

Cryptography and Architectures for Computer Security - Cheat Sheet

Information Theory

$$Pr(C = c) = \sum_{k: c \in \{\mathbb{E}_k(m), \forall m \in \mathcal{M}\}} Pr(K = k) Pr(P = \mathbb{D}_k(c))$$

$$\textbf{Perfect secrecy } Pr(P = m | C = c) = Pr(P = m) \implies |\mathcal{K}| = |\mathcal{C}| = |\mathcal{P}|$$

$$\textbf{Entropy } H(X) = - \sum_{i=1}^n p_i \log_2 p_i \quad (p_i \log_2 p_i = 0 \text{ for } p_i = 0)$$

$$H(X, Y) = - \sum_{i=1}^n \sum_{j=1}^m Pr(X = x_i, Y = y_j) \log_2 Pr(X = x_i, Y = y_j)$$

$$H(X|Y = y) = - \sum_{i=1}^n \sum_{j=1}^m Pr(X = x_i | Y = y) \log_2 Pr(X = x_i | Y = y)$$

$$H(X|Y) = - \sum_{i=1}^n \sum_{j=1}^m Pr(Y = y_j) Pr(X = x_i, Y = y_j) \log_2 Pr(X = x_i | Y = y_j)$$

$$H(X) + H(Y) \geq H(X, Y); H(X, Y) = H(Y) + H(X|Y); H(X|Y) \leq H(X)$$

$$\textbf{Key equivocation } H(K|C) = H(P) + H(K) - H(C)$$

$$\textbf{Language redundancy } R_L = 1 - \frac{H_L}{\log_2 |\mathcal{M}|}$$

$$\textbf{Spurious keys } \bar{s}_n \geq \frac{|\mathcal{K}|}{|\mathcal{M}|^{nR_L}} - 1$$

$$\textbf{Unicity distance } n_0 \approx \frac{\log_2 |\mathcal{K}|}{R_L \log_2 |\mathcal{M}|}$$

Symmetric Ciphers

Modes of Operation

$$\textbf{ECB } c_i = \mathbb{E}_k(m_i)$$

$$\textbf{CBC } c_0 = IV, c_i = \mathbb{E}_k(m_i \oplus c_{i-1})$$

$$\textbf{CFB/OFB } ISR_0 = IV, OSR_i = \mathbb{E}_k(ISR_{i-1}), c_i = m_i \oplus \text{j-th leftmost bits of } OSR_i$$

$$\textbf{CTR } ctr_i = IV + i, t_i = \mathbb{E}_k(ctr_i), c_i = t_i \oplus m_i$$

Cryptanalysis

$$\textbf{Pile-up lemma } Pr(Z_1 \oplus \dots \oplus Z_n = 0) = \frac{1}{2} + 2^{n-1} \prod_{i=1}^n \varepsilon_i$$

Hash Functions

$$\textbf{First preimage } Pr(m_i | d = h(m_i)) \approx \frac{q}{|D|}$$

$$\textbf{Second preimage } Pr(h(m_i) = h(m)) = \frac{q-1}{|D|}$$

$$\textbf{Collision } Pr(\text{no collisions}) = e^{-\frac{q(q-1)}{2|D|}} \implies q \leq 1.774\sqrt{|D|}$$

Algebraic Structures

Elliptic Curves

$$\text{For } \mathbb{K} = \mathbb{F}_p, p \geq 3: \Delta = 4a^3 + 27b^2 \neq 0$$

$$\mathbb{E}(\mathbb{F}_p) : y^2 = x^3 + ax + b \text{ for } a, b \in \mathbb{F}_p, p \geq 3$$

$$x_3 = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)^2 - x_1 - x_2, y_3 = \left(\frac{y_1 - y_2}{x_1 - x_2}\right)(x_1 - x_3) - y_1$$

$$x_4 = \left(\frac{3x_1^2 + a}{2y_1}\right)^2 - 2x_1, y_4 = -y_1 + \left(\frac{3x_1^2 + a}{2y_1}\right)(x_1 - x_4)$$

$$\begin{aligned}
-P_1 &= (x_1, -y_1) \\
\mathbb{E}(\mathbb{F}_{2^m}) : y^2 + xy &= x^3 + ax^2 + b \text{ for } a, b \in \mathbb{F}_{2^m}, \Delta = b \neq 0 \\
x_3 &= \left(\frac{y_1+y_2}{x_1+x_2}\right)^2 + \left(\frac{y_1+y_2}{x_1+x_2}\right) + x_1 + x_2 + a, y_3 = \left(\frac{y_1+y_2}{x_1+x_2}\right)(x_1 + x_3) + x_3 + y_1 \\
x_4 &= x_1^2 + \frac{b}{x_1^2}, y_4 = x_1^2 + \left(x_1 + \frac{y_1}{x_1}\right)x_3 + x_3 \\
-P_1 &= (x_1, x_1 + y_1)
\end{aligned}$$

Public Key Cryptosystems

RSA

$$\begin{aligned}
\textbf{Keys } k_{pub} &= \langle n, e \rangle, k_{priv} = \langle p, q, \varphi(n), d \rangle \\
n &= p \cdot q, \gcd(e, \varphi(n)) = 1, d = e^{-1} \pmod{\varphi(n)} \\
c &= m^e \pmod{\varphi(n)} \pmod{n}, m = c^d \pmod{\varphi(n)} \pmod{n} \\
\textbf{CRT } m_p &\equiv_p c^d \pmod{p-1}, m_q \equiv_q c^d \pmod{q-1}, m \equiv_n m_p q(q^{-1} \pmod{p}) + m_q p(p^{-1} \pmod{q})
\end{aligned}$$

ElGamal

$$\begin{aligned}
\textbf{Keys } k_{pub} &= \langle n, g, g^s \rangle, k_{priv} = \langle s \rangle \\
ctx &= \langle \gamma, \delta \rangle = \langle g^l, ptx(g^s)^l \rangle, ptx = \gamma^{n-s} \delta \\
sign &= \langle ptx, \langle \gamma, \delta \rangle \rangle = \langle ptx, g^l, l^{-1}(h(m) - s \cdot h(\gamma)) \pmod{n} \rangle
\end{aligned}$$

DSS-DSA

$$\textbf{Keys } k_{pub} = \langle p, q, g, g^s \rangle, k_{priv} = (s); q|(p-1), q = |\langle g \rangle|$$

Montgomery Multiplication

Number Theoretical Cryptanalysis

Primality test

$$\begin{aligned}
\textbf{Fermat } n \text{ is composite} &\implies a^{n-1} \not\equiv_n 1 \text{ with probability } > \frac{1}{2} \\
\textbf{Miller-Rabin } n-1 &= d2^r : a^d \not\equiv_n \pm 1 \text{ and } a^{d2^r} \not\equiv -1 \implies n \text{ is composite } (r \leq s-1)
\end{aligned}$$

Factoring

$$\begin{aligned}
\textbf{Fermat } x &= \lceil \sqrt{n} \rceil, y = x^2 - n, \text{ until } y \text{ is a perfect square } y = y + 2x + 1, x = x + 1, \text{ then the factors are } x \pm \sqrt{y} \\
\textbf{Pollard's } \rho &\text{ pick } a, b \text{ at random (e.g. } x_0 = 2, x_i = x_{i-1}^2 + 1 \pmod{n}), \text{ if } \gcd(a-b, n) \neq 1 \text{ the result is a factor} \\
\textbf{Pollard's p-1 } p &\text{ B-power-smooth, } a = 2^{B!}, \text{ so } p = \gcd(a-1, n)
\end{aligned}$$

DLog

$$\textbf{Polig-Hellman for each prime factor } \eta = g^{\frac{n}{p}}, \gamma_i = \gamma_{i-1} g^{l_{i-1} p^{i-1}}, \delta_i = (\beta \gamma_i^{-1})^{\frac{n}{p^{i+1}}}, l_i = \log_{\eta} \delta_i$$

Misc