The Measurement of Thermal Diffusivity

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Abstract

Sinusoidal temperature variations were applied to aluminium and copper rods to determine their thermal properties. The thermal diffusivity of copper and aluminium were found to be $1.27 \pm 0.07 \times 10^{-4}\ m^2 s^{-1}$ and $5.94 \pm 0.24 \times 10^{-5}\ m^2 s^{-1}$ respectively. The thermal conductivities of copper and aluminium were found to be $359 \pm 54\ Wm^{-1}K^{-1}$ and $153 \pm 27\ Wm^{-1}K^{-1}$ respectively. The electrical conductivity of the copper and aluminium were found to be $5.7 \pm 0.3 \times 10^7\ \Omega^{-1}m^{-1}$ and $2.9 \pm 0.1 \times 10^7\ \Omega^{-1}m^{-1}$ respectively. Testing of the Wiedemann-Franz relation was done by measuring the thermal and electrical conductivity of both copper and aluminium. This revealed a value of the Lorenz number for copper and aluminium to be $2.12 \pm 0.4 \times 10^{-8}\ W\Omega K^{-2}$ and $1.78 \pm 0.3 \times 10^{7}\ W\Omega K^{-2}$ respectively. The value of the Lorenz number for aluminium is lower than copper due to the temperature dependence of the relation.

Introduction

The thermal diffusivity of an object is the ratio of its thermal conductivity (κ) to its specific heat (c) and its density (ρ). This relationship is a measure of an objects ability to conduct heat relative to its ability to hold heat in and can be shown as below.

$$D_T = \frac{\kappa}{c\rho} \quad (1)$$

An experiment was conducted in order to determine the thermal diffusivity of copper and aluminium rods. The following expression can be used in this 1D case where sinusoidal temperature variations where applied to one end. x is the distance from the heat input and θ is the difference in temperature between the current temperature and the ambient temperature of the rod.

$$\frac{\partial\theta}{\partial t} = D_T \frac{\partial^2\theta}{\partial x^2} \quad (2)$$

The previous expression does not take into account the heat lost through thermal radiation from the sides of the rod to the air. To adjust for this the introduction of an extra term related to Newton's law of cooling is essential (μ is the radiation constant measured in s^{-1}).

$$\frac{\partial\theta}{\partial t} = D_T \frac{\partial^2\theta}{\partial x^2} - \mu\theta \quad (3)$$

A solution to this heat equation with a sinusoidal temperature input is as below where α and β represent the wave number and the phase shift specifically associated with the copper and aluminium bars and ω is the angular frequency of the temperature variations.

$$\theta(x,t) = \theta_o e^{-\alpha x}\sin(\omega t - \beta x) \quad (4)$$

From this an expression for the thermal diffusivity can be found.

$$D_T = \frac{\omega}{2\alpha\beta} \quad (5)$$

To get the heat capacity of the rods the following expression can be used with the use of a calorimeter to find the change in temperatures.

$$c_{rod} = \frac{m_{water} c_{water} \Delta T_{water}}{m_{rod} \Delta T_{rod}} \quad (6)$$

The Wiedemann-Franz law states that the ratio of the thermal conductivity to the electrical conductivity (σ) of a metal is proportional to the temperature. "This relationship is based upon the fact that the heat and electrical transport both involve the free electrons in the metal. The thermal conductivity increases with the average particle velocity since this increases the forward transport of energy. The electrical conductivity, on the other hand, decreases while particle velocity increases because the collisions divert the electrons from forward transport of charge." (physicshandbook, 2019). This relationship is shown below.

$$\kappa = LT\sigma \quad (7)$$

L is the Lorenz number which is a proportionality constant that can deviate based on the material and its temperature.

To determine the electrical conductivity of a cylindrical rod the following expression can be used where l is the length of the rod and d is its diameter.

$$\sigma = \frac{4l}{\pi d^2 R} = \frac{4lI}{\pi d^2 V} \quad (8)$$

With the electrical conductivity equation (7) can be used to test the Wiedemann-Franz law and determine the value of L for the rods.

Experimental Methods

The experimental setup is shown in **figure 1**, the angular frequency and sampling rate can be changed on the control unit. Measuring the sinusoidal temperature variations can begin once the Peltier and the reference temperature align.

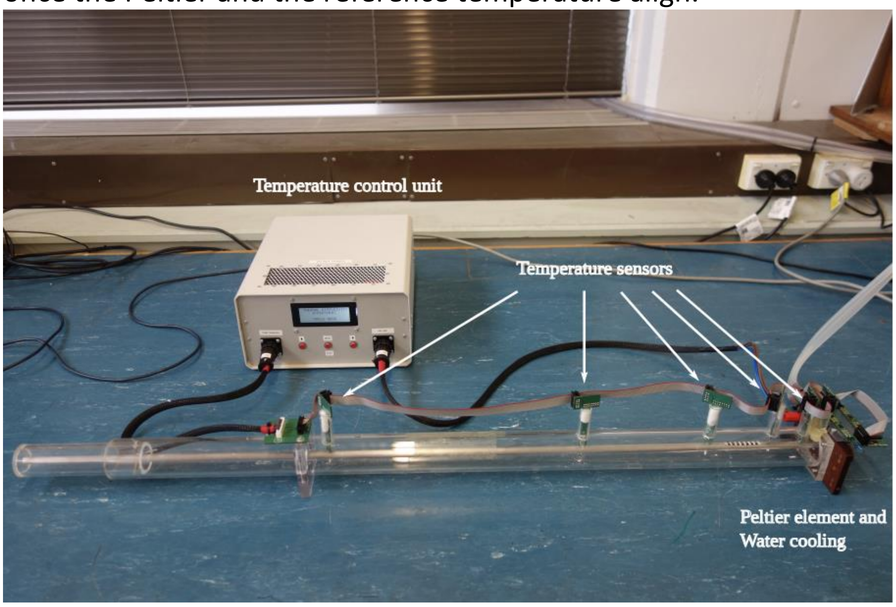

Figure 1: Experimental set-up (Script, 2019)

Uncertainty in measuring the distance of the sensors comes from difficulty in measuring the exact location of the sensor inside the plastic casing with a ruler.

To determine the electrical conductivity of the copper and aluminium, the rods are placed in a casing on two sharp contacts to break through the oxide layer on the outside of the rod. A voltmeter and an ammeter are connected to the rod. The resistance of each rod can be determined by applying a potential difference across the two contacts. From this the resistivity can be determined and therefore the electrical conductivity can be calculated.

The heat capacity of the copper and aluminium can be found using the method described in the lab book. The error in this comes from difficulty in finding accurate temperature changes. When measuring the temperature of the heated rod the thermocouple needs time to cool down before measuring the final temperature of the system.

Results

After running the experiment on the copper rod at one millihertz for five cycles **figure 2** was produced. The distance of the sensors from the heat source increases from the Peltier to the sixth sensor. As this distance increases it is clear from **figure 2** that the temperature of the rod decreases with distance from the rod. After running the experiment on the copper rod again at two millihertz and for one and two millihertz with the aluminium rod you can see the same trend. These plots are shown in the lab book. The temperature decreased with distance slightly faster with the aluminium rod compared to the copper.

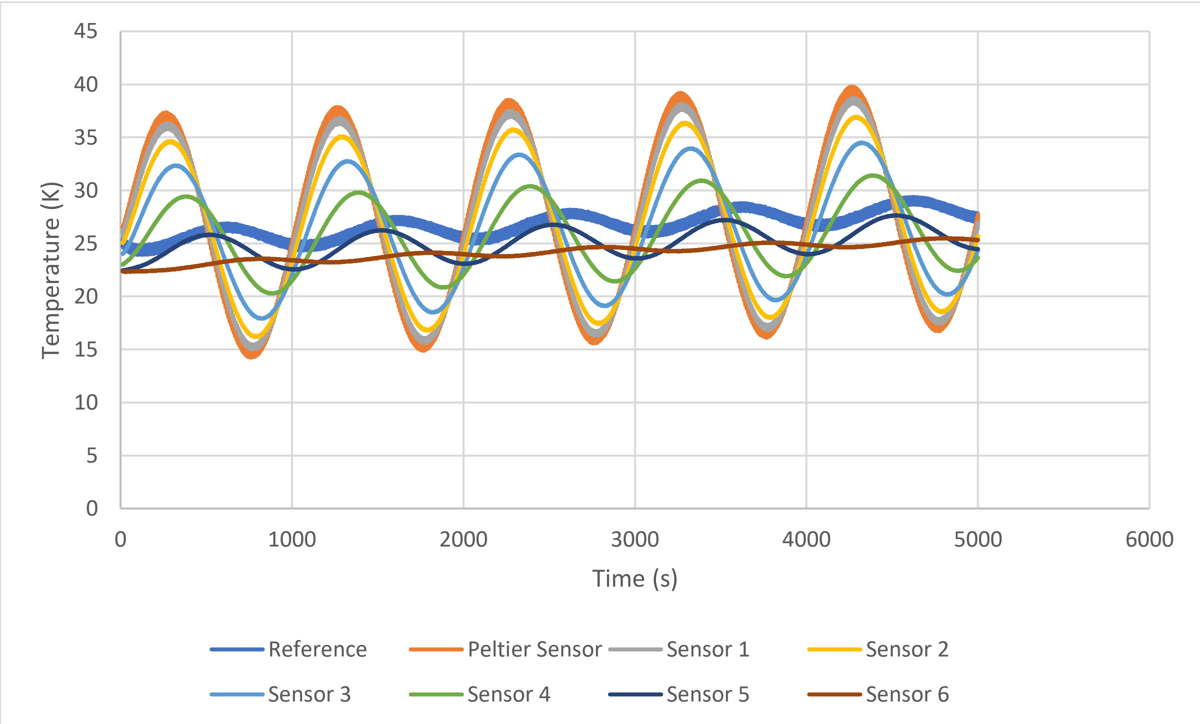

Figure 2: One millihertz temperature variations for five cycles of the copper rod

In order to find a value for alpha from equation (4) the amplitude of each peak at each distance was found. The natural logarithm of the average of these was plotted against the distance of each sensor from the source. The gradient of the slope produced gives a value of alpha. The plot for the copper rod at one millihertz is shown in **figure 3**. The uncertainty for each point is based on the range of amplitude values and the alpha uncertainty is based on the line of worst fit.

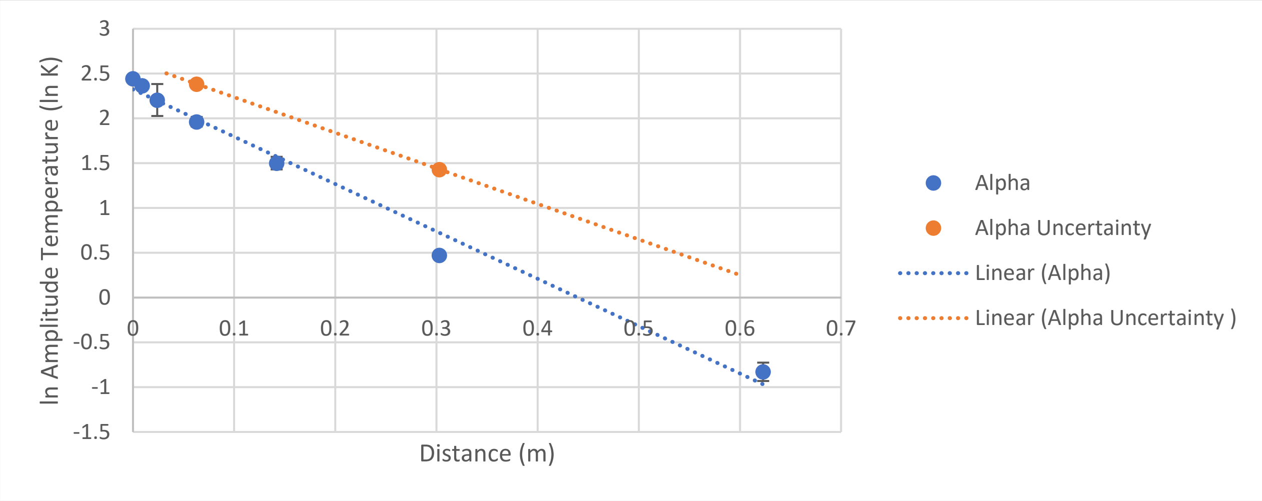

Figure 3: Alpha plot for the copper rod at one millihertz. Alpha = 6.3 ± 1.3

To find a value for beta from equation (4) the time difference between the peaks of each sensor at each cycle was found. The average of these was multiplied by the omega value specific to the experiment to find the phase shift and plotted against the distance of the sensors with the gradient of the liner plot produced being beta. Once again, the uncertainty for each point is based on the range of time values for each cycle. The plot for the copper rod at one millihertz is shown in **figure 4**.

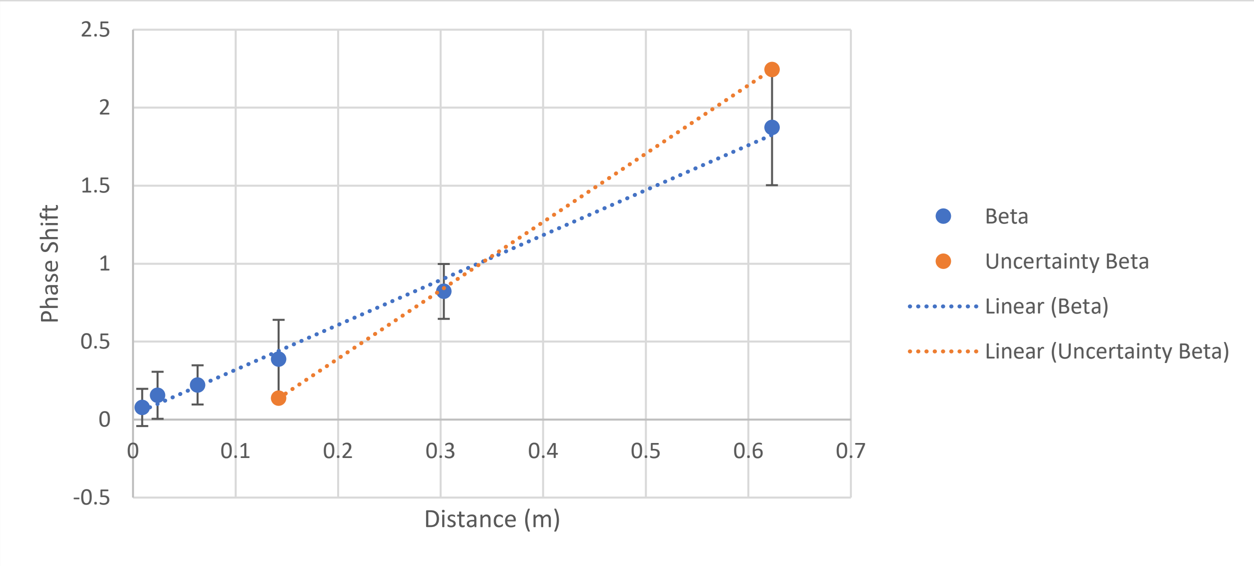

Figure 4: Beta plot for the copper rod at one millihertz. Beta = 3.9 ± 1.5

The alpha and beta plots for aluminium are shown in **figure 5** and **6**.

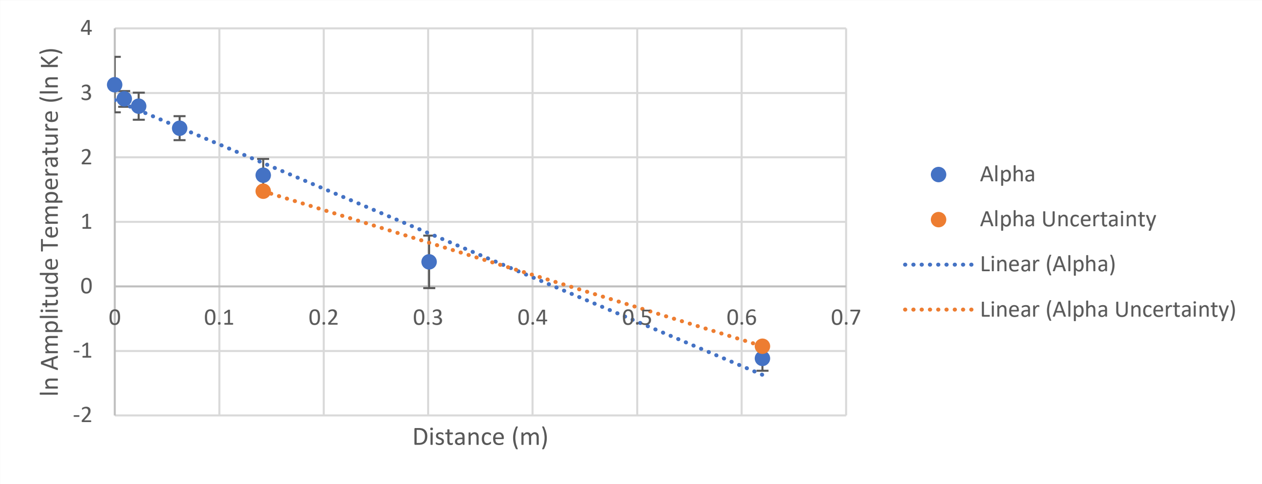

Figure 5: Alpha plot for the aluminium rod at one millihertz. Alpha = 9.9 ± 1.8

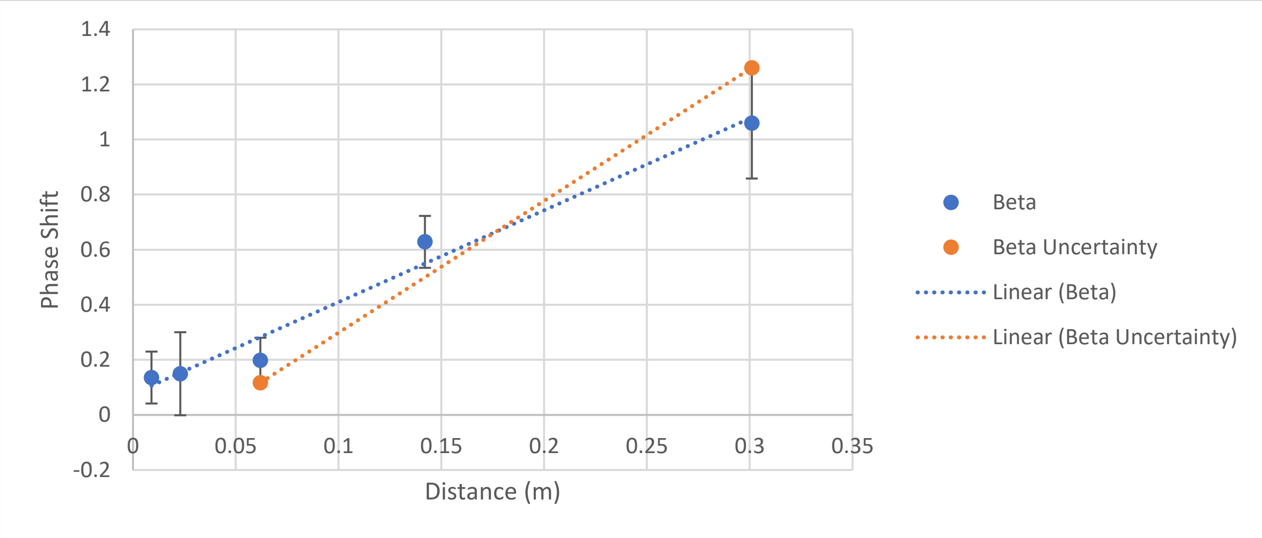

Figure 6: Beta plot for the aluminium rod at one millihertz. Beta = 5.3 ± 1.5

The plots for aluminium and copper at two millihertz are shown in the lab book.

Using the attained values for alpha and beta and equation (5) the thermal diffusivity of each rod can be found.

$$D_T = \frac{\omega}{2\alpha\beta}$$

$$D_{T_{Cu\ 1mHz}} = 1.28\ \pm 0.07\ \times\ 10^{-4}\ m^2 s^{-1}$$

$$D_{T_{Cu\ 2mHz}} = 1.25\ \pm 0.07 \times\ 10^{-4}\ m^2 s^{-1}$$

$$D_{T_{Al\ 1mHz}} = 5.95\ \pm 0.23 \times\ 10^{-5}\ m^2 s^{-1}$$

$$D_{T_{Al\ 2mHz}} = 5.92\ \pm 0.25 \times\ 10^{-5}\ m^2 s^{-1}$$

The density of the aluminium and copper rods were found by getting the ratio of the mass and volume of a sample. The uncertainty is based on the intervals of the ruler, scales and the Vernier callipers. The values were calculated to be;

$$\rho_{Cu} = 8879 \pm 110 \; kgm^{-3}$$

$$\rho_{Al} = 2820 \pm 60 \; kgm^{-3}$$

The heat capacities of the copper and aluminium rods were calculated using equation (6), uncertainty for this is based on the difficulties in trying to get accurate changes in temperature due to the thermocouple being so sensitive. These values were calculated to be;

$$c_{Cu} = 0.32 \pm 0.07 \, Jg^{-1}{}^{\circ}\mathrm{C}^{-1}$$

$$c_{Al} = 0.92 \pm 0.09 \, Jg^{-1}{}^{\circ}\mathrm{C}^{-1}$$

The average of the thermal diffusivity values, along with the density and heat capacities can be used to find the thermal conductivity of the metals with equation (1);

$$\kappa_{Cu} = 359 \pm 54 \;\; Wm^{-1}K^{-1}$$

$$\kappa_{Al} = 153 \pm 27 \, Wm^{-1}K^{-1}$$

The electrical conductivity of the copper and aluminium was found by measuring the resistance across two contact points on each rod when a potential difference was applied. The resistance was measured with the contacts being at different lengths along the rods. A graph of this was plotted (in lab book) with the gradient being equal to equation (8). From this we can calculate values of electrical conductivity to be;

$$\sigma_{Cu} = 5.7 \pm 0.3 \times 10^{7} \; \Omega^{-1}m^{-1}$$

$$\sigma_{Al} = 2.9 \pm 0.1 \times 10^{7} \; \Omega^{-1}m^{-1}$$

To test the Wiedemann-Franz relation the electrical conductivity and the thermal conductivity that was determined can be used in conjunction with equation (7) with the room temperature being measured at 297 K.

$$L_{Cu} = 2.12 \pm 0.4 \times 10^{-8} \; W\Omega K^{-2}$$

$$L_{Al} = 1.78 \pm 0.3 \times 10^{7} \; W\Omega K^{-2}$$

Discussion and Conclusion

The average thermal diffusivity value found for the copper rod was $1.27 \pm 0.07 \times 10^{-4} \; m^2 s^{-1}$. This does not agree within uncertainty value of the theoretical value of $1.11 \times 10^{-4} \; m^2 s^{-1}$ (engineersedge, 2019). This is due to the difficulty in finding the peaks on the temperature variation figures (**figure 2**) in order to find alpha and beta. The Peltier did not have smooth temperature increases and decreases and instead changed the temperature by small jumps. This made it difficult in determining exact peaks and hence the alpha and beta values might not be accurate due to repetitive small errors in finding the peaks for each sensor. The density of the copper rod was found to be $8879 \pm 110\ kgm^{-3}$ which agrees within uncertainty of the theoretical value of $8960\ kgm^{-3}$ (amesweb, 2019). The electrical conductivity value of $5.7 \pm 0.3 \times 10^{7}\ \Omega^{-1}m^{-1}$ agrees within uncertainty of $5.98 \times 10^{7}\ \Omega^{-1}m^{-1}$ (thebalance, 2019). The heat capacity value of $0.32 \pm 0.07\ Jg^{-1}°\mathrm{C}^{-1}$ agrees within uncertainty of $0.385\ Jg^{-1}°\mathrm{C}^{-1}$ (iun, 2019). Aside from the thermal diffusivity all of these values agree within uncertainty of the sourced theoretical values of pure copper suggesting the copper used was not an alloy.

The average thermal diffusivity value found for the aluminium rod was $5.94 \pm 0.24 \times 10^{-5}\ m^2s^{-1}$. This does not agree within uncertainty value of the theoretical value of $9.7 \times 10^{-5}\ m^2s^{-1}$ (engineersedge, 2019). However it is significantly closer to the value of the Aluminium 6061 T6 alloy of $6.4 \times 10^{-5}\ m^2s^{-1}$. This suggests the aluminium rod is not pure aluminium and instead an alloy. The value found is still outside of uncertainty of the alloy value, once again due to the difficulty in measuring the peaks. The density of the aluminium rod was found to be $2820 \pm 60\ kgm^{-3}$ which does not agree within uncertainty of the theoretical value of $2700\ kgm^{-3}$ (worldatlas, 2019). The electrical conductivity value of $2.9 \pm 0.1 \times 10^{7}\ \Omega^{-1}m^{-1}$ does not agree within uncertainty of $3.5 \times 10^{7}\ \Omega^{-1}m^{-1}$ $(\mathrm{thebalance}, 2019)$. The heat capacity value of $0.92 \pm 0.09\ Jg^{-1}°\mathrm{C}^{-1}$ agrees within uncertainty of $0.9\ Jg^{-1}°\mathrm{C}^{-1}$ $(\mathrm{chem.libretexts}, 2019)$. The majority of these value not agreeing with values fo pure aluminium suggest the rod used was an alloy of aluminium and not pure.

When testing the Wiedemann-Franz relation the lorenz number found for the copper rod was $2.12 \pm 0.4 \times 10^{-8}\ W\Omega K^{-2}$, which is within uncertainty of the theoretical value of $2.23 \times 10^{-8}\ W\Omega K^{-2}$ (RNave, 2019). The value found for the aluminium rod was $1.78 \pm 0.3 \times 10^{7}\ W\Omega K^{-2}$, which is not within uncertainty of the theoretical value. The lorenz number is based on the metal and its temperature. Aluminium is one of the few metals where the lorenz number decreases with temperature so this could be a reason for a number lower than expected (ipfs, 1990).

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