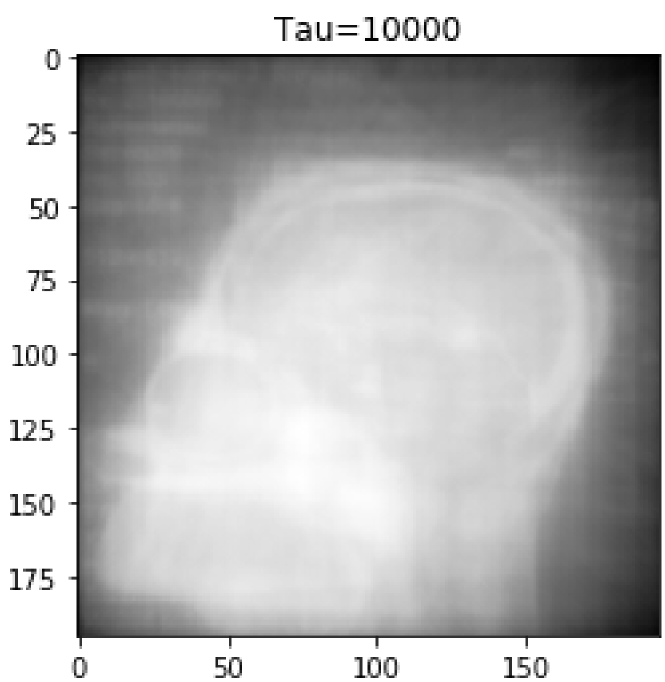

Tau=0
0
25
50
75
100
125
150
175
0
50
100
150


Tau=1
0
25
50
75
100
125
150
175
0
50
100
150


Tau=10
0
25
50
75
100
125
150
175
0
50
100
150

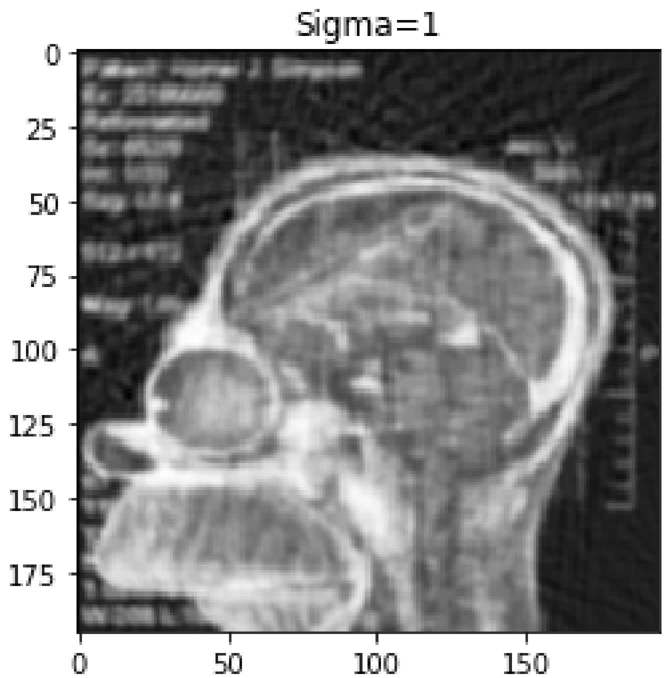

Tau=100
0
25
50
75
100
125
150
175
0
50
100
150

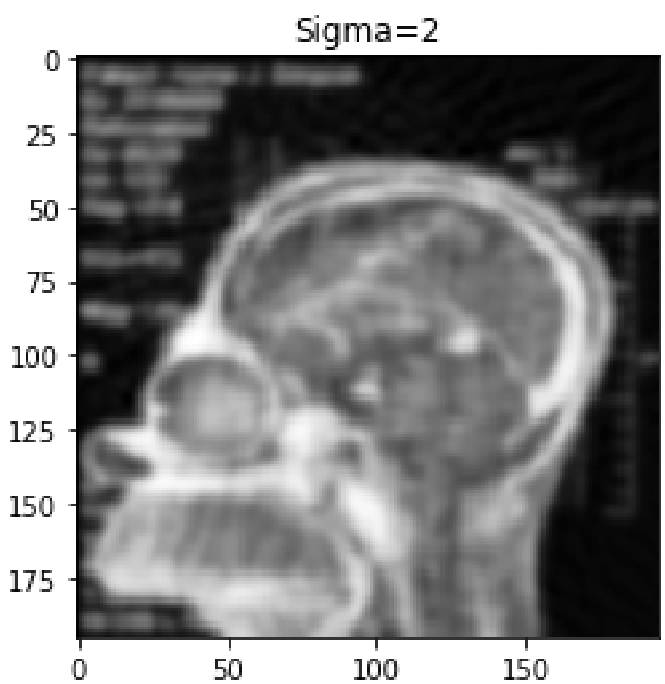


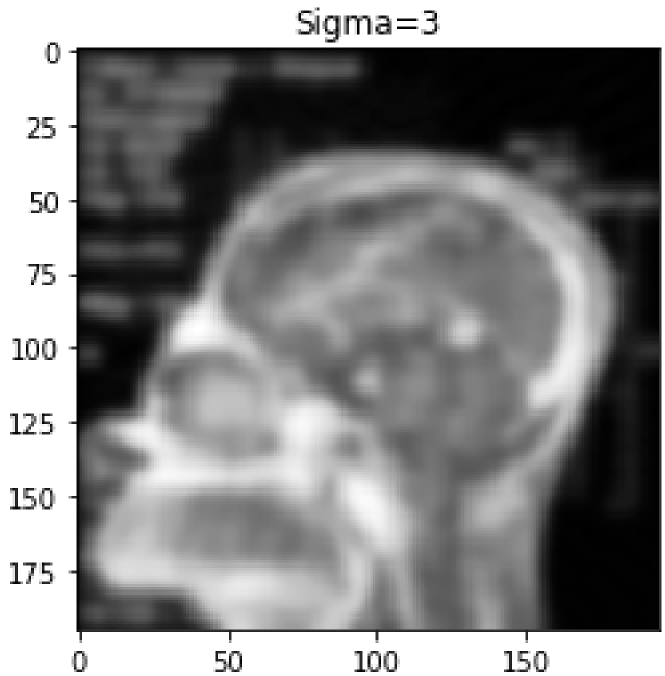



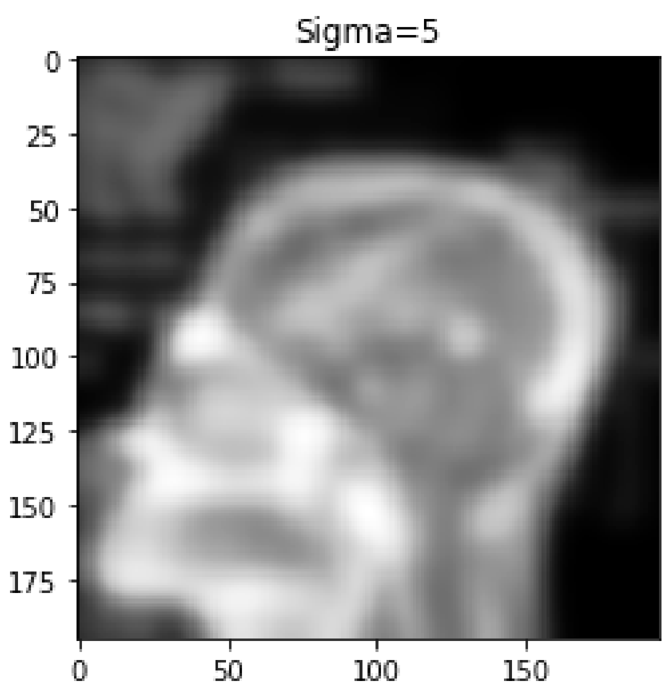

```
In [4]: # now do the Gaussian filtering
        sigmas = [1, 2, 3, 5, 7]
```

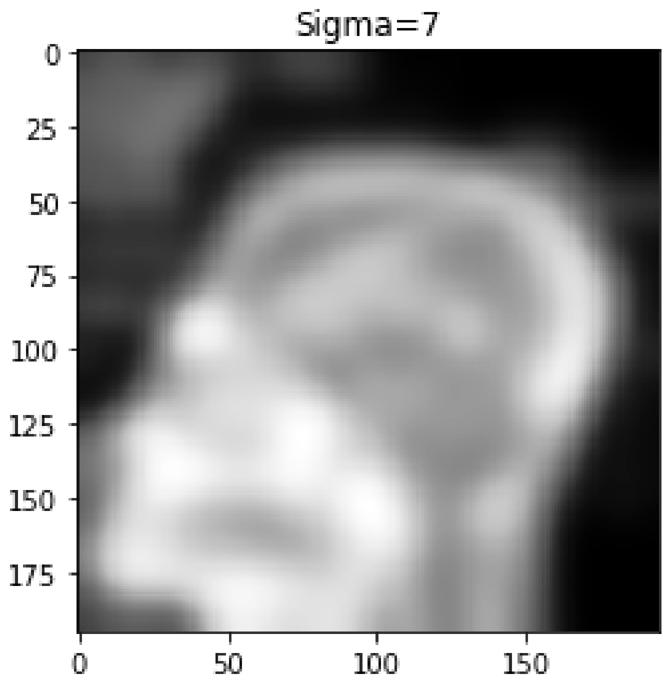
```python
for sigma in sigmas:
    filtered_image = gaussian_filter(img_tau_zero, sigma)
    plt.imshow(filtered_image, cmap=plt.cm.Greys_r)
    plt.title('Sigma=%i' % sigma)
    plt.show()
```




Sigma=3
0
25
50
75
100
125
150
175
0
50
100
150


Sigma=5
0
25
50
75
100
125
150
175
0
50
100
150




For both methods increasing tau respectively sigma improves the result only until a certain level later on the images get worse. For Gaussian filtering the image already starts to get blurrier when sigma is increased to 2. Using the ridge regression the image gets worse when increasing tau to 1000. The Gaussian filtering reduces the noise more than the Ridge Regression, but also looses contrast. The Ridge Regression images stay relatively sharp but at some point the lighter parts of the image seam smeared all over the image.

## 1.3 Automatic feature selection for regression

### 1.3.1 Implement Orthogonal Matching Pursuit

```
In [5]: def omp_regression(X, y, T):
            if T > X.shape[1]:
                T = X.shape[1]
            A = []
            B = list(range(0, X.shape[1]))
            residual = y
            # matrix to return filled with zeros
            betaMatrix = np.zeros((X.shape[1], T))
            for t in range(T):
                inactiveX = X[:, B]
                # get index of column having maximal correlation
                # with current residual
                j_index = np.argmax(inactiveX.T.dot(residual))
                # get corresponding j from list of inactive columns
                j = B[j_index]
```

```
            A.append(j)
            B.remove(j)
            activeX = X[:, A]
            beta = np.linalg.lstsq(activeX, y)[0]  # (activeX, y, atol=1e-05, btol=1e-05)[
            residual = y - activeX.dot(beta)
            # put non-zero elements for this iteration into
            # matrix to return
            betaMatrix[A, t] = beta
        return betaMatrix
```

### 1.3.2 Classification with sparse LDA

```
In [6]: digits = load_digits()
        data = digits["data"]
        images = digits["images"]
        target = digits["target"]
        target_names = digits["target_names"]
        # filter data for 1 and 7
        filter_mask = (target == 1) | (target == 7)
        data = data[filter_mask]
        images = images[filter_mask]
        target = target[filter_mask]
        # standardize the data
        std = np.std(data, axis=0)
        std = np.expand_dims(std, axis=0)
        mean = np.mean(data, axis=0)
        mean = np.expand_dims(mean, axis=0)
        # if std is zero change it to 1 avoid dividing by 0
        std[std == 0] = 1
        data = (data - mean) / std
        # now change target vector to the given desired response
        # 1 if instance is a 1 and -1 if instance is a 7
        target[target == 7] = -1
        # split in train and testset
        x_train, x_test, y_train, y_test = model_selection.train_test_split(data,
                                                                            target,
                                                                            test_size=0.33,
                                                                            random_state=0)
        betaMat = omp_regression(x_train, y_train, 64)
        # for every test instance and every t caculate
        # X*betaMat
        classification = x_test.dot(betaMat)
        # a negative response means class seven
        classification[classification < 0] = -1
        # a positive response means class 1
        classification[classification >= 0] = 1
        # now calculate the error rate for every t
        num_test_instances = x_test.shape[0]
```

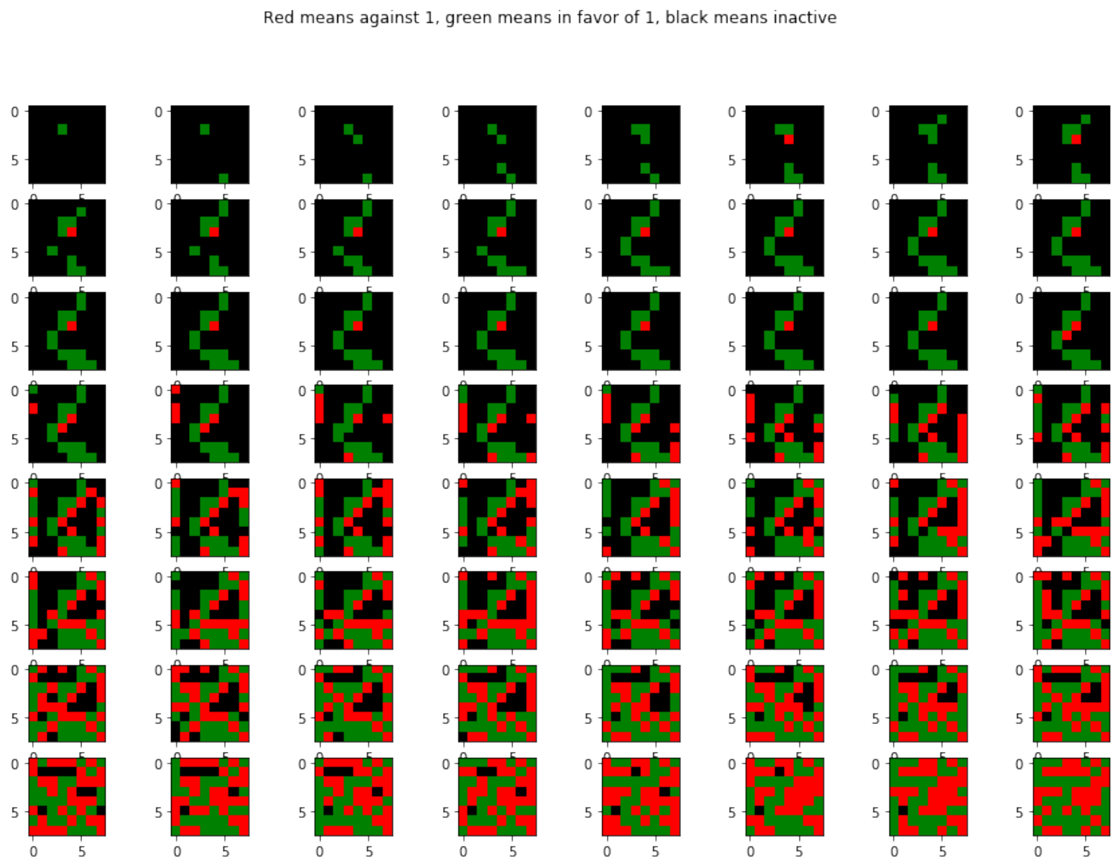
```python
for i in range(classification.shape[1]):
    wrong_elems = np.sum(classification[:, i] != y_test)
    error_rate = wrong_elems / num_test_instances * 100
    print('Error rate for t=%i is: %d%%' % ((i+1), error_rate))
```

```
Error rate for t=1 is: 3%
Error rate for t=2 is: 0%
Error rate for t=3 is: 0%
Error rate for t=4 is: 0%
Error rate for t=5 is: 0%
Error rate for t=6 is: 0%
Error rate for t=7 is: 0%
Error rate for t=8 is: 0%
Error rate for t=9 is: 0%
Error rate for t=10 is: 0%
Error rate for t=11 is: 0%
Error rate for t=12 is: 0%
Error rate for t=13 is: 0%
Error rate for t=14 is: 0%
Error rate for t=15 is: 0%
Error rate for t=16 is: 0%
Error rate for t=17 is: 0%
Error rate for t=18 is: 0%
Error rate for t=19 is: 0%
Error rate for t=20 is: 0%
Error rate for t=21 is: 0%
Error rate for t=22 is: 0%
Error rate for t=23 is: 0%
Error rate for t=24 is: 0%
Error rate for t=25 is: 0%
Error rate for t=26 is: 0%
Error rate for t=27 is: 0%
Error rate for t=28 is: 0%
Error rate for t=29 is: 0%
Error rate for t=30 is: 0%
Error rate for t=31 is: 0%
Error rate for t=32 is: 0%
Error rate for t=33 is: 0%
Error rate for t=34 is: 0%
Error rate for t=35 is: 0%
Error rate for t=36 is: 0%
Error rate for t=37 is: 0%
Error rate for t=38 is: 0%
Error rate for t=39 is: 0%
Error rate for t=40 is: 0%
Error rate for t=41 is: 0%
Error rate for t=42 is: 0%
Error rate for t=43 is: 0%
```

```
Error rate for t=44 is: 0%
Error rate for t=45 is: 0%
Error rate for t=46 is: 0%
Error rate for t=47 is: 0%
Error rate for t=48 is: 0%
Error rate for t=49 is: 0%
Error rate for t=50 is: 0%
Error rate for t=51 is: 0%
Error rate for t=52 is: 0%
Error rate for t=53 is: 0%
Error rate for t=54 is: 0%
Error rate for t=55 is: 0%
Error rate for t=56 is: 0%
Error rate for t=57 is: 0%
Error rate for t=58 is: 0%
Error rate for t=59 is: 0%
Error rate for t=60 is: 0%
Error rate for t=61 is: 0%
Error rate for t=62 is: 0%
Error rate for t=63 is: 0%
Error rate for t=64 is: 0%
```

For standardized data 2 pixels are enough to distinguish the two classes and get an test error of 0%. If the data is not standardized the classifier needs 19 pixels to classify less than 50% wrong and 32 pixels to classifiy less than 10% wrong.

```
In [7]: visualize_beta = omp_regression(x_train, y_train, 64)
        visualize_beta[visualize_beta < 0] = -1
        visualize_beta[visualize_beta > 0] = 1
        # make a color map of fixed colors
        cmap = colors.ListedColormap(['red', 'black', 'green'])
        bounds = [-1,-0.5,0.5,1]
        norm = colors.BoundaryNorm(bounds, cmap.N)
        fig, axes = plt.subplots(8,8, figsize=(15,10))
        axes = axes.ravel()
        plt.suptitle('Red means against 1, green means in favor of 1, black means inactive')
        for i in range(0, 64):
            axes[i].imshow(visualize_beta[:, i].reshape((8,8)), cmap=cmap, norm=norm)
        plt.show()
```




Pixels with negative weight vote against class 1 pixels with positive weight vote in favor of class one. The first two pixels that get activated are 19 and 61. In our handcrafted solution we chose pixels 19 and 60 so we are pretty close to the features selected by the sparse LDA.

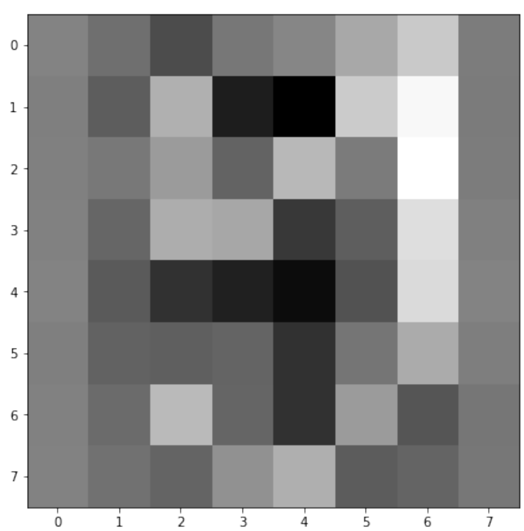
### 1.3.3 One-against-the-rest classification

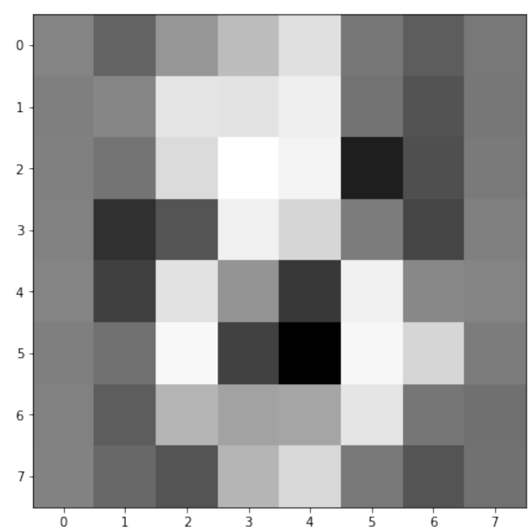
```
In [8]: # fix random seed for reproducible results
        random.seed(0)
        data = digits["data"]
        images = digits["images"]
        target = digits["target"]
        target_names = digits["target_names"]
        # standardize the data
        std = np.std(data, axis=0)
        std = np.expand_dims(std, axis=0)
        mean = np.mean(data, axis=0)
        mean = np.expand_dims(mean, axis=0)
        # if std is zero change it to 1 avoid dividing by 0
        std[std == 0] = 1
        data = (data - mean) / std
        # split in train and testset
```

```python
x_train, x_test, y_train, y_test = model_selection.train_test_split(data,
                                                                     target,
                                                                     test_size=0.33,
                                                                     random_state=0)
B = []
# for every class train a classifier
for k in range(10):
    # get all labels of target class
    sample_target = y_train[y_train == k]
    # get number of instances in target class
    N = len(sample_target)
    # get all instances of target class
    sample = x_train[y_train == k]
    # get all other classes
    rest = x_train[y_train != k]
    # now choose randomly N elements from the other classes
    rand = random.sample((range(0, len(rest))), N)
    # get the instances and labels of the other classes
    rest = rest[rand]
    rest_target = y_train[y_train != k]
    rest_target = rest_target[rand]
    # plug target class and the rest together
    sample = np.vstack((sample, rest))
    # plug the labels together
    sample_target = np.concatenate((sample_target, rest_target))
    # change labels to desired responses
    # 1 for target class, -1 for the others
    sample_target[sample_target != k] = -1
    sample_target[sample_target == k] = 1
    # now get the betas from the omp_regression
    b = omp_regression(sample, sample_target, 64)
    # only keep the last beta
    B.append(b[:, 63])
B = np.asarray(B)

# Testing
# empty list for all predictions of the classifiers
predict = []
# for every instance in the testset
for k in range(x_test.shape[0]):
    # get prediction of all classifiers
    y = B.dot(x_test[k, :].T)
    predict.append(y)
# check which classifier has the highest respons
predicted_label = np.argmax(predict, axis=1)
# now check if for an instance all responses were
# negative and if so assign the instance to class
# unknown
```

```python
max = np.max(predict, axis=1)
predicted_label[max < 0] = -1
# print the confusion matrix
print('Confusion matrix. Classes are: \n -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9')
print(confusion_matrix(y_test, predicted_label))
# get the indeces of the instances classified as unknwon
minus_ones = np.where(predicted_label == -1)[0]
# get the corresponding images
unknown_images = x_test[minus_ones]
nr_subplots = unknown_images.shape[0]
fig, axes = plt.subplots(1,nr_subplots, figsize=(15,10))
axes = axes.ravel()
for i in range(unknown_images.shape[0]):
    axes[i].imshow(unknown_images[i, :].reshape((8,8)), cmap=plt.cm.Greys_r)
plt.show()
```

```
Confusion matrix. Classes are:
 -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
[[ 0  0  0  0  0  0  0  0  0  0  0]
 [ 0 48  0  0  0  0  0  1  0  0  0]
 [ 0  0 50  0  1  2  1  3  0  1  3]
 [ 0  1  3 50  4  0  0  0  0  4  0]
 [ 0  0  0  1 46  0  1  1  1  2  3]
 [ 0  0  4  0  0 43  0  0  1  2  0]
 [ 0  0  1  0  0  0 59  1  0  0  4]
 [ 0  0  0  1  0  0  0 65  0  1  0]
 [ 0  0  2  0  4  1  0  0 49  0  0]
 [ 1  0  7  1  0  0  0  1  1 53  3]
 [ 1  0  0  0  1  0  0  0  0  3 57]]
```



Class unknown avoids the situation that an instance is assigned to a class even though all classifiers have negative response meaning that the instance probably does not belong to any of the known classes. Visualizing the images classified as unknwon one clearly sees that they hardly resemble any number and thus can not be classified.