# Statistics and Data Analysis Assignment

Domen Mohorčič, Larsen Cundrič

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## INTRODUCTION

Our goal was to…

## ROLLER COASTERS DATASET

The dataset Coaster2015 presents data from various roller coasters across the globe. It has 16 attributes: name, park city, state, country, type, construction, height, speed, length, inversions, numinversions, duration, geforce, opened and region.

### Summary Statistics

```
##
## -- Column specification --------------------------------------------------------
## cols(
##   Name = col_character(),
##   Park = col_character(),
##   City = col_character(),
##   State = col_character(),
##   Country = col_character(),
##   Type = col_character(),
##   Construction = col_character(),
##   Height = col_double(),
##   Speed = col_double(),
##   Length = col_double(),
##   Inversions = col_character(),
##   Numinversions = col_double(),
##   Duration = col_double(),
##   GForce = col_double(),
##   Opened = col_double(),
##   Region = col_character()
## )
```

Coaster2015 dataset has 408 instances. As we can see, some values are missing. Attributes name, park city, state, country, type, construction and regian are categorical, while others are numerical. Where type and construction are basically the same attributes as seen later on…

```r
# GForce to many missing values..
summary(roller_coasters_raw)
```

```
##      Name               Park               City              State          
##  Length:408         Length:408         Length:408         Length:408        
##  Class :character   Class :character   Class :character   Class :character  
##  Mode  :character   Mode  :character   Mode  :character   Mode  :character  
##                                                                             
##                                                                             
##                                                                             
##                                                                             
##    Country              Type           Construction           Height       
##  Length:408         Length:408         Length:408         Min.   :  2.438  
##  Class :character   Class :character   Class :character   1st Qu.:  8.651  
##  Mode  :character   Mode  :character   Mode  :character   Median : 18.288  
##                                                           Mean   : 23.125  
##                                                           3rd Qu.: 33.167  
##                                                           Max.   :128.016  
##                                                           NA's   :82       
##      Speed            Length          Inversions        Numinversions    
##  Min.   :  9.72   Min.   :  12.19   Length:408         Min.   : 0.0000  
##  1st Qu.: 45.00   1st Qu.: 291.00   Class :character   1st Qu.: 0.0000  
##  Median : 68.85   Median : 415.75   Mode  :character   Median : 0.0000  
##  Mean   : 69.36   Mean   : 597.04                      Mean   : 0.7843  
##  3rd Qu.: 88.95   3rd Qu.: 833.12                      3rd Qu.: 0.0000  
##  Max.   :194.40   Max.   :2243.02                      Max.   :10.0000  
##  NA's   :138      NA's   :90                                            
##     Duration         GForce          Opened        Region         
##  Min.   :  0.3   Min.   :2.100   Min.   :1924   Length:408        
##  1st Qu.: 75.0   1st Qu.:3.175   1st Qu.:1991   Class :character  
##  Median :108.0   Median :4.500   Median :1999   Mode  :character  
##  Mean   :112.5   Mean   :4.115   Mean   :1995                     
##  3rd Qu.:140.8   3rd Qu.:5.000   3rd Qu.:2004                     
##  Max.   :300.0   Max.   :6.200   Max.   :2014                     
##  NA's   :216     NA's   :348     NA's   :28                       
```

Lets have a look at the categorical variables first. We will skip the Name and Park since they have too many unique values:

```r
length(unique(roller_coasters_raw$Name))
```

```
## [1] 339
```

```r
length(unique(roller_coasters_raw$Park))
```

```
## [1] 168
```

```r
length(unique(roller_coasters_raw$City))
```

```
## [1] 150
```

As for the rest, look at the summary below:

```r
table(roller_coasters_raw$Country)
```

```
##
##   AR  BR  CL  CO  CR   D  EQ   F  GT  MX  PE  US  VE
##   10  19   3   7   5  82   2  44   3  17   2 213   1
```

```r
table(roller_coasters_raw$State)
```

```
##
## AR BR CA CL CO CR  D EQ  F GT IL IN MX OH OR PE TX VE WA
## 10 19 77  3 21  5 82  2 44  3 18 13 17 37  4  2 38  1 12
```

```r
table(roller_coasters_raw$Type) # same as construction
```

```
##
##   S   W
## 366  42
```

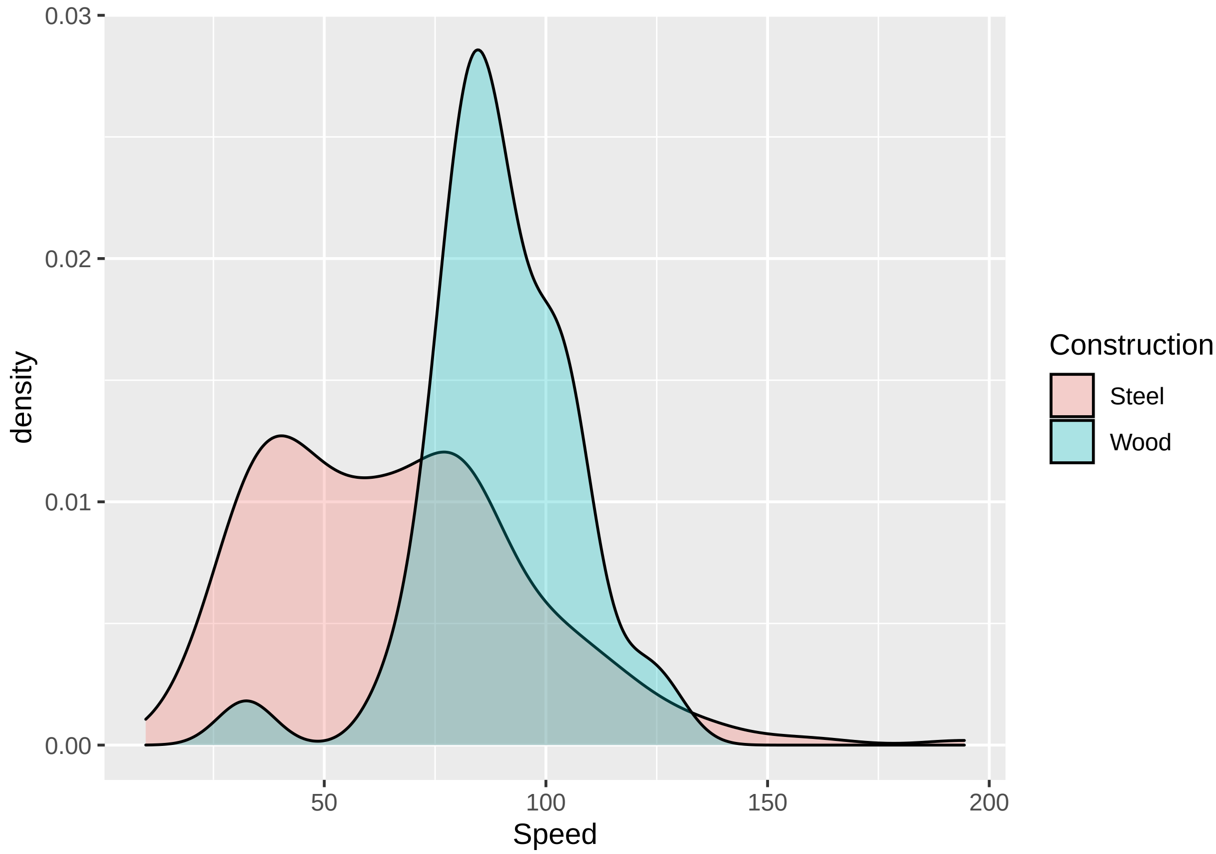
```r
table(roller_coasters_raw$Construction)
```

```
##
## Steel  Wood
##   366    42
```

As for the numerical ones, we are generally most interested in speed. So we present most data relative to the speed of coasters. The speed is measured in milles per hour (mph), and its distribution relative to Construction can be seen here:

```r
roller_coasters_raw %>% ggplot()+
  geom_density(aes(x = Speed, fill = Construction), alpha = 0.3)
```

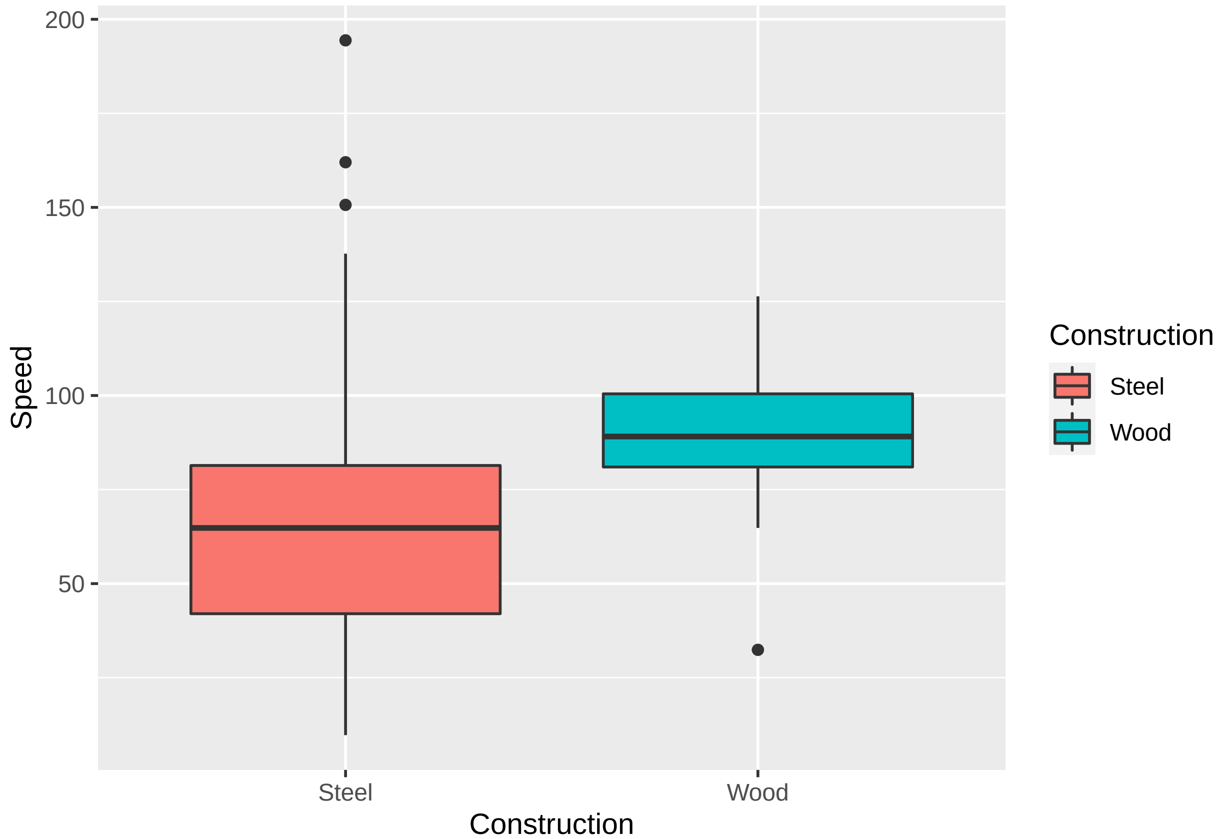
```
## Warning: Removed 138 rows containing non-finite values (stat_density).
```




We present the same data on a boxplot:

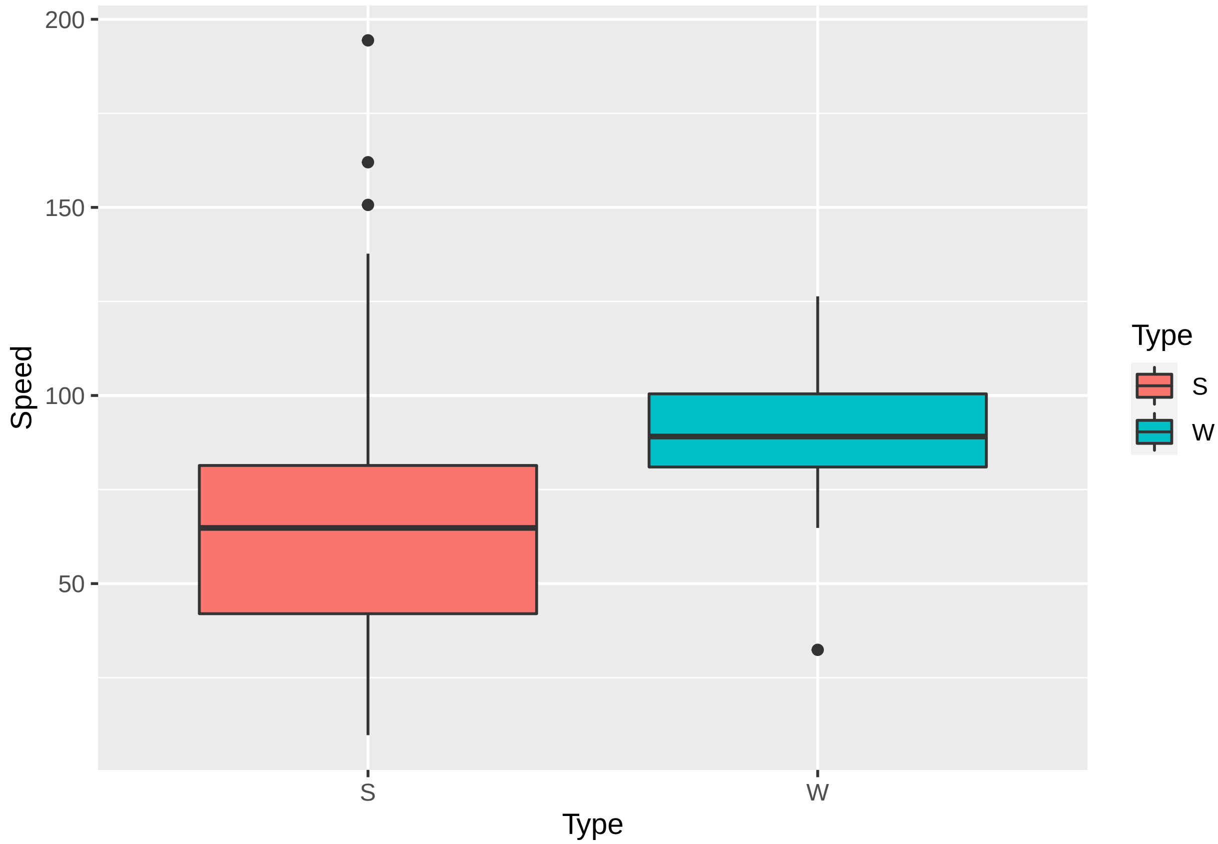
```
ggplot(data = roller_coasters_raw) +
  geom_boxplot(mapping = aes(x = Construction, y = Speed, fill = Construction))
```

```
## Warning: Removed 138 rows containing non-finite values (stat_boxplot).
```




```r
ggplot(data = roller_coasters_raw) +
  geom_boxplot(mapping = aes(x = Type, y = Speed, fill = Type))
```

```
## Warning: Removed 138 rows containing non-finite values (stat_boxplot).
```

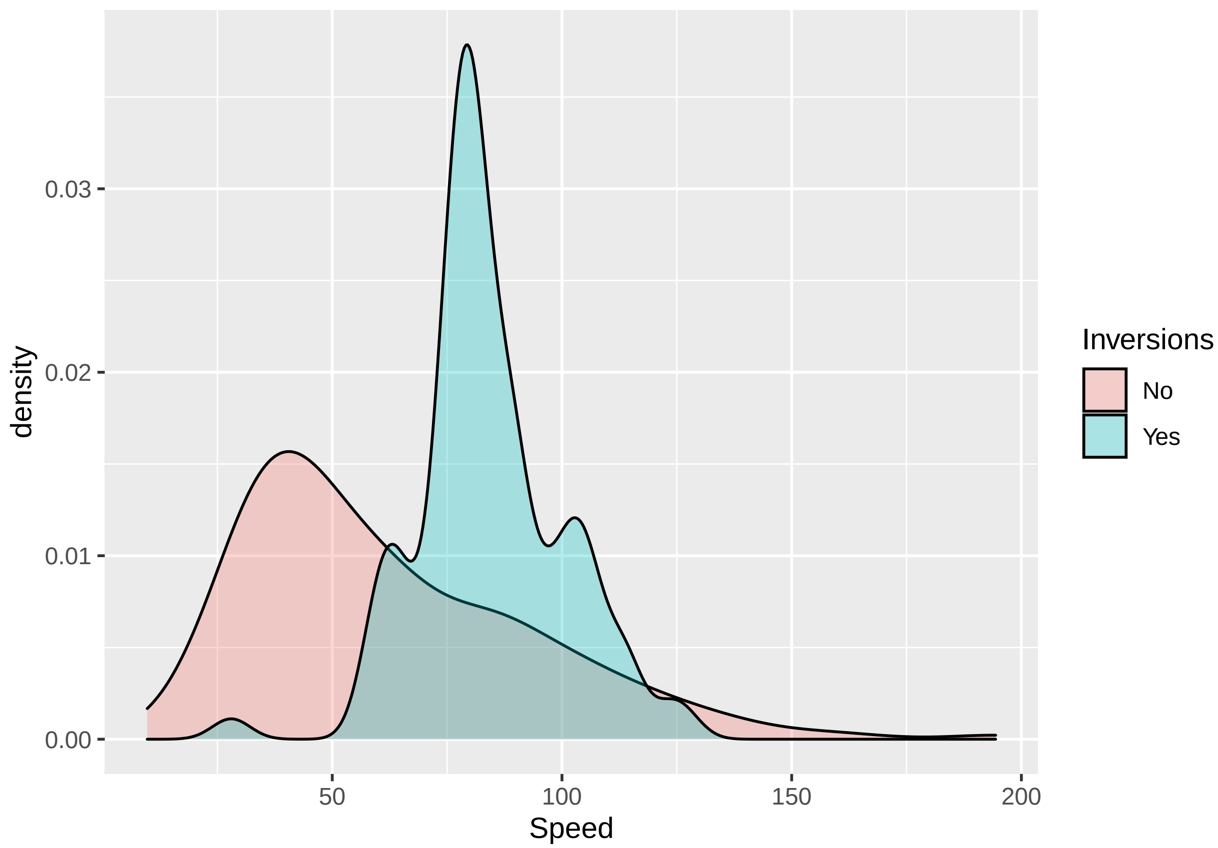



From the density plot and boxplot we can observe that wooden coaster are on average faster than the steel ones. This will also be one of the hypothesis tests later on to confirm our observations. Also note, that Type and Construction are the same attributes.

Inversions also present some interesting data. When we have inversions we tend to have higher speeds as shown below on a density plot and box plot:

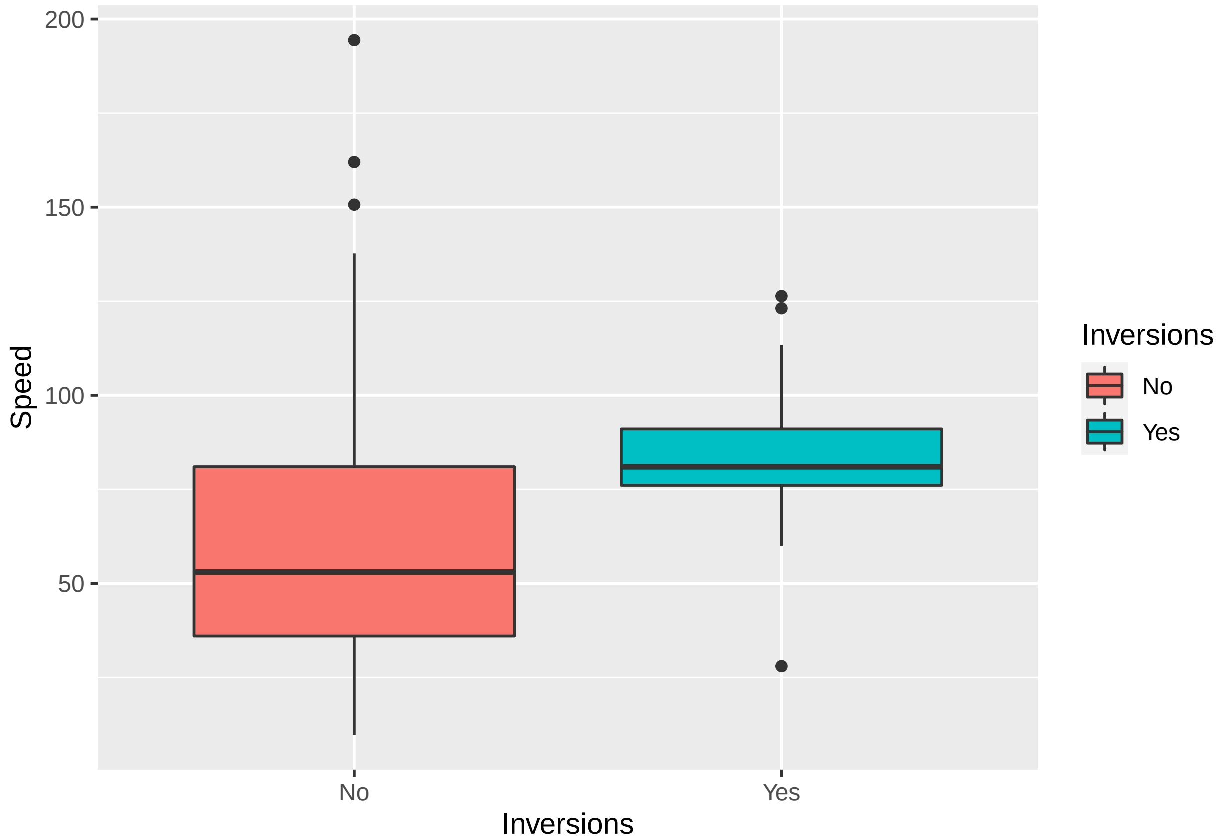
```
roller_coasters_raw %>% ggplot()+
  geom_density(aes(x = Speed, fill = Inversions), alpha = 0.3)
```

```
## Warning: Removed 138 rows containing non-finite values (stat_density).
```




```
ggplot(data = roller_coasters_raw) +
  geom_boxplot(mapping = aes(x = Inversions, y = Speed, fill = Inversions))
```

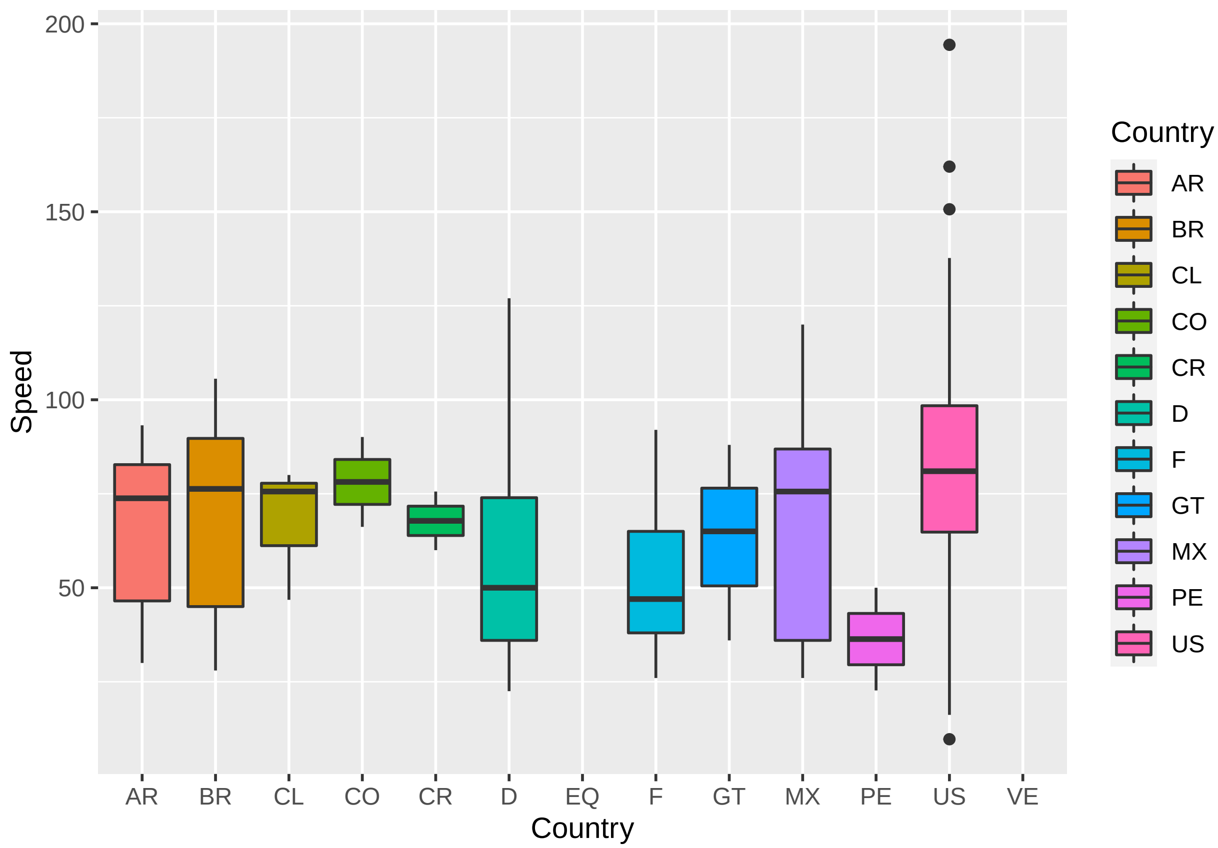
```
## Warning: Removed 138 rows containing non-finite values (stat_boxplot).
```




Last but not least, we compared the Countries and saw averages move from 50 to 75 mph, where US has the highest average:

```
ggplot(data = roller_coasters_raw) +
  geom_boxplot(mapping = aes(x = Country, y = Speed, fill = Country))
```

```
## Warning: Removed 138 rows containing non-finite values (stat_boxplot).
```




We also tested symmetry of Speed distributions for Steel and wood constructions:

```
symmetry.test(roller_coasters_raw$Speed)
```

```
##
##  m-out-of-n bootstrap symmetry test by Miao, Gel, and Gastwirth (2006)
##
## data:  roller_coasters_raw$Speed
## Test statistic = 0.37053, p-value = 0.776
## alternative hypothesis: the distribution is asymmetric.
## sample estimates:
## bootstrap optimal m
##                  52
```

```
symmetry.test(roller_coasters_raw[roller_coasters_raw$Construction == "Steel", ]$Speed)
```

```
##
##  m-out-of-n bootstrap symmetry test by Miao, Gel, and Gastwirth (2006)
##
## data:  roller_coasters_raw[roller_coasters_raw$Construction == "Steel",     ]$Speed
## Test statistic = 1.1388, p-value = 0.394
## alternative hypothesis: the distribution is asymmetric.
## sample estimates:
## bootstrap optimal m
##                  51
```

```r
symmetry.test(roller_coasters_raw[roller_coasters_raw$Construction == "Wood", ]$Speed)
```

```
##
##  m-out-of-n bootstrap symmetry test by Miao, Gel, and Gastwirth (2006)
##
## data:  roller_coasters_raw[roller_coasters_raw$Construction == "Wood",     ]$Speed
## Test statistic = 0.18255, p-value = 0.882
## alternative hypothesis: the distribution is asymmetric.
## sample estimates:
## bootstrap optimal m
##                  17
```

We see that Speed is a symmetric distribution and also both Steel and Wood have symmetric distributions which will help us later in the hypothesis testing.

Other scatterplots are presented in the Regression section…

## Inference and Hypothesis testing

The usual procedure for hypothesis testing is such:

0) Check CLT conditions Central limit theorem:

- Samples are independent,

- Sample size is bigger or equal to 30,
- Population distribution is not strongly skewed.

1) Set-up the hypothesis

2) Assume threshold values

- $\alpha - significancelevel$ - typically 0.05

3) Calculate the Results:

- point est.
- number of cases
- sd - standard deviation
- se - standard error
- df - degrees of freedom $df = n - 1$
- t-statistics
- p-value

4) Draw conclusions - Accept or reject hypothesis

If we meet those criteria, we can infer about the population based on the analysis we do on the sample. We firstly assume that all the instances are independant. We can also see that there are more than enough instances:

```
roller_coasters_raw %>%
  filter(!is.na(Speed)) %>%
  nrow()
```

```
## [1] 270
```

```
roller_coasters_raw %>%
  filter(!is.na(Height)) %>%
  nrow()
```

```
## [1] 326
```

```
roller_coasters_raw %>%
  filter(!is.na(Length)) %>%
  nrow()
```

```
## [1] 318
```

```
roller_coasters_raw %>%
  filter(!is.na(Numinversions)) %>%
  nrow()
```

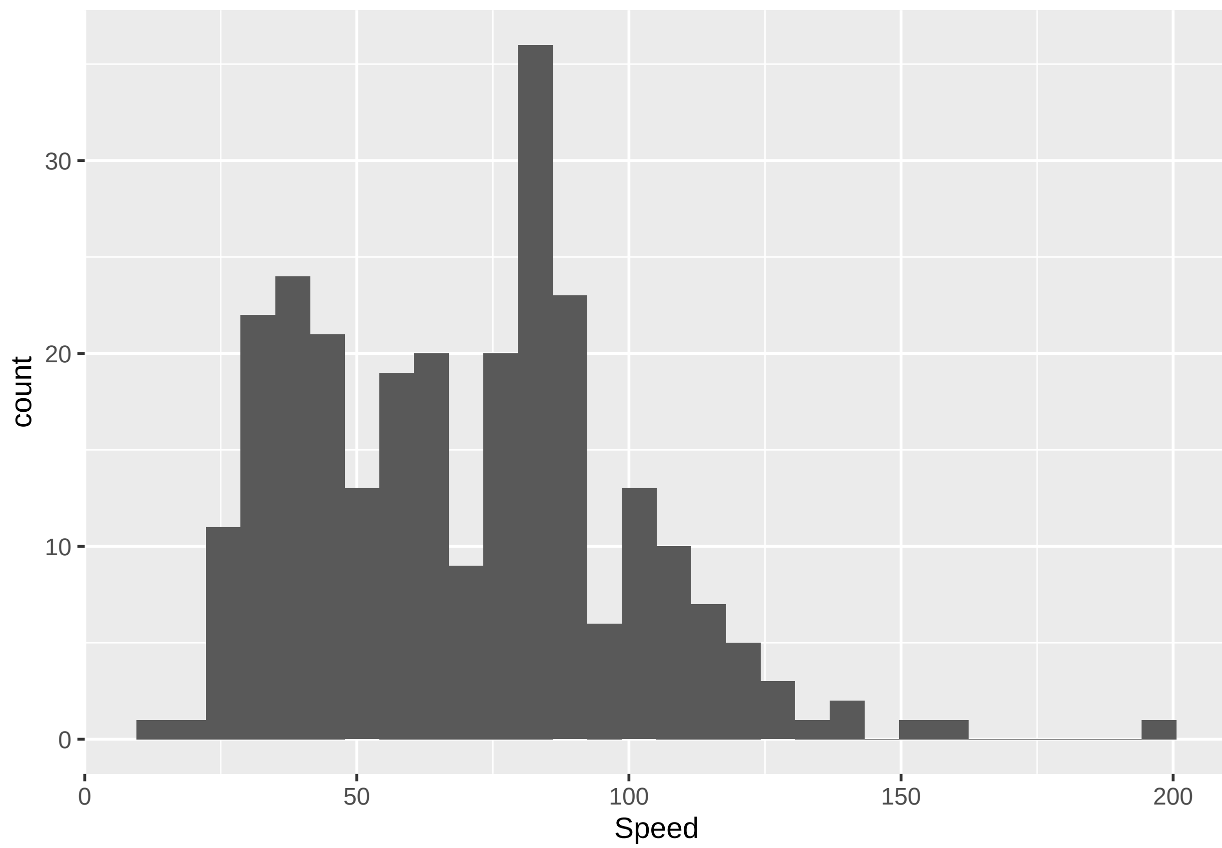
```
## [1] 408
```

Lastly, we want to see if the data is not heavily skewed:

```
ggplot(roller_coasters_raw) +
  geom_histogram(aes(x = Speed))
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

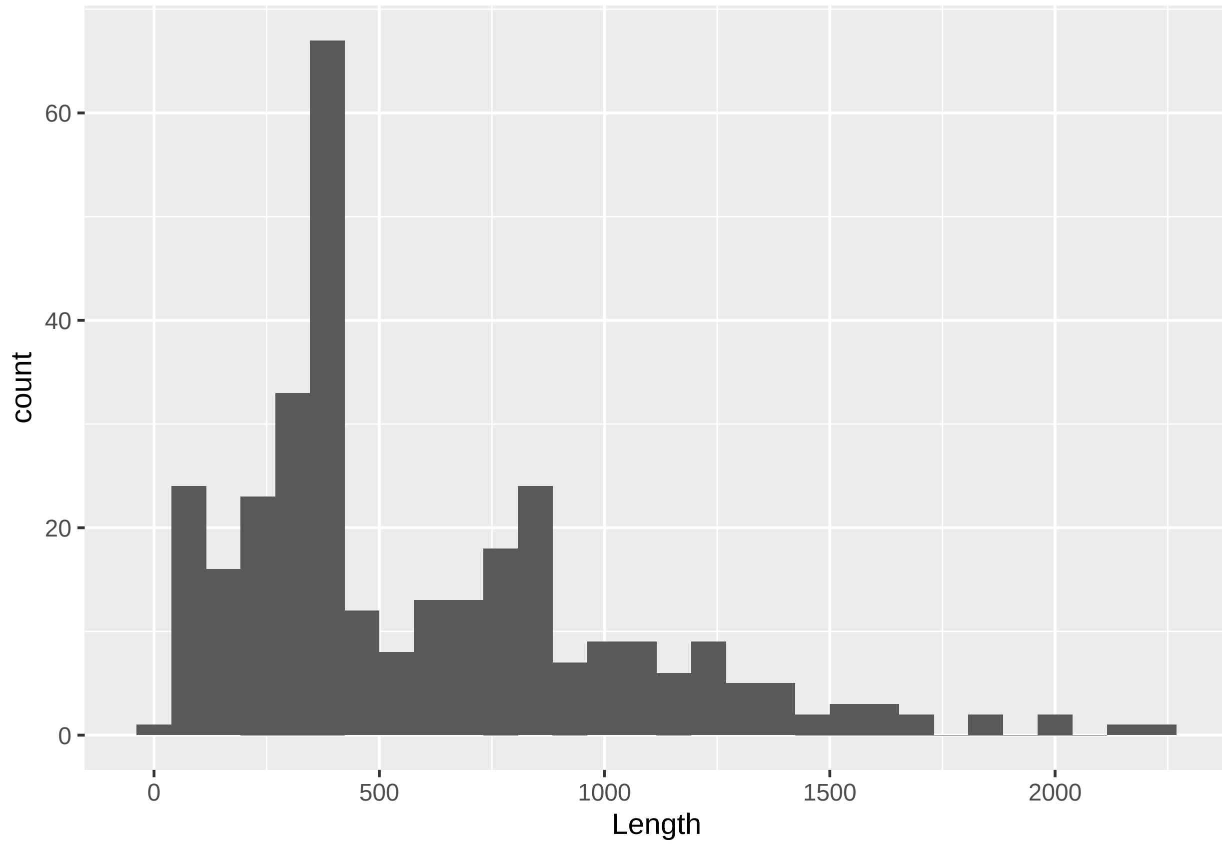
```
## Warning: Removed 138 rows containing non-finite values (stat_bin).
```




```
ggplot(roller_coasters_raw) +
  geom_histogram(aes(x = Length))
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

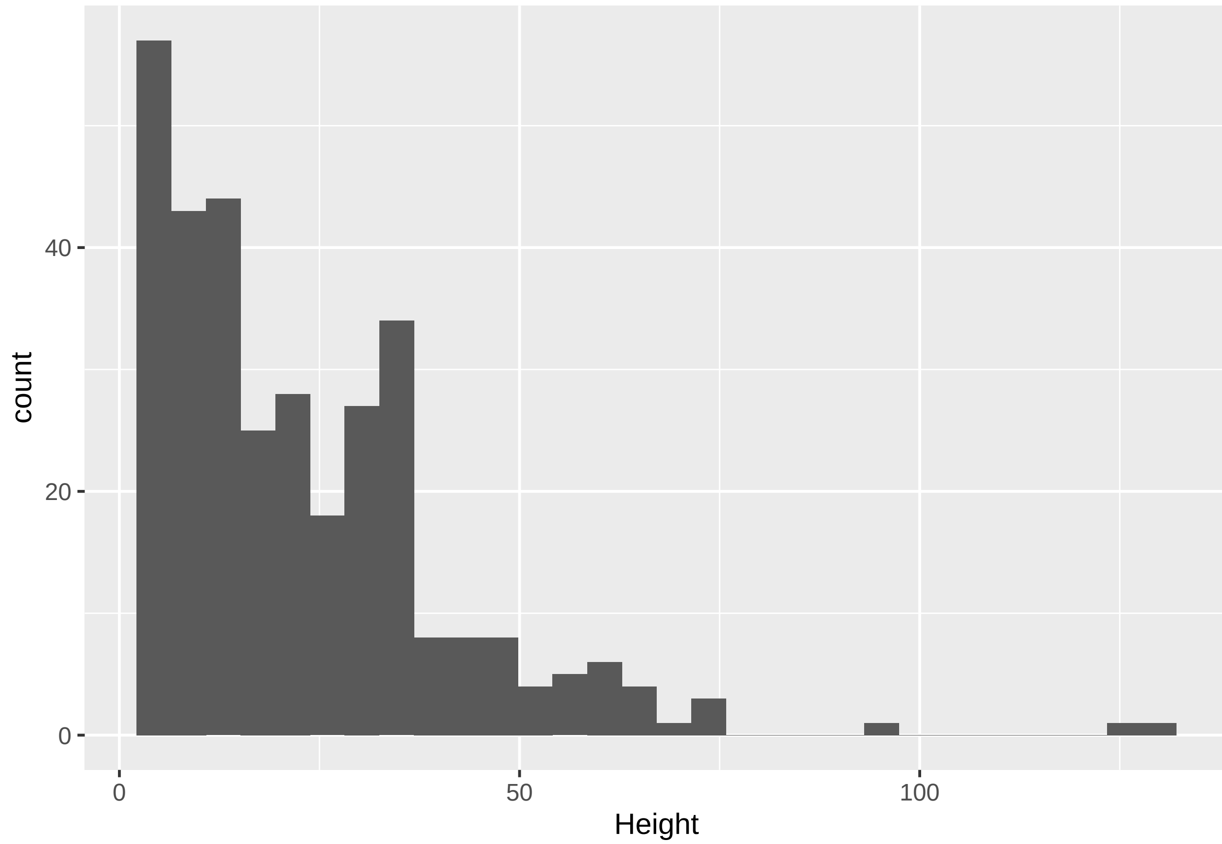
```
## Warning: Removed 90 rows containing non-finite values (stat_bin).
```




```r
ggplot(roller_coasters_raw) +
  geom_histogram(aes(x = Height))
```

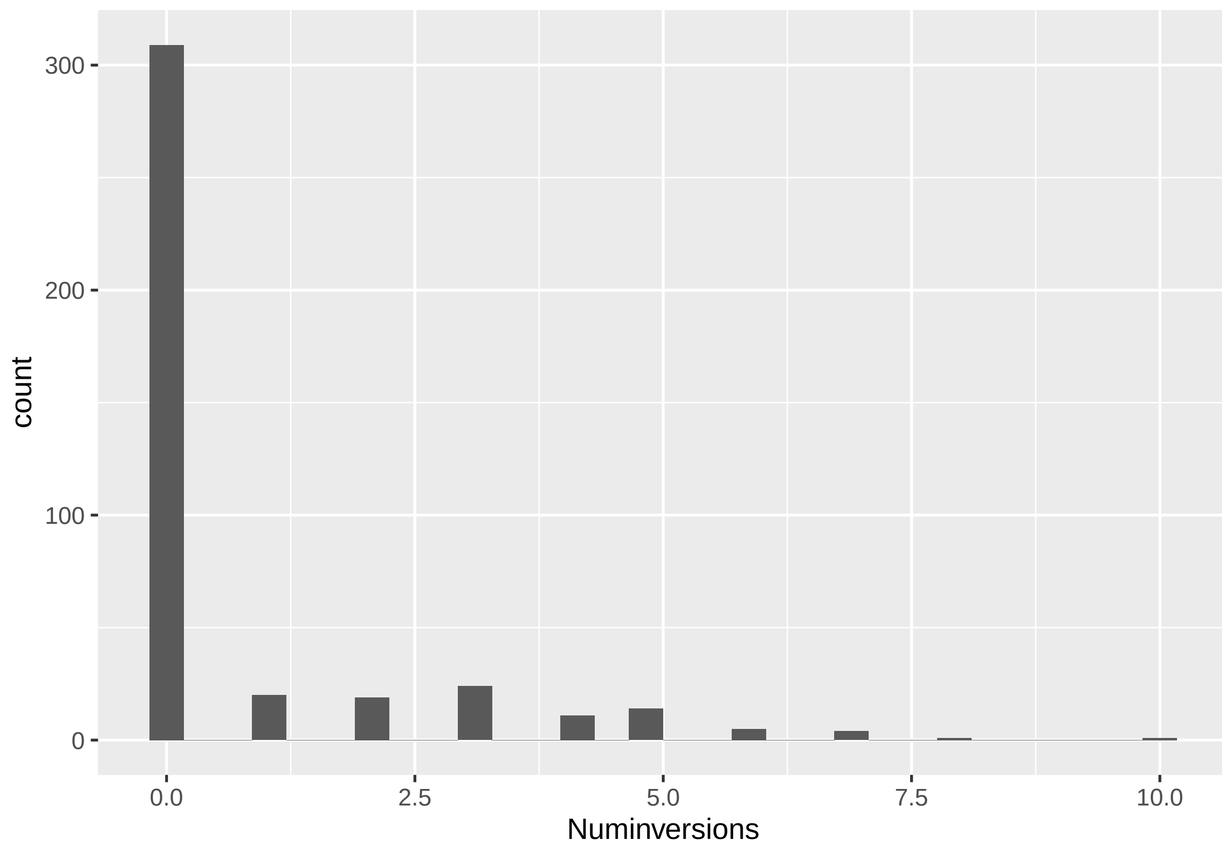
```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

```
## Warning: Removed 82 rows containing non-finite values (stat_bin).
```




```
ggplot(roller_coasters_raw) +
  geom_histogram(aes(x = Numinversions))
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




From the above distributions we can observe that the most suitable distribution to make hypothesis testing on is Speed. And its symmetry is already proven in the summary statistics section…

We can also prove that Height, Length and Numinversions are not normally distributed nor are they symmetric using the symmetry and shapiro test below:

```
(symmetry.test(roller_coasters_raw$Height))
```

```
##
##  m-out-of-n bootstrap symmetry test by Miao, Gel, and Gastwirth (2006)
##
## data:  roller_coasters_raw$Height
## Test statistic = 6.772, p-value < 2.2e-16
## alternative hypothesis: the distribution is asymmetric.
## sample estimates:
## bootstrap optimal m
##                  44
```

```
(shapiro.test(roller_coasters_raw$Height))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  roller_coasters_raw$Height
## W = 0.84671, p-value < 2.2e-16
```

```r
(symmetry.test(roller_coasters_raw$Length))
```

```
##
##  m-out-of-n bootstrap symmetry test by Miao, Gel, and Gastwirth (2006)
##
## data:  roller_coasters_raw$Length
## Test statistic = 10.298, p-value < 2.2e-16
## alternative hypothesis: the distribution is asymmetric.
## sample estimates:
## bootstrap optimal m
##                  61
```

```r
(shapiro.test(roller_coasters_raw$Length))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  roller_coasters_raw$Length
## W = 0.90217, p-value = 1.789e-13
```

```r
(symmetry.test(roller_coasters_raw$Numinversions))
```

```
##
##  m-out-of-n bootstrap symmetry test by Miao, Gel, and Gastwirth (2006)
##
## data:  roller_coasters_raw$Numinversions
## Test statistic = 21.332, p-value < 2.2e-16
## alternative hypothesis: the distribution is asymmetric.
## sample estimates:
## bootstrap optimal m
##                  44
```

```r
(shapiro.test(roller_coasters_raw$Numinversions))
```

```
##
##  Shapiro-Wilk normality test
##
## data:  roller_coasters_raw$Numinversions
## W = 0.54578, p-value < 2.2e-16
```

As such, we are allowed to infere and do hypothesis testing on Speed, since only Speed meets the Limit Theorem requirements…

```r
roller_coasters_speeds <- roller_coasters_raw %>%
  select(Speed) %>%
  filter(!is.na(Speed))
roller_coasters_speeds
```

```
## # A tibble: 270 x 1
##    Speed
```

```
##    <dbl>
##   1  194.
##   2  162
##   3  151.
##   4  138.
##   5  127
##   6  138.
##   7  130.
##   8  120
##   9  126.
## 10  113.
## # ... with 260 more rows
```

**Hypothesis 1 - One sample t-test**

Our hypothesis 1:

$H_0$ : population mean speed is 70mph $\mu = 70$ $H_A$ : population mean speed is not 70mph $\mu \neq 70$

$$\alpha = 0.05$$

Calculate the necessary variables:

```
(point_est_speed <- 70)
```

```
## [1] 70
```

```
(mean_speed <- mean(roller_coasters_speeds$Speed))
```

```
## [1] 69.36267
```

```
(sd_speed <- sd(roller_coasters_speeds$Speed)) # standard deviation
```

```
## [1] 29.32774
```

```
(sem_speed <- sd_speed / nrow(roller_coasters_speeds)) # standard error
```

```
## [1] 0.1086213
```

```
(df_speed <- nrow(roller_coasters_speeds) - 1)
```

```
## [1] 269
```

```
(t_speed <- (point_est_speed-mean_speed) / sem_speed)
```

```
## [1] 5.867482
```

p-value

```
(p_val <- 2*(1- pt(t_speed, df = df_speed)))
```

```
## [1] 1.296661e-08
```

95% confidence intervals

```
#lower limit
# mean_speed - 1.96 * sem_speed
mean_speed + qt(0.025, df = df_speed) * sem_speed
```

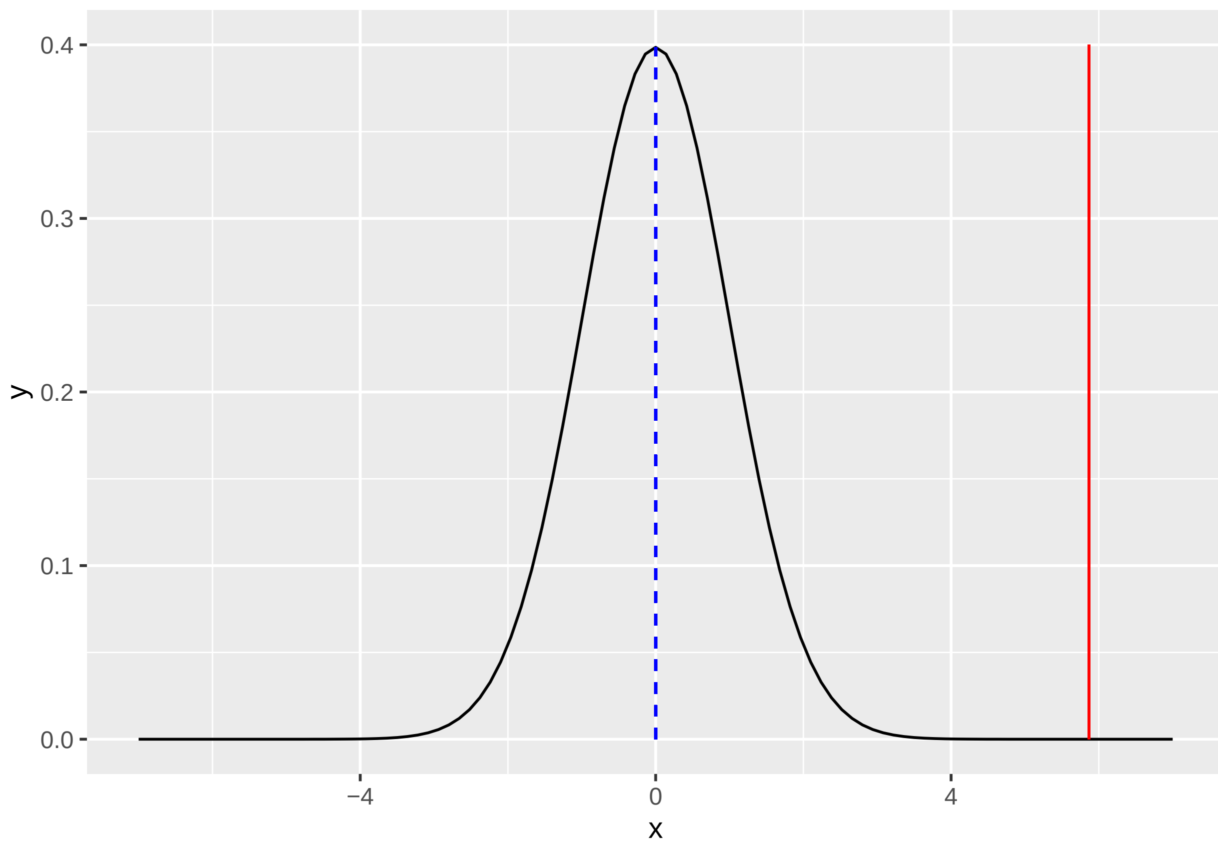
```
## [1] 69.14881
```

```
#upper limit
# mean_speed + 1.96 * sem_speed
mean_speed + qt(0.975, df = df_speed) * sem_speed
```

```
## [1] 69.57652
```

Let's plot our discovery...

```
xframe <- seq(-7, 7, length = 100)
ggplot(data.frame(x = xframe), aes(x = x)) +
  stat_function(fun = dt, args = list(df = df_speed)) +
  geom_segment(aes(x = 0, y = 0, xend = 0, yend = dt(0, df = df_speed)),
               color = 'blue',
               linetype = 'dashed') +
  geom_segment(aes(x = t_speed, y = 0, xend = t_speed, yend = 0.4),
               color = 'red')
```




We reject the null hypothesis in favor of the alternative. Mean roller coaster speed is not 70mph!

**Hypothesis 2 - Difference of two means t-test**

We want to check if the Wooden roller coasters are on average faster that the Steel ones.

```
roller_coasters_steel <- roller_coasters_raw %>%
  filter(Construction == "Steel" & !is.na(Speed))

roller_coasters_wood <- roller_coasters_raw %>%
  filter(Construction == "Wood" & !is.na(Speed))
```

Check number of instances:

```
nrow(roller_coasters_steel)
```

```
## [1] 236
```

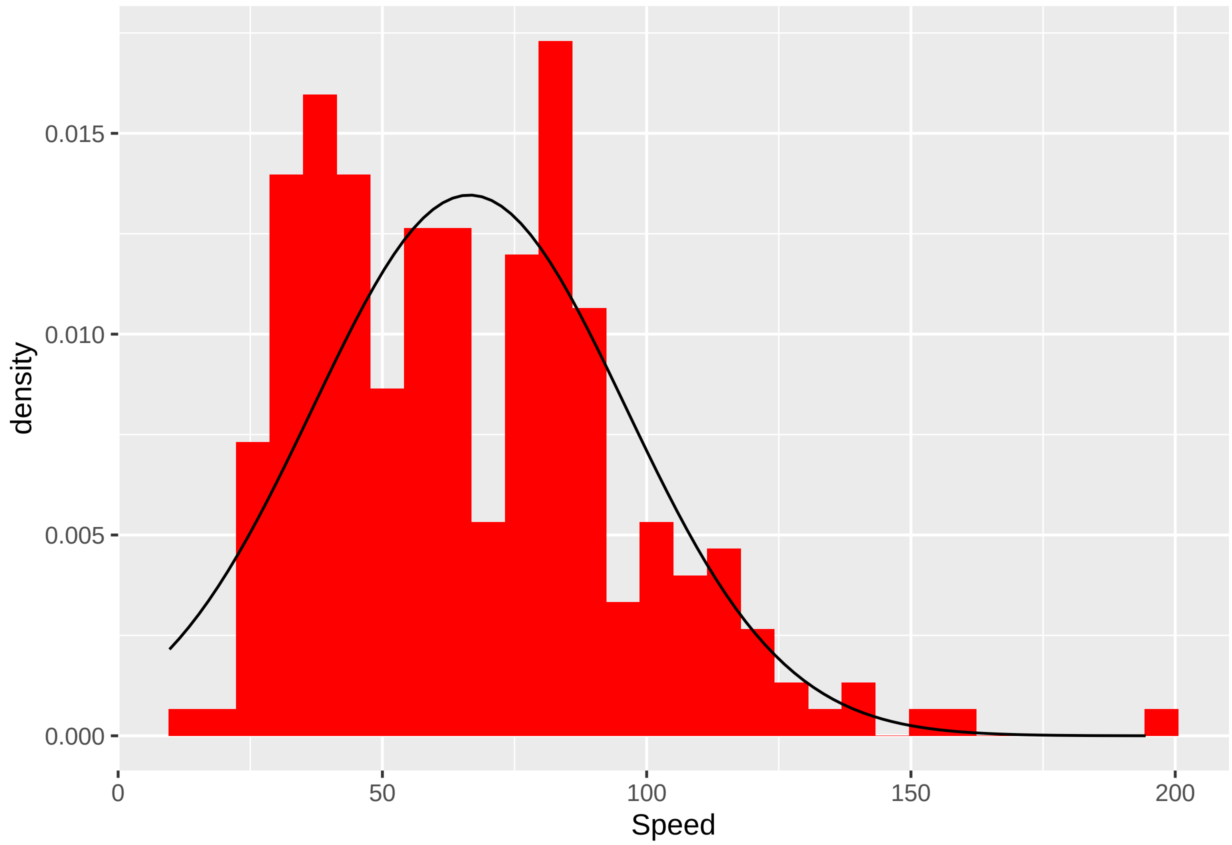
```
nrow(roller_coasters_wood)
```

```
## [1] 34
```

Although already proven with symmetry test in the summary statistics, let's have a look at our distribution plots and their skewness:

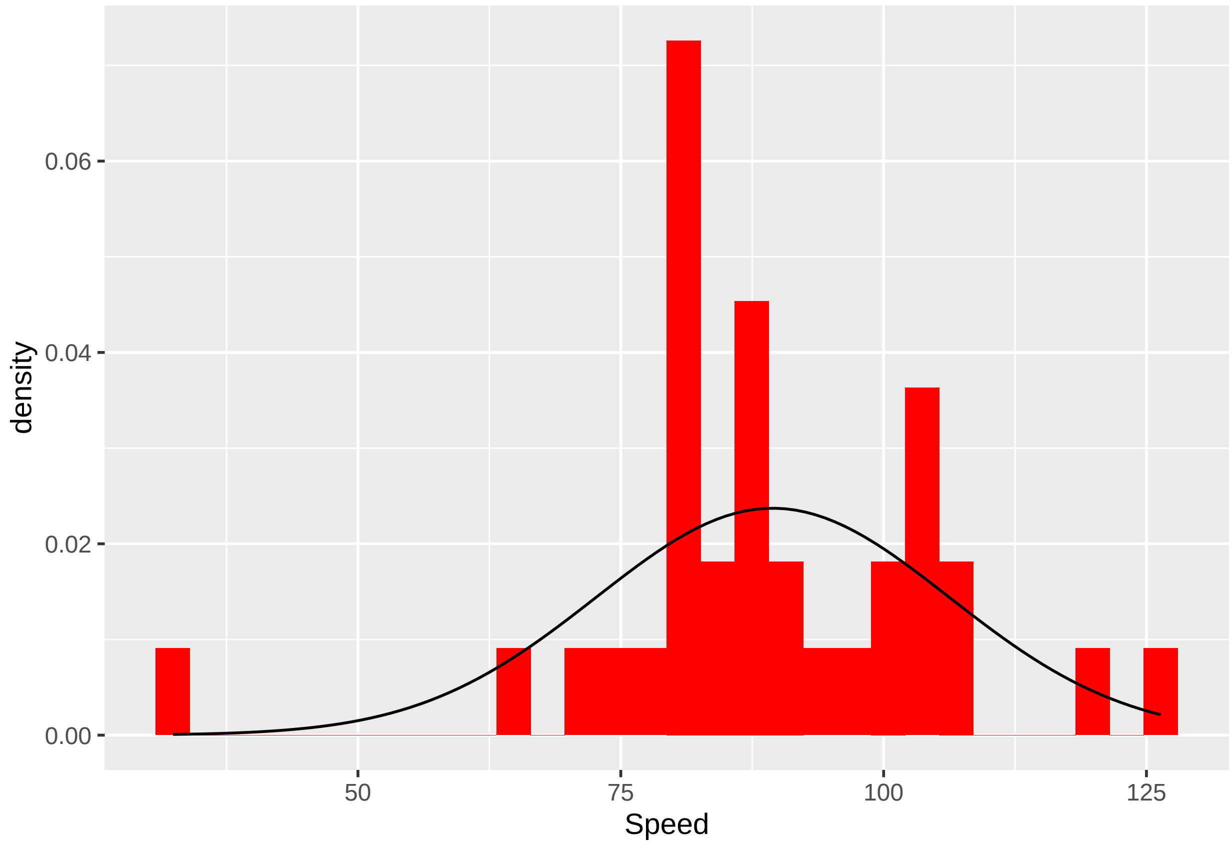
```
ggplot(roller_coasters_steel) +
  geom_histogram(aes(x = Speed, y = ..density..), fill ='red') +
  stat_function(fun = dnorm, args = list(mean = mean(roller_coasters_steel$Speed), sd = sd(roller_coast
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```
ggplot(roller_coasters_wood) +
  geom_histogram(aes(x = Speed, y = ..density..), fill ='red') +
  stat_function(fun = dnorm, args = list(mean = mean(roller_coasters_wood$Speed), sd = sd(roller_coaste
```

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




This is enough to assume we can proceede with our hypothesis testing.

Our hypothesis 2:

$$H_O : mean_{Wood} - mean_{Steel} = 0$$
$$H_A : mean_{Wood} - mean_{Steel} \neq 0$$

$$\alpha = 0.05$$

Calculate necessary variables:

```
(point_est_const <- mean(roller_coasters_wood$Speed) - mean(roller_coasters_steel$Speed))
```

```
## [1] 22.98329
```

```
# (sample_sd <- sd(kiwi_gs_m$height_cm))
(SE <- sqrt((sd(roller_coasters_wood$Speed)^2/nrow(roller_coasters_wood)) + sd(roller_coasters_steel$Spe
```

```
## [1] 3.470155
```

```
(df <- nrow(roller_coasters_wood) - 1) # less
```

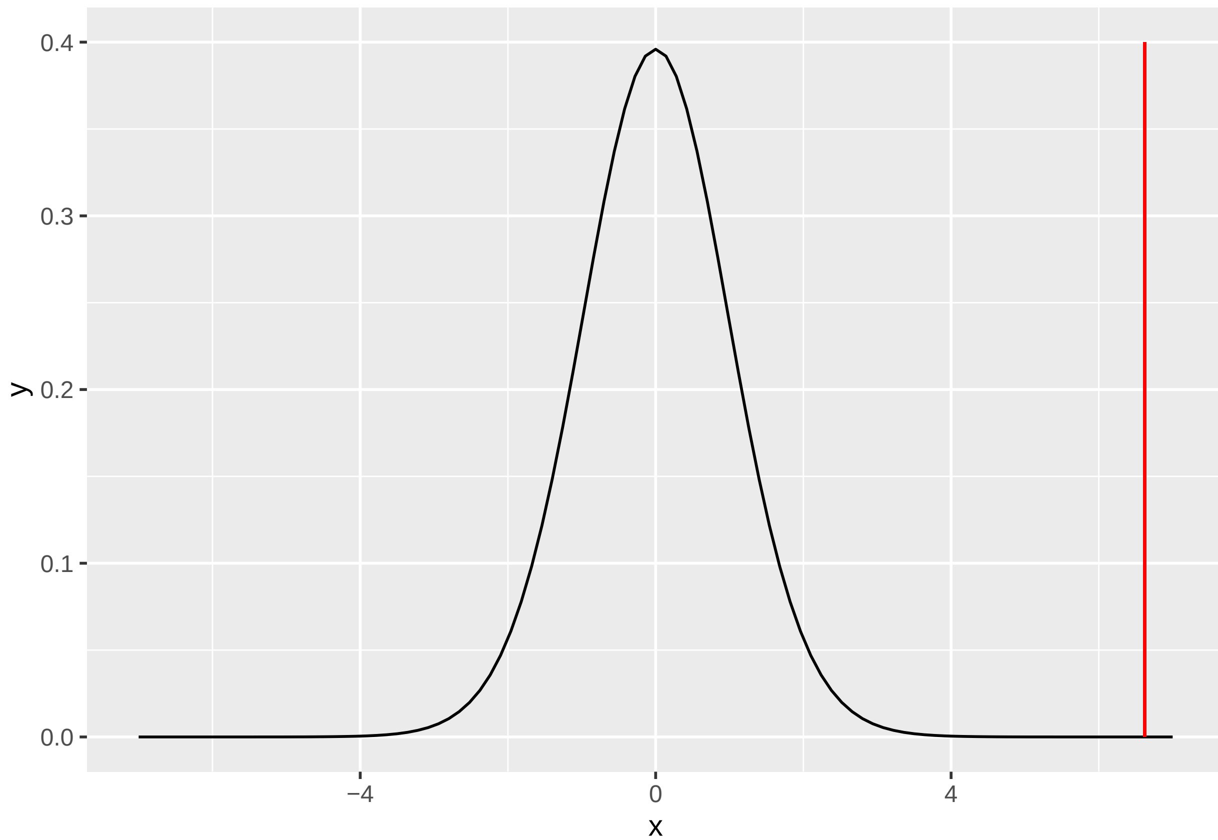
```
## [1] 33
```

```
(t_stat_const <- (point_est_const - 0) / SE) # t-score!
```

```
## [1] 6.62313
```

Plot our finding:

```
ggplot(data.frame(x = seq(-7, 7, length = 100)), aes(x = x)) +
  stat_function(fun = dt, args = list(df = df)) +
  geom_segment(aes(x = t_stat_const, y = 0, xend = t_stat_const, yend = 0.4), color = 'red')
```




p-value:

```
(p_val <- 2 * (1 - pt(t_stat_const, df)))
```

```
## [1] 1.560164e-07
```

We reject the NULL hypothesis in favour of the alternative. The difference in means is significant and wooden roller coasters go faster on average.

### Regression Analysis

Our goal is to make a linear regression model for prediction of a coasters Speed attribute.

### Correlation Analysis

Let's have a look at the correlations (Pearson) and see which are the best candidates.

```
# precej zanimivi so Height, Length, Numinversions
(cor.test(roller_coasters_raw$Height, roller_coasters_raw$Speed))
```

```
##
##  Pearson's product-moment correlation
##
## data:  roller_coasters_raw$Height and roller_coasters_raw$Speed
## t = 38.222, df = 256, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.9019179 0.9388051
## sample estimates:
##       cor
## 0.9224392
```

```
(cor.test(roller_coasters_raw$Length, roller_coasters_raw$Speed))
```

```
##
##  Pearson's product-moment correlation
##
## data:  roller_coasters_raw$Length and roller_coasters_raw$Speed
## t = 15.582, df = 258, p-value < 2.2e-16
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.6278199 0.7540719
## sample estimates:
##       cor
## 0.6962931
```

```
(cor.test(roller_coasters_raw$Numinversions, roller_coasters_raw$Speed))
```

```
##
##  Pearson's product-moment correlation
##
## data:  roller_coasters_raw$Numinversions and roller_coasters_raw$Speed
## t = 5.5742, df = 268, p-value = 6.061e-08
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.2110692 0.4253337
## sample estimates:
##       cor
## 0.3223236
```

```
(cor.test(roller_coasters_raw$Duration, roller_coasters_raw$Speed))
```

```
##
##  Pearson's product-moment correlation
```

```
##
## data:  roller_coasters_raw$Duration and roller_coasters_raw$Speed
## t = 3.9954, df = 162, p-value = 9.781e-05
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.1532868 0.4328823
## sample estimates:
##       cor
## 0.2995011
```

```
(cor.test(roller_coasters_raw$GForce, roller_coasters_raw$Speed))
```

```
##
##  Pearson's product-moment correlation
##
## data:  roller_coasters_raw$GForce and roller_coasters_raw$Speed
## t = 3.3676, df = 56, p-value = 0.001377
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.1701111 0.6045861
## sample estimates:
##       cor
## 0.4103754
```

```
(cor.test(roller_coasters_raw$Opened, roller_coasters_raw$Speed)) # not good
```

```
##
##  Pearson's product-moment correlation
##
## data:  roller_coasters_raw$Opened and roller_coasters_raw$Speed
## t = 0.26238, df = 260, p-value = 0.7932
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  -0.1051251  0.1371870
## sample estimates:
##        cor
## 0.01626982
```

We see that all are suitable for predition only Opened is not…

From the pairplot below, we can observe that there are some linear or non linear relationships between length, height and speed:

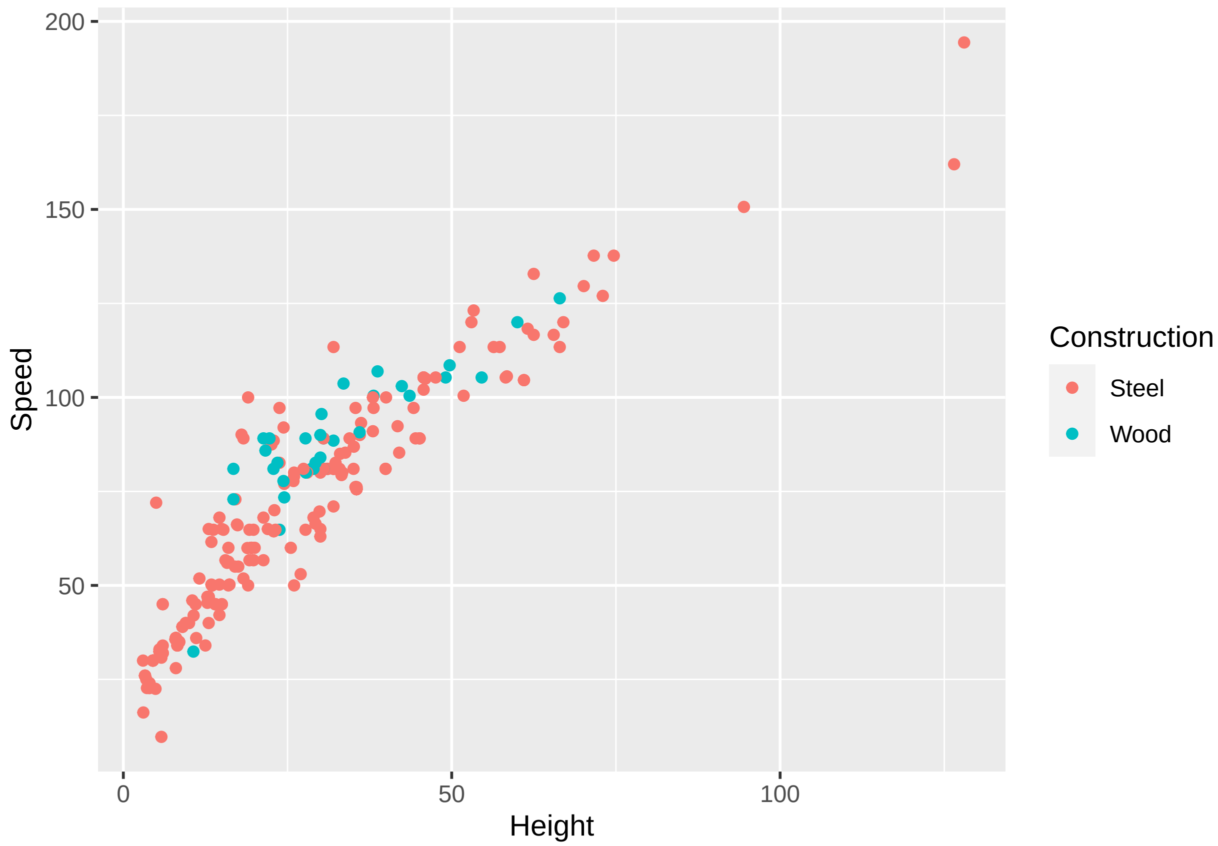
```
# pairs(roller_coasters, lower.panel = NULL)
```

### Regressoin Plots

To make sure we get the right attributes for our regression prediction of speed we wanted to take a look at the regression plots:

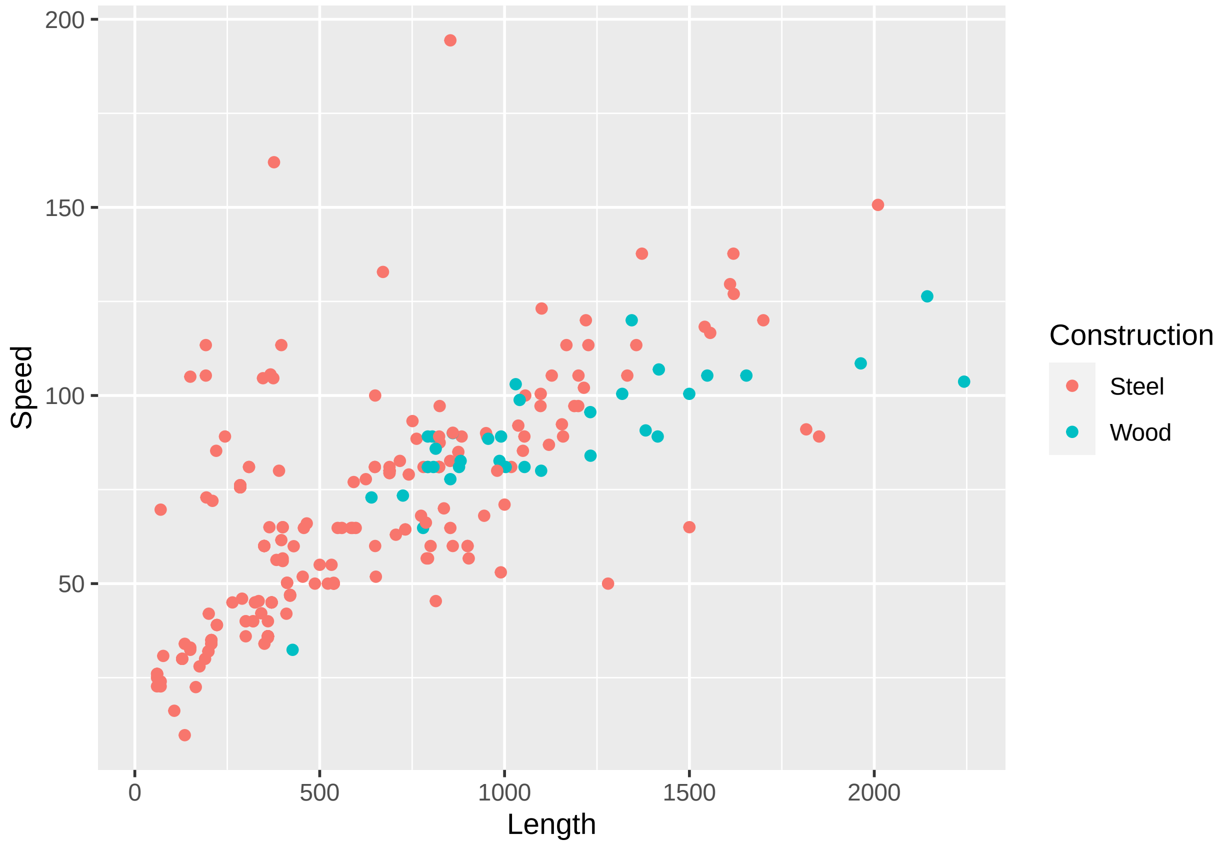
```r
roller_coasters_raw %>%
  ggplot() +
    geom_point(aes(x = Height, y = Speed, color = Construction))
```

```
## Warning: Removed 150 rows containing missing values (geom_point).
```



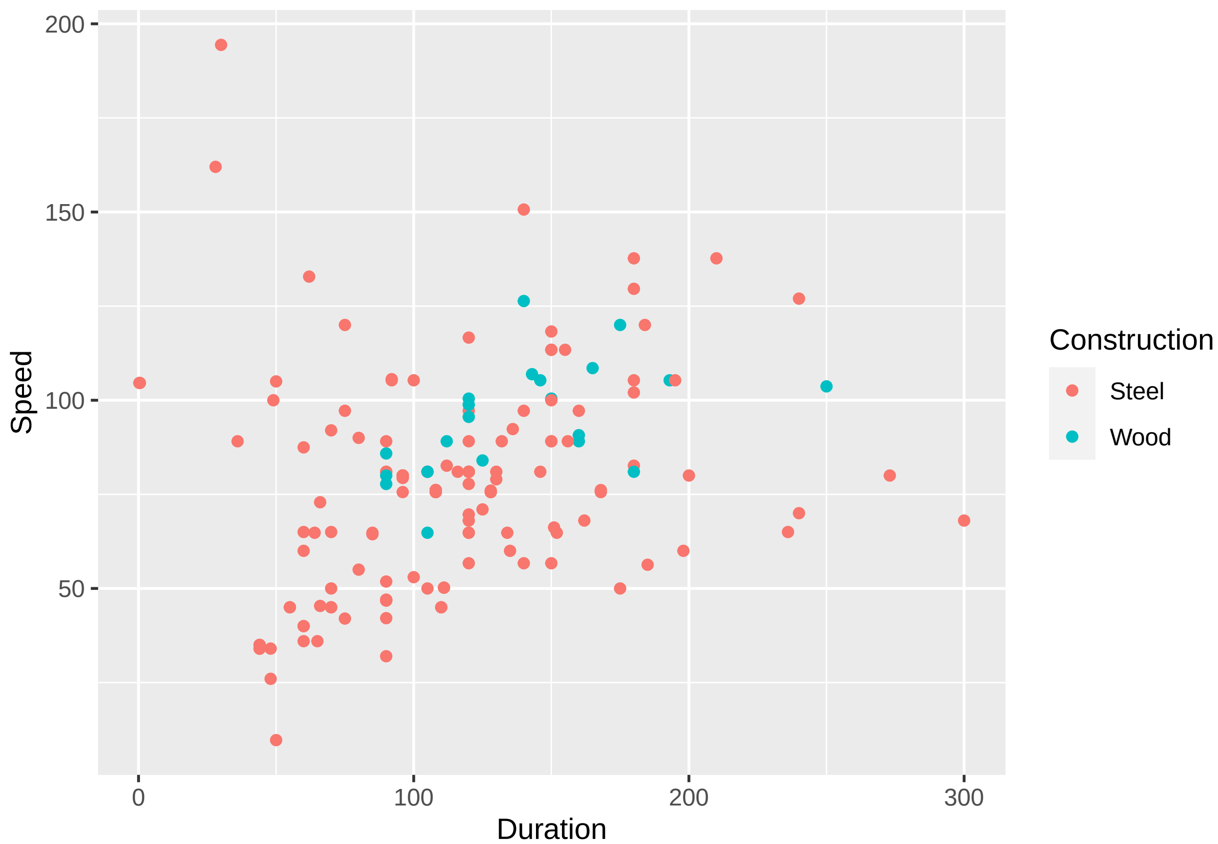

```r
roller_coasters_raw %>%
  ggplot() +
    geom_point(aes(x = Length, y = Speed, color = Construction))
```

```
## Warning: Removed 148 rows containing missing values (geom_point).
```




```
roller_coasters_raw %>%
  ggplot() +
    geom_point(aes(x = Duration, y = Speed, color = Construction))
```

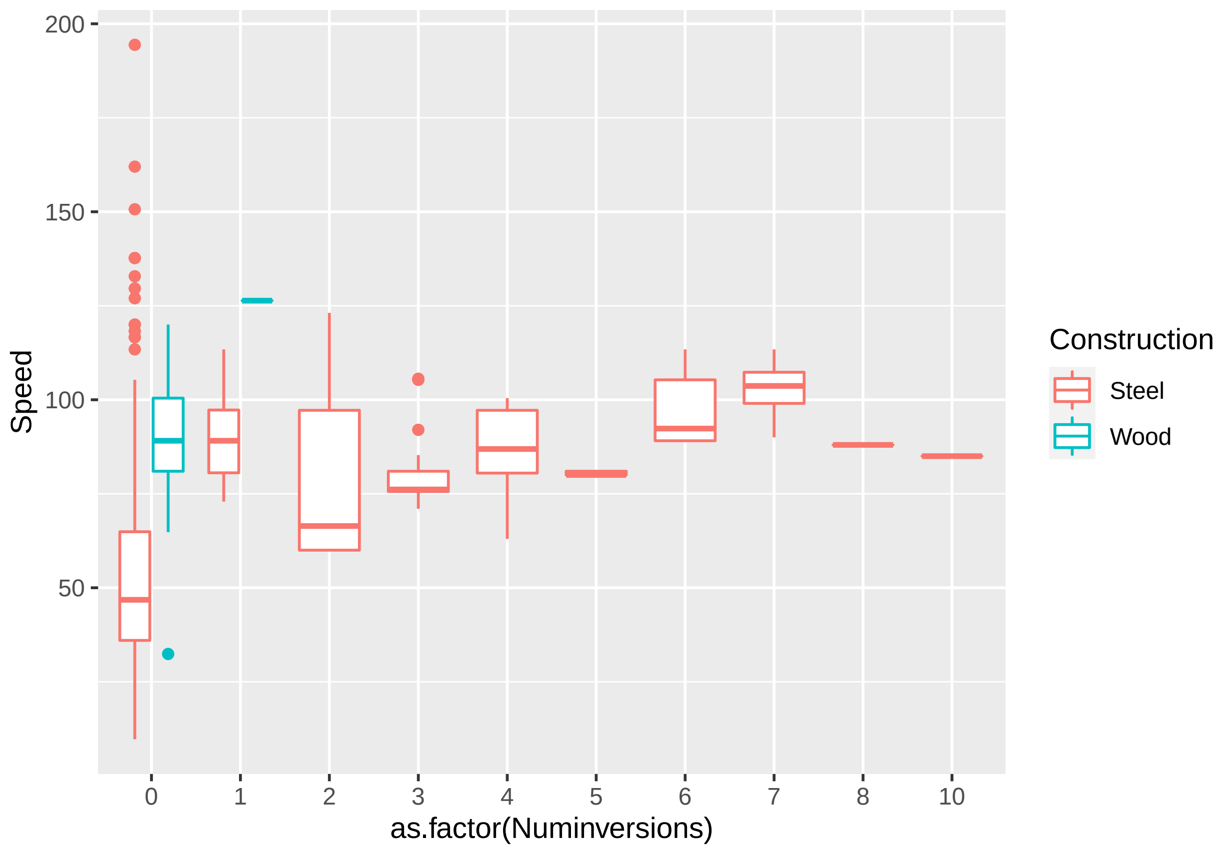
```
## Warning: Removed 244 rows containing missing values (geom_point).
```




We treated Numinversions as a categorical variable since it has too few values to make a proper regression plot.

```
roller_coasters_raw %>%
  ggplot() +
    geom_boxplot(aes(x = as.factor(Numinversions), y = Speed, color = Construction))
```

```
## Warning: Removed 138 rows containing non-finite values (stat_boxplot).
```

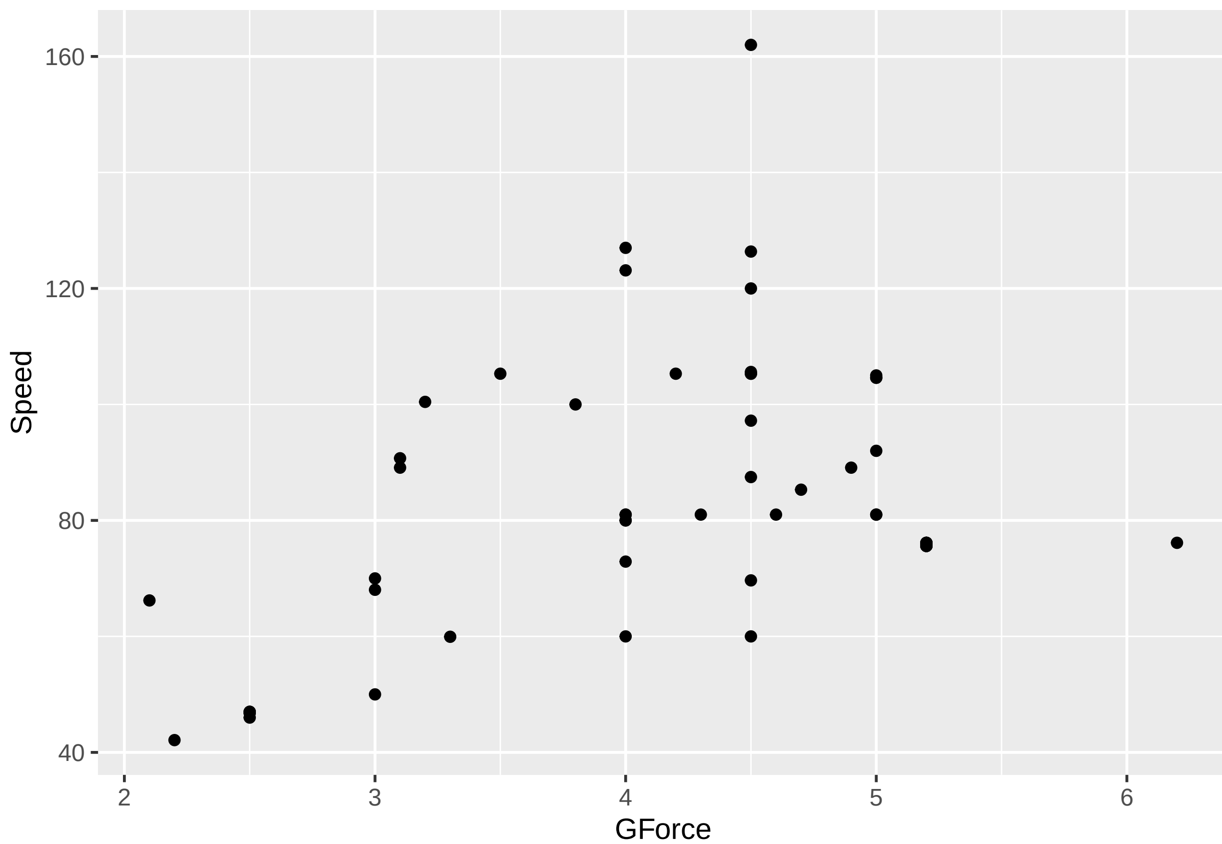



With every plot above we can see some linearity going on. But the best are definitely height, length, and categorical variable Construction, since the boxplot and hypothesis test clearly showed there is a significan difference between the average speeds.

We also wanted to show that GForce has to few values and not a good linear relationship, so that is why we won't include it into our prediction model:

```
roller_coasters_raw %>%
  filter(!is.na(GForce)) %>%
  ggplot() +
    geom_point(aes(x = GForce, y = Speed))
```

```
## Warning: Removed 2 rows containing missing values (geom_point).
```




**Regression**

We prepared a cleaned dataset with only the variables that are going to predict speed.

```
roller_coasters <- roller_coasters_raw %>%
  select(Construction, Length, Height, Speed) %>%
  filter(!is.na(Speed) & !is.na(Height) & !is.na(Length)) %>%
  mutate("Steel" = as.numeric(Construction == 'Steel')) %>%
  select(-Construction)
roller_coasters
```

```
## # A tibble: 252 x 4
##    Length Height Speed Steel
##     <dbl>  <dbl> <dbl> <dbl>
##  1   853.  128.   194.     1
##  2   376.  126.   162      1
##  3  2010.   94.5  151.     1
##  4  1619.   74.7  138.     1
##  5  1620    73    127      1
##  6  1372.   71.6  138.     1
##  7  1610.   70.1  130.     1
##  8  1700    67    120      1
##  9  2143.   66.4  126.     0
## 10   396.   66.4  113.     1
## # ... with 242 more rows
```

```r
## 75% of the sample size
smp_size <- floor(0.75 * nrow(roller_coasters))

## set the seed to make your partition reproducible
set.seed(123)
train_ind <- sample(seq_len(nrow(roller_coasters)), size = smp_size)

(train <- roller_coasters[train_ind, ])
```

```
## # A tibble: 189 x 4
##    Length Height Speed Steel
##     <dbl>  <dbl> <dbl> <dbl>
##  1   412.   16.2  50.2     1
##  2   207     8.5  34.9     1
##  3   538.   13.4  50.2     1
##  4   375.   61   105.      1
##  5   427.   10.7  32.4     0
##  6   774.   14.6  68.0     1
##  7   950    36    90       1
##  8   717.   23.8  82.6     1
##  9   309.   39.9  81       1
## 10   264     6    45       1
## # ... with 179 more rows
```

```r
(test <- roller_coasters[-train_ind, ])
```

```
## # A tibble: 63 x 4
##    Length Height Speed Steel
##     <dbl>  <dbl> <dbl> <dbl>
##  1   376.  126.   162      1
##  2  2010.   94.5  151.     1
##  3   671.   62.5  133.     1
##  4   347.   61    105.     1
##  5   367.   58.2  105.     1
##  6  1167.   57.3  113.     1
##  7  1654.   49.1  105.     0
##  8  1332.   47.5  105.     1
##  9   150    46    105      1
## 10   192.   45.7  105.     1
## # ... with 53 more rows
```

For linear models we have to take care that the following hold: 1) Linearity of the data 2) Nearly normal residuals also check for outliers, mostly influencial outliers 3) Constant variability 4) Independent observations

```r
lin_model <- lm(Speed ~ ., data = train)
(summary(lin_model))
```

```
##
## Call:
## lm(formula = Speed ~ ., data = train)
```

```
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -21.388  -6.020  -0.644   4.466  35.365
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.376471   2.513555  13.279  < 2e-16 ***
## Length       0.013174   0.002178   6.047 7.97e-09 ***
## Height       1.248969   0.047545  26.269  < 2e-16 ***
## Steel       -5.752860   2.109654  -2.727  0.00701 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 8.749 on 185 degrees of freedom
## Multiple R-squared:  0.9103, Adjusted R-squared:  0.9088
## F-statistic: 625.7 on 3 and 185 DF,  p-value: < 2.2e-16
```

```
(coef(lin_model))
```

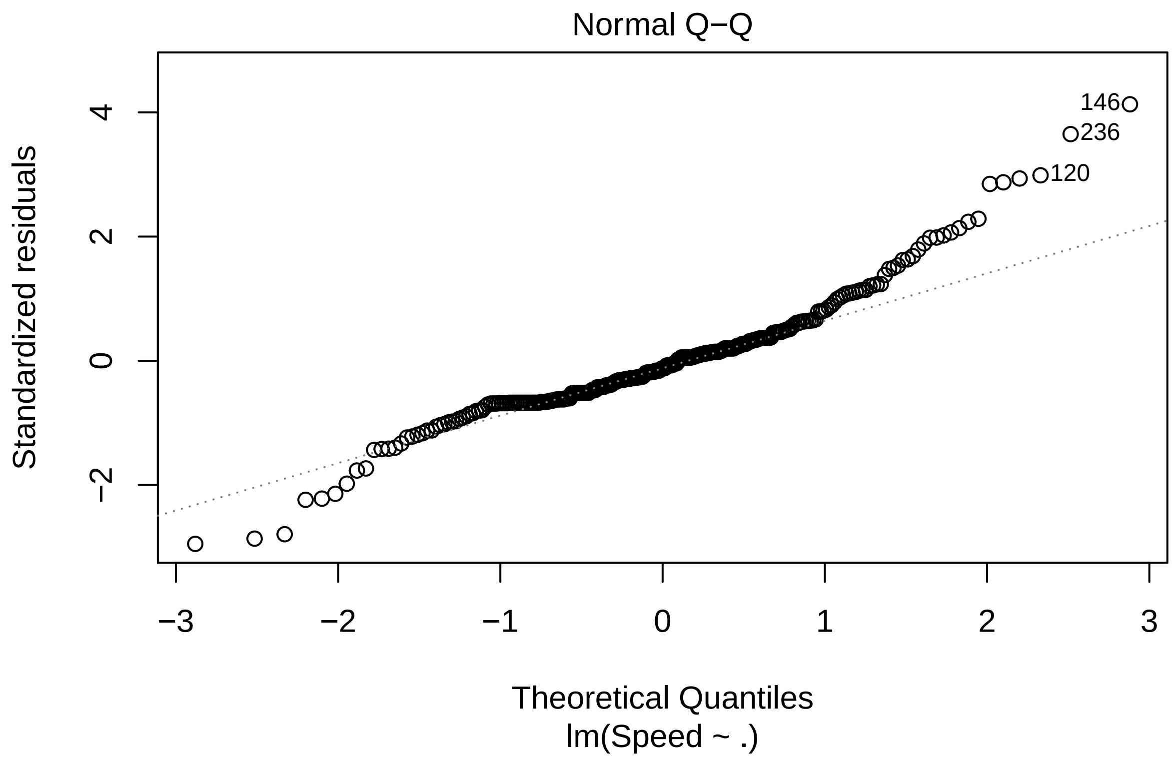
```
## (Intercept)      Length      Height       Steel
## 33.37647051  0.01317372  1.24896948 -5.75286049
```

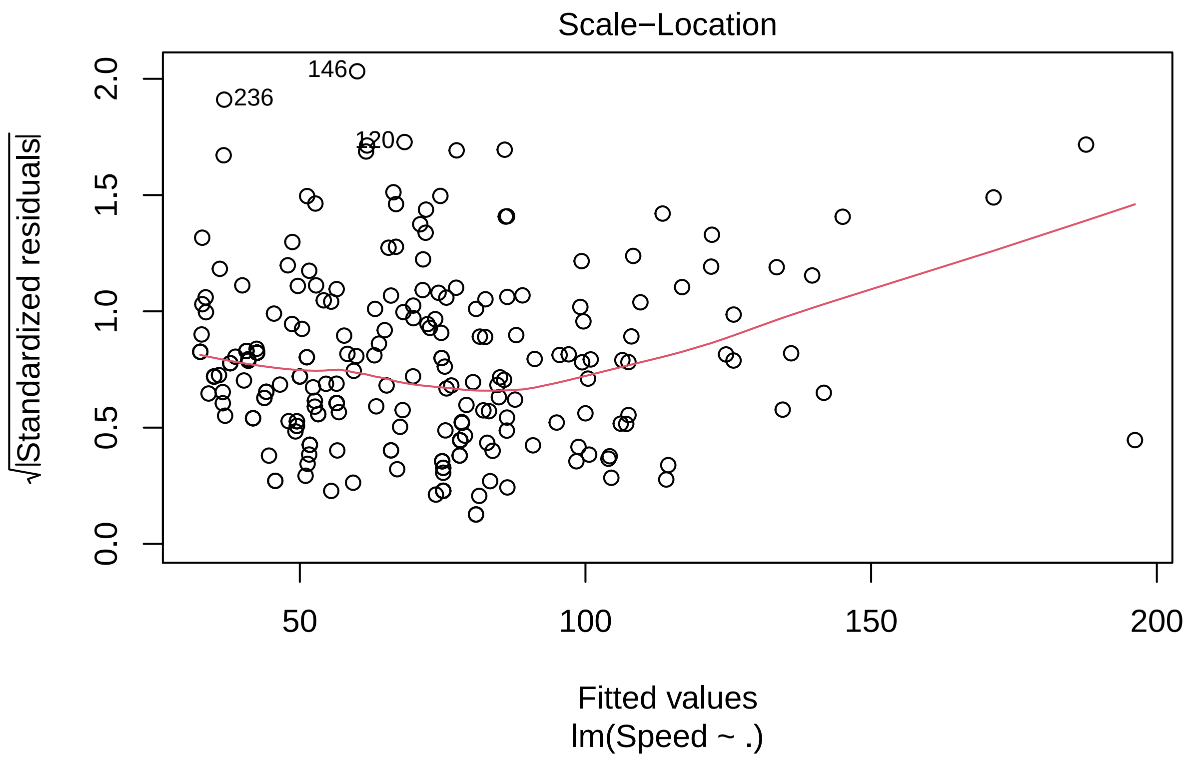
```
rc_all <- lm(Speed ~ ., data = roller_coasters)
(summary(rc_all))
```

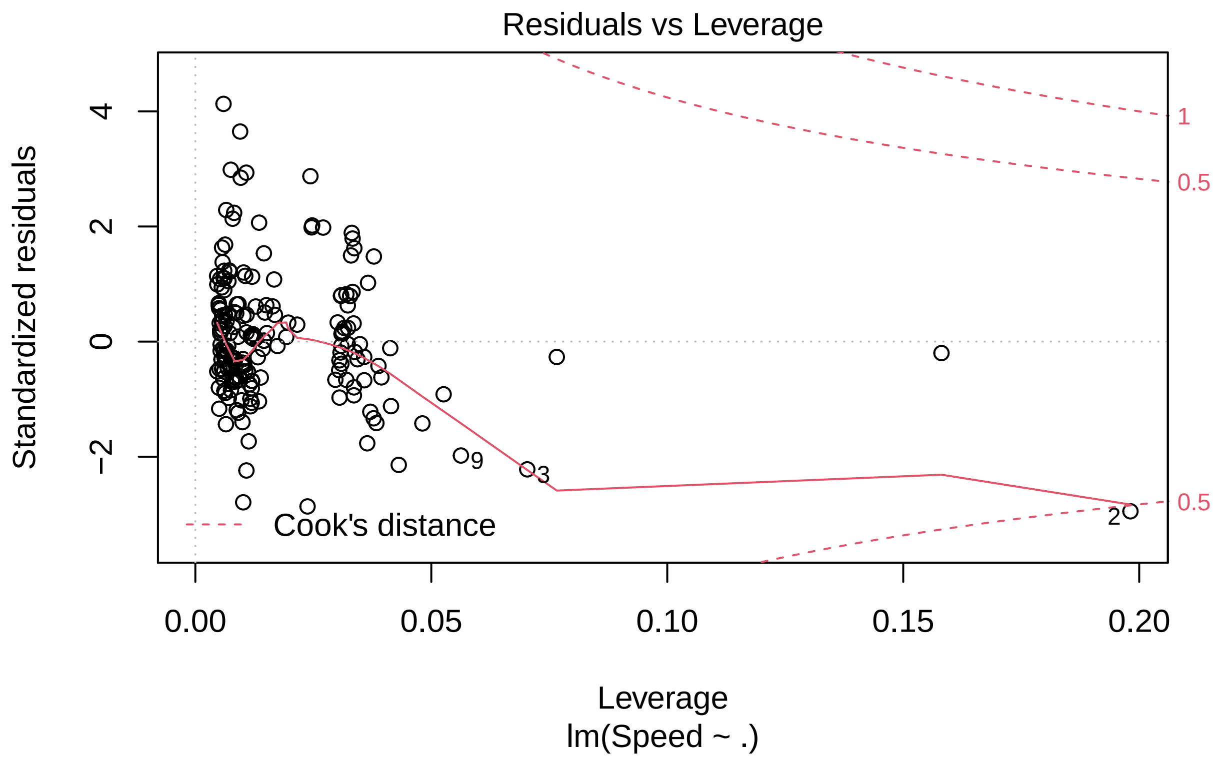
```
##
## Call:
## lm(formula = Speed ~ ., data = roller_coasters)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -27.452  -6.131  -1.180   3.826  39.948
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 33.699899   2.434493  13.843  < 2e-16 ***
## Length       0.014037   0.001915   7.329 3.26e-12 ***
## Height       1.222438   0.039889  30.646  < 2e-16 ***
## Steel       -5.998684   2.046036  -2.932  0.00368 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.701 on 248 degrees of freedom
## Multiple R-squared:  0.8946, Adjusted R-squared:  0.8933
## F-statistic: 701.3 on 3 and 248 DF,  p-value: < 2.2e-16
```

```
plot(rc_all)
```


Residuals vs Fitted
Residuals
40
20
0
−20
146
236
120
50
100
150
200
Fitted values
lm(Speed ~ .)

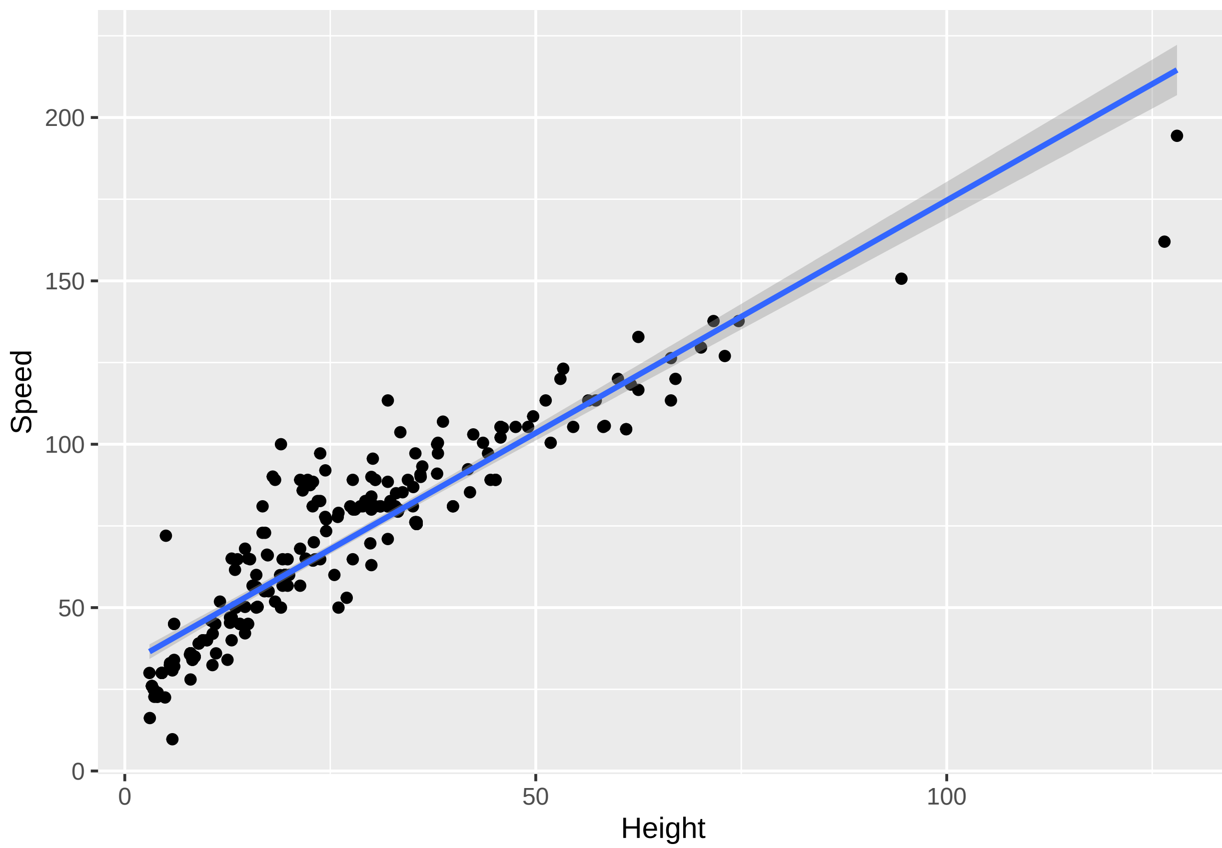

Normal Q−Q
Standardized residuals
4
2
0
−2
146
236
120
−3
−2
−1
0
1
2
3
Theoretical Quantiles
lm(Speed ~ .)


Scale−Location
√|Standardized residuals|
0.0
0.5
1.0
1.5
2.0
146
236
120
50
100
150
200
Fitted values
lm(Speed ~ .)




```
roller_coasters %>% ggplot()+
  geom_point(aes(x = Height, y = Speed))+
  geom_smooth(aes(x = Height, y = Speed), method = lm)
```

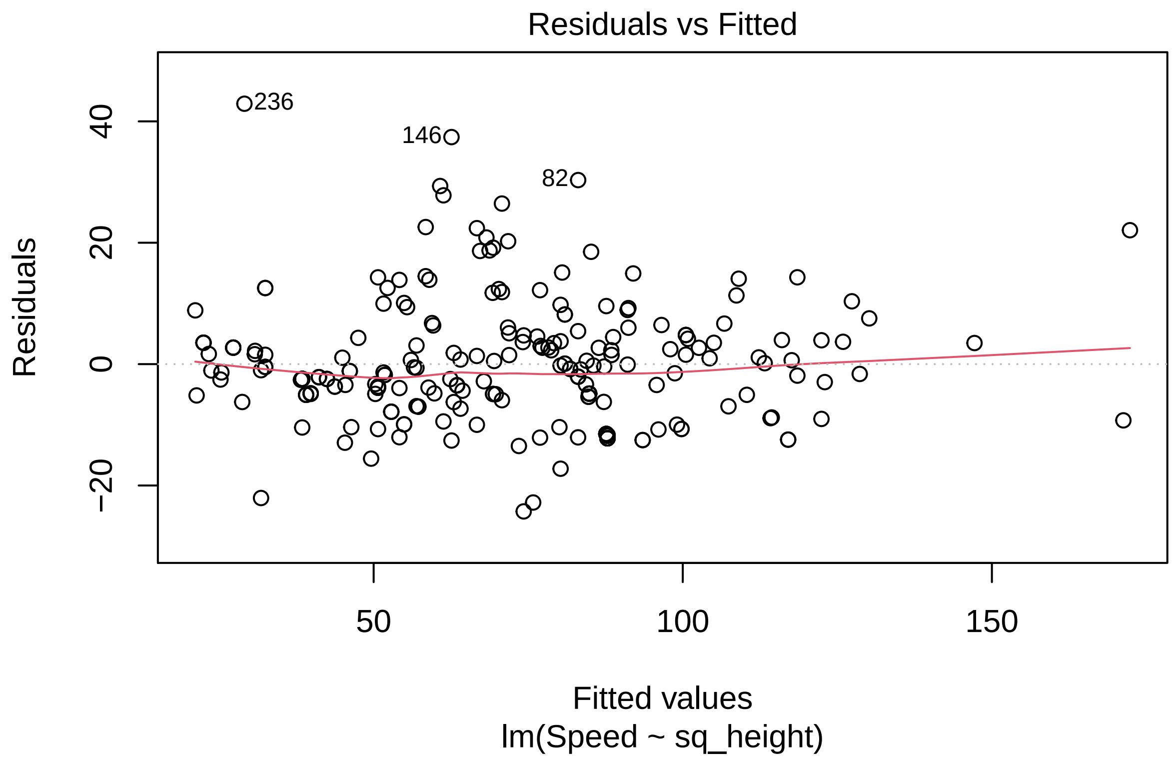
```
## `geom_smooth()` using formula 'y ~ x'
```



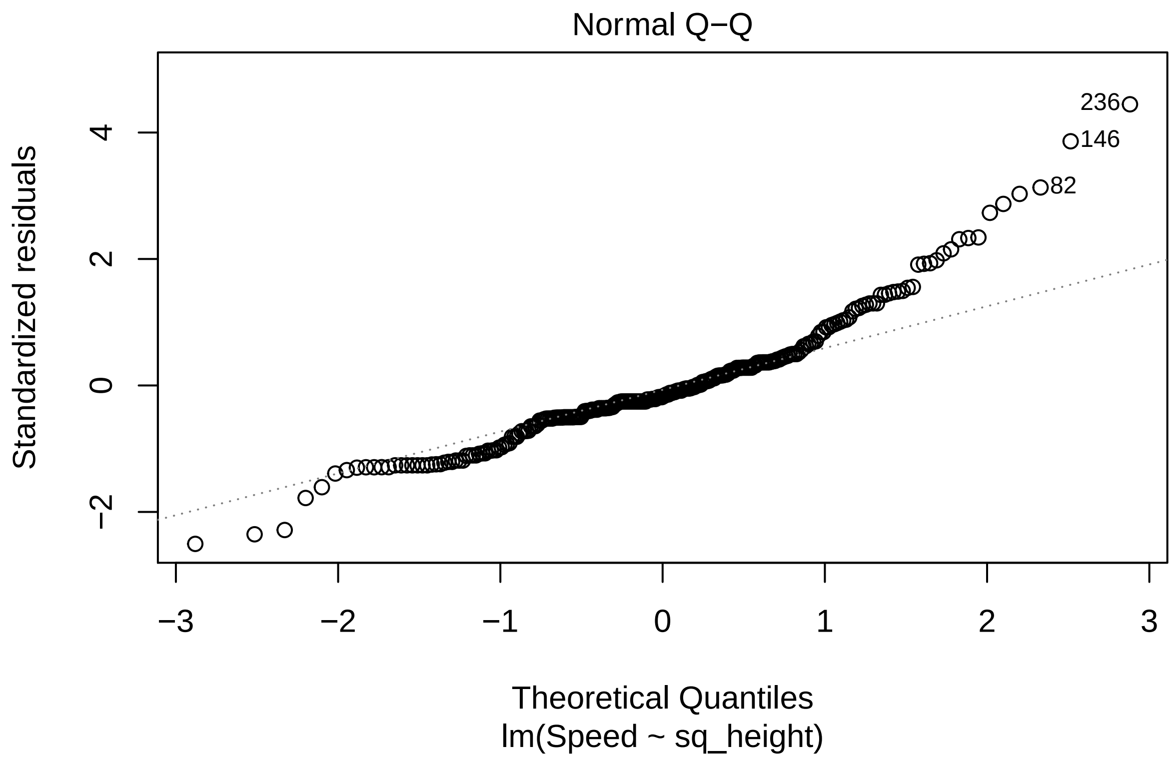

```
roller_coasters$log_height <- log(roller_coasters$Height)
roller_coasters$sq_height <- sqrt(roller_coasters$Height)

rc_all <- lm(Speed ~ sq_height, data = roller_coasters)
(summary(rc_all))
```

```
##
## Call:
## lm(formula = Speed ~ sq_height, data = roller_coasters)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -24.269  -4.981  -1.706   3.645  42.904
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -6.1846     1.7613  -3.511 0.000529 ***
## sq_height    15.7782     0.3444  45.813  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.708 on 250 degrees of freedom
## Multiple R-squared:  0.8936, Adjusted R-squared:  0.8931
## F-statistic:  2099 on 1 and 250 DF,  p-value: < 2.2e-16
```

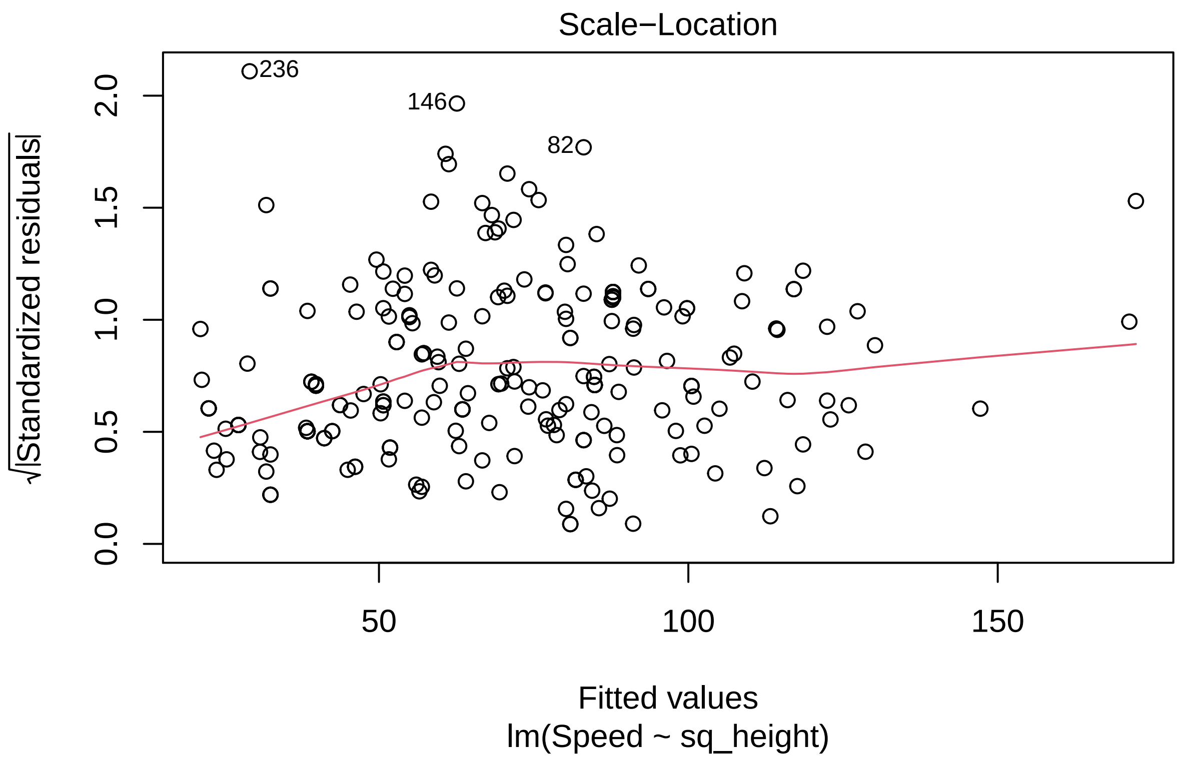
```
plot(rc_all)
```



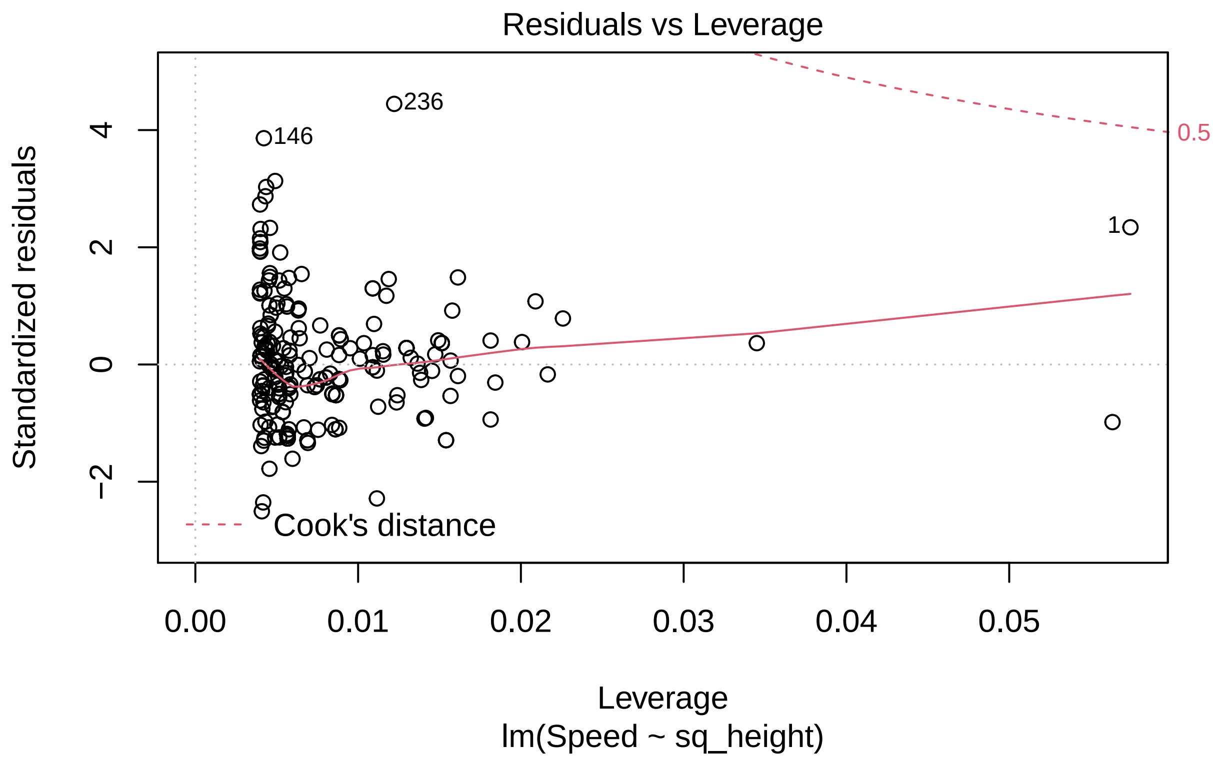
Residuals vs Fitted
Residuals
40
20
0
−20
236
146
82
50
100
150
Fitted values
lm(Speed ~ sq_height)



Normal Q−Q
Standardized residuals
Theoretical Quantiles
lm(Speed ~ sq_height)
236
146
82
4
2
0
−2
−3
−2
−1
0
1
2
3



Scale−Location
236
146
82
0.0
0.5
1.0
1.5
2.0
√|Standardized residuals|
50
100
150
Fitted values
lm(Speed ~ sq_height)




## Red billed seagulls

The dataset seagulls.csv represents the data collected about seagulls in Auckland, New Zeland. Dataset can be found here.

Data was collected on two seperate occasions (summer and winter) and on four different locations: Muriwai (a), Piha (b), Mareatai (c), and Waitawa (d).

They collected seagull's weight, length, and sex, as well as its location and season. Authors of the dataset also point out that none of the locations is a major breeding site.

We also cleaned dataset a bit. Some cases have misspelled "MURIWAI" as "MURWAI". Variables location, coast, season, and sex have been converted from strings to factors, and length was renamed to height, since that is more accurate varable description.

```
seagulls <- read.csv("datasets/seagulls.csv")
seagulls[seagulls$LOCATION == "MURWAI",]$LOCATION <- "MURIWAI"
colnames(seagulls)[2] <- "HEIGHT"
seagulls$LOCATION <- as.factor(seagulls$LOCATION)
seagulls$COAST <- as.factor(seagulls$COAST)
seagulls$SEASON <- as.factor(seagulls$SEASON)
seagulls$SEX <- as.factor(seagulls$SEX)
```


Figure 1: Auckland region

### Summary statistics

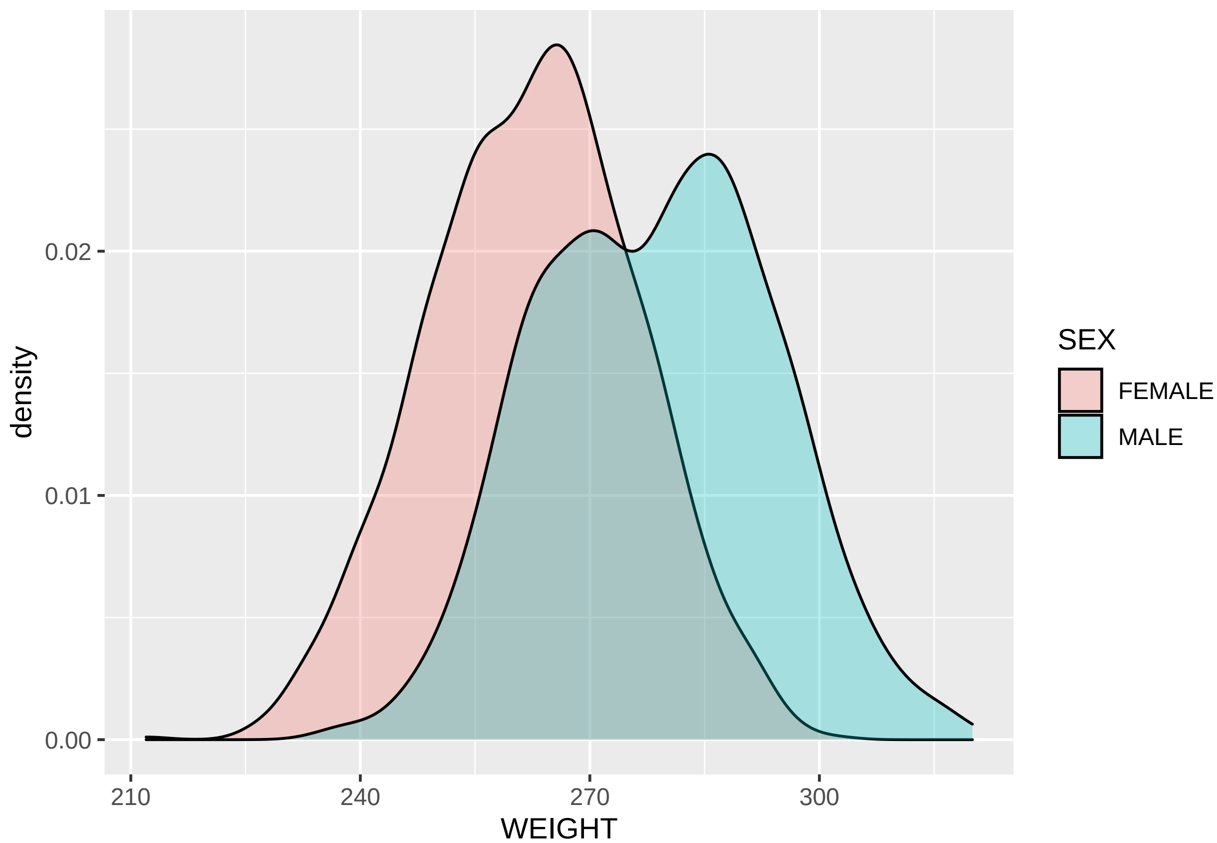
Seagulls dataset has 2487 cases and 6 variables: weight, height, location, coast, season, and sex. Weight and length are numerical, while location, coast, season, and sex are categorical.

```
summary(seagulls)
```

```
##      WEIGHT          HEIGHT          LOCATION      COAST        SEASON
##  Min.   :212.0   Min.   :28.5   MARAETAI:673   EAST:1251   SUMMER:1313
##  1st Qu.:259.0   1st Qu.:35.5   MURIWAI :589   WEST:1236   WINTER:1174
##  Median :269.0   Median :37.1   PIHA    :647
##  Mean   :270.4   Mean   :37.1   WAITAWA :578
##  3rd Qu.:282.0   3rd Qu.:38.8
##  Max.   :320.0   Max.   :44.8
##      SEX
##  FEMALE:1280
##  MALE  :1207
##
##
##
##
```

Weight of seagulls is in grams (g), and its distribution can be seen here:

```
seagulls %>% ggplot()+
  geom_density(aes(x = WEIGHT, fill = SEX), alpha = 0.3)
```




Average weight of males is 278.73g with minimum of 235g and maximum of 320g. Average weight of females is 262.49g with minimum of 212g and maximum of 302g. We can see that weights of males are not normally distributed, while weights of females could be. We can check this with normality test:

```
shapiro.test(seagulls[seagulls$SEX == "MALE",]$WEIGHT)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  seagulls[seagulls$SEX == "MALE", ]$WEIGHT
## W = 0.994, p-value = 8.841e-05
```

```
shapiro.test(seagulls[seagulls$SEX == "FEMALE",]$WEIGHT)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  seagulls[seagulls$SEX == "FEMALE", ]$WEIGHT
## W = 0.99724, p-value = 0.02575
```

We can see that weight is not normally distributed neither for males nor females, but latter are very close to passing the normallity test. We can also check if the distributions are at least symmetric:

```r
symmetry.test(seagulls[seagulls$SEX == "MALE",]$WEIGHT)
```

```
##
##  m-out-of-n bootstrap symmetry test by Miao, Gel, and Gastwirth (2006)
##
## data:  seagulls[seagulls$SEX == "MALE", ]$WEIGHT
## Test statistic = -0.80072, p-value = 0.458
## alternative hypothesis: the distribution is asymmetric.
## sample estimates:
## bootstrap optimal m
##                 330
```

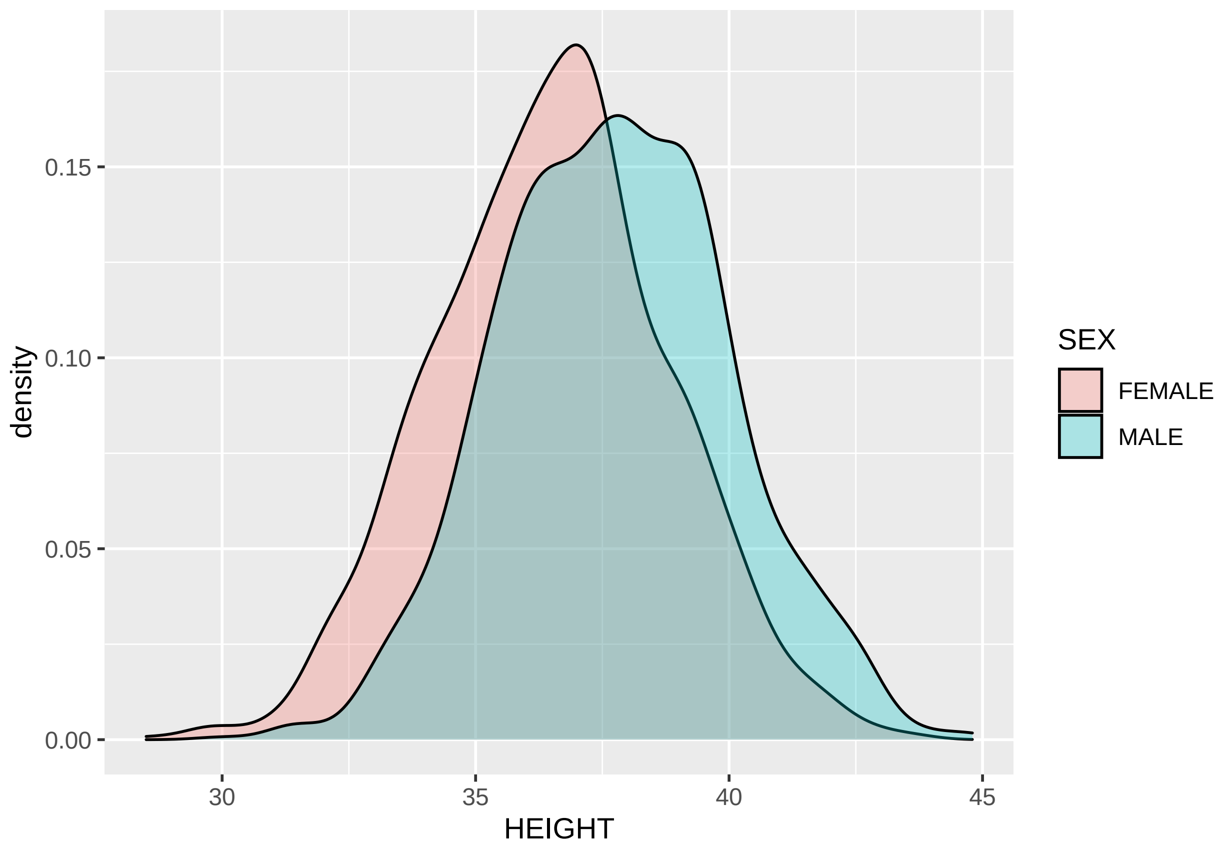
```r
symmetry.test(seagulls[seagulls$SEX == "FEMALE",]$WEIGHT)
```

```
##
##  m-out-of-n bootstrap symmetry test by Miao, Gel, and Gastwirth (2006)
##
## data:  seagulls[seagulls$SEX == "FEMALE", ]$WEIGHT
## Test statistic = -1.773, p-value = 0.14
## alternative hypothesis: the distribution is asymmetric.
## sample estimates:
## bootstrap optimal m
##                 631
```

Both pass symmetry test, meaning they are not strongly skewed and can be used later for inference.

Height of seagulls is in centimeters (cm):

```r
seagulls %>% ggplot()+
  geom_density(aes(x = HEIGHT, fill = SEX), alpha = 0.3)
```




Average height of males is 37.74cm. Smallest male's height is 30cm, while largest is 44.8cm. Female's average height is 36.5cm with minimum of 28.5cm and maximum of 43.7cm. Seagulls height seems more normally distributed than weight, but we can check:

```
shapiro.test(seagulls[seagulls$SEX == "MALE",]$HEIGHT)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  seagulls[seagulls$SEX == "MALE", ]$HEIGHT
## W = 0.9983, p-value = 0.2733
```

```
shapiro.test(seagulls[seagulls$SEX == "FEMALE",]$HEIGHT)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  seagulls[seagulls$SEX == "FEMALE", ]$HEIGHT
## W = 0.9989, p-value = 0.6345
```

We can see that height for both sexes passes as normally distributed.

We have four locations in our dataset: Maraetai, Waitawa, Muriwai, and Piha. Coast is either east or west and is a more broad description of location (Maraetai and Waitawa are under east coast and Muriwai and Piha are under west coast). Locations are almost equally represented in our dataset:

```
table(seagulls$LOCATION) / nrow(seagulls)
```

```
##
##  MARAETAI   MURIWAI      PIHA   WAITAWA
## 0.2706072 0.2368315 0.2601528 0.2324085
```

Coast variable is also equaly distributed:

```
table(seagulls$COAST) / nrow(seagulls)
```

```
##
##      EAST      WEST
## 0.5030157 0.4969843
```

Season is either winter or summer. There are a little more entries for summer than for winter, but the difference is miniscule:

```
table(seagulls$SEASON) / nrow(seagulls)
```

```
##
##    SUMMER    WINTER
## 0.5279453 0.4720547
```

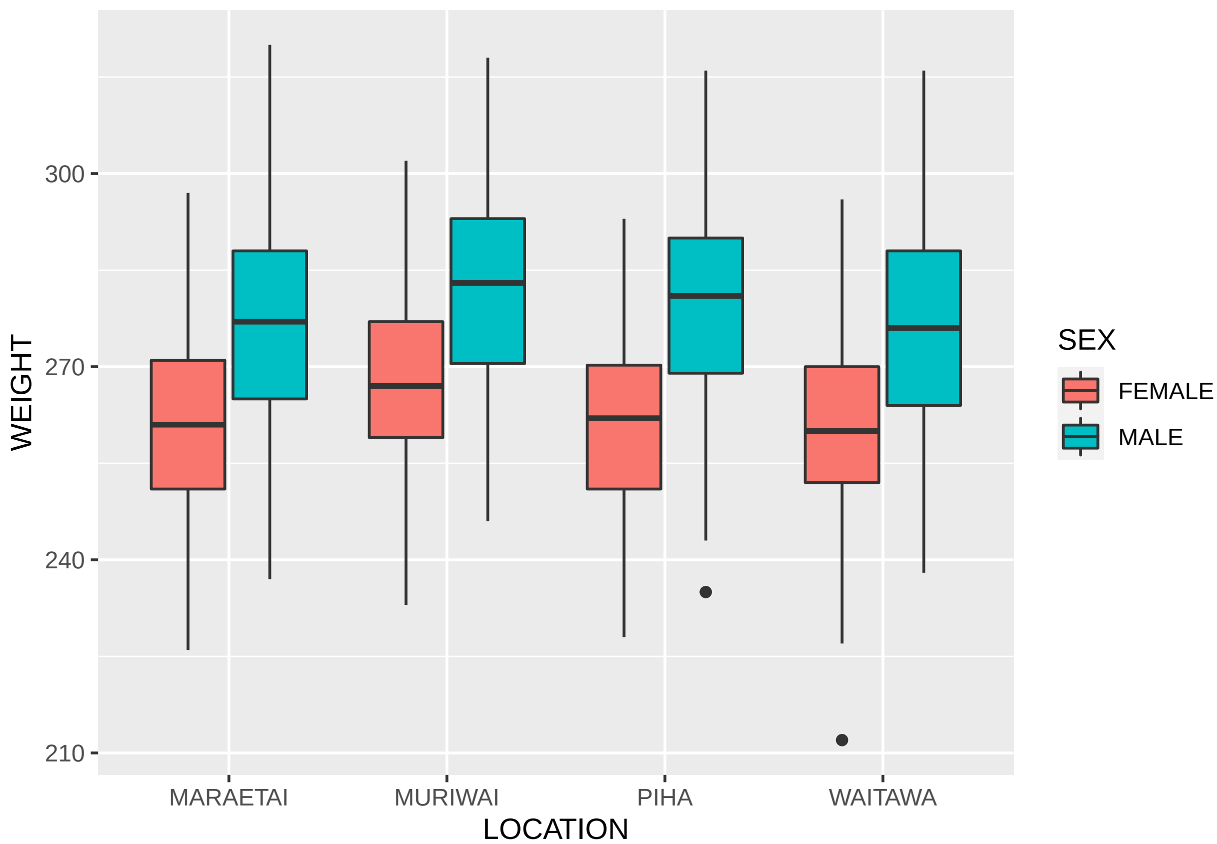
There are more females presented in our dataset but the difference can be ignored:

```
table(seagulls$SEX) / nrow(seagulls)
```

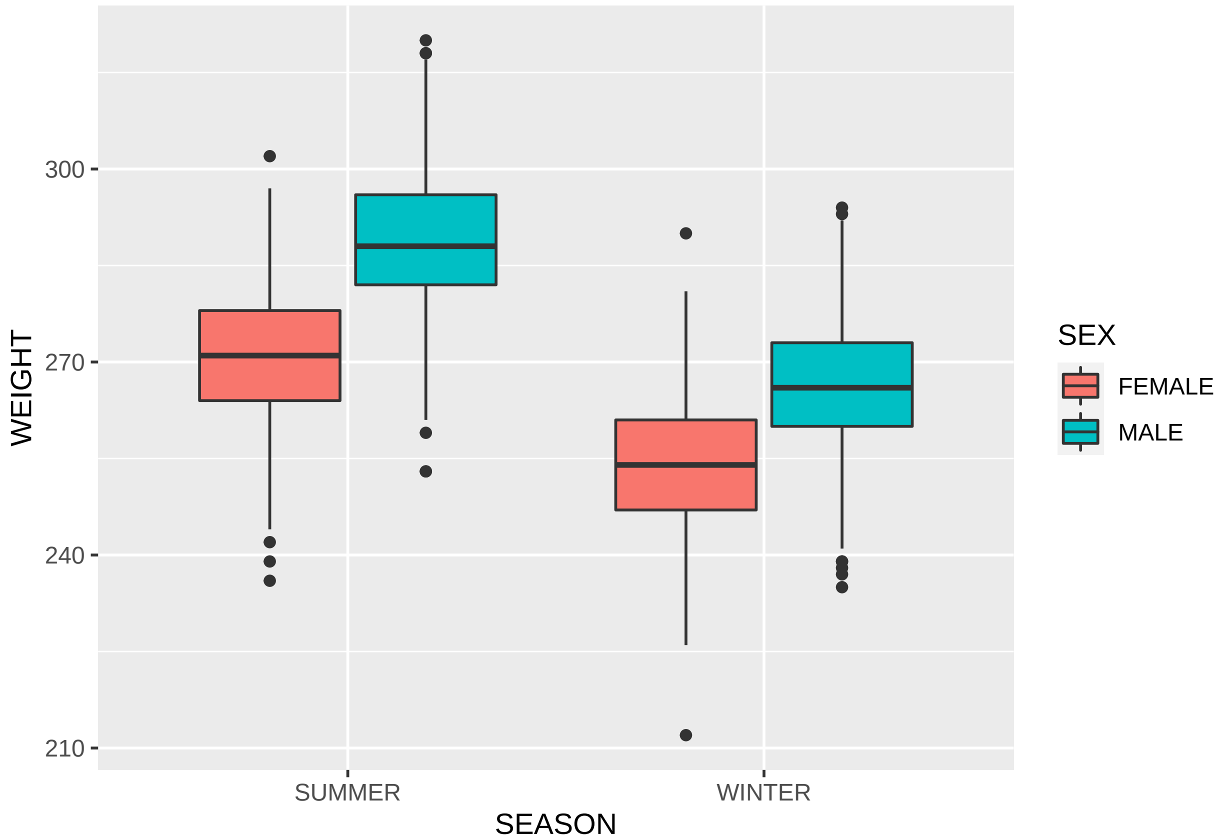
```
##
##    FEMALE      MALE
## 0.5146763 0.4853237
```

We also drew some other plots representing how different variables are connected:

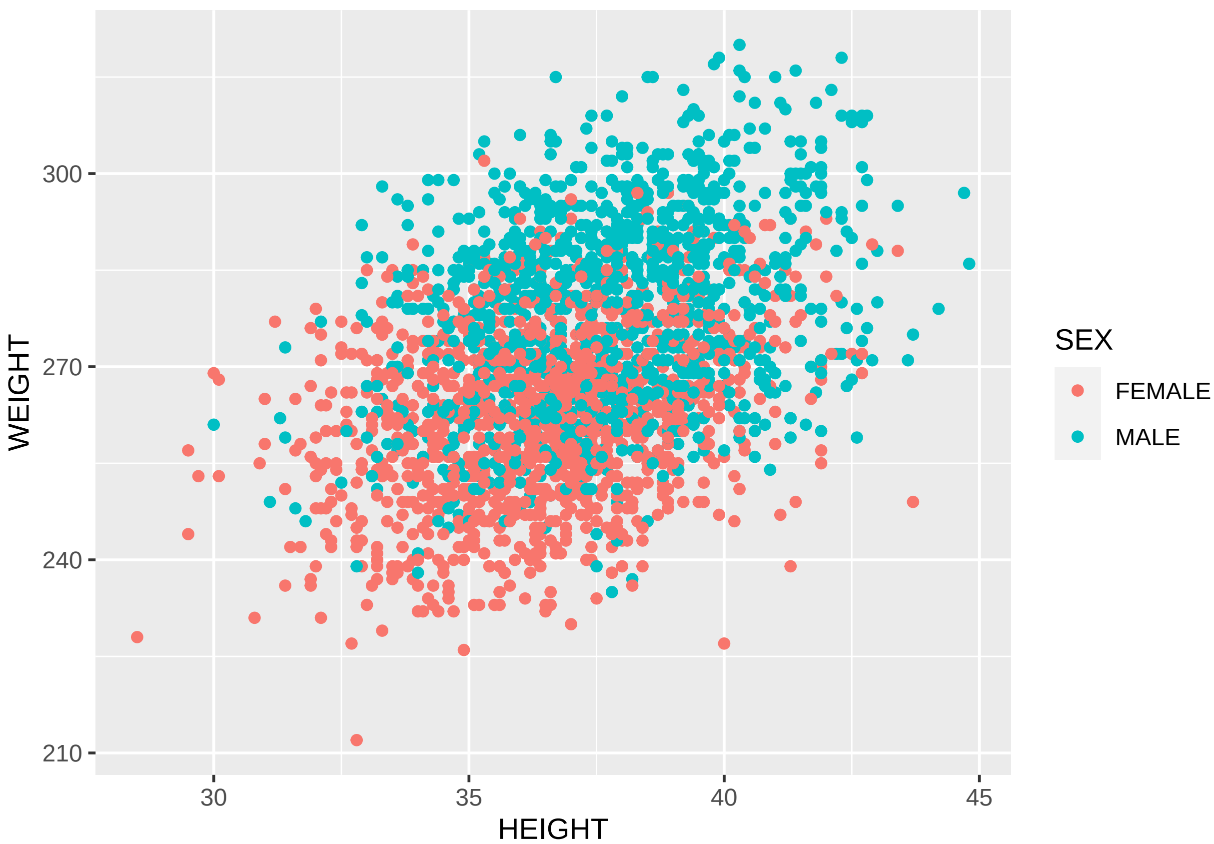
```
seagulls %>% ggplot()+
  geom_boxplot(aes(x = LOCATION, y = WEIGHT, fill = SEX))
```




```
seagulls %>% ggplot()+
  geom_boxplot(aes(x = SEASON, y = WEIGHT, fill = SEX))
```




```
seagulls %>% ggplot()+
  geom_point(aes(x = HEIGHT, y = WEIGHT, color = SEX))
```




### Inference

Since we can divide our datasets in many ways, we can also check many differnet hypothesis.

#### Is weight of males same on east and west coast?

We want to know if there is a difference between males in east and west coast.

$$H_0 : mean_{east} - mean_{west} = 0$$

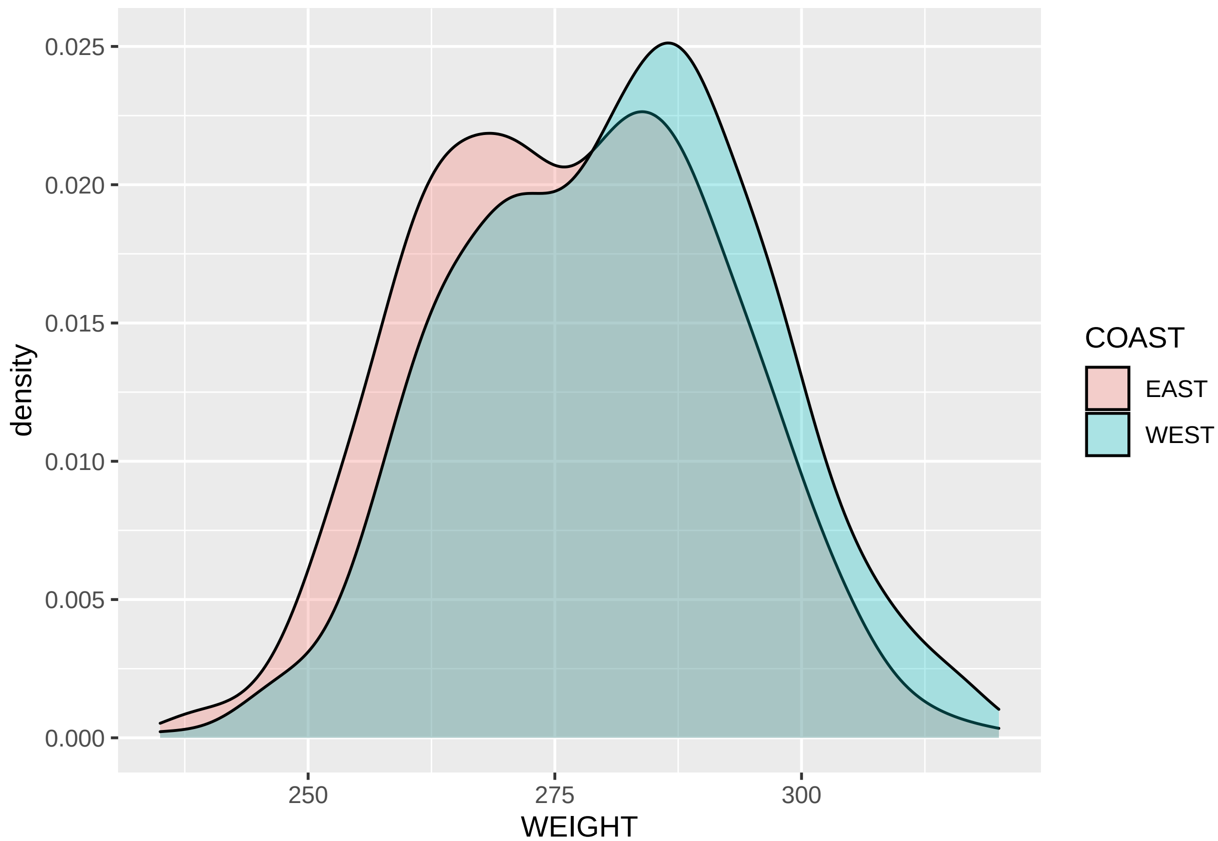
$$H_A : mean_{east} - mean_{west} \neq 0$$

We first divide our dataset into two smaller ones, which represent males from different coasts.

```
sg_east <- seagulls %>% filter(COAST == "EAST", SEX == "MALE")
sg_west <- seagulls %>% filter(COAST == "WEST", SEX == "MALE")
```

Next we need to check CLT conditions. Since samples were collected independently from one another, first condition is true. Next we need to check if both samples have sufficient size. There are 629 males from east and 578 males from west. Both samples are larger than 30, so second condition is also true. Then we need to check if any of samples is skewed. We can draw their distributions and see that they both are somewhat symmetrical.

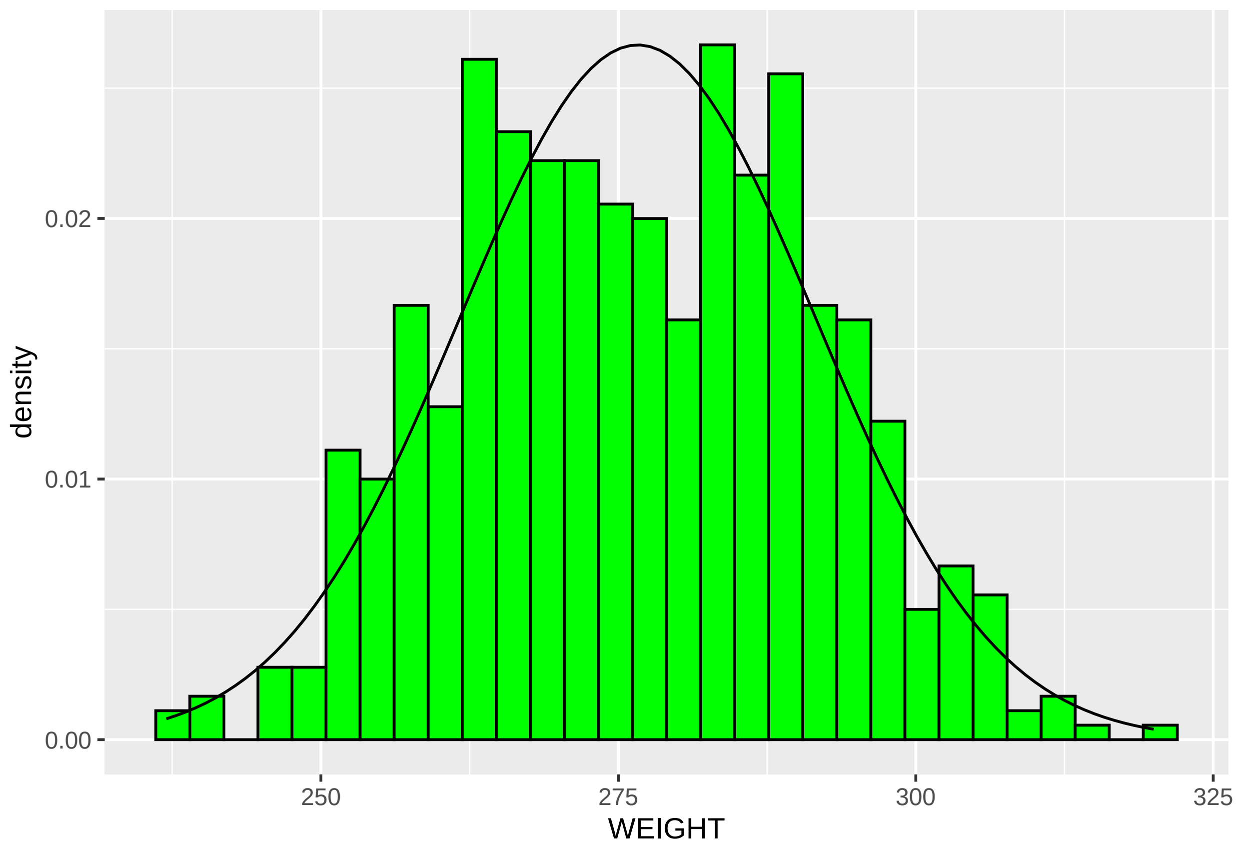
```
seagulls %>% filter(SEX == "MALE") %>% ggplot()+
  geom_density(aes(x = WEIGHT, fill = COAST), alpha = 0.3)
```



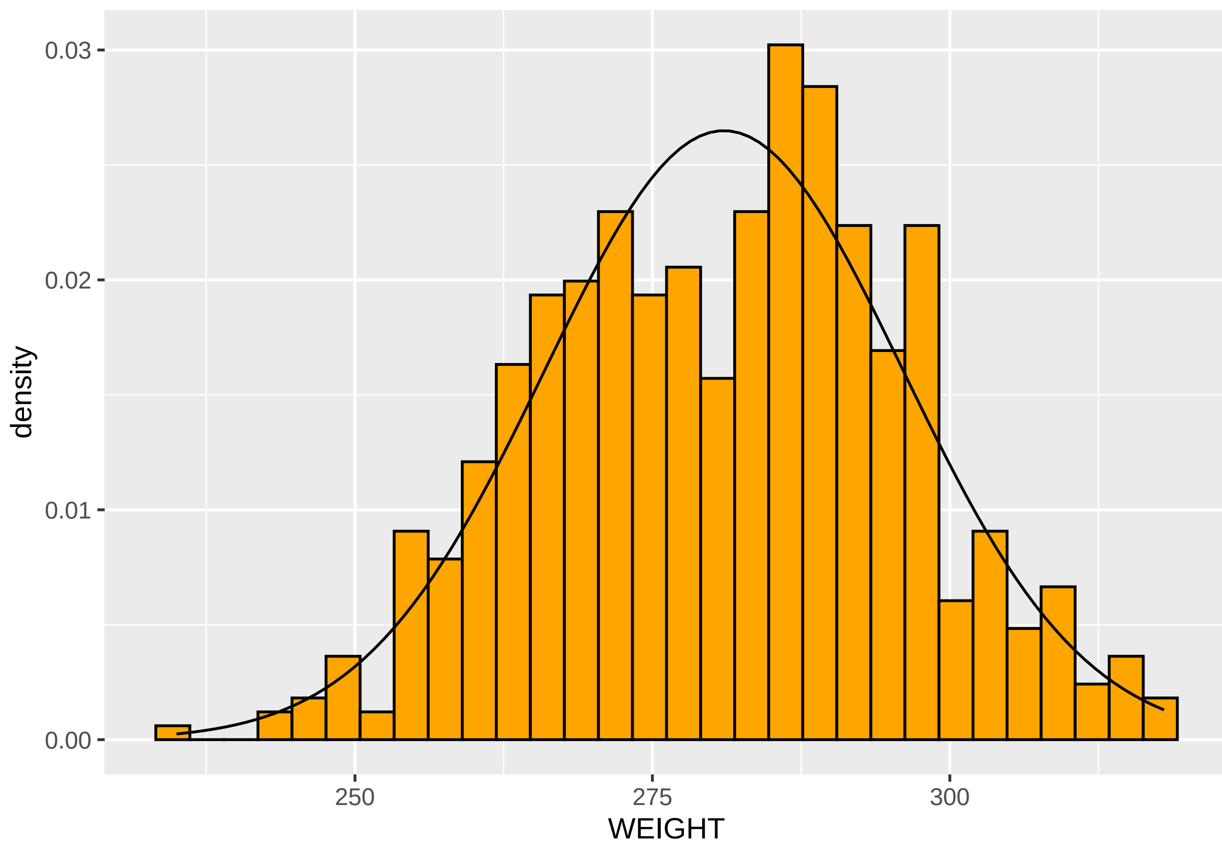

We can also calculate skewness of both distributions. Weight of males from east coast have skewness of 0.0370527 and males from west have skewness of -0.0483849. Both values are small, so we can safely say that neither distribution is strongly skewed.

We also need to check wether cases from groups are independant from each other. Since they were collected on different locations, they are independant. We can check if both groups are normally distributed. For that we can draw histogram of weights and overlay it with normal distribution with same average and standard deviation:

```
east.mean <- mean(sg_east$WEIGHT)
east.sd <- sd(sg_east$WEIGHT)
sg_east %>% ggplot()+
  geom_histogram(aes(x = WEIGHT, y = ..density..), fill = "green", color = "black")+
  stat_function(fun = dnorm, args = list(mean = east.mean, sd = east.sd))
```




```
west.mean <- mean(sg_west$WEIGHT)
west.sd <- sd(sg_west$WEIGHT)
sg_west %>% ggplot()+
  geom_histogram(aes(x = WEIGHT, y = ..density..), fill = "orange", color = "black")+
  stat_function(fun = dnorm, args = list(mean = west.mean, sd = west.sd))
```




Neither distribution seem normally distributed. We can further test that hypothesis with normallity test:

```
shapiro.test(sg_east$WEIGHT)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  sg_east$WEIGHT
## W = 0.99247, p-value = 0.002899
```

```
shapiro.test(sg_west$WEIGHT)
```

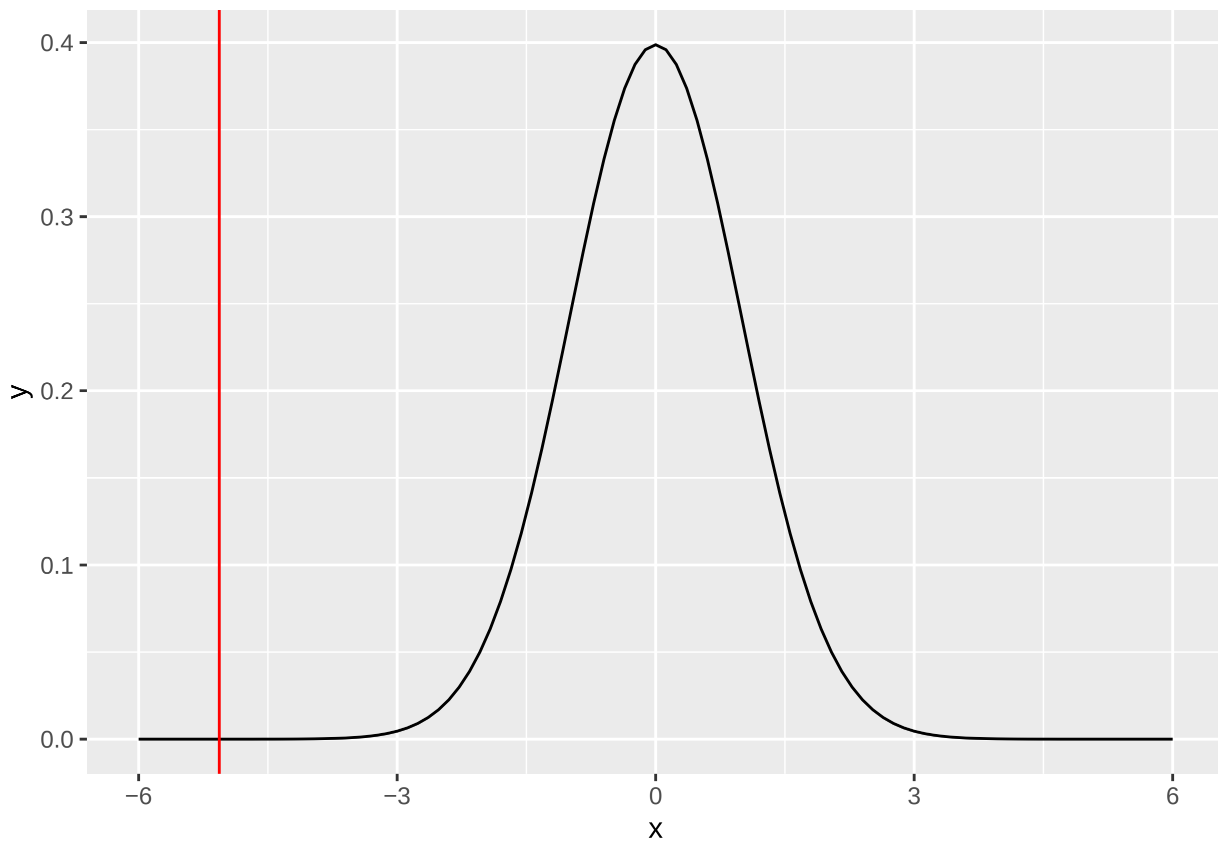
```
##
##  Shapiro-Wilk normality test
##
## data:  sg_west$WEIGHT
## W = 0.99417, p-value = 0.0257
```

Neither group has normal distribution, but they are symmetrical, so we will continue with our hypothesis testing.

We set a threshold value $\alpha = 0.05$.

We calculate our point estimate, standard error, and t-score and plot it:

```r
point_estimate <- east.mean - west.mean
SE <- sqrt(east.sd ^ 2 / nrow(sg_east) + west.sd ^ 2 / nrow(sg_west))
df <- min(nrow(sg_east) - 1, nrow(sg_west) - 1)
t_score <- point_estimate / SE

ggplot(data.frame(x = seq(-6, 6, length = 200)), aes(x = x))+
  stat_function(fun = dt, args = list(df = df))+
  geom_vline(xintercept = t_score, color = "red")
```




We can see that our t-score (red line) falls to the left of student's t-distribution, so our null hypothesis is very likely false. We can further confirm that with our p-value calculation:

```r
(p_value <- 2 * pt(t_score, df))
```

```
## [1] 5.528673e-07
```

Since p-value is smaller than $\alpha$ ($5.5286726 \times 10^{-7} < 0.05$), we reject $H_0$ in favor of $H_A$. Seagulls on east and west coast do not weight the same. Because our point estimate is negative, we can say that seagulls on west coast weight more than seagulls on east coast.

### Are males and females equaly represented?

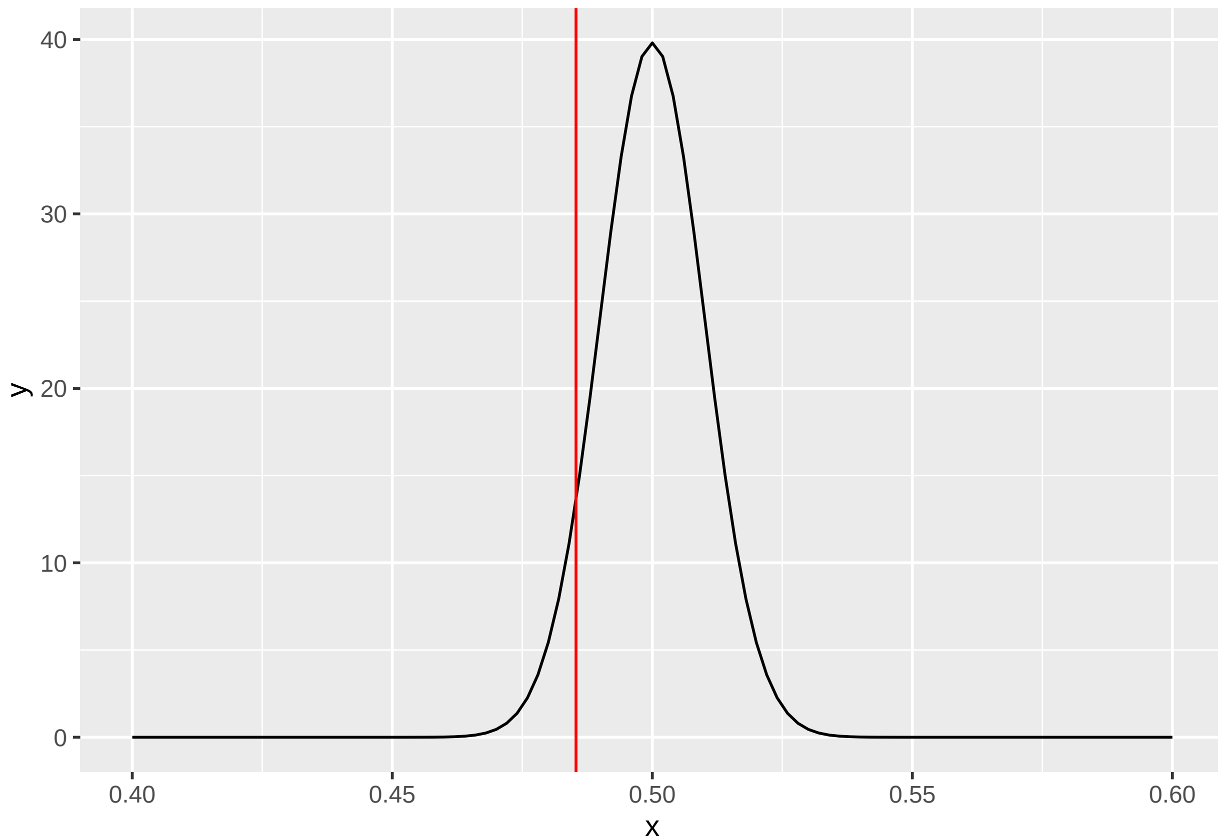
We want to know if males and females are equaly represented, that is, if ratio of males to entire population is 50%.

$$H_0 : p_{males} = 0.50$$

$$H_A : p_{males} \neq 0.50$$

Since samples in our dataset are independant observations, first CLT condition is satisfied. We also have 1207 males and 1280 females. Both numbers are greater than 10, so we can proceed with categorical inference on proportion testing.

```
ratio <- seagulls %>% filter(SEX == "MALE") %>% nrow() / nrow(seagulls)
SE <- sqrt(ratio * (1 - ratio) / nrow(seagulls))

ggplot(data.frame(x = seq(0.4, 0.6, length = 100)), aes(x = x))+
  stat_function(fun = dnorm, args = list(mean = 0.5, sd = SE))+
  geom_vline(xintercept = ratio, color = "red")
```




```
(p_value <- pnorm(ratio, mean = 0.5, sd = SE))
```

```
## [1] 0.07153663
```

Since p_value of 0.0715366 is greater than our threshold value of 0.05 we accept the null hypothesis. There is the same number of males and females in seagull population.

### Are the locations in our dataset representative?

We are interested if the locations in our dataset are represented equally.

$ H_0 $: Equal proportions of all locaitons

$ H_A $: Unequal proportions of all locations

We are goind to do a xhi-squared test for goodness of fit.

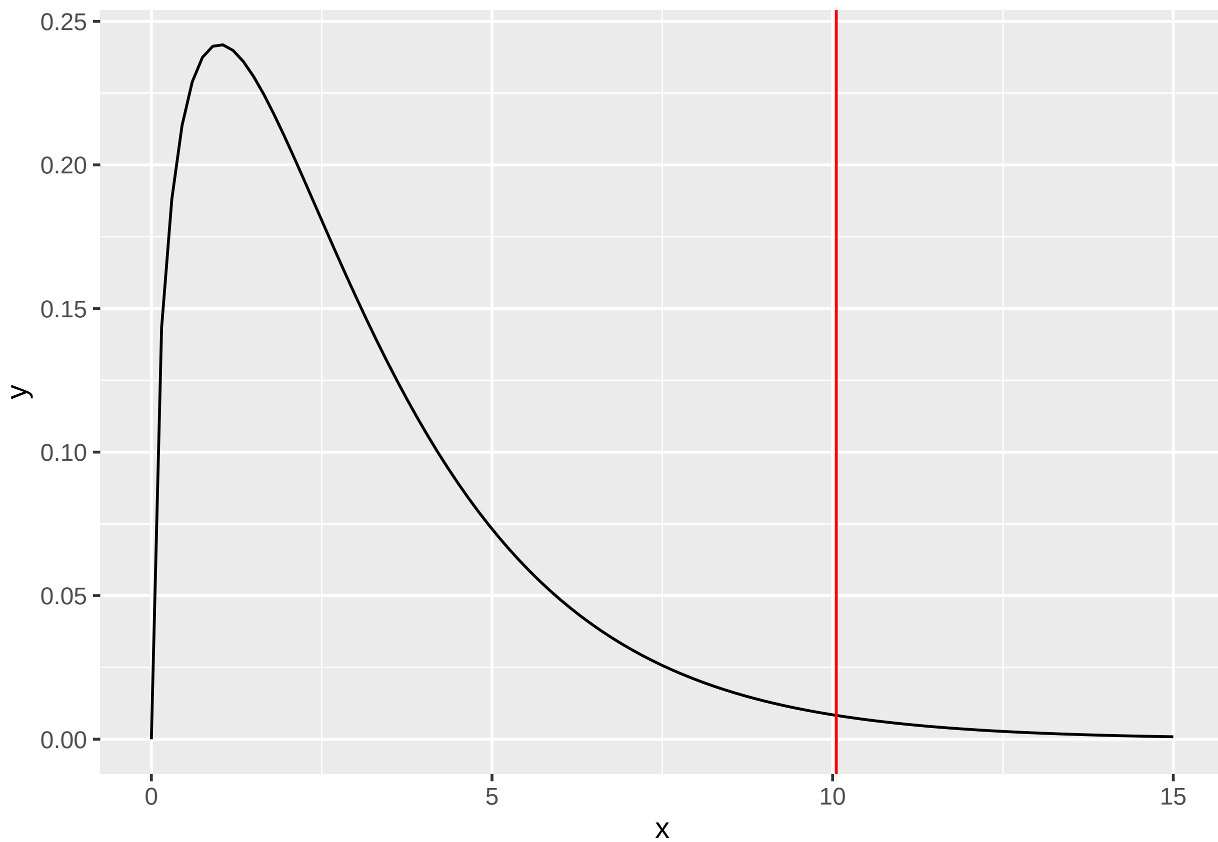
Collected data about locations is independant. All 4 categories also have at least 5 cases to them, so both chi-square test conditions are met.

```
table(seagulls$LOCATION)
```

```
##
## MARAETAI  MURIWAI     PIHA  WAITAWA
##      673      589      647      578
```

We need to calculate expected count for each category and for every category we calculate its Z score and then sum squares of Z scores together. Finally we check where on this chi-squared distribution lies our score.

```
num_classes <- length(unique(seagulls$LOCATION))
expected_location <- nrow(seagulls) / num_classes
z <- (table(seagulls$LOCATION) - expected_location) / sqrt(expected_location)
chi <- sum(z ^ 2)

df <- num_classes - 1

ggplot(data.frame(x = seq(0, 15, length = 100)), aes(x = x))+
  stat_function(fun = dchisq, args = list(df = df))+
  geom_vline(xintercept = chi, color = "red")
```




```
(p_value <- 1 - pchisq(chi, df))
```

```
## [1] 0.01811698
```

Since our p_value is smaller then $\alpha$ ($0.018117 < 0.05$), we can reject out null hypothesis in favor of alternative. Locations in our dataset are not equaly represented.