Solving multiple square jigsaw puzzles with missing pieces

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Abstract

Chapter 1

Introduction

Chapter 2

Theoretical Background

2.1 Automatic Jigsaw Puzzle Solvers

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The first jigsaw puzzles were produced around 1760 by John Spilsbury, a London engraver and mapmaker, and made out of wood [SDN13]. Hence the name "jigsaw" which refers to the jigsaws that were used to cut out the pieces of the puzzles. Modern jigsaws puzzles, where an image was printed on a cardboard sheet that was cut into a set of interlocking pieces were, introduced in the 1930s [WS04]. Even though puzzles are successfully solved by children worldwide, automatic puzzle solvers are a technically challenging problem. Demaine et al. [DD07] could show that the puzzle problem is NP-complete if the pairwise affinity among pieces is unreliable.
The first automatic jigsaw puzzle solver was proposed in 1964 by Freeman et al [FG64]. It was designed to solve jigsaw puzzles with pieces which are all uniformly gray and the only available information is the shape of the pieces and could handle up to nine-piece problems. Various other early works explore aspects of using piece shape information and contour matching to solve jigsaw puzzles. For example, Wolfson et al. [WSKL88] use an approach that is also widely used by humans. They first reconstruct the boundary of the jigsaw puzzle and then gradually fill the inside with the most reliably matching pieces. Webster et al. [WRLS90] introduce a methodology that derives a set of critical points which define a feature called an Isthmus. This Isthmus can be used in matching boundaries of planar regions. The approach of Kong et al. [KK01] consists of two steps. In the first step local shape matching is used to find likely candidate pairs for adjacent fragments. In the second step ambiguities resulting from local shape matching are resolved by finding a global solution and the pieces are merged together. They report successfully reassembling broken ceramic tiles and a map puzzle with their approach.
The first to not only consider information about the shape of a piece but also its color information were Kosiba et al. [KDB$^+$94]. The color information is

used differently depending on whether the original image is known or not. If the original image is known they calculate the overall color characteristic of each piece, meaning the mean and variance of the hue, saturation and intensity values. They then compare these features with color characteristics of various regions in the original image and try to reassemble the jigsaw puzzle in the same orientation as the original image. If the original image is unknown small color windows at regular intervals along the border of the pieces are sampled and color characteristics for each of these windows are calculated. These features are then, together with the shape information of the pieces, used to compare the borders of pieces and find likely matching pairs. Makridis et al. [MP06] consider a puzzle as an image that is divided into a number of subimages (pieces). For each piece they extract a set of boundary characteristic points and for each characteristic point a set of color and geometrical features is extracted. Then the sets of features are compared to decide whether two subimages match or not. Two matching pieces are then merged together to form a new subimage and the algorithm proceeds with the new subimage and the remaining subimages until either all subimages are merged to one image or no more matching subimages can be found. Instead of color information Sagiroglu et al. [SE06] use textural features and geometrical constraints to determine matching pieces. They propose a texture prediction algorithm which predicts pixel values in a band outside the border of a piece. The textural features of this predicted band are then correlated with the original pictorial specifications of possible neighboring pieces. They aim to maximize the matching and continuity of the texture while the geometrical constraints are satisfied.

Nielsen et al. [NDH08] are the first to only use image features to solve a jigsaw puzzle. They consider only a single-pixel wide continuous strip for each edge of a piece. Two pieces are considered a likely match if there is little to no gradient at their common edges. This approach allows them to solve not only jigsaw puzzles with many similar or identically shaped pieces, but also jigsaw puzzles that consist only of rectangular pieces. Cho et al. [CAF10] carry on this idea and only look at puzzles that consist of square pieces. They develop a graphical model to solve the jigsaw puzzles. The puzzle is built around a set of "anchor pieces" whose position is fixed at the correct location before the other pieces are placed. Fixing as few as 4 to 6 pieces at their correct location is enough to solve puzzles of over 400 pieces. Their dissimilarity based compatibility function to quantify the pairwise distance between two pieces became the basis of most future work. To determine how similar two pieces are the sum-of-squared color differences along the abutting boundary is calculated. Yang et al. [YAL11] improved the results of Cho et al. [CAF10] by using a particle filter. Additionally they do not assume any prior knowledge on the image layout or beforehand correct placed pieces.

Pomeranz et al. [PSBS11] are the first to propose an automatic square jigsaw puzzle solver which is based on a greedy placer and a novel prediction based dissimilarity. Similar to Yang et al. [YAL11] their solver does not require prior knowledge about the image or clues about the pieces' location. The only input to the solver are the pieces themselves, their orientation and the puzzle

dimensions. The greedy solver then solves the jigsaw puzzle in several steps. First the compatibility function is calculated to measure the affinity between pieces. Second the placement is executed. Given a single piece or a partial constructed puzzle, the placer tries to find the position of the remaining parts on the board. This is then followed by a segmentation step. Given the placements of all pieces on the board from the previous step this partial solution is divided into segments which are estimated to be assembled correctly, disregarding their absolute location. In a forth step shifting is performed. Given a set of puzzle segments these segments and remaining individual part are relocated on the board such that a better approximate solution is obtained. These steps are then repeated until the evaluation of the best buddies metric, a metric to determine how likely neighboring pieces are true neighbors, reaches a local maximum. Gallagher et al. [Gal12] generalize the approach of Pomeranz et al. [PSBS11] to also handle pieces of unknown orientation and puzzles with no information about their dimensions by using a greedy tree algorithm. Additionally they use a Mahalanobis-inspired jigsaw piece compatibility measure. Son et al. [SHC14] extend the approach of Gallagher et al. [Gal12] by adding loop constraints. They use a loop-based strategy to reconstruct jigsaw puzzles from the local matching candidates. Their algorithm seeks out and exploits loops as a form of outlier detection.
Sholomon et al. [SDN13], in turn, introduce a genetic algorithm as a strategy for piece placement. A genetic algorithm contains a population of chromosomes which in this case are placements of all puzzle pieces. They start with 1,000 chromosomes this means 1,000 random placements. In each generation the population is evaluated using a fitness function which is based on the pairwise compatibility of every pair of adjacent pieces. Then a new population is produced by the selection of chromosomes and the crossover of chromosome pairs. The probability with which a chromosome is chosen for either crossover or directly becoming part of the next generation is directly proportional to the value of its fitness function.
The approach of Paikin & Tal [PT15], which we implement in this project, is inspired by the work of Pomeranz et al. [PSBS11] and also uses a greedy algorithm. Similar to previous works the placement of the pieces is based on the compatibility between pieces. But they propose a more accurate and faster compatibility function that takes not only advantage of the similarity between pieces but also takes into account the reliability of this similarity. Since a greedy algorithm is extremely vulnerable to early errors they take special care when selecting the first piece to place. Unlike previous works, where the first piece is randomly selected, Paikin & Tal require the first piece to have distinctive borders and lie in a distinctive region. In addition, they do not choose the best piece for a specific location, but that piece that minimizes the likelihood of making a mistake, regardless of its position. By this approach they are able to also solve jigsaw puzzles with additional challenges like puzzles with missing pieces, puzzles of unknown size, puzzles with unknown orientation of the pieces and multiple puzzles whose pieces are mixed together and neither the size of the puzzles is known or information on possibly missing pieces are given. Only

the number of puzzles to solve is known.
A more detailed description of the puzzle solver by Paikin & Tal will be provided in the following chapter in which we elaborate on our implementation of the solver.

Chapter 3

Materials and Methods

Chapter 4

Results

Chapter 5

Discussion

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