

OMEN-SED 1.0: A new, numerically efficient sediment module for the coupling to Earth System Models

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Abstract. Here we describe the first version of the Organic Matter ENabled SEDiment model (OMEN-SED 1.0).

1 Introduction

DH: How to include comments.

Role of marine sediments for climate and global biogeochemical cycles:

- 5 Marine surface sediments are key components in the Earth system. They host the largest carbon reservoir within the surficial Earth system, provide the only long term sink for atmospheric CO₂, recycle nutrients and represent the most important geochemical archive used for deciphering past changes in biogeochemical cycles and climate. Physical and chemical processes in sediments (i.e. diagenetic processes) depend on the water column and vice versa: Diagenesis is mainly donor controlled, as it is fuelled by the external supply of solid material (e.g. organic matter, calcium carbonate, opal) from the water column and is affected by overlying bottom water concentrations of solutes. At the same time, diagenesis in the sediments transforms the deposited material and returns some of the resulting products (e.g. nutrients, DIC) to the water column. This so-called benthic-pelagic coupling is essential for understanding global biogeochemical cycles and climate (e.g. Archer and
- 10 Maier-Reimer, 1994; Soetaert et al., 2000; Archer et al., 2000; Mackenzie, 2005).
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Biological primary production of organic matter (OM, CH₂O in equation R1) and the reverse process of degradation can be written in a greatly simplified reaction as:



- On geological timescales production of OM is generally greater than degradation which results in
- 20 some organic matter being buried in marine sediments and oxygen accumulating in the atmosphere.

Thus burial of OM leads to net oxygen input to, and CO₂ removal from the atmosphere (Berner, 2004). On shorter timescales, the upper few meters of the sediments (i.e. early diagenesis) are specifically important as it is decided here if a substance is recycled to the water column or buried for a longer period of time in the deeper sediments (Hensen et al., 2006). Most biogeochemical cycles
25 and reactions in this part of marine sediments can be related either directly or indirectly to the degradation of organic matter (e.g Boudreau and Ruddick, 1991; Arndt et al., 2013). Organic matter degradation releases metabolic CO₂ to the pore water, causing it to have a lower pH and provoking the dissolution of CaCO₃ (Emerson and Bender, 1981). Oxygen and nitrate for instance, the most powerful electron acceptors, are consumed in the course of the degradation of organic matter, resulting
30 in the release of ammonium and phosphate to the pore water. As such, changes in the degradation rate of OM can profoundly affect the oxygen and nutrient inventory of the ocean and thus primary productivity (Van Cappellen and Ingall, 1994; Lenton and Watson, 2000).

TODO: Something on: Role of marine sediments for another time period , e.g. glacial/interglacial atmospheric pCO₂ cycles

35 Nutrient recycling from the sediments has been invoked by modelling and data studies to explain the occurrence of extreme events in Earth history, for instance Oceanic Anoxic Events (OAEs) (e.g. Mort et al., 2007; Tsandev and Slomp, 2009). OAEs represent severe disturbances of the global carbon, oxygen and nutrient cycles of the ocean and are usually characterized by widespread bottom water anoxia and photic zone euxinia (Jenkyns, 2010). One way to explain the genesis and retention
40 of OAEs is increased oxygen demand due to enhanced primary productivity (PP). Increased nutrient inputs to fuel PP may have come from marine sediments as the burial efficiency of phosphorus declines when bottom waters become anoxic (Ingall and Jahnke, 1994; Van Cappellen and Ingall, 1994). The recovery from OAE like conditions is thought to involve the permanent removal of excess CO₂ from the atmosphere and ocean by burying carbon in the form of organic matter in marine
45 sediments (e.g. Arthur et al., 1988; Jarvis et al., 2011), which is consistent with the geological record of widespread black shale formation (Stein et al., 1986). However, the overall amount, exact timing and the rate of organic matter burial remain a topic of an ongoing debate.

Diagenetic Models:

Therefore, globally quantifying the burial and degradation of organic matter in marine sediments and
50 related biogeochemical dynamics is important for understanding climate and the cycling of many chemical elements on various timescales. Such studies and quantifications are possible through the application of idealised mathematical representations of diagenesis, or so-called diagenetic models (e.g. Berner, 1980; Boudreau, 1997). The number of research questions that can be addressed with diagenetic models is infinite and a plethora of different approaches have been developed, mainly
55 following two distinct directions (Arndt et al., 2013). First, state-of-the art vertically resolved diagenetic models simulating all of the essential coupled redox and equilibrium reactions within marine sediments that control carbon burial and benthic recycling fluxes (e.g. BRNS, Aguilera et al., 2005;

MEDIA, Meysman et al., 2003; OMEXDIA, Soetaert et al., 1996b). These “complete”, non-steady-state models generally use a so-called multi-G approach, thus dividing the bulk organic carbon pool
60 into a number of compound classes that are characterised by different degradabilities k_i . Combining such a complex diagenetic model with an ocean biogeochemical model results in the most realistic benthic-pelagic coupling. However, the global applicability of this set-up is limited by the high computation cost of simulating all biogeochemical reactions in the sediments. The second group of models is less sophisticated and comprehensive than the “complete”, non-steady-state diagenetic
65 models and is used for the coupling to global Earth System Models (e.g. DCESS, Shaffer et al., 2008; MEDUSA, Munhoven, 2007 or the model of Heinze et al. (1999)). In particular, these analytic models consider fewer biogeochemical reactions and assume that the sedimentary organic matter pool is composed of just a single or two discrete compound classes (1G or 2G approach respectively).

70 **How are sediments resolved in Earth System models:**

Even though there is potential to use more appropriate global sediment representations, in most current Earth System models sediment-water exchange of OM and chemical elements is either neglected or treated in a very simplistic way (Soetaert et al., 2000; Hülse et al., 2016). Most Earth system Models of Intermediate Complexity (EMICs) and also some of the higher resolution global carbon
75 cycle models represent the sediment-water interface either as a reflective or a conservative/semi-reflective boundary (Hülse et al., 2016). Thus, all particulate material deposited on the seafloor is either instantaneously consumed (reflective boundary), or a fixed fraction is buried in the sediments (conservative/semi-reflective boundary). Both highly simplified approaches furthermore completely neglect the exchange of solute species through the sediment-water interface and, therefore, can-
80 not resolve the complex benthic-pelagic coupling. However, due to their computational efficiency, both representations are often used in global biogeochemical models (e.g. Najjar et al., 2007; Ridgwell et al., 2007; Goosse et al., 2010). A superior approach is the vertically integrated dynamic model, which represents the whole sediment column as a single box (Hülse et al., 2016). Here, OM deposited on the seafloor is added to the sediment box where it gets degraded and dissolved
85 species diffuse through the sediment-water interface in accordance with these transformations. This approach thus ignores the vertical extent of the sediments and the temporary storage of dissolved species (Soetaert et al., 2000). Yet, it is computationally efficient and also allows differentiating between various fractions of organic matter. Most EMICs incorporate a vertically integrated dynamic model for particulate inorganic carbon only (i.e. mainly CaCO_3) and just a few consider oxic-only
90 sediment degradation of organic matter (Hülse et al., 2016). The most complex description of marine sediments in Earth System models are vertically resolved diagenetic models, solving the one-dimensional, fully coupled reaction-transport equation for multiple solid and dissolved species (e.g. Berner, 1980; Boudreau, 1997). These models account for important transport processes (e.g. bioturbation, molecular diffusion and bioirrigation), resolve the coupled biogeochemical dynamics of

95 dissolved species and usually at least a part of the resulting characteristic redox-zonation of marine sediments (Soetaert et al., 2000; Hülse et al., 2016). Examples of global carbon cycle models employing a vertically resolved diagenetic model are PISCES (Gehlen et al., 2006) and CLIMBER 3 α (Ilyina et al., 2013), both using a version of Heinze et al. (1999), however, just utilizing a 1G approach with a constant degradation rate constant. None of the EMICs reviewed by Hülse et al.
100 (2016) use such a sediment representation.

Problem with that:

Obviously, such simplifications can neither account for the observed vast structural complexity in natural organic matter and its resulting different degradation rates nor for the rapid decrease in OM degradability in the uppermost centimeters of the sediments (Arndt et al., 2013). At least a 3G approach is necessary to accurately represent organic matter dynamics in this part of the sediments where most OM is degraded (e.g. Soetaert et al., 1996b). Furthermore, the spatial variability in benthic OM degradation kinetics is unknown at the global scale and reported rate constants can vary by almost 10 orders of magnitude (Arndt et al., 2013). Thus, a major challenge for diagenetic models is defining appropriate OM degradation rate constants which is either achieved through profile fitting for a specific site or, for global applications, the rate constants follow empirical relations with a related, readily available sediment characteristic such as water depth (Middelburg et al., 1997), sedimentation rate (Toth and Lerman, 1977; Tromp et al., 1995) or OM flux (Boudreau, 1997). However, these relationships are mostly based on simple fitting exercises to limited data sets and show at the most a very weak trend and no statistically significant relationship (Arndt et al., 2013). This creates considerable uncertainties when diagenetic models are coupled to global biogeochemical ocean models especially when applying the model for different geological timescales. Other model parameters implicitly accounting for processes not explicitly described are sedimentation and bioturbation rate and are generally related to water depth following Middelburg et al. (1997).

Solution presented here:

120 Here, we present the OrganicMatter ENabled SEDiment model (OMEN-SED 1.0), a new, one-dimensional, numerically efficient reactive transport model (RTM) that describes OM cycling as well as the associated dynamics of the most important terminal electron acceptors (TEAs, i.e. O₂, NO₃, SO₄, CH₄), related reduced substances (NH₄, H₂S), macronutrients (PO₄) and associated pore water quantities (ALK, DIC). OMEN-SED's computational efficiency allows its coupling to
125 Earth System Models of different complexities and therefore the investigation of coupled global biogeochemical dynamics over geological timescales. Here, the model is presented as a 2G-approach, however, a third, nondegradable OM pool can be added and furthermore OMEN-SED can easily be extended to a Multi-G approach.

2 Model Description

130 The following section provides a detailed description of the new model. Table 1 summarizes the biogeochemical reaction network and a glossary of parameters along with their respective units is provided in Tables 9 and 10.

2.1 General Model Approach

135 The calculation of benthic return/uptake and burial fluxes is based on the vertically resolved conservation equation for solid and dissolved species in porous media is given by (e.g. Berner, 1980; Boudreau, 1997):

$$\frac{\partial \xi C_i}{\partial t} = -\frac{\partial F}{\partial z} + \xi \sum_j R_i^j \quad (1)$$

140 where C_i is the concentration of the biogeochemical species i , ξ equals the porosity ϕ for solute species and $(1 - \phi)$ for solid species, hence represents the partitioning of species i into the solute and dissolved phase. The term z is the sediment depth, t denotes the time, F summarises the transport fluxes and $\sum_j R_i^j$ represents the sum of production/consumption rates j that affect species i . The reaction network accounts for the most important primary and secondary redox reactions, equilibrium reactions, mineral dissolution and precipitation, as well as adsorption and desorption processes that 145 affect the explicitly resolved chemical species.

State-of-the-art reaction-transport models generally solve the ordinary differential equation (ODE) (eq. 1) numerically and thus allow to account for transient dynamics, depth-varying parameters or a high degree of coupling between different chemical species (e.g. Soetaert et al., 1996b; Aguilera et al., 2005). Yet, numerical models are computational expensive, thus rendering their application in 150 an Earth System Model framework prohibitive. An analytical solution of equation (1) provides an alternative and computational more efficient approach which enjoyed great popularity in the early days of diagenetic modelling and computer technology. However, early analytical models were often very problem-specific and only considered one or two coupled species (e.g. Berner, 1964). ([Lehrman, Berner](#) ?? which other pub? Over the next decades, a number of more complex analytical 155 models describing the coupled dynamics of OM degradation, TEAs and reduced substances in sediments were developed (e.g. Billen, 1982; Goloway and Bender, 1982; Jahnke et al., 1982), before the boost in computing power enabled the development of fully-coupled, multi-species, numerical models (e.g. Van Cappellen and Wang, 1995; Soetaert et al., 1996b).

160 Finding an analytical solution to Eq. (1), especially when complex reaction networks are to be considered, is not straightforward and generally requires the assumption of steady state. In addition, the complexity of the reaction network can be reduced by dividing the sediment into distinct zones and accounting for the most pertinent biogeochemical processes within each zone, thus increasing the likelihood of finding an analytical solution to Eq. (1).



Figure 1. Schematic of the different modelled species and layers in our sediment model. Here showing the case $z_{\text{ox}} < z_{\text{bio}} < z_{\text{NO}_3} < z_{\text{SO}_4}$.

Therefore, OMEN-SED assumes that benthic dynamics can be represented by a series of steady-states. Because the Earth system model relevant variability in boundary conditions and fluxes is generally longer than the characteristic timescales of the reaction-transport processes, the sediment can be described by a series of pseudo steady-states. In addition, it divides the sediment into a bioturbated and a non-bioturbated zone defined by the constant bioturbation depth z_{bio} (see Fig. 1). Furthermore, it accounts for the dynamic redox zonation of marine sediments by dividing the sediment into: 2) an oxic zone delineated by the oxygen penetration depth z_{ox} , 3) a denitrification zone situated between z_{ox} and the nitrate penetration depth z_{NO_3} , 4) a sulfate reduction zone situated between z_{NO_3} and the sulfate penetration depth z_{SO_4} and 5) a methanogenic zone situated below z_{SO_4} (Fig. 1). All penetration depths are dynamically calculated by the model. Each zone is characterised by a set of diagenetic equations that encapsulate the most pertinent reaction and transport processes in this zone (see section 2.2 and 2.3 for more details).

OMEN calculates and feeds back to the Earth System model the fraction of POC preserved in the sediments and the sediment-water interface fluxes of the dissolved species C_i :

$$\text{Flux_SWI}(C_i) = \phi \cdot \left(D_i \frac{\partial C_i}{\partial z} + w C_i(0) \right) \quad (2)$$

where w is the deposition rate, D_i is the diffusion coefficient and $C_i(0)$ the sediment-water concentration of species i .

2.2 Transport

The model accounts for both the advective, as well as the diffusive transport of dissolved and solid species, assuming that sediment compaction is negligible (i.e. $\frac{\partial \phi}{\partial z} = 0$). The molecular diffusion of dissolved species is described via a species-specific apparent diffusion coefficient, $D_{\text{mol},i}$. In
185 addition, the activity of infaunal organisms in the bioturbated zone of the sediment ($z < z_{\text{bio}}$) that causes random displacements of sediments and porewaters is simulated using a diffusive term (e.g. Boudreau, 1986), with a constant bioturbation coefficient D_{bio} in the bioturbated zone. The pumping activity by burrow-dwelling animals and the resulting ventilation of tubes, the so-called bioirrigation, is encapsulated in a factor, f_{ir} that enhances the molecular diffusion coefficient (hence, $D_{i,0} =$
190 $D_{\text{mol},i} \cdot f_{ir}$, Soetaert et al., 1996a). The flux divergence can thus be formulated as:

$$\frac{\partial F}{\partial z} = -\frac{\partial}{\partial z} \left(-\xi D_i \frac{\partial C_i}{\partial z} + \xi w C_i \right) \quad (3)$$

where D_i is the diffusion coefficient of species i ($D_i = D_{i,0} + D_{\text{bio}} = D_{\text{mol},i} \cdot f_{ir} + D_{\text{bio}}$ for dissolved species and $D_i = D_{\text{bio}}$ for solid species) and w is the deposition rate. The bioturbation coefficient D_{bio} is set to zero below z_{bio} . In addition, infaunal activity ceases ($D_{\text{bio}} = 0$) once bottom waters
195 become anoxic ($O_2 < ??? \text{ mol cm}^{-3}$). check for good value + add if-query in code!!

2.3 Reaction Network

Earth System models generally track the biogeochemical dynamics of organic and inorganic carbon, essential nutrients (nitrogen, phosphorus) and oxygen with the aim of investigating the evolution of the ocean's redox structure and carbonate system and its feedbacks on global climate. This general
200 aim thus defines a minimum set of state variables and reaction processes that need to be resolved for an efficient representation of the benthic-pelagic coupling in Earth system models. A suitable sediment model must provide a robust quantification of organic (and inorganic) carbon burial fluxes, as well as the benthic return fluxes of growth-limiting nutrients, equilibrium invariant and reduced species, and oxygen uptake fluxes. As a consequence, the reaction network must account for the
205 most important primary and secondary redox reactions, equilibrium reactions, mineral precipitation/dissolution and adsorption/desorption, resulting in a complex set of coupled reaction-transport equations. The following subsections provide a short discussion of the reaction processes included in the model and give an overview of the vertically resolved conservation equations and boundary conditions for solid and dissolved species in each layer. Table 1 provides a summary of the reactions
210 and variables considered in the reaction network. Table 14 summarises their reaction stoichiometry and Table ?? provides an overview of their description in the model.

2.3.1 Organic matter

In marine sediments, organic matter (OM) is degraded by heterotrophic activity coupled to the sequential utilisation of terminal electron acceptors, typically in the order of O_2 , NO_3^- , $Mn(VI)$,

Table 1. Reactions and variables implemented in the Reaction Network of OMEN-SED (1.0). The primary and secondary redox reactions are listed in the sequence they occur with increasing sediment depth.

Description	
Primary redox reactions	Degradation of organic matter via aerobic respiration, denitrification, sulfate reduction, methanogenesis (implicit)
Secondary redox reactions	Oxidation of ammonium and sulfide by oxygen, anaerobic oxidation of methane by sulfate
Adsorption/Desorption	Ad-/Desorption of P on/from Fe(OH) ₃ , NH ₄ adsorption, PO ₄ adsorption
Mineral precipitation	Formation of authigenic P
Variables	Organic matter, oxygen, nitrate, ammonium, sulfate, sulfide (hydrogen sulfide), phosphate, Fe-bound P, DIC, ALK

Table 2. Boundary conditions for organic matter. For the boundaries we define: $z_{\text{bio}}^- := \lim_{h \rightarrow 0} (z_{\text{bio}} - h)$ and $z_{\text{bio}}^+ := \lim_{h \rightarrow 0} (z_{\text{bio}} + h)$.

Boundary	Condition	
$z = 0$	known concentration	1) $C_i(0) = C_{i0}$
$z = z_{\text{bio}}$	continuity	2) $C_i(z_{\text{bio}}^-) = C_i(z_{\text{bio}}^+)$ 3) $-D_{\text{bio}} \cdot \frac{\partial C_i}{\partial z} \Big _{z_{\text{bio}}^-} = 0$

215 Fe(III) and SO₄²⁻ followed by methanogenesis and/or fermentation. Here, organic matter degradation is described via a multi-G model approach (Arndt et al., 2013, and references therein), assuming that the bulk OM consists of a number of discrete compound classes C_i characterised by specific degradation rate constants k_i . Such a multi-G approach allows for selective preservation of compound classes according to their reactivity, k_i and, thus, accounts for the change in organic matter reactivity during burial. Each compound class is degraded according to first-order kinetics. The conservation equation for organic matter dynamics is thus given by:

$$\frac{\partial C_i}{\partial t} = 0 = D_{C_i} \frac{\partial^2 C_i}{\partial z^2} - w \frac{\partial C_i}{\partial z} - k_i \cdot C_i \quad (4)$$

The analytical solution of Eq. (4) (see section 2.5 for details) requires the definition of a set of boundary conditions (Table 2). The model assumes a known concentration/flux at the sediment-water interface and continuity across the bottom of the bioturbated zone, z_{bio} .

2.3.2 Oxygen

In marine sediments, oxygen is consumed through the aerobic degradation of organic matter and a number of secondary redox reactions. In the oxic layer ($z < z_{ox}$), the model explicitly accounts 230 for the aerobic degradation of OM, which consumes oxygen with a fixed O:C ratio (OC, Tab. 10)

Table 3. Boundary conditions for oxygen. For the boundaries we define: $z_{\text{bio}}^- := \lim_{h \rightarrow 0} (z_{\text{bio}} - h)$ and $z_{\text{bio}}^+ := \lim_{h \rightarrow 0} (z_{\text{bio}} + h)$.

Boundary	Condition	
$z = 0$	known concentration	1) $O_2(0) = O_{20}$
$z = z_{\text{bio}}$	continuity	2) $O_2(z_{\text{bio}}^-) = O_2(z_{\text{bio}}^+)$
$z = z_{\text{ox}}$	O_2 consumption ($z_{\text{ox}} = z_\infty$)	3) $-(D_{O_2,0} + D_{\text{bio}}) \cdot \frac{\partial O_2}{\partial z} \Big _{z_{\text{bio}}^-} = -D_{O_2,0} \cdot \frac{\partial O_2}{\partial z} \Big _{z_{\text{bio}}^+}$ 4) IF ($O_2(z_\infty) > 0$) $\frac{\partial O_2}{\partial z} \Big _{z_{\text{ox}}} = 0$ ELSE $O_2(z_{\text{ox}}) = 0 \quad \text{and} \quad -D_{O_2} \cdot \frac{\partial O_2}{\partial z} \Big _{z_{\text{ox}}} = F_{\text{red}}(z_{\text{ox}})$ $F_{\text{red}}(z_{\text{ox}}) = \frac{1-\phi}{\phi} \cdot \int_{z_{\text{ox}}}^\infty \sum_i (2\gamma_{\text{NH}_4} \text{NC}_i + \gamma_{\text{H}_2\text{S}} \text{SO}_4 \text{C}) k_i C_i dz$
	($z_{\text{ox}} < z_\infty$) with flux from below	

and produces ammonium, which is partially nitrified to nitrate (γ_{NH_4}). In addition, the oxygen consumption through the oxidation of reduced species (NH_4 , H_2S) produced in the suboxic and anoxic layers of the sediment is implicitly taken into account through the flux boundary condition at the dynamic oxygen penetration depth z_{ox} . This simplification can be justified as it has been shown that

235 these secondary redox reactions mainly occur at the oxic/suboxic interface (Soetaert et al., 1996b).

The factor $\frac{1-\phi}{\phi}$ accounts for the volume conversion from the solid to the dissolved phase. Oxygen dynamics are thus described by:

$$\frac{\partial O_2}{\partial t} = 0 = D_{O_2} \frac{\partial^2 O_2}{\partial z^2} - w \frac{\partial O_2}{\partial z} - \frac{1-\phi}{\phi} \sum_i k_i \cdot [\text{OC} + 2\gamma_{\text{NH}_4} \text{NC}_i] \cdot C_i(z) \quad (5)$$

240 The analytical solution of Eq. (5) requires the definition of boundary conditions (Table 3). The model assumes a known bottom water concentration and the complete consumption of oxygen at the oxygen penetration depth (or zero flux if $z_{\text{ox}} = z_\infty$). It considers equal oxygen concentration and diffusive flux above (z_{bio}^-) and below (z_{bio}^+) the bioturbation boundary. In addition, the model imposes a flux of reduced species through the bottom of the oxic zone that is calculated as the reduced substances 245 produced through anoxic mineralization of organic matter below z_{ox} . Thus, assuming that fractions (γ_{NH_4} and $\gamma_{\text{H}_2\text{S}}$) of these reduced species are oxidised at the oxic/suboxic interface.

2.3.3 Nitrate and Ammonium

To model nitrate and ammonium dynamics the sediment is partitioned into two geochemical layers (oxic and suboxic), where different equations describe the biogeochemical processes. Above the

250 oxygen penetration depth organic matter mineralization produces ammonium, which is partly nitrified to nitrate (the fraction γ_{NH_4}). In the suboxic zone ($z > z_{\text{ox}}$), oxygen concentration is zero and nitrate serves as the electron acceptor to respire organic matter, thus nitrate is consumed by denitrification and ammonium is produced. Below the nitrate penetration depth z_{NO_3} , ammonium is still produced through OM mineralization. Therefore the diagenetic equations for nitrate and ammonium

255 are given by:

1. Layer ($z \leq z_{\text{ox}}$)

$$\frac{\partial NO_3^I}{\partial t} = 0 = D_{NO_3} \frac{\partial^2 NO_3^I}{\partial z^2} - w \frac{\partial NO_3^I}{\partial z} + \gamma_{\text{NH}_4} \frac{1-\phi}{\phi} \cdot \sum_i NC_i \cdot k_i \cdot C_i(z) \quad (6)$$

$$\frac{\partial NH_4^I}{\partial t} = 0 = \frac{D_{\text{NH}_4}}{1+K_{\text{NH}_4}} \frac{\partial^2 NH_4^I}{\partial z^2} - w \frac{\partial NH_4^I}{\partial z} + \frac{1-\gamma_{\text{NH}_4}}{1+K_{\text{NH}_4}} \cdot \frac{1-\phi}{\phi} \cdot \sum_i NC_i \cdot k_i \cdot C_i(z) \quad (7)$$

260 2. Layer ($z_{\text{ox}} < z \leq z_{NO_3}$)

$$\frac{\partial NO_3^{II}}{\partial t} = 0 = D_{NO_3} \frac{\partial^2 NO_3^{II}}{\partial z^2} - w \frac{\partial NO_3^{II}}{\partial z} - \frac{1-\phi}{\phi} NO_3 C \cdot \sum_i k_i \cdot C_i(z) \quad (8)$$

$$\frac{\partial NH_4^{II}}{\partial t} = 0 = \frac{D_{\text{NH}_4}}{1+K_{\text{NH}_4}} \frac{\partial^2 NH_4^{II}}{\partial z^2} - w \frac{\partial NH_4^{II}}{\partial z} \quad (9)$$

265 3. Layer ($z_{NO_3} < z \leq z_\infty$)

$$\frac{\partial NH_4^{III}}{\partial t} = 0 = \frac{D_{\text{NH}_4}}{1+K_{\text{NH}_4}} \frac{\partial^2 NH_4^{III}}{\partial z^2} - w \frac{\partial NH_4^{III}}{\partial z} + \frac{1}{1+K_{\text{NH}_4}} \cdot \frac{1-\phi}{\phi} \cdot \sum_i NC_i \cdot k_i \cdot C_i(z) \quad (10)$$

The boundary conditions to solve Equations 6 - 10 are summarized in Table 4. The model assumes known bottom water concentrations for both species, the complete consumption of nitrate at the 270 nitrate penetration depth (or zero flux if $z_{NO_3} = z_\infty$) and no change in ammonium flux at z_∞ . It considers equal concentrations and diffusive fluxes at z_{bio} and z_{ox} . In addition, the re-oxidation of upward-diffusing reduced ammonium is considered in the oxic-suboxic boundary condition for nitrate and ammonium.

2.3.4 Sulfate and Sulfide

275 When nitrate is depleted, sulfate reduction is the pathway to mineralize organic matter, thus consuming sulfate (SO_4) and producing hydrogen sulfide (H_2S) until the sulfate penetration depth (z_{SO_4}). Sulfate and sulfide dynamics are thus described by:

1. Layer ($z \leq z_{NO_3}$)

$$\frac{\partial SO_4^I}{\partial t} = 0 = D_{SO_4} \frac{\partial^2 SO_4^I}{\partial z^2} - w \frac{\partial SO_4^I}{\partial z} \quad (11)$$

$$\frac{\partial H_2S^I}{\partial t} = 0 = D_{H_2S} \frac{\partial^2 H_2S^I}{\partial z^2} - w \frac{\partial H_2S^I}{\partial z} \quad (12)$$

2. Layer ($z_{NO_3} < z \leq z_{SO_4}$)

$$\frac{\partial SO_4^{II}}{\partial t} = 0 = D_{SO_4} \frac{\partial^2 SO_4^{II}}{\partial z^2} - w \frac{\partial SO_4^{II}}{\partial z} - \frac{1-\phi}{\phi} \cdot \sum_i SO_4 C \cdot k_i \cdot C_i(z) \quad (13)$$

$$\frac{\partial H_2S^{II}}{\partial t} = 0 = D_{H_2S} \frac{\partial^2 H_2S^{II}}{\partial z^2} - w \frac{\partial H_2S^{II}}{\partial z} + \frac{1-\phi}{\phi} \cdot \sum_i SO_4 C \cdot k_i \cdot C_i(z) \quad (14)$$

Table 4. Boundary conditions for nitrate and ammonium. For the boundaries we define: $z_-^- := \lim_{h \rightarrow 0} (z_- - h)$ and $z_+^+ := \lim_{h \rightarrow 0} (z_- + h)$.

Boundary	Condition	
$z = 0$	known concentration	1) $NO_3(0) = NO_{30}$
$z = z_{\text{bio}}$	continuity	2) $NO_3(z_{\text{bio}}^-) = NO_3(z_{\text{bio}}^+)$
$z = z_{\text{ox}}$	continuity	3) $-(D_{NO_3,0} + D_{\text{bio}}) \cdot \frac{\partial NO_3}{\partial z} _{z_{\text{bio}}^-} = -D_{NO_3,0} \cdot \frac{\partial NO_3}{\partial z} _{z_{\text{bio}}^+}$
	where:	4) $NO_3(z_{\text{ox}}^-) = NO_3(z_{\text{ox}}^+)$
$z = z_{NO_3}$	NO ₃ consumption ($z_{NO_3} = z_\infty$)	5) $-D_{NO_3} \cdot \frac{\partial NO_3}{\partial z} _{z_{\text{ox}}^-} + \gamma_{NH_4} \cdot F_{NH_4}(z_{\text{ox}}) = -D_{NO_3} \cdot \frac{\partial NO_3}{\partial z} _{z_{\text{ox}}^+}$ $F_{NH_4}(z_{\text{ox}}) = \frac{1}{1+K_{NH_4}} \cdot \frac{1-\phi}{\phi} \cdot \int_{z_{NO_3}}^\infty \sum_i k_i \cdot NC_i \cdot C_i dz$
	($z_{NO_3} < z_\infty$)	6) IF ($NO_3(z_\infty) > 0$) $\frac{\partial NO_3}{\partial z} _{z_{NO_3}} = 0$ ELSE $NO_3(z_{NO_3}) = 0$
$z = 0$	known concentration	1) $NH_4(0) = NH_{40}$
$z = z_{\text{bio}}$	continuity	2) $NH_4(z_{\text{bio}}^-) = NH_4(z_{\text{bio}}^+)$
$z = z_{\text{ox}}$	continuity	3) $-\frac{D_{NH_4,0} + D_{\text{bio}}}{1+K_{NH_4}} \cdot \frac{\partial NH_4}{\partial z} _{z_{\text{bio}}^-} = -\frac{D_{NH_4,0}}{1+K_{NH_4}} \cdot \frac{\partial NH_4}{\partial z} _{z_{\text{bio}}^+}$
	where:	4) $NH_4(z_{\text{ox}}^-) = NH_4(z_{\text{ox}}^+)$
$z = z_{NO_3}$	continuity flux	5) $-\frac{D_{NH_4}}{1+K_{NH_4}} \cdot \frac{\partial NH_4}{\partial z} _{z_{\text{ox}}^-} - \gamma_{NH_4} \cdot F_{NH_4}(z_{\text{ox}}) = -\frac{D_{NH_4}}{1+K_{NH_4}} \cdot \frac{\partial NH_4}{\partial z} _{z_{\text{ox}}^+}$ $F_{NH_4}(z_{\text{ox}}) = \frac{1}{1+K_{NH_4}} \cdot \frac{1-\phi}{\phi} \cdot \int_{z_{NO_3}}^\infty \sum_i k_i \cdot NC_i \cdot C_i dz$
$z = z_\infty$	zero NH ₄ flux	6) $NH_4(z_{NO_3}^-) = NH_4(z_{NO_3}^+)$
		7) $-\frac{D_{NH_4}}{1+K_{NH_4}} \cdot \frac{\partial NH_4}{\partial z} _{z_{NO_3}^-} = -\frac{D_{NH_4}}{1+K_{NH_4}} \cdot \frac{\partial NH_4}{\partial z} _{z_{NO_3}^+}$
		8) $\frac{\partial NH_4}{\partial z} _{z_\infty} = 0$

3. Layer ($z_{SO_4} < z \leq z_\infty$)

$$\frac{\partial H_2S^{III}}{\partial t} = 0 = D_{H_2S} \frac{\partial^2 H_2S^{III}}{\partial z^2} - w \frac{\partial H_2S^{III}}{\partial z} \quad (15)$$

- 290 To solve equations 11 - 15 the model assumes known concentrations at the sediment-water interface and continuity across the bioturbation depth and the nitrate penetration depth (see Table 5). The re-oxidation of reduced H₂S to SO₄ is considered in the oxic-suboxic boundary condition for both species, here including the methanic zone, as H₂S is also produced during anaerobic oxidation of methane (AOM). Furthermore, sulfate is used at z_{SO_4} to oxidize methane from below and thus 295 producing H₂S. In case $z_{SO_4} < z_\infty$, sulfate concentration is zero at z_{SO_4} and its diffusive flux must equal the amount of methane produced below; or, in case $z_{SO_4} = z_\infty$, a zero flux condition for sulfate is considered. At lower boundary (z_∞) zero flux of H₂S is considered. **correct??**

Table 5. Boundary conditions for sulfate and sulfide. For the boundaries we define: $z_-^- := \lim_{h \rightarrow 0} (z_- - h)$ and $z_-^+ := \lim_{h \rightarrow 0} (z_- + h)$.

Boundary	Condition	
$z = 0$	known concentration	1) $\text{SO}_4(0) = \text{SO}_{40}$
$z = z_{\text{bio}}$	continuity	2) $\text{SO}_4(z_{\text{bio}}^-) = \text{SO}_4(z_{\text{bio}}^+)$
	flux	3) $-(D_{\text{SO}_4,0} + D_{\text{bio}}) \cdot \frac{\partial \text{SO}_4}{\partial z} _{z_{\text{bio}}^-} = -D_{\text{SO}_4,0} \cdot \frac{\partial \text{SO}_4}{\partial z} _{z_{\text{bio}}^+}$
$z = z_{\text{ox}}$	continuity	4) $\text{SO}_4(z_{\text{ox}}^-) = \text{SO}_4(z_{\text{ox}}^+)$
	flux	5) $-D_{\text{SO}_4} \cdot \frac{\partial \text{SO}_4}{\partial z} _{z_{\text{ox}}^-} + \gamma_{\text{H}_2\text{S}} \cdot F_{\text{H}_2\text{S}}(z_{\text{ox}}) = -D_{\text{SO}_4} \cdot \frac{\partial \text{SO}_4}{\partial z} _{z_{\text{ox}}^+}$
	where:	$F_{\text{H}_2\text{S}}(z_{\text{ox}}) = \frac{1-\phi}{\phi} \cdot \left(\int_{z_{\text{NO}_3}}^{\text{SO}_4} \sum_i \text{SO}_4 \cdot k_i \cdot C_i dz + \gamma_{\text{CH}_4} \cdot \int_{z_{\text{SO}_4}}^{\infty} \sum_i \text{MC} \cdot k_i \cdot C_i dz \right)$
$z = z_{\text{NO}_3}$	continuity	6) $\text{SO}_4(z_{\text{NO}_3}^-) = \text{SO}_4(z_{\text{NO}_3}^+)$
	flux	7) $-D_{\text{SO}_4} \cdot \frac{\partial \text{SO}_4}{\partial z} _{z_{\text{NO}_3}^-} = -D_{\text{SO}_4} \cdot \frac{\partial \text{SO}_4}{\partial z} _{z_{\text{NO}_3}^+}$
$z = z_{\text{SO}_4}$	SO_4 consumption ($z_{\text{SO}_4} = z_\infty$)	8) IF ($\text{SO}_4(z_\infty) > 0$) $\frac{\partial \text{SO}_4}{\partial z} _{z_{\text{SO}_4}} = 0$
	$(z_{\text{SO}_4} < z_\infty)$	ELSE $\text{SO}_4(z_{\text{SO}_4}) = 0$ and $-D_{\text{SO}_4} \cdot \frac{\partial \text{SO}_4}{\partial z} _{z_{\text{SO}_4}} = \gamma_{\text{CH}_4} \cdot F_{\text{CH}_4}(z_{\text{SO}_4})$ $F_{\text{CH}_4}(z_{\text{SO}_4}) = \frac{1-\phi}{\phi} \cdot \int_{z_{\text{SO}_4}}^{\infty} \sum_i \text{MC} \cdot k_i \cdot C_i dz$
$z = 0$	known concentration	1) $\text{H}_2\text{S}(0) = \text{H}_2\text{S}_0$
$z = z_{\text{bio}}$	continuity	2) $\text{H}_2\text{S}(z_{\text{bio}}^-) = \text{H}_2\text{S}(z_{\text{bio}}^+)$
	flux	3) $-(D_{\text{H}_2\text{S},0} + D_{\text{bio}}) \cdot \frac{\partial \text{H}_2\text{S}}{\partial z} _{z_{\text{bio}}^-} = -D_{\text{H}_2\text{S},0} \cdot \frac{\partial \text{H}_2\text{S}}{\partial z} _{z_{\text{bio}}^+}$
$z = z_{\text{ox}}$	continuity	4) $\text{H}_2\text{S}(z_{\text{ox}}^-) = \text{H}_2\text{S}(z_{\text{ox}}^+)$
	flux	5) $-D_{\text{H}_2\text{S}} \cdot \frac{\partial \text{H}_2\text{S}}{\partial z} _{z_{\text{ox}}^-} - \gamma_{\text{H}_2\text{S}} F_{\text{H}_2\text{S}}(z_{\text{ox}}) = -D_{\text{H}_2\text{S}} \cdot \frac{\partial \text{H}_2\text{S}}{\partial z} _{z_{\text{ox}}^+}$
	where:	$F_{\text{H}_2\text{S}}(z_{\text{ox}}) = \frac{1-\phi}{\phi} \cdot \left(\int_{z_{\text{NO}_3}}^{\text{SO}_4} \sum_i \text{SO}_4 \cdot k_i \cdot C_i dz + \gamma_{\text{CH}_4} \cdot \int_{z_{\text{SO}_4}}^{\infty} \sum_i \text{MC} \cdot k_i \cdot C_i dz \right)$
$z = z_{\text{NO}_3}$	continuity	6) $\text{H}_2\text{S}(z_{\text{NO}_3}^-) = \text{H}_2\text{S}(z_{\text{NO}_3}^+)$
	flux	7) $-D_{\text{H}_2\text{S}} \cdot \frac{\partial \text{H}_2\text{S}}{\partial z} _{z_{\text{NO}_3}^-} = -D_{\text{H}_2\text{S}} \cdot \frac{\partial \text{H}_2\text{S}}{\partial z} _{z_{\text{NO}_3}^+}$
$z = z_{\text{SO}_4}$	continuity	8) $\text{H}_2\text{S}(z_{\text{SO}_4}^-) = \text{H}_2\text{S}(z_{\text{SO}_4}^+)$
	flux (with AOM)	9) $-D_{\text{H}_2\text{S}} \cdot \frac{\partial \text{H}_2\text{S}}{\partial z} _{z_{\text{SO}_4}^-} + \gamma_{\text{CH}_4} \cdot F_{\text{CH}_4}(z_{\text{SO}_4}) = -D_{\text{H}_2\text{S}} \cdot \frac{\partial \text{H}_2\text{S}}{\partial z} _{z_{\text{SO}_4}^+}$
	where:	$F_{\text{CH}_4}(z_{\text{SO}_4}) = \frac{1-\phi}{\phi} \cdot \int_{z_{\text{SO}_4}}^{\infty} \sum_i \text{MC} \cdot k_i \cdot C_i dz$
$z = z_\infty$	zero H_2S flux	10) $\frac{\partial \text{H}_2\text{S}}{\partial z} _{z_\infty} = 0$

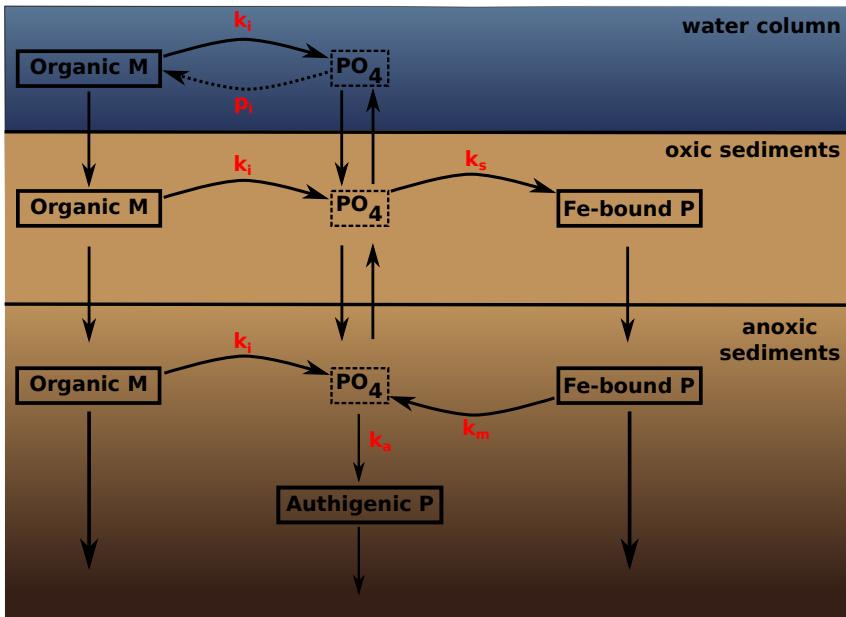


Figure 2. A schematic of the sedimentary P cycle in OMEN-SED (1.0). Red numbers represent kinetic rate constants for phosphorus dynamics (compare Table 10; p_i represents uptake rate of PO_4 via primary production in shallow environments). Adapted from Slomp et al. (1996).

2.3.5 Phosphate

300 To model phosphorus (P) in the sediments the model takes into account the change with depth of phosphate (PO_4) and iron-bound P, thereby mainly following the description of Slomp et al. (1996) and Gypens et al. (2008). Throughout the sediment column organic matter is mineralized resulting in a release of phosphate to the pore water. In the oxic part of the sediment, this PO_4 either diffuses upward to the water column or is adsorbed to Fe oxides forming Fe-bound P (or M) (Slomp et al.,
305 1998). In the suboxic/anoxic zone, PO_4 is not only produced through organic matter degradation but is also released from the Fe-bound P pool due to the reduction of Fe oxides. Furthermore, phosphate concentrations can become high enough in this layer for authigenic mineral formation to occur (Van Cappellen and Berner, 1988). This phosphorus bound in authigenic minerals represents a permanent sink for reactive phosphorus (Slomp et al., 1996). See Figure 2 for a schematic overview of
310 the sedimentary P cycle. Therefore the diagenetic equations for phosphorus are written:

1. Layer ($z \leq z_{\text{ox}}$)

$$\frac{\partial \text{PO}_4^I}{\partial t} = \frac{D_{\text{PO}_4}}{1 + K_{\text{PO}_4}^I} \frac{\partial^2 \text{PO}_4^I}{\partial z^2} - w \frac{\partial \text{PO}_4^I}{\partial z} + \frac{1-\phi}{\phi} \frac{1}{1 + K_{\text{PO}_4}^I} \sum_i (\text{PC}_i \cdot k_i \cdot C_i(z)) - \frac{k_s}{1 + K_{\text{PO}_4}^I} (\text{PO}_4^I - \text{PO}_4^s) \quad (16)$$

$$315 \quad \frac{\partial M^I}{\partial t} = D_M \frac{\partial^2 M^I}{\partial z^2} - w \frac{\partial M^I}{\partial z} + \frac{\phi}{1-\phi} k_s (\text{PO}_4^I - \text{PO}_4^s) \quad (17)$$

2. Layer ($z_{\text{ox}} < z$)

$$\frac{\partial M^{II}}{\partial t} = D_M \frac{\partial^2 M^{II}}{\partial z^2} - w \frac{\partial M^{II}}{\partial z} - k_m (M^{II} - M^\infty) \quad (18)$$

$$320 \quad \frac{\partial \text{PO}_4^{II}}{\partial t} = \frac{D_{\text{PO}_4}}{1 + K_{\text{PO}_4}^{II}} \frac{\partial^2 \text{PO}_4^{II}}{\partial z^2} - w \frac{\partial \text{PO}_4^{II}}{\partial z} + \frac{1-\phi}{\phi} \frac{1}{1 + K_{\text{PO}_4}^{II}} \sum_i (\text{PC}_i \cdot k_i \cdot C_i(z)) - \frac{k_a}{1 + K_{\text{PO}_4}^{II}} (\text{PO}_4^{II} - \text{PO}_4^a) + \frac{(1-\phi)}{\phi} \frac{k_m}{1 + K_{\text{PO}_4}^{II}} (M^{II} - M^\infty) \quad (19)$$

(20)

The boundary conditions to solve Equations 16 - 19 are summarized in Table 6. The model assumes known bottom water concentrations and equal concentrations and diffusive fluxes at z_{bio} and z_{ox} for

325 both species. Additionally it considers no change in phosphate flux and an asymptotic Fe-bound P concentration at z_∞ .

@ Sandra: SWI Flux for M does not exist, right???

2.3.6 Dissolved Inorganic Carbon (DIC)

330 Dissolved inorganic carbon (DIC) is produced by when organic matter is degraded. Explain different ratios for the two layers.

1. Layer ($z \leq z_{\text{SO}_4}$)

$$335 \quad \frac{\partial \text{DIC}^I}{\partial t} = 0 = D_{\text{DIC}} \frac{\partial^2 \text{DIC}^I}{\partial z^2} - w \frac{\partial \text{DIC}^I}{\partial z} + \frac{1-\phi}{\phi} \cdot \sum_i \text{DICC}^I \cdot k_i \cdot C_i(z) \quad (21)$$

2. Layer ($z_{\text{SO}_4} < z \leq z_\infty$)

$$\frac{\partial \text{DIC}^{II}}{\partial t} = 0 = D_{\text{DIC}} \frac{\partial^2 \text{DIC}^{II}}{\partial z^2} - w \frac{\partial \text{DIC}^{II}}{\partial z} + \frac{1-\phi}{\phi} \cdot \sum_i \text{DICC}^{II} \cdot k_i \cdot C_i(z) \quad (22)$$

To solve equations 21 and 22 the model assumes the boundary conditions summarised in Table 7.

Table 6. Boundary conditions for phosphate and Fe-bound P (M). For the boundaries we define: $z_-^- := \lim_{h \rightarrow 0} (z_- - h)$ and $z_-^+ := \lim_{h \rightarrow 0} (z_- + h)$.

Boundary	Condition	
$z = 0$	known concentration	1) $\text{PO}_4(0) = \text{PO}_{40}$
$z = z_{\text{bio}}$	continuity	2) $\text{PO}_4(z_{\text{bio}}^-) = \text{PO}_4(z_{\text{bio}}^+)$
	flux	3) $(D_{\text{PO}_4,0} + D_{\text{bio}}) \cdot \frac{\partial \text{PO}_4}{\partial z} _{z_{\text{bio}}^-} = D_{\text{PO}_4,0} \cdot \frac{\partial \text{PO}_4}{\partial z} _{z_{\text{bio}}^+}$
$z = z_{\text{ox}}$	continuity	4) $\text{PO}_4(z_{\text{ox}}^-) = \text{PO}_4(z_{\text{ox}}^+)$
	flux	5) $-\frac{D_{\text{PO}_4}}{1 + K_{\text{PO}_4}^I} \cdot \frac{\partial \text{PO}_4}{\partial z} _{z_{\text{ox}}^-} = -\frac{D_{\text{PO}_4}}{1 + K_{\text{PO}_4}^I} \cdot \frac{\partial \text{PO}_4}{\partial z} _{z_{\text{ox}}^+}$
$z = z_\infty$	flux	10) $\frac{\partial \text{PO}_4}{\partial z} _{z_\infty} = 0$
$z = 0$	known concentration	1) $M(0) = M_0$
$z = z_{\text{bio}}$	continuity	2) $M(z_{\text{bio}}^-) = M(z_{\text{bio}}^+)$
	flux	3) $\frac{\partial M}{\partial z} _{z_{\text{bio}}^-} = \frac{\partial M}{\partial z} _{z_{\text{bio}}^+}$
$z = z_{\text{ox}}$	continuity	4) $M(z_{\text{ox}}^-) = M(z_{\text{ox}}^+)$
	flux	5) $\frac{\partial M}{\partial z} _{z_{\text{ox}}^-} = \frac{\partial M}{\partial z} _{z_{\text{ox}}^+}$
$z = z_\infty$	assymptotic concentration	10) $M(z_\infty) = M_\infty$

Table 7. Boundary conditions for DIC. For the boundaries we define: $z_-^- := \lim_{h \rightarrow 0} (z_- - h)$ and $z_-^+ := \lim_{h \rightarrow 0} (z_- + h)$.

Boundary	Condition	
$z = 0$	known concentration	1) $\text{DIC}(0) = \text{DIC}_0$
$z = z_{\text{bio}}$	continuity	2) $\text{DIC}(z_{\text{bio}}^-) = \text{DIC}(z_{\text{bio}}^+)$
	flux	3) $-(D_{\text{DIC},0} + D_{\text{bio}}) \cdot \frac{\partial \text{DIC}}{\partial z} _{z_{\text{bio}}^-} = -D_{\text{DIC},0} \cdot \frac{\partial \text{DIC}}{\partial z} _{z_{\text{bio}}^+}$
$z = z_{\text{SO}_4}$	continuity	8) $\text{DIC}(z_{\text{SO}_4}^-) = \text{DIC}(z_{\text{SO}_4}^+)$
	flux (with AOM)	9) $-D_{\text{DIC}} \cdot \frac{\partial \text{DIC}}{\partial z} _{z_{\text{SO}_4}^-} + \gamma_{\text{CH}_4} \cdot F_{\text{CH}_4}(z_{\text{SO}_4}) = -D_{\text{DIC}} \cdot \frac{\partial \text{DIC}}{\partial z} _{z_{\text{SO}_4}^+}$
	where:	$F_{\text{CH}_4}(z_{\text{SO}_4}) = \frac{1-\phi}{\phi} \cdot \int_{z_{\text{SO}_4}}^\infty \sum_i \text{MC} \cdot k_i \cdot C_i dz$
$z = z_\infty$	zero DIC flux	10) $\frac{\partial \text{DIC}}{\partial z} _{z_\infty} = 0$

340 2.3.7 Alkalinity

Alkalinity is produced and consumed by when organic matter is degraded. Explain why it is produced/consumed, where and how in the different layers $j \in \{I, II, III, IV\}$ and talk earlier on in the model description why it matters.

345 1. Layer ($z \leq z_{\text{ox}}$)

$$\begin{aligned} \frac{\partial \text{ALK}^I}{\partial t} = 0 &= D_{\text{ALK}} \frac{\partial^2 \text{ALK}^I}{\partial z^2} - w \frac{\partial \text{ALK}^I}{\partial z} \\ &+ \frac{1-\phi}{\phi} \cdot \sum_i \left(\text{ALK}^{\text{NIT}} \cdot \frac{\gamma_{\text{NH}_4}}{1 + K_{\text{NH}_4}} \text{NC}_i + \text{ALK}^{\text{OX}} \cdot \text{OC} \right) \cdot k_i \cdot C_i(z) \end{aligned} \quad (23)$$

Table 8. Boundary conditions for alkalinity. For the boundaries we define: $z_-^- := \lim_{h \rightarrow 0} (z_- - h)$ and $z_-^+ := \lim_{h \rightarrow 0} (z_- + h)$.

Boundary	Condition	
$z = 0$	known concentration	1) $\text{ALK}(0) = \text{ALK}_0$
$z = z_{\text{bio}}$	continuity	2) $\text{ALK}(z_{\text{bio}}^-) = \text{ALK}(z_{\text{bio}}^+)$
	flux	3) $-(D_{\text{ALK},0} + D_{\text{bio}}) \cdot \frac{\partial \text{ALK}}{\partial z} _{z_{\text{bio}}^-} = -D_{\text{ALK},0} \cdot \frac{\partial \text{ALK}}{\partial z} _{z_{\text{bio}}^+}$
$z = z_{\text{ox}}$	continuity	4) $\text{ALK}(z_{\text{ox}}^-) = \text{ALK}(z_{\text{ox}}^+)$
	flux	5) $-D_{\text{ALK}} \cdot \frac{\partial \text{ALK}}{\partial z} _{z_{\text{ox}}^-} + F_{\text{ALK}}(z_{\text{ox}}) = -D_{\text{ALK}} \cdot \frac{\partial \text{ALK}}{\partial z} _{z_{\text{ox}}^+}$ $F_{\text{ALK}}(z_{\text{ox}}) = \frac{1-\phi}{\phi} \cdot \left(\text{ALK}^{\text{H}_2\text{S}} \cdot \gamma_{\text{H}_2\text{S}} \int_{z_{\text{NO}_3}}^{\text{SO}_4} \sum_i \text{SO}_4 \text{C} \cdot k_i \cdot C_i dz \right)$ $+ \frac{1-\phi}{\phi} \cdot \left(\text{ALK}^{\text{NIT}} \frac{\gamma_{\text{NH}_4}}{1+k_{\text{NH}_4}} \int_{z_{\text{NO}_3}}^{\infty} \sum_i \text{NC}_i \cdot k_i \cdot C_i dz \right)$
	where:	
$z = z_{\text{NO}_3}$	continuity	6) $\text{ALK}(z_{\text{NO}_3}^-) = \text{ALK}(z_{\text{NO}_3}^+)$
	flux	7) $-D_{\text{ALK}} \cdot \frac{\partial \text{ALK}}{\partial z} _{z_{\text{NO}_3}^-} = -D_{\text{ALK}} \cdot \frac{\partial \text{ALK}}{\partial z} _{z_{\text{NO}_3}^+}$
$z = z_{\text{SO}_4}$	continuity	8) $\text{ALK}(z_{\text{SO}_4}^-) = \text{ALK}(z_{\text{SO}_4}^+)$
	flux (with AOM)	9) $-D_{\text{ALK}} \cdot \frac{\partial \text{ALK}}{\partial z} _{z_{\text{SO}_4}^-} + F_{\text{ALK}}(z_{\text{SO}_4}) = -D_{\text{ALK}} \cdot \frac{\partial \text{ALK}}{\partial z} _{z_{\text{SO}_4}^+}$ $F_{\text{ALK}}(z_{\text{SO}_4}) = \frac{1-\phi}{\phi} \cdot \left(\text{ALK}^{\text{AOM}} \gamma_{\text{CH}_4} \cdot \int_{z_{\text{SO}_4}}^{\infty} \sum_i \text{MC} \cdot k_i \cdot C_i dz \right)$
	where:	
$z = z_{\infty}$	zero ALK flux	10) $\frac{\partial \text{ALK}}{\partial z} _{z_{\infty}} = 0$

2. Layer ($z_{\text{ox}} < z \leq z_{\text{NO}_3}$)

$$350 \quad \frac{\partial \text{ALK}^{II}}{\partial t} = 0 = D_{\text{ALK}} \frac{\partial^2 \text{ALK}^{II}}{\partial z^2} - w \frac{\partial \text{ALK}^{II}}{\partial z} + \frac{1-\phi}{\phi} \cdot \sum_i \text{ALK}^{\text{DEN}} \cdot \text{NC}_i \cdot k_i \cdot C_i(z) \quad (24)$$

3. Layer ($z_{\text{NO}_3} < z \leq z_{\text{SO}_4}$)

$$\frac{\partial \text{ALK}^{III}}{\partial t} = 0 = D_{\text{ALK}} \frac{\partial^2 \text{ALK}^{III}}{\partial z^2} - w \frac{\partial \text{ALK}^{III}}{\partial z} + \frac{1-\phi}{\phi} \cdot \sum_i \text{ALK}^{\text{SUL}} \cdot \text{SO}_4 \text{C} \cdot k_i \cdot C_i(z) \quad (25)$$

355 4. Layer ($z_{\text{SO}_4} < z \leq z_{\infty}$)

$$\frac{\partial \text{ALK}^{IV}}{\partial t} = 0 = D_{\text{ALK}} \frac{\partial^2 \text{ALK}^{IV}}{\partial z^2} - w \frac{\partial \text{ALK}^{IV}}{\partial z} + \frac{1-\phi}{\phi} \cdot \sum_i \text{ALK}^{\text{MET}} \cdot \text{MC} \cdot k_i \cdot C_i(z) \quad (26)$$

To solve equations 23 and 26 the model assumes the boundary conditions summarised in Table 8.

2.4 Model Parameters

360 This section describes the parameters used in OMEN-SED (1.0) to describe sediment transport and biogeochemical reactions related to the burial and mineralization of organic matter under a wide range of environmental conditions. Table 9 states the parameters for sediment characteristics and Table 10 summarizes the stoichiometric factors and secondary reaction parameters used in the model.

2.4.1 Transport Parameters

365 Advection is the bulk flow of sediments and can be directly related to the accumulation of new material on the seafloor (i.e. sedimentation, Burdige, 2006). This results in a downward flux of older sediment material and porewater in relation to the sediment-water interface. When coupled to an ocean model, its sedimentation flux can be readily used in OMEN-SED 1.0. The stand-alone version of OMEN-SED 1.0 uses the empirical global relationship between sediment accumulation rate (cm yr^{-1}) and seafloor depth (m) of Middelburg et al. (1997):
370

$$w = 3.3 \cdot 10^{-0.87478367 - 0.00043512 \cdot \text{depth}}, \quad (27)$$

As discussed before (Sec. 2.2), the diffusion coefficient of species i is calculated as $D_i = D_{i,0} + D_{\text{bio}} = D_{\text{mol},i} \cdot f_{ir} + D_{\text{bio}}$ for dissolved species and $D_i = D_{\text{bio}}$ for solid species. The bioturbation coefficient D_{bio} ($\text{cm}^2 \text{ yr}^{-1}$) is constant in the bioturbated zone and also follows the empirical relationship by Middelburg et al. (1997):
375

$$D_{\text{bio}} = 5.2 \cdot 10^{0.76241122 - 0.00039724 \cdot \text{depth}} \quad (28)$$

Studies showed that bioturbational effects on a global scale are largely restricted to the upper 10 cm of the sediments and are only marginally related to seafloor depth (e.g. Boudreau, 1998; Teal et al., 2010). Therefore, OMEN-SED (1.0) imposes a globally invariant bioturbation depth of 10
380 cm. Bioirrigation can enhance the molecular diffusion coefficient $D_{i,0} = D_{\text{mol},i} \cdot f_{ir}$ (Soetaert et al., 1996a). However, here we do not consider this effect and set f_{ir} to a constant value of 1. The specific molecular diffusion coefficients $D_{\text{mol},i}$ are corrected for sediment porosity ϕ , tortuosity F and are linearly interpolated for an ambient temperature T using zero-degree coefficients D_i^0 and temperature dependent diffusion coefficients D_i^T (compare Gypens et al., 2008):

$$385 D_{\text{mol},i} = (D_i^0 + D_i^T \cdot T) \cdot \frac{1}{\phi \cdot F}$$

Tortuosity can be expressed in terms of porosity as $F = \frac{1}{\phi^m}$ (Ullman and Aller, 1982) with the exponent m varying according to the type of sediment (here we use $m=3$). Values for D_i^T and D_i^0 are summarized in Table 9 and are adapted from Li and Gregory (1974) and Gypens et al. (2008).

2.4.2 Reaction Parameters

390 The applied multi-G approach for organic matter degradation considers specific degradation rate constants k_i (yr^{-1}) for each compound class. **TODO:** *The degradation constants are generally taken from the coupled Earth System model* and are assumed invariant along the sediment column, therefore independent of the nature of the terminal electron acceptor. The stoichiometry of organic matter is represented by the factors NC_i and PC_i denoting the molecular nitrogen to carbon and
395 phosphorus to carbon ratio. In the sulfidic and methanic zone the reduction of 1 mol organic matter

Table 9. Fixed sediment characteristics and transport parameters. **TODO:** Update table!

Parameter	Unit	Value	Description/Source
ρ_{sed}	g cm^{-3}	2.5	Sediment density
w	cm yr^{-1}	Fct. of seafloor depth	Advection/Sediment accumulation rate Middelburg et al. (1997)
z_{bio}	cm	10	Bioturbation depth Boudreau (1998); Teal et al. (2010)
D_{bio}	$\text{cm}^2 \text{yr}^{-1}$	Fct. of seafloor depth	Bioturbation coefficient Middelburg et al. (1997)
ϕ	-	0.8	Porosity
F	-	$\frac{1}{\phi^m}$	Tortuosity, here $m=3$
$f_i r$	-	1	Irrigation factor
PO_4^s	mol cm^{-3}	$1 \cdot 10^{-9}$	equilibrium conc. for P sorption Slomp et al. (1996)
PO_4^a	mol cm^{-3}	$3.7 \cdot 10^{-9}$	equilibrium conc. for authigenic P precipitation Slomp et al. (1996)
M^∞	mol cm^{-3}	$1.99 \cdot 10^{-9}$	asymptotic concentration for Fe-bound P Slomp et al. (1996)
Diffusion coefficients (Li and Gregory, 1974; Gypens et al., 2008)			
$D_{\text{O}_2}^0$	$\text{cm}^2 \text{yr}^{-1}$	348.62172	Molecular diffusion coefficient of oxygen at 0°C
$D_{\text{O}_2}^T$	$\text{cm}^2 \text{yr}^{-1} \text{°C}^{-1}$	14.08608	Diffusion coefficient for linear temp. dependence of oxygen
$D_{\text{NO}_3}^0$	$\text{cm}^2 \text{yr}^{-1}$	308.42208	Molecular diffusion coefficient of nitrate at 0°C
$D_{\text{NO}_3}^T$	$\text{cm}^2 \text{yr}^{-1} \text{°C}^{-1}$	12.2640	Diffusion coefficient for linear temp. dependence of nitrate
$D_{\text{NH}_4}^0$	$\text{cm}^2 \text{yr}^{-1}$	308.42208	Molecular diffusion coefficient of ammonium at 0°C
$D_{\text{NH}_4}^T$	$\text{cm}^2 \text{yr}^{-1} \text{°C}^{-1}$	12.2640	Diffusion coefficient for linear temp. dependence of ammonium
$D_{\text{SO}_4}^0$	$\text{cm}^2 \text{yr}^{-1}$	157.68	Molecular diffusion coefficient of sulfate at 0°C
$D_{\text{SO}_4}^T$	$\text{cm}^2 \text{yr}^{-1} \text{°C}^{-1}$	7.884	Diffusion coefficient for linear temp. dependence of sulfate
$D_{\text{H}_2\text{S}}^0$	$\text{cm}^2 \text{yr}^{-1}$	307.476	Molecular diffusion coefficient of sulfide at 0°C
$D_{\text{H}_2\text{S}}^T$	$\text{cm}^2 \text{yr}^{-1} \text{°C}^{-1}$	9.636	Diffusion coefficient for linear temp. dependence of sulfide
$D_{\text{PO}_4}^0$	$\text{cm}^2 \text{yr}^{-1}$	112.90764	Molecular diffusion coefficient of phosphate at 0°C
$D_{\text{PO}_4}^T$	$\text{cm}^2 \text{yr}^{-1} \text{°C}^{-1}$	5.586252	Diffusion coefficient for linear temp. dependence of phosphate

Table 10. Values for biogeochemical parameters used in OMEN-SED (1.0).

Parameter/Variable	Unit	Value	Description
Stoichiometric factors and molecular ratios			
NC ₁	mol/mol	0.1509	nitrogen to carbon ratio refractory fraction, two different ones? why these values?
NC ₂	mol/mol	0.13333	nitrogen to carbon ratio labile fraction
PC _i	mol/mol	0.0094	phosphorus to carbon ratio
MC	mol/mol	0.5	methane to carbon ratio produced during methanogenesis
DICC ^I	mol/mol	1.0	DIC to carbon ratio until z_{SO_4}
DICC ^{II}	mol/mol	0.5	DIC to carbon ratio below z_{SO_4}
OC	mol/mol	1.0	oxygen to carbon ratio
NO ₃ C	mol/mol	94.4/106	nitrate to carbon ratio
SO ₄ C	mol/mol	0.5	sulfate to carbon ratio
ALK ^{OX}	mol/mol	15	ALK from aerobic degradation
ALK ^{NIT}	mol/mol	-2	ALK from nitrification
ALK ^{DEN}	mol/mol	93.4	ALK from denitrification
ALK ^{SUL}	mol/mol	15	ALK from sulfate reduction
ALK ^{MET}	mol/mol	14	ALK from methanogenesis
ALK ^{H2S}	mol/mol	-2	ALK from H ₂ S oxidation
ALK ^{AOM}	mol/mol	2	ALK from methanogenesis
Secondary reaction parameters			
γ_{NH_4}	-	0.8	fraction of NH ₄ that is oxidised in oxic layer
γ_{H_2S}	-	0.95	fraction of H ₂ S that is oxidised in oxic layer
γ_{CH_4}	-	1.0	fraction of CH ₄ that is oxidised at z_{SO_4}
Rate constants			
k_i	yr ⁻¹	from Earth System Model	OM degradation rate constants
k_s	yr ⁻¹	???	rate constant for P sorption
k_m	yr ⁻¹	???	rate constant for Fe-bound P release
k_a	yr ⁻¹	???	rate constant for authigenic P formation

additionally produces 0.5 mol of hydrogen sulfide (SO₄C) and 0.5 mol of methane (MC). In the total sediment column organic matter mineralization consumes the specific TEA with a fixed ratio (OC, NO₃C and SO₄C respectively). See Table 10 for a complete summary of the parameters and their values.

400 **2.5 Module Structure**

TODO: An analytical steady-state solution is found for the reaction-transport equation of each chemical species in each layer. At each boundary (i.e. $z_{\text{ox}}, z_{\text{bio}}, z_{\text{NO}_3}$ and z_{SO_4}) the model has to match continuity and flux for different ODE solutions of the layer above and below the specific boundary. In particular the bioturbation boundary is problematic as it can theoretically occur in any geochemical layer (compare Fig. 3). In order to simplify this recurring boundary matching problem it is implemented in an independent algorithm which is described in Section 2.5.1. Instructions and requirements for coupling OMEN-SED (1.0) to a global Earth System Model are given in Section 2.5.2.

2.5.1 Generic boundary condition matching (GBCM)

410 A general steady-state advection-diffusion-reaction (ADR) diagenetic equation looks like:

$$\frac{\partial C}{\partial t} = 0 = D \frac{\partial^2 C}{\partial z^2} - w \frac{\partial C}{\partial z} - \sum_i \alpha_i \exp(-\beta_i z) - k \cdot C + Q. \quad (29)$$

where z is the sediment depth, t the time, D is the diffusion coefficient and w is the advection rate.

The ODE solution is of the general form:

$$415 \quad C(z) = A \exp(az) + B \exp(bz) + \sum_i \frac{\alpha_i}{D\beta_i^2 - w\beta_i - k} \cdot \exp(-\beta_i z) + \frac{Q}{k} \quad (30)$$

and can therefore be expressed as:

$$C(z) = A \cdot E(z) + B \cdot F(z) + G(z) \quad (31)$$

420 where $E(z), F(z)$ are the homogeneous solutions of the ODE, $G(z)$ the particular integral, and A, B are the integration constants (compare Fig. 3 for the whole sediment column).

Each boundary matching problem involves matching continuity and flux for the two solutions $C_U(z)$ (= 'upper') and $C_L(z)$ (= 'lower') across a boundary at $z = z_b$. Therefore, we get two ODE solutions of the genral form:

$$425 \quad C_U(z) = A_U \cdot E_U(z) + B_U \cdot F_U(z) + G_U(z) \quad (32)$$

$$C_L(z) = A_L \cdot E_L(z) + B_L \cdot F_L(z) + G_L(z). \quad (33)$$

The two boundary conditions are: for continuity (where for generality we allow a discontinuity V_b)

$$430 \quad C_U(z_b) = C_L(z_b) + V_b \quad (34)$$

and for flux

$$D_U C'_U(z_b) + w C_U(z_b) = D_L C'_L(z_b) + w C_L(z_b) + F_b \quad (35)$$

where w is advection, D are the diffusion coefficients and F_b is any flux discontinuity.

- 435 In terms of the ODE solutions (32), (33), the boundary conditions represent two equations connecting the four integration constants:

$$\begin{pmatrix} E_U & F_U \\ D_U E'_U & D_U F'_U \end{pmatrix} \begin{pmatrix} A_U \\ B_U \end{pmatrix} = \begin{pmatrix} E_L & F_L \\ D_L E'_L & D_L F'_L \end{pmatrix} \begin{pmatrix} A_L \\ B_L \end{pmatrix} + \begin{pmatrix} G_L - G_U + V_b \\ D_L G'_L - D_U G'_U + F_b - w V_b \end{pmatrix} \quad (36)$$

where the ODE solutions E , F , G are all evaluated at z_b .

- 440 Equation (36) can be solved to give A_U and B_U as a function of the integration constants from the layer below (A_L and B_L), thereby constructing a piecewise solution for the whole region, with now just two integration constants A_L and B_L .

In the code the function **benthic_utils.matchsoln** provides this solution in the form:

$$\begin{pmatrix} A_U \\ B_U \end{pmatrix} = \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix} \begin{pmatrix} A_L \\ B_L \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}. \quad (37)$$

- 445 Using (37) we can now rewrite $C_U(z)$ in (32) as a function of A_L and B_L :

$$C_U(z) = (c_1 A_L + c_2 B_L + d_1) \cdot E_U(z) + (c_3 A_L + c_4 B_L + d_2) \cdot F_U(z) + G_U(z)$$

and hence define the “transformed” basis functions $E_U^*(z)$, $F_U^*(z)$, $G_U^*(z)$ such that:

$$C_U(z) = A_L \cdot E_U^*(z) + B_L \cdot F_U^*(z) + G_U^*(z) \quad (38)$$

where

$$450 \quad E_U^*(z) = c_1 E_U(z) + c_3 F_U(z)$$

$$F_U^*(z) = c_2 E_U(z) + c_4 F_U(z) \quad (39)$$

$$G_U^*(z) = G_U(z) + d_1 E_U(z) + d_2 F_U(z)$$

(in the code this is done by **benthic_utils.xformsoln**).

- 455 **Solving the sediment layer stack**

Equations (37), (38) and (39) can now be applied for each layer boundary, working up from the bottom of the sediments. The net result is to construct a piecewise solution with just two integration constants (coming from the lowest layer), which can then be solved for by applying one boundary condition for the sediment-water interface and one for the bottom of the sediments (e.g. a concentration condition at the bottom of the sediments, and a flux condition at the SWI).

TODO: Add figure, illustrating this e.g. for nitrate...

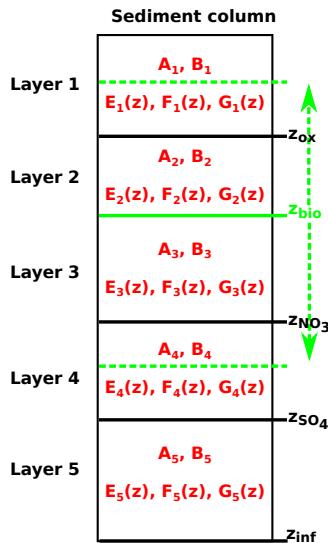


Figure 3. Schematic of the generic boundary condition matching (GBCM) problem. Showing the resulting integration constants (A_i, B_i) and ODE solutions (E_i, F_i, G_i) for the different sediment layers and the variable bioturbation boundary.

Abstracting out the bioturbation boundary

The bioturbation boundary affects the diffusion coefficient of the modelled solutes and the conservation equation of organic matter which is available for mineralization. The boundary is particularly
465 inconvenient as it can in principle occur in the middle of any “geochemical” layer and therefore generates multiple cases (compare Fig. 3). To simplify this for solutes, the “piecewise solution construction” above is used to abstract out the bioturbation boundary. An initial test for each layer is made to identify its “bioturbation-status” (fully bioturbated, fully non-bioturbated or crossing the bioturbation boundary) and (if needed) a piecewise solution is constructed by matching boundary
470 conditions across the bioturbation boundary. The “outside” code therefore never needs to know whether it is dealing with a piecewise solution (i.e. matched across a bioturbation boundary) or a “simple” solution (i.e. the layer is fully bioturbated or fully non-bioturbated).

In the code, this is performed by **zTOC.prepfg_I12** which hands back a structure **ls** containing the “bioturbation-status” for each layer and (if needed) the description of the piecewise solution (coefficients $c_1, c_2, c_3, c_4, d_1, d_2$ as above). So e.g. for sulfate, **zTOC.prepfg_I12** is called three times at the beginning of **zSO4.calcbc** (one for each “geochemical” layer: oxic, suboxic, sulfidic) handing back three structures **ls** describing the layer’s “bioturbation-status”, abstracting away the bioturbation boundary and all associated conditional logic. When calculating the solutions for the different
475

layers, the pre-calculated structure `ls` is passed to the function `zTOC.calcfg_l12` which sorts out the
480 correct solution type to use.

2.5.2 Coupling to an Earth System Model

Describe what needs to be done/checked. Compare/use as example what I did with GENIE coupling.
What are the if-clauses used? Input to OMEN-SED, output of OMEN-SED.

3 Test Cases

485 3.1 Sediment profiles

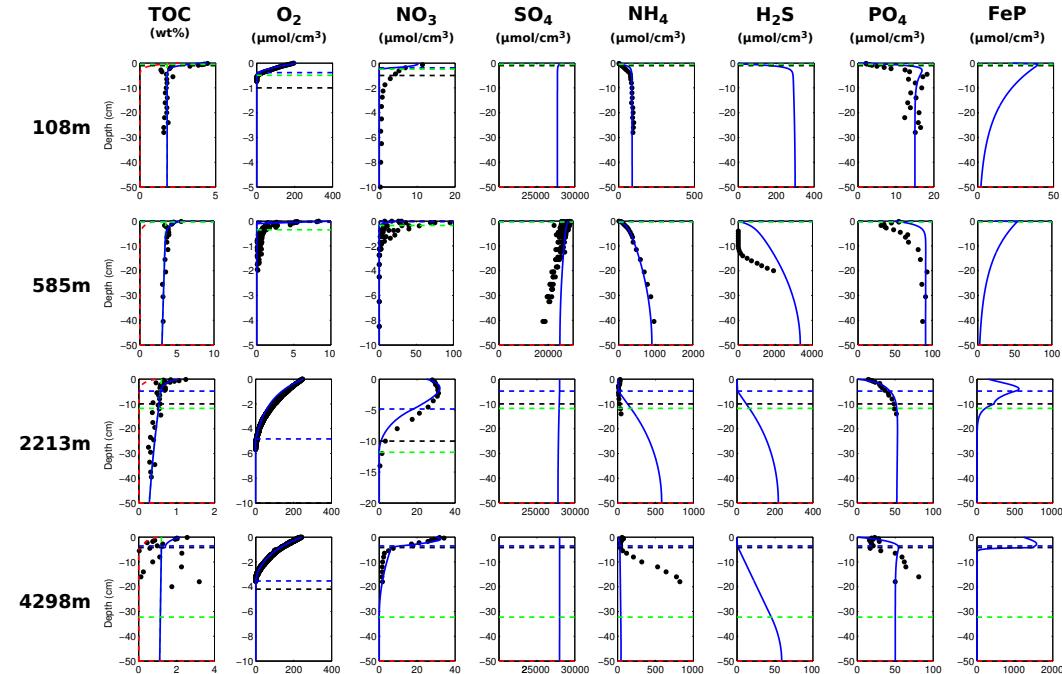


Figure 4. Modelled (curves) and measured (filled dots) dissolved and solid phase pore water profiles for four different sediment cores. Note that different scales are used for different stations. The blue TOC curve represents the sum of the refractory (green) and labile (red) TOC fraction.

3.2 Sensitivity Analysis

Model parameters implicitly account for processes not explicitly described, they are notoriously difficult to constrain and a source of uncertainty for a numerical model. Sensitivity Analysis (SA) is a term used for mathematical techniques to investigate how the variations in the outputs (y_1, \dots, y_N)
490 of a numerical model can be attributed to variations in the different input parameters (x_1, \dots, x_M)

Table 11. Model boundary conditions for the sampling stations in Figure 4.

Sediment characteristics:							
Depth (m)	Temp. (°C)	z_{bio} (cm)	D_{bio} ($\text{cm}^2 \text{yr}^{-1}$)	TOC ₁ (wt%)	TOC ₂ (wt%)	k_1 (yr^{-1})	k_2 (yr^{-1})
108	12.5	1.0	0.02	2.64	1.8	0.65	$1.0e^{-5}$
585	5.85	0.01	0.02	2.0	3.5	0.2	$8.0e^{-4}$
2213	3.2	10.0	0.17	0.45	0.6	0.15	$9.0e^{-4}$
4298	2.5	4.2	0.18	1.0	1.2	0.055	$1e^{-5}$

Bottom water concentrations of solutes (all in $\mu\text{mol cm}^{-3}$):							
Depth	O ₂	NO ₃	SO ₄	NH ₄	H ₂ S	PO ₄	PO ₄ ^a
108	210.0	9.6	28,000	0.4	0.0	0.0	15.0
585	10.0	25.0	28,000	0.0	0.0	50.0	90.0
2213	250	25.0	28,000	0.6	0.0	0.0	5.0
4298	243	30.1	28,000	0.22	0.0	5.0	

(Pianosi et al., 2016). Different types of sensitivity indices, which quantify this relative influence with a scalar S_i , can be calculated, ranging from simple one-at-a-time methods to statistical evaluations of the output distribution (e.g. variance-based or density-based approaches Pianosi et al., 2016). Especially the latter indices are easy to interpret and to compare across different parameters
495 and/or different model outputs as they generally take values between zero and one ($S_i \in [0, 1]$). An index of zero indicates a non-influential parameter and a higher value a more influential parameter. Here, we use SA mainly to identify which parameters have the largest impact on the different model outputs and therefore require careful calibration. As the probability density functions of our model outputs (i.e. the resulting SWI-fluxes) are generally highly-skewed towards extreme organic
500 matter degradation rates (not shown) variance-based sensitivity indices are not very reliable uncertainty indicators (Pianosi et al., 2016). Rather than just considering the variance we employ the novel density-based PAWN method by (Pianosi and Wagener, 2015) which considers the entire conditional and unconditional Cumulative Distribution Function (CDF) of the model output. The sensitivity index of parameter i is calculated as the difference between the two CDFs, i.e.

$$505 \quad S_i = \max_{x_i} \max_y |F_y(y) - F_{y|x_i}(y)| \quad (40)$$

where $F_y(y)$ is the unconditional CDF of the output y and $F_{y|x_i}(y)$ is the conditional CDF when the i -th parameter is fixed to x_i . For a more detailed description of the method we refer the interested reader to Pianosi and Wagener (2015). Due to the model complexity it is impossible to compute the sensitivity indices analytically therefore they are approximated from a Latin-Hypercube sampling of
510 parameter inputs and calculated outputs. We use PAWN as implemented within the Sensitivity Analysis for Everyone (SAFE) matlab toolbox (Pianosi et al., 2015) to investigate 11 model parameters for ranges specified in Table 12. We calculate sensitivity indices for all resulting SWI-fluxes from

Table 12. Range of model parameters used for sensitivity analysis of model predicted output.

Parameter	Description	Units	Minimum	Maximum	Source
k_1	labile OM degradation constant	yr^{-1}	$1e^{-4}$	5.0	(1)
\tilde{k}_2	order of refractory OM degradation constant ($k_2 = \tilde{k}_2 \cdot k_1$)	-	$1e^{-4}$	$1e^{-1}$	(1)
f_1	fraction of labile OM	-	0.02	0.98	-
K_{NH_4}	Adsorption coefficient	-	0.8	1.7	(2)
γ_{NH_4}	NH_4 fraction oxidised		0.5	1.0	-
$\gamma_{\text{H}_2\text{S}}$	H_2S fraction oxidised		0.5	1.0	-
$K_{\text{PO}_4}^I$	Adsorption coeff. oxic	-	100.0	400.0	(3)
$K_{\text{PO}_4}^{II}$	Adsorption coeff. anoxic	-	1.3	2.0	(3)
k_s	kinetic P sorption	yr^{-1}	0.1	100.0	(4, 5)
k_m	Fe-bound P release	yr^{-1}	0.015	0.02	(4, 5)
k_a	authigenic P formation	yr^{-1}	0.001	10.0	(4, 6)

Sources: (1) Arndt et al. (2013); (2): Van Cappellen and Wang (1996); (3): Krom and Berner (1980)
(4): Gypens et al. (2008); (5): Slomp et al. (1996); (6): Van Cappellen and Berner (1988)

Table 13. Maybe in Suppl. Model boundary conditions for the two idealised sediment conditions used for the sensitivity analysis (Figure 6). All solute concentrations are in $\mu\text{mol cm}^{-3}$.

Depth (m)	Temp. ($^{\circ}\text{C}$)	OC (wt%)	O_2	NO_3	SO_4	PO_4	z_{bio} (cm)
400	8.0	2.0	0.0	40.0	28,000	40.0	0.001
4000	1.5	1.0	300.0	20.0	28,000	40.0	10.0

OMEN-SED for two idealised sediment conditions (see Table 13). The resulting indices (compare Fig. 5 for two exemplary cases) are then translated into a color code and summarised in a pattern 515 plot (Fig. 6) to simplify comparison. The most significant parameters for all model outputs are the degradation rate for the labile OM part (k_1) and its share in the total OM pool (f_1). For the anoxic setup, where no oxidation occurs, the secondary redox parameters (i.e. γ_{NH_4} , $\gamma_{\text{H}_2\text{S}}$) are essentially non-influential, however, they are getting more important for the oxic scenario. The PO_4 SWI-flux appears to be insensitive under oxic conditions as the majority is absorbed to Fe-oxides correct....?

To test if OMEN-SED (1.0) is able to reproduce observed SWI-fluxes, the Latin-Hypercube sampling outputs are compared with a global database of benthic fluxes of O_2 and NO_3 (Bohlen et al., 2012). The coloured scatter plots (Figure 7) show that the observed fluxes fall well in the range of SWI-fluxes calculated with OMEN-SED (1.0). Also highlighted by the emergence of colour patterns in Figure 7 A+B are the strong interactions between the amount of labile OM f_1 and its degradation 525 rate k_1 for the resulting SWI-flux of the most powerful TEA available.

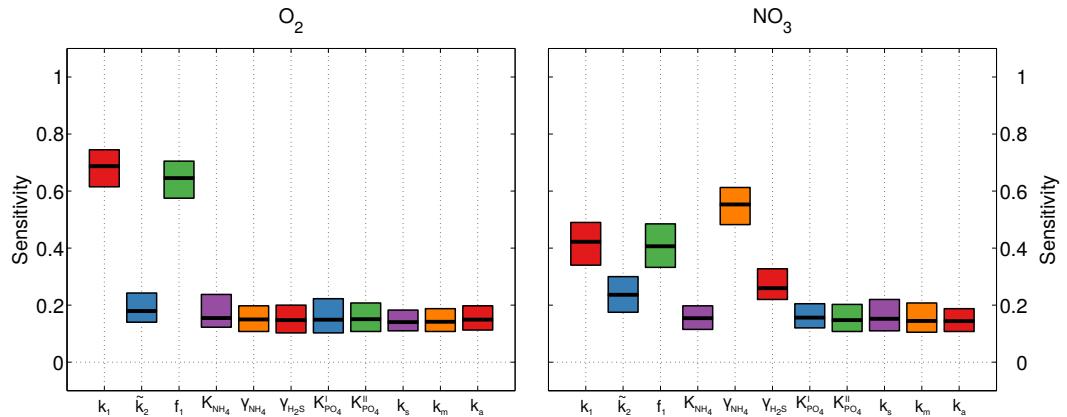


Figure 5. Maybe in Suppl. Box plot of parameter sensitivities for the O_2 and NO_3 SWI-flux for the 4000m oxic condition. Average sensitivities (black lines) and 90% confidence intervals using $N = 11200$ model evaluations and $N_{boot} = 100$ bootstrap resamples.

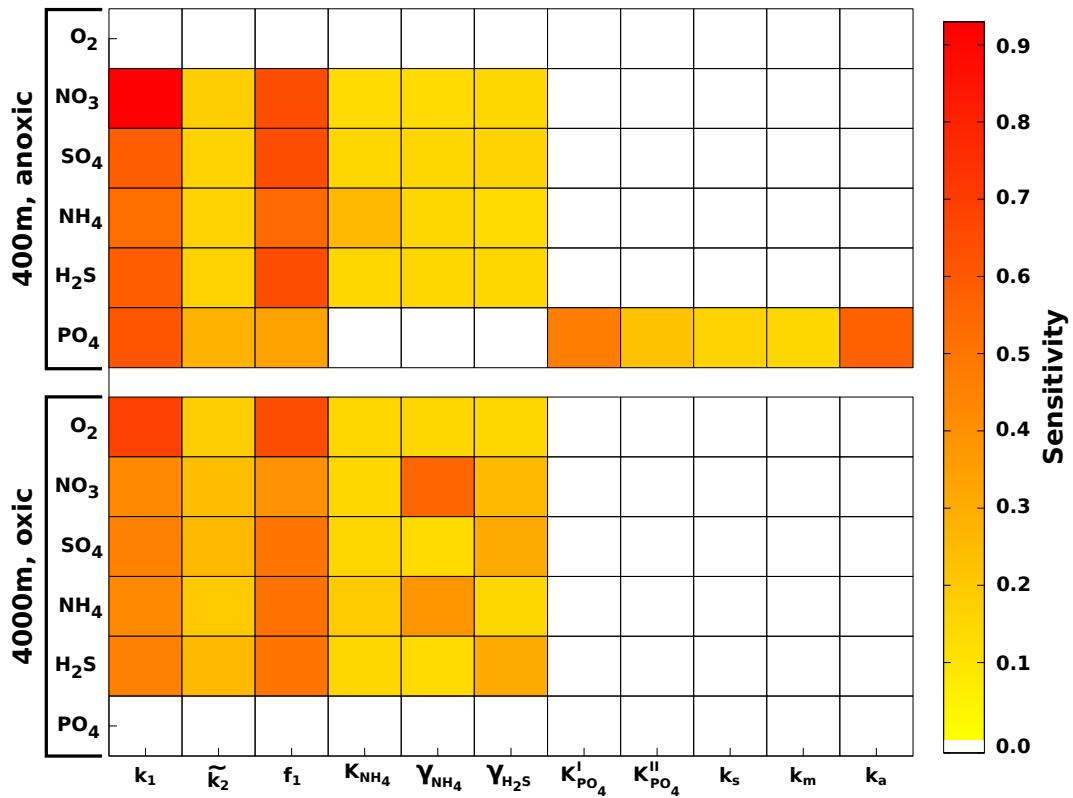


Figure 6. Pattern plot, showing the output sensitivity for each SWI flux (i.e. the chemical compounds on the horizontal axis) and each input factor (i.e. the model parameters on the vertical axis) for two idealised sediment cores.

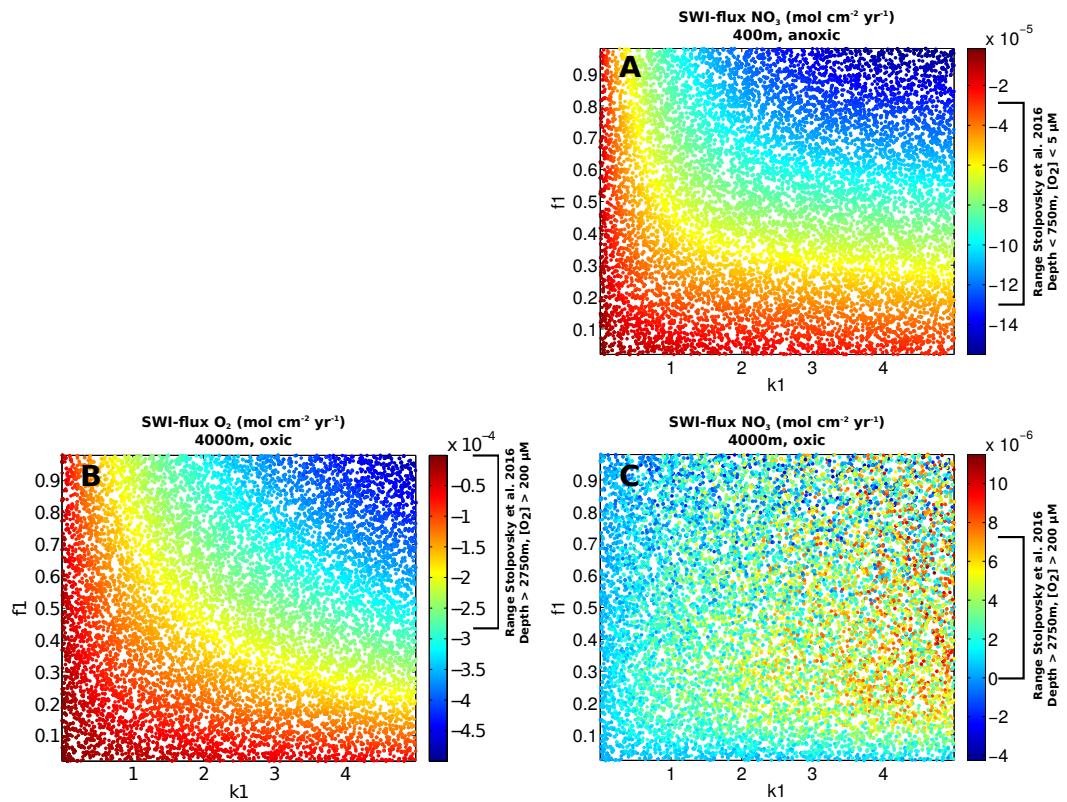


Figure 7. Coloured scatter plots (k_1 vs f_1) of resulting OMEN-SED (1.0) SWI-fluxes for the 400m anoxic (A: NO_3) and 4000m oxic (B: O_2 , C: NO_3) scenario. Negative values representing flux from the water column into the sediments. Indicated area at the respective colour scale represents range of benthic fluxes in the global database of Bohlen et al. (2012). **TODO:** Add N and swap colour scale + add text: Low influx, High influx...

3.3 Benthic fluxes on a global scale

Application to Seitert, 2004 OM, burwiczk see rate data and evaluation based on global data (Archer)

3.4 GENIE-Cretaceous test?

4 Scope of applicability and model limitations

5 Conclusions

TEXT

Table 14. Primary pathways of organic matter degradation, secondary redox reactions and stoichiometries implemented in the reaction network.

Pathway	Stoichiometry
Primary Redox Reactions	
Aerobic degradation	$(\text{CH}_2\text{O})_x(\text{NH}_3)_y(\text{H}_3\text{PO}_4)_z + (\text{x} + 2\text{y})\text{O}_2 + (\text{y} + 2\text{z})\text{HCO}_3^- \rightarrow (\text{x} + \text{y} + 2\text{z})\text{CO}_2 + \text{yNO}_3^- + \text{zHPO}_4^{2-} + (\text{x} + 2\text{y} + 2\text{z})\text{H}_2\text{O}$
Denitrification	$5(\text{CH}_2\text{O})_x(\text{NH}_3)_y(\text{H}_3\text{PO}_4)_z + (\text{4x} + 3\text{y})\text{NO}_3^- \rightarrow (\text{2x} + 4\text{y})\text{N}_2 + (\text{x} - 3\text{y} + 10\text{z})\text{CO}_2 + (\text{4x} + 3\text{y} - 10\text{z})\text{HCO}_3^- + 5\text{zHPO}_4^{2-} + (\text{3x} + 6\text{y} + 10\text{z})\text{H}_2\text{O}$
Sulfate reduction	$2(\text{CH}_2\text{O})_x(\text{NH}_3)_y(\text{H}_3\text{PO}_4)_z + \text{xSO}_4^{2-} + 2(\text{y} - 2\text{z})\text{CO}_2 + 2(\text{y} - 2\text{z})\text{H}_2\text{S} \rightarrow \text{xH}_2\text{O} + 2(\text{x} + \text{y} - 2\text{z})\text{HCO}_3^- + 2\text{yNH}_4^+ + 2\text{zHPO}_4^{2-}$
Methanogenesis	—
Secondary Redox Reactions	
Nitrification	$\text{NH}_4^+ + 2\text{O}_2 + 2\text{HCO}_3^- \rightarrow \text{NO}_3^- + 2\text{CO}_2 + 3\text{H}_2\text{O}$
Sulfide oxidation	$\text{H}_2\text{S} + 2\text{O}_2 + 2\text{HCO}_3^- \rightarrow \text{SO}_4^{2-} + 2\text{CO}_2 + 2\text{H}_2\text{O}$
AOM	$\text{CH}_4 + \text{SO}_4^{2-} \rightarrow \text{HCO}_3^- + \text{HS}^- + \text{H}_2\text{O}$ how get H_2S? $\rightarrow \text{CO}_3^{2-} + \text{H}_2\text{S} + \text{H}_2\text{O}$

6 Code Availability

Appendix A: Reaction Network

A1

535 *Acknowledgements.* TEXT

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