Teoria delle strutture - PROBLEMA 4

```
% serbatoio pieno d'acqua shematizzato come guscio sottile
%% Clear workspace and close any open windows
clear all
close all
```

SOLUZIONE PUNTO 1 & 2

```
% 1) Assumendo che il serbatoio possa essere schematizzato come un guscio sottile, scrivere
%l’equazione differenziale e le condizioni al bordo che permettono di determinare la soluzione
%di questo problema di equilibrio.

% 2) Determinare le espressioni dello spostamento trasversale e delle caratteristiche della
%sollecitazione (semplificare il problema relativamente al calcolo delle componenti flessionali
%della sollecitazione, assumendo nel calcolo che il guscio sia di lunghezza infinita).
```

simbolico

```
syms z  A1 A2 E R nu t h gamma beta D real

p=-gamma*(h-z) % Carico idrostatico
```

p = $-\gamma\ (h-z)$

Variabili numeriche

```
R_n=1000; % raggio serbatoio [mm]
E_n=210000; % Young's modulus steel [N/mm^2] se.. 2.1E11[N/m^2]
nu_n=0.3; % coefficiente di Poisson acciaio
t_n=R_n/50; %spessore serbatoio
h_n=5*R_n; %altezza serbatoio
gamma_n=0.000001; % acqua N/mm3
p_n=-gamma_n*(h_n-z); % Carico idrostatico
D_n=(E_n*t_n^3)/(12*(1-nu_n^2)); % rigidezza
beta_n=((E_n*t_n*R_n^2)/D_n)^(1/4);
```

Soluzione omogenea

```
wo=A1*exp(-(beta*z)/(R*sqrt(2)))*cos((beta*z)/(R*sqrt(2)))...
    +A2*exp(-(beta*z)/(R*sqrt(2)))*sin((beta*z)/(R*sqrt(2)))
```

wo =

$$A_1\,\mathrm{e}^{-\frac{\sqrt{2}\,\beta\,z}{2\,R}}\cos\left(\frac{\sqrt{2}\,\beta\,z}{2\,R}\right)+A_2\,\mathrm{e}^{-\frac{\sqrt{2}\,\beta\,z}{2\,R}}\sin\left(\frac{\sqrt{2}\,\beta\,z}{2\,R}\right)$$

Soluzione particolare

```
wp=(p*R^2)/(E*t)
```

wp =

$$-\frac{R^2\,\gamma\ (h-z)}{\mathrm{E}\,t}$$

Soluzione

```
w(z)=wo+wp
```

w(z) =

$$A_1\,\mathrm{e}^{-\frac{\sqrt{2}\,\beta z}{2R}}\cos\left(\frac{\sqrt{2}\ \beta z}{2R}\right)+A_2\,\mathrm{e}^{-\frac{\sqrt{2}\,\beta z}{2R}}\sin\left(\frac{\sqrt{2}\ \beta z}{2R}\right)-\frac{R^2\gamma\ (h-z)}{\mathrm{E}\,t}$$

```
% Derivate
w_z(z)=diff(w,z) % derivata prima
```

w_z(z) =

$$\frac{R^2\gamma}{\mathrm{E}\,t}-\frac{\sqrt{2}\ A_1\,\beta\,\mathrm{e}^{-\sigma_1}\cos(\sigma_1)}{2R}+\frac{\sqrt{2}\ A_2\,\beta\,\mathrm{e}^{-\sigma_1}\cos(\sigma_1)}{2R}-\frac{\sqrt{2}\ A_1\,\beta\,\mathrm{e}^{-\sigma_1}\sin(\sigma_1)}{2R}-\frac{\sqrt{2}\ A_2\,\beta\,\mathrm{e}^{-\sigma_1}\sin(\sigma_1)}{2R}$$

where

$$\sigma_1=\frac{\sqrt{2}\ \beta z}{2R}$$

Applico le Condizioni al contorno:

```
cc1=w(0)==0
```

cc1 =

$$A_1-\frac{R^2\gamma\,h}{\mathrm{E}\,t}=0$$

```
cc2=w_z(0)==0
```

cc2 =

$$\frac{\sqrt{2}\ A_2\,\beta}{2R}-\frac{\sqrt{2}\ A_1\,\beta}{2R}+\frac{R^2\gamma}{\mathrm{E}\,t}=0$$

risolvo il sistema per trovare A1 e A2

```
S=solve([cc1 cc2],[A1 A2]);
A_1=S.A1(1)
```

A_1 =

$$\frac{R^2\gamma\,h}{\mathrm{E}\,t}$$

```
A_2=simplify(S.A2(1))
```

A_2 =

$$-\frac{R^2\gamma\ (\sqrt{2}\ R-\beta\,h)}{\mathrm{E}\,\beta\,t}$$

spostamento

```
w1(z)=simplify(subs(w,[A1 A2],[A_1 A_2]))
```

w1(z) =

$$\frac{R^2\gamma\,h\,\mathrm{e}^{-\sigma_1}\cos(\sigma_1)}{\mathrm{E}\,t}-\frac{R^2\gamma\ (h-z)}{\mathrm{E}\,t}-\frac{R^2\gamma\,\mathrm{e}^{-\sigma_1}\sin(\sigma_1)\ (\sqrt{2}\ R-\beta\,h)}{\mathrm{E}\,\beta\,t}$$

where

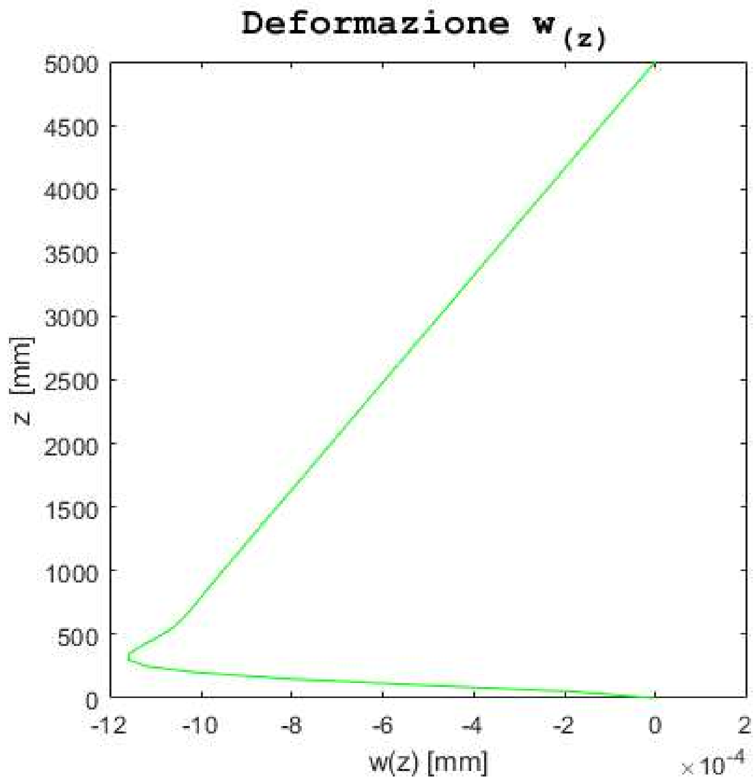
$$\sigma_1=\frac{\sqrt{2}\ \beta z}{2R}$$

soluzione numerica

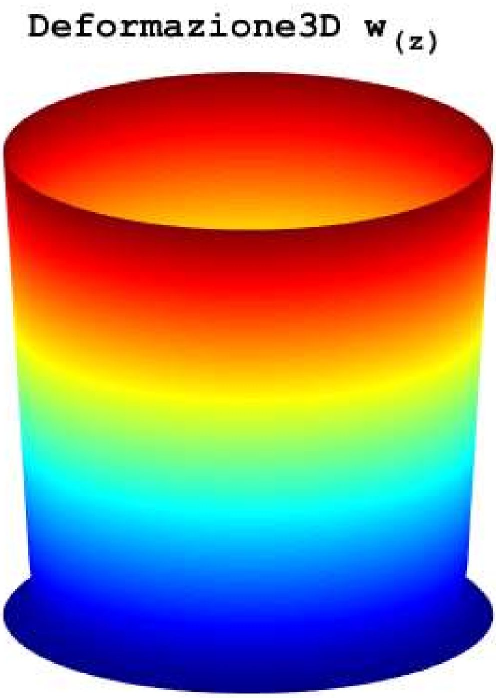
```
A_11=subs(A_1,[R gamma h E t],[R_n gamma_n h_n E_n t_n]);
```

```
A_22=subs(A_2,[R gamma h E t beta],[R_n gamma_n h_n E_n t_n beta_n]);
w_n(z)=A_11*exp(-(beta_n*z)/(R_n*sqrt(2)))*cos((beta_n*z)/(R_n*sqrt(2)))...
    +A_22*exp(-(beta_n*z)/(R_n*sqrt(2)))*sin((beta_n*z)/(R_n*sqrt(2)))...
    +(p_n*R_n^2)/(E_n*t_n);
wn_z(z)=diff(w_n,z); %derivata di w_n
wn_zz(z)=diff(wn_z,z); %derivata di wn_z

% Plot
%% Soluzione
a=0:h_n/(100-1):h_n;
figure(1)
plot(double(w_n(a)),a,'g')
title(['\fontname{Courier}\fontsize{15}Deformazione w_(_z_)'],'color','K');
ylabel('z  [mm]');
xlabel('w(z) [mm]');
axis square
```




```
figure(2)
fig = gcf; % current figure handle
fig.Color = [1 1 1];
fig.ToolBar = 'none';
colormap(jet(100))
c=100000; % coefficiente che amplifica la deformazione
w1_n=R_n+(double(w_n(a))*c);
[X,Y,Z]=cylinder(w1_n,100);
s1=surface(X,Y,Z,'FaceAlpha',1,'EdgeColor','k','LineWidth',0.01);
title(['\fontname{Courier}\fontsize{15}Deformazione3D w_(_z_) '],'color','K');
axis off
axis square
shading interp;
view([5.700 18.800])
rotate3d
```



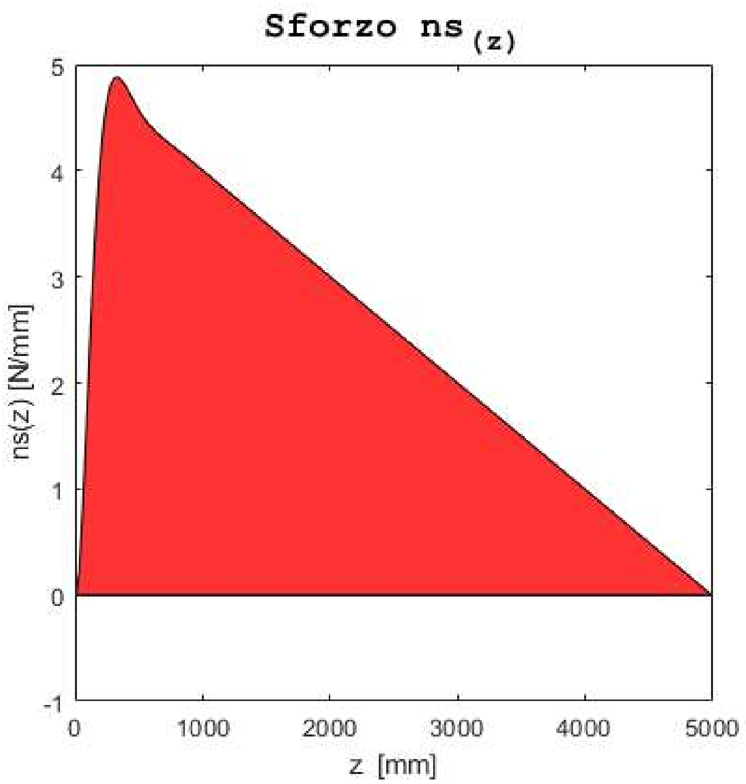
Calcolo CDS

```
ns(z)=simplify(-w*E*t/R)
```

ns(z) =

$$-\frac{\mathrm{E}\,t\left(A_1\,\mathrm{e}^{-\frac{\sqrt{2}\,\beta\,z}{2\,R}}\cos\left(\frac{\sqrt{2}\,\beta\,z}{2\,R}\right)+A_2\,\mathrm{e}^{-\frac{\sqrt{2}\,\beta\,z}{2\,R}}\sin\left(\frac{\sqrt{2}\,\beta\,z}{2\,R}\right)-\frac{R^2\,\gamma\,(h-z)}{\mathrm{E}\,t}\right)}{R}$$

```
%% Plot the ns(z) stresses
a=0:h_n/(200-1):h_n;
figure(3)
area(a,double(-w_n(a)*E_n*t_n/R_n),'FaceColor','r','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo ns_(_z_)'],'color','K');
xlabel('z  [mm]');
ylabel('ns(z) [N/mm]');
axis square
```




```
w_zz(z)=simplify(diff(w_z,z))
```

w_zz(z) =

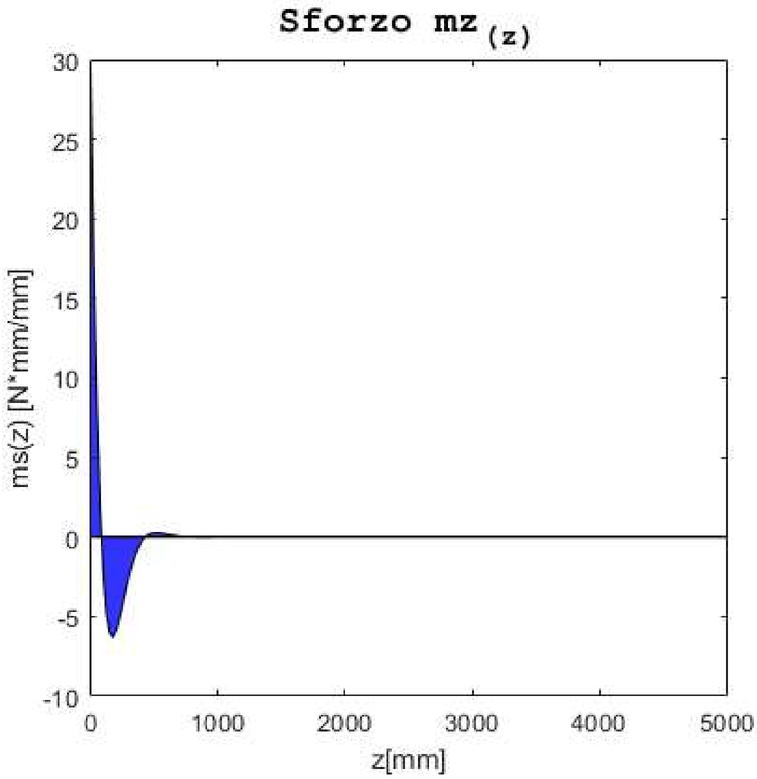
$$-\frac{\beta^2 \mathrm{e}^{-\frac{\sqrt{2}\,\beta z}{2R}} \left(A_2 \cos\left(\frac{\sqrt{2}\,\beta z}{2R}\right) - A_1 \sin\left(\frac{\sqrt{2}\,\beta z}{2R}\right)\right)}{R^2}$$

```
mz(z)=simplify(-D*w_zz)
```

mz(z) =

$$\frac{\mathrm{D}\,\beta^2 \mathrm{e}^{-\frac{\sqrt{2}\,\beta z}{2R}} \left(A_2 \cos\left(\frac{\sqrt{2}\,\beta z}{2R}\right) - A_1 \sin\left(\frac{\sqrt{2}\,\beta z}{2R}\right)\right)}{R^2}$$

```
%% Plot the mz(z) stresses
figure(4)
mz_n(z)=(-wn_zz*D_n);
area(a,double(mz_n(a)),'FaceColor','b','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo mz_(_z_)'],'color','K');
xlabel('z[mm]');
ylabel('ms(z) [N*mm/mm]');
axis square
```




```
tz(z)=simplify(diff(mz))
```

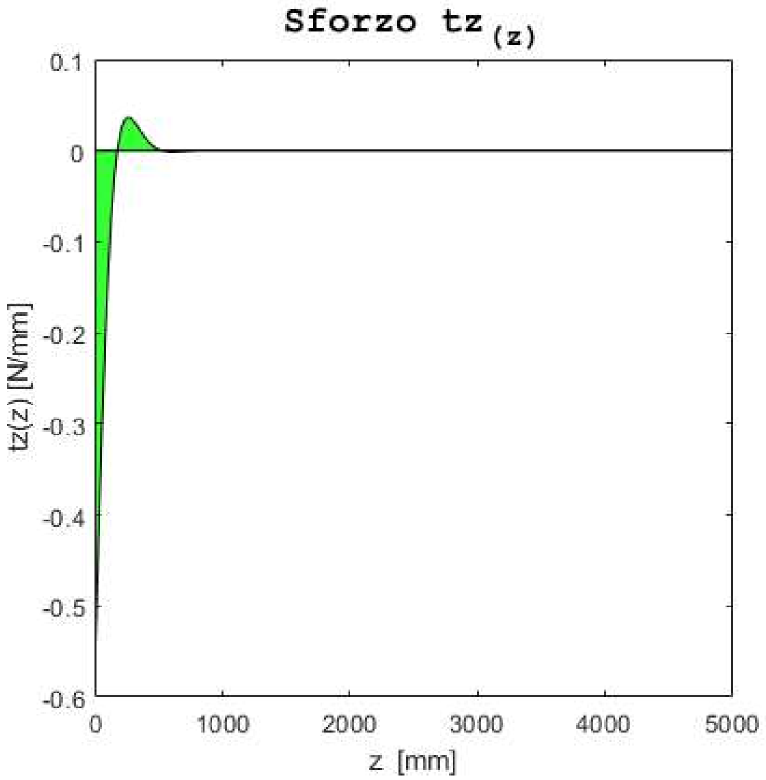
tz(z) =

$$-\frac{\sqrt{2}\ \mathrm{D}\ \beta^3\ \mathrm{e}^{-\sigma_1}\ (A_1 \cos(\sigma_1) + A_2 \cos(\sigma_1) - A_1 \sin(\sigma_1) + A_2 \sin(\sigma_1))}{2\ R^3}$$

where

$$\sigma_1 = \frac{\sqrt{2}\ \beta\ z}{2\ R}$$

```
%% Plot the ns(z) stresses
figure(5)
tz_n(z)=diff(mz_n);
area(a,double(tz_n(a)),'FaceColor','g','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo tz_(_z_)'],'color','K');
xlabel('z  [mm]');
ylabel('tz(z) [N/mm]');
axis square
```


Sforzo tz(z)
tz(z) [N/mm]
z [mm]
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0
1000
2000
3000
4000
5000