

Esercizio 2

$x = R \equiv \text{Randomukee}$

$x = B \equiv \text{Bugiardo kee}$

$y = R_y \equiv \text{risposta corretta destra}$

$y = L_f \equiv \text{risposta corretta sinistra}$

$z = R_y \equiv \text{risposta destra}$

$z = L_f \equiv \text{risposta sinistra}$

$$P(x = R) = 2/3, \quad P(x = B) = 1/3, \quad P(y = R_y) = 1/2, \quad P(y = L_f) = 1/2.$$

$$P(z = R_y | y = R_y, x = R) = 3/4, \quad P(z = R_y | y = L_f, x = R) = 1/4$$

$$P(z = R_y | y = R_y, x = B) = 0, \quad P(z = R_y | y = L_f, x = B) = 1$$

$$\textcircled{2} P(z = R_y | y = R_y) = P(z = R_y | y = R_y, x = R) P(x = R) + P(z = R_y | y = R_y, x = B) P(x = B) = \frac{3}{4} \cdot \frac{2}{3} + 0 \cdot \frac{1}{3} = \frac{1}{2}$$

$$\begin{aligned} P(z = R_y) &= P(z = R_y | y = R_y, x = R) \cdot P(y = R_y) \cdot P(x = R) + P(z = R_y | y = R_y, x = B) \cdot P(y = R_y) \cdot P(x = B) \\ &\quad + P(z = R_y | y = L_f, x = R) \cdot P(y = L_f) \cdot P(x = R) + P(z = R_y | y = L_f, x = B) \cdot P(y = L_f) \cdot P(x = B) \\ &= \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{2}{3} + 0 \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{2}{3} + 1 \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{2} \end{aligned}$$

$$P(y = R_y | z = R_y) = \frac{P(z = R_y | y = R_y) \cdot P(y = R_y)}{P(z = R_y)} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{1/2} = \frac{1}{2}$$

③

$$P(z=R_y, z_2=R_y | y=R_y, x=R) = 9/16 = \frac{3}{4} \cdot \frac{3}{4}$$

$$P(z=R_y, z_2=R_y | y=L_f, x=R) = 1/16 = \frac{1}{4} \cdot \frac{1}{4}$$

$$P(z=R_y, z_2=R_y | y=R_y, x=B) = 0$$

$$P(z=R_y, z_2=R_y | y=L_f, x=B) = 1$$

$$\begin{aligned} P(z=R_y, z_2=R_y | y=R_y) &= P(z=R_y, z_2=R_y | y=R_y, x=R) \cdot P(x=R) + \\ &+ P(z=R_y, z_2=R_y | y=R_y, x=B) \cdot P(x=B) = \frac{9}{16} \cdot \frac{2}{3} + 0 \cdot \frac{1}{3} \\ &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} P(z=R_y, z_2=R_y) &= P(z=R_y, z_2=R_y | y=R_y, x=R) \cdot P(x=R) \cdot P(y=R_y) + \\ &+ P(z=R_y, z_2=R_y | y=R_y, x=B) \cdot P(x=B) \cdot P(y=R_y) + \\ &+ P(z=R_y, z_2=R_y | y=L_f, x=R) \cdot P(x=R) \cdot P(y=L_f) + \\ &+ P(z=R_y, z_2=R_y | y=L_f, x=B) \cdot P(x=B) \cdot P(y=L_f) = \end{aligned}$$

$$= \frac{9}{16} \cdot \frac{1}{2} \cdot \frac{2}{3} + 0 \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{16} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{9}{48} + \frac{1}{48} + \frac{1}{6} = \frac{18}{48} = \frac{3}{8}$$

$$\begin{aligned} P(y=R_y | z=R_y, z_2=R_y) &= \frac{P(z=R_y, z_2=R_y | y=R_y) \cdot P(y=R_y)}{P(z=R_y, z_2=R_y)} = \frac{\frac{3}{8} \cdot \frac{1}{2}}{\frac{3}{8}} = \frac{1}{2} \end{aligned}$$

$$(4) P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y = R_y, x = R) = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$$

$$P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y = L_f, x = R) = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{64}$$

$$P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y = R_y, x = B) = 0$$

$$P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y = L_f, x = R) = 1$$

$$\begin{aligned} P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y_1 = R_y) &= P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y_1 = R_y, x = R) \cdot P(x = R) + \\ &+ P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y_1 = R_y, x = B) \cdot P(x = B) = \frac{27}{64} \cdot \frac{2}{3} + 0 \cdot \frac{1}{3} \\ &= \frac{54}{192} \end{aligned}$$

$$\begin{aligned} P(z_1 = R_y, z_2 = R_y, z_3 = R_y) &= P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y = R_y, x = R) \cdot P(x = R) \cdot P(y = R_y) + \\ &+ P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y = R_y, x = B) \cdot P(x = B) \cdot P(y = R_y) + \\ &+ P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y = L_f, x = R) \cdot P(x = R) \cdot P(y = L_f) + \\ &+ P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y = L_f, x = B) \cdot P(x = B) \cdot P(y = L_f) = \\ &= \frac{27}{64} \cdot \frac{1}{2} \cdot \frac{2}{3} + 0 \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{64} \cdot \frac{1}{2} \cdot \frac{2}{3} + 1 \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{27}{192} + \frac{1}{192} + \frac{1}{6} = \\ &= \frac{60}{192} \end{aligned}$$

$$\begin{aligned} P(y = R_y | z_1 = R_y, z_2 = R_y, z_3 = R_y) &= \frac{P(z_1 = R_y, z_2 = R_y, z_3 = R_y | y = R_y) \cdot P(y = R_y)}{P(z_1 = R_y, z_2 = R_y, z_3 = R_y)} = \frac{\frac{54}{192} \cdot \frac{1}{2}}{\frac{60}{192}} = \\ &= \frac{9}{20} \end{aligned}$$

$$⑤ P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=R_y, x=R) = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{81}{256}$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=L_f, x=R) = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{256}$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=R_y, x=B) = 0$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=L_f, x=B) = 1$$

$$\begin{aligned} P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=R_y) &= P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=R_y, x=R) \cdot P(x=R) + \\ &+ P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=R_y, x=B) \cdot P(x=B) = \\ &= \frac{81}{256} \cdot \frac{2}{3} + 0 \cdot \frac{1}{3} = \frac{162}{768} \end{aligned}$$

$$\begin{aligned} P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y) &= P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=R_y, x=R) \cdot P(y=R_y) \cdot P(x=R) + \\ &+ P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=R_y, x=B) \cdot P(y=R_y) \cdot P(x=B) + \\ &+ P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=L_f, x=R) \cdot P(y=L_f) \cdot P(x=R) + \\ &+ P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=L_f, x=B) \cdot P(y=L_f) \cdot P(x=B) = \\ &= \frac{81}{256} \cdot \frac{1}{2} \cdot \frac{2}{3} + 0 \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{256} \cdot \frac{1}{2} \cdot \frac{1}{3} + 1 \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{81}{768} + \frac{1}{768} + \frac{1}{6} = \\ &= \frac{210}{768} \end{aligned}$$

$$P(y=R_y | z=R_y, z_1=R_y, z_2=R_y, z_3=R_y) = \frac{P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y | y=R_y) \cdot P(y=R_y)}{P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y)}$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y)$$

$$= \frac{162}{768} \cdot \frac{1}{2} \cdot \frac{768}{210} = \frac{27}{70}$$

$$(6) P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=R_y, x=R) = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{4} = \frac{27}{256}$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=L_f, x=R) = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{256}$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=R_y, x=B) = 0$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=L_f, x=B) = 0$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=R_y) \text{ total} = P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=R_y, x=R) \cdot P(x=R) +$$

$$+ P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=R_y, x=B) \cdot P(x=B) =$$

$$= \frac{27}{256} \cdot \frac{2}{3} + 0 \cdot \frac{1}{3} = 54/768$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f) = P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=R_y, x=R) \cdot P(x=R) \cdot P(y=R_y) +$$

$$+ P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=L_f, x=R) \cdot P(x=R) \cdot P(y=L_f) +$$

$$+ P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=R_y, x=B) \cdot P(x=B) \cdot P(y=R_y) +$$

$$+ P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=L_f, x=B) \cdot P(x=B) \cdot P(y=L_f) =$$

$$= \frac{27}{256} \cdot \frac{1}{2} \cdot \frac{1}{3} + \frac{3}{256} \cdot \frac{1}{2} \cdot \frac{2}{3} + 0 \cdot \frac{1}{2} \cdot \frac{1}{3} + 0 \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{27}{768} + \frac{3}{768} =$$

$$= 30/768$$

$$P(y=R_y | z=R_y, z_1=R_y, z_2=R_y, z_3=L_f) = \frac{P(z=R_y, z_1=R_y, z_2=R_y, z_3=L_f | y=R_y) \cdot P(y=R_y)}{P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y)}$$

$$P(z=R_y, z_1=R_y, z_2=R_y, z_3=R_y)$$

$$= \frac{54}{768} \cdot \frac{1}{2} \cdot \frac{768}{30} = 9/10$$