X : Banach , M : finite dimensional subspace of X 22. Oct. 2015

Example: $X := C^0([0,1])$, $\| \cdot \|_X := \| \cdot \|_\infty$

$$M := \mathbb{P}^n : \quad \dim(M) = n+1$$

$$\Rightarrow \mathbb{P}^n = \operatorname{span}\{x^i\}_{i=0}^n$$

Interpolation: Linear Operator on $X \longmapsto M$

Given $\{x_i\}_{i=0}^n$

$$\mathcal{L}p = p \quad \text{for all } p \in \mathbb{P}^n \equiv M$$

$$\mathcal{L}f := e^i(f) e_i(x) = \sum_{i=0}^n f(x_i)\, e_i(x)$$ example

$$e^i \in X^*, \quad e_i \in M$$

Linear function from $X \to \mathbb{R}$ — "Alternative" basis for $\mathbb{P}^n$

$$\mathcal{L}f := e^i(f)\, e_i(x) \qquad M := \operatorname{span}\{e_i(x)\}_{i=0}^n$$

$$e^i(e_j) := \delta^i_j \qquad M^* := \operatorname{span}\{e^i\}_{i=0}^n$$

→ Lagrange interpolation

Definition := $(\mathcal{L}f)(x_i) = f(x_i)$ Given $x_0 < x_1 < \dots < x_n$

The choice of the basis determines the algorithm

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In general, given a basis: $\underline{e_i(x)}$

$$\mathcal{L} f := e^i(f)\, e_i(x)$$

$$(\mathcal{L} f)(x_i) = f(x_i) \Rightarrow e^j(f)\, e_j(x_i) = f(x_i)$$

Define $\underline{\underline{e^i(f) =: f^i}}$ $\Rightarrow$ $e_J(x_i)\, f^J = f(x_i)$

$$\underline{\underline{B_{iJ} f^J = f(x_i)}}$$

$$(\mathcal{L} f)(x) = \underline{\underline{f^J}}\ \underline{\underline{e_J(x)}}$$

When is the algorithm well posed?

$\Rightarrow$ condition number of $\underline{\underline{B}}$:

B identity $\Rightarrow \delta_{iJ} f^J = f^i = f(x_i) =: \underline{\underline{e^i(f)}}$

$\Rightarrow$ $e^i(f)$ is the point evaluation of f at x_i