
# Protecting HQC against differential power analysis attacks

Cryptography and Architectures for Computer Security project

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# Introduction

## What is a quantum computer?

A device that performs computations by exploiting the properties of quantum states:

- **Superposition**: the combination of quantum states is a quantum state
- **Entanglement**: the quantum state of a particle depends on the state of other particles

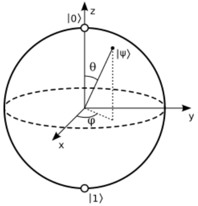
## Quantum Computing 101



### What is a qubit?

- The basic unit of quantum information
- Represented as a quantum superposition of two basis states

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



# Classic computers *vs* classic cryptography



## Symmetric cryptosystems

**Key enumeration**: perform an exhaustive search on the whole keyspace, $\mathcal{O}(n)$ time

## Asymmetric cryptosystems

**General number field sieve**: factor a composite number $n$ is $\mathcal{O}(e^{1.9(\log n)^{1/3}(\log\log n)^{2/3}})$ (subexponential) time

# Quantum computers *vs* classic cryptography



## Symmetric cryptosystems

**Grover's algorithm**: search in a database of $n$ elements in $\mathcal{O}(n^{1/2})$ time and $\mathcal{O}(\log n)$ space

## Asymmetric cryptosystems

**Shor's algorithm**: factor a composite number $n$, on a quantum computer, in $\mathcal{O}((\log n)^2(\log\log n)(\log\log\log n))$ (polynomial) time

# NIST Post-Quantum Cryptography Standardization



## Timeline

- 26/02/16: Announcement of NIST's call for submissions
- 21/12/17: Announcement of the first round candidates (69)
- 30/01/19: Announcement of the second round candidates (26)
- 22/07/20: Announcement of the third round candidates (7 finalists, 8 alternatives)
- 2022-2024: Publication of the standard drafts

# NIST Post-Quantum Cryptography Standardization



## *Classes* of cryptosystems

- **Lattice**: KYBER, FrodoKEM, NTRU Prime, SABER, ...
- **Code-based**: BIKE, Classic McElliece, HQC, LEDAcrypt, ...
- **Supersingular elliptic curve isogeny**: SIKE
- **Mutivariate**: CFPKM, Giophantus

We now focus on code-based cryptosystems.

# Coding Theory 101

## Linear codes



Let $\mathcal{V}$ be a vector space of dimension $n$ over $\mathbb{F}_2$.

### Linear codes

A **linear code** $\mathcal{C}$ of length $n$ and dimension $k$ is a vector subspace of $\mathcal{V}$ of size $k$ (such a code is also denoted by $[n, k]$).
Elements of $\mathcal{C}$ are called **codewords**.

# Generating the code



Let $\mathcal{C}$ be a $[n, k]$ linear code.

## Generator Matrix

A **generator matrix** of $\mathcal{C}$ is $\mathbf{G} \in \mathbb{F}_2^{k \times n}$ if

$$\mathcal{C} = \{\mathbf{mG} \ , \ \mathbf{m} \in \mathbb{F}_2^k\}$$

Codewords are linear combinations of the rows of $\mathbf{G}$, so the rows of the generator matrix form a base of the vector subspace $\mathcal{C}$

## Parity check Matrix

A **parity check matrix** of $\mathcal{C}$ is $\mathbf{H} \in \mathbb{F}_2^{(n-k) \times n}$ if

$$\mathcal{C} = \{\mathbf{v} \in \mathbb{F}_2^n \mid \mathbf{H}\mathbf{v}^\top = \mathbf{0}\}$$

# Dual codes



Let $\mathcal{C}$ be a $[n, k]$ code, $\mathbf{G}$ and $\mathbf{H}$ its generator and parity check matrices, and $\mathbf{v}$ a codeword of $\mathcal{C}$

## Dual code

$\mathbf{H}$ is the generator of the dual code $\mathcal{C}^{\perp}$

$$\mathbf{G} = \left[ I_k \mid P \right] \qquad \mathbf{H} = \left[ -P^{\top} \mid I_{n-k} \right]$$

## Syndrome

We call **syndrome** of $\mathbf{v}$ the product $\mathbf{H}\mathbf{v}^{\top}$. From the definition, we have:

$$\mathbf{v} \in \mathcal{C} \Leftrightarrow \mathbf{H}\mathbf{v}^{\top} = \mathbf{0}$$

# Circulant Matrices



## Circulant matrix

Let $\mathbf{v} \in \mathbb{F}_2^p = (v_0, \cdots, v_{p-1})$. The circulant matrix induced by $\mathbf{v}$ is defined as:

$$\mathbf{circ}(\mathbf{v}) = \begin{pmatrix} v_0 & v_1 & \cdots & v_{p-1} \\ v_{p-1} & v_0 & \cdots & v_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ v_1 & v_2 & \cdots & v_0 \end{pmatrix} \in \mathbb{F}_2^{p \times p}$$

The algebra of $p \times p$ circulant matrices over $\mathbb{F}_2$ is isomorphic to the algebra of polynomials in the ring $\mathbb{F}_2[x]/(x^p - 1)$

$$\mathbf{circ}(\mathbf{v}) \simeq v_0 + v_1 x^1 + \cdots + v_{p-1} x^{p-1}$$

## Quasi-cyclic codes

A **quasi-cyclic code** $\mathcal{C}$ is a linear code $[n, k]$, with $n = pn_0$ and $k = pk_0$, such that every cyclic shift of a codeword by $n_0$ symbols yields another codeword

A generator matrix $\mathbf{G}$ of the QC-code $\mathcal{C}$ has the form:

$$\mathbf{G} = \begin{bmatrix} C_{1,1} & C_{1,2} & \cdots & C_{1,n_0} \\ C_{2,1} & C_{2,2} & \cdots & C_{2,n_0} \\ \vdots & \vdots & \ddots & \vdots \\ C_{k_0,1} & C_{k_0,2} & \cdots & C_{k_0,n_0} \end{bmatrix} \in \mathbb{F}_2^{k \times n}$$

where each entry $C_{i,j}$ is a $p \times p$ circulant matrix.

## Metrics



Let $\mathcal{C}$ be a $[n, k]$ code, and $\mathbf{v}, \mathbf{w}$ codewords of $\mathcal{C}$

### Hamming distance

The **Hamming distance** between two vectors is the Hamming weight of the difference of the vectors

$$d(\mathbf{v}, \mathbf{w}) = \mathtt{wt}(\mathbf{v} - \mathbf{w})$$

### Minimum distance

The **minimum distance** of a code is the minimum distance among any couple of codewords

$$d_{min} = \min_{\mathbf{v}, \mathbf{w} \in \mathcal{C}, \mathbf{v} \neq \mathbf{w}} \mathtt{wt}(\mathbf{v} - \mathbf{w})$$

## Transmission errors

When travelling on a channel, the original codeword $\mathbf{x}$ might be corrupted, so that the received codeword is

$$\mathbf{y} = \mathbf{x} + \mathbf{e}, \text{ with } \mathbf{e} \text{ error vector}$$

For $\mathtt{wt}(\mathbf{e}) \leq \delta = \lfloor \frac{d_{min}-1}{2} \rfloor$ we are able to recover the original codeword

# Decoding a code



Let $\mathbf{y} = \mathbf{x} + \mathbf{e}$ be the received codeword

## Minimum distance decoding

Pick a codeword $\mathbf{u}$ that minimizes the Hamming distance $d(\mathbf{u}, \mathbf{y})$

$$\text{Decode}(\mathbf{y}) = \arg\min_{\mathbf{u} \in \mathcal{C}} d(\mathbf{u}, \mathbf{y})$$

## Syndrome decoding

By the definition of syndrome of a codeword we have:

$$\mathbf{Hy} = \mathbf{H}(\mathbf{x} + \mathbf{e}) = \mathbf{Hx} + \mathbf{He} = \mathbf{0} + \mathbf{He} = \mathbf{He}$$

# Hard problems of coding theory



## Syndrome decoding problem [BMVT78]

Given a $[n, k]$ code $\mathcal{C}$ with parity matrix $\mathbf{H}$, the syndrome $\mathbf{y} \in \mathbb{F}_2^{n-k}$ and $w \leq n$, find the codeword $\mathbf{x} \in \mathbb{F}_2^n$ such that $\mathbf{H}\mathbf{x}^\top = \mathbf{y}^\top$ and $\mathtt{wt}(\mathbf{x}) = w$.

# Hamming Quasi-Cyclic

# Overview



## HQC

One of the third round alternative candidates

- IND-CPA Public Key Encryption
- IND-CCA2 Key Encapsulation Mechanism
- Based on a variant of the syndrome decoding problem
- Employs quasi-cyclic codes to shorten key sizes

## Codes

HQC employs two codes:

- a decodable $\mathcal{C}[n, k]$ generated by $\mathbf{G} \in \mathbb{F}_2^{k \times n}$ (public)
- a random $[2n, n]$ code with parity-check matrix $\mathbf{H} = (\mathbf{I}_n | \mathbf{circ}(\mathbf{h}))$

# Public Key Encryption



## Setup

Generate the parameters $(n, k, \delta, w, w_r, w_e)$ for the corresponding security level $\lambda$

## Key Generation

- Generate $seed_h \xleftarrow{\$} \{0,1\}^\lambda$
- Generate $\mathbf{h} \xleftarrow{\mathrm{PRNG}(seed_h)} \mathbb{F}_2^n$
- Generate $(\mathbf{x}, \mathbf{y}) \xleftarrow{\$} \mathbb{F}_2^n$, with $\mathtt{wt}(\mathbf{x}) = \mathtt{wt}(\mathbf{y}) = w$
- Calculate $\mathbf{s} = \mathbf{x} + \mathbf{hy}$

$\mathsf{pk} = (\mathbf{h}, \mathbf{s})$ $\mathsf{sk} = (\mathbf{x}, \mathbf{y})$

# Public Key Encryption



To encrypt a message $\mathbf{m} \in \mathbb{F}_2^k$ :

## Encryption

- Generate $(\mathbf{r_1}, \mathbf{r_2}) \xleftarrow{\$} \mathbb{F}_2^n$, with $\mathtt{wt}(\mathbf{r_1}) = \mathtt{wt}(\mathbf{r_2}) = w_r$
- Generate $\mathbf{e} \xleftarrow{\$} \mathbb{F}_2^n$, with $\mathtt{wt}(\mathbf{e}) = w_e$
- Calculate $\mathbf{u} = \mathbf{r_1} + \mathbf{hr_2}$
- Calculate $\mathbf{v} = \mathbf{mG} + \mathbf{sr_2} + \mathbf{e}$

$$\mathrm{ctx} = (\mathbf{u}, \mathbf{v})$$

## Decryption

From the ctx $= (\mathbf{u}, \mathbf{v})$ and the private key pk $= (\mathbf{x}, \mathbf{y})$:

$$\begin{aligned}\mathbf{v} - \mathbf{u} \cdot \mathbf{y} &= (\mathbf{mG} + \mathbf{s} \cdot \mathbf{r_2} + \mathbf{e}) - (\mathbf{r_1} + \mathbf{h} \cdot \mathbf{r_2})\mathbf{y} = \mathbf{mG} + \mathbf{x} \cdot \mathbf{r_2} - \mathbf{r_1} \cdot \mathbf{y} + \mathbf{e} \\ &= \mathbf{mG} + (\mathbf{x} + \mathbf{h} \cdot \mathbf{y})\mathbf{r_2} + \mathbf{e} - (\mathbf{r_1} \cdot \mathbf{y} + \mathbf{h} \cdot \mathbf{r_2} \cdot \mathbf{y}) \\ &= \mathbf{mG} + \mathbf{x} \cdot \mathbf{r_2} + \mathbf{h} \cdot \mathbf{y} \cdot \mathbf{r_2} + \mathbf{e} - \mathbf{r_1} \cdot \mathbf{y} - \mathbf{h} \cdot \mathbf{r_2} \cdot \mathbf{y} \\ &= \mathbf{mG} + \mathbf{x_2} - \mathbf{r_1} \cdot \mathbf{y} + \mathbf{e}\end{aligned}$$

So we have $\mathbf{m} = \mathcal{C}.\text{Decode}(\mathbf{v} - \mathbf{u} \cdot \mathbf{y})$ whenever

$$\mathtt{wt}(\mathbf{x} \cdot \mathbf{r_2} - \mathbf{r_1} \cdot \mathbf{y} + \mathbf{e}) \leq \delta$$

# Key Encapsulation Mechanism



By applying the Fujisaki-Okamoto transformation it is possible to build a IND-CCA2 KEM. Let $\mathcal{G}, \mathcal{H}, \mathcal{K}$ be hash functions and $\mathcal{E}$ an instance of HQC.PKE

- **Setup**: generate the parameters $\lambda$ as in HQC.PKE, except that k will be the length of the key to be exchanged
- **KeyGen**: as in HQC.PKE

# Key Encapsulation Mechanism



Let $\mathcal{G}, \mathcal{H}, \mathcal{K}$ be hash functions and $\mathcal{E}$ an instance of HQC.PKE

## Encapsulation

- Generate the seed $\mathbf{m} \xleftarrow{\$} \mathbb{F}_2^k$ and the randomness $\theta \leftarrow \mathcal{G}(\mathbf{m})$
- Generate the ciphertext $\mathbf{c} \leftarrow (\mathbf{u}, \mathbf{v}) = \mathcal{E}.\mathsf{Encrypt}(\mathsf{pk}, \mathbf{m}, \theta)$
- Derive the symmetric key $K = \mathcal{K}(\mathbf{m}, \mathbf{c})$
- Calculate $\mathbf{d} \leftarrow \mathcal{H}(\mathbf{m})$
- Send the pair $(\mathbf{c}, \mathbf{d})$

# Key Encapsulation Mechanism



Let $\mathcal{G}, \mathcal{H}, \mathcal{K}$ be hash functions and $\mathcal{E}$ an instance of HQC.PKE

## Decapsulation

- Decrypt $\mathbf{m}' \leftarrow \mathcal{E}.\mathrm{Decrypt}(\mathrm{sk}, \mathbf{c})$
- Compute $\theta' \leftarrow \mathcal{G}(\mathbf{m}')$ and $\mathbf{c}' \leftarrow \mathcal{E}.\mathrm{Encrypt}(\mathrm{pk}, \mathbf{m}', \theta')$
- Check that $\mathbf{c} = \mathbf{c}'$ and $\mathbf{d} = \mathcal{H}(\mathbf{m}')$
- Derive the shared key $K \leftarrow \mathcal{K}(\mathbf{m}, \mathbf{c})$

# Masking

# Side Channel Attacks



Implementations of theoretically secure cryptosystems may *leak* some information, allowing an attacker to extract secret data.

## Side Channel Vulnerabilities

Side channel attacks take use of different leakage sources, such as:

- Execution time
- Power consumption
- Electromagnetic radiation
- Differential faults

# Power Analysis



### Simple Power Analysis

Analyze the electrical current drawn by the device over time: different operations may have different power consumption.

### Differential Power Analysis

Statistically analyze the power consumption of the device over different executions: detect biases between, for example, a known secret key and a unknown one.

# Power Analysis



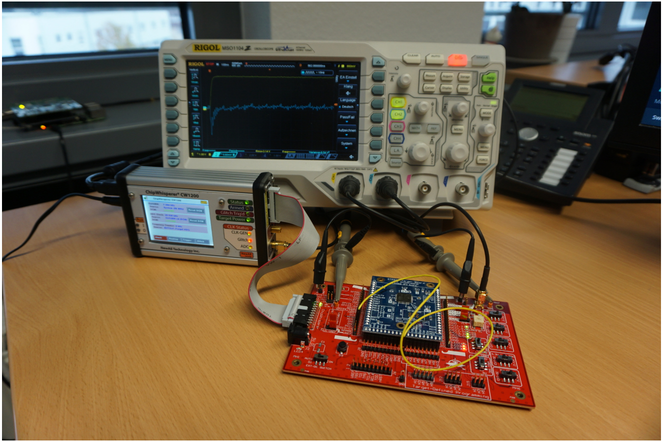



Figure: ChipWhisperer

# Countermeasures to Power Analysis



Power analysis attacks are usually passive, so they cannot be detected by the device.

To mitigate the effectiveness of these attacks, it is possible to:

- **SPA**: avoid branches with conditions that depend on secret data
- **DPA**: *mask* the secret data when performing operations on them

## Masking

Given a secret $x$, we can split it into $d$ *shares* in such a way that

$$x = x_0 \odot x_1 \cdots \odot x_{d-1}$$

for some operation $\odot$ (e.g. XOR in binary fields).

# Securing AND gates



## Probing Model [ISW03]

Given an algorithm that operates on data split among $d$ shares, we say that the algorithm is secure against a $(d-1)$-th order probing attack if, on input $x = (x_1, \cdots, x_d)$, it admits no tuple of $d-1$ (or less) shares that depends on $x$

## Ishai-Sahai-Wagner's Scheme

Let $x = (x_1, \cdots, x_d), y = (y_1, \cdots, y_d)$ binary variables: to securely compute $x \wedge y$ at order $\lfloor d/2 \rfloor$:

- Pick a random bit $r_{i,j}$
- Compute $r_{j,i} = (r_{i,j} + (x_i \wedge y_j)) + (x_j \wedge y_i)$
- Compute $c_i = (x_i \wedge y_i) + \sum_{j \neq i} r_{i,j}$

## Securing multiplications



The ISW scheme can be extended from $\mathbb{F}_2$ to $\mathbb{F}_2^n$, increasing its security to $d$-th order attacks

### Rivain-Prouff's Scheme

Let $a = (a_1, \cdots, a_d), b = (b_1, \cdots, b_d)$, with $a_i, b_i \in \mathbb{F}_2^n$

- Calculate $c_i \leftarrow a_i \cdot b_i$ for each share

For $i$ from 1 to $d$ and $j$ from $i+1$ to $d$:

- Extract a random value $s \xleftarrow{\$} \mathbb{F}_2^n$
- Calculate $s' \leftarrow (s + (a_i \cdot b_j)) + (a_j \cdot b_i)$
- Calculate $c_i \leftarrow c_i + s$ and $c_j \leftarrow c_j + s'$

The sum of the shares $(c_1, \cdots, c_d)$ yields the product $a \cdot b$

# Implementation

## Starting point



HQC has two software implementations

### HQC implementations

- `reference`: implemented in C, no optimizations, not constant-time, sparse-dense multiplications
- `optimized`: implemented using AVX2 instructions, vectorized decoding, Karatsuba dense-dense multiplications

Our implementation starts from the `reference`.

# Target architecture

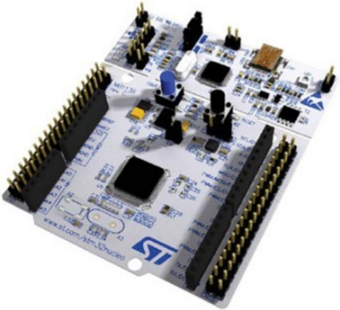


Our implementation is oriented torwards embedded devices

## Target

Our target is the STM32F401RE board:

- Cortex-M4 processor (`ARMv7E` architecture)
- 512KB Flash Memory
- 96KB RAM
- No builtin RNG



# Objects representations



- Random values: generated from a 40B seed with `seedexpander` functions
- Codewords: as polynomials (through arrays), in two possible forms:
  - dense: each bit represents a coefficient of the corresponding degree, implemented with `uint64_t` arrays of `VEC_N_SIZE_64` elements
  - sparse: elements correspond to the degrees with nonzero coefficient, implemented with `uint32_t` arrays of `PARAM_OMEGA` elements

# Masking



## Shares representations

The number of shares is a parameter of the cryptosystem, defined at compile time.
Shares are defined via the `shares_t` struct, containing a number of dense vectors equal to the number of shares

## Splitting secrets into shares

The arrays containing secret data are divided in a number of blocks equal to the number of shares.
Each block is copied in the corresponding share, and elements outside the block are zeroed.

# Operations on shares



## Reducing shares to an array

When operations on secret data are completed, the shares of the secret element are added together with a XOR.

## Adding shares

When adding up two `shares_t` a random mask is generated and *distributed* among the shares, so that by reducing to a single element the random mask cancels out.

# Multiplication



In the optimized version the authors employ a Karatsuba dense-dense multiplication strategy

- Makes the operation time-constant (with some tweaks)
- Is significantly slower than the sparse-dense strategy

The sparsity of the operands (17669 vs 75) makes the preexisting sparse-dense multiplication more suitable in our case.

## Sparse-dense multiplication

Variation of the schoolbook shift-and-add strategy.
Let $A$ be the dense polynomial, $B$ the sparse one, we have
$A \cdot X^{B_i} = A \cdot X^{B_i \bmod ts} \cdot X^{\lfloor \frac{B_i}{ts} \rfloor}$

# Multiplication masking



## `safe_mul`

Application of the Rivain-Prouff scheme:

- Operand size is reduced
- Apply polynomial reduction modulo $x^n + 1$ to multiplication results
- Loop unrolling

## Where to apply `safe_mul`

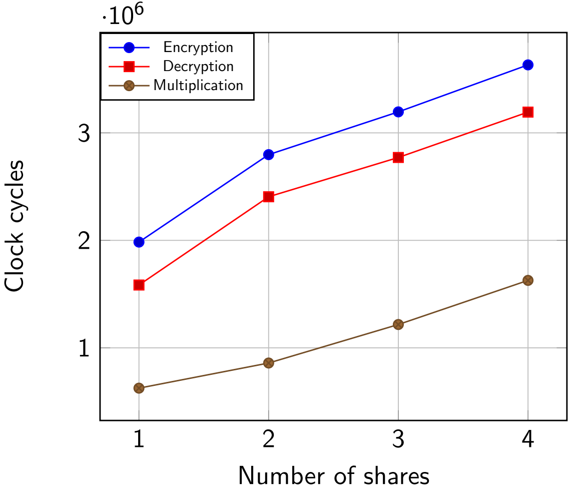
The masked multiplication is more expensive, so it is applied only to calculate:

- Encryption: $\mathbf{s} \cdot \mathbf{r}_2$ (to calculate $\mathbf{v}$)
- Decryption: $\mathbf{u} \cdot \mathbf{y}$ (to calculate the codeword to decode)

# Results

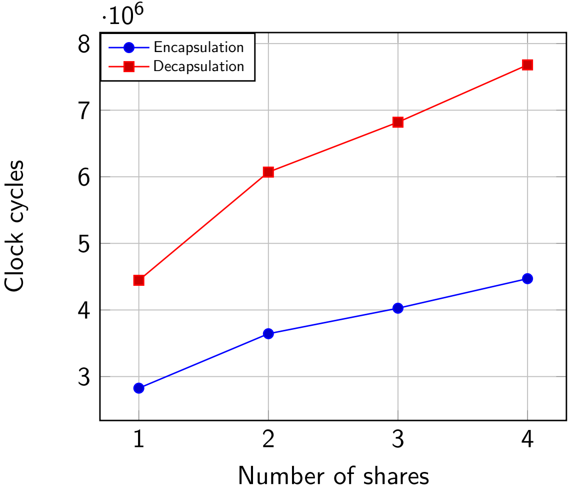
# Execution time





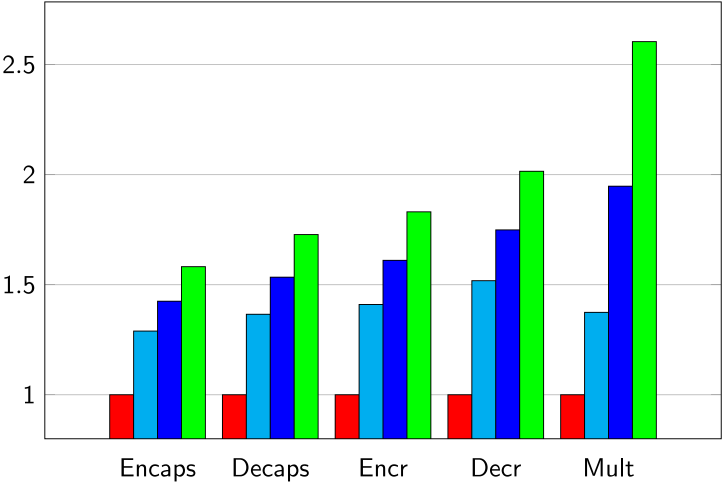
# Execution time





# Performance loss





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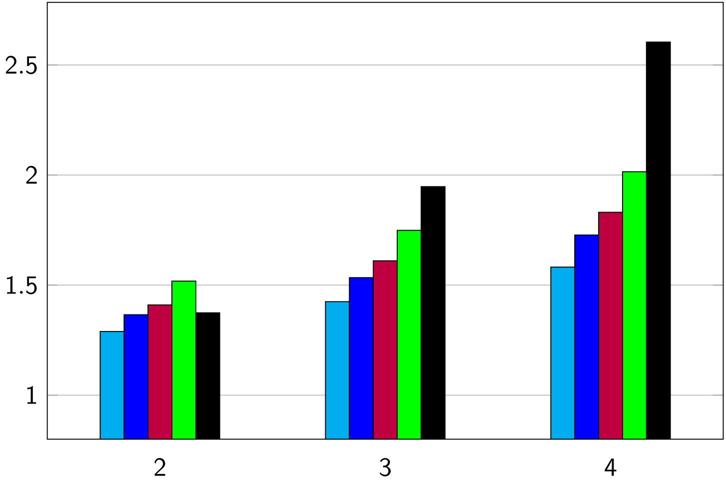

## Welch's t-test

Given two statistical populations $X_1$ and $X_2$ of $N_1$ and $N_2$ samples respectively:

$$t = \frac{\bar{X_1} - \bar{X_2}}{\sqrt{\frac{s^2_{X_1}}{N_1} + \frac{s^2_{X_2}}{N_2}}}$$

Can be used to verify that the means of the distributions are equal (null hypothesis).
Setting the confidence to 99.999%, we accept $H_0$ for $|t| \leq 4.5$

### Test Vector Leakage Assessment

Based on the Welch's t-test, defines the two groups as:

- Random: use different inputs for each computation
- Fixed: use the same inputs for all computations

In case of random data (*e.g.* masks), these are always generated randomly.

# Constant-time execution



## Welch's t

| # | encaps | decaps | enc | dec | mul |
|---|---|---|---|---|---|
| 1 | 0.94 | 4.57 | 25.66 | 10.26 | 30.09 |
| 2 | 6.00 | 3.36 | 4.62 | 5.96 | 4.96 |
| 3 | 0.29 | 1.69 | 1.60 | 5.88 | 3.51 |
| 4 | 0.13 | NA[a] | 3.30 | 4.01 | 0.22 |

[a] Not enough memory

# References I




- Elwyn Berlekamp, Robert McEliece, and Henk Van Tilborg, *On the inherent intractability of certain coding problems (corresp.)*, IEEE Transactions on Information Theory **24** (1978), no. 3, 384–386.
- Yuval Ishai, Amit Sahai, and David Wagner, *Private circuits: Securing hardware against probing attacks*, Annual International Cryptology Conference, Springer, 2003, pp. 463–481.