Billiard Table Simulation

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Abstract

A game of Billiard or Pool looks fairly simple but the mechanics behind the motion of the balls around the table is very interesting. By a computational model of the table we can observe what all the important factors that one needs to consider when playing a game. Friction slows the balls down as they travel. The walls almost conserve all linear momentum when impacted with. A collision of the balls can modeled as total momentum and energy conservation. The rotation of the ball can change the trajectory on the table as it interacts with the friction between ball and table. The walls were viewed with 2 extreme scenarios of angular momentum conservation.

Introduction

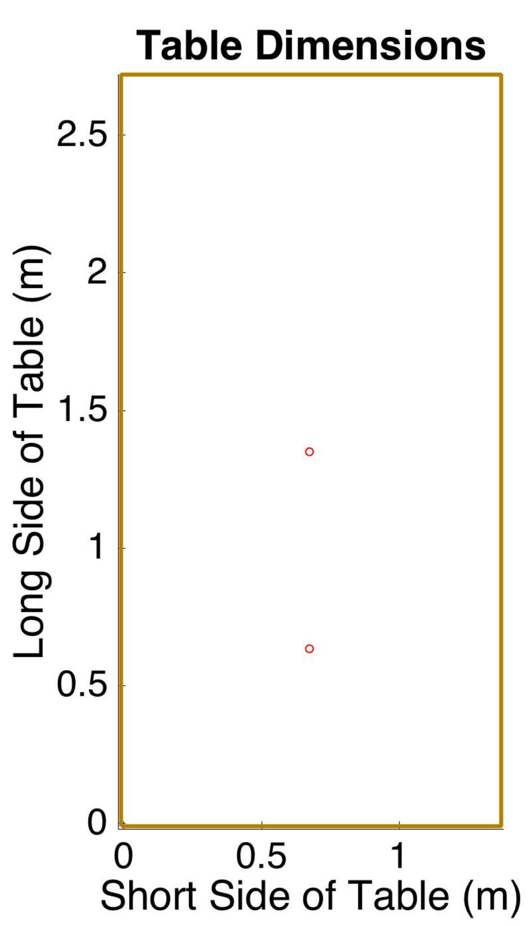


Figure1: Initial Set up the Table with balls placed as indicated and dimensions laid out

A standard Billiard Table has the dimensions shown in Figure 1. We have considered a few parameters to focus on in this simulation.

- Friction
- Linear Momentum
- Angular Momentum
- Rotational Force

A game of Billiard highly depends on the interaction between balls and walls. Where a more experienced player will also use the spin of the ball as a major contributor to their game [2]. Knowing the exact behavior of all the balls once a shot is made is a crucial part to being a good player.

Methods

We are modeling all the forces on both balls as they are moving throughout the table until the total force goes to zero. We are calculating the total acceleration at each step which we then progress with one step of Euler approximation a numerical differential equation solver [3]. The equation below calculates the acceleration .

$$\ddot{r} = -[(\frac{C_D\rho A(\vec{v}.\vec{v})}{2m})(v_x\hat{i}+v_y\hat{j}+v_z\hat{k})] - [g\hat{k}] + \qquad (1)$$

$$(\frac{C_L\rho A(\vec{v}.\vec{v})}{2m})\frac{1}{||w||||v||}([w_yv_z - w_zv_y]\hat{i} - [w_xv_z - w_zv_x]\hat{j} + [w_xv_y - w_yv_x]\hat{k}) - \frac{mguv}{A}]$$

[1]

This model allows for 3 dimensional motion which we need for the spin component but we are limiting the Z-Direction for linear motion.

Results

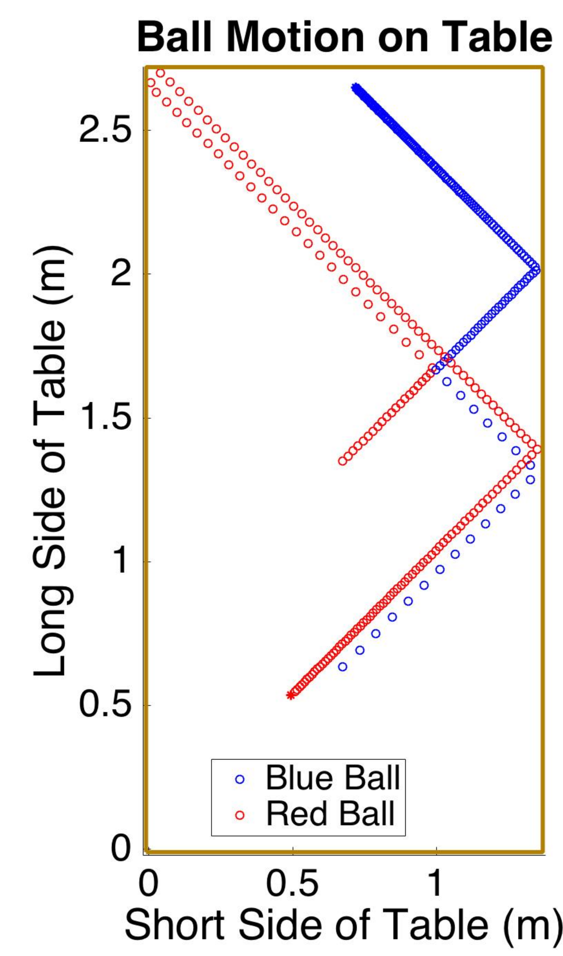


Figure 2: Simulation of a 2 ball collision where momentum and kinetic energy is conserved

As seen in Figure 1 two balls are starting at different velocities as ball 1(Blue Ball) has a collision with the wall before it collides with the red ball. The collision of the two balls completely conservers energy and momentum. The speed of the balls can be seen with the distance in between plotting points. Upon impact the blue ball looses velocity and the red one gains velocity as an effect from conservation of momentum and them being the same mass, this can also be seen in the Figure 3.

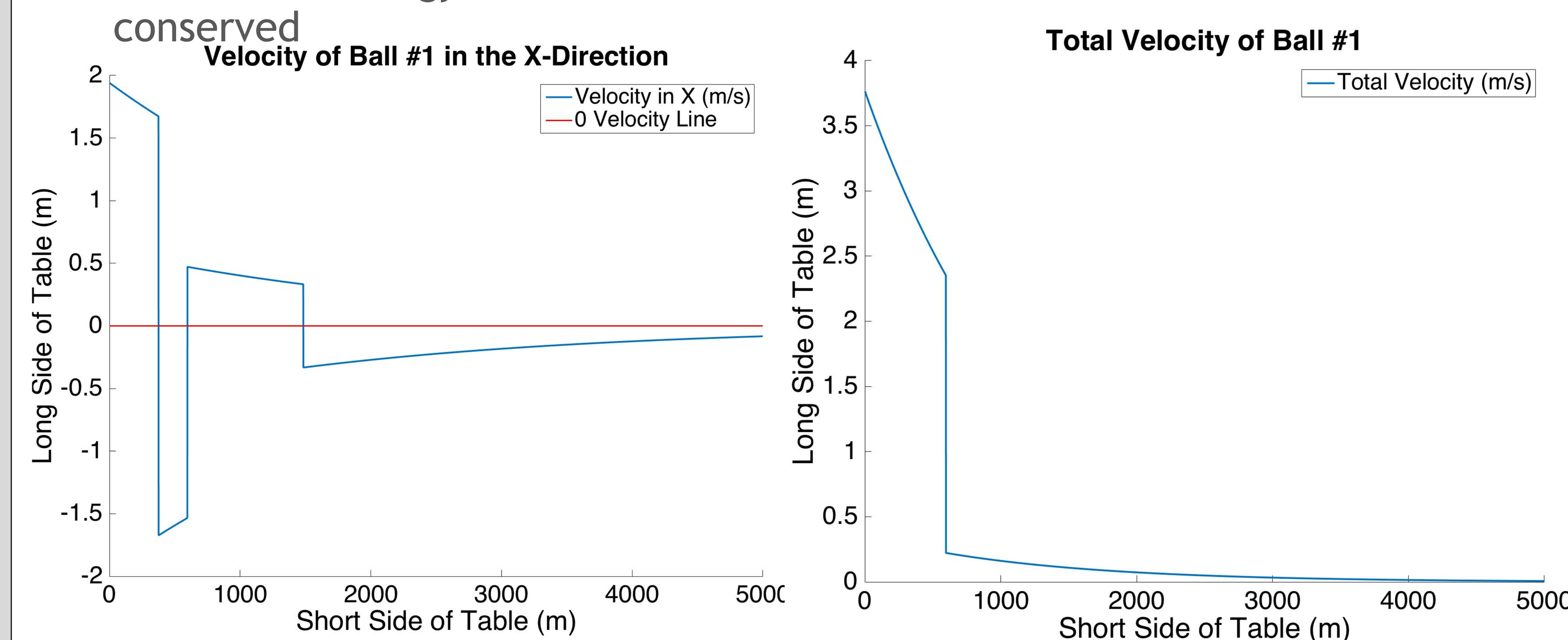
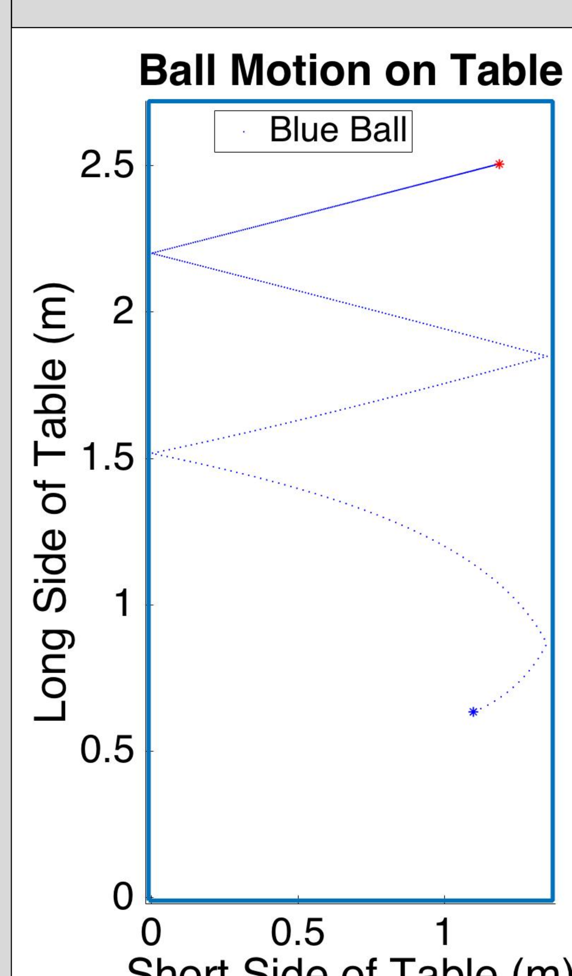


Figure 3: The velocities of Ball #1(Blue Ball) . (a) Velocity in the X-Direction where impact and change in momentum can be seen. (b) Total Velocity where impact can clearly be seen in the drop of velocity with the other ball

Angular Momentum Conservation



For looking at the Rotation of the ball we are only considering the Blue ball. A rotation of the ball around the Z-Axis allows the ball to take curved paths across the table. The increase of the rotation increases the amount of curve as well. In this first case the ball conserves all angular momentum and only changes linear direction when it encounters the wall as can be seen in Figure 4. The Spin is held in the same direction.

Figure 4: All angular momentum is conserved and rotation is kept the same after impact (blue * =start, red * =finish)

Results

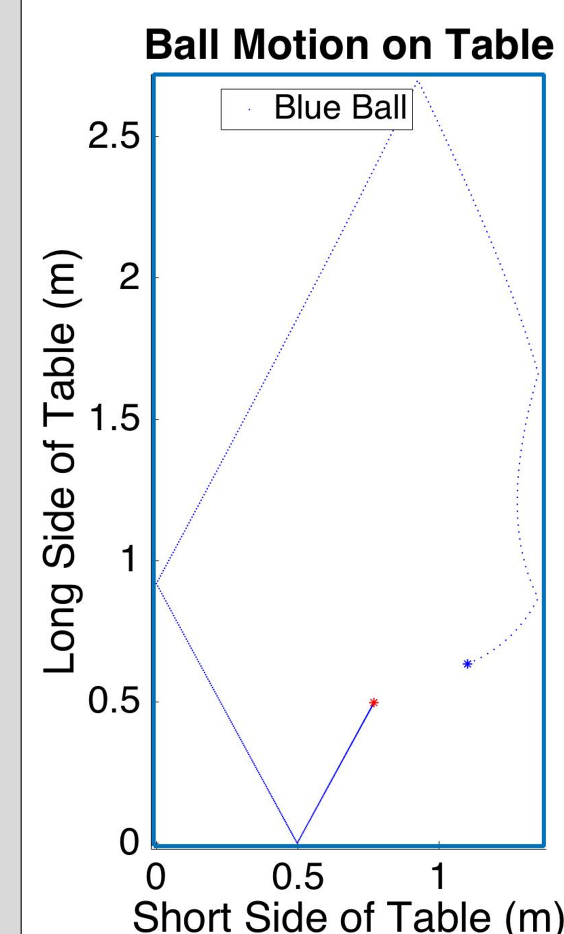
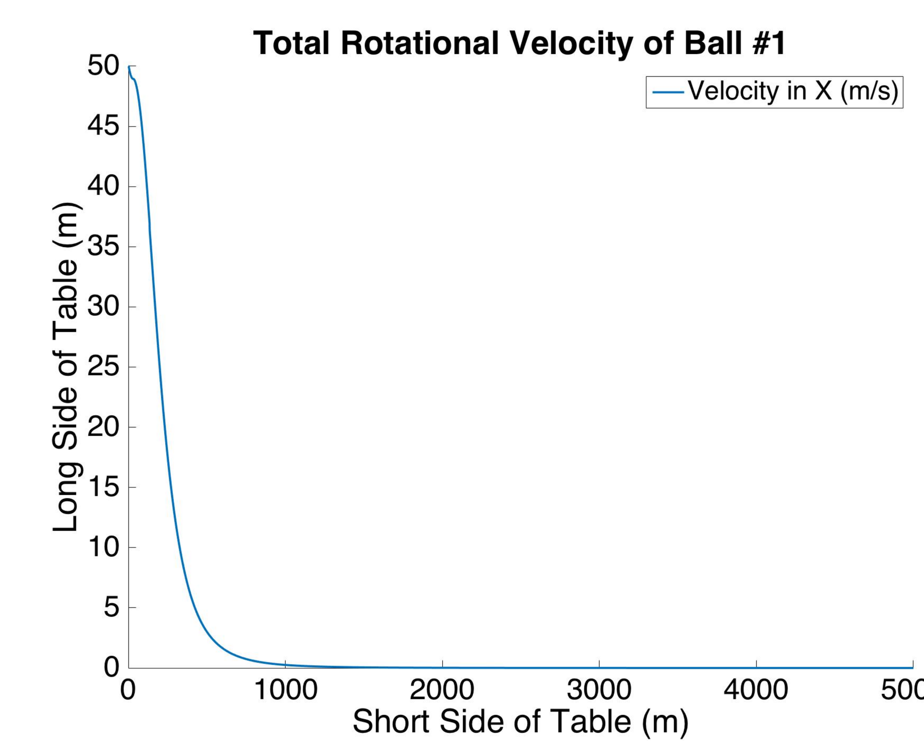


Figure 5: Rotation was reversed at impact with wall, as if coefficient of friction was just before getting sticky. (blue * =start, red * =finish)

The second case we observe the other extreme of the coefficient of friction between the wall being very high. If the coefficient of friction is just before becoming sticky the ball experiences a change in rotational velocity. In this case the magnitude stays the same but the direction changes, which can be seen in Figure 5. This change completely alters the final outcome of the trajectory and the position.



The rotation also undergoes friction which slows it down overtime. In this simulation we made it exponentially proportional to the linear velocity as can be seen in Figure 6. This seemed to model the ball most accurately. The decrease in spin can also be observed in the other rotational Figures

Figure 6: Total rotational velocity of the ball as the spin slows down exponentially with the linear velocity

Conclusions

The simple looking game of Billiard has endless parameters which make the game more interesting. By assuming conservation of momentum and energy and taking into account friction a linear game can be modeled very well. Rotation on the ball makes this more challenging. Where making a few assumptions and looking at the extremes we can predict some paths around the table even for different types of friction between the ball and wall.

References

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2. David G. Alciatore, The Amazing World of Billiards Physics, Colorado State University, May 2007
3. Faires Douglas, Burden Richard, Numerical Methods, Brooks/Cole, Fourth Edition, 2013