Teoria delle strutture - PROBLEMA 4

```
% serbatoio pieno d'acqua shematizzato come guscio sottile
%% Clear workspace and close any open windows
clear all
close all
```

SOLUZIONE PUNTO 3

```
% 3) Assumendo che il serbatoio possa essere schematizzato come un guscio sottile, scrivere
%l’equazione differenziale e le condizioni al bordo che permettono di determinare la soluzione
%di questo problema di equilibrio.
```

SIMBOLICO (per risolvere il guscio)

```
syms z  A1 A2 E R nu t h gamma beta D real

p=-gamma*(h-z) % Carico idrostatico
```

p = $-\gamma\ (h - z)$

Soluzione omogenea

```
wo=A1*exp(-(beta*z)/(R*sqrt(2)))*cos((beta*z)/(R*sqrt(2)))...
    +A2*exp(-(beta*z)/(R*sqrt(2)))*sin((beta*z)/(R*sqrt(2)))
```

wo =

$$A_1\, \mathrm{e}^{-\frac{\sqrt{2}\,\beta z}{2R}} \cos\left(\frac{\sqrt{2}\ \beta z}{2\,R}\right) + A_2\, \mathrm{e}^{-\frac{\sqrt{2}\,\beta z}{2R}} \sin\left(\frac{\sqrt{2}\ \beta z}{2\,R}\right)$$

Soluzione particolare

```
wp=(p*R^2)/(E*t)
```

wp =

$$-\frac{R^2\,\gamma\ (h-z)}{\mathrm{E}\,t}$$

Soluzione

```
w(z)=wo+wp
```

w(z) =

$$A_1\, \mathrm{e}^{-\frac{\sqrt{2}\,\beta z}{2R}} \cos\left(\frac{\sqrt{2}\ \beta z}{2\,R}\right) + A_2\, \mathrm{e}^{-\frac{\sqrt{2}\,\beta z}{2R}} \sin\left(\frac{\sqrt{2}\ \beta z}{2\,R}\right) - \frac{R^2\,\gamma\ (h-z)}{\mathrm{E}\,t}$$

```
% Derivate
w_z(z)=diff(w,z) % derivata prima
```

w_z(z) =

$$\frac{R^2\,\gamma}{\mathrm{E}\,t} - \frac{\sqrt{2}\ A_1\,\beta\,\mathrm{e}^{-\sigma_1}\cos(\sigma_1)}{2\,R} + \frac{\sqrt{2}\ A_2\,\beta\,\mathrm{e}^{-\sigma_1}\cos(\sigma_1)}{2\,R} - \frac{\sqrt{2}\ A_1\,\beta\,\mathrm{e}^{-\sigma_1}\sin(\sigma_1)}{2\,R} - \frac{\sqrt{2}\ A_2\,\beta\,\mathrm{e}^{-\sigma_1}\sin(\sigma_1)}{2\,R}$$

where

$$\sigma_1 = \frac{\sqrt{2}\ \beta z}{2\,R}$$

```
w_zz(z)=diff(w_z,z) % derivata seconda
```

```
w_zz(z) =
```

$$\frac{A_1\,\beta^2\,\mathrm{e}^{-\frac{\sqrt{2}\,\beta\,z}{2\,R}}\sin\left(\frac{\sqrt{2}\,\beta\,z}{2\,R}\right)}{R^2}-\frac{A_2\,\beta^2\,\mathrm{e}^{-\frac{\sqrt{2}\,\beta\,z}{2\,R}}\cos\left(\frac{\sqrt{2}\,\beta\,z}{2\,R}\right)}{R^2}$$

CDS

```
% lo ricavo perchè mi serve per imporre una condizione al contorno
 mz(z)=simplify(-D*w_zz)
```

```
mz(z) =
```

$$\frac{\mathrm{D}\,\beta^2\,\mathrm{e}^{-\frac{\sqrt{2}\,\beta\,z}{2\,R}}\left(A_2\cos\left(\frac{\sqrt{2}\,\beta\,z}{2\,R}\right)-A_1\sin\left(\frac{\sqrt{2}\,\beta\,z}{2\,R}\right)\right)}{R^2}$$

SIMBOLICO (per risolvere la piastra)

```
syms c1 c2 c3 r  real
```

Soluzione piastra circolare

```
p2=-gamma*h %carico idrostatico sulla piastra
```

`p2 =` $-\gamma\,h$

```
w2(r)=p2*r^4/(64*D)+c1*r^2/4+c3
```

```
w2(r) =
```

$$c_3+\frac{c_1\,r^2}{4}-\frac{\gamma\,h\,r^4}{64\,\mathrm{D}}$$

derivate di w2(r)

```
w2_r(r)=diff(w2,r) % derivata prima
```

```
w2_r(r) =
```

$$\frac{c_1\,r}{2}-\frac{\gamma\,h\,r^3}{16\,\mathrm{D}}$$

```
w2_rr(r)=diff(w2_r,r) % derivata seconda
```

```
w2_rr(r) =
```

$$\frac{c_1}{2}-\frac{3\,\gamma\,h\,r^2}{16\,\mathrm{D}}$$

Ricavo mr

```
% lo ricavo perchè mi serve per imporre una condizione al contorno
mr(r)=D*(-w2_rr-nu*w2_r/r)
```

```
mr(r) =
```

$$-\mathrm{D}\left(\frac{c_1}{2}+\frac{\nu\left(\frac{c_1\,r}{2}-\frac{\gamma\,h\,r^3}{16\,\mathrm{D}}\right)}{r}-\frac{3\,\gamma\,h\,r^2}{16\,\mathrm{D}}\right)$$

Applico le Condizioni al contorno:

```
cc1=w(0)==0            % spostamento del guscio nullo
```

```
cc1 =
```

$$A_1-\frac{R^2\,\gamma\,h}{\mathrm{E}\,t}=0$$

```
cc2=w_z(0)==w2_r(-R)  % rotazioni uguali
```

cc2 =

$$\frac{\sqrt{2}\,A_2\,\beta}{2\,R}-\frac{\sqrt{2}\,A_1\,\beta}{2\,R}+\frac{R^2\,\gamma}{\mathrm{E}\,t}=\frac{R^3\,\gamma\,h}{16\,\mathrm{D}}-\frac{R\,c_1}{2}$$

```
cc3=w2(-R)==0          % spostamento piastra nullo
```

cc3 =

$$c_3+\frac{R^2\,c_1}{4}-\frac{R^4\,\gamma\,h}{64\,\mathrm{D}}=0$$

```
cc4=mr(-R)==mz(0)     % momenti uguali
```

cc4 =

$$-\mathrm{D}\left(\frac{c_1}{2}+\frac{\nu\,\left(\frac{R\,c_1}{2}-\frac{R^3\,\gamma\,h}{16\,\mathrm{D}}\right)}{R}-\frac{3\,R^2\,\gamma\,h}{16\,\mathrm{D}}\right)=\frac{A_2\,\mathrm{D}\,\beta^2}{R^2}$$

risolvo il sistema per trovare A1 , A2 , c1, c3

```
S=solve([cc1 cc2 cc3 cc4],[A1 A2 c1 c3]);
A_1=simplify(S.A1(1))
```

A_1 =

$$\frac{R^2\,\gamma\,h}{\mathrm{E}\,t}$$

```
A_2=simplify(S.A2(1))
```

A_2 =

$$-\frac{\sqrt{2}\,R^2\,\gamma\,\left(8\,\mathrm{D}\,R+8\,\mathrm{D}\,R\,\nu+\mathrm{E}\,R^2\,h\,t-4\,\sqrt{2}\,\mathrm{D}\,\beta\,h-4\,\sqrt{2}\,\mathrm{D}\,\beta\,h\,\nu\right)}{8\,\mathrm{D}\,\mathrm{E}\,\beta\,t\,\left(\nu-\sqrt{2}\,\beta+1\right)}$$

```
c_1=simplify(S.c1(1))
```

c_1 =

$$\frac{\gamma\,\left(3\,\mathrm{E}\,R^2\,h\,t-16\,\mathrm{D}\,\beta^2\,h+16\,\sqrt{2}\,\mathrm{D}\,R\,\beta+\mathrm{E}\,R^2\,h\,\nu\,t-\sqrt{2}\,\mathrm{E}\,R^2\,\beta\,h\,t\right)}{8\,\mathrm{D}\,\mathrm{E}\,t\,\left(\nu-\sqrt{2}\,\beta+1\right)}$$

```
c_3=simplify(S.c3(1))
```

c_3 =

$$-\frac{R^2\,\gamma\,\left(5\,\mathrm{E}\,R^2\,h\,t-32\,\mathrm{D}\,\beta^2\,h+32\,\sqrt{2}\,\mathrm{D}\,R\,\beta+\mathrm{E}\,R^2\,h\,\nu\,t-\sqrt{2}\,\mathrm{E}\,R^2\,\beta\,h\,t\right)}{64\,\mathrm{D}\,\mathrm{E}\,t\,\left(\nu-\sqrt{2}\,\beta+1\right)}$$

spostamento piastra

```
w22(r)=simplify(subs(w2,[c1  c3],[c_1  c_3]))
```

w22(r) =

$$\frac{\gamma\,\left(R^2-r^2\right)\,\left(32\,\mathrm{D}\,\beta^2\,h-5\,\mathrm{E}\,R^2\,h\,t+\mathrm{E}\,h\,r^2\,t-32\,\sqrt{2}\,\mathrm{D}\,R\,\beta-\mathrm{E}\,R^2\,h\,\nu\,t+\mathrm{E}\,h\,\nu\,r^2\,t-\sqrt{2}\,\mathrm{E}\,\beta\,h\,r^2\,t+\sqrt{2}\,\mathrm{E}\,R^2\,\beta\,h\,t}{64\,\mathrm{D}\,\mathrm{E}\,t\,\left(\nu-\sqrt{2}\,\beta+1\right)}$$

```
w22_r(r)=diff(w22,r) % derivata prima
```

w22_r(r) =

$$\frac{\gamma\ (R^2-r^2)\ (2\,\mathrm{E}\,h\,r\,t+2\,\mathrm{E}\,h\,\nu\,r\,t-2\ \sqrt{2}\ \mathrm{E}\,\beta\,h\,r\,t)}{64\,\mathrm{D}\,\mathrm{E}\,t\,\sigma_1}-\frac{\gamma\,r\ (32\,\mathrm{D}\,\beta^2\,h-5\,\mathrm{E}\,R^2\,h\,t+\mathrm{E}\,h\,r^2\,t-32\ \sqrt{2}\ \mathrm{D}\,R\,\beta-\mathrm{E}\,R^2}{32\,\mathrm{D}\,\mathrm{E}\,t\,\sigma_1}$$

where

$$\sigma_1=\nu-\sqrt{2}\ \beta+1$$

```
w22_rr(r)=diff(w22_r,r) % derivata seconda
```

w22_rr(r) =

$$\frac{\gamma\ (R^2-r^2)\ (2\,\mathrm{E}\,h\,t+2\,\mathrm{E}\,h\,\nu\,t-2\ \sqrt{2}\ \mathrm{E}\,\beta\,h\,t)}{64\,\mathrm{D}\,\mathrm{E}\,t\,\sigma_1}-\frac{\gamma\,r\ (2\,\mathrm{E}\,h\,r\,t+2\,\mathrm{E}\,h\,\nu\,r\,t-2\ \sqrt{2}\ \mathrm{E}\,\beta\,h\,r\,t)}{16\,\mathrm{D}\,\mathrm{E}\,t\,\sigma_1}-\frac{\gamma\ (32\,\mathrm{D}\,\beta^2\,h-5\,\mathrm{E}\,R^2}{}$$

where

$$\sigma_1=\nu-\sqrt{2}\ \beta+1$$

spostamento guscio

```
w11(z)=simplify(subs(w,[A1 A2],[A_1 A_2]))
```

w11(z) =

$$\frac{R^2\,\gamma\,h\,\mathrm{e}^{-\sigma_1}\cos(\sigma_1)}{\mathrm{E}\,t}-\frac{R^2\,\gamma\ (h-z)}{\mathrm{E}\,t}+\frac{R^2\,\gamma\,\mathrm{e}^{-\sigma_1}\sin(\sigma_1)\ (8\,\mathrm{D}\,\beta\,h-\sqrt{2}\ (8\,\mathrm{D}\,R+8\,\mathrm{D}\,R\,\nu+\mathrm{E}\,R^2\,h\,t)+8\,\mathrm{D}\,\beta\,h\,\nu)}{8\,\mathrm{D}\,\mathrm{E}\,\beta\,t\ (\nu-\sqrt{2}\ \beta+1)}$$

where

$$\sigma_1=\frac{\sqrt{2}\ \beta\,z}{2\,R}$$

```
w11_z(z)=diff(w11,z) % derivata prima
```

w11_z(z) =

$$\frac{R^2\,\gamma}{\mathrm{E}\,t}-\frac{\sqrt{2}\ R\,\beta\,\gamma\,h\,\mathrm{e}^{-\sigma_2}\cos(\sigma_2)}{2\,\mathrm{E}\,t}-\frac{\sqrt{2}\ R\,\beta\,\gamma\,h\,\mathrm{e}^{-\sigma_2}\sin(\sigma_2)}{2\,\mathrm{E}\,t}+\frac{\sqrt{2}\ R\,\gamma\,\mathrm{e}^{-\sigma_2}\cos(\sigma_2)\,\sigma_1}{\sigma_3}-\frac{\sqrt{2}\ R\,\gamma\,\mathrm{e}^{-\sigma_2}\sin(\sigma_2)\,\sigma_1}{\sigma_3}$$

where

$$\sigma_1=8\,\mathrm{D}\,\beta\,h-\sqrt{2}\ (8\,\mathrm{D}\,R+8\,\mathrm{D}\,R\,\nu+\mathrm{E}\,R^2\,h\,t)+8\,\mathrm{D}\,\beta\,h\,\nu$$

$$\sigma_2=\frac{\sqrt{2}\ \beta\,z}{2\,R}$$

$$\sigma_3=16\,\mathrm{D}\,\mathrm{E}\,t\ (\nu-\sqrt{2}\ \beta+1)$$

```
w11_zz(r)=diff(w11_z,z) % derivata seconda
```

w11_zz(r) =

$$\frac{\beta^2\,\gamma\,h\,\mathrm{e}^{-\sigma_1}\sin(\sigma_1)}{\mathrm{E}\,t}-\frac{\beta\,\gamma\,\mathrm{e}^{-\sigma_1}\cos(\sigma_1)\ (8\,\mathrm{D}\,\beta\,h-\sqrt{2}\ (8\,\mathrm{D}\,R+8\,\mathrm{D}\,R\,\nu+\mathrm{E}\,R^2\,h\,t)+8\,\mathrm{D}\,\beta\,h\,\nu)}{8\,\mathrm{D}\,\mathrm{E}\,t\ (\nu-\sqrt{2}\ \beta+1)}$$

where

$$\sigma_1=\frac{\sqrt{2}\ \beta\,z}{2\,R}$$

```
u(z)=simplify(int(w11,z)*nu/R)
```

u(z) =

$$\frac{\nu \left(\frac{4 R^2 \beta \gamma z^2 \sigma_1 + \frac{8 R^3 \gamma e^{-\sigma_2} \sin(\sigma_2) \left(-h \beta^2 + R + R \nu\right)}{\beta} - 8 R^2 \beta \gamma h z \sigma_1 - \frac{8 R^3 \gamma e^{-\sigma_2} \cos(\sigma_2) \left(\sqrt{2} R \beta - 2 R \nu - R \ldots\right.}{}}{8 E \beta t \sigma_1} \right.}{}$$

where

$$\sigma_1 = \nu - \sqrt{2} \beta + 1$$

$$\sigma_2 = \frac{\sqrt{2} \beta z}{2 R}$$

Variabili numeriche

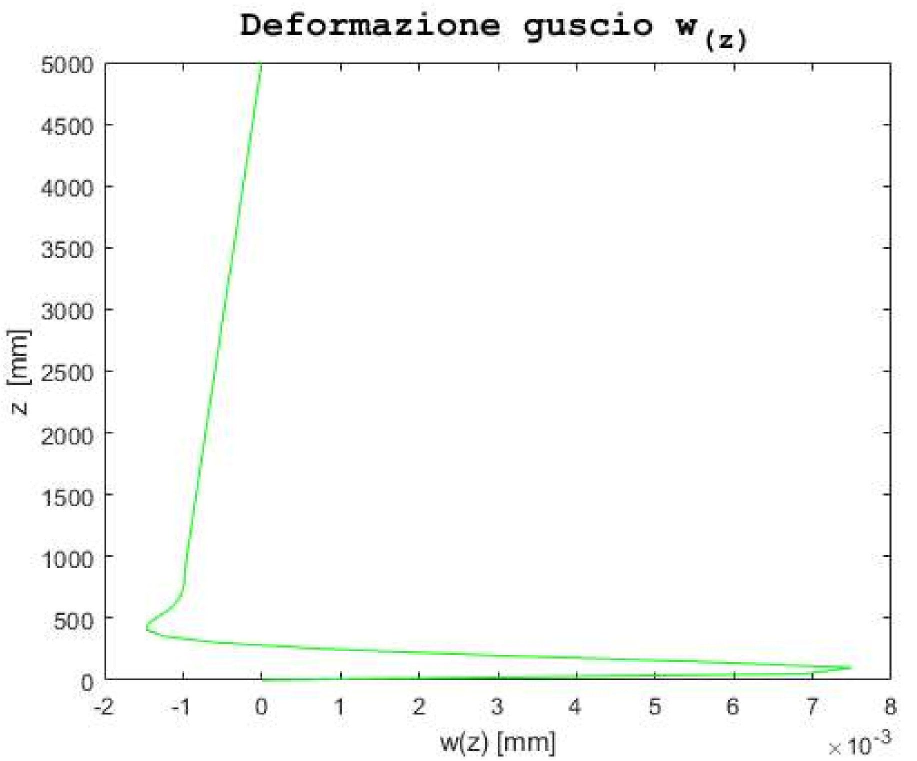
```
R_n=1000; % raggio serbatoio [mm]
E_n=210000; % Young's modulus steel [N/mm^2] se.. 2.1E11[N/m^2]
nu_n=0.3; % coefficiente di Poisson acciaio
t_n=R_n/50; %spessore serbatoio
h_n=5*R_n; %altezza serbatoio
gamma_n=0.000001; % acqua N/mm3
p_n=-gamma_n*(h_n-z); % Carico idrostatico
D_n=(E_n*t_n^3)/(12*(1-nu_n^2)); % rigidezza
beta_n=((E_n*t_n*R_n^2)/D_n)^(1/4);
p2_n=-gamma_n*h_n;
```

Problema in numerico

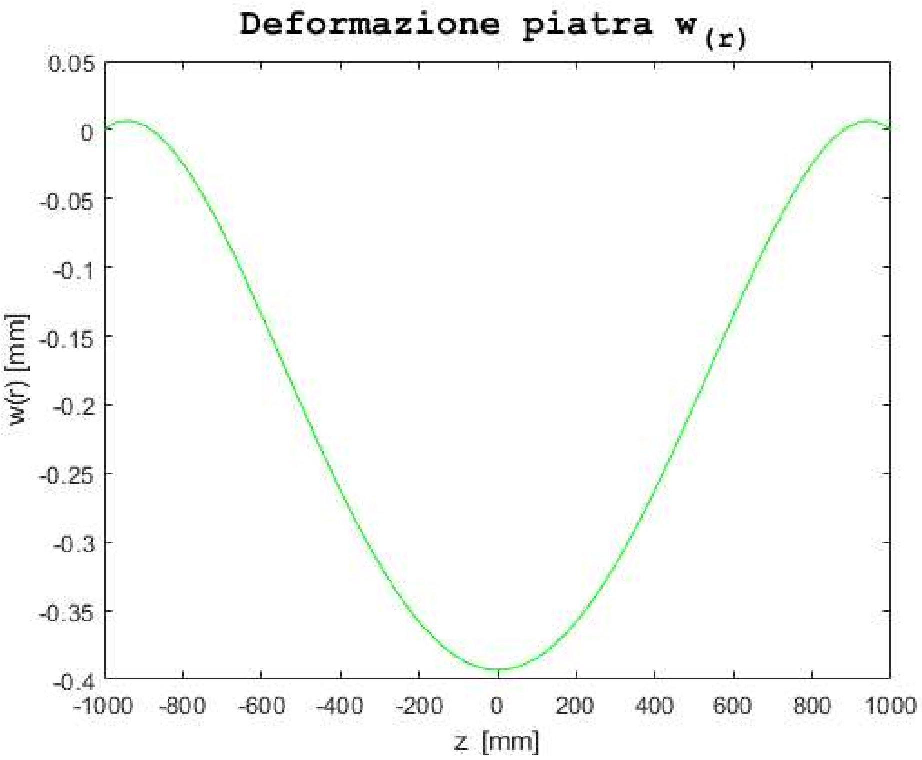
```
A1n=double(subs(A_1,[R gamma h E t],[R_n gamma_n h_n E_n t_n]));
A2n=double(subs(A_2,[R gamma h E t D beta nu],[R_n gamma_n h_n E_n t_n D_n beta_n nu_n]));
c1n=double(subs(c_1,[R gamma h E t D beta nu],[R_n gamma_n h_n E_n t_n D_n beta_n nu_n]));
c3n=double(subs(c_3,[R gamma h E t D beta nu],[R_n gamma_n h_n E_n t_n D_n beta_n nu_n]));

wgn(z)=simplify(subs(w,[A1 A2 beta R gamma h E t],[A1n A2n beta_n R_n gamma_n h_n E_n t_n]));
wpn(r)=simplify(subs(w2,[c1 c3 h D gamma],[c1n c3n h_n D_n gamma_n]));
% DERIVATE
wgn_z(z)=diff(wgn,z); % derivata prima
wgn_zz(z)=diff(wgn_z,z); % derivata seconda
wpn_r(r)=diff(wpn,r); % derivata prima
wpn_rr(r)=diff(wpn_r,r); % derivata seconda
```

Plot Soluzione

```
% Deformazione guscio
a=0:h_n/(100-1):h_n;
figure(1)
plot(double(wgn(a)),a,'g')
title(['\fontname{Courier}\fontsize{15}Deformazione guscio w_(_z_)'],'color','K');
ylabel('z  [mm]');
xlabel('w(z) [mm]');
hold on
```

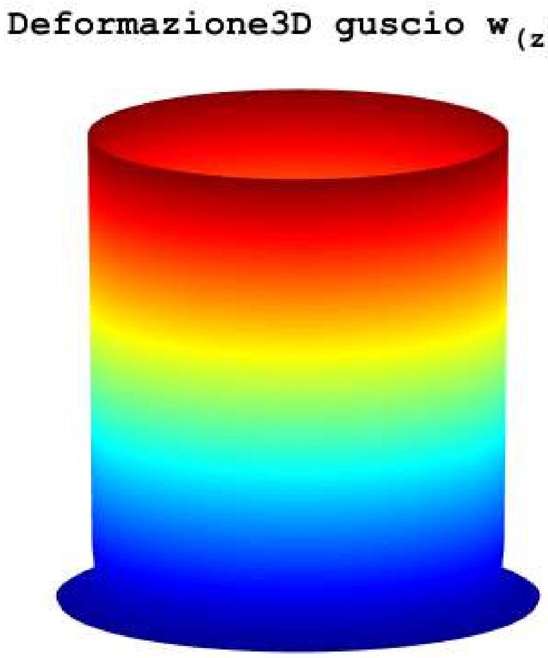



```
% Deformazione piastra
figure(2)
a2=-R_n:R_n/(100-1):R_n;
plot(a2,double(wpn(a2)),'g')
title(['\fontname{Courier}\fontsize{15}Deformazione piatra w_(_r_)'],'color','K');
xlabel('z  [mm]');
ylabel('w(r) [mm]');
hold on
```

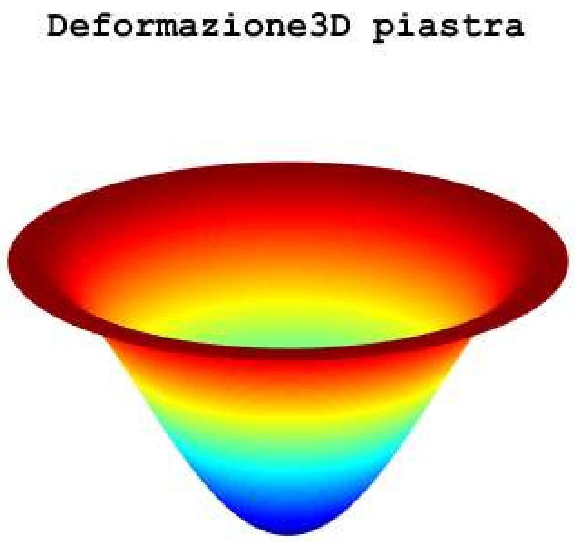



```
%Deformazione3D guscio
figure(3)
fig = gcf; % current figure handle
fig.Color = [1 1 1];
fig.ToolBar = 'none';
```

```
colormap(jet(100))
c=20000; % coefficiente che amplifica la deformazione
w1_n=R_n+(double(wgn(a))*c);
[X,Y,Z]=cylinder(w1_n,100);
s1=surface(X,Y,Z,'FaceAlpha',1,'EdgeColor','k','LineWidth',0.01);
title(['\fontname{Courier}\fontsize{15}Deformazione3D guscio w_(_z_) '],'color','K');
axis off
shading interp;
axis square
rotate3d;
view([39.700 12.400])
```




```
figure(10)
colormap(jet(100))
fig = gcf; % current figure handle
fig.Color = [1 1 1];
fig.ToolBar = 'none';
%Deformazione3D piastra
a3 = linspace(0, 2*pi, 100);      % Angle
rn = linspace(0, R_n, 100);       % Radius
c=30;
[A,R2]=meshgrid(a3, rn);       % Creat Mesh Matrices
fr=double(wpn(R2));            % Evaluate Function Of ‘R2’ Matrix
[xm,ym,zm]=pol2cart(A,R2,fr);  % Convert To Cartesian
surface(xm, ym, zm)
title(['\fontname{Courier}\fontsize{15}Deformazione3D piastra'],'color','K');
axis off
shading interp;
hold on
view(3)
```



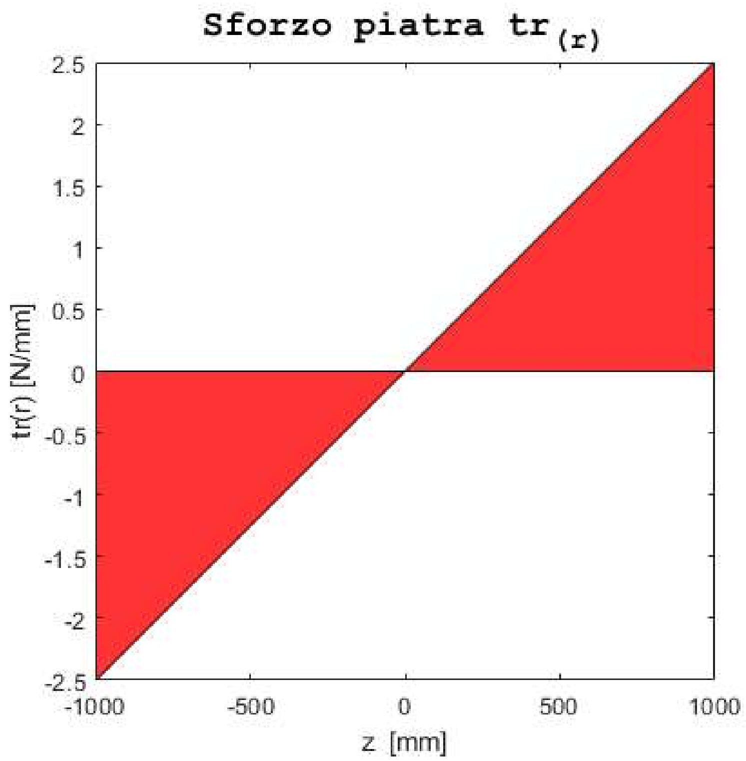
CDS piastra

```
tr(r)=simplify(-p2*r/2)
```

tr(r) =

$$\frac{\gamma\, h\, r}{2}$$

```
%% Plot the ns(z) stresses
a2=-R_n:R_n/(100-1):R_n;
figure(4)
area(a2,double((-p2_n*a2/2)),'FaceColor','r','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo piatra tr_(_r_)'],'color','K');
xlabel('z  [mm]');
ylabel('tr(r) [N/mm]');
axis square
```

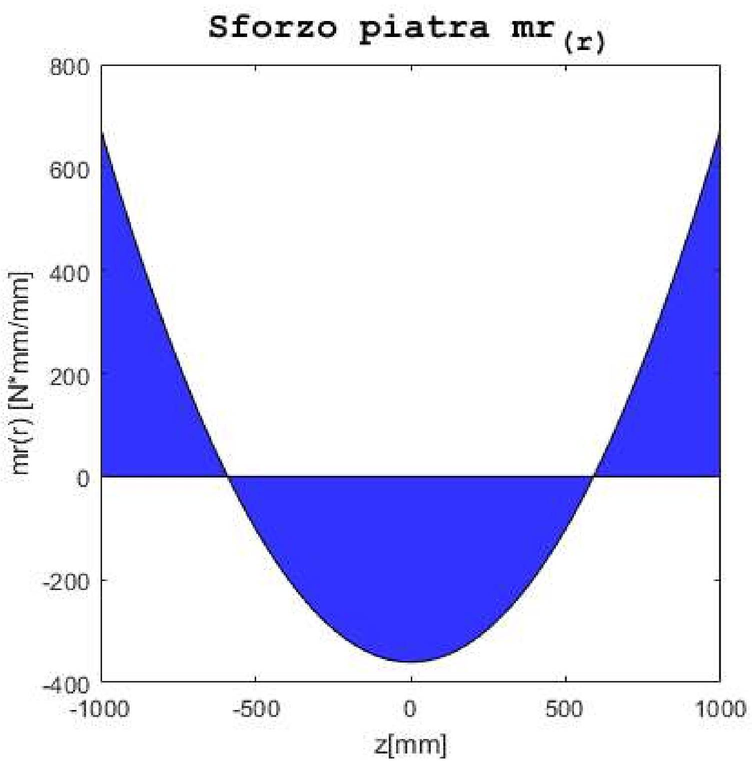



```
mr(r)=simplify(D*(-w2_rr(r)-nu*w2_r(r)/r))
```

mr(r) =

$$\frac{3\,\gamma\,h\,r^2}{16} - \frac{D\,c_1}{2} - \frac{D\,c_1\,\nu}{2} + \frac{\gamma\,h\,\nu\,r^2}{16}$$

```
%% Plot the mr(r) stresses
figure(5)
mr_n(r)=simplify((D_n*(-wpn_rr-nu_n*wpn_r/r)));
area(a2,double(mr_n(a2)),'FaceColor','b','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo piatra mr_(_r_)'],'color','K');
xlabel('z[mm]');
ylabel('mr(r) [N*mm/mm]');
axis square
```

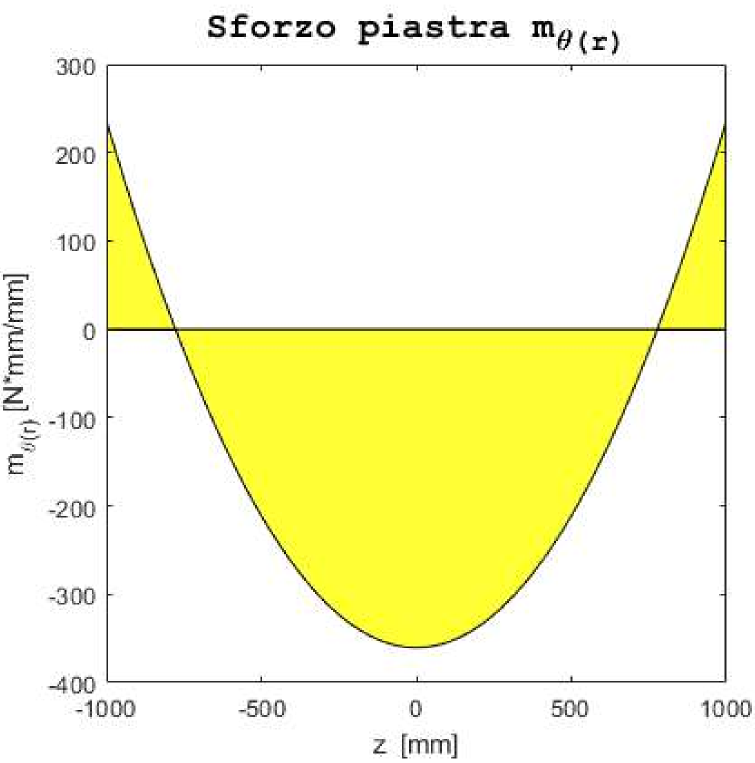


```
m_theta(r)=simplify(D*(-w2_r(r)/r-nu*w2_rr(r)))
```

m_theta(r) =

$$\frac{\gamma\, h\, r^2}{16} - \frac{\mathrm{D}\, c_1}{2} - \frac{\mathrm{D}\, c_1\, \nu}{2} + \frac{3\, \gamma\, h\, \nu\, r^2}{16}$$

```
%% Plot the mtheta(r) stresses
figure(6)
mn_theta(r)=simplify(D_n*(-wpn_r(r)/r-nu_n*wpn_rr(r)));
area(a2,double(mn_theta(a2)),'FaceColor','y','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo piastra m_\theta_(_r_)'],'color','K');
xlabel('z  [mm]');
ylabel('m_\theta_(_r_) [N*mm/mm]');
axis square
```



CDS guscio

```
ns(z)=simplify(-w11*E*t/R)
```

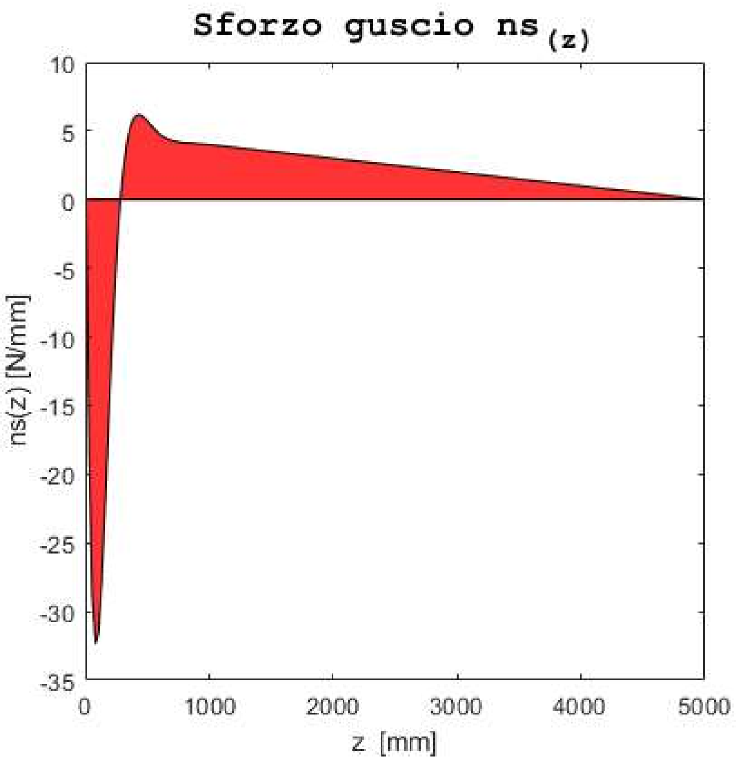
ns(z) =

$$-\frac{\mathrm{E}\, t \left(\frac{R^2\, \gamma\, h\, \mathrm{e}^{-\sigma_1} \cos(\sigma_1)}{\mathrm{E}\, t} - \frac{R^2\, \gamma\, (h - z)}{\mathrm{E}\, t} + \frac{R^2\, \gamma\, \mathrm{e}^{-\sigma_1} \sin(\sigma_1)\, (8\, \mathrm{D}\, \beta\, h - \sqrt{2}\, (8\, \mathrm{D}\, R + 8\, \mathrm{D}\, R\, \nu + \mathrm{E}\, R^2\, h\, t) + 8\, \mathrm{D}\, \beta\, h\, \nu)}{8\, \mathrm{D}\, \mathrm{E}\, \beta\, t\, (\nu - \sqrt{2}\, \beta + 1)}\right.}{R}$$

where

$$\sigma_1 = \frac{\sqrt{2}\, \beta\, z}{2\, R}$$

```
%% Plot the ns(z) stresses
a=0:h_n/(200-1):h_n;
figure(7)
area(a,double(-wgn(a)*E_n*t_n/R_n),'FaceColor','r','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio ns_(_z_)'],'color','K');
xlabel('z  [mm]');
ylabel('ns(z) [N/mm]');
axis square
```




```
mz(z)=simplify(-D*w11_zz)
```

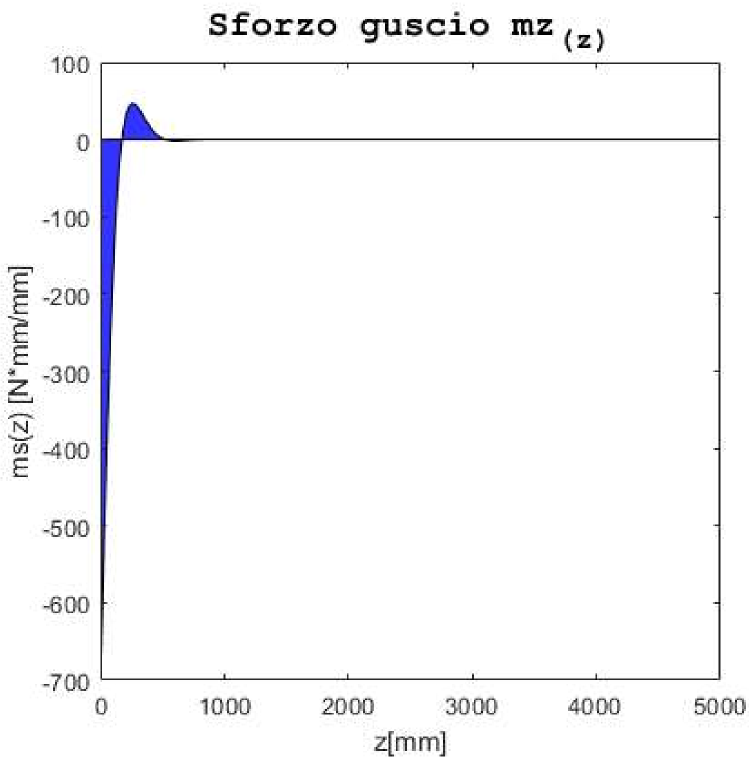
mz(z) =

$$-\mathrm{D}\left(\frac{\beta^2\,\gamma\,h\,\mathrm{e}^{-\sigma_1}\,\sin(\sigma_1)}{\mathrm{E}\,t}-\frac{\beta\,\gamma\,\mathrm{e}^{-\sigma_1}\,\cos(\sigma_1)\,\left(8\,\mathrm{D}\,\beta\,h-\sqrt{2}\,\left(8\,\mathrm{D}\,R+8\,\mathrm{D}\,R\,\nu+\mathrm{E}\,R^2\,h\,t\right)+8\,\mathrm{D}\,\beta\,h\,\nu\right)}{8\,\mathrm{D}\,\mathrm{E}\,t\,\left(\nu-\sqrt{2}\,\beta+1\right)}\right)$$

where

$$\sigma_1=\frac{\sqrt{2}\,\beta\,z}{2\,R}$$

```
%% Plot the mz(z) stresses
figure(8)
mz_n(z)=(wgn_zz*D_n);
area(a,double(mz_n(a)),'FaceColor','b','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio mz_(_z_)'],'color','K');
xlabel('z[mm]');
ylabel('ms(z) [N*mm/mm]');
axis square
```




```
tz(z)=simplify(diff(mz))
```

tz(z) =

$$-\mathrm{D}\left(\frac{\sqrt{2}\;\beta^3\,\gamma\,h\,\mathrm{e}^{-\sigma_2}\cos(\sigma_2)}{\sigma_4}-\frac{\sqrt{2}\;\beta^3\,\gamma\,h\,\mathrm{e}^{-\sigma_2}\sin(\sigma_2)}{\sigma_4}+\frac{\sqrt{2}\;\beta^2\,\gamma\,\mathrm{e}^{-\sigma_2}\cos(\sigma_2)\,\sigma_1}{\sigma_3}+\frac{\sqrt{2}\;\beta^2\,\gamma\,\mathrm{e}^{-\sigma_2}\sin(\sigma_2)\,\sigma_1}{\sigma_3}\right)$$

where

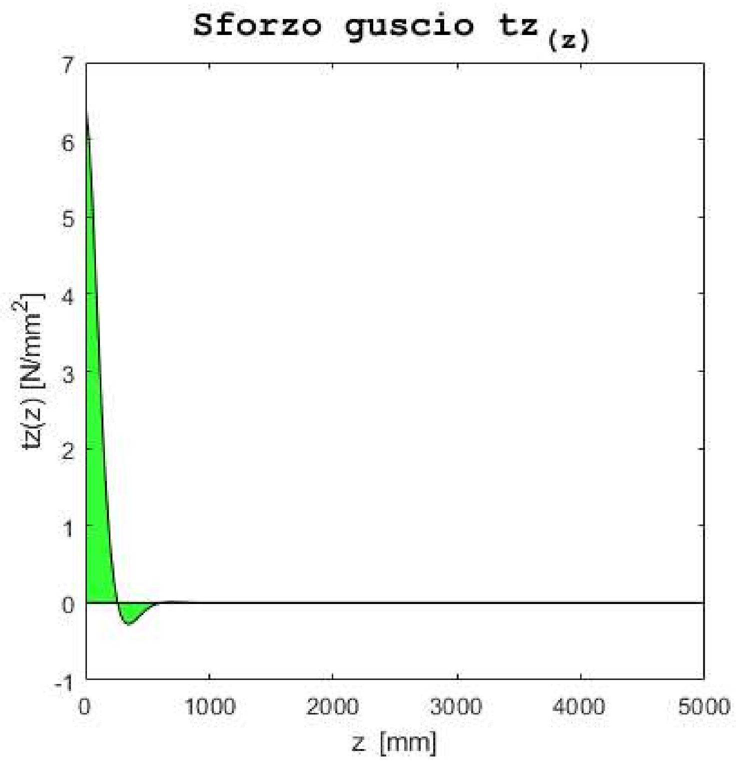
$$\sigma_1 = 8\,\mathrm{D}\,\beta\,h-\sqrt{2}\;(8\,\mathrm{D}\,R+8\,\mathrm{D}\,R\,\nu+\mathrm{E}\,R^2\,h\,t)+8\,\mathrm{D}\,\beta\,h\,\nu$$

$$\sigma_2 = \frac{\sqrt{2}\;\beta\,z}{2\,R}$$

$$\sigma_3 = 16\,\mathrm{D}\,\mathrm{E}\,R\,t\;(\nu-\sqrt{2}\;\beta+1)$$

$$\sigma_4 = 2\,\mathrm{E}\,R\,t$$

```
%% Plot the ns(z) stresses
figure(9)
tz_n(z)=diff(mz_n);
area(a,double(tz_n(a)),'FaceColor','g','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio tz_(_z_)'],'color','K');
xlabel('z  [mm]');
ylabel('tz(z) [N/mm^2]');
axis square
```



SOLUZIONE PUNTO 4

```
% 4) Determinare le espressioni dello spostamento trasversale e delle caratteristiche della
%sollecitazione (semplificare il problema relativamente al calcolo delle componenti flessionali
%della sollecitazione, assumendo nel calcolo che il guscio sia di lunghezza infinita).
```

Calcolo theta(-R) della piastra

```
theta(r)=simplify(w22_r(r))
```

theta(r) =

$$-\frac{\gamma\, r\, (16\,\mathrm{D}\,\beta^2\, h - 3\,\mathrm{E}\,R^2\, h\, t + \mathrm{E}\, h\, r^2\, t - 16\,\sqrt{2}\,\mathrm{D}\,R\,\beta - \mathrm{E}\,R^2\, h\,\nu\, t + \mathrm{E}\, h\,\nu\, r^2\, t - \sqrt{2}\,\mathrm{E}\,\beta\, h\, r^2\, t + \sqrt{2}\,\mathrm{E}\,R^2\,\beta\, h\, t)}{16\,\mathrm{D}\,\mathrm{E}\, t\,(\nu - \sqrt{2}\,\beta + 1)}$$

```
k=simplify(mr(-R)/theta(-R)) % costante della molla
```

k =

$$\frac{16\,\mathrm{D}\,\mathrm{E}\, t\,(\nu - \sqrt{2}\,\beta + 1)\left(\frac{\mathrm{D}\, c_1}{2} + \frac{\mathrm{D}\, c_1\,\nu}{2} - \frac{3\, R^2\,\gamma\, h}{16} - \frac{R^2\,\gamma\, h\,\nu}{16}\right)}{R\,\gamma\,(2\,\mathrm{E}\, h\, t\, R^2 + 16\,\sqrt{2}\,\mathrm{D}\, R\,\beta - 16\,\mathrm{D}\, h\,\beta^2)}$$

Valutazione numerica

```
Kn=double(mr_n(-R_n)/wpn_r(-R_n))
```

Kn =
    2925748.79

```
%% rimposto il problema con le condizioni al contorno dovute alla molla
```

Applico le Condizioni al contorno:

```
cc1=w(0)==0                              % spostamento del guscio nullo
```

cc1 =

$$A_1 - \frac{R^2\,\gamma\, h}{\mathrm{E}\, t} = 0$$

```
cc2=w_z(0)==double(mr_n(-R_n)/Kn)  % rotazioni
```

```
cc2 =
```

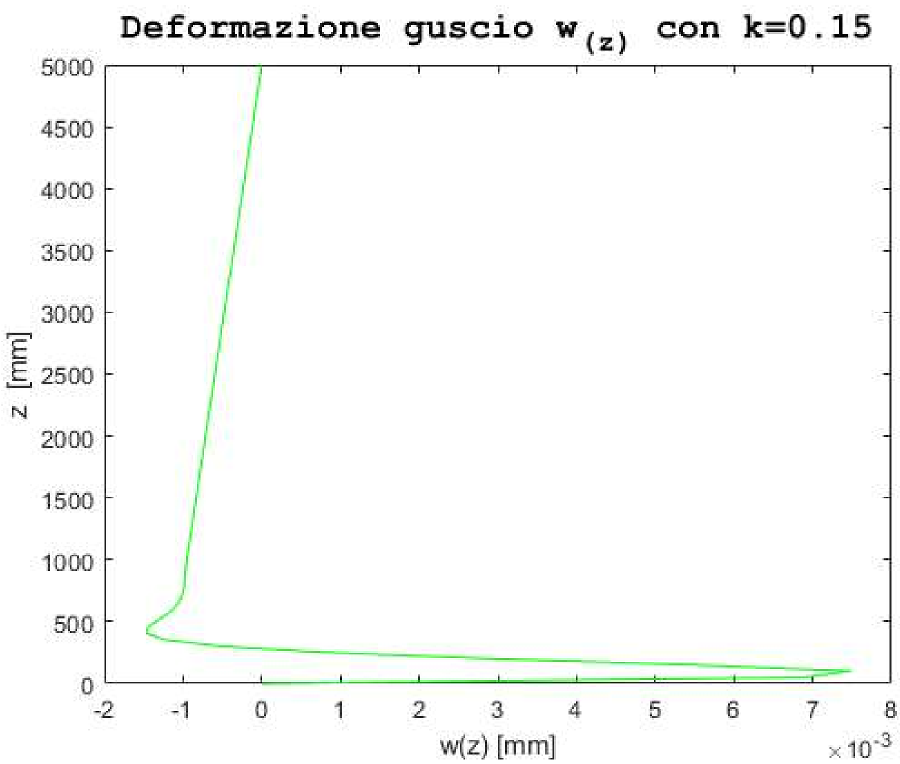
$$\frac{\sqrt{2}\,A_2\,\beta}{2\,R}-\frac{\sqrt{2}\,A_1\,\beta}{2\,R}+\frac{R^2\,\gamma}{\mathrm{E}\,t}=\frac{8459484659262361}{36893488147419103232}$$

```
S=solve([cc1 cc2],[A1 A2]);
A_11=simplify(S.A1(1));
A_22=simplify(S.A2(1));
An_11=double(subs(A_11,[R gamma h E t],[R_n gamma_n h_n E_n t_n]));
An_22=double(subs(A_22,[R gamma h E t beta],[R_n gamma_n h_n E_n t_n beta_n]));
%Spostamento
wgn2(z)=simplify(subs(w,[A1 A2 beta R gamma h E t],[An_11 An_22 beta_n R_n gamma_n h_n E_n t_n]));
% Derivate
wgn2_z(z)=diff(wgn2,z); % derivata prima
wgn2_zz(z)=diff(wgn2_z,z); % derivata seconda
% CDS
ns2(z)=simplify(-wgn2*E_n*t_n/R_n);
mz2_n(z)=(-wgn2_zz*D_n);
tz2_n(z)=diff(mz2_n);
```

Plot con K=0.15

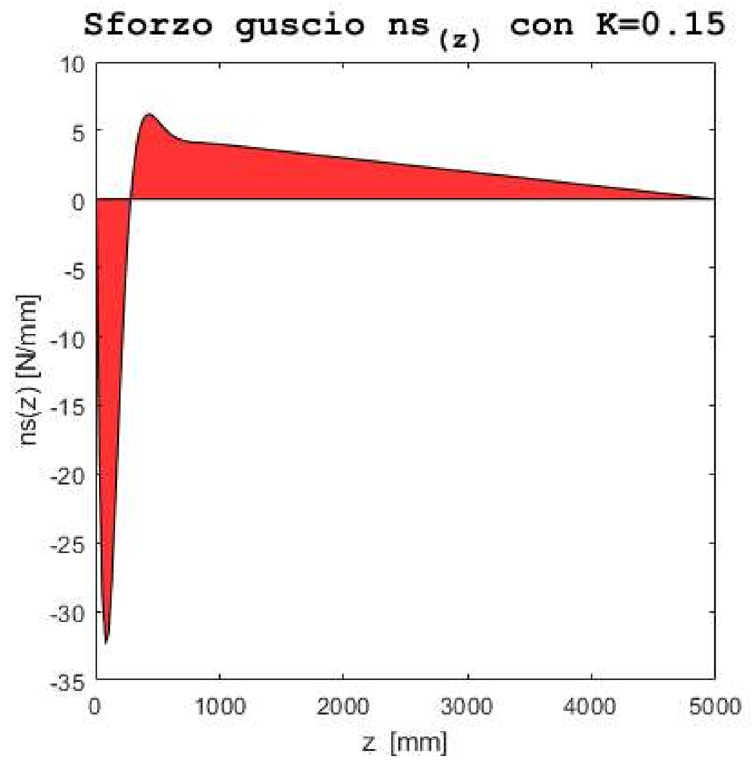
caso in cui le deformazioni e le CDS sono uguali a quelle ottenute nel punto 3

```
a=0:h_n/(100-1):h_n;
% DEFORMAZIONE
figure(11)
plot(double(wgn2(a)),a,'g')
title(['\fontname{Courier}\fontsize{15}Deformazione guscio w_(_z_) con k=0.15'],'color','K');
ylabel('z  [mm]');
xlabel('w(z) [mm]');
hold on
```

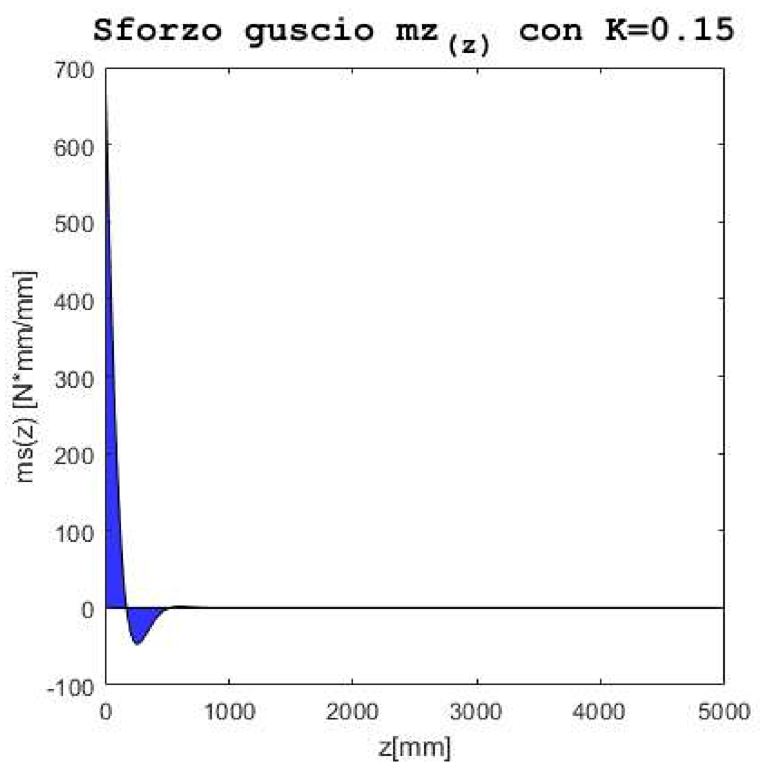



```
% CDS
%% Plot the ns(z) stresses
a=0:h_n/(200-1):h_n;
figure(7)
area(a,double(ns2(a)),'FaceColor','r','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio ns_(_z_) con K=0.15'],'color','K');
xlabel('z  [mm]');
ylabel('ns(z) [N/mm]');
```

```
axis square
```

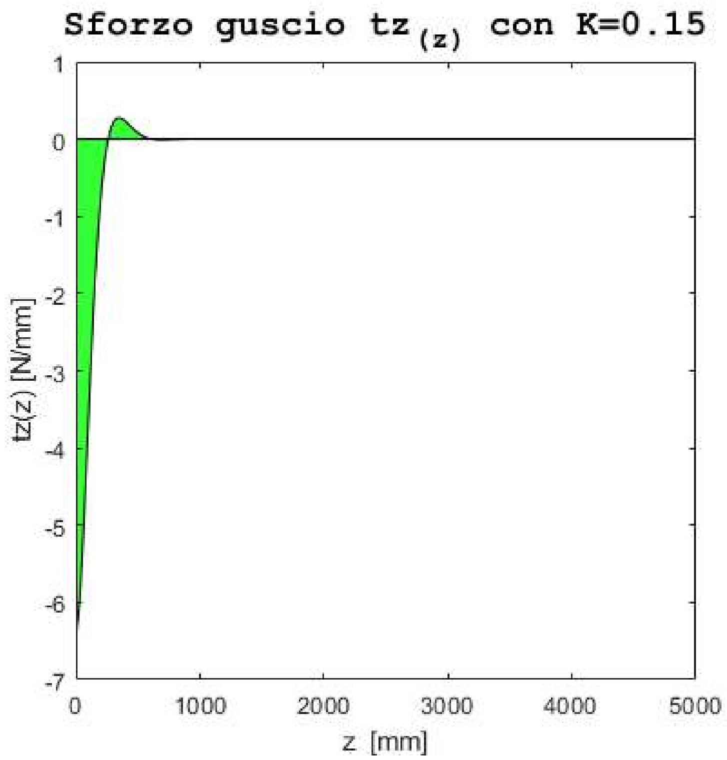



```
%% Plot the mz(z) stresses
figure(12)
area(a,double(mz2_n(a)),'FaceColor','b','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio mz_(_z_) con K=0.15'],'color','K');
xlabel('z[mm]');
ylabel('ms(z) [N*mm/mm]');
axis square
```




```
%% Plot the ns(z) stresses
figure(13)
area(a,double(tz2_n(a)),'FaceColor','g','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio tz_(_z_) con K=0.15'],'color','K');
xlabel('z  [mm]');
ylabel('tz(z) [N/mm]');
```

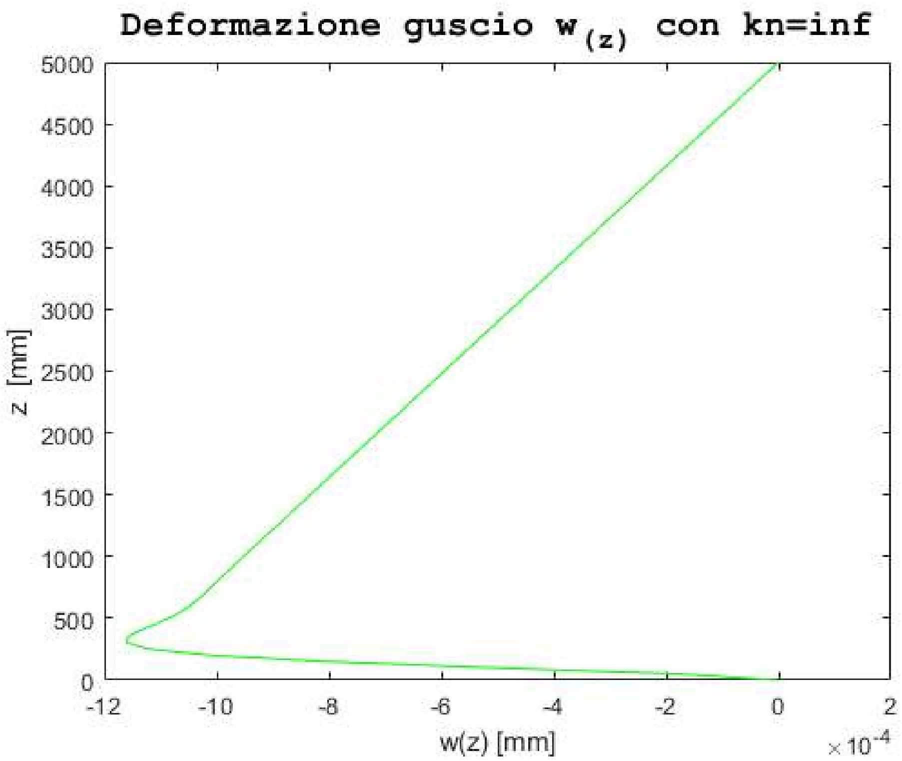
```
axis square
```



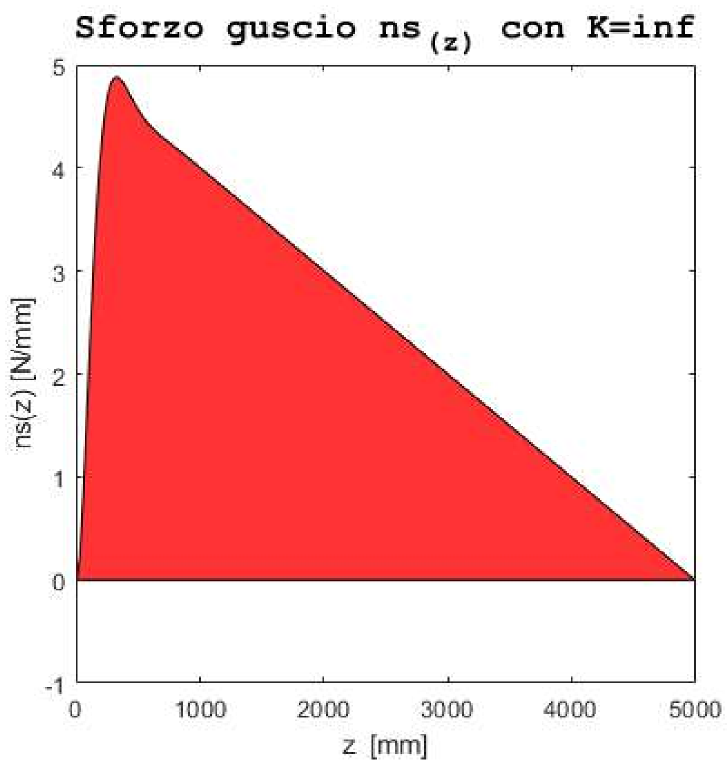
Plot con K=inf

caso in cui le deformazioni e le CDS sono uguali a quelle ottenute nei punti 1 & 2

```
% Risolvo il sistema con Kn=0
Kn=inf;
cc1=w(0)==0;                         % spostamento del guscio nullo
cc2=w_z(0)==double(mr_n(-R_n)/Kn);  % rotazione
S=solve([cc1 cc2],[A1 A2]);
A_11=simplify(S.A1(1));
A_22=simplify(S.A2(1));
An_11=double(subs(A_11,[R gamma h E t],[R_n gamma_n h_n E_n t_n]));
An_22=double(subs(A_22,[R gamma h E t beta],[R_n gamma_n h_n E_n t_n beta_n]));
%Spostamento
wgn2(z)=simplify(subs(w,[A1 A2 beta R gamma h E t],[An_11 An_22 beta_n R_n gamma_n h_n E_n t_n]));
% Derivate
wgn2_z(z)=diff(wgn2,z); % derivata prima
wgn2_zz(z)=diff(wgn2_z,z); % derivata seconda
% CDS
ns2(z)=simplify(-wgn2*E_n*t_n/R_n);
mz2_n(z)=(-wgn2_zz*D_n);
tz2_n(z)=diff(mz2_n);
%Plot
a=0:h_n/(100-1):h_n;
% DEFORMAZIONE
figure(14)
plot(double(wgn2(a)),a,'g')
title(['\fontname{Courier}\fontsize{15}Deformazione guscio w_(_z_) con kn=inf'],'color','K');
ylabel('z  [mm]');
xlabel('w(z) [mm]');
hold on
```

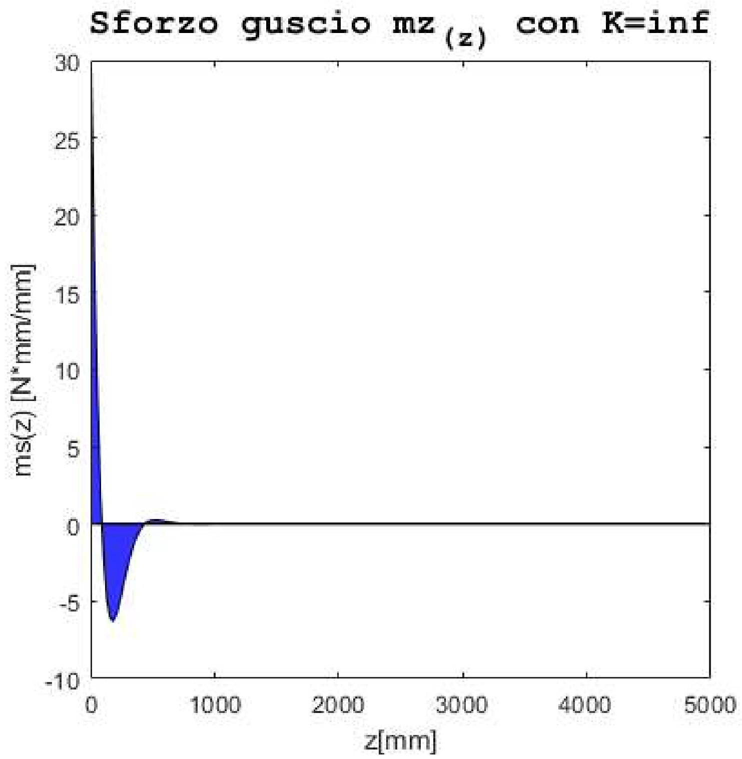



```
% CDS
%% Plot the ns(z) stresses
a=0:h_n/(200-1):h_n;
figure(7)
area(a,double(ns2(a)),'FaceColor','r','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio ns_(_z_) con K=inf'],'color','K');
xlabel('z  [mm]');
ylabel('ns(z) [N/mm]');
axis square
```

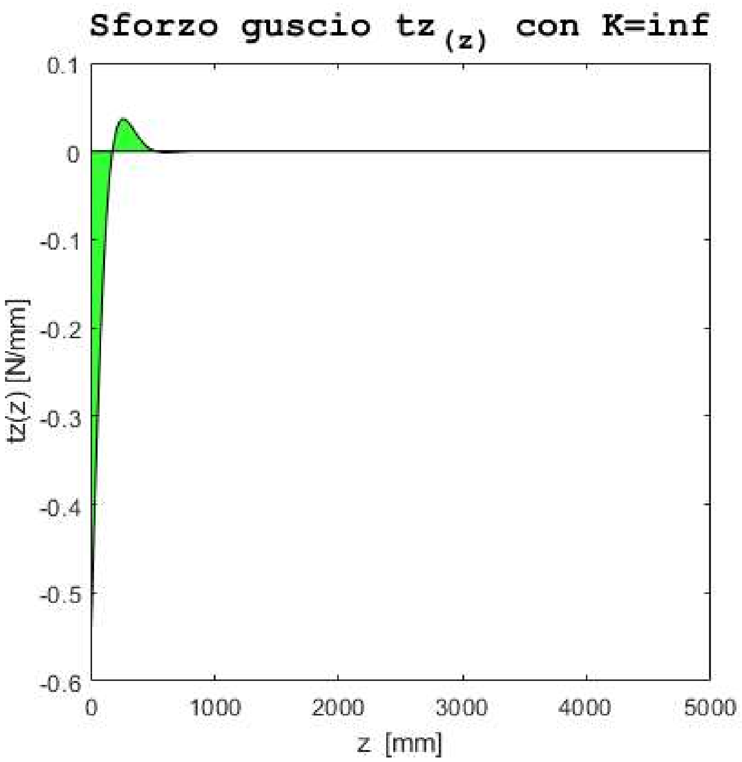



```
%% Plot the mz(z) stresses
figure(15)
area(a,double(mz2_n(a)),'FaceColor','b','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio mz_(_z_) con K=inf'],'color','K');
xlabel('z[mm]');
```

```
ylabel('ms(z) [N*mm/mm]');
axis square
```



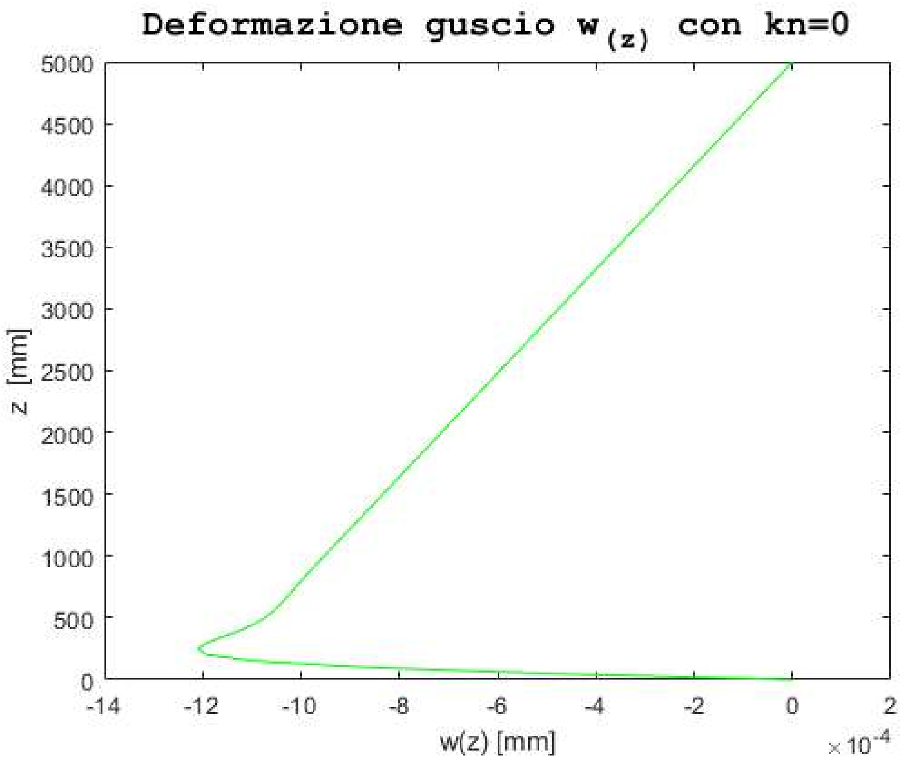

```
%% Plot the ns(z) stresses
figure(16)
area(a,double(tz2_n(a)),'FaceColor','g','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio tz_(_z_) con K=inf'],'color','K');
xlabel('z  [mm]');
ylabel('tz(z) [N/mm]');
axis square
```



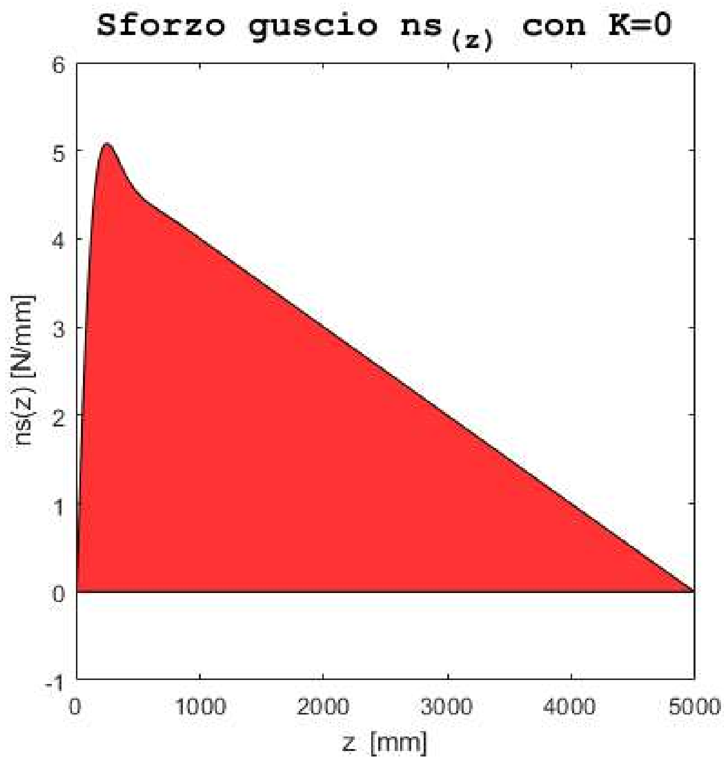
Plot con K=0

caso in cui le deformazioni e le CDS sono uguali a quelle di un guscio semplicemente appoggiato, senza piastra circolare in fondo

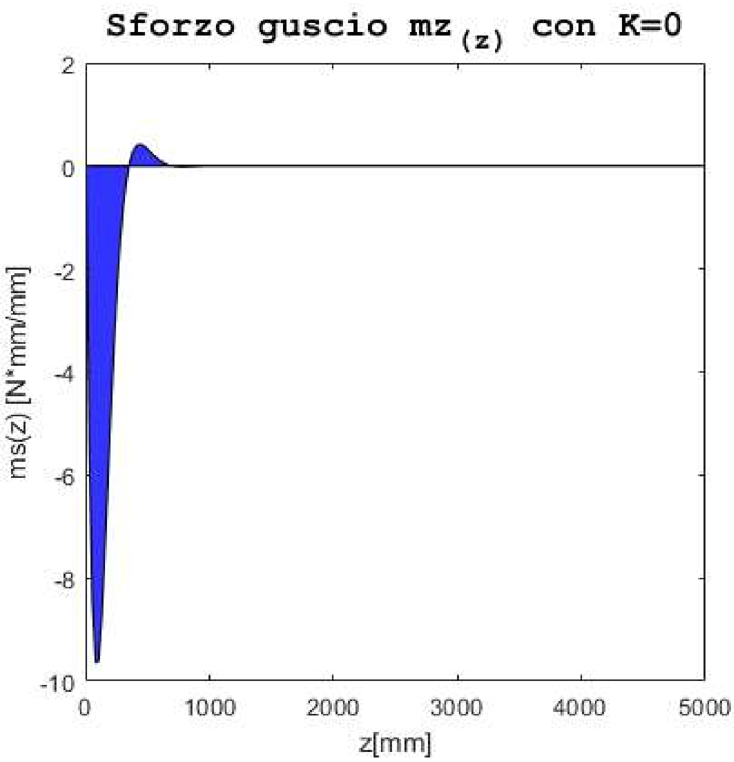
```
% Risolvo il sistema con Kn=0
Kn=0;
cc1=w(0)==0;                          % spostamento del guscio nullo
cc2=-w_zz(0)*D_n==double(Kn*wpn_r(-R_n));  % momento
S=solve([cc1 cc2],[A1 A2]);
A_11=simplify(S.A1(1));
A_22=simplify(S.A2(1));
An_11=double(subs(A_11,[R gamma h E t],[R_n gamma_n h_n E_n t_n]));
An_22=double(subs(A_22,[R gamma h E t beta],[R_n gamma_n h_n E_n t_n beta_n]));
%Spostamento
wgn2(z)=simplify(subs(w,[A1 A2 beta R gamma h E t],[An_11 An_22 beta_n R_n gamma_n h_n E_n t_n]));
% Derivate
wgn2_z(z)=diff(wgn2,z); % derivata prima
wgn2_zz(z)=diff(wgn2_z,z); % derivata seconda
% CDS
ns2(z)=simplify(-wgn2*E_n*t_n/R_n);
mz2_n(z)=(-wgn2_zz*D_n);
tz2_n(z)=diff(mz2_n);
%Plot
a=0:h_n/(100-1):h_n;
% DEFORMAZIONE
figure(17)
plot(double(wgn2(a)),a,'g')
title(['\fontname{Courier}\fontsize{15}Deformazione guscio w_(_z_) con kn=0'],'color','K');
ylabel('z  [mm]');
xlabel('w(z) [mm]');
hold on
```



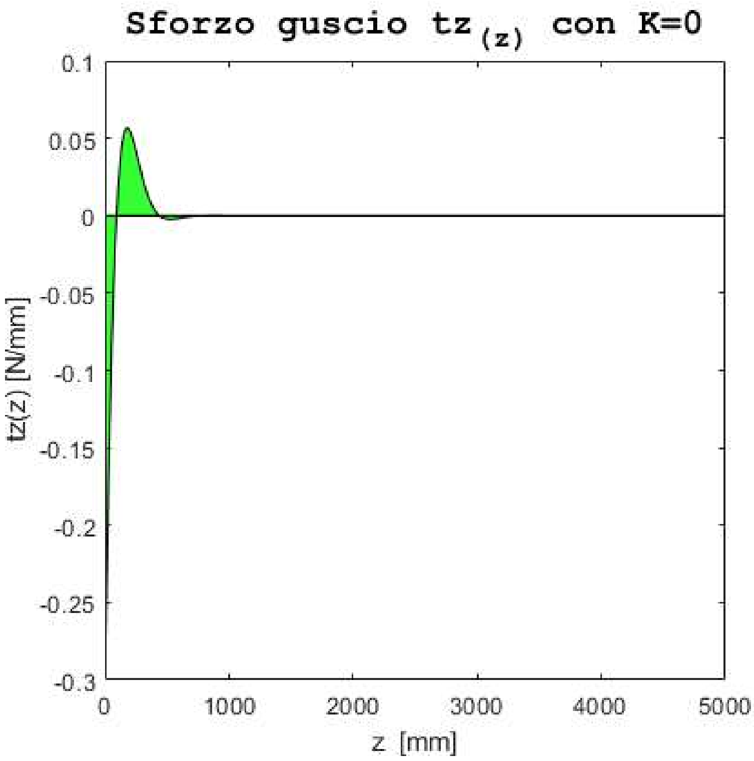

```
% CDS
%% Plot the ns(z) stresses
a=0:h_n/(200-1):h_n;
figure(7)
area(a,double(ns2(a)),'FaceColor','r','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio ns_(_z_) con K=0'],'color','K');
xlabel('z  [mm]');
ylabel('ns(z) [N/mm]');
axis square
```




```
%% Plot the mz(z) stresses
figure(18)
area(a,double(mz2_n(a)),'FaceColor','b','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio mz_(_z_) con K=0'],'color','K');
xlabel('z[mm]');
ylabel('ms(z) [N*mm/mm]');
axis square
```




```
%% Plot the ns(z) stresses
figure(19)
area(a,double(tz2_n(a)),'FaceColor','g','Facealpha',0.8)
title(['\fontname{Courier}\fontsize{15}Sforzo guscio tz_(_z_) con K=0'],'color','K');
xlabel('z  [mm]');
ylabel('tz(z) [N/mm]');
axis square
```



Sforzo guscio tz(z) con K=0
tz(z) [N/mm]
z [mm]
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
0
1000
2000
3000
4000
5000