linear-regression

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1 Fundamentals of Machine Learning - Exercise 5

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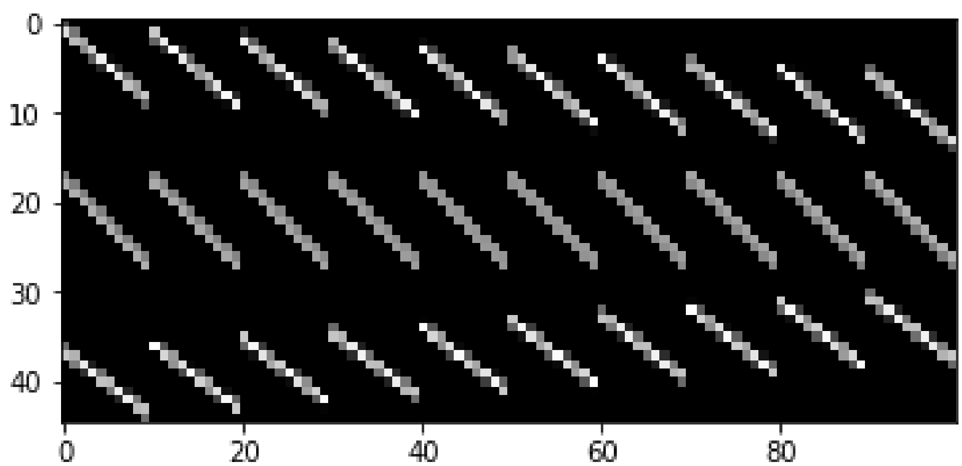
```
In [1]: import numpy as np
        from scipy.sparse import linalg
        from scipy.sparse import coo_matrix
        from scipy.sparse import csc_matrix
        import matplotlib.pyplot as plt
```

```
In [2]: def construct_X(M, alphas, Np=None):
            if not Np:
                Np = np.ceil(np.sqrt(2) * M).astype(int)
            if Np % 2 == 0:
                Np += 1
            D = M * M
            No = len(alphas)
            N = Np * No
            C1 = (np.mgrid[0:M, 0:M][0]).flatten()
            C2 = (np.mgrid[0:M, 0:M][1]).flatten()
            C = np.vstack((C1, C2))
            # centralise the coordinates in C
            C = C - (M - 1) / 2
            # convert alphas to radian
            alphas_rad = np.radians(alphas)
            # now make vectors out of the angles
            n = np.zeros((2, No))
            n[0] = -np.sin(alphas_rad)
            n[1] = np.cos(alphas_rad)
            p = n.T.dot(C) + (Np - 1) / 2
            lower_element = np.floor(p).astype(int)
            upper_element = np.ceil(p).astype(int)
            lower_value = upper_element - p
            upper_value = p - lower_element
            weights = []
            i_indices = []
            j_indices = []
```

```
    for i in range(No):
        for j in range(len(lower_element[i])):
            if (lower_element[i][j] == upper_element[i][j]):
                weights.append(1)
                i_indices.append(i*Np+lower_element[i][j])
                j_indices.append(j)
            else:
                weights.append(lower_value[i][j])
                i_indices.append((i*Np+lower_element[i][j]))
                j_indices.append(j)
                weights.append(upper_value[i][j])
                i_indices.append(((i*Np+upper_element[i][j])))
                j_indices.append(j)
    X = coo_matrix((weights, (i_indices, j_indices)), shape=(N, D), dtype=np.float32)
#    print('Matrix is ready')
    return X
```

In [3]:
```
X_testing = construct_X(10, [-33, 1, 42])

X_dense = X_testing.todense()
plt.imshow(X_dense, interpolation="nearest", cmap=plt.cm.Greys_r)
plt.show()
```

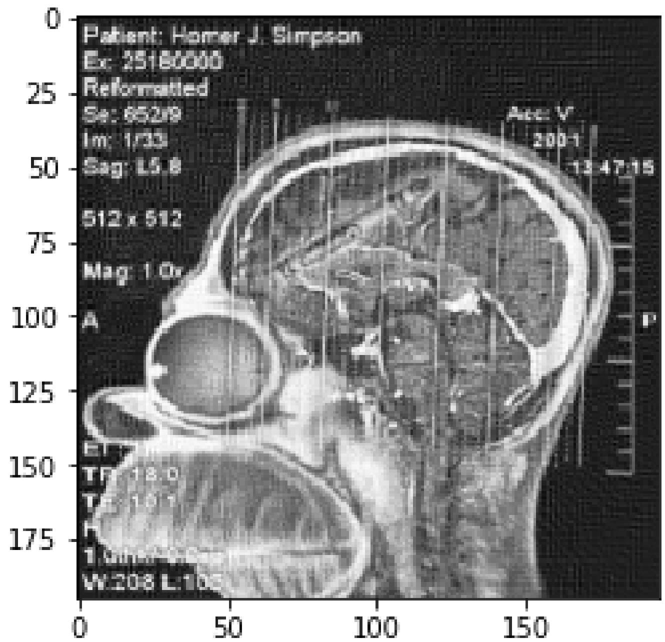


1.1 2 Recovering the image

In [4]:
```
alphas195 = np.load('hs_tomography/alphas_195.npy')
X = construct_X(195, alphas195, 275)
print('Number of non-zero elements: ' + str(X.nnz))
total_elements = X.shape[0] * X.shape[1]
zeros = total_elements - X.nnz
sparsity = zeros/total_elements
```

```
print('Sparsity: ' + str(sparsity))
X_csc = X.tocsc()
y = np.load('hs_tomography/y_195.npy')
img = linalg.lsqr(X_csc, y, atol=1e-05, btol=1e-05)
img = img[0]
img = img.reshape((195, 195))
plt.imshow(img, cmap=plt.cm.Greys_r)
plt.show()
```

```
Number of non-zero elements: 13535957
Sparsity: 0.992768406286928
```

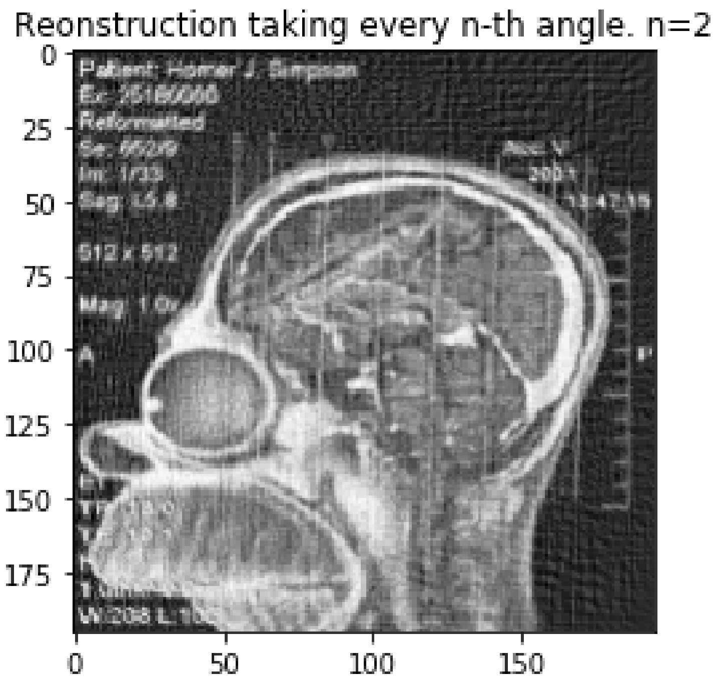


Diagnosis: He has a crayon in his brain. Proposed Treatment: Melt the crayon and then withdraw the molten wax by suction. Make a candle out of the wax as a souvenir.

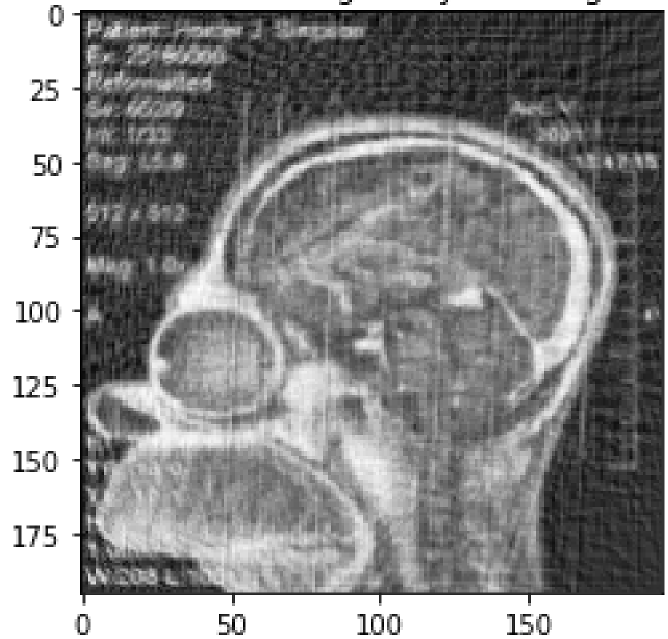
1.2 3 Minimizing the radiation dose

```
In [5]: alphas = np.load('hs_tomography/alphas_195.npy')
        y = np.load('hs_tomography/y_195.npy')
        Np = 275
        length = len(alphas)
        stepsizes = [2, 3, 4, 5]
        for j in range(len(stepsizes)):
            step = stepsizes[j]
            newalphas = []
            newy = np.array([])
```

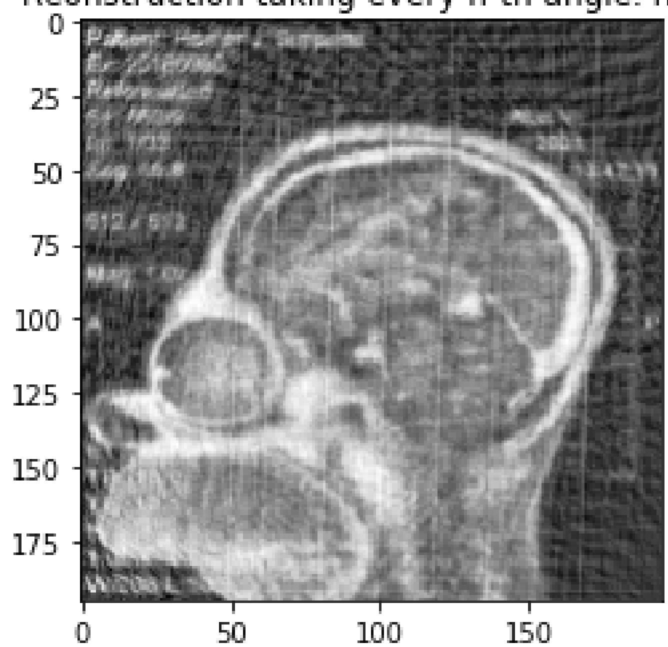
```python
for i in range(0, length, step):
    newalphas.append(alphas[i])
    data = y[np.arange(i*Np, i*Np+Np)]
    newy = np.concatenate((newy, data))
reduced_X = construct_X(195, newalphas, 275)
reduced_X_csc = reduced_X.tocsc()
img = linalg.lsqr(reduced_X_csc, newy, atol=1e-05, btol=1e-05)
img = img[0]
img = img.reshape((195, 195))
#plt.figure(figsize=(5,5))
plt.imshow(img, cmap=plt.cm.Greys_r)
plt.title("Reonstruction taking every n-th angle. n=" +str(step))
plt.show()
```

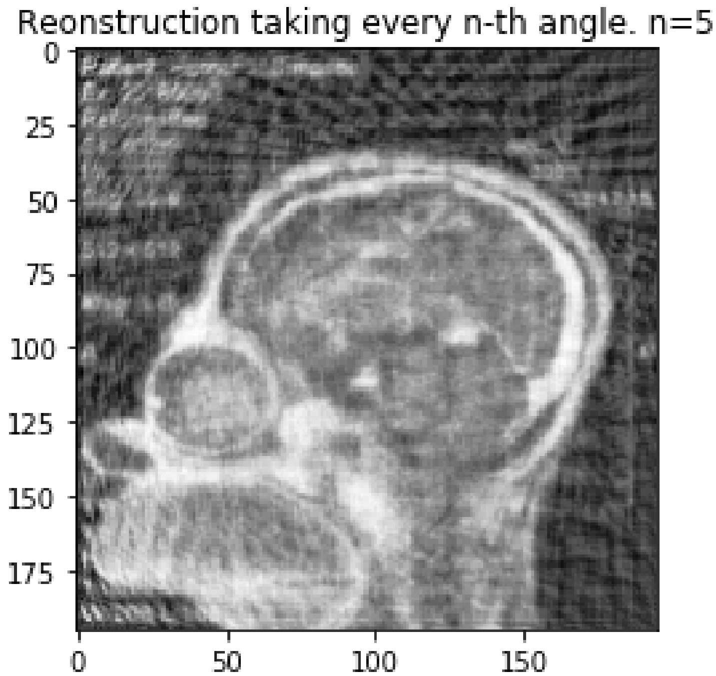


Reonstruction taking every n-th angle. n=3



Reonstruction taking every n-th angle. n=4





Taking every 5th angle of the original dataset one can see the head, but not really what causes the headache. Taking every 4th angle one starts to see that there ist a foreign object inside the brain of the patient. Taking every 3rd angle and being a good radiologist makes it possible to identify the object as a crayon. A bad radiologist might need the reconstruction taking every second angle. So to determine that the headache is caused by a foreign object in the patient's brain one needs at least 44 projections. To determine the nature of the foreign object one needs at least 59 projections and a good radiologist. If you don't have a good radiologist at hand the minimum number of projections lies inbetween 59 and 89.