OMEN-SED 1.0: A new, numerically efficient sediment module for the coupling to Earth System Models

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Abstract. Here we describe the first version of the Organic Matter ENabled SEDiment model (OMEN-SED 1.0).

1 Introduction

DH: How to include comments.

Marine surface sediments are key components in the Earth system. They host the largest carbon reservoir within the surficial Earth system, provide the only long term sink for atmospheric CO₂, recycle nutrients and represent the most important geochemical archive used for deciphering past changes in biogeochemical cycles and climate. Physical and chemical processes in sediments (i.e. diagenetic processes) depend on the water column and vice versa: Diagenesis is mainly donor controlled, as it is fuelled by the external supply of solid material (e.g. organic matter, calcium carbonate, opal) from the water column and is affected by overlying bottom water concentrations of solutes. At the same time, diagenesis in the sediments transforms the deposited material and returns some of the resulting products (e.g. nutrients, DIC) to the water column. This so-called benthic-pelagic coupling is essential for understanding global biogeochemical cycles and climate (e.g. Archer and Maier-Reimer, 1994; Soetaert et al., 2000; Archer et al., 2000; Mackenzie, 2005).

Biological primary production of organic matter (OM, CH₂O in equation R1) and the reverse process of degradation can be written in a greatly simplified reaction as:

$$CO_2 + H_2O \rightleftharpoons CH_2O + O_2.$$
 (R1)

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On geological timescales production of OM is generally greater than degradation which results in some organic matter being buried in marine sediments and oxygen accumulating in the atmosphere. Thus burial of OM leads to net oxygen input to, and CO_2 removal from the atmosphere (Berner, 2004). On shorter timescales, the upper few meters of the sediments (i.e. early diagenesis) are specifically important as it is decided here if a substance is recycled to the water column or buried for a longer period of time in the deeper sediments (Hensen et al., 2006). Most biogeochemical cycles and reactions in this part of marine sediments can be related either directly or indirectly to the degradation of organic matter (e.g Boudreau and Ruddick, 1991; Arndt et al., 2013). Organic matter degradation releases metabolic CO_2 to the pore water, causing it to have a lower pH and provoking the dissolution of $CaCO_3$ (Emerson and Bender, 1981). Oxygen and nitrate for instance, the most powerful electron acceptors, are consumed in the course of the degradation of organic matter, resulting in the release of ammonium and phosphate to the pore water. As such, changes in the degradation rate of OM can profoundly affect the oxygen and nutrient inventory of the ocean and thus primary productivity (Van Cappellen and Ingall, 1994; Lenton and Watson, 2000).

Nutrient recycling from the sediments has been invoked by modelling and data studies to explain the occurance of extreme events in Earth history, for instance Oceanic Anoxic Events (OAEs) (e.g. Mort et al., 2007; Tsandev and Slomp, 2009). OAEs represent severe disturbances of the global carbon, oxygen and nutrient cycles of the ocean and are usually characterized by widespread bottom water anoxia and photic zone euxinia (Jenkyns, 2010). One way to explain the genesis and persistence of OAEs is increased oxygen demand due to enhanced primary productivity (PP). Increased nutrient inputs to fuel PP may have come from marine sediments as the burial efficiency of phosphorus declines when bottom waters become anoxic (Ingall and Jahnke, 1994; Van Cappellen and Ingall, 1994). The recovery from OAE like conditions is thought to involve the permanent removal of excess CO₂ from the atmosphere and ocean by burying carbon in the form of organic matter in marine sediments (e.g. Arthur et al., 1988; Jarvis et al., 2011), which is consistent with the geological record of widespread black shale formation (Stein et al., 1986). However, the overall amount, exact timing and the rate of organic matter burial remain a topic of an ongoing debate.

Therefore, globally quantifying the burial and degradation of organic matter in marine sediments and related biogeochemical dynamics is important for understanding climate and the cycling of many chemical elements on various timescales. Such studies and quantifications are possible through the application of idealised mathematical representations of diagenesis, or so-called diagenetic models (e.g. Berner, 1980; Boudreau, 1997). The number of research questions that can be addressed with diagenetic models is infinite and a plethora of different approaches have been developed, mainly following two distinct directions (Arndt et al., 2013). First, state-of-the art vertically resolved diagenetic models simulating all of the essential coupled redox and equilibrium reactions within marine sediments that control carbon burial and benthic recycling fluxes (e.g. BRNS, Aguilera et al., 2005; MEDIA, Meysman et al., 2003; OMEXDIA, Soetaert et al., 1996b). These "complete", non-steady-

state models generally use a so-called multi-G approach, thus dividing the bulk organic carbon pool into a number of compound classes that are characterised by different degradabilities k_i . Combining such a complex diagenetic model with an ocean biogeochemical model results in the most realistic benthic-pelagic coupling. However, the global applicability of this set-up is limited by the high computation cost of simulating all biogeochemical reactions in the sediments. The second group of models is less sophisticated and comprehensive than the "complete", non-steady-state diagenetic models and is used for the coupling to global Earth System Models (e.g. DCESS, Shaffer et al., 2008; MEDUSA, Munhoven, 2007 or the model of Heinze et al. (1999)). In particular, these analytic models consider fewer biogeochemical reactions and assume that the sedimentary organic matter pool is composed of just a single or two discrete compound classes (1G or 2G approach respectively).

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Even though there is potential to use more appropriate global sediment representations, in most current Earth System models sediment-water exchange of OM and chemical elements is either neglected or treated in a very simplistic way (Soetaert et al., 2000; Hülse et al., 2017). Most Earth system Models of Intermediate Complexity (EMICs) and also some of the higher resolution global carbon cycle models represent the sediment-water interface either as a reflective or a conservative/semireflective boundary (Hülse et al., 2017). Thus, all particulate material deposited on the seafloor is either instantaneously consumed (reflective boundary), or a fixed fraction is buried in the sediments (conservative/semi-reflective boundary). Both highly simplified approaches furthermore completely neglect the exchange of solute species through the sediment-water interface and, therefore, cannot resolve the complex benthic-pelagic coupling. However, due to their computational efficiency, both representations are often used in global biogeochemical models (e.g. Najjar et al., 2007; Ridgwell et al., 2007; Goosse et al., 2010). A superior approach is the vertically integrated dynamic model, which represents the whole sediment column as a single box (Hülse et al., 2017). Here, OM deposited on the seafloor is added to the sediment box where it gets degraded and dissolved species diffuse through the sediment-water interface in accordance with these transformations. This approach thus ignores the vertical extent of the sediments and the temporary storage of dissolved species (Soetaert et al., 2000). Yet, it is computationally efficient and also allows differentiating between various fractions of organic matter. Most EMICs incorporate a vertically integrated dynamic model for particulate inorganic carbon only (i.e. mainly CaCO₃) and just a few consider oxic-only sediment degradation of organic matter (Hülse et al., 2017). The most complex description of marine sediments in Earth System models are vertically resolved diagenetic models, solving the onedimensional, fully coupled reaction-transport equation for multiple solid and dissolved species (e.g. Berner, 1980; Boudreau, 1997). These models account for important transport processes (e.g. bioturbation, molecular diffusion and bioirrigation), resolve the coupled biogeochemical dynamics of dissolved species and usually at least a part of the resulting characteristic redox-zonation of marine sediments (Soetaert et al., 2000). Examples of global carbon cycle models employing a vertically resolved diagenetic model are PISCES (Gehlen et al., 2006) and CLIMBER 3α (Ilyina et al., 2013),

both using a version of Heinze et al. (1999), however, just utilizing a 1G approach with a constant degradation rate constant. None of the EMICs reviewed by Hülse et al. (2017) use such a sediment representation.

Obviously, such simplifications can neither account for the observed vast structural complexity in natural organic matter and its resulting different degradation rates nor for the rapid decrease in OM degradability in the uppermost centimeters of the sediments (Arndt et al., 2013). At least a 3G approach is necessary to accurately represent organic matter dynamics in this part of the sediments where most OM is degraded (e.g. Soetaert et al., 1996b). Furthermore, the spatial variability in benthic OM degradation kinetics is unknown at the global scale and reported rate constants can vary by almost 10 orders of magnitude (Arndt et al., 2013). Thus, a major challenge for diagenetic models is defining appropriate OM degradation rate constants which is either achieved through profile fitting for a specific site or, for global applications, the rate constants follow empirical relations with a related, readily available sediment characteristic such as water depth (Middelburg et al., 1997), sedimentation rate (Toth and Lerman, 1977; Tromp et al., 1995) or OM flux (Boudreau, 1997). However, these relationships are mostly based on simple fitting exercises to limited data sets and show at the most a very weak trend and no statistically significant relationship (Arndt et al., 2013). This creates considerable uncertainties when diagenetic models are coupled to global biogeochemical ocean models especially when applying the model for different geological timescales. Other model parameters implicitly accounting for processes not explicitly described are sedimentation and bioturbation rate and are generally related to water depth following Middelburg et al. (1997).

Here, we present the OrganicMatter ENabled SEDiment model (OMEN-SED 1.0), a new, onedimensional, numerically efficient reactive transport model (RTM) that describes OM cycling as well as the associated dynamics of the most important terminal electron acceptors (TEAs, i.e. O₂, NO₃, 115 SO₄, CH₄), related reduced substances (NH₄ H₂S), macronutrients (PO₄) and associated pore water quantities (ALK, DIC). OMEN-SED's computational efficiency allows its coupling to Earth System Models of different complexities and therefore the investigation of coupled global biogeochemical dynamics over geological timescales. Here, the model is presented as a 2G-approach, however, a third, nondegradable OM pool can be added and furthermore OMEN-SED can easily be extended to a Multi-G approach.

2 Model Description

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The following section provides a detailed description of the new model. Table 1 summarizes the biogeochemical reaction network and a glossary of parameters along with their respective units is provided in Tables 9 and 10.

125 2.1 General Model Approach

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The calculation of benthic return/uptake and burial fluxes is based on the vertically resolved conservation equation for solid and dissolved species in porous media (e.g. Berner, 1980; Boudreau, 1997):

$$\frac{\partial \xi C_i}{\partial t} = -\frac{\partial F}{\partial z} + \xi \sum_j R_i^j \tag{1}$$

where C_i is the concentration of the biogeochemical species i, ξ equals the porosity ϕ for solute species and $(1-\phi)$ for solid species, hence represents the partitioning of species i into the solute and dissolved phase. The term z is the sediment depth, t denotes the time, F summarises the transport fluxes and $\sum_j R_i^j$ represents the sum of production/consumption rates j that affect species i. The reaction network accounts for the most important primary and secondary redox reactions, equilibrium reactions, mineral dissolution and precipitation, as well as adsorption and desorption processes that affect the explicitly resolved chemical species.

State-of-the-art reaction-transport models generally solve the ordinary differential equation (ODE, Eq. 1) numerically and thus allow to account for transient dynamics, depth-varying parameters or a high degree of coupling between different chemical species (e.g. Soetaert et al., 1996b; Aguilera et al., 2005). Yet, numerical models are computational expensive, thus rendering their application in an Earth System Model framework prohibitive. An analytical solution of equation (1) provides an alternative and computational more efficient approach which enjoyed great popularity in the early days of diagenetic modelling and computer technology. However, early analytical models were often very problem-specific and only considered one or two coupled species (e.g. Berner, 1964). (e.g. Lehrman, Berner) ?? which other pub? Over the next decades, a number of more complex analytical models describing the coupled dynamics of OM degradation, TEAs and rediced substances in sediments were developed (e.g. Billen, 1982; Goloway and Bender, 1982; Jahnke et al., 1982), before the boost in computing power enabled the development of fully-coupled, multi-species, numerical models (e.g. Van Cappellen and Wang, 1995; Soetaert et al., 1996b).

Finding an analytical solution to Eq. (1), especially when complex reaction networks are to be considered, is not straightforward and generally requires the assumption of steady state. In addition, the complexity of the reaction network can be reduced by dividing the sediment into distinct zones and accounting for the most pertinent biogeochemical processes within each zone, thus increasing the likelihood of finding an analytical solution to Eq. (1) (see Eq. (30) in section 2.5.1 for the general steady-state solution).

Therefore, OMEN-SED assumes that benthic dynamics can be represented by a series of steadystates. Because the Earth system model relevant variability in boundary conditions and fluxes is generally longer than the characteristic timescales of the reaction-transport processes, the sediment can be described by a series of pseudo steady-states. In addition, it divides the sediment into a bio-

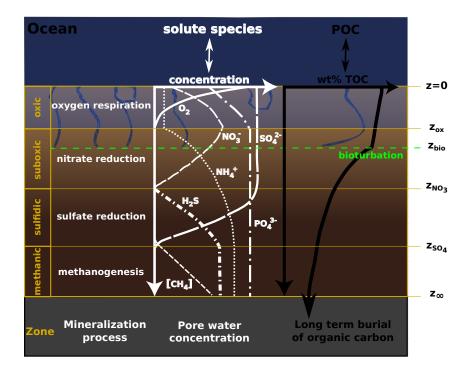


Figure 1. Schematic of the different modelled species and layers in OMEN-SED. Here showing the case $z_{\rm ox} < z_{\rm bio} < z_{\rm NO_3} < z_{\rm SO_4}$.

turbated and a non-bioturbated zone defined by the constant bioturbation depth $z_{\rm bio}$ (see Fig. 1). Furthermore, it accounts for the dynamic redox zonation of marine sediments by dividing the sediment into: 2) an oxic zone delineated by the oxygen penetration depth $z_{\rm ox}$, 3) a denitrification zone situated between $z_{\rm ox}$ and the nitrate penetration depth $z_{\rm NO_3}$, 4) a sulfate reduction zone situated between $z_{\rm NO_3}$ and the sulfate penetration depth $z_{\rm SO_4}$ and 5) a methanogenic zone situated below $z_{\rm SO_4}$ (Fig. 1). All penetration depths are dynamically calculated by the model. Each zone is characterised by a set of diagenetic equations that encapsulate the most pertinent reaction and transport processes in this zone (see section 2.2 and 2.3 for more details).

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OMEN calculates and feeds back to the Earth System model the fraction of POC preserved in the sediments and the sediment-water interface fluxes of the dissolved species C_i (in mol cm⁻² year⁻¹):

$$Flux_SWI(C_i) = \phi \left(D_i \frac{\partial C_i(z)}{\partial z} \bigg|_{0} - w \left[C_i(0) - C_i(z_{\infty}) \right] \right)$$
 (2)

where w is the deposition rate, D_i is the diffusion coefficient and $C_i(0)$, $C_i(z_{\infty})$ the concentration of species i at the SWI and at the lower sediment boundary.

175 2.2 Transport

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The model accounts for both the advective, as well as the diffusive transport of dissolved and solid species, assuming that sediment compaction is negligible (i.e. $\frac{\partial \phi}{\partial z} = 0$). The molecular diffusion of dissolved species is described via a species-specific apparent diffusion coefficient, $D_{\mathrm{mol},i}$. In addition, the activity of infaunal organisms in the bioturbated zone of the sediment ($z < z_{\mathrm{bio}}$) that causes random displacements of sediments and porewaters is simulated using a diffusive term (e.g. Boudreau,1986), with a constant bioturbation coefficient D_{bio} in the bioturbated zone. The pumping activity by burrow-dwelling animals and the resulting ventilation of tubes, the so-called bioirrigation, is encapsulated in a factor, f_{ir} that enhances the molecular diffusion coefficient (hence, $D_{i,0} = D_{\mathrm{mol},i} \cdot f_{ir}$, Soetaert et al., 1996a). The flux divergence can thus be formulated as:

$$\frac{\partial F}{\partial z} = -\frac{\partial}{\partial z} \left(-\xi D_i \frac{\partial C_i}{\partial z} + \xi w C_i \right) \tag{3}$$

where D_i is the diffusion coefficient of species i ($D_i = D_{i,0} + D_{bio} = D_{mol,i} \cdot f_{ir} + D_{bio}$ for dissolved species and $D_i = D_{bio}$ for solid species) and w is the deposition rate. The bioturbation coefficient D_{bio} is set to zero below z_{bio} .

2.3 Reaction Network

Earth System models generally track the biogeochemical dynamics of organic and inorganic carbon, essential nutrients (nitrogen, phosphorus) and oxygen with the aim of investigating the evolution of the ocean's redox structure and carbonate system and its feedbacks on global climate. This general aim thus defines a minimum set of state variables and reaction processes that need to be resolved for an efficient representation of the benthic-pelagic coupling in Earth system models. A suitable sediment model must provide a robust quantification of organic (and inorganic) carbon burial fluxes, as well as the benthic return fluxes of growth-limiting nutrients, equilibrium invariant and reduced species, and oxygen uptake fluxes. As a consequence, the reaction network must account for the most important primary and secondary redox reactions, equilibrium reactions, mineral precipitation/dissolution and adsorption/desorption, resulting in a complex set of coupled reaction-transport equations. The following subsections provide a short discussion of the reaction processes included in the model and give an overview of the vertically resolved conservation equations and boundary conditions for solid and dissolved species in each layer. Table 1 provides a summary of the reactions and biogeochemical tracers considered in the reaction network. Table 14 summarises their reaction stoichiometry and Table ?? provides an overview of their description in the model.

205 2.3.1 Organic matter or Particulate Organic Carbon (POC)

In marine sediments, organic matter (OM) is degraded by heterotrophic activity coupled to the sequential utilisation of terminal electron acceptors, typically in the order of O_2 , NO_3^- , Mn(VI),

Table 1. Reactions and biogeochemical tracers implemented in the Reaction Network of OMEN-SED. The primary and secondary redox reactions are listed in the sequence they occur with increasing sediment depth.

	Description
Primary redox reactions	Degradation of organic matter via aerobic respiration, denitrification,
	sulfate reduction, methanogenesis (implicit)
Secondary redox reactions	Oxidation of ammonium and sulfide by oxygen, anaerobic oxidation
	of methane by sulfate
Adsorption/Desorption	Ad-/Desorption of P on/from $\mathrm{Fe}(\mathrm{OH})_3, \mathrm{NH}_4$ adsorption, PO_4 adsorption
Mineral precipitation	Formation of authigenic P
Biogeochemical tracers	Organic matter, oxygen, nitrate, ammonium, sulfate, sulfide (hydrogen sulfide),
	phosphate, Fe-bound P, DIC, ALK

Table 2. Boundary conditions for organic matter. For the boundaries we define: $z_{\text{bio}}^- := \lim_{h \to 0} (z_{\text{bio}} - h)$ and $z_{\text{bio}}^+ := \lim_{h \to 0} (z_{\text{bio}} + h)$.

Boundary	Condition		
z = 0	known concentration	1)	$C_i(0) = C_{i0}$
$z = z_{\rm bio}$	continuity		$C_i(z_{\mathrm{bio}}^-) = C_i(z_{\mathrm{bio}}^+)$
		3)	$-D_{\text{bio}} \cdot \frac{\partial C_i}{\partial z} \big _{z_{\text{bio}}} = 0$

Fe(III) and SO_4^{2-} followed by methanogenesis and/or fermentation. Here, organic matter degradation is described via a multi-G model approach (Arndt et al., 2013, and references therein), assuming that the bulk OM consists of a number of discrete compound classes C_i characterised by specific degradation rate constants k_i . Such a multi-G approach allows for selective preservation of compound classes according to their reactivity, k_i and, thus, accounts for the change in organic matter reactivity during burial. Each compound class is degraded according to first-order kinetics. The conservation equation for organic matter dynamics is thus given by:

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$$\frac{\partial C_i}{\partial t} = 0 = D_{C_i} \frac{\partial^2 C_i}{\partial z^2} - w \frac{\partial C_i}{\partial z} - k_i \cdot C_i \tag{4}$$

The analytical solution of Eq. (4) (see section 2.5 for details) requires the definition of a set of boundary conditions (Table 2). The model assumes a known concentration/flux at the sediment-water interface and continuity across the bottom of the bioturbated zone, $z_{\rm bio}$.

220 2.3.2 Oxygen

In marine sediments, oxygen is consumed via aerobic degradation of organic matter and a number of secondary redox reactions. In the oxic layer ($z < z_{ox}$), the model explicitly accounts for the aerobic degradation of OM, which consumes oxygen with a fixed O_2 : C ratio (O_2 C, Tab. 10) and produces

Table 3. Boundary conditions for oxygen. For the boundaries we define: $z_{\text{bio}}^- := \lim_{h \to 0} (z_{\text{bio}} - h)$ and $z_{\text{bio}}^+ := \lim_{h \to 0} (z_{\text{bio}} + h)$.

Boundary	Condition	
z = 0	known concentration	1) $O_2(0) = O_{20}$
$z = z_{\rm bio}$	continuity	2) $O_2(z_{\text{bio}}^-) = O_2(z_{\text{bio}}^+)$
		3) $-(D_{\text{O}_2,0} + D_{\text{bio}}) \cdot \frac{\partial \text{O}_2}{\partial z} _{z_{\text{bio}}^-} = -D_{\text{O}_2,0} \cdot \frac{\partial \text{O}_2}{\partial z} _{z_{\text{bio}}^+}$
$z = z_{\rm ox}$	O ₂ consumption	4) IF $(O_2(z_\infty) > 0)$
	O_2 consumption $(z_{ m ox}=z_{\infty})$	$\frac{\partial \mathcal{O}_2}{\partial z} _{z_{\text{ox}}} = 0$
		ELSE
	$(z_{\rm ox} < z_{\infty})$	$O_2(z_{ox}) = 0$ and $-D_{O_2} \cdot \frac{\partial O_2}{\partial z} _{z_{ox}} = F_{red}(z_{ox})$
	with flux from below	$F_{red}(z_{\text{ox}}) = \frac{1-\phi}{\phi} \cdot \int_{z_{\text{ox}}}^{\infty} \sum_{i} (2\gamma_{\text{NH}_4} \text{NC}_i + 2\gamma_{\text{H}_2} \text{sSO}_4 \text{C}) k_i C_i dz$

ammonium, which is partially nitrified to nitrate (γ_{NH_4}) . In addition, the oxygen consumption due to oxidation of reduced species (NH_4, H_2S) produced in the suboxic and anoxic layers of the sediment is implicitly taken into account through the flux boundary condition at the dynamic oxygen penetration depth z_{ox} . This simplification can be justified as it has been shown that these secondary redox reactions mainly occur at the oxic/suboxic interface (Soetaert et al., 1996b). The factor $\frac{1-\phi}{\phi}$ accounts for the volume conversion from the solid to the dissolved phase. Oxygen dynamics are thus described by:

$$\frac{\partial \mathcal{O}_2}{\partial t} = 0 = D_{\mathcal{O}_2} \frac{\partial^2 \mathcal{O}_2}{\partial z^2} - w \frac{\partial \mathcal{O}_2}{\partial z} - \frac{1 - \phi}{\phi} \sum_i k_i \cdot \left[\mathcal{O}_2 \mathcal{C} + 2\gamma_{\mathrm{NH}_4} \mathcal{N} \mathcal{C}_i \right] \cdot C_i(z)$$
 (5)

The analytical solution of Eq. (5) requires the definition of boundary conditions (Table 3). OMEN-SED assumes a known bottom water concentration and the complete consumption of oxygen at the oxygen penetration depth (or zero flux if $z_{\rm ox}=z_{\infty}$). Equal oxygen concentration and diffusive flux above ($z_{\rm bio}^-$) and below ($z_{\rm bio}^+$) the bioturbation boundary is considered. In addition, the model accounts for reduced species produced by anaerobic mineralization diffusing into the oxic layer from below, assuming that respective fractions ($\gamma_{\rm NH_4}$ and $\gamma_{\rm H_2S}$) are re-oxidised at the oxic/suboxic interface.

240 2.3.3 Nitrate and Ammonium

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To model nitrate and ammonium dynamics the sediment is partitioned into two geochemical layers (oxic and suboxic), where different equations describe the biogeochemical processes. Above the oxygen penetration depth organic matter mineralization produces ammonium, which is partly nitrified to nitrate (the fraction $\gamma_{\rm NH_4}$). In the suboxic zone ($z>z_{\rm ox}$), oxygen concentration is zero and nitrate serves as the electron acceptor to respire organic matter, thus nitrate is consumed by denitrification and ammonium is produced. Below the nitrate penetration depth $z_{\rm NO_3}$, ammonium is still produced via OM mineralization. The model assumes that adsorption of ammonium to sediment

particles is fast compared with the characteristic transport time scales. Thus, a constant equilibrium adsorption coefficient $K_{\rm NH_4}$ is used to parameterize the loss of dissolved NH₄ to adsorped NH₄ 250 (Wang and Van Cappellen, 1996). Therefore the diagenetic equations for nitrate and ammonium are given by:

1. Layer $(z \le z_{ox})$

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$$\frac{\partial NO_3^I}{\partial t} = 0 = D_{NO_3} \frac{\partial^2 NO_3^I}{\partial z^2} - w \frac{\partial NO_3^I}{\partial z} + \gamma_{\text{NH}_4} \frac{1 - \phi}{\phi} \cdot \sum_i \text{NC}_i \cdot k_i \cdot C_i(z)$$
 (6)

$$\frac{\partial \mathrm{NH_4}^I}{\partial t} = 0 = \frac{D_{\mathrm{NH_4}}}{1 + K_{\mathrm{NH_4}}} \frac{\partial^2 \mathrm{NH_4}^I}{\partial z^2} - w \frac{\partial \mathrm{NH_4}^I}{\partial z} + \frac{1 - \gamma_{\mathrm{NH_4}}}{1 + K_{\mathrm{NH_4}}} \cdot \frac{1 - \phi}{\phi} \cdot \sum_i \mathrm{NC_i} \cdot k_i \cdot C_i(z) \tag{7}$$

2. Layer $(z_{\text{ox}} < z \le z_{\text{NO}_3})$

$$\frac{\partial NO_3^{II}}{\partial t} = 0 = D_{NO_3} \frac{\partial^2 NO_3^{II}}{\partial z^2} - w \frac{\partial NO_3^{II}}{\partial z} - \frac{1 - \phi}{\phi} NO_3 C \cdot \sum_i k_i \cdot C_i(z) \tag{8}$$

$$\frac{\partial \text{NH}_4^{II}}{\partial t} = 0 = \frac{D_{\text{NH}_4}}{1 + K_{\text{NH}_4}} \frac{\partial^2 \text{NH}_4^{II}}{\partial z^2} - w \frac{\partial \text{NH}_4^{II}}{\partial z}$$
(9)

3. Layer $(z_{NO_3} < z \le z_{\infty})$

$$\frac{\partial \mathrm{NH_4}^{III}}{\partial t} = 0 = \frac{D_{\mathrm{NH_4}}}{1 + K_{\mathrm{NH_4}}} \frac{\partial^2 \mathrm{NH_4}^{III}}{\partial z^2} - w \frac{\partial \mathrm{NH_4}^{III}}{\partial z} + \frac{1}{1 + K_{\mathrm{NH_4}}} \cdot \frac{1 - \phi}{\phi} \cdot \sum_{i} \mathrm{NC_i} \cdot k_i \cdot C_i(z)$$

$$\tag{10}$$

The boundary conditions to solve Equations 6 - 10 are summarized in Table 4. The model assumes known bottom water concentrations for both species, the complete consumption of nitrate at the nitrate penetration depth (or zero flux if $z_{\rm NO_3}=z_{\infty}$) and no change in ammonium flux at z_{∞} . It considers equal concentrations and diffusive fluxes at $z_{\rm bio}$ and $z_{\rm ox}$. In addition, the re-oxidation of upward-diffusing reduced ammonium is considered in the oxic-suboxic boundary condition for nitrate and ammonium.

270 2.3.4 Sulfate and Sulfide

When nitrate is depleted, sulfate reduction is the pathway to mineralize organic matter, thus consuming sulfate (SO₄) and producing hydrogen sulfide (H₂S) until the sulfate penetration depth (z_{SO_4}). Sulfate and sulfide dynamics are thus described by:

275 1. Layer ($z \le z_{NO_3}$)

$$\frac{\partial SO_4^I}{\partial t} = 0 = D_{SO_4} \frac{\partial^2 SO_4^I}{\partial z^2} - w \frac{\partial SO_4^I}{\partial z}$$
(11)

$$\frac{\partial \mathbf{H}_2 \mathbf{S}^I}{\partial t} = 0 = D_{\mathbf{H}_2 \mathbf{S}} \frac{\partial^2 \mathbf{H}_2 \mathbf{S}^I}{\partial z^2} - w \frac{\partial \mathbf{H}_2 \mathbf{S}^I}{\partial z} \tag{12}$$

Table 4. Boundary conditions for nitrate and ammonium. For the boundaries we define: $z_-^- := \lim_{h \to 0} (z_- - h)$ and $z_+^+ := \lim_{h \to 0} (z_- + h)$.

Boundary	Condition		
z = 0	known concentration	1)	$NO_3(0) = NO_{30}$
$z = z_{\rm bio}$	continuity	2)	$NO_3(z_{ m bio}^-)$ = $NO_3(z_{ m bio}^+)$
		3)	$-\left(D_{NO_3,0}+D_{\mathrm{bio}}\right)\cdot\frac{\partial^{NO_3}}{\partial z} _{z_{\mathrm{bio}}^-} = -D_{NO_3,0}\cdot\frac{\partial^{NO_3}}{\partial z} _{z_{\mathrm{bio}}^+}$
$z = z_{\rm ox}$	continuity	4)	$NO_{3}(z_{ m ox}^{-})$ = $NO_{3}(z_{ m ox}^{+})$
		5)	$-D_{NO_3} \cdot \tfrac{\partial NO_3}{\partial z} _{z_{\rm ox}^-} + \gamma_{\rm NH_4} \cdot F_{\rm NH_4}(z_{\rm ox}) = -D_{NO_3} \cdot \tfrac{\partial NO_3}{\partial z} _{z_{\rm ox}^+}$
	where:		$F_{\rm NH_4}(z_{\rm ox}) = \frac{1}{1 + K_{\rm NH_4}} \cdot \frac{1 - \phi}{\phi} \cdot \int_{z_{\rm NO_3}}^{\infty} \sum_i k_i \cdot \rm NC_i \cdot C_i dz$
$z = z_{NO_3}$	NO ₃ consumption	6)	$\mathbf{IF} (\mathrm{NO}_3(z_{\infty}) > 0)$
	$(z_{\rm NO_3} = z_{\infty})$		$\frac{\partial NO_3}{\partial z} _{z_{\text{NO}_3}} = 0$
			ELSE
	$(z_{ m NO_3} < z_{\infty})$		$NO_3(z_{\mathrm{NO}_3}) = 0$
z = 0	known concentration	1)	$\mathrm{NH_4}(0) = \mathrm{NH_{40}}$
$z = z_{\rm bio}$	continuity	2)	$\mathrm{NH_4}(z_\mathrm{bio}^-) = \mathrm{NH_4}(z_\mathrm{bio}^+)$
		3)	$-\frac{D_{\mathrm{NH_4,0}} + D_{\mathrm{bio}}}{1 + K_{\mathrm{NH_4}}} \cdot \frac{\partial \mathrm{NH_4}}{\partial z}\big _{z_{\mathrm{bio}}^-} = -\frac{D_{\mathrm{NH_4,0}}}{1 + K_{\mathrm{NH_4}}} \cdot \frac{\partial \mathrm{NH_4}}{\partial z}\big _{z_{\mathrm{bio}}^+}$
$z = z_{\rm ox}$	continuity	4)	$NH_4(z_{ox}^-)=NH_4(z_{ox}^+)$
		5)	$-\frac{D_{\rm NH_4}}{1+K_{\rm NH_4}} \cdot \frac{\partial {\rm NH_4}}{\partial z} _{z_{\rm ox}^-} - \gamma_{\rm NH_4} \cdot F_{\rm NH_4}(z_{\rm ox}) = -\frac{D_{\rm NH_4}}{1+K_{\rm NH_4}} \cdot \frac{\partial {\rm NH_4}}{\partial z} _{z_{\rm ox}^+}$
	where:		$F_{\mathrm{NH_4}}(z_{\mathrm{ox}}) = \frac{1}{1 + K_{\mathrm{NH_4}}} \cdot \frac{1 - \phi}{\phi} \cdot \int_{z_{\mathrm{NO_3}}}^{\infty} \sum_{i} k_i \cdot \mathrm{NC_i} \cdot C_i dz$
$z = z_{NO_3}$	continuity		$NH_4(z_{NO_3}^-)=NH_4(z_{NO_3}^+)$
	flux	7)	$-\frac{D_{\mathrm{NH_4}}}{1+K_{\mathrm{NH_4}}}\cdot\frac{\partial \mathrm{NH_4}}{\partial z} _{z_{\mathrm{NO_3}}^-} = -\frac{D_{\mathrm{NH_4}}}{1+K_{\mathrm{NH_4}}}\cdot\frac{\partial \mathrm{NH_4}}{\partial z} _{z_{\mathrm{NO_3}}^+}$
$z=z_{\infty}$	zero NH4 flux	8)	$\frac{\partial \text{NH}_4}{\partial z} _{z_\infty} = 0$

2. Layer $(z_{NO_3} < z \le z_{SO_4})$

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$$\frac{\partial SO_4^{II}}{\partial t} = 0 = D_{SO_4} \frac{\partial^2 SO_4^{II}}{\partial z^2} - w \frac{\partial SO_4^{II}}{\partial z} - \frac{1 - \phi}{\phi} \cdot \sum_i SO_4 C \cdot k_i \cdot C_i(z)$$
(13)

$$\frac{\partial \mathbf{H}_2 \mathbf{S}^{II}}{\partial t} = 0 = D_{\mathbf{H}_2 \mathbf{S}} \frac{\partial^2 \mathbf{H}_2 \mathbf{S}^{II}}{\partial z^2} - w \frac{\partial \mathbf{H}_2 \mathbf{S}^{II}}{\partial z} + \frac{1 - \phi}{\phi} \cdot \sum_{i} \mathbf{SO}_4 \mathbf{C} \cdot k_i \cdot C_i(z)$$
(14)

3. Layer $(z_{SO_4} < z \le z_{\infty})$

$$\frac{\partial \mathbf{H}_2 \mathbf{S}^{III}}{\partial t} = 0 = D_{\mathbf{H}_2 \mathbf{S}} \frac{\partial^2 \mathbf{H}_2 \mathbf{S}^{III}}{\partial z^2} - w \frac{\partial \mathbf{H}_2 \mathbf{S}^{III}}{\partial z}$$
(15)

To solve equations 11 - 15 the model assumes known concentrations at the sediment-water interface and continuity across the bioturbation depth and the nitrate penetration depth (see Table 5). The re-oxidation of reduced $\rm H_2S$ to $\rm SO_4$ is considered in the oxic-suboxic boundary condition for both species, here including the methanic zone, as $\rm H_2S$ is also produced during anaerobic oxidation of methane (AOM). Furthermore, sulfate is used at $z_{\rm SO_4}$ to oxidize methane from below and thus producing $\rm H_2S$. In case $z_{\rm SO_4} < z_{\infty}$, sulfate concentration is zero at $z_{\rm SO_4}$ and its diffusive flux must

Table 5. Boundary conditions for sulfate and sulfide. For the boundaries we define: $z_-^- := \lim_{h \to 0} (z_- - h)$ and $z_+^+ := \lim_{h \to 0} (z_- + h)$.

Boundary	Condition			
z = 0	known concentration	1)	$SO_4(0)=SO_{40}$	
$z = z_{\rm bio}$	continuity	2)	$\mathrm{SO_4}(z_\mathrm{bio}^-) = \mathrm{SO_4}(z_\mathrm{bio}^+)$	
	flux	3)	$-\left(D_{\mathrm{SO}_{4},0}+D_{\mathrm{bio}}\right)\cdot\frac{\partial \mathrm{SO}_{4}}{\partial z}\big _{z_{\mathrm{bio}}^{-}}=-D_{\mathrm{SO}_{4},0}\cdot\frac{\partial \mathrm{SO}_{4}}{\partial z}\big _{z_{\mathrm{bio}}^{+}}$	DH: @Sandra: BC 5)
$z = z_{\rm ox}$	continuity	4)	$SO_4(z_{ox}^-)=SO_4(z_{ox}^+)$	Include $\int_{z_{\text{SO},4}}^{\infty}$ here?
	flux	5)	$-D_{\mathrm{SO}_4} \cdot \frac{\partial \mathrm{SO}_4}{\partial z}\big _{z_{\mathrm{ox}}^-} + \gamma_{\mathrm{H}_2\mathrm{S}} \cdot F_{\mathrm{H}_2\mathrm{S}}(z_{\mathrm{ox}}) = -D_{\mathrm{SO}_4} \cdot \frac{\partial \mathrm{SO}_4}{\partial z}\big _{z_{\mathrm{ox}}^+}$	1504
	where:		$F_{\rm H_2S}(z_{\rm ox}) = \frac{1-\phi}{\phi} \cdot \left(\int_{z_{\rm NO_3}}^{\rm SO_4} \sum_i {\rm SO_4C} \cdot k_i \cdot C_i dz + \gamma_{\rm CH_4} \cdot C_i dz + \gamma_{\rm $	$\int_{z_{SO_4}}^{\infty} \sum_{i} MC \cdot k_i \cdot C_i \ dz \bigg) \ \bigg $
$z = z_{NO_3}$	continuity	6)	$SO_4(z_{NO_3}^-) = SO_4(z_{NO_3}^+)$	DII. @Candon dialana
	flux	7)	$-D_{\mathrm{SO}_4} \cdot \frac{\partial \mathrm{SO}_4}{\partial z}\big _{z_{\mathrm{NO}_2}^-} = -D_{\mathrm{SO}_4} \cdot \frac{\partial \mathrm{SO}_4}{\partial z}\big _{z_{\mathrm{NO}_2}^+}$	DH : @Sandra: think yes,
$z = z_{SO_4}$	SO ₄ consumption	8)	$\mathbf{IF}\left(\mathrm{SO}_4(z_\infty)>0\right)$	because at 8) CH ₄ from $\int_{-\infty}^{\infty}$ is oxidised to H ₂ St
	$(z_{\mathrm{SO}_4} = z_{\infty})$		$\frac{\partial SO_4}{\partial z} _{z_{SO_4}} = 0$	$\int_{z_{SO_4}}^{\infty}$ is oxidised to H ₂ S; at 5) this H ₂ S to SO ₄
			ELSE	at 3) tills 1125 to 504
	$(z_{\mathrm{SO}_4} < z_{\infty})$		$SO_4(z_{SO_4}) = 0$ and $-D_{SO_4} \cdot \frac{\partial SO_4}{\partial z} _{z_{SO_4}} = \gamma_{CH_4} \cdot F_{CH_4}$	$_{ m H_4}(z_{ m SO_4})$
	with flux from below:		$F_{\text{CH}_4}(z_{\text{SO}_4}) = \frac{1-\phi}{\phi} \cdot \int_{z_{\text{SO}_4}}^{\infty} \sum_i \text{MC} \cdot k_i \cdot C_i dz$	
z = 0	known concentration	1)	$H_2S(0) = H_2S_0$	
$z = z_{\rm bio}$	continuity	2)	${ m H_2S}(z_{ m bio}^-) = { m H_2S}(z_{ m bio}^+)$	
	flux	3)	$-\left(D_{\rm H_2S,0}+D_{\rm bio}\right)\cdot\frac{\partial \rm H_2S}{\partial z}\big _{z_{\rm bio}^-} = -D_{\rm H_2S,0}\cdot\frac{\partial \rm H_2S}{\partial z}\big _{z_{\rm bio}^+}$	
$z = z_{\rm ox}$	continuity	4)	$H_2S(z_{ox}^-)=H_2S(z_{ox}^+)$	
	flux	5)	$-D_{\rm H_2S} \cdot \frac{\partial \rm H_2S}{\partial z} _{z_{\rm ox}^-} - \gamma_{\rm H_2S} F_{\rm H_2S}(z_{\rm ox}) = -D_{\rm H_2S} \cdot \frac{\partial \rm H_2S}{\partial z} _{z_{\rm ox}^+}$	
	where:		$F_{\rm H_2S}(z_{\rm ox}) = \frac{1-\phi}{\phi} \cdot \left(\int_{z_{\rm NO_3}}^{\rm SO_4} \sum_i {\rm SO_4C} \cdot k_i \cdot C_i dz + \gamma_{\rm CH_4} \cdot C_i dz + \gamma_{\rm $	$\int_{z_{SO_4}}^{\infty} \sum_{i} MC \cdot k_i \cdot C_i \ dz \bigg) \ \bigg $
$z = z_{\text{NO}_3}$	continuity	6)	${ m H_2S}(z_{{ m NO}_3}^-) = { m H_2S}(z_{{ m NO}_3}^+)$	
	flux	7)	$-D_{\rm H_2S} \cdot \frac{\partial \rm H_2S}{\partial z} _{z_{\rm NO_2}^-} = -D_{\rm H_2S} \cdot \frac{\partial \rm H_2S}{\partial z} _{z_{\rm NO_2}^+}$	
$z = z_{SO_4}$	continuity	8)	$H_2S(z_{SO_4}^-)=H_2S(z_{SO_4}^+)$	
	flux (with AOM)	9)	$-D_{\rm H_2S} \cdot \frac{\partial \rm H_2S}{\partial z} _{z_{\rm SO_4}^-} + \gamma_{\rm CH_4} \cdot F_{\rm CH_4}(z_{\rm SO_4}) = -D_{\rm H_2S} \cdot \frac{\partial \rm H_2S}{\partial z} _{z_{\rm SO_4}^-}$	z+ SO4
	where:		$F_{\text{CH}_4}(z_{\text{SO}_4}) = \frac{1-\phi}{\phi} \cdot \int_{z_{\text{SO}_4}}^{\infty} \sum_i \text{MC} \cdot k_i \cdot C_i dz$	· · · · · · ·
$z = z_{\infty}$	zero $ m H_2S$ flux	10)	$\frac{\partial \mathbf{H}_2 \mathbf{S}}{\partial z} _{z_{\infty}} = 0$	

equal the amount of methane produced below; or, in case $z_{\rm SO_4}=z_{\infty}$, a zero flux condition for sulfate is considered. At the lower boundary z_{∞} zero flux of $\rm H_2S$ is considered. correct??

295 **2.3.5 Phosphate**

To model phosphorus (P) dynamics in the sediments OMEN-SED takes into account the change with depth of phosphate (PO_4) and iron-bound P, thereby mainly following the description of Slomp et al. (1996) and Gypens et al. (2008). Throughout the sediment column organic matter is mineralized

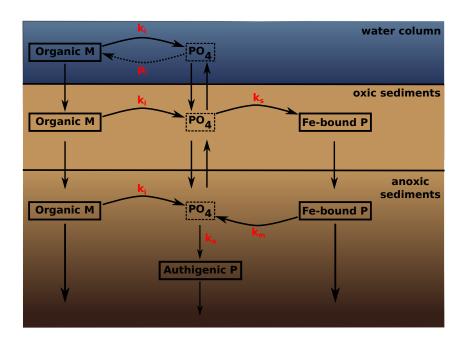


Figure 2. A schematic of the sedimentary P cycle in OMEN-SED. Red numbers represent kinetic rate constants for phosphorus dynamics (compare Table 10; p_i represents uptake rate of PO₄ via primary production in shallow environments). Adapted from Slomp et al. (1996).

resulting in a release of phosphate to the pore water. In the oxic part of the sediment, this PO_4 either diffuses upward to the water column or is adsorped to Fe oxides forming Fe-bound P (or M) (Slomp et al., 1998). In the suboxic/anoxic zone, PO_4 is not only produced via organic matter degradation but is also released from the Fe-bound P pool due to the reduction of Fe oxides. Furthermore, phosphate concentrations can become high enough in this layer for authigenic mineral formation to occur (Van Cappellen and Berner, 1988). This phosphorus bound in authigenic minerals represents a permanent sink for reactive phosphorus (Slomp et al., 1996). See Figure 2 for a schematic overview of the sedimentary P cycle. As for ammonium, the adsorption of P to the sediment matrix is treated as an equilibrium processes, parameterized with dimensionless adsorption coefficients for the oxic and anoxic zone ($K_{PO_4}^I$, $K_{PO_4}^{II}$ Slomp et al., 1998). Therefore the diagenetic equations for phosphate are written as:

1. Layer $(z \le z_{ox})$

$$\frac{\partial PO_4^I}{\partial t} = \frac{D_{PO_4}}{1 + K_{PO_4}^I} \frac{\partial^2 PO_4^I}{\partial z^2} - w \frac{\partial PO_4^I}{\partial z} + \frac{1 - \phi}{\phi} \frac{1}{1 + K_{PO_4}^I} \sum_i (PC_i \cdot k_i \cdot C_i(z))$$

$$- \frac{k_s}{1 + K_{PO_4}^I} (PO_4^I - PO_4^s) \tag{16}$$

$$\frac{\partial M^{I}}{\partial t} = D_{M} \frac{\partial^{2} M^{I}}{\partial z^{2}} - w \frac{\partial M^{I}}{\partial z} + \frac{\phi}{1 - \phi} k_{s} (PO_{4}^{I} - PO_{4}^{s})$$

$$(17)$$

Table 6. Boundary conditions for phosphate and Fe-bound P (M). For the boundaries we define: $z_-^- := \lim_{h\to 0} (z_- - h)$ and $z_-^+ := \lim_{h\to 0} (z_- + h)$.

Boundary	Condition	
z = 0	known concentration	1) PO ₄ (0)=PO ₄₀
$z = z_{\rm bio}$	continuity	2) $PO_4(z_{bio}^-) = PO_4(z_{bio}^+)$
	flux	3) $(D_{PO_4,0} + D_{bio}) \cdot \frac{\partial PO_4}{\partial z} _{z_{bio}^-} = D_{PO_4,0} \cdot \frac{\partial PO_4}{\partial z} _{z_{bio}^+}$
$z = z_{\text{ox}}$	continuity	4) $PO_4(z_{ox}^-) = PO_4(z_{ox}^+)$
	flux	$\int 1 - \frac{D_{\rm PO_4}}{1 + K_{\rm PO_4}^I} \cdot \frac{\partial {\rm PO_4}}{\partial z} \big _{z_{\rm ox}} = -\frac{D_{\rm PO_4}}{1 + K_{\rm PO_4}^{II}} \cdot \frac{\partial {\rm PO_4}}{\partial z} \big _{z_{\rm ox}}$
$z=z_{\infty}$	flux	$10) \frac{\partial PO_4}{\partial z} _{z_{\infty}} = 0$
z = 0	known concentration	$1) M(0) = M_0$
$z = z_{\rm bio}$	continuity	2) $M(z_{ m bio}^{-})$ = $M(z_{ m bio}^{+})$
	flux	3) $\frac{\partial M}{\partial z} _{z_{\text{bio}}^-} = \frac{\partial M}{\partial z} _{z_{\text{bio}}^+}$
$z = z_{\rm ox}$	continuity	4) $M(z_{ox}^{-})=M(z_{ox}^{+})$
	flux	$\int \frac{\partial M}{\partial z} \Big _{z_{\text{ox}}^-} = \frac{\partial M}{\partial z} \Big _{z_{\text{ox}}^+}$
$z=z_{\infty}$	assymptotic concentration	$10) M(z_{\infty}) = M_{\infty}$

2. Layer $(z_{ox} < z)$

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$$\frac{\partial M^{II}}{\partial t} = D_{M} \frac{\partial^{2} M^{II}}{\partial z^{2}} - w \frac{\partial M^{II}}{\partial z} - k_{m} (M^{II} - M^{\infty})$$

$$\frac{\partial PO_{4}^{II}}{\partial t} = \frac{D_{PO_{4}}}{1 + K_{PO_{4}}^{II}} \frac{\partial^{2} PO_{4}^{II}}{\partial z^{2}} - w \frac{\partial PO_{4}^{II}}{\partial z} + \frac{1 - \phi}{\phi} \frac{1}{1 + K_{PO_{4}}^{II}} \sum_{i} (PC_{i} \cdot k_{i} \cdot C_{i}(z))$$

$$- \frac{k_{a}}{1 + K_{PO_{4}}^{II}} (PO_{4}^{II} - PO_{4}^{a}) + \frac{(1 - \phi)}{\phi} \frac{k_{m}}{1 + K_{PO_{4}}^{II}} (M^{II} - M^{\infty})$$
(19)

The boundary conditions to solve Equations 16 - 19 are summarized in Table 6. The model assumes known bottom water concentrations and equal concentrations and diffusive fluxes at $z_{\rm bio}$ and $z_{\rm ox}$ for both species. Additionally it considers no change in phosphate flux and an assymptotic Fe-bound P concentration at z_{∞} .

2.3.6 Dissolved Inorganic Carbon (DIC)

Organic matter degradation produces dissolved inorganic carbon (DIC) with a stoichiometric DIC:C ratio of 1:2 in the methanic zone and 1:1 in the rest of the sediment column. DIC dynamics in OMEN-SED are thus described by equations 21 and 21 and boundary conditions as summarized in Table 7. The model assumes a known DIC concentration at the sediment-water interface, a zero flux condition at the lower boundary z_{∞} and continuity across the bioturbation and sulfate penetration depth. In

Table 7. Boundary conditions for DIC. For the boundaries we define: $z_{-}^{-} := \lim_{h\to 0} (z_{-} - h)$ and $z_{-}^{+} := \lim_{h\to 0} (z_{-} + h)$.

Boundary	Condition	
z = 0	known concentration	1) $\operatorname{DIC}(0) = \operatorname{DIC}_0$
$z = z_{\rm bio}$	continuity	2) $\operatorname{DIC}(z_{\operatorname{bio}}^{-}) = \operatorname{DIC}(z_{\operatorname{bio}}^{+})$
	flux	3) $-(D_{\text{DIC},0} + D_{\text{bio}}) \cdot \frac{\partial \text{DIC}}{\partial z} _{z_{\text{bio}}^{-}} = -D_{\text{DIC},0} \cdot \frac{\partial \text{DIC}}{\partial z} _{z_{\text{bio}}^{+}}$
$z = z_{SO_4}$	continuity	4) $\operatorname{DIC}(z_{\operatorname{SO}_4}^-) = \operatorname{DIC}(z_{\operatorname{SO}_4}^+)$
	flux (with AOM)	5) $-D_{\mathrm{DIC}} \cdot \frac{\partial \mathrm{DIC}}{\partial z} _{z_{\mathrm{SO}_{4}}^{-}} + \gamma_{\mathrm{CH}_{4}} \cdot F_{\mathrm{CH}_{4}}(z_{\mathrm{SO}_{4}}) = -D_{\mathrm{DIC}} \cdot \frac{\partial \mathrm{DIC}}{\partial z} _{z_{\mathrm{SO}_{4}}^{+}}$
	where:	$F_{\text{CH}_4}(z_{\text{SO}_4}) = \frac{1-\phi}{\phi} \cdot \int_{z_{\text{SO}_4}}^{\infty} \sum_i \text{MC} \cdot k_i \cdot C_i dz$
$z=z_{\infty}$	zero DIC flux	$6) \frac{\partial \text{DIC}}{\partial z} _{z_{\infty}} = 0$

addition, the anaerobic oxidation of methane at $z_{\rm SO_4}$ produces DIC (with 1:1 stoichiometry) which is accounted for through the flux boundary condition at $z_{\rm SO_4}$ (Table 7 eq. 5)

1. Layer $(z \le z_{SO_4})$

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$$\frac{\partial DIC^{I}}{\partial t} = 0 = D_{DIC} \frac{\partial^{2} DIC^{I}}{\partial z^{2}} - w \frac{\partial DIC^{I}}{\partial z} + \frac{1 - \phi}{\phi} \cdot \sum_{i} \text{DICC}^{I} \cdot k_{i} \cdot C_{i}(z)$$
(21)

340 2. Layer $(z_{SO_4} < z \le z_{\infty})$

$$\frac{\partial DIC^{II}}{\partial t} = 0 = D_{DIC} \frac{\partial^2 DIC^{II}}{\partial z^2} - w \frac{\partial DIC^{II}}{\partial z} + \frac{1 - \phi}{\phi} \cdot \sum_{i} \text{DICC}^{II} \cdot k_i \cdot C_i(z)$$
 (22)

2.3.7 Alkalinity

Organic matter degradation and secondary redox reactions exert a complex influence on alkalinity with opposite effects depending on the TEA involved (Wolf-Gladrow et al., 2007). To model alkalinity in OMEN-SED the sediment column is partitioned into four geochemical layers, where different equations describe the biogeochemical processes using variable stoichiometric coefficients (compare Tables 10 and 14). Above $z_{\rm ox}$, the combined effects of NH₄ and P release due to aerobic OM mineralization increases alkalinity according to ALK^{OX} whereas nitrification decreases alkalinity with stoichiometry ALK^{NIT}. In the remaining three zones anaerobic OM mineralization increases alkalinity with variable stoichiometric coefficients (i.e. ALK^{DEN}, ALK^{SUL}, ALK^{MET}). Thus, the diagenetic equations for alkalinity are written as:

1. Layer $(z \le z_{ox})$

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$$\frac{\partial ALK^{I}}{\partial t} = 0 = D_{ALK} \frac{\partial^{2} ALK^{I}}{\partial z^{2}} - w \frac{\partial ALK^{I}}{\partial z} + \frac{1 - \phi}{\phi} \cdot \sum_{i} \left(ALK^{NIT} \cdot \frac{\gamma_{NH_{4}}}{1 + K_{NH_{4}}} NC_{i} + ALK^{OX} \right) \cdot k_{i} \cdot C_{i}(z)$$
(23)

2. Layer $(z_{\text{ox}} < z \le z_{\text{NO}_3})$

$$\frac{\partial ALK^{II}}{\partial t} = 0 = D_{ALK} \frac{\partial^2 ALK^{II}}{\partial z^2} - w \frac{\partial ALK^{II}}{\partial z} + \frac{1 - \phi}{\phi} \cdot \sum_{i} ALK^{DEN} \cdot k_i \cdot C_i(z)$$
 (24)

3. Layer $(z_{NO_3} < z \le z_{SO_4})$

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$$\frac{\partial \text{ALK}^{III}}{\partial t} = 0 = D_{\text{ALK}} \frac{\partial^2 \text{ALK}^{III}}{\partial z^2} - w \frac{\partial \text{ALK}^{III}}{\partial z} + \frac{1 - \phi}{\phi} \cdot \sum_{i} \text{ALK}^{\text{SUL}} \cdot k_i \cdot C_i(z)$$
 (25)

4. Layer $(z_{SO_4} < z \le z_{\infty})$

$$\frac{\partial ALK^{IV}}{\partial t} = 0 = D_{ALK} \frac{\partial^2 ALK^{IV}}{\partial z^2} - w \frac{\partial ALK^{IV}}{\partial z} + \frac{1 - \phi}{\phi} \cdot \sum_{i} ALK^{MET} \cdot k_i \cdot C_i(z)$$
 (26)

To solve equations 23 - 26 the model assumes a known concentration at the sediment-water interface and continuity across the bioturbation depth and the penetration depths of O_2 , NO_3 and SO_4 (see Table 8). The decrease of alkalinity due to oxidation of reduced species produced in the suboxic and anoxic layers (with stoichiometry ALK^{NIT} and ALK^{H_2S}) is implicitly taken into account through the flux boundary condition at z_{ox} (Table 8 Eq. 5). Furthermore, the oxidation of methane by sulphate reduction increases alkalinity with stoichiometry ALK^{AOM} which is accounted for through the flux boundary condition at z_{SO_4} (Table 8 eq. 9). At the lower boundary z_{∞} a zero flux condition is applied.

375 2.4 Model Parameters

This section describes the parameters used in OMEN-SED to describe sediment transport and biogeochemical reactions related to the burial and mineralization of organic matter under a wide range of environmental conditions. Table 9 states the parameters for sediment characteristics and table 10 summarizes the stoichiometric factors and secondary reaction parameters used in the model.

380 **2.4.1** Transport Parameters

Advection is the bulk flow of sediments and can be directly related to the accumulation of new material on the seafloor (i.e. sedimentation, Burdige, 2006). This results in a downward flux of older sediment material and porewater in relation to the sediment-water interface. When coupled to an ocean model, its sedimentation flux can be readily used in OMEN-SED 1.0. The stand-alone version

Table 8. Boundary conditions for alkalinity. For the boundaries we define: $z_-^- := \lim_{h \to 0} (z_- - h)$ and $z_-^+ := \lim_{h \to 0} (z_- + h)$.

Boundary	Condition		
z = 0	known concentration	1)	$ALK(0) = ALK_0$
$z = z_{\rm bio}$	continuity	2)	$ALK(z_{bio}^-) = ALK(z_{bio}^+)$
	flux	3)	$-\left(D_{\mathrm{ALK},0} + D_{\mathrm{bio}}\right) \cdot \frac{\partial \mathrm{ALK}}{\partial z}\big _{z_{\mathrm{bio}}^{-}} = -D_{\mathrm{ALK},0} \cdot \frac{\partial \mathrm{ALK}}{\partial z}\big _{z_{\mathrm{bio}}^{+}}$
$z = z_{\rm ox}$	continuity	4)	$ALK(z_{ox}^{-})=ALK(z_{ox}^{+})$
	flux	5)	$-D_{\text{ALK}} \cdot \frac{\partial \text{ALK}}{\partial z}\big _{z_{\text{ox}}^{-}} + F_{\text{ALK}}(z_{\text{ox}}) = -D_{\text{ALK}} \cdot \frac{\partial \text{ALK}}{\partial z}\big _{z_{\text{ox}}^{+}}$
	where:		$F_{\rm ALK}(z_{\rm ox}) = \frac{1-\phi}{\phi} \cdot \left({\rm ALK^{H_2S} \cdot \gamma_{H_2S} \int_{z_{\rm NO_3}}^{SO_4} \sum_{i} {\rm SO_4C} \cdot k_i \cdot C_i \ dz} \right)$
			$+\frac{1-\phi}{\phi}\cdot\left(\mathrm{ALK^{NIT}} \frac{\gamma_{\mathrm{NH_4}}}{1+k_{\mathrm{NH_4}}} \int_{z_{\mathrm{NO_3}}}^{\infty} \sum_{i} \mathrm{NC_i} \cdot k_i \cdot C_i \; dz\right)$
$z = z_{NO_3}$	continuity	6)	$\mathrm{ALK}(z_{\mathrm{NO}_{3}}^{-}) = \mathrm{ALK}(z_{\mathrm{NO}_{3}}^{+})$
	flux	7)	$-D_{\rm ALK} \cdot \frac{\partial \rm ALK}{\partial z} _{z_{\rm NO_3}^-} = -D_{\rm ALK} \cdot \frac{\partial \rm ALK}{\partial z} _{z_{\rm NO_3}^+}$
$z = z_{SO_4}$	continuity	8)	$ALK(z_{SO_4}^-) = ALK(z_{SO_4}^+)$
	flux (with AOM)	9)	$-D_{\mathrm{ALK}} \cdot \frac{\partial \mathrm{ALK}}{\partial z} _{z_{\mathrm{SO}_{4}}^{-}} + F_{\mathrm{ALK}}(z_{\mathrm{SO}_{4}}) = -D_{\mathrm{ALK}} \cdot \frac{\partial \mathrm{ALK}}{\partial z} _{z_{\mathrm{SO}_{4}}^{+}}$
	where:		$F_{ m ALK}(z_{ m SO_4}) = rac{1-\phi}{\phi} \cdot \left({ m ALK}^{ m AOM} \gamma_{ m CH_4} \cdot \int_{z_{ m SO_4}}^{\infty} \sum_i k_i \cdot C_i \; dz \right)$
$z = z_{\infty}$	zero ALK flux	10)	$\frac{\partial ALK}{\partial z} _{z_{\infty}} = 0$

of OMEN-SED 1.0 uses the empirical global relationship between sediment accumulation rate (cm yr^{-1}) and seafloor depth (m) of Middelburg et al. (1997):

$$w = 3.3 \cdot 10^{-0.87478367 - 0.00043512 \cdot \text{depth}},\tag{27}$$

As discussed before (Sec. 2.2), the diffusion coefficient of species i is calculated as $D_i = D_{i,0} + D_{\text{bio}} = D_{\text{mol},i} \cdot f_{ir} + D_{\text{bio}}$ for dissolved species and $D_i = D_{\text{bio}}$ for solid species. The bioturbation coefficient D_{bio} (cm² yr⁻1) is constant in the bioturbated zone and also follows the empirical relationship by Middelburg et al. (1997):

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$$D_{\text{bio}} = 5.2 \cdot 10^{0.76241122 - 0.00039724 \cdot \text{depth}} \tag{28}$$

Studies showed that bioturbational effects on a global scale are largely restricted to the upper 10 cm of the sediments and are only marginally related to seafloor depth (e.g. Boudreau, 1998; Teal et al., 2010). Therefore, OMEN-SED imposes a globally invariant bioturbation depth $z_{\rm bio}$ of 10 cm. In case the bottom water oxygen concentration is below 5 nanomole cm⁻³ infaunnal activity ceases and $z_{\rm bio} = 0.01$ cm.

Bioirrigation can enhance the molecular diffusion coefficient $D_{i,0} = D_{\text{mol},i} \cdot f_{ir}$ (Soetaert et al., 1996a). However, here we do not consider this effect and set f_{ir} to a constant value of 1. The specific molecular diffusion coefficients $D_{\text{mol},i}$ are corrected for sediment porosity ϕ , tortuosity F and are linearly interpolated for an ambient temperature T using zero-degree coefficients D_i^0 and

temperature dependent diffusion coefficients D_i^T (compare Gypens et al., 2008):

$$D_{\mathrm{mol},i} = (D_i^0 + D_i^T \cdot T) \cdot \frac{1}{\phi \cdot F}$$

Tortuosity can be expressed in terms of porosity as $F = \frac{1}{\phi^m}$ (Ullman and Aller, 1982) with the exponent m varying according to the type of sediment (here we use m=3). Values for D_i^T and D_i^0 are summarized in Table 9 and are adapted from Li and Gregory (1974) and Gypens et al. (2008).

Table 9. Sediment characteristics and transport parameters. TODO: Update table!

Parameter	Unit	Value	Description/Source
$ ho_{ m sed}$	g cm ⁻³	2.6	Sediment density
w	${\rm cm}{\rm yr}^{-1}$	Fct. of seafloor	Advection/Sediment accumulation rate
		depth or from ESM	(Middelburg et al., 1997)
$z_{ m bio}$	cm	10 or 0.01	Bioturbation depth
			(Boudreau, 1998; Teal et al., 2010)
D_{bio}	$\mathrm{cm^2yr^{-1}}$	Fct. of seafloor	Bioturbation coefficient
		depth	(Middelburg et al., 1997)
ϕ	-	0.85	Porosity
F	-	$\frac{1}{\phi^m}$	Tortuosity, here m=3
f_{ir}	-	1	Irrigation factor
Adsorption	n coefficients		
$K_{ m NH_4}$	-	1.3	NH ₄ adsorption coefficient, (see Wang and Van Cappellen, 1996)
$K_{\mathrm{PO_4}}^{ox}$	-	200.0	PO ₄ adsorption coefficient (oxic), (see Slomp et al., 1998)
$K_{\mathrm{PO_4}}^{anox}$	-	1.3	PO ₄ adsorption coefficient (anoxic), (see Slomp et al., 1998)
Diffusion of	coefficients (Li and	Gregory, 1974; Gypen	s et al., 2008)
$D_{\mathrm{O}_2}^0$	${ m cm}^2{ m yr}^{-1}$	348.62172	Molecular diffusion coefficient of oxygen at 0°C
$D_{\mathrm{O}_2}^T$	$\mathrm{cm}^2\mathrm{yr}^{-1}^{\circ}\mathrm{C}^{-1}$	14.08608	Diffusion coefficient for linear temp. dependence of oxygen
$D_{\mathrm{NO_{3}}}^{0}$	$\mathrm{cm}^2\mathrm{yr}^{-1}$	308.42208	Molecular diffusion coefficient of nitrate at 0°C
$D_{\mathrm{NO}_{3}}^{T}$	$\mathrm{cm}^2\mathrm{yr}^{-1}^{\circ}\mathrm{C}^{-1}$	12.2640	Diffusion coefficient for linear temp. dependence of nitrate
$D^0_{ m NH_4}$	$\mathrm{cm}^2\mathrm{yr}^{-1}$	309.0528	Molecular diffusion coefficient of ammonium at $0^{\circ}\mathrm{C}$
$D_{\mathrm{NH_{4}}}^{T}$	$\mathrm{cm}^2\mathrm{yr}^{-1}^{\circ}\mathrm{C}^{-1}$	12.2640	Diffusion coefficient for linear temp. dependence of ammonium
$D^0_{\mathrm{SO}_4}$	${\rm cm}^2{\rm yr}^{-1}$	157.68	Molecular diffusion coefficient of sulfate at $0^{\circ}\mathrm{C}$
$D_{\mathrm{SO}_4}^T$	$\mathrm{cm}^2\mathrm{yr}^{-1}^{\circ}\mathrm{C}^{-1}$	7.884	Diffusion coefficient for linear temp. dependence of sulfate
$D_{ m H_2S}^0$	$\mathrm{cm}^2\mathrm{yr}^{-1}$	307.476	Molecular diffusion coefficient of sulfide at 0° C
$D_{\rm H_2S}^T$	$\mathrm{cm}^2\mathrm{yr}^{-1}^{\circ}\mathrm{C}^{-1}$	9.636	Diffusion coefficient for linear temp. dependence of sulfide
$D^0_{\mathrm{PO}_4}$	$\mathrm{cm}^2\mathrm{yr}^{-1}$	112.90764	Molecular diffusion coefficient of phosphate at $0^{\circ}\mathrm{C}$
$D_{\mathrm{PO}_{4}}^{T}$	$cm^2 yr^{-1} \circ C^{-1}$	5.586252	Diffusion coefficient for linear temp. dependence of phosphate

2.4.2 Reaction Parameters and Stoichiometries

The applied multi-G approach for organic matter degradation considers specific degradation rate constants k_i (yr⁻¹) for each compound class which are assumed invariant along the sediment col-410 umn and therefore independent of the nature of the terminal electron acceptor. The rate constants can be altered manually to fit observed sediment profiles (compare Section 3.1) or related to a master variable (e.g. sedimentation rate or POC-flux) provided by a coupled Earth System Model (compare Section 2.5.2 and 3.3). The rate constants for P related processes (i.e. sorption of PO₄ to Fe oxides, release of PO₄ from Fe-bound P due to Fe-oxide reduction and authigenic CFA precipitation) are k_s , k_m , k_a respectively. The pore water equilibrium concentrations for P sorption and 415 CFA precipation (PO_4^s, PO_4^a) and the asymptotic concentration for Fe-bound $P(M^\infty)$ are taken from the Slomp et al. (1996). The stoichiometry of organic matter is represented by the factors NC_i and PC_i denoting the molecular nitrogen to carbon and phosphorus to carbon ratio. In the sulfidic and methanic zone the reduction of 1 mol organic matter additionally produces $SO_4C = \frac{138}{212}$ mol 420 of hydrogen sulfide and MC = 0.5 mol of methane. In the total sediment column organic matter mineralization consumes the specific TEA with a fixed ratio (O₂C, NO₃C and SO₄C respectively). See Table 10 for a complete summary of the parameters and their values.

2.5 Module Structure

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An analytical steady-state solution is found for the reaction-transport equation of each chemical species for every layer. At each boundary (i.e. $z_{\rm ox}, z_{\rm bio}, z_{\rm NO_3}$ and $z_{\rm SO_4}$) the model has to match continuity and flux for the ODE solutions of the layer above and below the specific boundary. In particular the bioturbation boundary is problematic as it can theoretically occur in any geochemical layer (compare Fig. 3). In order to simplify this recurring boundary matching problem it is implemented in an independent algorithm which is described in Section 2.5.1. Instructions and requirements for coupling OMEN-SED to a global Earth Sytem Model are given in Section 2.5.2.

2.5.1 Generic Boundary Condition Matching (GBCM)

A general steady-state advection-diffusion-reaction (ADR) diagenetic equation looks like:

$$\frac{\partial C}{\partial t} = 0 = D \frac{\partial^2 C}{\partial z^2} - w \frac{\partial C}{\partial z} - \sum_i \alpha_i \exp(-\beta_i z) - k \cdot C + Q. \tag{29}$$

where z is the sediment depth, t the time, D is the diffusion coefficient and w is the advection rate. The ODE solution is of the general form:

$$C(z) = A\exp(az) + B\exp(bz) + \sum_{i} \frac{\alpha_i}{D\beta_i^2 - w\beta_i - k} \cdot \exp(-\beta_i z) + \frac{Q}{k}$$
(30)

and can therefore be expressed as:

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$$C(z) = A \cdot E(z) + B \cdot F(z) + G(z)$$
 (31)

Table 10. Values for biogeochemical parameters used in OMEN-SED. The variables x, y and z denote the atomic ratio of carbon, nitrogen and phosphorous of the degrading organic matter (here set to C: N: P = 106: 16: 1).

Parameter/Variable	Unit	Value	Description
Stoichiometric facto	ors and mole	cular ratios	
NC_{i}	mol/mol	$\frac{y}{x} = \frac{16}{106}$	nitrogen to carbon ratio
PC_{i}	mol/mol	$\frac{z}{x} = \frac{1}{106}$	phosphorus to carbon ratio
MC	mol/mol	0.5	methane to carbon ratio
			produced during methanogenesis
$\mathrm{DICC}^{\mathrm{I}}$	mol/mol	1.0	DIC to carbon ratio until $z_{\mathrm{SO_4}}$
$\mathrm{DICC^{II}}$	mol/mol	0.5	DIC to carbon ratio below $z_{\mathrm{SO_4}}$
O_2C	mol/mol	$\frac{x+2y}{x} = \frac{138}{106}$	oxygen to carbon ratio
NO_3C	mol/mol	$\frac{4x+3y}{5x} = \frac{94.4}{106}$	nitrate to carbon ratio
$\mathrm{SO_4C}$	mol/mol	$\frac{1}{2}$ O ₂ C = $\frac{138}{212}$	sulfate to carbon ratio
$\mathrm{ALK}^{\mathrm{OX}}$	mol/mol	$\frac{y-2z}{x} = \frac{14}{106}$	ALK from aerobic degradation
$\mathrm{ALK}^{\mathrm{NIT}}$	mol/mol	-2	ALK from nitrification
$\mathrm{ALK}^{\mathrm{DEN}}$	mol/mol	$\frac{4x + 3y - 10z}{5x} = \frac{92.4}{106}$	ALK from denitrification
$\mathrm{ALK}^{\mathrm{SUL}}$	mol/mol	$\frac{x+y-2z}{x} = \frac{120}{106}$	ALK from sulfate reduction
$\mathrm{ALK}^{\mathrm{MET}}$	mol/mol	$\frac{y-2z}{x} = \frac{14}{106}$	ALK from methanogenesis
$\mathrm{ALK}^{\mathrm{H2S}}$	mol/mol	-2	ALK from H ₂ S oxidation
$\mathrm{ALK}^{\mathrm{AOM}}$	mol/mol	2	ALK from AOM
Secondary reaction	parameters		
$\gamma_{ m NH_4}$	-	0.9	fraction of NH ₄ that is oxidised
			in oxic layer
$\gamma_{ m H_2S}$	-	0.95	fraction of H_2S that is oxidised
			in oxic layer
$\gamma_{\mathrm{CH_4}}$	-	0.99	fraction of CH ₄ that is oxidised
			at $z_{\mathrm{SO_4}}$
P related paramete	rs		
k_s	yr^{-1}	1.0	Rate constant for PO ₄ sorption
k_m	yr^{-1}	0.02	Rate constant for Fe-bound P release
k_a	yr^{-1}	10.0	Rate constant for authigenic CFA precipitation
$PO_4^{\ s}$	${\rm molcm^{-3}}$	$1\cdot 10^{-9}$	equilibrium conc. for P sorption
			(Slomp et al., 1996)
$PO_4{}^a$	${\rm molcm^{-3}}$	$3.7\cdot10^{-9}$	equilibrium conc. for authigenic P precipitation
			(Slomp et al., 1996)
M^{∞}	${\rm molcm^{-3}}$	$1.99 \cdot 10^{-10}$	asymptotic concentration for Fe-bound P
			(Slomp et al., 1996)

DH: ALK^{OX} correct?: y=NH4 prod.; -2z=P release

where E(z), F(z) are the homogeneous solutions of the ODE, G(z) the particular integral, and A, B are the integration constants (compare Fig. 3 for the whole sediment column).

Each boundary matching problem involves matching continuity and flux for the two solutions $C_U(z)$ (= 'upper') and $C_L(z)$ (= 'lower') across a boundary at $z=z_b$. Therefore, we get two ODE solutions of the genral form:

$$C_U(z) = A_U \cdot E_U(z) + B_U \cdot F_U(z) + G_U(z) \tag{32}$$

$$C_L(z) = A_L \cdot E_L(z) + B_L \cdot F_L(z) + G_L(z). \tag{33}$$

450 The two boundary conditions are: for continuity (where for generality we allow a discontinuity V_b)

$$C_U(z_b) = C_L(z_b) + V_b \tag{34}$$

and for flux

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$$D_U C_U'(z_b) + w C_U(z_b) = D_L C_L'(z_b) + w C_L(z_b) + F_b$$
 (35)

where w is advection, D are the diffusion coefficients and F_b is any flux discontinuity.

In terms of the ODE solutions (32), (33), the boundary conditions represent two equations connecting the four integration constants:

$$\begin{pmatrix}
E_{U} & F_{U} \\
D_{U}E'_{U} & D_{U}F'_{U}
\end{pmatrix}
\begin{pmatrix}
A_{U} \\
B_{U}
\end{pmatrix} =
\begin{pmatrix}
E_{L} & F_{L} \\
D_{L}E'_{L} & D_{L}F'_{L}
\end{pmatrix}
\begin{pmatrix}
A_{L} \\
B_{L}
\end{pmatrix} +
\begin{pmatrix}
G_{L} - G_{U} + V_{b} \\
D_{L}G'_{L} - D_{U}G'_{U} + F_{b} - wV_{b}
\end{pmatrix}$$
(36)

where the ODE solutions E, F, G are all evaluated at z_b .

Equation (36) can be solved to give A_U and B_U as a function of the integration constants from the layer below (A_L and B_L), thereby constructing a piecewise solution for both layers, with just two integration constants.

In the code the function **benthic_utils.matchsoln** provides this solution in the form:

$$\begin{pmatrix} A_U \\ B_U \end{pmatrix} = \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix} \begin{pmatrix} A_L \\ B_L \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}. \tag{37}$$

Using (37) we can now rewrite $C_U(z)$ in (32) as a function of A_L and B_L :

$$C_U(z) = (c_1 A_L + c_2 B_L + d_1) \cdot E_U(z) + (c_3 A_L + c_4 B_L + d_2) \cdot F_U(z) + G_U(z)$$

and hence define the "transformed" basis functions $E_U^*(z)$, $F_U^*(z)$, $G_U^*(z)$ such that:

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$$C_U(z) = A_L \cdot E_U^*(z) + B_L \cdot F_U^*(z) + G_U^*(z)$$
 (38)

where

$$E_{U}^{*}(z) = c_{1}E_{U}(z) + c_{3}F_{U}(z)$$

$$F_{U}^{*}(z) = c_{2}E_{U}(z) + c_{4}F_{U}(z)$$

$$G_{U}^{*}(z) = G_{U}(z) + d_{1}E_{U}(z) + d_{2}F_{U}(z)$$
(39)

(in the code this is done by **benthic utils.xformsoln**).

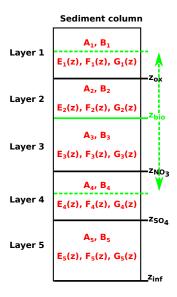


Figure 3. Schematic of the generic boundary condition matching (GBCM) problem. Showing the resulting integration constants (A_i, B_i) and ODE solutions (E_i, F_i, G_i) for the different sediment layers and the variable bioturbation boundary.

Solving the sediment layer stack

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Equations (37), (38) and (39) can now be applied for each layer boundary, working up from the bottom of the sediments. The net result is a piecewise solution of the whole sediment column with just two integration constants (coming from the lowest layer), which can then be solved for by applying one boundary condition for the sediment-water interface and one for the bottom of the sediments (e.g. a concentration condition at the SWI and a zero flux condition at z_{∞}).

Abstracting out the bioturbation boundary

The bioturbation boundary affects the diffusion coefficient of the modelled solutes and the conservation equation of organic matter. The boundary is particularly inconvenient as it can, in principle, occur in the middle of any "geochemical" layer and therefore generates multiple cases (compare Fig. 3). To simplify this for solutes, the GBCM algorithm above is used to construct a piecewise solution

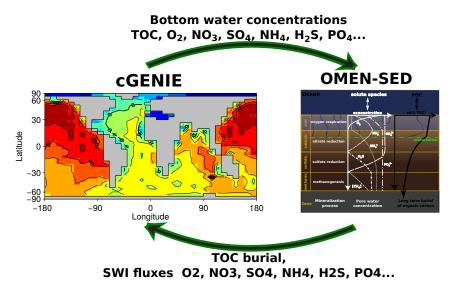


Figure 4. Schematic of the relationship between OMEN-SED and cGENIE. Arrows and accompanied text represent the information transferred between models.

and to abstract out the bioturbation boundary. An initial test for each "geochemical" layer is made to identify its "bioturbation-status" (fully bioturbated, fully non-bioturbated or crossing the bioturbation boundary) and (if needed) a piecewise solution is constructed by matching boundary conditions across the bioturbation boundary. The "outside" code therefore never needs to know whether it is dealing with a piecewise solution (i.e. matched across a bioturbation boundary) or a "simple" solution (i.e. the layer is fully bioturbated or fully non-bioturbated).

In the code, this is performed by **zTOC.prepfg_I12** which hands back a structure Is containing the "bioturbation-status" for each layer and (if needed) the description of the piecewise solution (coeffcients $c_1, c_2, c_3, c_4, d_1, d_2$ as above). For example in the case of sulfate, **zTOC.prepfg_I12** is called three times before the actual profile is calculated (once per layer: oxic, suboxic, sulfidic) and hands back three structures Is with information about the layer's "bioturbation-status" and all associated conditional logic. When calculating the solutions for the different layers, the pre-calculated structure Is is passed to the function **zTOC.calcfg_I12** which sorts out the correct solution type to use.

2.5.2 Coupling to an Earth System Model

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Here the coupling of OMEN-SED to the carbon-centric version of the "GENIE" Earth system model (cGENIE Ridgwell et al., 2007) is described (Fig. 4). Results from pre-industrial experiments are presented in Section 3.3. cGENIE is a model of Intermediate Complexity based on the efficient climate model "C-GOLDSTEIN" of Edwards and Marsh (2005), featuring a frictional-geostrophic 3D-ocean circulation model coupled to a fast Energy-Moisture Balance 2D-atmosphere together with a dynamic-thermodynamic sea-ice component. The version of cGENIE used here includes the marine

geochemical cycling of carbon, oxygen, phosphorus and sulfur (Ridgwell et al., 2007), preservation of carbonates in deep-sea sediments (Ridgwell and Hargreaves, 2007) and terrestrial weathering (Colbourn et al., 2013). The ocean model is implemented on a 36×36 equal-area horizontal grid with 16 vertical levels using the pre-industrial continental configuration and bathymetry as in Archer et al. (2009). In contrast to Archer et al. (2009) the same grid resolution (36×36) is used for the sediment geochemistry model SEDGEM. Instead of completely remineralising POC at the seafloor, OMEN-SED is called by SEDGEM for each grid point. Depending on the overlying biogeochemical ocean model processes can be included or excluded in OMEN-SED and stoichiometric factors need to be adjusted to ensure preservation of mass. As nitrogen is not modelled explicitly in the employed cGENIE configuration the related stoichiometries in OMEN-SED are set to zero (i.e. NC_i , ALK^{NIT} and ALK^{DEN}). cGENIE, however, implicitly includes the effects of NH_4 release and its nitrification on Alkalinity and neglects the impact of P release; therefore, related stoichiometries are changed to $ALK^{OX} = -16/106$ and $ALK^{SUL} = 122/106$.

Several biogeochemical tracers and parameters are transferred from SEDGEM to OMEN-SED and have to be converted into the required units. Bottom water concentrations of solutes are converted from $mol kg^{-1}$ to $mol cm^{-3}$ and the depositional flux of POC (POC_{flux}) is converted from cm³ cm⁻² yr⁻¹ to mol cm⁻² yr⁻¹ assuming an average density of POC of 1.0 cm³ g⁻¹. Other parameters used from cGENIE are seafloor depth, local temperature and the partitioning of bulk POC into the slower and faster degrading pool (as cGENIE represents a labile and a refractory POC fraction, see Ridgwell et al. (2007)). The advection/sediment accumulation rate is generally taken from cGENIE, however, a minimum values of $w = 0.5 \,\mathrm{cm}\,\mathrm{kyrs}^{-1}$ is imposed as OMEN-SED tends to be unstable for lower values. The bulk POC_{flux} is seperated into the labile and refractory component and the routine to calculate the sedimentary POC profiles is called. Here, the two POC depositional fluxes are first converted into SWI concentrations (in mol cm⁻³) by solving Eq. (3) for z=0. OMEN-SED computes the resulting SWI-fluxes of solutes (in mol cm⁻² yr⁻¹) and the fraction of POC preserved in the sediment at a depth of 100 cm (POC_{pres}) and returns the results to cGENIE. In case no POC is deposited on the seafloor (i.e. $POC_{flux} = 0$), OMEN-SED is not called and the SWI-fluxes of solutes and POC_{pres} are set to zero. In order to reduce memory requirements the sediment profiles (e.g. as shown in Fig. 5) are not calculated in the FORTRAN version of OMEN-SED, however, the boundary conditions are saved at the end of the experiment and sediment profiles for specific grid-cells, ocean basins and/or ocean transects can be plotted using the stand-alone matlab version of OMEN-SED.

540 3 Test Cases

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TODO: Check Intro sentences To validate our approach, model results were compared with observed pore water profiles (Section 3.1), an extensive sensitivity analysis for the most important model pa-

rameters was performed and resulting sediment-water interface fluxes were compared with a global database (Section 3.2). Furthermore, OMEN-SED was coupled to the cGENIE Earth System model and different published parameterisations for the OM degradation rate constants are tested on a global scale (Section 3.3).

3.1 Sediment profiles

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Modelled profiles were compared with measured pore water data from different ocean depths (Figure 5) including the continental shelf (108 m), upper slope (585 m), lower slope (2213 m) and the deep sea (4298 m). The site at 585 m depth located in the Santa Barbara Basin is characterised by anoxic bottom waters and high POC concentrations (POC ~5.5 wt%, Reimers et al., 1996), whereas the other sites at the Iberian margin (108 and 2213 m) and the Nazaré Canyon (4298 m, Epping et al., 2002) are oxic. Sediment-water interface characteristics and concentrations of POC and dissolved species in OMEN-SED were set to the observed values where available (Table 11). The POC and pore water profiles were fitted by optimizing the POC partitioning into the fast and slow degrading pool and their respective first order degradation rate constants (priority given to reproduce the POC and O_2 profiles). For phosphorus the equilibrium concentration for authigenic P formation (PO₄^a) was adjusted to fit the PO_4 concentration at z_{∞} . For the two open Iberian margin stations (108 and 2213 m) OMEN-SED fits all observations well. OMEN-SED does especially well at depth 2213m by reproducing the deep O_2 penetration and the subsurface maximum in NO_3 concentration due to the nitrification of NH_4 . For the Santa Barbara basin (585 m) a misfit is observed for H_2S and PO_4 in the upper 20 cm of the sediment. This can be explained by the presence of $\mathrm{Mn^{2+}}$, $\mathrm{Fe^{2+}}$ and dissolved Fe at this site which are either reduced to degrade POC and/or react with H₂S to form iron sulfides, therefore inhibiting the rise in concentration of H₂S (Reimers et al., 1996). Phosphorus is adsorbed to Fe oxides and incorporated into carbonate fluorapatite (CFA) which is highly parameterised in OMEN-SED and not modelled explicitly. For the Nazaé Canyon station (4298 m) satisfactory fits could be realised apart from NH₄. However, also the original study (Epping et al., 2002) had the same problem using a more complex diagenetic model and suggested non-local solute exchange being responsible for the higher NH₄ concentrations at this site.

DH: P explanation correct/okay?
Also, say something about [ALK] at 585m?

570 3.2 Sensitivity Analysis

Model parameters implicitly account for processes not explicitly described, they are notoriously difficult to constrain and a source of uncertainty for numerical and analytical models. One strategy to explore and quantify this model uncertainty that can always be applied is sensitivity analysis (SA). SA is a term used for mathematical techniques to investigate how the variations in the outputs $(y_1, ..., y_N)$ of a model can be attributed to variations in the different input parameters $(x_1, ..., x_M)$ (Pianosi et al., 2016). Different types of sensitivity indices, which quantify this relative influence with a scalar S_i , can be calculated, ranging from simple one-at-a-time methods to statistical evalua-

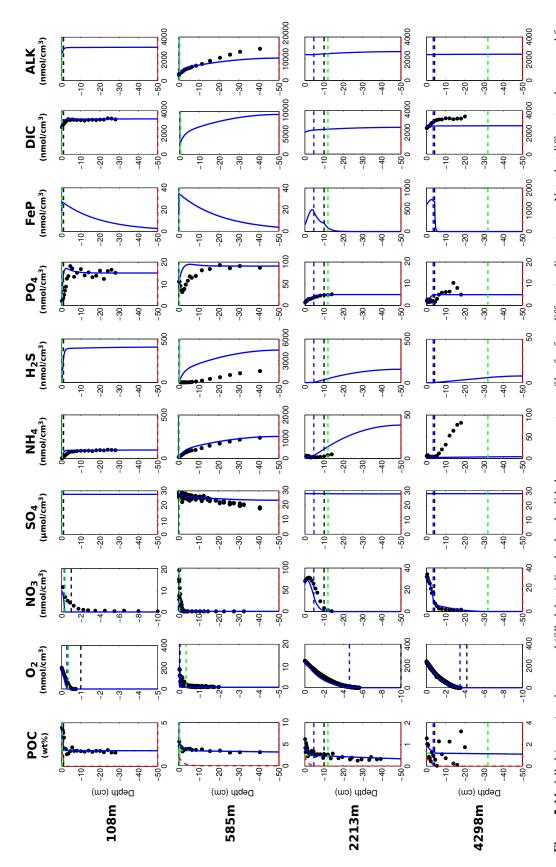


Figure 5. Modelled (curves) and measured (filled dots) dissolved and solid phase pore water profiles for four different sediment cores. Note that different scales are used for different stations. The blue POC curve represents the sum of the refractory (green) and labile (red) POC fraction.

Table 11. Model boundary conditions for the sampling stations in Figure 5. (For all sites DIC bottom water concentration of 2.400 nanomole cm⁻³ is assumed.)

Sedime	Sediment characteristics:									
Depth	Temp.	$z_{ m bio}$	$D_{ m bio}$	POC_1	POC_2	$\mathbf{k_1}$	k_2			
(m)	$(^{\circ}C)$	(cm)	(cm^2yr^{-1})	(wt%)	(wt%)	(yr^{-1})	(yr^{-1})			
108	12.5	1.0	0.02	2.64	1.8	0.65	$1.0e^{-5}$			
585	5.85	0.01	0.02	2.0	3.5	0.2	$8.0e^{-4}$			
2213	3.2	10.0	0.17	0.45	0.5	0.1	$4.0e^{-4}$			
4298	2.5	4.2	0.18	0.83	1.2	0.052	$1e^{-5}$			
Bottom	water co	ncentra	tions of solut	es (all in n	anomole c	m^{-3}):				
Depth	O_2	NO_3	SO_4	$\mathrm{NH_4}$	H_2S	PO_4	PO_4^{a}	Alkalinity		
108	210	9.6	28,000	0.4	0.0	0.0	15.0	2,400		
585	10	25.0	28,000	0.0	0.0	50.0	90.0	2,480		
2213	250	25.0	28,000	0.6	0.0	0.0	5.0	2,400		
4298	243	30.1	28,000	0.22	0.0	0.0	5.0	2,400		

tions of the output distribution (e.g. variance-based or density-based approaches Pianosi et al., 2016). Especially the latter indices are easy to interpret and can be compared across different parameters and/or different model outputs as they generally take values between zero and one $(S_i \in [0,1])$. An index of zero indicates a non-influential parameter and a higher value a more influential parameter. Here, we use SA mainly to identify which parameters have the largest impact on the different model outputs and therefore require careful calibration. As the probability density functions of our model outputs (i.e. the resulting SWI-fluxes) are generally highly-skewed towards extreme organic matter degradation rates (not shown) variance-based sensitivity indices are not very reliable uncertainty indicators (Pianosi et al., 2016). Rather than just considering the variance we employ the novel density-based PAWN method by (Pianosi and Wagener, 2015) which considers the entire conditional and unconditional Cumulative Destribution Function (CDF) of the model output. The sensitivity index of parameter i is calculated as the difference between the two CDFs, i.e.

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$$S_i = \max_{x_i} \max_{y} |F_y(y) - F_{y|x_i}(y)| \tag{40}$$

where $F_y(y)$ is the unconditional CDF of the output y and $F_{y|x_i}(y)$ represents the conditional CDF when the i-th parameter is fixed to x_i . For a more detailed description of the method we refer the interested reader to Pianosi and Wagener (2015).

Due to the model complexity it is impossible to compute the sensitivity indices analytically therefore they are approximated from a Latin-Hypercube sampling of parameter inputs and calculated outputs. The PAWN method, as implemented within the Sensitivity Analysis for Everyone (SAFE) matlab toolbox (Pianosi et al., 2015), is used to investigate 11 model parameters for ranges as specified in Table 12. Sensitivity indices for all resulting SWI-fluxes for two idealised sediment conditions

Table 12. Range of model parameters used for sensitivity analysis of model predicted output.

Parameter	Description	Units	Minimum	Maximum	Source
k_1	labile OM degradation constant	yr^{-1}	$1e^{-4}$	5.0	(1)
$\widetilde{\mathrm{k}_2}$	order of refractory OM degradation	-	$1e^{-4}$	$1e^{-1}$	(1)
	constant $(k_2 = \widetilde{k_2} \cdot k_1)$				
f_1	fraction of labile OM	-	0.02	0.98	-
$K_{ m NH_4}$	Adsorption coefficient	-	0.8	1.7	(2)
$\gamma_{ m NH_4}$	NH_4 fraction oxidised		0.5	1.0	-
$\gamma_{\rm H_2S}$	H_2S fraction oxidised		0.5	1.0	-
$K_{\mathrm{PO_4}}^{ox}$	Adsorption coeff. oxic	-	100.0	400.0	(3)
$K_{\mathrm{PO_4}}^{anox}$	Adsorption coeff. anoxic	-	1.3	2.0	(3)
k_s	kinetic P sorption	yr^{-1}	0.1	100.0	(4, 5)
k_m	Fe-bound P release	yr^{-1}	0.015	0.02	(4, 5)
k_a	authigenic P formation	yr^{-1}	0.001	10.0	(4, 6)

Sources:

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Table 13. Model boundary conditions for the two idealised sediment conditions used for the sensitivity analysis (Fig. 6 and 7). All solute concentrations are in nanomole cm^{-3} .

Depth (m)	Temp. (°C)	OC (wt%)	O_2	NO_3	SO_4	PO_4	$z_{ m bio}$ (cm)
400	8.0	2.0	0.0	40.0	28,000	40.0	0.001
4000	1.5	1.0	300.0	20.0	28,000	40.0	10.0

(i.e. anoxic at 400 m and oxic at 4000 m, see Table 13) are calculated. We use 200 samples to estimate the unconditional CDF, 100 samples to estimate the conditional CDFs and 10 conditioning points, thus 11,200 evaluations are performed for each sediment condition. The resulting indices (compare Fig. 9 in the Appendix) are then translated into a color code and summarised in a pattern plot to simplify comparison (Fig. 6). The most significant parameters for all model outputs are the degradation rate for the labile OM part (k_1) and its share in the total OM pool (f_1). For the anoxic setup, where no oxidation occurs, the secondary redox parameters (i.e. $\gamma_{\rm NH_4}$, $\gamma_{\rm H_2S}$) are essentially non-influential. Whereas in the oxic scenario, SWI-fluxes of NH₄, SO₄ and H₂S are very sensitive to changes in the secondary redox parameters. The PO₄ SWI-flux appears to be insensitive especially under for the oxic condition as the majority is absorbed to Fe-oxides. The sensitivities change if other PO₄ related equilibrium concentrations PO₄ PO₄ and M[∞] are used (not shown).

To test if OMEN-SED is able to reproduce the magnitude of observed SWI-fluxes, another set of Latin-Hypercube samplings is produced for the two idealised sediment conditions (sample sizes N=3500). Here the two most sensitive parameters \mathbf{k}_1 and \mathbf{f}_1 and also $\widetilde{\mathbf{k}_2}$ are varied, the other

DH: explanation for small P sensitivity correct?

⁽¹⁾ Arndt et al. (2013); (2): Van Cappellen and Wang (1996); (3): Krom and Berner (1980)

^{(4):} Gypens et al. (2008); (5): Slomp et al. (1996); (6): Van Cappellen and Berner (1988)

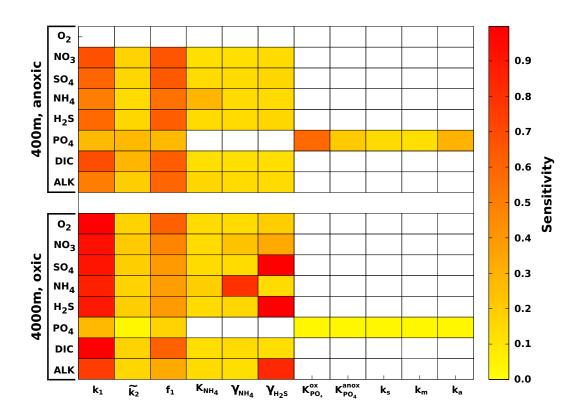


Figure 6. Pattern plot, showing the output sensitivity for each SWI flux (i.e. the chemical compounds on the horizontal axis) and each input factor (i.e. the model parameters on the vertical axis) for two idealised sediment cores. White patterns are assigned where the SWI flux is independent of the specific parameter.

DH: Rather low PO4 sensitivity - bc of Equil. concentr.?

parameters are set to their default values (Tables 9 and 10). For the deep sea condition we account for the presence of more refractory OM by sampling $f_1 \in [0.02, 0.3]$. Minimum and Maximum values for the other parameters are as in Table 12. The results are compared with a global database of benthic fluxes of O_2 and NO_3 (Bohlen et al., 2012). The coloured scatter plots in Figure 7 show that the observed fluxes fall well in the range of SWI-fluxes calculated with OMEN-SED. Also highlighted by the emergence of colour patterns in Figure 7 A+B are the strong interactions between the amount of labile OM f_1 and its degradation rate k_1 for the resulting SWI-fluxes of the most powerful TEA available. In general, a higher degradation rate in combination with more labile OM available leads to a higher SWI-flux.

3.3 cGENIE pre-industrial

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TODO: WRITE this section!

We test different published parameterisations for the OM degradation rate constants by coupling OMEN SED to the pre-industrial cGENIE model setup. Model is run for 20,000 years to steady-

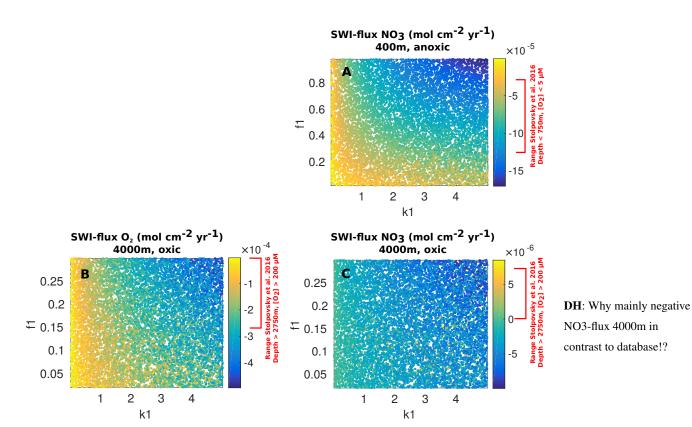


Figure 7. Coloured scatter plots $(k_1 \text{ vs } f_1)$ of resulting OMEN-SED SWI-fluxes for the 400m anoxic (A: NO₃) and 4000m oxic (B: O₂, C: NO₃) scenario. Negative values representing a flux from the water column into the sediments. Indicated area in red at the respective colour scale represents the range of benthic fluxes in the global database of Bohlen et al. (2012).

state. OMEN-SED is called for each grid-cell in every timestep, feeding back to cGENIE the resulting SWI-fluxes and the fraction of POC preserved in the sediments. Describe Figure 8 and compare with what we know. E.g. Thullner et al. 2009!

630 4 Scope of applicability and model limitations

5 Conclusions

Not too bad this model...

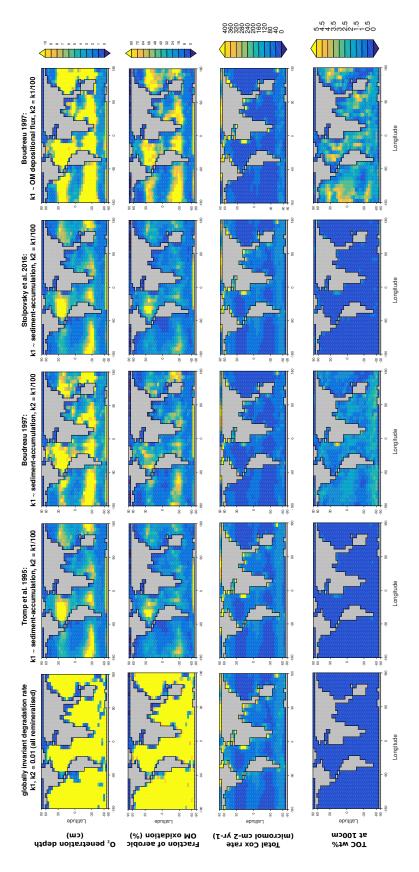


Figure 8. Results of OMEN-cGENIE coupling using different, published parameterisations for the OM degradation rates (k1, k2, compare Table ??) coupled to the same cGENIE ocean setup. All results shown are sediment characteristics calculated by OMEN-SED. NOTE: Preliminary results (here frac2 still constant!) TODO: plot wt% just until 3%.

6 Code Availability

Appendix A: Reaction Network

635 Appendix B: Sensitivity Analysis

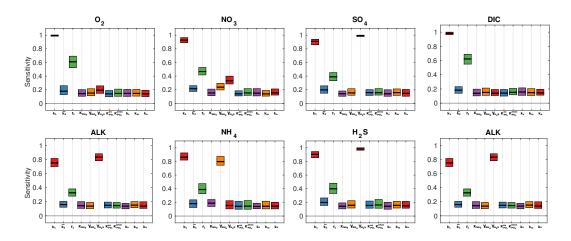


Figure 9. Move to Appendix Box plot of parameter sensitivities for the calculated SWI-fluxes for the 4000m oxic condition. Average sensitivities (black lines) and 90% confidence intervals using N=11200 model evaluations and Nboot=100 bootstrap resamples.

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Acknowledgements. Thank you...

Table 14. Primary pathways of organic matter degradation, secondary redox reactions and stoichiometries implemented in the reaction network.

Pathway	Stoichiometry
	Primary Redox reactions
Aerobic degradation	$(CH_2O)_x(NH_3)_y(H_3PO_4)_z + (x+2y)O_2 + (y+2z)HCO_3^- \rightarrow (x+y+2z)CO_2 + yNO_3^- + zHPO_4^2 - + (x+2y+2z)H_2O_3 + zHPO_4^2 - zHPO_4^2 + zHPO_4^2 - zHPO_4^2 + zHPO$
Denitrification	$(\mathrm{CH_2O})_x(\mathrm{NH_3})_y(\mathrm{H_3PO_4})_z + \tfrac{(4x + 3y)}{5}\mathrm{NO_3} \to \tfrac{(2x + 4y)}{5}\mathrm{NO_3} + \tfrac{(2x + 3y + 10z)}{5}\mathrm{N_2} + \tfrac{(4x + 3y + 10z)}{5}\mathrm{CO_2} + \tfrac{(4x + 3y - 10z)}{5}\mathrm{HCO_3} + \mathrm{zHPO_4^2} + \tfrac{(3x + 6y + 10z)}{5}\mathrm{H_2O_3} + \tfrac{(4x + 3y + 10z)}{5}\mathrm{H_2O_3} +$
Sulfate reduction	$(\mathrm{CH_2O})_x(\mathrm{NH_3})_y(\mathrm{H_3PO_4})_z + \tfrac{x}{2}\mathrm{SO_4^{2-}} + (y-2z)\mathrm{CO_2} + (y-2z)\mathrm{H_2O} \to \tfrac{x}{2}\mathrm{H_2S} + (x+y-2z)\mathrm{HCO_3^{-}} + y\mathrm{NH_4^+} + z\mathrm{HPO_4^{2-}}$
Methanogenesis	$(\mathrm{CH_2O})_x(\mathrm{NH_3})_y(\mathrm{H_3PO_4})_z + (y-2z)\mathrm{H_2O} \to \frac{x}{2}\mathrm{CH_4} + \frac{x-2y+4z}{2}\mathrm{CO_2} + (x-2z)\mathrm{HCO_3}^- + y\mathrm{NH_4}^+ + z\mathrm{HPO_4}^{2-}$
	Secondary Redox reactions
Nitrification	$NH_4^+ + 2O_2 + 2HCO_3^- \rightarrow NO_3^- + 2CO_2 + 3H_2O$
Sulfide oxidation	$H_2S + 2O_2 + 2HCO_3^- \rightarrow SO_4^{2-} + 2CO_2 + 2H_2O$
AOM	$CH_4 + CO_2 + SO_4^{2-} \rightarrow 2HCO_3^- + H_2S$
	Adsorption reactions and mineral precipitation
$\mathrm{NH_4}$ adsorption	$NH_4^+ \xrightarrow{KNH_4} NH_4^+ (ads)$
P ad-/desorption ??? $PO_4^{2-} \xrightarrow{K_PO_4}$	$PO_4^{2-} \xrightarrow{KPO_4^2} PO_4^{2-} (ads); \qquad HPO_4^{2-} \xrightarrow{k_s} Fe - bound P \xrightarrow{km} HPO_4^{2-}$
CFA precipitation	$\mathrm{PO}_4^{2-} \stackrel{\mathrm{k_a}}{\longrightarrow} \mathrm{CFA}$

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