Task 1

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Consider a following model:

$$f(x_1, x_2) = (x_1 + x_2)^2$$

Assume that $x_1, x_2 \sim \mathrm{U}[-1,1]$ and $x_1 = x_2$ (full dependency).
Calculate PD profile for variable x_1 in this model.

$$g^1_{PD}(z) = \mathbb{E}_{x_2 \sim \mathrm{U}[-1,1]} f(z, x_2) = \mathbb{E}_{x_2 \sim \mathrm{U}[-1,1]} (z + x_2)^2 =$$

$$= z^2 + 2z\mathbb{E}_{x_2 \sim \mathrm{U}[-1,1]} x_2 + \mathbb{E}_{x_2 \sim \mathrm{U}[-1,1]} x_2^2 = z^2 + 2z \int_{-1}^{1} \frac{x_2}{2} \mathrm{d}x_2 + \int_{-1}^{1} \frac{x_2^2}{2} \mathrm{d}x_2 =$$

$$= z^2 + \frac{1}{2} z(1-1) + \frac{1}{6}(1+1) = z^2 + \frac{1}{3}$$

Calculate ME and ALE profiles for variable x_1 in this model.

$$g^1_{ME}(z) = \mathbb{E}_{x_2|x_1=z} f(z, x_2) = \mathbb{E}_{x_2|x_1=z} (z + x_2)^2 =$$

$$= z^2 + 2z\mathbb{E}_{x_2|x_1=z} x_2 + \mathbb{E}_{x_2|x_1=z} x_2^2 = z^2 + 2z^2 + z^2 = 4z^2$$

$$g^1_{ALE}(z) = \int_{1}^{z} \mathbb{E}_{x_2|x_1=v} \frac{\partial (x_1 + x_2)^2}{\partial x_1} \mathrm{d}v = \int_{1}^{z} \mathbb{E}_{x_2|x_1=v} (2x_1 + 2x_2) \mathrm{d}v =$$

$$= 4 \int_{1}^{z} v \mathrm{d}v = 2(z^2 - 1)$$